

Sheet (2)... (Solution)

1. A hypothetical isotropic antenna is radiating in free-space. At a distance of 100 m from the antenna, the total electric field (E_0) is measured to be 5 V/m. Find the

- (a) Power density (W_{rad})
- (b) Power radiated (P_{rad})

$$(a) W_{rad} = \frac{1}{2} [E \times H^*] = \frac{E^2}{2\eta} \hat{A}_r = \frac{5^2 \cdot \pi}{2(120\pi)} = 0.03315 \hat{A}_r \text{ Watts/m}^2$$

$$\begin{aligned} (b) P_{rad} &= \int_S W_{rad} dS = \int_0^{2\pi} \int_0^{\pi} (0.03315)(r^2 \sin\theta d\theta d\phi) \\ &= \int_0^{2\pi} \int_0^{\pi} (0.03315)(100)^2 \cdot \sin\theta d\theta d\phi \\ &= 2\pi (0.03315)(100)^2 \cdot \int_0^{\pi} \sin\theta d\theta = 2\pi (0.03315)(100)^2 \cdot 2 \\ &= 4165.75 \text{ watts} \end{aligned}$$

2. Estimate the directivity of an antenna with $\Theta_{HP} = 2^\circ$ and $\Phi_{HP} = 1^\circ$.

$$D \text{ approximate} = \frac{41253}{\theta_{HP}\phi_{HP}} = \frac{41253}{2*1} = 20627.$$

3. Find the number of square degrees in the solid angle Ω on a spherical surface that is between ($\theta = 20^\circ$ and $\theta = 40^\circ$), and ($\phi = 30^\circ$ and $\phi = 70^\circ$).

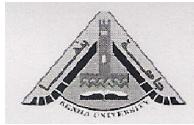
$$\Omega = \int_{30}^{70} d\phi \int_{20}^{40} \sin\theta d\theta = (70-30) * \left(\frac{180}{\pi}\right) * (-\cos\theta)_{20}^{40} = 398.17 \text{ deg}^2.$$

4. The radiation intensity of antenna is given by $U = B_0 \cos\theta$. U exists only in the upper hemisphere, Find

- a. The exact directivity.
- b. The approximate directivity.
- c. The decibel difference.

$$U = U_n = P_n = \cos\theta.$$

$$(a) D_{exact} = \frac{4\pi}{\iint_0^{\pi} \int_0^{\pi} \cos\theta \sin\theta d\theta d\phi} = \frac{4\pi}{(2\pi)(-\frac{\cos^2\theta}{2})_0^{\frac{\pi}{2}}} = 4.$$



$$(b) D \text{ approximate} = \frac{4\pi}{\theta_{HP}\phi_{HP}} = \frac{4\pi}{\theta_{HP}\phi_{HP}} = \frac{4\pi}{(\theta_{HP})^2} |_{sr} = \frac{41253}{(\theta_{HP})^2} |_{deg2}$$

We calculate $\theta_{max} \rightarrow (\cos\theta_{max} = 1) \rightarrow \text{at } \theta_{max} = 0^\circ$,

We calculate $\theta_h \rightarrow (\cos\theta_h = \frac{1}{2}) \rightarrow \theta_h = 60^\circ$

$$\theta_{HP} = 2 * |\theta_{max} - \theta_h| = 2 * |0^\circ - 60^\circ| = 120^\circ$$

$$\text{so : } D_{approx.} = \frac{41253}{(\theta_{HP})^2} |_{deg2} = \frac{41253}{(120)^2} = 2.86.$$

$$(c) \text{ Decibel difference} = 10 \log \frac{4}{2.86} = 1.46db.$$

- 5.** An antenna has a field pattern given by $E(\theta) = \cos^2\theta$, For $0 \leq \theta \leq 90^\circ$. Find the beam area of this pattern.

$$1- \text{Exact... } \Omega_A = \int_0^{2\pi} \int_0^{\pi/2} \cos^4\theta \sin\theta d\theta d\phi = -2\pi * \left(\frac{1}{5} \cos^5\theta\right)_0^{\pi/2} = 1.26Sr.$$

$$2- \text{Approximate... } \Omega_A = \theta_{HP}\phi_{HP} = (\theta_{HP})^2$$

To obtain θ_{HP} ... we must firstly calculate θ_{max} , θ_h

To Obtain θ_{max} ... the angle of which $P_n = E_n^2$ maximum.

