New Numeracies for a Technological Culture [1]

RICHARD NOSS

1. The culture of utility in mathematics education

1982 was the year which saw the publication in Britain of the Cockcroft report, emerging from a government committee which had been given a simple yet compelling brief:

> To consider the teaching of mathematics in primary and secondary schools in England and Wales, with particular regard to the mathematics required in further and higher education, employment and adult life generally, and to make recommendations. (D.E.S., 1982, p. ix)

The report also attempted a redefinition of the word 'numeracy', a term first coined in the Crowther Report of 1959, which had defined it as:

> an understanding of the scientific approach to the study of phenomena - observation, hypothesis, experiment, verification [...] the need in the modern world to think quantitatively, to realise how far our problems are problems of degree even when they appear as problems of kind. (Ministry of Education, 1959, p. 270)

By 1982, the word 'numeracy' had lost much of the richness with which it had been invested two decades earlier. Cockcroft noted that most of those submitting evidence to the committee had used the word in the narrow sense of the ability to perform basic arithmetic operations, and he tried to broaden the idea a little by arguing not only for an 'at-homeness' with numbers, but also for:

> some appreciation and understanding of information which is presented in mathematical terms, for instance in graphs, charts or tables. (p. 11)

This is essentially the view of numeracy which, some fifteen years later, has been used as a working definition in the *Framework for Numeracy,* a document of the National Project for Literacy and Numeracy set up by the outgoing Conservative government in 1996, and continued under the present administration. In their formulation, there is similarly an attempt to broaden the concept of numeracy beyond merely knowing about numbers and operations, to include the need to 'make sense of numerical problems' (Straker, 1997, p. 4). Nonetheless, in this new formulation, as clearly as in its predecessor of 1982, much of the original depth of the idea of 'numeracy' has been discarded in favour of a relatively narrow, number-based conception.

The trend looks set to continue: the recent U.K. government White Paper *Excellence in Schools* bases its campaign for literacy and numeracy by noting that:

> the first task of the education service is to ensure that every child is taught to read, write and *add up.* (DfEE, 1997, p. 9 - *emphasis added*)

It may be that this formulation is merely a convenient turn of phrase. Nevertheless, it serves to focus attention on the ubiquitous analogy between literacy and numeracy, and on the relationship between numeracy and mathematics. We talk about literacy in the context of learning to read and write. But we also refer to a *literate* individual, one who has a familiarity with plays, poetry, novels, and so on. The etymology of the words is illustrative: *literacy* maintains its connection with *literature*, it is an idea with depth. Numeracy, on the other hand, shares its root only with that of number. In this article, I try to broaden the notion of numeracy to comprise the mathematical, rather than to discard the word altogether.

It may be that the drawing of ever-narrower boundaries around 'numeracy' has been entirely justified, and that there is no cause to sound the educational alarm. Nevertheless, there has been little serious discussion of what constitutes numeracy within the educational community. What, if anything, has been lost by the gradual erosion of broader mathematical connections in favour of basic number skills? What effects have there been on our perception of mathematical attainment as a result, and whom has it affected? More generally, what is the theoretical and practical rationale which has driven this narrowing of the idea of numeracy, and what are its potential effects for the mathematical knowledge of the citizens of the next millennium?

It is unusual for a government report to contain an explicit formulation of its theoretical antecedents, and Cockcroft is no exception. But there is, nonetheless, an implicit theoretical framework which underpins its vision of mathematics, and the kinds of cultural assumptions which frame it. It is the culture of *utility*. Mathematics should be taught to the extent that it is useful 'in the workplace, and in adult life'.

If that is so, there are a number of obvious questions. Useful to whom? For what purposes? Equally, if the definition of utility is based on what is 'seen' in workplace practices, it is legitimate to enquire how these may have changed, and in what direction. We should wonder to what extent mathematics in practice is, in any case, 'visible' in the sense that its presence or absence is unproblematically evident.

Charged with looking to see how mathematics is used in the workplace, Cockcroft did just that. The committee took evidence from a variety of sources and their report has much that remains interesting to say about the range of mathe-

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matics required for diverse occupations: *operatives* ('... whose jobs do not appear to require any formal application of mathematics'), *craftsmen* ('He [*sic*] may also need to ... estimate or calculate areas and volumes of non-rectilinear shapes ...'), *engineering technicians* ('variable, but roughly at the level of grade 3 CSE'), *clerks* ('most of whom are female [...] predominantly arithmetic'), *retail workers* ('only a limited range of arithmetical skills'), *hotel and catering* ('concerned with calculations involving money, with weighing and measuring') and lastly *nurses* ('concerned with measurement and recording, often in graphical form').

The report noted that workplace practices seldom demand standard arithmetic operations such as $\frac{2}{5} + \frac{3}{7}$ (para. 75, p. 22), and it failed to locate much need for algebra (para. 77, p. 22), let alone ideas such as proof, modelling and mathematical rigour. Accordingly, it allocates these kinds of mathematical notions little if any role in the curriculum, a reasonable conclusion based on the assumption that mathematics at work is defined by the presence of numerical or algebraic calculation, and on the utilitarian framework of the report as a whole. This framework is pressed to its logical conclusion: 'those who do not travel by bus or train probably have no need to consult timetables' (p. 2), and those who do not eat in restaurants have 'no need to calculate a service charge'. Summing up, the report puts it succinctly:

> We believe that it is possible to summarise a very large part of the mathematical needs of employment as a feeling for measurement. (p. 24)

With the benefit of hindsight, we can see just how clearly the working practices of, say, a modern supermarket checkout operator differ from those of a technician, or just how misleading it might be now to subsume a broad range of jobs under the title 'operative'. The picture painted by Cockcroft is one in which the mathematical needs of working life are fundamentally basic for all but a few, and in which most employers were relatively content with the qualifications of the youngsters they employed. (Brown (1996) cites rising unemployment as a contributory factor to employers' short-lived satisfaction with their recruits' qualifications.)

