Names: _____ **PantherIDs:** _____

Homework 3 - Graph Theory - Spring '22 **Due Thursday, March 10**

1. There are various notions of connectivity for a digraph G. Here are three of them (see also the top of page 56 textbook):

– a digraph G is said to be *weakly connected* if its supporting graph $\tilde{G}$ is connected. (This is equivalent with the definition in textbook using semipaths; see there what's a semipath.)
Recall that the supporting graph of a digraph is obtained by erasing all orientations on edges (make all edges undirected).

– a digraph G is said to be *unilaterally connected* if for any two vertices u, v there exists a directed $u - v$ path OR a directed $v - u$ path.

– a digraph G is said to be *strongly connected* if for any two vertices u, v there exists a directed $u - v$ path AND a directed $v - u$ path.

(a) It is clear that the following implications hold:

$$(G \text{ strongly connected }) \Rightarrow (G \text{ unilaterally connected }) \Rightarrow (G \text{ weakly connected })$$

Show that none of the converse implications are true by providing an example of a digraph with three vertices in each case. That is, you are asked to give an example of a digraph G_1 with three vertices which is weakly connected but is not unilaterally connected and an example of a digraph G_2 with three vertices which is unilaterally connected but is not strongly connected.

(b) For the digraph of Problem 3 in your Exam 1, decide if the digraph is weakly connected, unilaterally connected, or strongly connected. Briefly justify your answer.

2. Read Professor Ram's notes of Section 1 of Chapter 5 (the section titled Euler circuits and Euler trails), and understand the proof of Euler's Circuit Theorem for general graphs and the following corollary about Euler trails. Then give a proof for Theorem 3 in Prof. Ram's notes - both parts (the Euler's Circuit and Euler's trail theorem for digraphs). Of course, you should adapt the proof from the case of graphs.