It occurs when $\cos^2\theta = 1$... at $\theta_{max} = 0^\circ$.

$$\text{To obtain } \theta_h \dots \text{the angle of which } P_n = \frac{1}{2}. \text{ (Or } E_n = \frac{1}{\sqrt{2}}\text{)}.$$

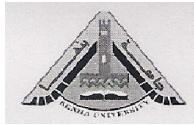
$$\text{It occurs when } \cos^2\theta_h = \frac{1}{\sqrt{2}}.$$

$$\text{So: } \theta_h = 32.76^\circ \approx 33^\circ.$$

$$\text{Now: } \theta_{HP} = 2 * |\theta_{max} - \theta_h| = 2 * |0 - 33| = 66^\circ.$$

$$\text{So: } \Omega_A = (\theta_{HP})^2 = (66)^2 = 4356 \text{ deg}^2 = 4356 * (\pi/180)^2 = 1.33Sr.$$

- 6.** The normalized field pattern of an antenna is given by $E(\theta) = \sin\theta \sin\phi$. E_n has a value only for $0 \leq \theta \leq \pi$ & $0 \leq \phi \leq \pi$, and zero elsewhere , Find
- The exact directivity.
 - The approximate directivity.
 - The decibel difference.



$$(a) \text{ Dexact} = \frac{4\pi}{\int_0^{\pi} \int_0^{\pi} \sin^3 \theta \sin^2 \phi d\theta d\phi} = \frac{4\pi}{\int_0^{\pi} \sin \theta * (1 - \cos^2 \theta) d\theta \int_0^{\pi} \frac{(1 - \cos 2\phi)}{2} d\phi} =$$

$$\frac{4\pi}{\left(\frac{\phi}{2} - \frac{\sin 2\phi}{2}\right)_0^{\pi} \int_0^{\pi} (\sin \theta d\theta - \sin \theta \cos^2 \theta d\theta)} = \frac{4\pi}{\left(\frac{\pi}{2}\right)[(-\cos \theta)_0^{\pi} + \left(\frac{\cos^3 \theta}{3}\right)_0^{\pi}]} =$$

$$\frac{4\pi}{\left(\frac{\pi}{2}\right)\left(\frac{4}{3}\right)} = 6.$$

$$(b) \text{ D approximate} = \frac{4\pi}{\theta_{HP} \phi_{HP}}.$$

We calculate $\theta_{max} \rightarrow (\sin \theta_{max} = 1..(max)) \rightarrow \text{at } \theta_{max} = 90^\circ$,
 We calculate $\Phi_{max} \rightarrow (\sin \Phi_{max} = 1..(max)) \rightarrow \text{at } \Phi_{max} = 90^\circ$,

We calculate $\theta_h \rightarrow (\sin \theta_h = \frac{1}{\sqrt{2}}) \rightarrow \theta_h = 45^\circ$

We calculate $\Phi_h \rightarrow (\sin \Phi_h = \frac{1}{\sqrt{2}}) \rightarrow \Phi_h = 45^\circ$

$$\text{So: } \theta_{HP} = 2 * |90 - 45^\circ| = 90^\circ = \frac{\pi}{2} \text{ (rad)}$$

By the same way

$$\text{We calculate } \Phi_{HP} = 2 * |90^\circ - 45^\circ| = 90^\circ = \frac{\pi}{2} \text{ (rad)}$$

$$\text{So: } D \text{ approximate} = \frac{4\pi}{\theta_{HP} \phi_{HP}} = \frac{4\pi}{\left(\frac{\pi}{2}\right)\left(\frac{\pi}{2}\right)} = 5.1.$$

$$(C) \text{ Decibel difference} = 10 \log \frac{6}{5.1} = 0.7db.$$

7. The power radiated by a lossless antenna is 10 watts. The directional characteristics of the antenna are represented by the radiation intensity of $U = B_0 \cos^3 \theta$ (watts/unit solid angle) ($0 \leq \theta \leq \pi/2$, $0 \leq \phi \leq 2\pi$)

Find the

- (a) Maximum power density (in watts/square meter) at a distance of 1,000 m (assume far-field distance). Specify the angle where this occurs.
 (b) Exact and approximate beam solid angle Ω_A .