In the last twenty years, the structure of the labour market has changed radically: demographic patterns, working practices and required skills have been transformed in complex ways. It is still true that many people appear to use practically no mathematics in their working and adult life generally. Yet the situation is not nearly so simple as it seems. For example, by carefully studying what kinds of problems people actually solve in a variety of workplaces, Harris (1991) has shown that there exists a rich source of mathematical activities which people exhibit in their working lives, even though these are flatly denied by those involved. Similarly, Wolf (1984) has amply illustrated that individuals are most reluctant to admit that they use mathematics in work, even when a mathematically-attuned observer can point to various practices which would count as mathematical. Quite simply, mathematics is not always visible: it lies beneath the surface of practices and cultures.

Here is a first paradox. If we, like Cockcroft and his successors, look at the surface of arithmetical activities in adults' working lives, we are bound to find only traces and shadows of mathematics, and we may conclude that the mathematical needs of adult life are both insignificant in quantity and trivial in quality. [2] Once it is accepted that people tend to use little mathematics in work, the utilitarian imperative necessarily redraws the boundaries of the mathematical in equally restricted terms. As the mathematics in working practices become less visible, so the mathematical knowledge of the school curriculum becomes less applicable. The utilitarian perspective gives rise to a recursive cycle, in which what is taught at school becomes less and less relevant to working practices, as working practices show less and less evidence of making use of what is taught at school.

The legacy of Cockcroft can be put quite simply. It laid the foundation - on an epistemological level - for what followed in the U.K. throughout the decade and into the nineties; mathematics defined by its use, cast in a variety of disguises: geometry as 'shape, space and measures', algebra subsumed into 'number', statistics renamed 'handling data', and proof distilled out for all but the few. Mathematics - the subject that dare not speak its name.

Here is a second paradox. Educationalists have been right to identify lack of mathematical confidence and alienation on the part of the many. But in attempting to alleviate this problem, we have risked divorcing school mathematics from its broader roots in science and technology, and ultimately, cast a fundamental question-mark over its place in the curriculum. In trying to connect mathematics with what is learnable, we have disconnected school mathematics from what is genuinely useful.

So the culture of utility has left us with two paradoxical situations: the first arising from the assumption that mathematical knowledge is visible (and broadly static) within the cultures of work; the second from the apparent impossibility of broadening the appeal of mathematics without progressively restricting its content to simple calculation.

I close this section with a final comment on Cockcroft. It concerns one of the very few references to the role of computers, a subject about which I shall have much more to say in the remainder of this article.

> Relatively few school leavers are likely to work directly with a computer. Their work will usually be at clerical or operator level dealing with the input and output of data, though some leavers with A-level [18+] qualifications obtain posts as junior programmers. The preparation of data for input to a computer entails the strict discipline of presenting data accurately in the required format; the handling of computer output often involves extracting data from tables which contain more information or more figures than are needed at that moment. These tasks demand little in the way of mathematical expertise apart from the need to feel 'at home' with the handling of numerical information. In some cases it is also necessary to be able to carry out straightforward arithmetical calculations which may involve the use of decimals and percentages. (D.E.S., 1982, para. 144, p. 40)

It is chastening to reflect that the first sentence was written as recently as fifteen years ago. Less obviously, I think this paragraph sums up the difficulties which bedevil the culture of utility. In casting the computer in the role of thinker, it assigns to the worker only a marginal role in interpreting numerical information, and assumes that the interpretation of computer output will hardly ever involve understanding what the computer has done to the input. In a pre-computer epoch, where useful mathematics was equated with calculation, the advent of new technologies served only to reinforce the view that working practices would inevitably be de-mathematised. The educational corollaries of this argument flow smoothly from the culture of utility; and it is to these, and the assumptions on which they are based, to which I now turn.

2. Cultures of the workplace

A parable

I open the door of the glistening bank and advance to the red-and-grey, vinyl-covered desk marked ENQUIRIES. *I smile at the red-and-grey-uniformed clerk, whose badge neatly proclaims her first name and she smiles at the computer screen which is neatly angled so as not quite to separate us. I start to enquire, but it seems that nothing can be done until the computer has been gratified. I hand over my passbook and she enters 13 alphanumeric characters expertly. Now Chris knows my name, where I live, how much money I have, how much I owe; now the computer will talk to her and she will talk to me.*

I begin my first request. "Bear with me", Chris interjects. She walks over to a desk marked ACCOUNTS *where Tammy is on the phone. Chris waits until the call is finished, and asks her something I cannot hear. When she returns, both Chris and the computer are happy with the answer. I ask a second question. This, it turns out, necessitates a call to one of the help-lines, whose codes are pre-programmed into the multi-buttoned telephone in front of her. "Bear with me", she says. After a short call, Chris knows the answer, tells me, smiles and punches some keys on the computer.*

My third and fourth enquiries generate a second call (to a different help-line) and a visit to the supervisor, apparently because my request falls outside the limit which Chris is allowed to handle unsupervised. The supervisor – whose uniform is several shades darker than Chris's, and who sports both her first and family names on her badge – looks at me, at my cheque, then back to me and, without speaking, nods. Chris places the passbook on a printer which is attached to the computer, and the machine punches information onto the book (by some miracle, it knows where to start typing on the page).

I am sent to queue for a teller, each of whom sits in a line behind a glass panel with their own computer. Their uniforms are like Chris's, but unlike her, the tellers are sedentary – they are tied to their machines. They chat to each other, crack jokes while their printers print, help each other with troublesome computers.

> *"The computer won't let me do that", my teller says, smiling.*
> *"Why not?", I ask.*
> *"Bear with me", she says, and calls the supervisor who comes to talk to me through the glass.*
> *"We can't do that", explains the supervisor.*
> *"Why not?", I ask.*
> *"It's the computer", she explains again, "It won't let us ... look".*

She types the numbers on the keyboard: they disappear as she types enter. *She is right: the computer* won't *let her. The computer doesn't explain why. The supervisor doesn't ask. Neither does Chris. Neither do I.*

Technology has transformed work, and Chris's role is as an appendage to technology, deskilled, alienated. But Chris is using mathematics to an unprecedented degree; it is hidden in the chips of her computer terminal, in the underlying models which have been programmed into them. For Chris, this mathematics is completely invisible: she has, it seems, no need to understand, to disinter the relationships and structures from beneath the surface of her practice.

