Benha University
Faculty Of Engineering at Shoubra

ECE 411
Antennas & Wave propagations
(2016/2017)

Lecture (2)
Antenna Parameters

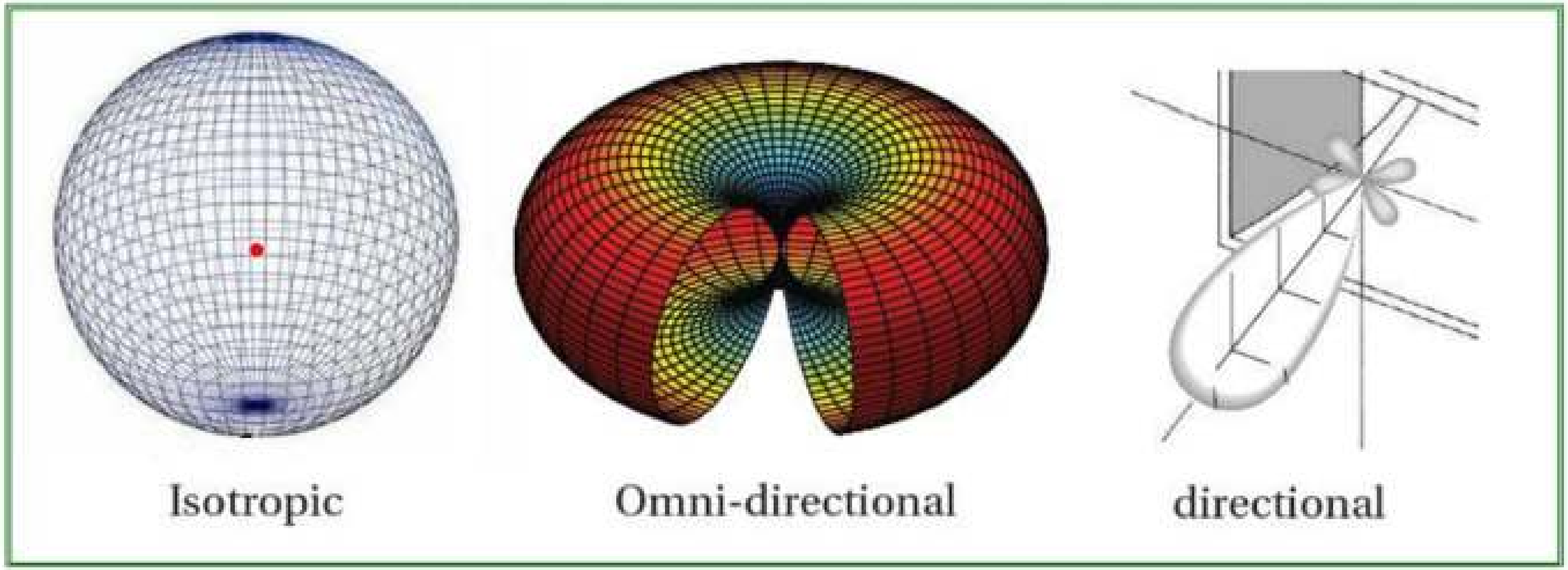
Prepared By :
Dr. Moataz Elsherbini
motaz.ali@feng.bu.edu.eg

Agenda

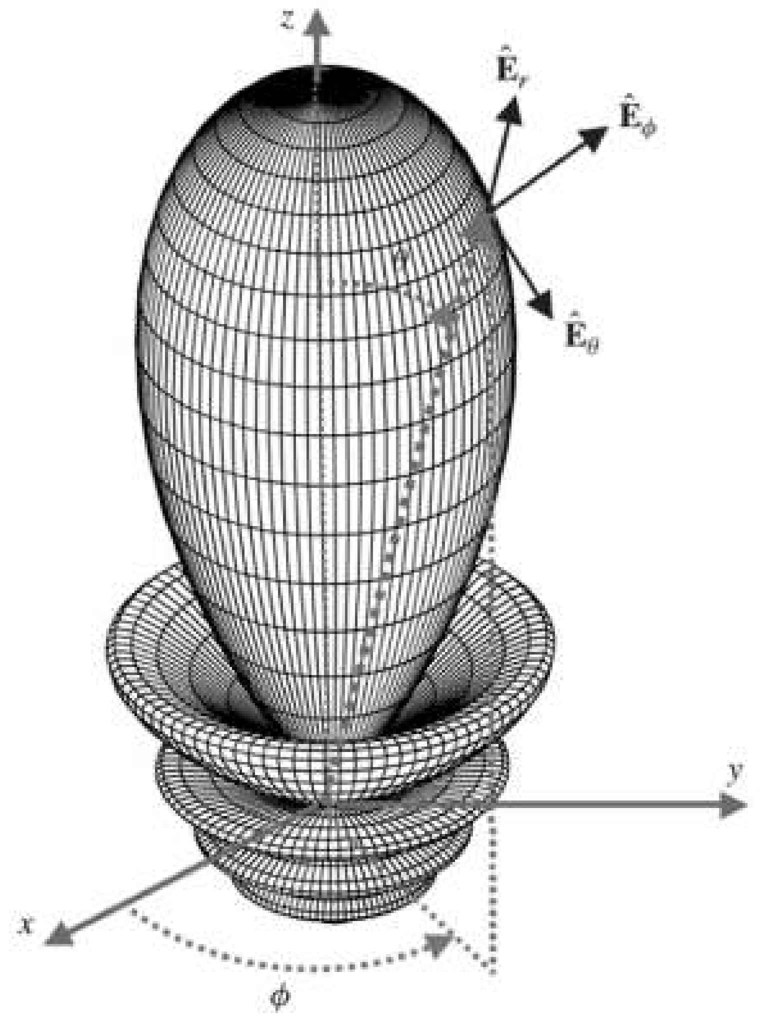
- Remember (Radiation Pattern)
- Beam Area , Beam Solid Angle, HPBW
- Directivity (Exact – Approximate)
- Examples

1 - Remember (Radiation Pattern)

1 - Remember (Radiation Pattern)