(c) Directivity, exact and approximate, of the antenna (dimensionless and in dB).

(d) Gain, exact and approximate, of the antenna (dimensionless and in dB).

$$U = B_0 \cos^3 \theta$$

$$(a) P_{rad} = \int_0^{2\pi} \int_0^{\pi/2} U \sin \theta d\theta d\phi = B_0 \int_0^{2\pi} \int_0^{\pi/2} \cos^3 \theta \sin \theta d\theta d\phi \\ = 2\pi B_0 \int_0^{\pi/2} \cos^3 \theta \sin \theta d\theta$$

$$P_{rad} = 2\pi B_0 \left(-\frac{\cos^4 \theta}{4} \right) \Big|_0^{\pi/2} = \frac{\pi}{2} B_0 = 10 \Rightarrow B_0 = \frac{20}{\pi} = 6.3662$$

$$U = 6.3662 \cos^3 \theta$$

$$W = \frac{U}{r^2} = \frac{6.3662}{r^2} \cos^3 \theta = \frac{6.3662}{(10^3)^2} \cdot \cos^3 \theta = 6.3662 \times 10^{-6} \cos^3 \theta$$

$$W|_{max} = 6.3662 \times 10^{-6} \cos^3 \theta \Big|_{max} = 6.3662 \times 10^{-6} \text{ Watts/m}^2$$

$$(b) D_0 = \frac{4\pi U_{max}}{P_{rad}} = \frac{4\pi (6.3662)}{10} = 8 = 9 \text{ dB}$$

$$(c) G_0 = e_t D_0 = 8 = 9 \text{ dB}$$

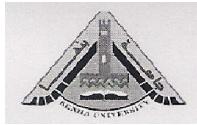
8. Calculate the $D_{approx.}$ from the HPBW of a unidirectional antenna if the power pattern is given by :

$$E(\theta, \phi) = 30 \cos^2 \theta \sin^{3/2} \phi$$

$$0 \leq \theta \leq \pi \quad 0 \leq \phi \leq \pi \quad \text{and zero otherwise.}$$

Then repeat by calculating D_{exact} for the previous pattern. Finally calculate the db difference between the exact and approximate records.

$$D_{exact} = \frac{4\pi}{\iint_{0,0}^{\pi,\pi} E_n^2(\theta, \phi) * \sin \theta d\theta d\phi} = \frac{4\pi}{\iint_{0,0}^{\pi,\pi} \cos^4 \theta * \sin^3 \phi * \sin \theta d\theta d\phi} \\ = \frac{4\pi}{\int_0^{\pi} \sin \phi * (1 - \cos^2 \phi) d\phi \int_0^{\pi} \cos^4 \theta * \sin \theta d\theta} = \frac{4\pi}{(-\frac{\cos^5 \theta}{5})_0^{\pi} \int_0^{\pi} (\sin \phi d\phi - \sin \phi \cos^2 \phi d\phi)}$$



$$= \frac{4\pi}{\left(\frac{2}{5}\right)\left[\left(-\cos\phi\right)_0^\pi + \left(\frac{\cos^3\phi}{3}\right)_0^\pi\right]} = \frac{4\pi}{\left(\frac{2}{5}\right)\left(\frac{4}{3}\right)} = 23.56$$

(b) To calculate D approximate.

We calculate $\theta_{max} \rightarrow (\cos^2\theta_{max} = 1..(max)) \rightarrow$ at $\theta_{max} = 0^\circ$,

We calculate $\Phi_{max} \rightarrow (\sin^{3/2}\Phi_{max} = 1..(max)) \rightarrow$ at $\Phi_{max} = 90^\circ$,

We calculate $\theta_h \rightarrow (\cos^2\theta_h = \frac{1}{\sqrt{2}}) \rightarrow \theta_h = 33^\circ$

We calculate $\Phi_h \rightarrow (\sin^{3/2}\Phi_h = \frac{1}{\sqrt{2}}) \rightarrow \Phi_h = 52.5^\circ$

So: $\theta_{HP} = 2 * |0 - 33^\circ| = 66^\circ$.