Chris (and her customers) are trapped within a Fordist nightmare, a modern variant of the automobile magnate's system of controlled and authoritarian production which was so chillingly captured in Chaplin's film *Modern Times*. Central to Ford's vision, and that of Frederick Taylor, his ideological counterpart, is the idea that effective management of the labour process demands the separation of conception from execution, the removal of human intellect from the working process, and the fragmentation and gradual removal of skills and craft knowledge. [3] The tendency to try to routinise and deskill is ubiquitous, even for those who program the machines in which knowledge is invested. [4] As Straesser *et al.* (1991) point out, the more Taylorised a working practice, the less mathematics its practitioners require.

Not all occupations, of course, have been deskilled in this way. Yet if ever there were a straightforward relationship between understanding a mathematical tool and using it in application, technology has made that relation problematic. If it used to be the case, for example, that engineering workers could make use of their simultaneous equations in the turning of gears, that engineers could use their slide rules as points of discussion and tools for appreciation of their materials, that bank clerks could put to good use their drilling in commercial arithmetic, and that draughtsmen and -women were able unproblematically to exploit their Euclidean geometry in the service of their drawings, it is true no longer. And it is this tendency, the deskilling of the labour process, which - if not providing a *rationale* for the deskilling of the mathematics curriculum - gave rise to a culture in the U.K. which allowed it to occur.

Now, as a mathematician, I would be happy to rest my case for the inclusion of mathematics as a school subject on aesthetic and cultural grounds alone. But I know, in this post-Thatcherite era, that that would not be enough. In any case, it is important to know whether the aesthetic and utilitarian are antithetical, and more important still, whether the gap between them is widening or narrowing. It is to this end that I, together with my colleagues Celia Hoyles and Stefano Pozzi, have recently been studying how different groups of employees - clerical and technical workers in an investment bank, paediatric nurses and airline pilots - make use of mathematics in their professional practices.

I will illustrate this work by sketching two episodes. The first concerns Peter, in charge of support and maintenance of computer equipment in a major European investment bank. His operating budget was some one or two million pounds a year, and one of his tasks was to assess the relative merits of buying or leasing computer equipment based on its 'present value' - how much it would be worth now given knowledge of its value some time in the future. To do this, he entered various parameters on a spreadsheet model with which he had been provided. Peter explained: "I press the button marked *present value* and see what it says". We asked Peter what *present value* did, how it worked, how it had been programmed. He didn't know. He did, he told us, have an infallible procedure which he invariably followed: "I look at the answer. If it says what I think we should do, I use the number to justify my decision. If not, I ignore it, or put in figures which will support my hunch."

The second episode concerns a pair of nurses in a paediatric ward. [5] The ward is, like Peter's bank, technology rich, containing a vast array of electronic apparatus for measuring patients' conditions and for administering a range of therapies. Wanda and Betty are looking after a renal transplant patient who has been prescribed *vancomycin,* an antibiotic. This had been prescribed at regular intervals - 600 mg every six hours for 24 hours. Then, as sometimes happens, the doctor had prescribed a change of frequency: from four times a day ('QDS' in nursing jargon) to twice a day ('BD'). The new dose was 1200 mg and it was to be administered every twelve hours. We encountered Wanda and Betty at midday, as they were discussing when to give the first 12-hourly dose: the last 6-hourly dose had been given at 6 a.m. that morning. The problem was this: should the first double dose be given now (Wanda's strategy) or should it be postponed until 6 p.m. (Betty's preference)? The two strategies are represented diagrammatically below.

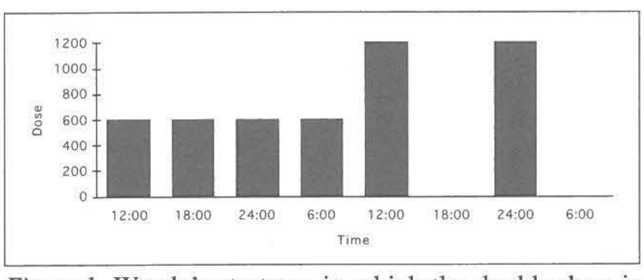

Figure 1 Wanda's strategy, in which the double dose is given straight away

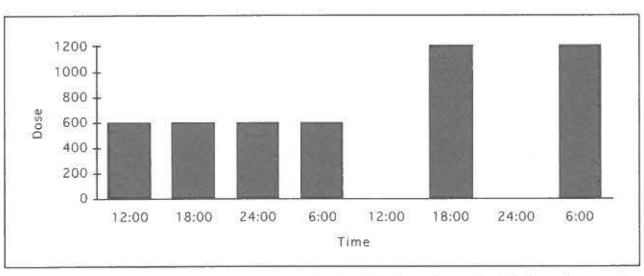

Figure 2 Betty's strategy, in which the double dose is delayed for six hours

The discussion that ensued was wide-ranging. Changing drug times is relatively common, as drug administration needs to fit with the effective use of nurses' time. But in this case, there were clinical considerations. Wanda was worried that if the level of drug in the blood fell too low it would cease to be therapeutic. On the other hand, Betty was well aware that too high a level of vancomycin is known to produce deafness. Clearly the model of drug-level employed can be a matter of life and death.

When the decision had been made, we asked other nurses what they thought. Cathy agreed with Wanda, but for a different reason: she argued that the QDS and BD doses should be thought of as separate regimes, and that the 1200 mg doses should be started right away as the low doses had been 'completed'. Amy advocated delaying the dose, so that the 'maximum daily dose' was not exceeded in *any* 24-hour period. Francis proposed a delay: she was concerned that renal patients 'clear' drugs more slowly than normal ones, and that a midday dosage might produce a dangerously high level of drug in the body.

The details of these different positions are not important. What is fascinating is that each was underpinned by a more or less explicit *model* of drug level. All five nurses mentioned that the problem could be solved empirically, by checking the drug level with a blood test. But such tests are not always taken (for various reasons, including the need not to disturb the patient unnecessarily) and sometimes the only course of action left open is to try to take a decision on the basis of a conceptual model of drug level over time. These models were surprisingly diverse, and their complexity was given expression by the nurses' language which involved mathematical terms like 'peaks' and 'gradients' - in some real sense, they were thinking about graphical representations of the patient's drug concentration over time.

