Statistical Engineering Approach to Improve the Realism of Computer-Simulated Experiments with Aircraft Trajectory Clustering

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Outline

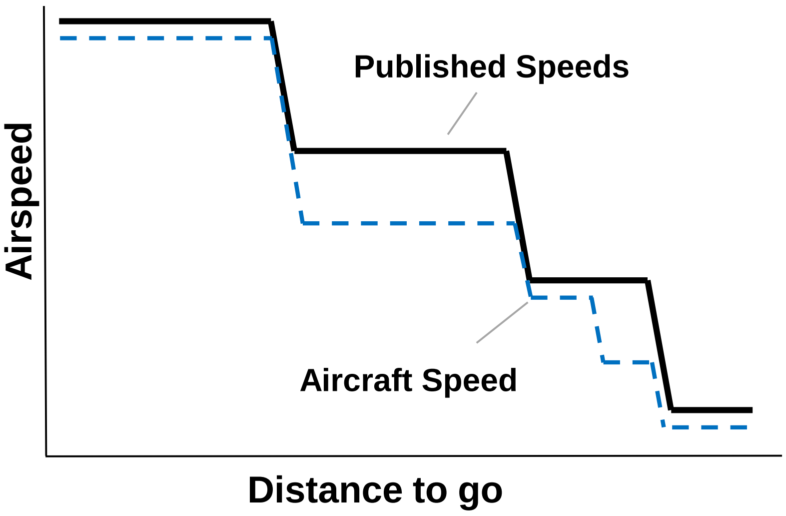
- Background and Motivating Example
- Cluster Analysis
- Challenges
- Trajectory Clustering Methodology
- Application of Trajectory Clustering
- Conclusions

NASA Interval Management Technology

- **NASA is developing and demonstrating technologies that enable the use of efficient aircraft arrival procedures during high demand operations**
- **Interval Management is a technology that enables an aircraft equipped with NASA's spacing algorithm and avionics to achieve precise spacing behind a lead aircraft**
- **Fast-time simulation was planned to evaluate recent modifications to the spacing algorithm**

NASA Interval Management Technology

- **A variety of realistic aircraft trajectories were needed for this fast-time simulation**
- **Previous fast-time simulations used trajectories created by subject matter experts**
- **These trajectories did not represent the variability expected during actual operations**
- **Data collected during a recent human-in-the-loop experiment provided an opportunity to increase the level of realism of the fast-time simulation by incorporating a greater variety of and more accurate trajectories**



Statistical Engineering

- **Development of a methodology to simulate more realistic aircraft trajectories required the use of statistical engineering**

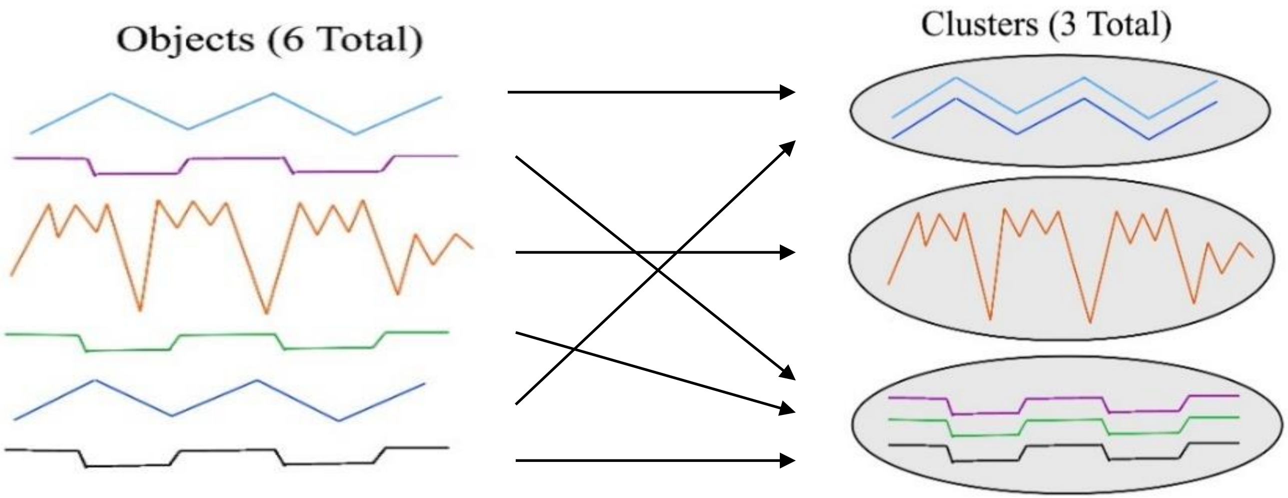
- **Goal of implementing a statistical engineering approach was to develop a methodology that could be utilized in future datasets and simulations**

