Fundamental Parameters of Antennas

- The time average Poynting vector (average power density) does not depend on time, and can be written as

$$\mathbf{W}_{\text{av}}(x, y, z) = [\mathscr{W}(x, y, z; t)]_{\text{av}} = \frac{1}{2}\text{Re}[\mathbf{E} \times \mathbf{H}^*] \quad (\text{W/m}^2)$$

Where, average power density is obtained by integrating the instantaneous Poynting vector over one period and dividing by the period.

E and **H** fields represent peak values, complex fields

Instantaneous Poynting vector represents the energy transfer per unit area per unit time of an electromagnetic field.

$$\mathscr{W} = \mathscr{E} \times \mathscr{H} = \frac{1}{2}\text{Re}[\mathbf{E} \times \mathbf{H}^*] + \frac{1}{2}\text{Re}[\mathbf{E} \times \mathbf{H}e^{j2\omega t}]$$

Avg=0

Fundamental Parameters of Antennas

Average power radiated by an antenna (radiated power)

$$P_{rad} = \iint W_{rad}\, ds$$

$$P_{rad} = P_{ave} = \int_0^{2\pi}\int_0^{\pi}\left(\frac{1}{2}\mathrm{Re}\left[\underline{E}\times\underline{H}^*\right]\right)\cdot \hat{a}_r r^2 \sin\theta\, d\theta\, d\phi \quad (2\text{-}9)$$

For Isotropic radiator which radiates equally in all directions(not exist but used as reference to compared with other antenna), the power density equal to

$$\mathbf{W}_0 = \hat{\mathbf{a}}_r W_0 = \hat{\mathbf{a}}_r \left(\frac{P_{rad}}{4\pi r^2}\right) \quad (\mathrm{W/m^2})$$

Fundamental Parameters of Antennas

Example 1

A hypothetical isotropic antenna is radiating in free pace. At a distance of 100 m from the antenna, the total electric field (E_θ) is measured to be 5 V/m. Find the

(a) Power density (W_{rad})

(b) Power radiated (P_{rad})

(a) $$\underline{W}_{rad} = \frac{1}{2}[\underline{E} \times \underline{H}^*] = \frac{E^2}{2\eta}\hat{a}_r = \frac{5^2\hat{a}_r}{2(120\pi)} = 0.03315\ \hat{a}_r \text{ Watts/m}^2$$

(b) Since isotropic:

Prad = w_{rad}*4πr^2

=0.03315*4π*100^2 =4165.75 watts

Note E depend on r so prad always const and wrad decrease as r increase

Fundamental Parameters of Antennas

Example 2

Example 2.2

$$\underline{W}_{ave} = \underline{W}_{rad} = \hat{a}_r A_0 \frac{\sin\theta}{r^2}$$

$$P_{ave} = P_{rad} = \oiint_S \underline{W}_{ave} \cdot d\,\underline{s}$$

$$= \int_0^{2\pi}\int_0^{\pi} \left[\hat{a}_r A_0 \frac{\sin\theta}{r^2}\right] \cdot \left[\hat{a}_r r^2 \sin\theta\, d\theta\, d\phi\right]$$

$$P_{ave} = P_{rad} = A_0 \int_0^{2\pi}\int_0^{\pi} \sin^2\theta\, d\theta\, d\phi = 2\pi A_0 \int_0^{\pi} \sin^2\theta\, d\theta$$

$$P_{rad} = A_0 \pi^2$$

Fundamental Parameters of Antennas

Radiation intensity: defined as

The power radiated from antenna per unit solid angle U(ɵ,ϕ)

$$U(\theta,\phi) = \frac{r^2}{2\eta}|\mathbf{E}(r,\theta,\phi)|^2 \simeq \frac{r^2}{2\eta}\left[|E_\theta(r,\theta,\phi)|^2 + |E_\phi(r,\theta,\phi)|^2\right] \quad \text{(2-12a)}$$

$W_{rad} \Rightarrow$ watt/m^2

$U \Rightarrow$ watt/solid angle = watt/m^2/r^2

$U = r^2 W_{rad}$

where

$\mathbf{E}(r,\theta,\phi)$ = far-zone electric-field intensity of the antenna

E_θ, E_ϕ = far-zone electric-field components of the antenna

η = intrinsic impedance of the medium

U = radiation intensity (W/unit solid angle)