By the same way

We calculate $\Phi_{HP} = 2 * |90^\circ - 52.5^\circ| = 75^\circ$

So: D approximate $= \frac{41253}{\theta_{HP}\phi_{HP}} = \frac{41253}{66 * 75} = 8.33$.

(c) decibel difference $= 10\log\frac{23.5}{8.33} = 4.5$

9. In target-search ground-mapping radars it is desirable to have echo power received from a target, of constant cross section, to be independent of its range. For one such application, the desirable radiation intensity of the antenna is given by

$$U(\theta, \phi) = \begin{cases} 1 & 0^\circ \leq \theta < 20^\circ \\ 0.342 \csc(\theta) & 20^\circ \leq \theta < 60^\circ \\ 0 & 60^\circ \leq \theta \leq 180^\circ \end{cases} \quad 0^\circ \leq \phi \leq 360^\circ$$

Find the directivity (in dB) using the exact formula.



$$U(\theta, \phi) = \begin{cases} 1 & 0^\circ \leq \theta \leq 20^\circ \\ 0.342 \csc(\theta) & 20^\circ \leq \theta \leq 60^\circ \\ 0 & 60^\circ \leq \theta \leq 180^\circ \end{cases} \quad 0^\circ \leq \phi \leq 360^\circ$$

$$\begin{aligned} P_{\text{rad}} &= \int_0^{2\pi} \int_0^{\pi} U(\theta, \phi) \sin \theta d\theta d\phi = 2\pi \left[\int_0^{20^\circ} \sin \theta d\theta + \int_{20^\circ}^{60^\circ} 0.342 \csc(\theta) \times \right. \\ &\quad \left. \sin \theta d\theta \right] = 2\pi \left\{ -\cos \theta \Big|_0^{20^\circ} + 0.342 \cdot \theta \Big|_{\pi/9}^{\pi/3} \right\} \\ &= 2\pi \left\{ [-\cos(\frac{\pi}{9}) + 1] + 0.342 (\frac{\pi}{3} - \frac{\pi}{9}) \right\} \\ &= 2\pi \left\{ [-0.93969 + 1] + 0.342 \pi (\frac{2}{9}) \right\} \\ &= 2\pi \left\{ 0.06031 + 0.23876 \right\} = 1.07912 \end{aligned}$$

$$D_0 = \frac{4\pi U_{\text{max}}}{P_{\text{rad}}} = \frac{4\pi (1)}{1.07912} = 6.68737 = 8.25255 \text{ dB.}$$

10. The normalized radiation intensity of a given antenna is given by

(a) $U = \sin \theta \sin \phi$, (b) $U = \sin \theta \sin^2 \phi$, (C) $U = \sin^2 \theta \sin^3 \phi$

The intensity exists only in the $0 \leq \theta \leq \pi$, $0 \leq \phi \leq \pi$ region, and it is zero elsewhere. Find the

(a) Exact directivity (dimensionless and in dB).

(b) Azimuthal and elevation plane half-power beam widths (in degrees).



$$D_0 = \frac{4\pi U_{max}}{P_{rad}}$$

(a) $U = \sin\theta \sin\phi$ for $0 \leq \theta \leq \pi$, $0 \leq \phi \leq \pi$

$U|_{max} = 1$ and it occurs when $\theta = \phi = \pi/2$.

$$P_{rad} = \int_0^\pi \int_0^\pi U \sin\theta d\theta d\phi = \int_0^\pi \sin\theta d\theta \int_0^\pi \sin^2\theta d\theta = 2(\frac{\pi}{2}) = \pi.$$

Thus $D_0 = \frac{4\pi(1)}{\pi} = 4 = 6.02 \text{ dB}$

The half-power beamwidths are equal to

$$\text{HPBW (az.)} = 2[90^\circ - \sin^{-1}(1/2)] = 2(90^\circ - 30^\circ) = 120^\circ$$

$$\text{HPBW (el.)} = 2[90^\circ - \sin^{-1}(1/2)] = 2(90^\circ - 30^\circ) = 120^\circ$$