- **Statisticians and aerospace engineers collaborated throughout the process to develop a methodological framework that is repeatable and scalable**

- **Cluster analysis was adapted and extended to identify patterns in recorded aircraft trajectories and results were incorporated into the fast-time simulation**

Cluster Analysis

- **Goal of cluster analysis is to group similar objects together to discover natural underlying behavior**
- **Clustering is an unsupervised learning approach, which means that the number of groups and group membership are not known *a priori***
- **Number of groups and how items are assigned to each group is based on how one defines similarity with regards to the data items**



Cluster Analysis

- Cluster analysis involves two steps:

  1) Quantify the sameness between objects using a similarity measure

  2) Place objects that are considered similar into groups using a clustering algorithm

- Similarity measure is a function that quantifies the degree of uniformity or sameness between a pair of objects

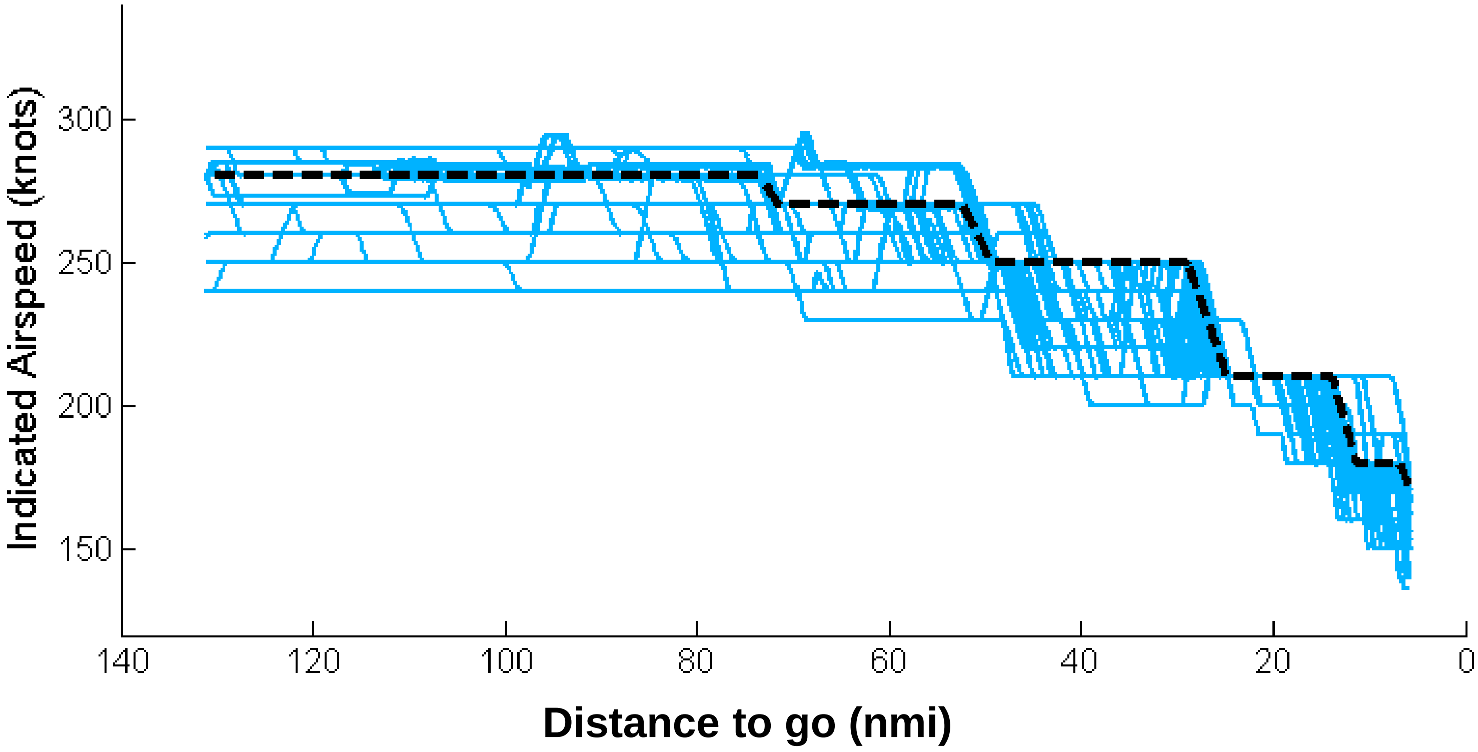
- Clustering algorithm is a procedure capable of placing similar objects together into groups