The total power is obtained by integrating the radiation intensity, as given by (2-12), over the entire solid angle of 4π. Thus

$$P_{rad} = \oiint_\Omega U\, d\Omega = \int_0^{2\pi}\int_0^{\pi} U \sin\theta\, d\theta\, d\phi \quad \text{(2-13)}$$

radiation intensity of an isotropic source $U_0 = \frac{P_{rad}}{4\pi}$

Fundamental Parameters of Antennas

Example 3

Compute Beam width, Beam solid angle and the power radiated through HPBW for normalized field pattern Fn=sinθ

Beam Width

Approximation can not be used here
As it is not one main lobe

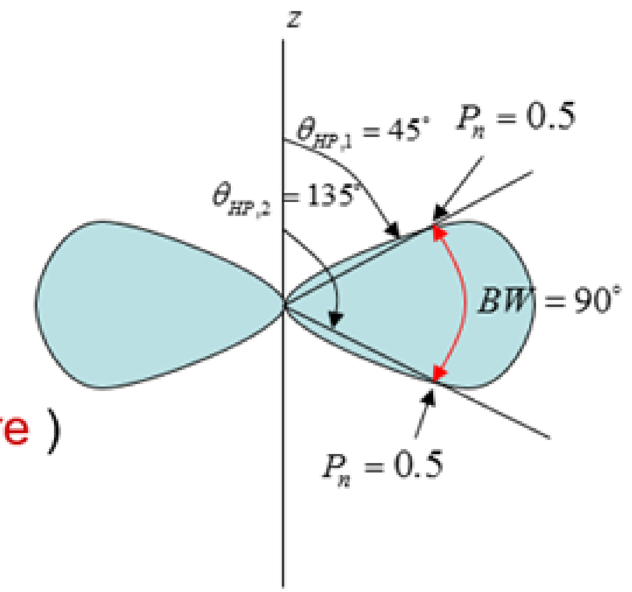
$$P_n(\theta,\phi) = \sin^2\theta \;\Rightarrow\; \frac{1}{2} = \sin^2\theta_{HP} \qquad \sin\theta_{HP} = \pm\frac{1}{\sqrt{2}}$$

$$\sin\theta_{HP} = +\frac{1}{\sqrt{2}}$$

A

$\theta_{HP,1} = 45^\circ$ and $\theta_{HP,2} = 135^\circ$

$$Beamwidth(BW) = 135^\circ - 45^\circ = 90^\circ$$



Pattern Solid Angle Ω_A (Integrate over the entire sphere)

$$\Omega_A = \iint P_n(\theta,\phi)\,d\Omega$$

$$\Omega_A = \iint \sin^2\theta \sin\theta\, d\theta\, d\phi = \int_0^{2\pi}\int_0^{\pi} \sin^3\theta\, d\theta\, d\phi = \left(\int_0^{\pi} \sin^3\theta\, d\theta\right)\left(\int_0^{2\pi} d\phi\right) = \left(\frac{4}{3}\right)(2\pi) = \frac{8\pi}{3}$$

Fundamental Parameters of Antennas

Half-power Pattern Solid Angle $\Omega_{A,HP}$ (*integrate over half power Beam width*)

$$\boxed{\Omega_{A,HP} = \int\int P_n(\theta,\phi)\, d\Omega}$$

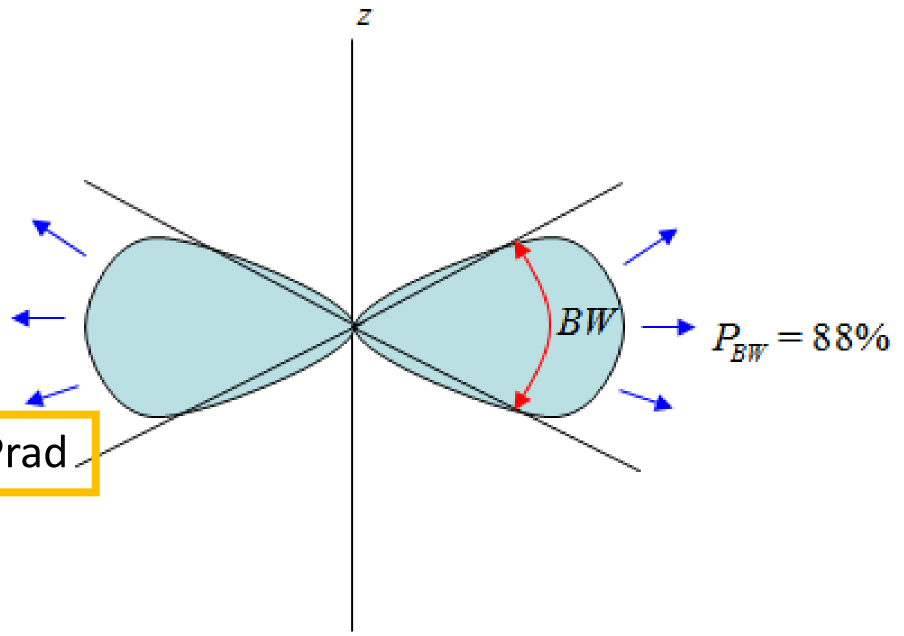
$$\Omega_{A,HP} = \iint \sin^2\theta \sin\theta\, d\theta\, d\phi = \int_0^{2\pi}\int_{45^\circ}^{135^\circ} \sin^3\theta\; d\theta\; d\phi = \left(\int_{45^\circ}^{135^\circ} \sin^3\theta\; d\theta\right)\left(\int_0^{2\pi} d\phi\right) = \left(\frac{5}{3\sqrt{2}}\right)(2\pi) = \frac{5\pi\sqrt{2}}{3}$$