In a similar manner, it can be shown that for

(b) $U = \sin\theta \sin^2\phi \Rightarrow D_0 = 5.09 = 7.07 \text{ dB}$

$$\text{HPBW (el.)} = 120^\circ, \text{ HPBW (az.)} = 90^\circ$$

(C)

$$U = \sin^2\theta \sin^3\phi \Rightarrow D_0 = 9\pi/4 = 7.07 = 8.49 \text{ dB}$$

$$\text{HPBW (el.)} = 90^\circ, \text{ HPBW (az.)} = 74.93^\circ$$

11. Find the directivity (dimensionless and in dB) for the antenna of Problem 4 using Kraus' approximate formula.

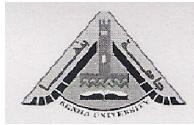
(a)

$$U = \sin\theta \cdot \sin\phi ; (a) D_0 \approx \frac{41253}{\Theta_{1d} \Theta_{2d}} = \frac{41253}{120 \times 120} = 2.86 = 4.57 \text{ dB}$$

(b) $D_0 \approx 3.82 = 5.82 \text{ dB}$

(c) $D_0 \approx 6.12 = 7.87 \text{ dB}$

12. The normalized radiation intensity of an antenna is rotationally symmetric in ϕ , and it is represented by



$$U = \begin{cases} 1 & 0^\circ \leq \theta < 30^\circ \\ 0.5 & 30^\circ \leq \theta < 60^\circ \\ 0.1 & 60^\circ \leq \theta < 90^\circ \\ 0 & 90^\circ \leq \theta \leq 180^\circ \end{cases}$$

(a) What is the directivity (above isotropic) of the antenna (in dB)?

$$(a) D_0 = \frac{4\pi U_{max}}{P_{rad}} = \frac{U_{max}}{U_0}$$

$$P_{rad} = \int_0^{2\pi} \int_0^{\pi} U \sin\theta d\theta d\phi = 2\pi \int_0^{\pi} U \sin\theta d\theta = 2\pi \left\{ \int_{30^\circ}^{30^\circ} \sin\theta d\theta + \int_{30^\circ}^{60^\circ} (0.5) \sin\theta d\theta + \int_{60^\circ}^{90^\circ} (0.1) \sin\theta d\theta \right\} = 2\pi \left\{ (-\cos\theta) \Big|_{30^\circ}^{30^\circ} + \left(-\frac{\cos\theta}{2}\right) \Big|_{30^\circ}^{60^\circ} + (-0.1\cos\theta) \Big|_{60^\circ}^{90^\circ} \right\}$$

$$(Cont'd) = 2\pi \left\{ (-0.866+1) + \left(\frac{-0.5+0.866}{2} \right) + \left(\frac{-0+0.5}{10} \right) \right\}$$

$$P_{rad} = 2\pi \left\{ -0.866 + 1 - 0.25 + 0.433 + 0.05 \right\} = 2\pi (0.367)$$

$$= 0.734 \cdot \pi \approx 2.3059$$

$$D_0 = \frac{1(4\pi)}{2.3059} = 5.4496 = 7.3636 \text{ dB}$$

13. The radiation intensity of an antenna is given by $U(\theta, \phi) = \cos^4 \theta \sin^2 \phi$, for $0 \leq \theta \leq \pi/2$ and $0 \leq \phi \leq 2\pi$ (i.e., in the upper half-space). It is zero in the lower half-space.