Challenges

- Formulating an acceptable similarity measure for aircraft trajectories and what constituted an acceptable cluster is challenging
- Would like the sequences of data points that discretize the trajectories to represent the paths of the original trajectories, but in practice these sequences are less then ideal
- Identifying similarities between trajectories is challenging in the presence of:
  - Missing data points and interruptions in trajectories
  - Trajectories of different lengths
  - Trajectories with different starting locations
- Unequal numbers of observations per trajectory add complexity

Challenges

- **Large amount of variability makes it challenging to visually distinguish between underlying groups and identify outliers**



Trajectory Clustering Methodology

- **The trajectory clustering methodology developed incorporates**

  1) **Dynamic Time Warping to form similarity measures**

  2) **k-Means algorithm applied to the similarity measures to determine the clusters**

- **The resulting clusters are used to identify patterns in aircraft trajectory data**

Dynamic Time Warping

- Similarity measure is a distance function that quantifies the degree of sameness between two objects

- Dynamic Time Warping was used because it provides a similarity measure that is
  - Definable for sequences of differing lengths
  - Robust to short interruptions
  - Insensitive to high variability in our trajectory dataset

Dynamic Time Warping

1. **For trajectories $Q = \{q_i\}_{i=1}^{n}$ and $S = \{s_j\}_{j=1}^{m}$, generate the matrix D with $n \times m$ entries $d(q_i, s_j)$.**

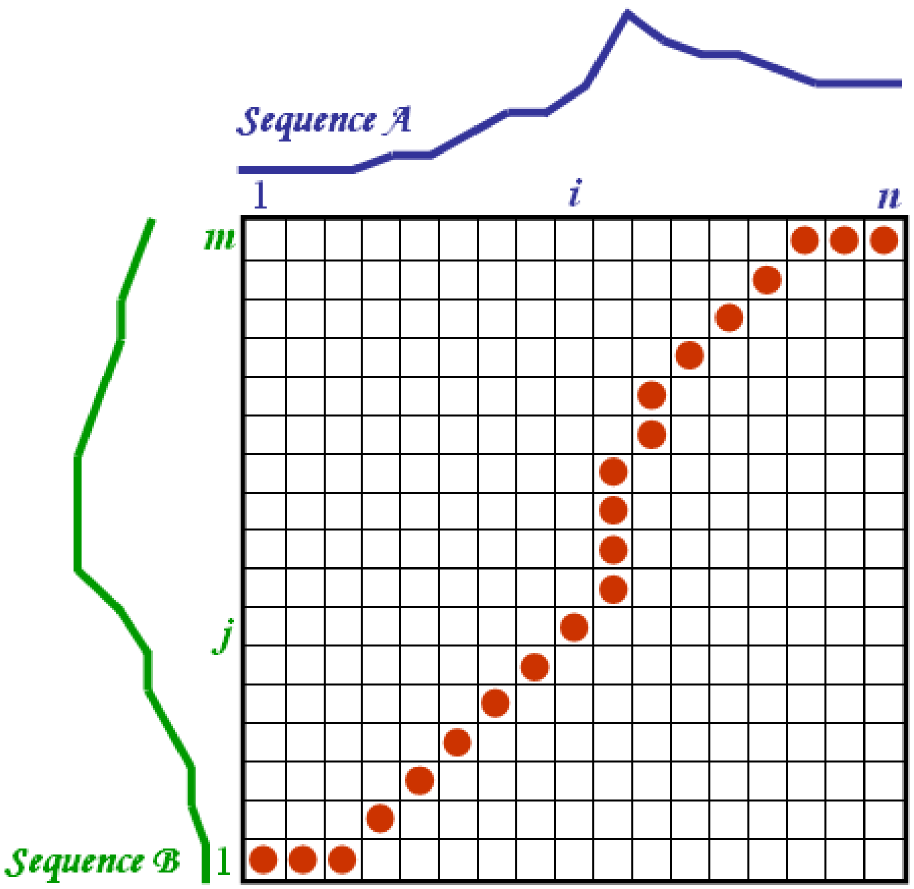
2. **Calculate the entries of the cumulative distance matrix Γ.**