I want to draw attention to two issues which emerge from these episodes. First, they describe how people try to make sense out of complex situations by building models, or, if they do not have access to the raw material of model-building, by circumventing them as Peter did. Peter could not open up his model, he could not make sense of it or fine-tune it to his purposes: so he was left in the position merely of accepting or ignoring the computer's output as best he could. To gain access to underlying models, to make them *visible*, is to focus on the quantities which matter, and on the relationships among them. But in order to think at that level, one needs two further elements: tools which bring the model to life (like graphs, variables and parameters) and the means to express its structure (like numerical, algebraic or geometrical tools).

The second issue concerns the complexity of interaction between professional and mathematical considerations. We saw how professional expertise and intuitions were mobilised in powerful ways to make sense of the situations. Clearly, the problem would not be resolved by building more intelligence into the technology, leaving still less room for intuition. But neither would Peter or the nurses want to be in situations without computational support, without some ways of processing the mass of information at their disposal. Decision-making seems most likely to benefit from models which can be fine-tuned by individuals and groups, situations in which professional knowledge and intuition can be webbed together with mathematical understanding and computational support. [6]

At the fringes of industrial and commercial practices, this lesson is being learned. In certain kinds of occupations, particularly those in which technology plays an important part, it is no longer the case that the human employer can be seen as merely an adjunct of the computer system. On the contrary, even judged on the criterion of efficiency alone, more and more people are having to make sense of the models which underpin the systems they use. In the words of a recent document on social exclusion, published by IBM:

> Demand is rapidly shifting in favour of people with skills and against those without them. (IBM, 1996, p. 4)

This trend is not a new one. Since the nineteen-eighties, certain sections of industry have been arguing that there are disadvantages to firms that maintain Fordist or Taylorist strategies of dividing tasks and divorcing responsibility from job execution, that there is, as Mathews (1989) puts it, a post-Fordist option. As banking is one area in which I have done some research, it is instructive to look more closely at the roles that technology is playing there, as well as throwing a little more light on Chris's plight. Mathews observes:

> most banks have proceeded down the Taylorist road, equating on-line access with deskilling, and assuming that counter staff would need less and less training as the terminal took over more and more of their functions. However, this deskilling approach [...] is now coming up against the same sort of limits [...] Banks are finding that their entire financial networks are becoming vulnerable to input errors perpetrated by unskilled counter operators. (p. 62)

Put simply, the problem is *not* that the counter operators are unable to do arithmetic: that is one aspect that the computer systems can do admirably. It is that they have no understanding of what it is they are doing, what the computers do with their inputs, and how they might make sense of the output the computer gives them. Their problem is not that they cannot calculate, not that they cannot 'add up', but that they have no *model* of the system. Even more crucially, they have no sense of what it means to construct such a model. And in some contexts, the educational implications are far-reaching.

> The importance of understanding the firm's system, in both the operating procedures and the processing steps these procedures trigger, calls for a new and broader type of training. Previously, it was by the successive apprenticeship of several departments that upwardly mobile personnel learned most of their banking. Now experience cannot suffice, as the critical operating procedures have been internalised in the computer system. (Adler, quoted in Mathews, 1989, p. 63)

There are signs around the edges of the system that the personal and social needs of individuals can sometimes converge: there is evidence that job satisfaction and personal empowerment are not necessarily antithetical to the efficiency of large-scale systems. More surprising still, it is the computer which points to this possibility: the evidence is that it *is* possible for computer systems to treat human beings as partners rather than inconveniently expensive appendages. The sense of loss, of alienation from computer systems which leave no room for human intervention, can sometimes be simultaneously alienating for individuals and inefficient for the purposes for which they were designed. [7] The key issue here is that judgement and calculation are often conceived as opposed, and in such cases, the outcome for the individual (and the system) is always negative. [8] The divorce between experiential, tacit knowledge and intellectual, 'scientific' knowledge is made more intense; it results in individuals losing their sense of intimacy with the tools of their practice.

The effects of designing out human beings from computer systems can have much greater than mere personal consequences: they threaten the very existence of life itself. In the report into the nuclear accident at Three Mile Island, the President's commission stated:

> Operator trainees were not provided with a fundamental, comprehensive understanding of their reactor plant design and operation which would enable them to recognise the significance of a set of circumstances not explicitly predicted by the operation procedures and which would lead them to place the plant in a safe condition. Essentially, operators learned how various pieces of equipment worked, how plant operating and safety systems worked, and how to apply preconceived, stepwise procedures to various abnormal and emergency situations. In other words, they were dependent on 'the manual'. They were not taught or expected to know how or why a specific nuclear power plant would respond to different types of failures and lacked the basic understanding necessary to deal safely with what was, in fact, a relatively minor failure when it occurred. (Raizen, 1994, p. 91)

There is no question that the dominant philosophy of commerce and industry is still to 'vest intelligence and control in technical systems, and treat workers like donkeys' (Mathews, 1989, p. 184). But there is a tension in the professional air, a gradual acknowledgement that the deskilling design strategy is robbing computerised systems of their potential, both individual and social. I have no idea whether Western economic systems will choose the post-Fordist future for the productive process. That is not my concern here. My interest is to understand how some signposts in the world of work are pointing towards new, post-Fordist futures for learning.

There is a ubiquitous tendency for computers to lull people into a state of complacent ignorance which leaves all understanding to the programmers. We in the social sciences are hardly immune.

> Many scientists use the widespread computerized statistical packages, which alleviate the need for computation. However, people who use statistical packages are often shaky in their understanding of the basic principles involved and often apply statistical tests or graphical displays inappropriately.
> (Thurston, 1990, p. 848)

Increasingly, not only scientists and social scientists, but technicians, clerks and health workers will need to understand such basic principles; they will need to sort out what has gone wrong, what mathematical knowledge has been buried invisibly beneath the surface of their computers, and how to dig it out. As the demands of workplace practices point beyond mere pattern recognition, and beyond that which can be grasped by any one individual (however well-educated), solutions to problems will need to draw on precisely the kind of mathematisation which is embodied in computational models. People will need to represent to themselves what has happened, particularly whenever the activities in which they are involved become in some way non-routine. (See Suchman (1987) for a fascinating analysis of these issues in the context of photocopier design.) Our own work with nurses and bankers indicates that the need to represent models occurs in non-routine situations, and that a workplace mathematics far broader than basic numeracy is required when decisions become contested or problematic (Pozzi, Noss and Hoyles, 1998).