$$\int_{45^\circ}^{135^\circ} \sin^3\theta\; d\theta = \left[-\cos\theta + \frac{\cos^3\theta}{3}\right]_{45^\circ}^{135^\circ} = \left[\left(-\cos(135^\circ) + \frac{\cos^3(135^\circ)}{3}\right) - \left(-\cos(45^\circ) + \frac{\cos^3(45^\circ)}{3}\right)\right]$$

$$= \left[\left(\frac{1}{\sqrt{2}} - \frac{1}{6\sqrt{2}}\right) - \left(-\frac{1}{\sqrt{2}} + \frac{1}{6\sqrt{2}}\right)\right] = \frac{2}{\sqrt{2}} - \frac{2}{6\sqrt{2}} = \frac{10}{6\sqrt{2}} = \frac{5}{3\sqrt{2}}$$

Power radiated through the beam width

$$P_{BW} = \frac{\Omega_{A,HP}}{\Omega_A} = \frac{\frac{5\pi\sqrt{2}}{3}}{\frac{8\pi}{3}} = \frac{5\sqrt{2}}{8} \cong 0.88 \text{ (or) } 88\% \text{ of Prad}$$

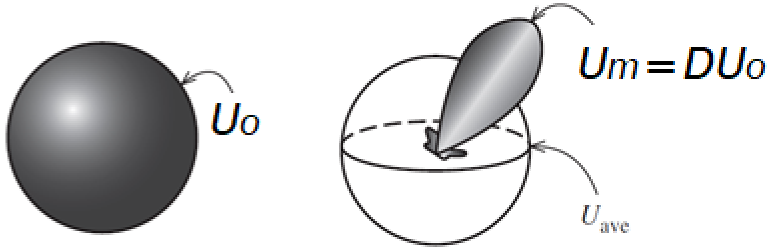


Directivity

is the ratio of radiation intensity in a given direction to isotropic radiation intensity

$$D = \frac{U}{U_0} = \frac{4\pi U}{P_{rad}}$$

Illustration of directivity



(a) Radiation intensity distributed isotropically.

(b) Radiation intensity from an actual antenna.

If the direction is not specified → (maximum directivity)

$$D_{max} = D_0 = \frac{U|_{max}}{U_0} = \frac{U_{max}}{U_0} = \frac{4\pi U_{max}}{P_{rad}}$$

Example 4

Example 2.5

find the maximum directivity of the antenna whose radiation intensity is $U = A_0 \sin\theta$ Write an expression for the directivity as a function of the directional angles θ and ϕ.

$$P_{rad} = \int_0^{2\pi}\int_0^{\pi} U \sin\theta \, d\theta \, d\phi = A_0 \int_0^{2\pi}\int_0^{\pi} \sin^2\theta \, d\theta \, d\phi = \pi^2 A_0$$

Using (2-16a), we find that the maximum directivity is equal to

$$D_0 = \frac{4\pi U_{max}}{P_{rad}} = \frac{4}{\pi} = 1.27$$

Since the radiation intensity is only a function of θ, the directivity as a function of the directional angles is represented by

$$D = D_0 \sin\theta = 1.27 \sin\theta$$

Fundamental Parameters of Antennas

Approximated Directivity (by Kraus)

For antennas with one narrow major lobe and very negligible minor lobes, the beam solid angle is approximately equal to the product of the half-power beam widths in two perpendicular planes