   a. **Set $\gamma(1,1) \equiv d(q_1, s_1)$ and $\gamma(0,0) = \gamma(0, j) = \gamma(i, 0) \equiv 0$.**

   b. **Calculate the remaining entries of Γ using the elements of D and $\gamma(i,j) = d(q_i, s_j) + \min\{\gamma(i-1, j-1), \gamma(i-1, j), \gamma(i, j-1)\}$**

3. **Retain $\gamma(n, m)$ as the optimal solution to $DTW(Q, S) = min \sum_{p=1}^{P} w_{(i,j),p}$ where $W = \{w_{(i,j),p}\}_{p=1}^{P}$ is a warping path through the matrix D. This is the Dynamic Time Warping similarity measure.**

Dynamic Time Warping



Three requirements of Dynamic Time Warping:

1. **Respective endpoints have to map to each other**

2. **Every point has to be mapped to some other point on the other sequence**

3. **Mappings are not allowed to cross each other**

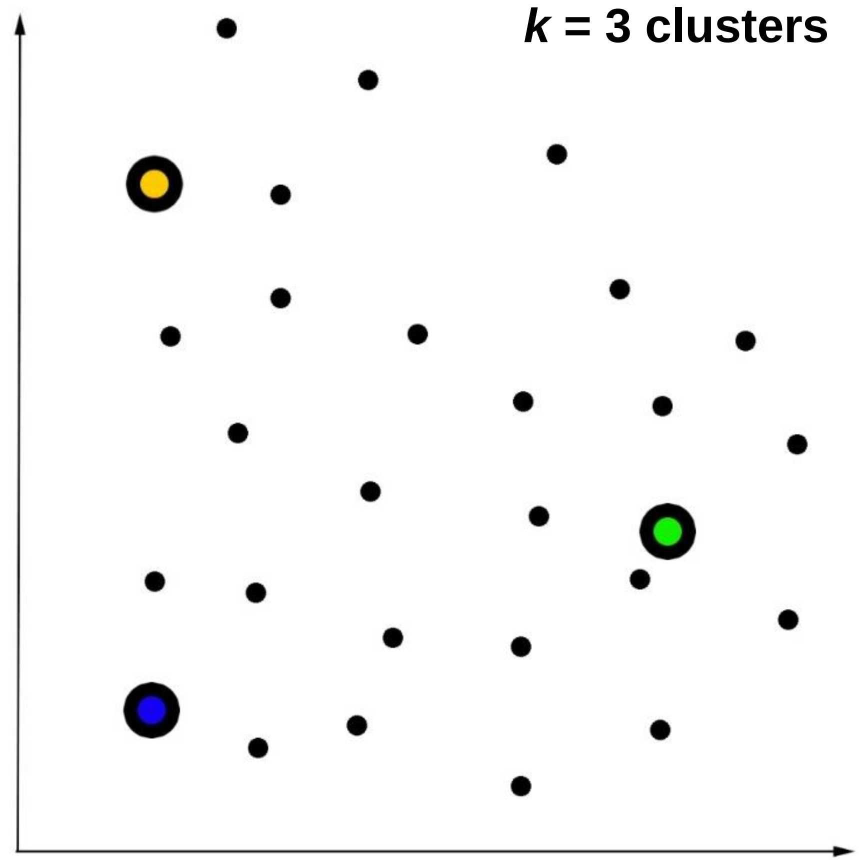
k-Means Clustering Algorithm

<u>Step 1:</u> Randomly choose *k* points from the dataset as the initial centroids.

<u>Step 2:</u> For each centroid, a cluster is formed by the set of objects that are closer to that centroid than to any other centroid.

<u>Step 3:</u> Calculate the within cluster means.