Find the

- (a) Exact directivity (dimensionless and in dB)
- (b) Elevation plane half-power beam width (in degrees).



$$(a) P_{\text{rad}} = \int_0^{2\pi} \int_0^{\pi} U(\theta, \phi) \sin \theta d\theta d\phi = \int_0^{2\pi} \sin^2 \phi d\phi \cdot \int_0^{\pi/2} \cos^4 \theta \sin \theta d\theta \\ = (\pi)(\frac{1}{5}) = \frac{\pi}{5}.$$

$$U_{\max} = U(\theta=0^\circ, \phi=\pi/2) = 1.$$

$$D_o = \frac{4\pi U_{\max}}{P_{\text{rad}}} = \frac{4\pi}{(\pi/5)} = 20 = 13.0 \text{ dB}$$

(b) Elevation Plane: θ varies, ϕ fixed

→ choose $\phi = \pi/2$.

$$U(\theta, \phi=\pi/2) = \cos^4 \theta, \quad 0 \leq \theta \leq \pi/2.$$

$$\cos^4 \left[\frac{\text{HPBW(el.)}}{2} \right] = \frac{1}{2}$$

$$\text{HPBW(el.)} = 2 \cdot \cos^{-1} \{ \sqrt{0.5} \} = 65.5^\circ.$$

14. The far-zone electric-field intensity (array factor) of an end-fire two-element array antenna, placed along the z-axis and radiating into free-space, is given by

$$E = \cos \left[\frac{\pi}{4} (\cos \theta - 1) \right] \frac{e^{-jkr}}{r}, \quad 0 \leq \theta \leq \pi$$

Find the directivity using Kraus' approximate formula

$$(a). E|_{\max} = \cos \left[\frac{\pi}{4} (\cos \theta - 1) \right] \Big|_{\max} = 1 \quad \text{at } \theta = 0^\circ.$$

$$0.707 E_{\max} = 0.707 \cdot (1) = \cos \left[\frac{\pi}{4} (\cos \theta_1 - 1) \right]$$

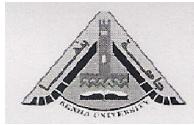
$$\frac{\pi}{4} (\cos \theta_1 - 1) = \pm \frac{\pi}{4} \Rightarrow \theta_1 = \begin{cases} \cos^{-1}(2) = \text{does not exist} \\ \cos^{-1}(0) = 90^\circ = \frac{\pi}{2} \text{ rad.} \end{cases}$$

$$\Theta_{1r} = \Theta_{2r} = 2 \left(\frac{\pi}{2} \right) = \pi$$

$$D_o \approx \frac{4\pi}{\Theta_{1r} \Theta_{2r}} = \frac{4\pi}{\pi^2} = \frac{4}{\pi} = 1.273 = 1.049 \text{ dB}$$

15. The normalized far-zone field pattern of an antenna is given by

$$E = \begin{cases} (\sin \theta \cos^2 \phi)^{1/2} & 0 \leq \theta \leq \pi \text{ and } 0 \leq \phi \leq \pi/2, 3\pi/2 \leq \phi \leq 2\pi \\ 0 & \text{elsewhere} \end{cases}$$



Find the directivity using

- (a) The exact expression
- (b) Kraus' approximate formula

$$U = \frac{1}{2\eta} |E|^2 = \frac{1}{2\eta} \sin\theta \cos^2\phi \Rightarrow U_{max} = \frac{1}{2\eta}$$

$$(a). P_{rad} = 2 \cdot \int_0^{\pi/2} \int_0^\pi \frac{1}{2\eta} \sin^2\theta \cos^2\phi d\theta d\phi = \frac{1}{\eta} \left(\frac{\pi}{4}\right) \left(\frac{\pi}{2}\right) = \frac{\pi^2}{8\eta}$$

$$D_o = \frac{4\pi U_{max}}{P_{rad}} = \frac{4\pi \left(\frac{1}{2\eta}\right)}{\frac{\pi^2}{8\eta}} = \frac{16}{\pi} = 5.09 = 7.07 \text{ dB}$$

$$(b). U_{max} = \frac{1}{2\eta} \text{ at } \theta = \pi/2, \phi = 0$$

In the elevation plane through the maximum $\phi=0$ and $U = \frac{1}{2\eta} \sin\theta$.