Representations are crucial to non-routine practice, and representations need a language, a set of intellectual tools which can reliably represent what is happening. At the same time, our research testifies to a communicative role for representations. For example, the 'rocket scientists' who build the complex models on which a bank's operations rely are increasingly unable to talk to those who use them. They have no language in common, no shared set of representational tools with which to understand each other's problems and practices. Building systems for others with whom you are unable to communicate, and whose problems you are unable to grasp, seems bound to end in disaster.

The essence of professional practice is difference and diversity: the efficiency of routine lies in the specificity of its language and conventions. But the essence of mathematics lies in its sameness: in the search for common structures, invariants and equivalencies. If the foreign exchange department cannot communicate with the options department, this does not matter until someone (but who?) gains a sufficiently broad view to spot common errors, or possible inefficiencies. If nurses from paediatric and geriatric wards cannot understand each others' charts, it will not matter until a nurse moves wards and a breakdown in routine occurs. There are horizontal and vertical failures of communication, and mathematical expression is one way in which they can be overcome.

There are possibilities in technology-based working practices which suggest an increasing need to make underlying structures visible to those who work within them. These, in turn, will involve people in the need to express themselves mathematically, to gain ways to think about the geometry and algebra of situations. We will need to think carefully about how to make mathematics visible in ways which are at once accessible and intellectually honest - i.e. ways which do not rob mathematics of its essence and which do not open still further the gaps between the mathematics of school and the mathematics of work, science and technology.

It will not be easy to tap the emerging cultures of the workplace to transform schools. There is no wave of post-Fordism to ride, and little hope of any spontaneous transformation of the working lives of Chris and her counterparts in the near future. Neither, it must be said, are there any signs in the U.K. that government or its advisors understand the need to consider what kinds of new mathematical knowledge are required, rather than simply more 'effective' ways to transmit old knowledge.

If we are to address the problem, it will need imaginative and creative thinking, and it will involve collaboration between mathematicians, mathematics educators, employees and employers, as well as others involved in learning and teaching. It will involve choosing the high-skill option for education, against the tide which runs in favour of the deskilled future for work. The computer, the epitome of deskilled and dehumanised social relations in so many contexts, may be part of the problem. Yet in the third and final section, I turn to outline why, surprisingly perhaps, I believe that the computer may also prove to be part of the solution.

3. New cultures of expression

In the workplace, the computer presence is two-edged. On the one hand, it is a potential instrument for deskilling and isolation, and on the other, a putative instrument of intellectual liberation and collaboration, perhaps lending to an unprecedented degree a human element to the working process. What determines this choice? A complete answer would involve the realm of sociology and politics, for technology is not neutral, and the uses to which it is put and the social forces which shape it are complex. [9] But whatever the explanation, there is no question that *design* is a fundamental issue: computer cultures do not just happen, they are constructed. Underlying that design are *epistemological* principles, congealed knowledge built into structures and intended applications. It is these principles, as much as the social and political context of their use, which determine choices among the relationships human beings develop with computers. [10]

These choices confront us in education too. Computers have only been present in any substantial numbers in classrooms for less than two decades, but the choices have been present since their introduction. They are, in essence, choices about knowledge, about skilling or deskilling, closure or openness, calculation or modelling - just as they are in the workplace. And, as in the workplace, they structure the relationships individuals will need to have with the principles which underlie their design. Does it suffice for students to be consumers of programs, to learn to punch in answers or vary parameters in models which have been built by software developers? Or will they need to gain insight into how the programs are built, what it means to build a mathematical model, how the construction of algorithms provides a rich source of metaphors and knowledge on which to develop mathematical understandings? Will students merely need to know how to run other people's programs, or will they need to reconstruct them for themselves in order to read and interpret the output creatively?

The former, Fordist future for computers in mathematical learning has recently re-emerged in the form of Integrated Learning Systems, or ILS. I will demonstrate the approach by choosing one out of the thousands of tasks a typical system can 'deliver'; the following example is contained in a publicity video of one of them, offered as an exemplar of the system. [11]

The student sits at a computer terminal, one of a line of computer terminals. She is wearing a pair of headphones, so that she can hear the voice which reads the text on the screen (one of ILS's selling points is its claim to motivate students in "today's multimedia world"). A question appears (Figure 3).

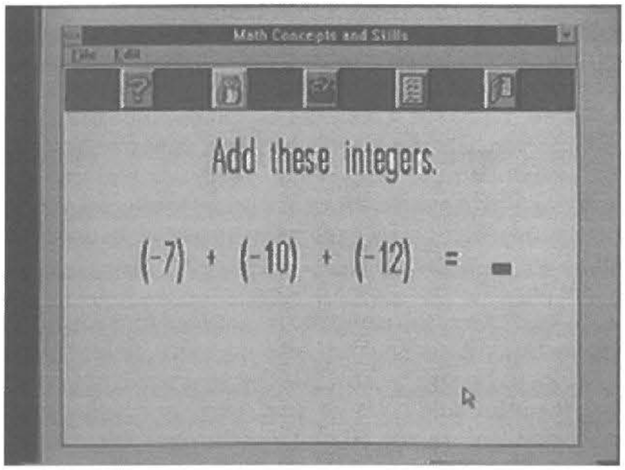

Figure 3 The initial screen

The student enters the answer, 29. 'Wrong', says the system. Help is required, so the student pushes the 'help' button and help materialises.