<u>Step 4:</u> Repeat Steps 2 and 3 until the process converges.



k-Means Clustering Algorithm

- **k-Means algorithm finds a user-specified *k* number of clusters for a given set of objects**
- **Determining a “best” choice of *k* can be determined through trial and error or using a systematic procedure such as the Gap Statistic**
- **For a range of possible values of *k*, the Gap Statistic calculates the expected number of clusters for each *k* using a comparison distribution and compares the differences of within cluster variation of these results to those when randomly sampling from the data**
- **A plot of these differences, or Gap Statistics, over the range of *k* can be visually inspected to help determine an appropriate choice**

Integration of Dynamic Time Warping and k-Means

- **Apply Dynamic Time Warping to the trajectory dataset to compute the similarity measures**

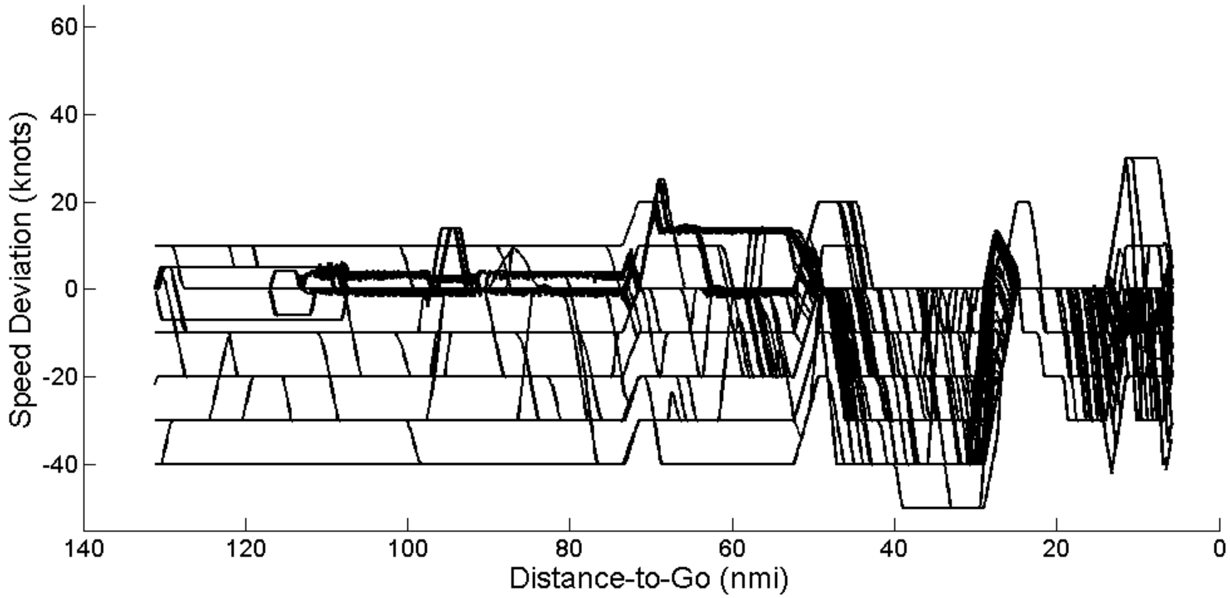
- **Select a value for *k* using the Gap Statistic applied to the new dataset of similarity measures**

- **Apply k-Means algorithm to the dataset of similarity measures**

- **The *k* clusters of Dynamic Time Warping similarity measures represent the clusters of trajectories**

Application of Trajectory Clustering

- **Dataset containing 164 trajectories collected during a human-in-the-loop experiment**
- **Lengths of trajectories ranged from approximately 600 to 1200 data points**
- **Patterns appear to exist but cannot visually determine which trajectories are similar**



Application of Trajectory Clustering

- **Interested in incorporating both distance and speed deviation so pairwise Euclidean distance was used to calculate the Dynamic Time Warping similarity measures**