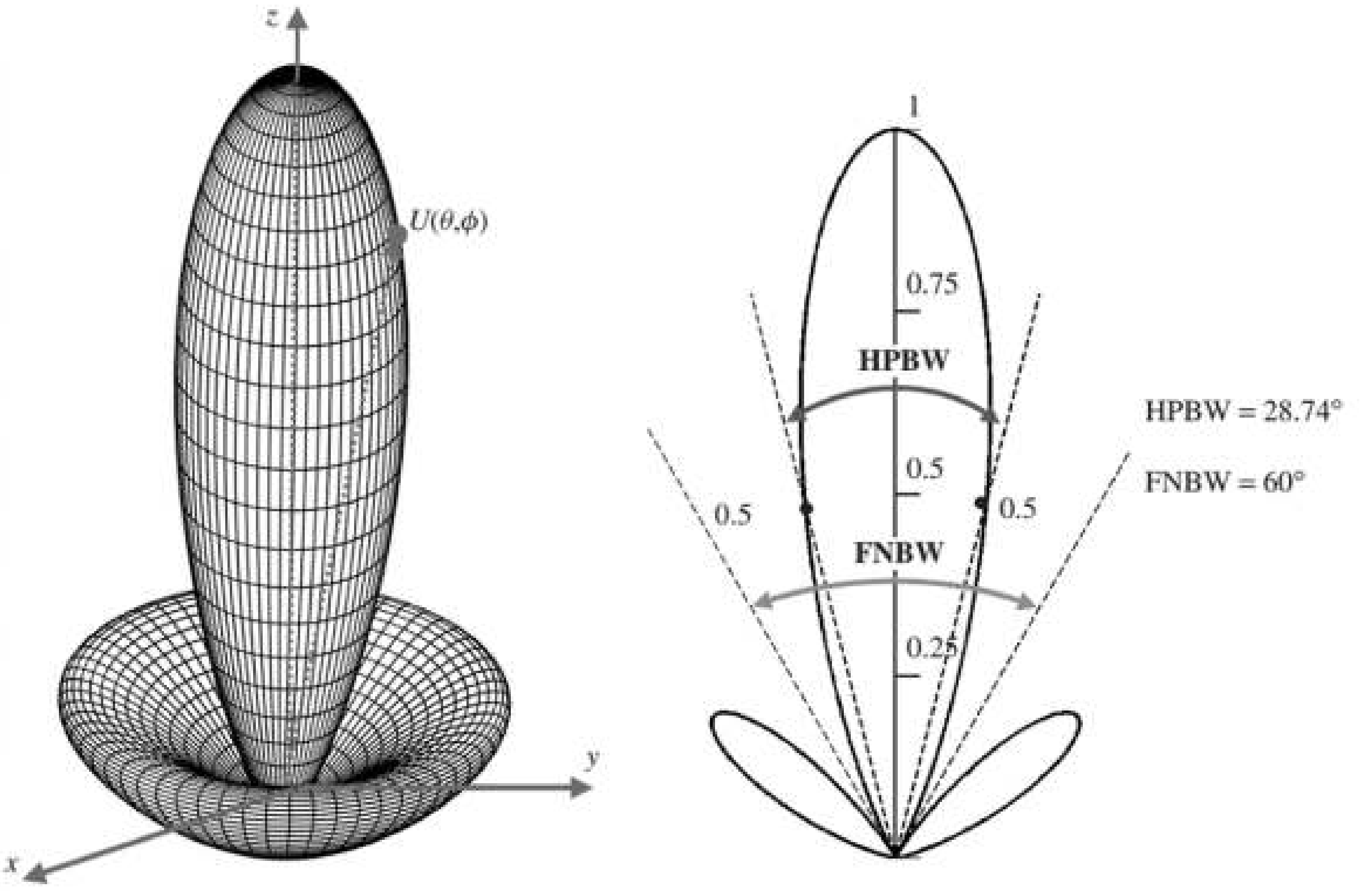


3D Field/Power pattern



Normalized three-dimensional amplitude field pattern (in linear scale)

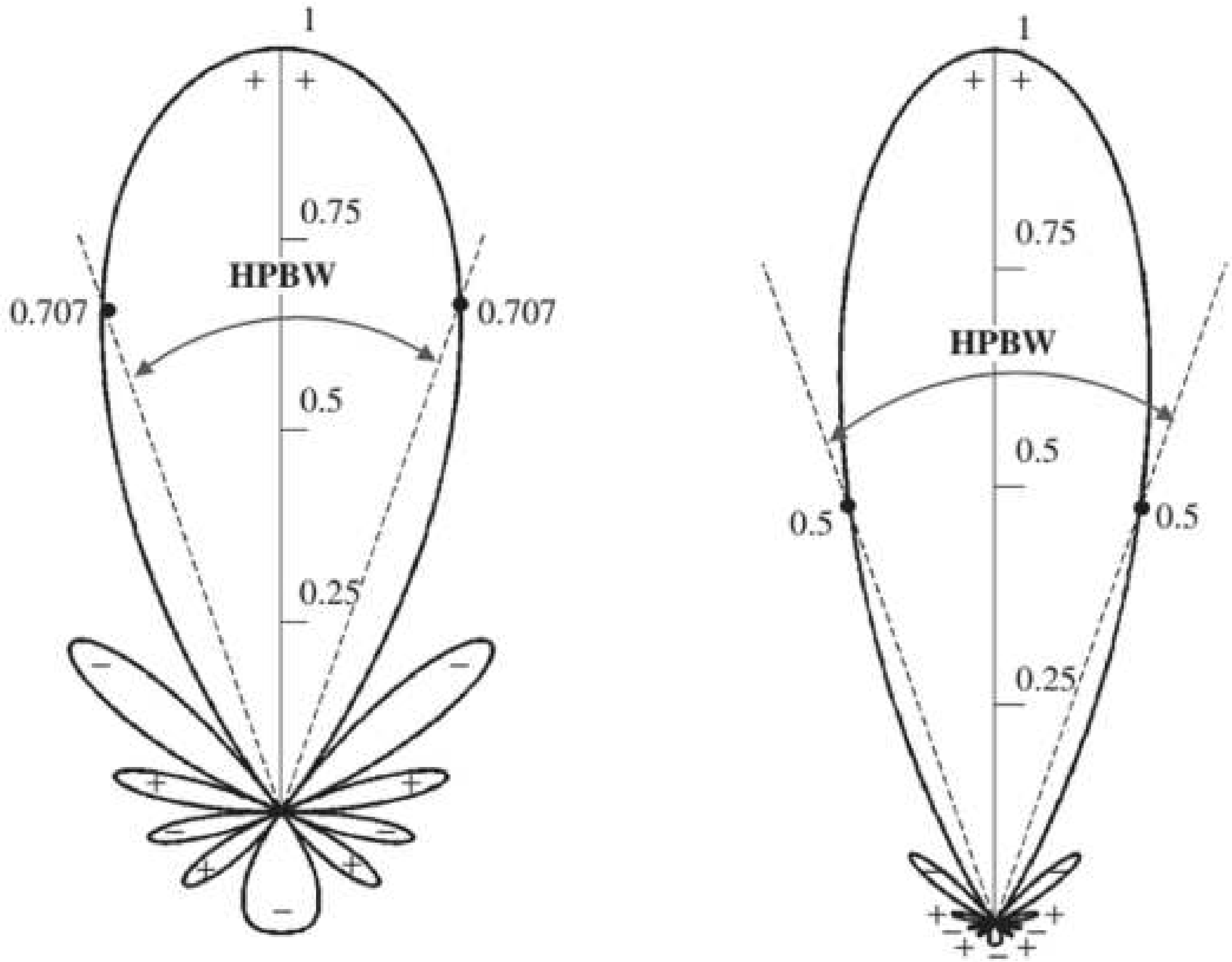
3D & 2D power pattern



(a) Three dimensional (b) Two dimensional

Three- and two-dimensional power patterns (in linear scale) of $U(\theta) = \cos^2(\theta)\cos^2(3\theta)$.