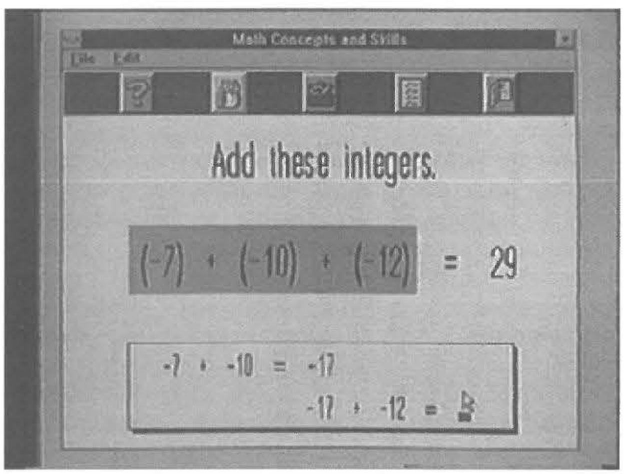

Figure 4 A help screen

This 'help' is accompanied by a spoken statement of what appears as screen text. But, if the student persists in not understanding, there is no cause for concern; for as the video clip intones: "If the student still gets it wrong, the correct answer will be highlighted".

There are many points which I could make about this tiny fragment, and I am aware that it is, perhaps, unfair to judge a system by inspecting one ten-thousandth of its functionality. So I will concentrate on just one question which I think is amply demonstrated in the extract: what kind of knowledge are students supposed to appropriate from interacting with this system?

The principle underlying the system's design is this: if the student is unable to add three negative numbers, it is because she cannot add two negative numbers. Adding three negative numbers is the result of adding pairs of negative numbers twice. So by breaking the target skill into two subskills the task can be taught satisfactorily. And if that fails, well ... the computer will provide the answer anyway.

The psychological and pedagogical rationales for these assumptions are, to say the least, questionable. They date back to the principles of programmed learning, briefly attempted and abandoned as a failure in the nineteen-sixties, and based on an unreconstructed behaviourist psychology. But my concern, as I have said, is epistemological. The assumption is that mathematical knowledge is strictly hierarchically ordered: that pieces of mathematical knowledge can always be broken down into smaller pieces. [12]

I would not deny for a moment that students need, as part of their mathematical learning, to acquire the routine skills of calculation, the facts of numbers. On the contrary, I *want* them to learn them so that they can forget about them. The point is that while it is important to understand the little bits and pieces of numerical facts, such knowledge is only a very small part of what mathematics is about, and what it is for. My criticism of Integrated Learning Systems is not that all of the material is ill-conceived: it is that it points in the wrong direction, towards the smallest and least interesting twigs, when the technology could be used to give students a view of the forest. Worse still, focusing on the acquisition of fragmented skills can be counterproductive.

> It is harder, not easier, to understand something broken down into all the precise little rules than to grasp it as a whole. (Thurston, 1990, p. 849)

ILS and their counterparts are a Fordist choice for education. They waste technological potential, they are hugely expensive and they squander the opportunity to reconsider what should be taught, never mind how and to whom. I have no doubt that children will learn something by using the systems: it would be strange indeed if children taught basic numerical techniques with an ILS did not achieve better scores than those who spent less time on basic numerical techniques or were hardly taught them at all. [13] But I want to use the computer to build more learnable mathematics, not merely to recapitulate more effectively the fragments of numerical skills and techniques that happen to be teachable without the computer.

Mathematics itself is in a state of change. Boundaries between pure and applied mathematics are shifting, and the computer has entered firmly into the mathematical arena where once it was shunned. Fierce debates rage on the computer's role, its place in the sanctity of mathematical proof, its position as an exploratory tool, its contested role in turning parts of mathematics into experimental rather than solely theoretical domains. Geometry, once the epitome of the abstract and formal, is becoming, in part, an experimental science under the influence of powerful computational systems which allow the manipulation and investigation of geometric structures in insightful ways. [14]

To say that this redefinition of mathematical boundaries is a result of the computer's presence is to avoid engaging with its most important facet. This is its ability to offer alternative means to express mathematical relationships, novel kinds of symbolism and innovative ways to manipulate mathematical objects: in short, the emergence of new mathematical cultures. The computer points to new ways to say mathematical things, as well as to new mathematical things to say.

For many years, the Mathematical Sciences Group at the Institute of Education in London where I work has been trying to construct computational tools of these kinds, to provide a window onto the ways students of all ages can be involved in making mathematical meanings. In a recent project, Celia Hoyles, Lulu Healy and Paul Clifford have been devising tasks for adolescent students, in order to study their understandings of proof and to build bridges between experimentation and proving. [15]

Here is an example:

> *Prove that the sum of any three consecutive integers is divisible by 3.*

A standard way to think about questions like this is to represent them algebraically. For example, I might jot down:

$$p+(p+1)+(p+2)$$

which I could immediately see to be $3p + 3$ or $3(p + 1)$ which is obviously (to me!) divisible by 3. From there, it seems natural (to me) to generalise, and to think about the divisibility of the sum of n consecutive integers.

$$\sum_{i=0}^{n-1}(p+i)$$

This I can see as:

$$np+\sum_{i=0}^{n-1} i$$

whose second term is an old acquaintance that I recognise as the sum of an 'arithmetic progression', giving:

$$np+\frac{(n-1)n}{2}$$

So the general divisibility problem seems to split into cases when n is odd or even. And so on.

The problem is not difficult, even in a quite general form, particularly for someone who has at their disposal a reasonable range of algebraic notation and convention, not to mention an understanding of what the concept of mathematical proof involves.

But now consider a student who does not yet have access to this expressive power, someone who has been taught very little about proof. How might we proceed to help her? One approach, suggested by Hoyles and Healy, might be to make some kind of visual or geometrical representation of the numbers, like the one in Figure 5. The hope is that we might encourage our student to see that moving the bottom right dot to the bottom left, would 'even up' the three columns, showing convincingly that the conjecture is true for 6, 7 and 8 dots, and that there may be a generality perceivable in this particular situation.

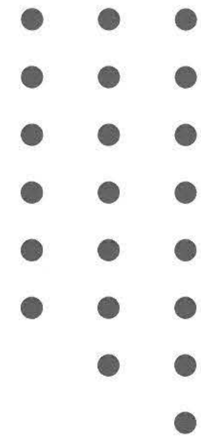

Figure 5 A way to represent three consecutive integers

This identification of number with column is certainly not automatic. And it is a double-edged metaphor. On the one hand, it provides a concrete representation of a number, and a visual image of its relationship with other numbers. On the other, it runs the risk of cutting individual numbers off from the *number system*, of suggesting to the student that numbers are *about* counters rather than powerful abstractions.