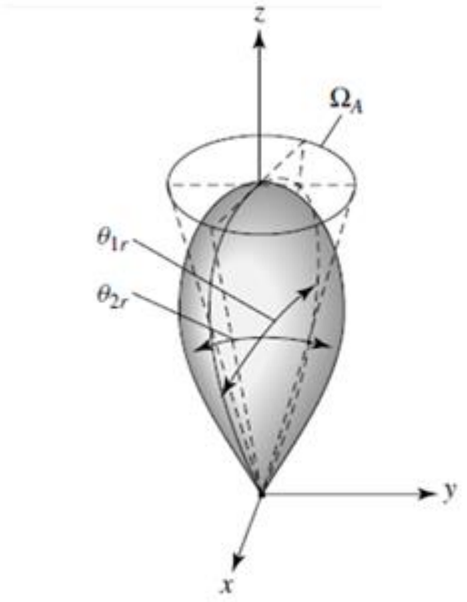
For radiation intensity

$U(\theta,\phi) = B_0 \cos^n(\theta)$ at $0<\theta<\pi/2$, $0<\phi<2\pi$ and 0 elsewhere

$$D_0 = \frac{4\pi}{\Omega_A} \simeq \frac{4\pi}{\theta_{1r} \cdot \theta_{2r}} \quad \text{(Kraus)}$$

θ_{1r} = half-power beamwidth in one plane (rad)

θ_{2r} = half-power beamwidth in a plane at a right angle to the other (rad)



Beam solid angles $\Omega_A = \theta_{1r} \cdot \theta_{2r}$

Fundamental Parameters of Antennas

Example 5

The normalized field pattern of an antenna is given by $E_n = \sin\theta \sin\phi$, where θ = zenith angle (measured from z axis) and ϕ = azimuth angle (measured from x axis). E_n has a value only for $0 \le \theta \le \pi$ and $0 \le \phi \le \pi$ and is zero elsewhere (pattern is unidirectional with maximum in $+y$ direction). Find (a) the exact directivity, (b) the approximate directivity and (c) the decibel difference.

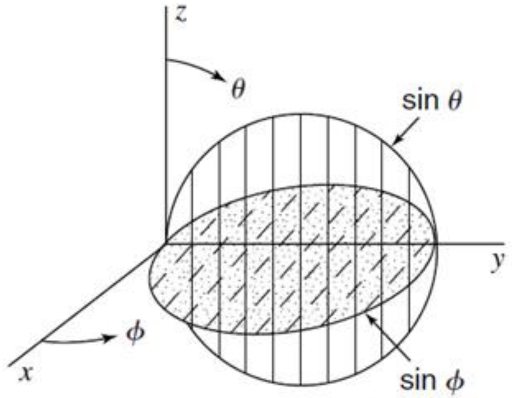
Solution

(a) $$G = \frac{4\pi}{\iint_{4\pi} P_n(\theta,\phi)d\Omega} = \frac{4\pi}{\int_0^\pi\int_0^\pi (\sin\theta\sin\phi)^2 \sin\theta d\theta d\phi}$$

$$G = \frac{4\pi}{\int_0^\pi\int_0^\pi \sin^3\theta \sin^2\phi d\theta d\phi} = \frac{4\pi}{2\pi/3} = 6$$

(b) $$D = \frac{41{,}253}{\theta^o_{HP}\phi^o_{HP}} = \frac{41{,}253}{(90^o)(90^o)} = 5.1$$

(c) $$\Delta D(dB) = 10\log\frac{6}{5.1} = 0.7dB$$



Unidirectional $\sin\theta$ and $\sin\phi$ field patterns.

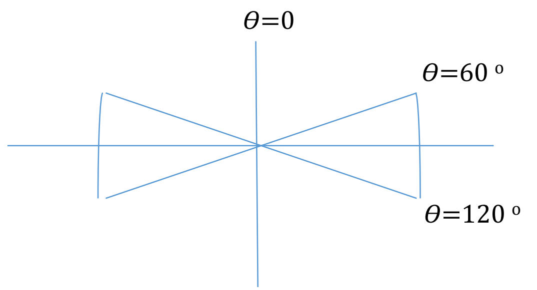
Fundamental Parameters of Antennas

Example 6 *Directivity of a Sector Omnidirectional Pattern*

An ideal omnidirectional antenna would have constant radiation in the horizontal plane (θ =90) and would fall rapidly to zero outside that plane. if power pattern in the vertical plane is constant out to ±30° **from horizontal**. find Directivity.