Field Versus Power Pattern

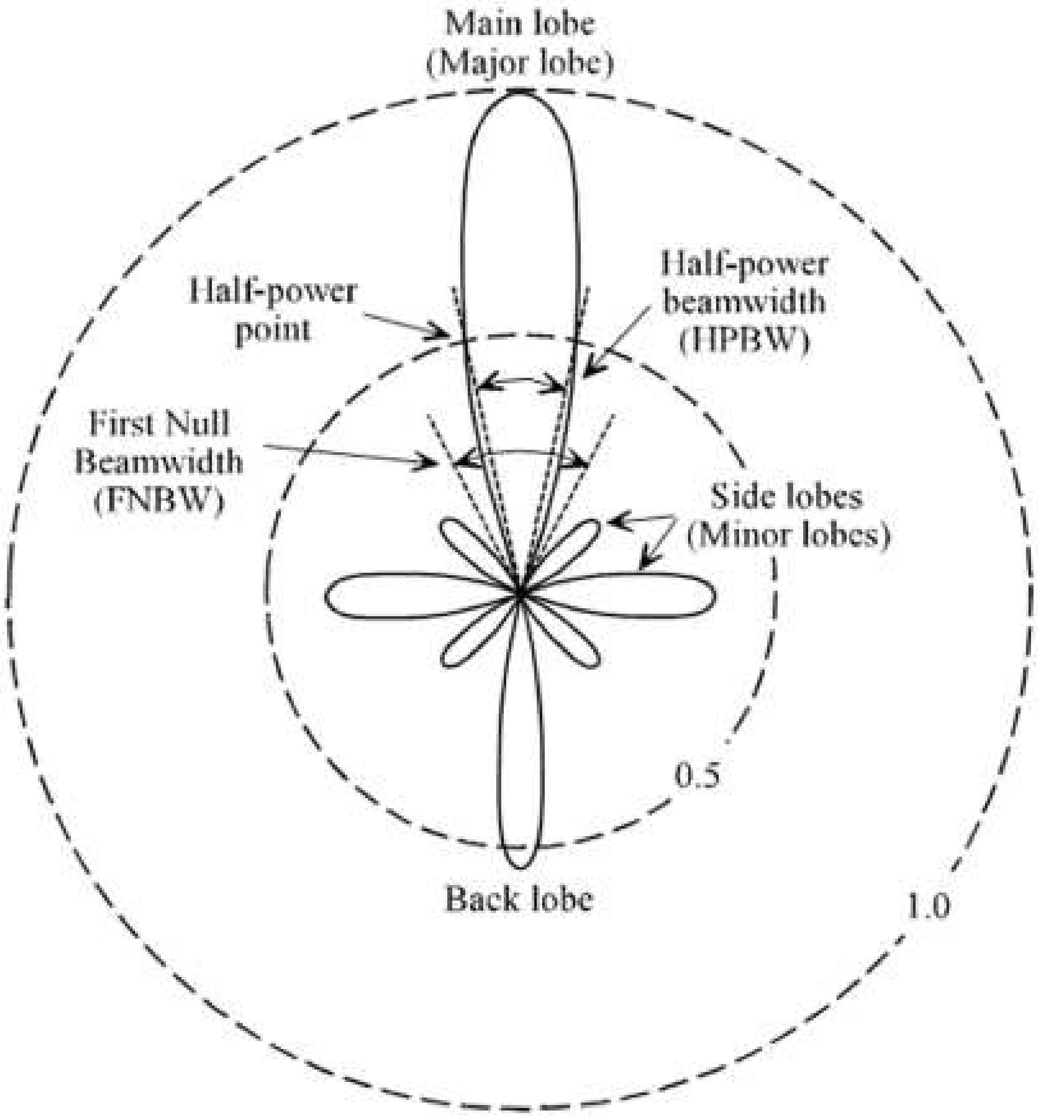


(a) Field pattern (*in linear scale*)

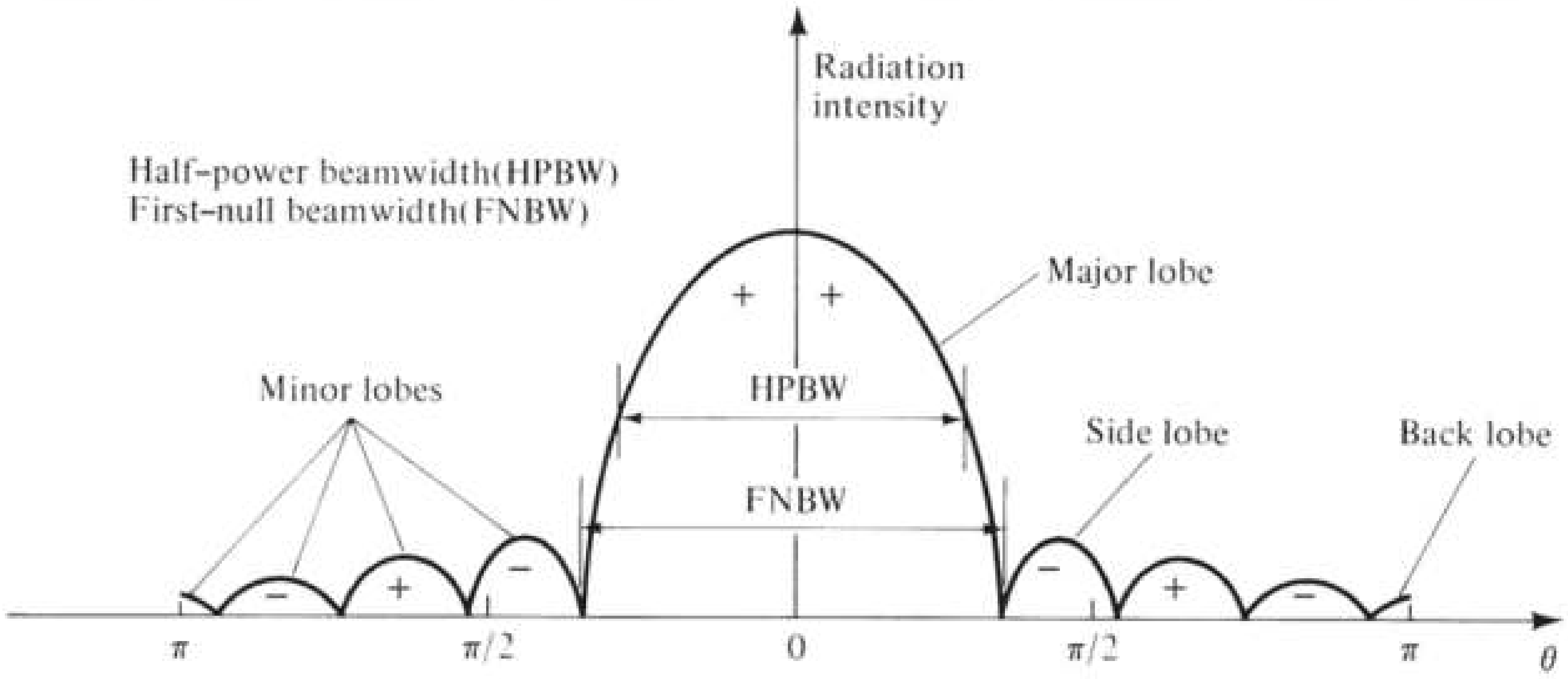
(b) Power pattern (*in linear scale*)

Radiation Lobes

Radiation lobes and beamwidths of an antenna pattern



Linear Plot for Radiation pattern



Linear plot of power pattern and its associated lobes and beamwidths

Example

For example, the radiation pattern of the Hertzian dipole can be plotted using the following steps.