The use of objects such as counters (or rods, or cubes, etc.) - 'manipulatives' in the educational vernacular - can be quite successful in generating a feeling for relationships in students and is ubiquitous in many U.K. mathematics classrooms. But the assumption that it necessarily provides a path to the rigour of algebra is mostly false, and it is often the case that this invitation to pattern spotting becomes just another kind of empirical activity. This is, quite simply, an instance of my second paradox: there is no straightforward connection between *action* and *expression*, and the former can - unless we are very careful - act as a substitute for - rather than offer access to - mathematical rigour.

We have recently developed a computational tool kit called *Expressor,* which Hoyles and Healy are using to explore children's expression of relationships, and to try to open up routes to algebraic expression and proof. In *Expressor*, the dots (and other objects) can be placed arbitrarily on the screen, or arranged in columns in various ways. Individual or groups of dots may be moved and coloured appropriately, and new configurations of columns may be produced to check various properties (such as looking for divisibility in terms of equal 'lengths' of columns). Dots can be placed by dragging them into position with a mouse, or by generating them with pieces of computer program entered as commands to the 'turtle'. Figure 6 overleaf gives a flavour of the idea.

There are many features of *Expressor* which I am unable to discuss here. [16] But there is one important facet which sets it apart from any 'manipulative' version that could be undertaken with counters or smarties. In the 'three numbers' task, the dots can be dragged into columns as with real counters; but as this is done, a recorded 'history' of the actions is stored (see the history box in Figure 6) in the form of fragments of computer program. This code is *executable*: that is, it can be 'run' to produce the output (or part of the output) which produced it.

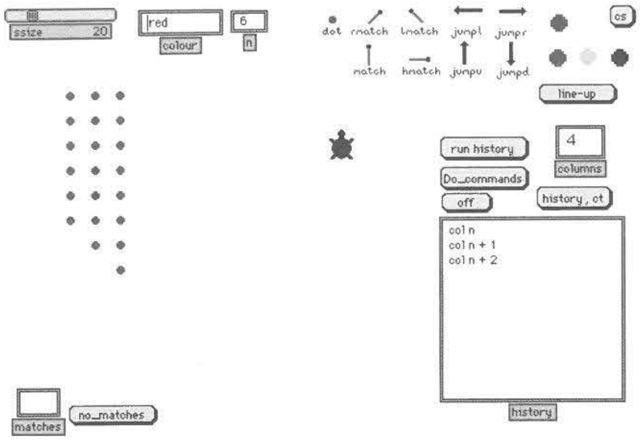

Figure 6 A typical *Expressor* screen - in this scenario, a program col has been written to generate 6 (n), 7 and 8 columns, and a box n is used to store the smallest of the three numbers

There is, therefore, a duality between the code and the graphical output of the dots. The action (on the dots) and the expression (in the form of pieces of program) are essentially interchangeable: the code is a rigorous description of the student's action, and her actions are executable as computer programs. Now our student might see that what is in the box called n hardly matters, and therefore that the theorem is independent of the first number. Perhaps she might be able to conjecture about the sum of four consecutive integers (what stays true and what does not), by shifting her attention onto the pieces of computer program and away from manipulation of dots.

Chloe, a 15-year-old student, provides an example of what I mean. [17] She is writing about her generalisation of the three-number result, and has formulated a new conjecture for four consecutive numbers. Her theorem and its proof are given in Figure 7.

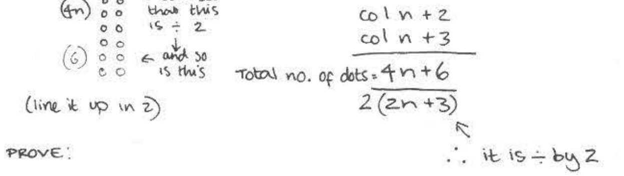

Figure 7 Chloe's proof of the 4 consecutive numbers theorem

The details of Chloe's proof are not important. What is interesting is that she uses the 'dots' idea as a way to express her proof. She seems to be thinking about 'columns' of dots, and she has a way to express that idea using the name of her program *col* and *n*, the number of dots - both of these names are borrowed from her *Expressor* activities. Her proof lives in a world somewhere between natural language and mathematical formalism; a world which allows abstractions to be formulated and explored by those who have not yet completed their journey into the rigorous world of mathematics, but which carries them a little nearer to it. For Chloe, the algebra is alive in *col* and *n*.

The key point about this example, and about computational objects in general, is that they offer a channel of access to the world of formal systems. It is this curious mixture of concrete and abstract, the dialectic between informal and formal, intuitive and rigorous, which is so fascinating to me. It points towards a resolution of the second paradox I raised earlier - the tendency to whittle away at the knowledge structures of school mathematics in a continuing attempt to make it learnable.

It is hard to understand a mathematical idea until you have used it, until you have seen its connection with other mathematical ideas, and possible applications. And so, as I have said, there is a tendency in educational circles to postpone rigour in the name of intuition, to delay formality as it seems to stand in the way of appropriation, and to hesitate on the road to abstraction, in favour of the practical and concrete. This dichotomising of concrete and abstract has recently come under sustained criticism (see, for example, Noss and Hoyles, 1996; Wilensky, 1991).

On the other hand, it is hard to use a mathematical idea until you have understood it: at the very least, you need to be able to express it in some formal way, to appreciate why it works, how it works. Hence the paradox, and the downward spiral we have come to see as inevitable, in which less and less mathematics is taught to more and more children. Manipulating the variables instantiated in boxes or procedures on the screen is a kind of algebra. It may not quite be algebra in its accepted sense, but it opens up the possibilities of new cultures of mathematical expression which can allow students to think about abstraction and generalisation in powerful ways. Here is the essential difference which the technology brings: the seeds of the abstract are sown by actions with the concrete.

Using the computer in carefully designed ways, it is possible simultaneously to use and come to understand; to build and use, or to put it more succinctly, to build *in* use. In short, I contend that the computer offers us a means to build numeracies in which learnability and knowledge are not antagonistic.