The power pattern expression is then written as:

$$F(\theta) = \begin{cases} 1 & \frac{1}{3}\pi < \theta < \frac{2}{3}\pi \\ 0 & \text{elsewhere} \end{cases}$$



Solution:

The solid angle of the pattern is

$$\Omega_A = \int\int |F(\theta,\phi)|\,d\Omega = \int_0^{2\pi}\int_{\pi/3}^{2\pi/3} \sin\theta\,d\theta\,d\phi$$

$$= (2\pi)[-\cos\,\theta]_{\pi/3}^{2\pi/3} = (2\pi)(0.5+0.5) = 2\pi$$

The directivity is

$$D = \frac{4\pi}{\Omega_A} = \frac{4\pi}{2\pi} = 2.$$

Fundamental Parameters of Antennas

Example 7

Example 2.8

Design an antenna with omnidirectional amplitude pattern with a half-power beamwidth of 90°. Express its radiation intensity by $U = \sin^n \theta$. Determine the value of n, max directivity

Solution: Since the half-power beamwidth is 90°, the angle at which the half-power point occurs is $\theta = 45°$. Thus

$$U(\theta = 45^\circ) = 0.5 = \sin^n(45^\circ) = (0.707)^n$$

or

$$n = 2$$

Therefore, the radiation intensity of the omnidirectional antenna is represented by $U = \sin^2 \theta$. An infinitesimal dipole (see Chapter 4) or a small circular loop (see Chapter 5) are two antennas which possess such a pattern.

Using the definition of (2-16a), the exact directivity is

$$U_{\max} = 1$$

$$P_{\text{rad}} = \int_0^{2\pi} \int_0^{\pi} \sin^2 \theta \, \sin\theta \, d\theta \, d\phi = \frac{8\pi}{3}$$

$$D_0 = \frac{4\pi}{8\pi/3} = \frac{3}{2} = 1.761 \text{ dB}$$

$$\int_0^{\pi} \sin^3 \theta \, d\theta = \frac{4}{3}$$

Fundamental Parameters of Antennas

Example 8

The antenna is a lossless end-fire array of 10 isotropic point sources spaced $\lambda/4$ and operating with increased directivity. The normalized field pattern is

$$E_n = \sin\left(\frac{\pi}{2n}\right)\frac{\sin(n\psi/2)}{\sin(\psi/2)}$$

where

$$\psi = d_r(\cos\phi - 1) - \frac{\pi}{n} \qquad d_r = \pi/2 \qquad n = 10$$

Since the antenna is lossless, gain = directivity.

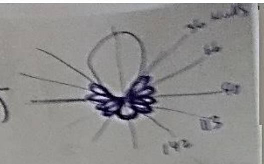
(a) Calculate the gain G.

(b) Calculate the approximate gain

(c) What is the difference?

Solution

$$E_n = \sin\left(\frac{\pi}{20}\right) \frac{\sin(2.5\pi(\cos\theta - 1) - 0.5\pi)}{\sin(0.25\pi(\cos\theta-1) - 0.05\pi)}$$



To Find HPBW Use Table

θ°	0	10	20	30	40	50	60	70	80	90
E_n	1	0.92	0.68	0.29	−0.12	−0.34	−0.21	0.11	0.2	0

θ°	100	110	120	130	140	150	160	170	180
E_n	−0.17	−0.07	0.11	0.14	0.04	−0.07	−0.14	−0.15	−0.15

So θ_h between 10° and 20° and $\theta_{max} = 0^\circ$

Try and Error [or table between $10^\circ \to 20^\circ$ step 1°]

$$\theta_h = 19.3^\circ$$

$$\therefore HPBW = 2|\theta_{max} - \theta_h| = 2 \times 19.3^\circ = 38.6^\circ$$

Approximate Directivity:

$$D_{app} = \frac{4\pi}{(HPBW)^2} = \frac{4\pi}{\left(\frac{38.6}{180}\pi\right)^2} = 27.7 \text{ or } 14.4\text{ dB}$$

Exact Directivity:

$$D = \frac{4\pi}{\Omega_A} = \frac{4\pi}{\int_0^{2\pi}\int_0^{\pi}\left[0.1564 \frac{(\sin(2.5\pi(\cos\theta-1)-0.5\pi))}{(\sin(0.25\pi(\cos\theta-1)-0.05\pi))}\right]^2 \sin\theta\, d\theta\, d\phi}$$

$$= \frac{4\pi}{(2\pi)\int_0^{\pi}\left[\quad\right]^2 \sin\theta\, d\theta} = \frac{4\pi}{2\pi \times (0.1564)^2 \times 4.59}$$

$$D_{exact} = 17.8 \text{ or } 12.5\text{ dB} \quad \therefore \Delta D = 1.89\text{ dB}$$