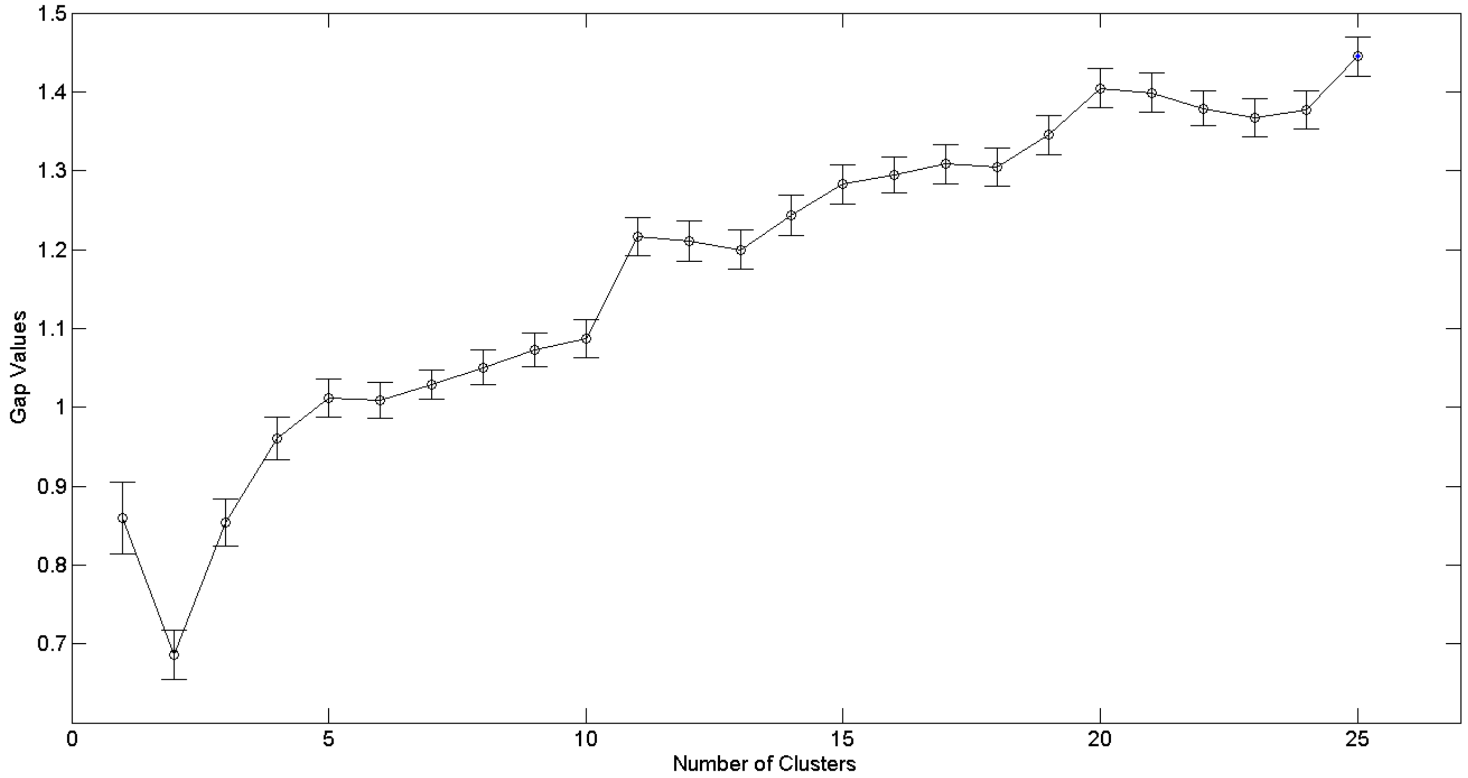
$$d\big((a,c),(b,d)\big) = \sqrt{(a-b)^2 + (c-d)^2}$$

- **Data were normalized to scale between zero and one to permit equal influence of each variable when calculating the similarity measures**