I would like to illustrate with a final example, this time from an area of non-elementary mathematics. I have recently been working with a group of mathematicians and chemists at Imperial College, London [18] who are trying to develop novel approaches to teaching undergraduate mathematics to chemistry students using the computer system *Mathematica*. Their work is based on the assumption that chemistry students need to understand and build models of their chemical knowledge, rather than simply use mathematics as a set of learned techniques which can be 'applied' when necessary.

In contrast, a related project in a second university has adopted an alternative approach, one similarly based on *Mathematica* but with an entirely different philosophy. The mathematicians there have chosen a tutorial approach, in some respects not unlike an Integrated Learning System, in which material is presented like an electronic book, and the role of *Mathematica* is restricted to checking students' responses to computer-generated questions.

I recently had the opportunity to study the two projects at close hand. It emerged that the difference in approaches - one design based on an open set of mathematical tools and the other a set of mathematical topics embedded in a tutorial program - was at root a difference in mathematical epistemology. The open system was designed for understanding, for providing students with a mathematical route for exploring and making sense of chemistry. The tutorial approach had entirely different objectives: students would be shielded from the mathematical structures underpinning their scientific learning, as this would stand in the way of their acquisition of the necessary skills and techniques. Here, mathematical tools were to be learned as tools only, not tools-in-use.

I tried to compare the respective success of the two projects. Gradually, I came to realise that the difference was not in degree of success: it would, in any case, be impossible to compare outcomes. What would I measure? At each site, different epistemologies were in play, students were expected to learn different things. It is likely that on a test of mathematical model-construction the tool-kit students would have won; and on a test of skills and techniques, it is entirely possible that the tutorial approach would have been more successful, at least in the short term. It is knowledge which determines design, as much as design which determines knowledge. The key question is: what kind of knowledge do we want to teach?

The moral of the story is clear: we do not have to make a choice between knowledge and pedagogy. But we will surely be led down this road if we concentrate our attention on teaching methods rather than what is to be taught. It will lead us back to educational cultures which proffer Frederick Taylor's system of scientific management reincarnated in the U.K. classroom as Key Stage tests, OFSTED [19] school inspections and ILS monitoring. It will return us to the logic of Fordism, in which schools and teachers will be encouraged to teach fragmented and isolated numerical skills, instead of assisting in the task of redefining the boundaries of what needs to be understood as a mathematical whole.

We need new pedagogies to teach old knowledge in accessible ways, but we need to consider how technologies can help us build new curricula as well. The limitations of old technologies have hung around the necks of mathematics classrooms for two thousand years, shaping what it was possible to teach, what it was possible to learn. Thanks to the barely-tapped potential of new technologies, all of us engaged in the educational enterprise have an opportunity to recapture the spirit of Crowther in our understanding of numeracies, to devise a breadth to the concept which compares equally with the notion of literacy. We can exploit technologies to develop new learning cultures, ones which challenge our notions of schools and classrooms, and support rather than supplant the professionalism of teachers. And we can construct new kinds of mathematical knowledge for children and adults to learn, new intellectual tools designed to make visible the mathematics which lies beneath our social and working lives. We can build new numeracies.

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Notes

[1] This article is an abridged and amended version of the author's Inaugural Lecture 'New Cultures, New Numeracies', presented on October 7th, 1997 at the Institute of Education, University of London.

[2] Lave (1988) has shown that numerical calculations play a surprisingly small part in most people's lives, and even when they appear to do so, people often choose to make decisions based on criteria which are not solely based on arithmetic. This makes it all the more dangerous to conclude that 'mathematics = visible calculation'.

[3] This is the scenario so prophetically described by Harry Braverman (1974) in his book, *Labor and Monopoly Capital.*

[4] For some considerable time, there has been a drive on the part of large organisations to deskill the work of programmers and to demystify their craft. The era of the lone hacker has long passed in favour of teams of people working on independent modules. This has, incidentally, been one of the social mechanisms for the development and ubiquity of object-oriented programming languages, which allow programs to be built without investing knowledge of their complexity in any single expensive and crucial individual.

[5] The episode outlined here is based on Hoyles, Noss and Pozzi (in press). This research is part of a project funded by the Economic and Social Research Council entitled *Towards a Mathematical Orientation through Computational Modelling*, Grant No. R022250004.

[6] In our book, *Windows on Mathematical Meanings* (Noss and Hoyles, 1996), Hoyles and I consider this idea of 'webbing' - the interweaving of intra- and extra-mathematical meanings - as fundamental to the learning of mathematics.

[7] For example, Göranzon (1993) provides a fascinating example of the ways in which forest rangers in Sweden felt robbed of their professional expertise by the computerisation of their professional practice.

[8] See Weizenbaum (1984) for an analysis of the ways in which judgement has been replaced by calculation and its effects.

[9] I have made some attempt to look at these issues in, for example, Noss (1991, 1995).

[10] Seminal contributions on these issues have been made by diSessa (1985, 1988) and Papert (1987, 1996).

[11] *Success Maker: Creating Success where It Counts* (1995) RM Learning Systems.

[12] The ILS in question measures progress on precisely this basis: the publicity boasts that it is capable of generating fifty different kinds of assessment reports based on fine judgements of progression, carefully ordered sequences in a linear model of learning and knowledge structure. A recent conversation with a programmer on a different ILS confessed to the casual and arbitrary nature of these judgements which he referred to as 'a joke'.

[13] Although it must be said that the evidence is far from convincing: see Underwood *et al.* (1995).

[14] Some idea of this power may be gained from running the *applets* available from the Minnesota Geometry Center: http://www.geom.umn.edu

[15] This project is entitled *Justifying and Proving in School Mathematics*, and is funded by the Economic and Social Research Council (Grant No. R00236178).

[16] Hoyles, Healy and myself have outlined the design of this microworld more elaborately in Noss *et al.* (1997)

[17] I am grateful to Hoyles and Healy for providing me with this example.

[18] See Kent, Ramsden and Wood (1996) for an outline of the*METRIC* project at Imperial College.

[19] Office for Standards in Education.

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