(1) Far field:

$$E_\theta = j\frac{\eta k I d\ell}{4\pi}\left(\frac{e^{-jkr}}{r}\right)\sin\theta, \quad \begin{cases} 0 \le \theta \le \pi \\ 0 \le \phi \le 2\pi \\ r \text{ fixed} \end{cases}$$

(2) Far field magnitude:

$$|E_\theta| = \frac{\eta k I d\ell}{4\pi r}|\sin\theta|, \quad \begin{cases} 0 \le \theta \le \pi \\ 0 \le \phi \le 2\pi \\ r \text{ fixed} \end{cases}$$

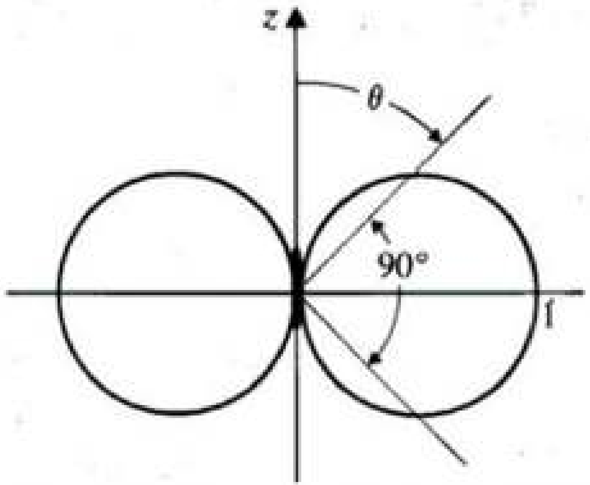
Example

(3) Normalization:

$$\left|E_\theta\right|_n = \frac{\dfrac{\eta k I d\ell}{4\pi r}\left|\sin\theta\right|}{\dfrac{\eta k I d\ell}{4\pi r}} = \left|\sin\theta\right|, \begin{cases} 0 \le \theta \le \pi \\ 0 \le \phi \le 2\pi \\ r \text{ fixed} \end{cases}$$

(4) Plot θ–plane pattern (fix ϕ at a chosen value, for example $\phi = 0°$)

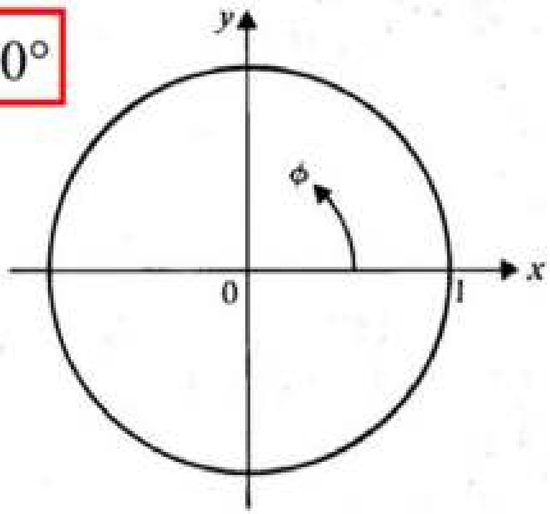
Nulls: at 0 , 180
Maxima: at 90 , - 90



Example

(5) Plot ϕ–plane pattern (fix θ at a chosen value, for example $\theta = 90°$)

$|E_\theta|_n$ with ϕ at $\theta = 90°$



Example 2

Draw the radiation pattern for an antenna has a field pattern of :

$E_n = \cos^2\theta$, for $0 \le \theta \le 90^o$, $0 \le \phi \le 360^o$

And Show the nulls, Maxima, FNBW, and HPBW

(1) nulls $E_n = 0$ $\cos^2\theta = 0$

$\theta = \pm 90^\circ$

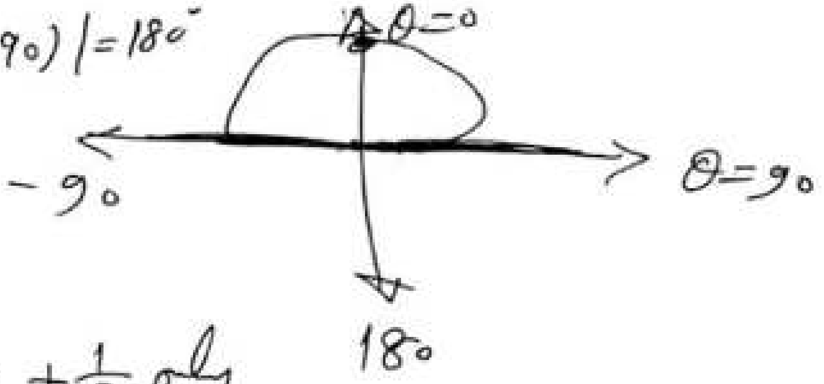
(2) Max. $E_n = \pm 1$ $\cos^2\theta = \pm 1$ $\therefore \cos^2\theta = +1$ only

$\therefore \theta_{max} = 0^\circ$, (180°) X refused (out of Range)

Example 2

(3) $FNBW = 2|\theta_{max} - \theta_{1n}| = 2|0 - 90| = 180^\circ$

or $|90 - (-90)| = 180^\circ$



(4) HPBW

let $E_n = \pm\frac{1}{\sqrt{2}}$

$\therefore \cos^2\theta_h = \pm\frac{1}{\sqrt{2}} = +\frac{1}{\sqrt{2}}$ only

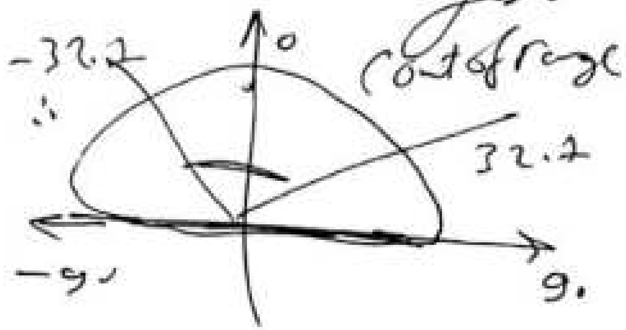
$\therefore \cos\theta_h = \pm\sqrt{\frac{1}{\sqrt{2}}}$