The 3-dB point occurs when

$$U = 0.5 U_{max} = 0.5 \left(\frac{1}{2\eta}\right) = \frac{1}{2\eta} \sin\theta_1 \Rightarrow \theta_1 = \sin^{-1}(0.5) \approx 30^\circ$$

$$\text{Therefore } \Theta_{3d} = 2(90 - 30) = 120^\circ$$

In the azimuth plane through the maximum $\theta = \pi/2$ and $U = \frac{1}{2\eta} \cos^2\phi$.

$$\text{The 3-dB point occurs when } U = 0.5 U_{max} = 0.5 \left(\frac{1}{2\eta}\right) = \frac{1}{2\eta} \cos^2\theta_1 \Rightarrow \phi_1 = \cos^{-1}(0.707) = 45^\circ, \quad \Theta_{3d} = 2(90^\circ - 45^\circ) = 90^\circ.$$

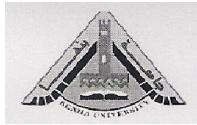
$$\text{Therefore using Kraus' formula } D_o \approx \frac{41,253}{120 \cdot (90)} = 3.82 = 5.82 \text{ dB}$$

16. Estimate the directivity for a source with relative field pattern

a. $E = \cos 2\theta \cos \theta$. Assume a unidirectional pattern.

b. $E = \sin\left(\frac{\Pi}{2} \cos \theta\right)$. Assume $0 \leq \theta \leq \Pi$ & $0 \leq \phi \leq 2\Pi$.

$$\begin{aligned}
 (a) \quad D_{exact} &= \frac{4\Pi}{\iint_0^{\Pi} (Cos2\theta Cos\theta)^2 * Sin\theta d\theta d\phi} \\
 &= \frac{4\Pi}{\Pi * \int_0^{\Pi} (Cos2\theta Cos\theta)^2 * Sin\theta d\theta}
 \end{aligned}$$



$$\begin{aligned}
 &= \frac{4}{\int_0^\pi (2\cos^2\theta - 1)^2 \cos^2\theta * \sin\theta d\theta} = \frac{4}{\int_0^\pi (4\cos^4\theta - 4\cos^2\theta + 1) \cos^2\theta * \sin\theta d\theta} \\
 &= \frac{4}{\int_0^\pi (4\cos^6\theta - 4\cos^4\theta + \cos^2\theta) * \sin\theta d\theta} = \\
 &\frac{4}{-\left(\frac{4\cos^7\theta}{7} - 4\frac{\cos^5\theta}{5} + \frac{\cos^3\theta}{3}\right)_0^\pi} = 19.1
 \end{aligned}$$

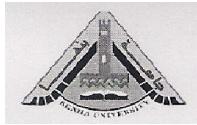
$$\begin{aligned}
 (b) Dexact &= \frac{4\pi}{\iint_0^\pi (\sin^2(\frac{\pi}{2}\cos\theta) * \sin\theta d\theta d\phi)} = \\
 &\frac{4\pi}{2\pi \int_0^\pi (\sin^2(\frac{\pi}{2}\cos\theta) * \sin\theta d\theta d\phi)} \\
 &= \frac{2}{\int_0^\pi (\sin^2(\frac{\pi}{2}\cos\theta) * \sin\theta d\theta d\phi)}
 \end{aligned}$$

$$Let \ (\frac{\pi}{2}\cos\theta) = x \rightarrow (1)$$

$$So \quad \frac{dx}{d\theta} = -\frac{\pi}{2}\sin\theta \Rightarrow \sin\theta d\theta = \frac{-2}{\pi} dx \rightarrow (2)$$

Replace (θ from 0 to π) to (x from $\frac{\pi}{2}$ to $-\frac{\pi}{2}$). By substituting in (1).

$$Dexact = \frac{2}{\int_{\frac{\pi}{2}}^{-\pi} (\sin^2(x) * \frac{-2}{\pi} dx)} = \frac{-\pi}{\int_{\frac{\pi}{2}}^{-\pi} (\sin^2(x) * dx)} = \frac{-\pi}{\int_{\frac{\pi}{2}}^{-\pi} \frac{1}{2}(1 - \cos 2x) dx} =$$



$$\frac{-\Pi}{\frac{1}{2}(x - \frac{\sin 2x}{2})^{\frac{-\Pi}{2}}} = 2.$$

(REPORT)

1. The normalized radiation intensity of an antenna is symmetric, and it can be approximated by

$$U(\theta) = \begin{cases} 1 & 0^\circ \leq \theta < 30^\circ \\ \frac{\cos(\theta)}{0.866} & 30^\circ \leq \theta < 90^\circ \\ 0 & 90^\circ \leq \theta \leq 180^\circ \end{cases}$$

And it is independent of ϕ . Find the

- (a) Exact directivity by integrating the function
- (b) Approximate directivity using Kraus' formula.