- **Calculations were carried out using MATLAB®**

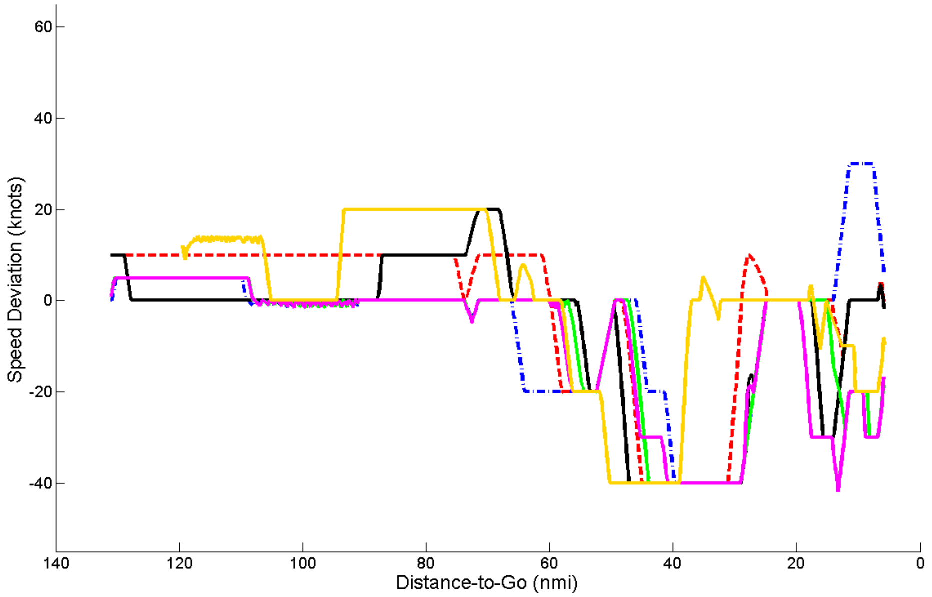
Application of Trajectory Clustering

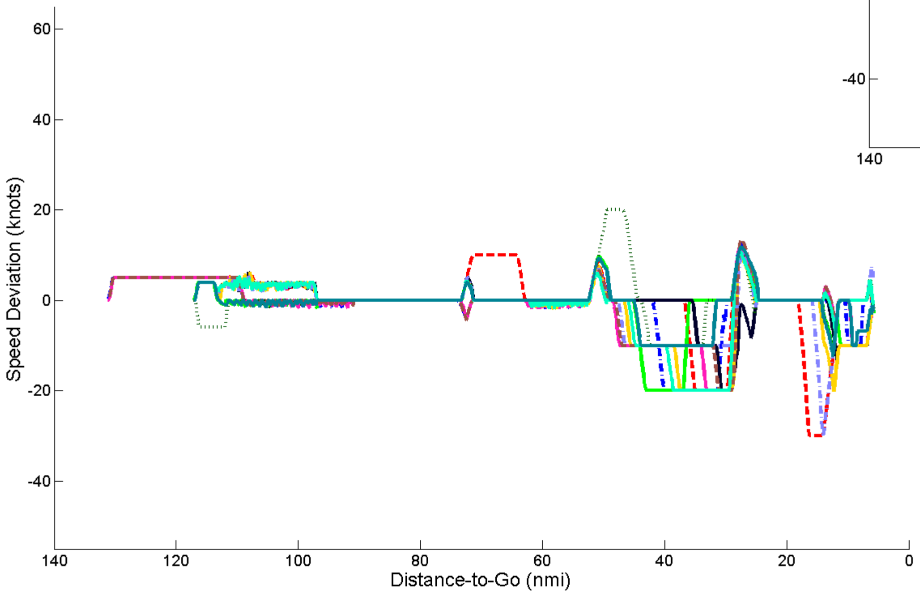
- Gap Statistic was implemented to determine an appropriate value of *k* for the k-Means algorithm
- Uniform comparison distribution was used
- *k* = 20 suggests a good starting place



Application of Trajectory Clustering

k-Means algorithm was applied with *k* = 20





Application of Trajectory Clustering

- **Within the general patterns of behavior, researchers were able to discern useful within cluster variability**
- **Of the 20 clusters, one contained a single trajectory and one contained two trajectories, indicating these as potential outliers**
- **Resulting clusters were determined by subject matter experts to identify realistic patterns in the data**
- **A sample of trajectories from each of the remaining 18 clusters could be selected and incorporated into the fast-time simulation to provide a more realistic variety of aircraft trajectories**

Conclusions

- **Successfully developed and implemented a methodology to uncover patterns within trajectory datasets**
- **Dynamic Time Warping similarity measure performed well despite challenges encountered**
- **Achieved a deeper understanding of the underlying behavior for aircraft trajectories**
- **Increased the level of realism and therefore the knowledge gained from the fast-time computer simulation**
- **Statistical engineering approach was a collaborative effort within a multi-disciplinary team**
- **Methodology developed can be applied in future simulations to investigate other air traffic management technologies or adapted for other types of problems involving trajectories**

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