- $+\sqrt{\frac{1}{\sqrt{2}}} \rightarrow \theta = \pm 32.7^\circ$
- $-\sqrt{\frac{1}{\sqrt{2}}} \rightarrow \theta = \pm 147.3^\circ$ X refused (out of range)

$\therefore HPBW = (32.7 - (-32.7)) = 65.4^\circ$

or $2|\theta_{max} - \theta_{1h}|$

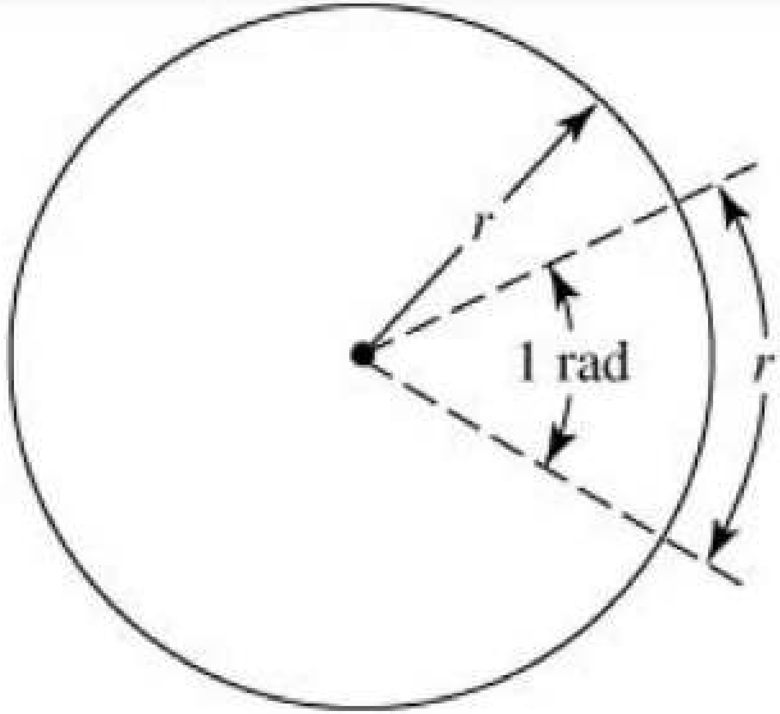
$\therefore 2|0 - 32.7| = 65.4^\circ$



2 - Beam Area , Beam Solid Angle, HPBW

Radian

Radian



$$C = 2\pi r$$

$$\text{Rads} = \frac{C}{r} = \frac{2\pi r}{r}$$

$$\text{Rads} = 2\pi$$

Steradian

Steradian

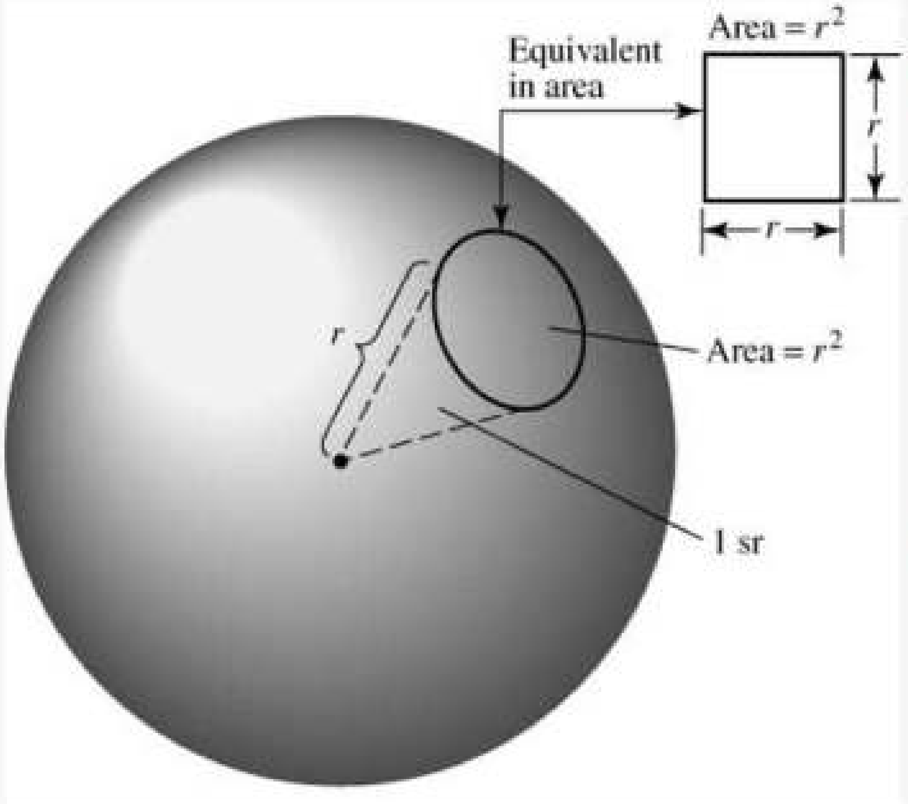


Fig. 2.10(b)