2. (a) $P_{rad} = \int_0^{2\pi} \int_0^{\pi} U(\theta, \phi) \sin \theta d\theta d\phi = 2\pi \cdot \left\{ \int_0^{30^\circ} \sin \theta d\theta + \int_{30^\circ}^{90^\circ} \frac{\cos \theta \sin \theta d\theta}{0.866} \right\}$

$$= 2\pi \left[\int_0^{\pi/6} \sin \theta d\theta + \int_{\pi/6}^{\pi/2} \frac{1}{0.866} \cos \theta \sin \theta d\theta \right]$$

$$= 2\pi \left[-\cos \theta \Big|_0^{\pi/6} + \frac{1}{0.866} \left(-\frac{\cos^2 \theta}{2} \right) \Big|_{\pi/6}^{\pi/2} \right] = 2\pi [-0.866 + 1 + 0.433] = 3.5626$$

$$D_0 = \frac{4\pi U_{max}}{P_{rad}} = \frac{4\pi (1)}{3.5626} = 3.5273 = 5.4745 \text{ dB}$$

(b) $U = \frac{\cos(\theta)}{0.866} = 0.5 \Rightarrow \cos \theta = 0.5(0.866) = 0.433, \theta = \arcsin(0.433) = 64.34^\circ$

$$\Theta_{1r} = 2(64.34) = 128.68^\circ = 2.246 \text{ rad} = \Theta_{2r}$$

$$D_0 \approx \frac{4\pi}{\Theta_{1r} \Theta_{2r}} = \frac{4\pi}{(2.246)^2} = 3.9641 \text{ dB}$$

2. Repeat Problem 8 when

$$E = \cos \left[\frac{\pi}{4} (\cos \theta + 1) \right] \frac{e^{-jkr}}{r}, \quad 0 \leq \theta \leq \pi$$



a. $E_{1\max} = \cos(\frac{\pi}{4}(\cos\theta + 1))|_{\max} = 1 \quad \text{at } \theta = \pi$,
 $0.707 = \cos(\frac{\pi}{4}(\cos\theta_1 + 1))$
 $\frac{\pi}{4}(\cos\theta_1 + 1) = \pm\frac{\pi}{4} \Rightarrow \theta_1 = \begin{cases} \cos^{-1}(-2) \rightarrow \text{does not exist.} \\ \cos^{-1}(0) \rightarrow 90^\circ \rightarrow \frac{\pi}{2} \text{ rad} \end{cases}$
 $\theta_{1r} = \theta_{2r} = 2(\frac{\pi}{2}) = \pi$.
 $D_0 \approx \frac{4\pi}{\pi^2} = \frac{4}{\pi} = 1.273 = 1.048 \text{ dB}$

3. The normalized radiation intensity of an antenna is represented by

$$U(\theta) = \cos^2(\theta) \cos^2(3\theta), \quad (0 \leq \theta \leq 90^\circ, \quad 0^\circ \leq \phi \leq 360^\circ)$$

Find the exact and approximate directivity.

- a. Since the $U(\theta)$ represents the *power* pattern, to find the half-power beamwidth you set the function equal to half of its maximum, or

$$U(\theta)|_{\theta=\theta_h} = \cos^2(\theta) \cos^2(3\theta)|_{\theta=\theta_h} = 0.5 \Rightarrow \cos \theta_h \cos 3\theta_h = 0.707$$

$$\theta_h = \cos^{-1} \left(\frac{0.707}{\cos 3\theta_h} \right)$$

Since this is an equation with transcendental functions, it can be solved iteratively. After a few iterations, it is found that

$$\theta_h \approx 0.25 \text{ radians} = 14.325^\circ$$

Since the function $U(\theta)$ is symmetrical about the maximum at $\theta = 0$, then the HPBW is

$$\text{HPBW} = 2\theta_h \approx 0.50 \text{ radians} = 28.65^\circ$$

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Good Luck

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