$$d\Omega = \frac{dA}{r^2} \quad (2\text{-}1)$$

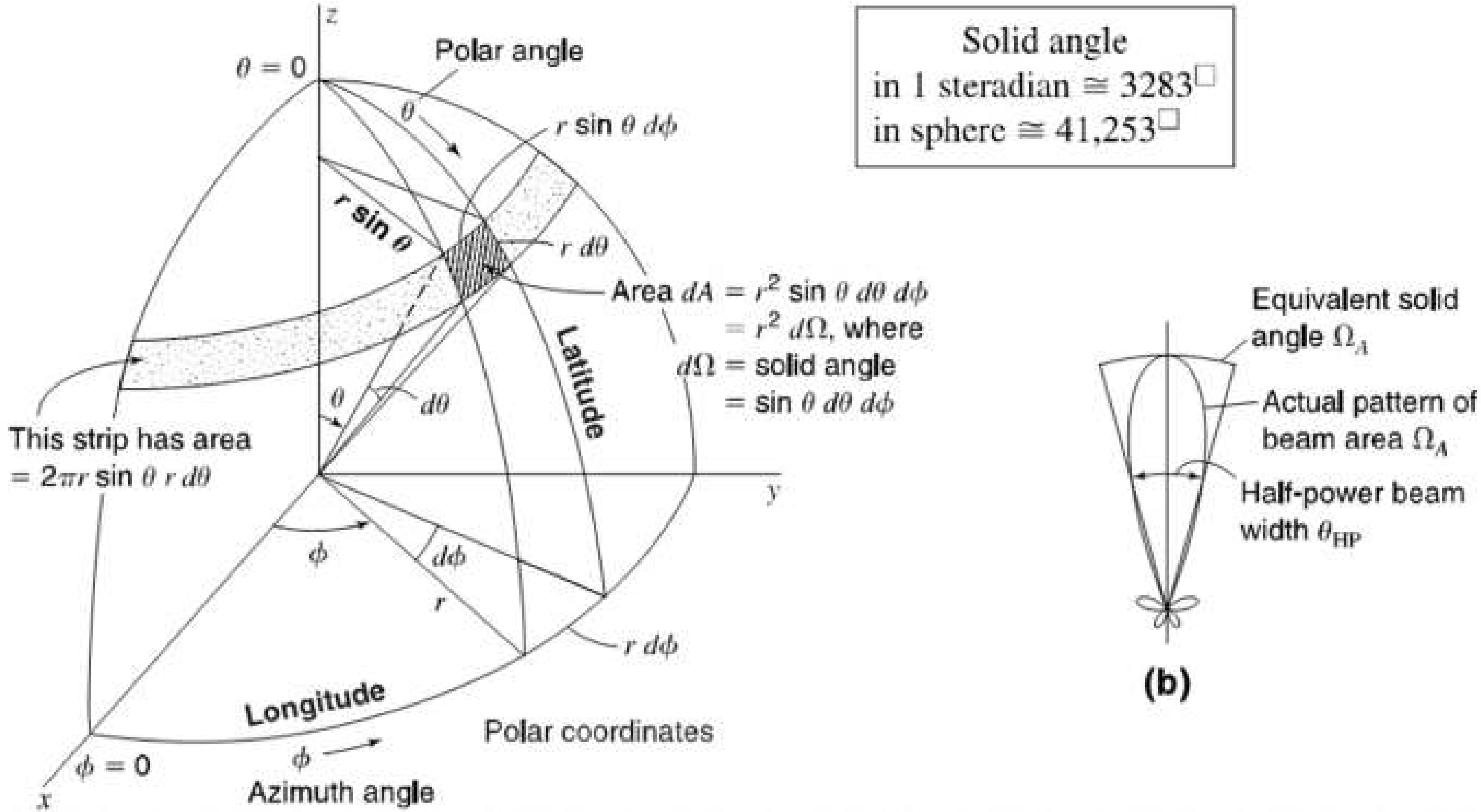
$$= \frac{r^2 \sin\theta d\theta d\phi}{r^2}$$

$$d\Omega = \sin\theta d\theta d\phi \quad (2\text{-}2)$$

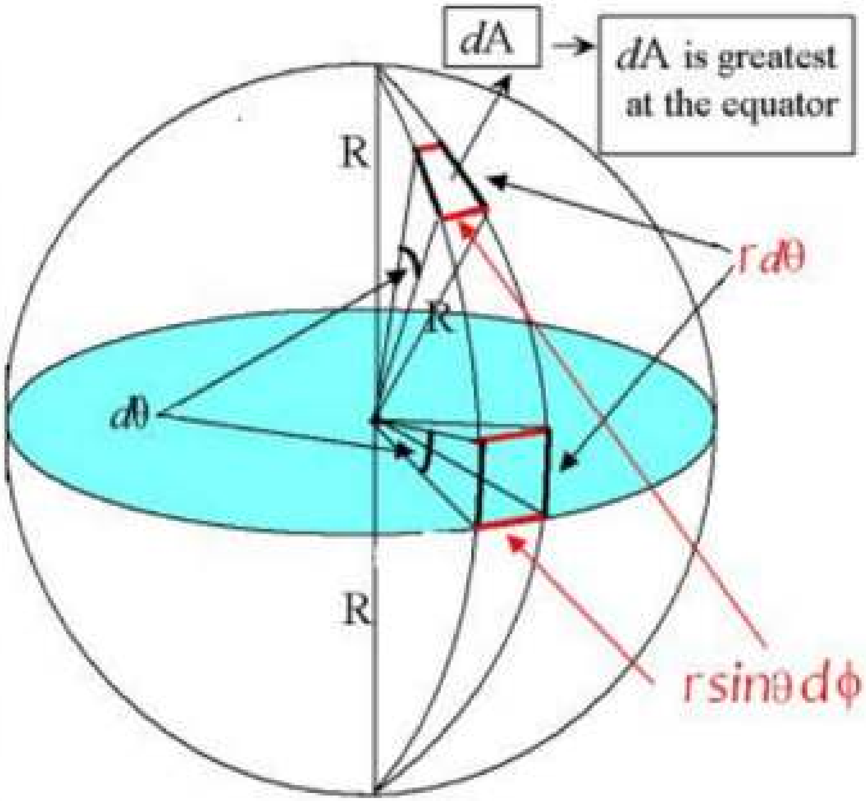
$$0 \le \theta \le \pi$$

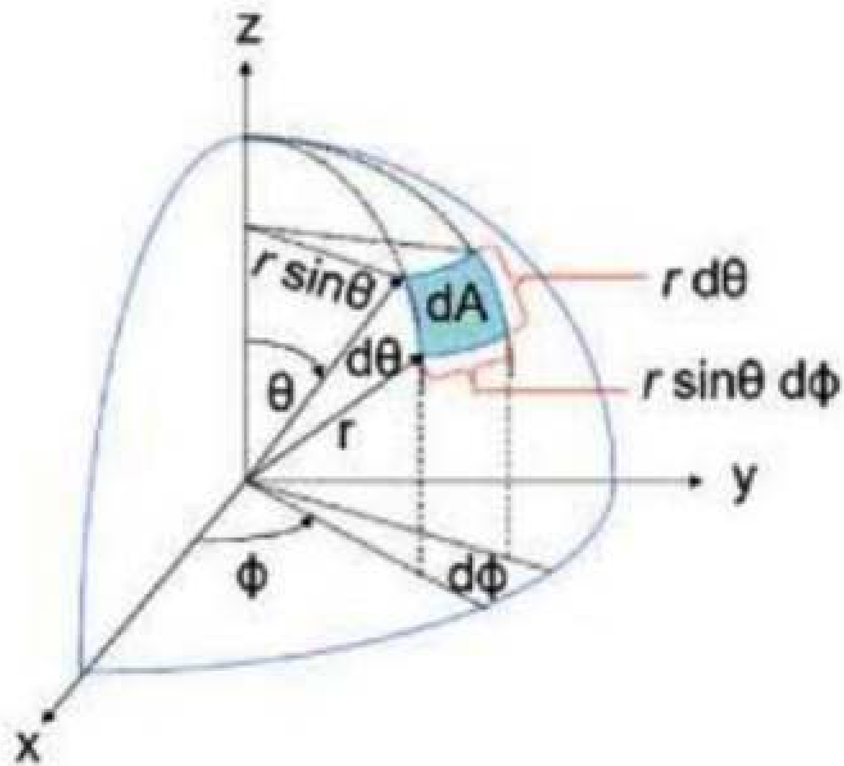
$$0 \le \phi \le 2\pi$$

Beam Solid Angle



Beam Solid Angle





$$dA = (r\,d\theta)(r\sin\theta\,d\phi) = r^2\,d\Omega$$

Beam Solid Angle

$$dA = (r\,d\theta)(r\sin\theta\,d\phi) = r^2\,d\Omega$$

where

$d\Omega$ = *solid angle* expressed in steradians (sr) or square degrees (□)
$d\Omega$ = solid angle subtended by the area dA

A differential solid angle **dΩ** in **sr** is:

$$d\Omega = \sin\theta d\theta d\phi$$

For **sphere**, the solid angle **dΩ** and the total area **A**

$$\Omega = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} \sin\theta d\theta d\phi = 4\pi \textbf{ sr}$$

$$A = \int_{\varphi=0}^{2\pi}\int_{\theta=0}^{\pi} r^2 \sin\theta d\theta d\varphi = 4\pi r^2$$

Steradian

where 4π = solid angle subtended by a sphere, sr

Thus,

$$1 \text{ steradian} = 1 \text{ sr} = (\text{solid angle of sphere})/(4\pi)$$

$$= 1 \text{ rad}^2 = \left(\frac{180}{\pi}\right)^2 (\text{deg}^2) = 3282.8064 \text{ square degrees}$$

$$4\pi \text{ steradians} = 3282.8064 \times 4\pi = 41{,}252.96 \cong 41{,}253 \text{ square degrees} = 41{,}253^{\square}$$

$$= \text{solid angle in a sphere}$$

Beam Area (for any Radiation Pattern)

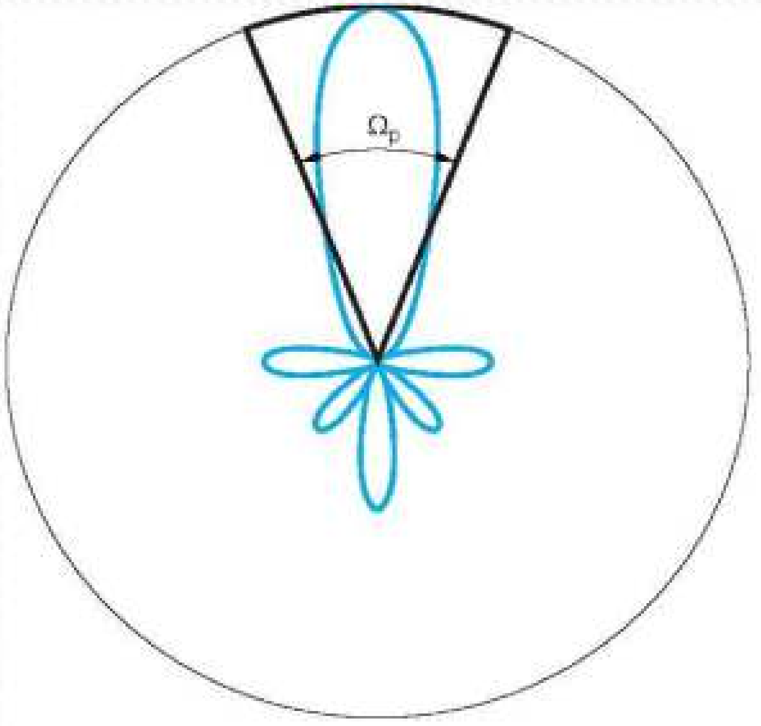
Beam area (Exact)

$$\Omega_A = \int_0^{2\pi}\int_0^{\pi} P_n(\theta,\phi)\cdot\sin(\theta)\,d\theta d\phi = \iint_{4\pi} P_n(\theta,\phi)\,d\Omega$$

$$d\Omega = \sin\theta \, d\theta \, d\phi, \text{ sr.}$$

Beam area (Approximate)

$$\Omega_A \cong \theta_{HP}\phi_{HP} \qquad \text{(sr)}$$



3 - Directivity

Directivity

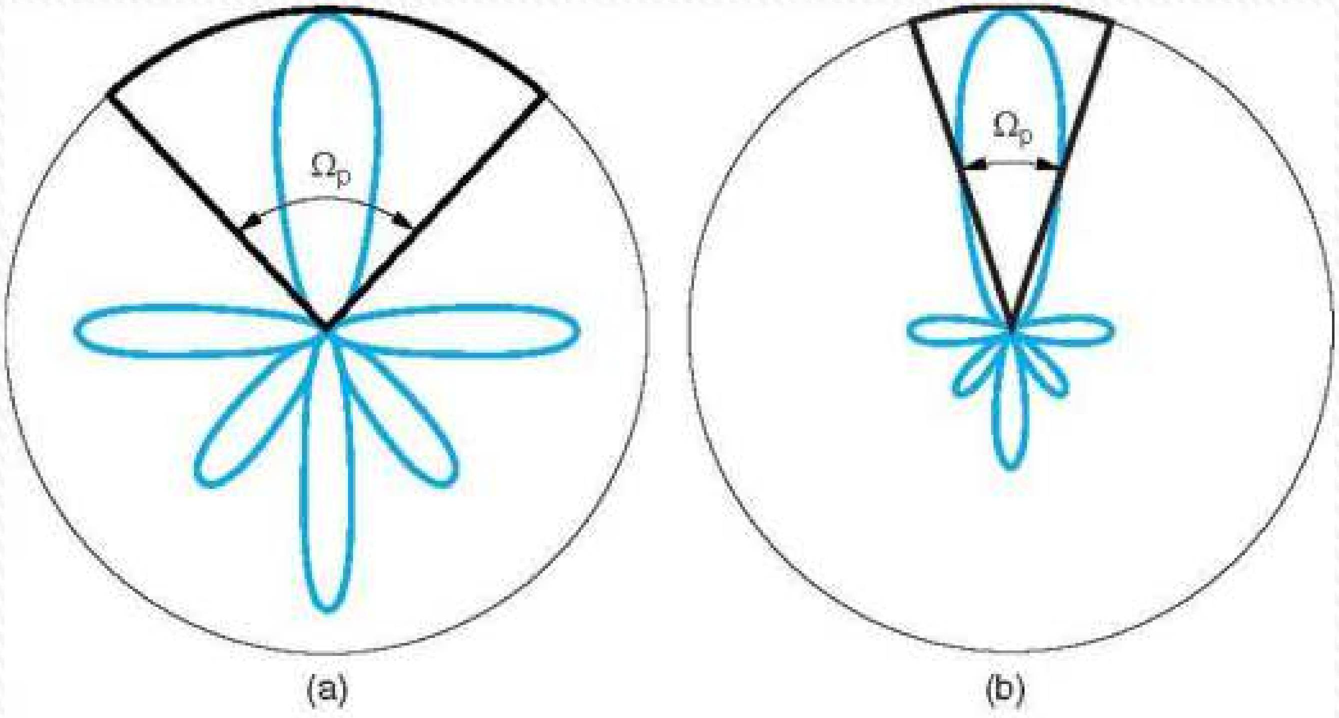
The maximum ***Directivity*** of an antenna is the ratio of the maximum power in particular direction to the average normalized power OR (the ratio of the radiation intensity in a given direction from the antenna to the radiation intensity averaged over all directions)

$$D = \frac{P(\theta,\varphi)_{\max}}{P(\theta,\varphi)_{average}} = \frac{4\pi}{\Omega_A} \geq 1$$

$$D(\theta,\phi) = \frac{U_{\max}(\theta,\phi)}{U_{iso}}$$

$$D(dB) = 10\log(D)$$

Directivity



For (a), power gets radiated to the side and back lobes, so the **pattern solid angle** is **large** and the **directivity** is **small**. For (b), almost all the power gets radiated to the main beam, so **pattern solid angle** is **small** and **directivity** is **high**.

Directivity

From pattern D (Exact)

$$D = \frac{4\pi}{\Omega_A} = \frac{4\pi}{\iint_{4\pi} P_n(\theta,\varphi)\, d\Omega}$$

$$P_n(\theta,\varphi) = E_n^2(\theta,\varphi) = U_n(\theta,\varphi)$$

$$D = \frac{4\pi}{\Omega_A} = \frac{4\pi}{\iint_{4\pi} P_n(\theta,\varphi)\, d\Omega} = \frac{4\pi}{\iint_{4\pi} E_n^2(\theta,\varphi)\, d\Omega} = \frac{4\pi}{\iint_{4\pi} U_n(\theta,\varphi)\, d\Omega}$$

D (Approximate)

$$D = \frac{4\pi}{\Omega_A} = \frac{4\pi}{\Theta_{HP}(rad) * \Phi_{HP}(rad)} = \frac{41253}{\Theta^o_{HP} * \Phi^o_{HP}}$$

Zatoona

Zatoona

$$D = \frac{4\pi}{\Omega_A}$$

Exact

$$\Omega_A = \iint_{4\pi} P_n(\theta,\varphi)\,d\Omega = \iint_{4\pi} U_n(\theta,\varphi)\,d\Omega = \iint_{4\pi} E_n^2(\theta,\varphi)\,d\Omega$$

Approximate (Sr)

$$\Omega_A = \Theta_{HP}(rad) * \Phi_{HP}(rad)$$

Approximate (degree square)

$$D = \frac{41253}{\Theta^o_{HP} * \Phi^o_{HP}}$$

4 - Examples

Example (1)

Estimate the directivity of an antenna with Θ_{HP} = 2º and Φ_{HP}=1º

$$D\ approximate = \frac{41253}{\theta_{HP}\phi_{HP}} = \frac{41253}{2*1} = 20627.$$

Example (2)

Find the number of square degrees in the solid angle Ω on a spherical surface that is between (θ=20º and θ=40º), and (ϕ=30º and ϕ=70º).

$$\Omega = \int_{30}^{70} d\phi \int_{20}^{40} \sin\theta d\theta . = (70\text{-}30)*(\frac{180}{\pi}) * (-Cos\theta)_{20}^{40} = 398.17\ deg^2.$$

Example (3)

The normalized field pattern of an antenna is given by **E(θ)=sinθ sinϕ**. E_n has a value only for 0≤ θ ≤ π & 0≤ ϕ ≤ π, and zero elsewhere , Find

The exact directivity.

The approximate directivity.

The decibel difference.

$$(a)\quad Dexact = \frac{4\Pi}{\int_0^{\Pi}\int_0^{\Pi} Sin^3\theta Sin^2\phi d\theta d\phi} = \frac{4\Pi}{\int_0^{\Pi} \sin\theta * (1-Cos^2\theta)d\theta \int_0^{\Pi} \frac{(1-Cos2\Phi)}{2} d\phi} =$$

$$\frac{4\Pi}{(\frac{\phi}{2}-\frac{Sin2\phi}{2})_0^{\Pi} \int_0^{\Pi} (Sin\theta d\theta - Sin\theta Cos^2\theta d\theta)} = \frac{4\Pi}{(\frac{\Pi}{2})[(-Cos\theta)_0^{\Pi} + (\frac{Cos^3\theta}{3})_0^{\Pi}]} =$$

$$\frac{4\Pi}{(\frac{\Pi}{2})(\frac{4}{3})} = 6.$$

Example (3)

***(b)**.. D approximate* $= \dfrac{4\Pi}{\theta_{HP}\phi_{HP}}$.

We calculate θ_{max} ➔ $(Sin\theta_{max} = 1..(max))$ ➔ *at* $\theta_{max} = 90^o$,
We calculate Φ_{max} ➔ $(Sin\Phi_{max} = 1..(max))$ ➔ *at* $\Phi_{max} = 90^o$,

We calculate θ_h ➔ $(Sin\theta_h = \dfrac{1}{\sqrt{2}})$ ➔ $\theta_h = 45^o$

We calculate Φ_h ➔ $(Sin\Phi_h = \dfrac{1}{\sqrt{2}})$ ➔ $\Phi_{h=} 45^o$

So: $\theta_{HP} = 2*|90-45^o| = 90^o = \dfrac{\Pi}{2}(rad)$

By the same way

We calculate $\Phi_{HP} = 2*|90^o - 45^o| = 90^o = \dfrac{\Pi}{2}(rad)$

So: D approximate $= \dfrac{4\Pi}{\theta_{HP}\phi_{HP}}. = \dfrac{4\Pi}{(\frac{\Pi}{2})(\frac{\Pi}{2})} = 5.1.$

***(C)** Decibel difference* $= 10 \log \dfrac{6}{5.1} = 0.7db.$

Example (4)

For this normalized radiation intensity,

$$P_n(\theta,\phi) = \sin^2\theta \sin^3\phi \text{ for } 0 \le \phi \le \pi,$$

$$0 \text{ otherwise.}$$

Find the solid angle and the directivity.

Solution // The pattern solid angle is:

$$\Omega_A = \iint P_n \, d\Omega = \iint \left(\sin^2\theta \sin^3\phi\right) \sin\theta \, d\theta \, d\phi,$$

$$\Omega_A = \int_0^{\pi} \sin^3\theta \, d\theta \int_0^{\pi} \sin^3\phi \, d\phi, \quad \text{(note limits on } \phi\text{)}$$

Example (4)

Where each integral is solved as follows:

$$y = \int_0^{\pi} \sin^3 x dx = \int_0^{\pi} \left(1 - \cos^2 x\right) \sin x dx = \int_0^{\pi} \sin x dx - \int_0^{\pi} \cos^2 x \sin x dx.$$

Please continue on your own!!

$$\Omega_A = \int_0^{\pi} \sin^3 \theta d\theta \int_0^{\pi} \sin^3 \varphi d\varphi = \left(\frac{4}{3}\right)\left(\frac{4}{3}\right)$$

Finally,

$$\Omega_A = 1.78 sr$$

and the directivity,

$$D_{\max} = \frac{4\pi}{\Omega_P} = \frac{4\pi}{1.78} = 7.1$$

How to Calculate HPBW in two perpendicular Planes ?

Next Lecture

Antenna parameters (Cont.)

Radiation Intensity

Power density

Beam efficiency

Gain

Effective and Physical Aperture

Dr. Moataz Elsherbini

motaz.ali@feng.bu.edu.eg

Thank You