

A Practical Guide to Rotor Dynamics

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Introduction

Rotor dynamics is a very interesting and complicated subject. The importance of this subject has increased over the last few decades as machine speeds have increased and higher flows and efficiencies have had the side effect of introducing problems with critical speeds, unbalance response and rotor stability. Some of these problems are related to the economics of the capital expenditures where a machine may be bought on a cost basis. Far too often this results in excessive and expensive rebuilds and costly lost production. This paper will introduce the basics of rotor dynamics and lead into some of the more advanced concepts. The mathematics will be kept to a minimum and as many helpful "rules of thumb" will be included as this subject allows. Units are shown in detail for each equation.

At times, rotor dynamics can be a very controversial subject from the nomenclature used to the question of the degree of accuracy needed to model a rotor dynamic system. There have been many simplifications and assumptions made in this paper and the author offers some opinions that some people may disagree with, but the approach here is quite conservative and has proven successful over the last 25 years of my career. The approaches and guidance offered here are based on experience of many people and many years of analyzing and testing machinery. Analytical and actual test data will be used as examples.

A nomenclature page is included and

whenever possible, the symbols used for various factors are identified in the body of the paper. At the end of the paper is a list of references that the author has found very valuable and are recommended for further reading.

Rules of thumb are fine but the reader should come to the conclusion that the only way to do a meaningful rotor dynamics analysis on a machine train will be to procure the necessary computer programs to do the analysis. Some sources for programs used in the preparation of this paper are noted in the acknowledgments. There are many sources for computer programs that will run well on personal computers. While some are easier to use than others and some use differing analytical techniques, like any tool, these programs are dependent on the skills of the user. GIGO remains a problem today and the more powerful the program, the more pitfalls there are waiting. Thus, the main purposes of this paper are to introduce the basic concepts of rotor dynamics and to serve as a reality check guide. I always ask myself at each stage of analysis "...can that be right?" and if it fits logically, I proceed. If not, I backup and examine the input or my assumptions.

Basic Concepts

A compressor rotor, pump impeller, steel structure and a tuning fork all have something in common: they all have resonances. When a tuning fork is struck, it emits a tone (actually many tones, but one principal tone). Strong winds may "ring" a building's natural frequencies or cause large motions on a bridge. A rotor may have its natural frequencies excited by many sources: rotating imbalance, rubs, or process changes such as surge. The first objective of rotor dynamics is to identify the resonant frequencies present in a system, determine their severity and, if necessary, design the system around them. A typical machinery train may consist of a driver, a gear and a driven such as

a compressor. It is not atypical to have 10 or more system resonances to design around. So how does one begin? Do you need a complicated computer analysis to identify these resonances and do a complete system analysis? The answer is, yes, you probably do, however, there are many things that can be looked at quickly and easily with an eye toward general trends and design practices. In some cases, this may suffice, particularly if the vendor is capable of good rotor dynamics analysis, and most are.

A rotor, supported in bearings of some sort, is analogous to the familiar spring-mass-damper system shown in figure 1.

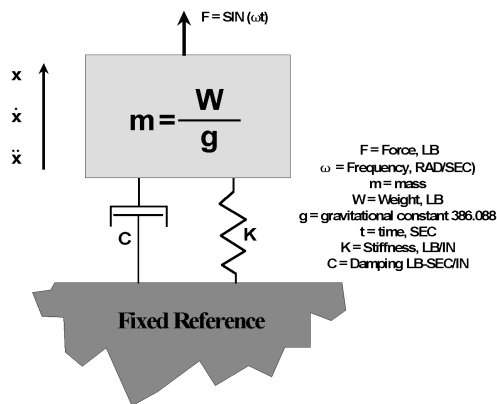


Figure 1 - SPRING-MASS-DAMPER

The governing equation for the motion of such a system is:

$$m \ddot{x} + c \dot{x} + k x = [the \textit{forcing function}]$$

It is absolutely crucial to understand the implications of this equation. It means that the forcing function (which in a rotating element usually means the imbalance forces but can be other things like misalignment, rubs, aerodynamic forces, etc.) is opposed by only three things: the system inertia (mass), the system damping and the system stiffness. These three things are frequency dependent. At low frequencies, below resonance, the stiffness is the primary control. When the frequency of a forcing function coincides with

a natural frequency of the rotor, we encounter what is commonly called a critical speed for rotating equipment. Here the damping controls the motion. At high frequencies, the inertia tends to control motion. This is analogous to vibration measurements where displacement sensors are most sensitive at lower frequencies and accelerometers are most sensitive at higher frequencies.

If we ignore the damping term for the moment and set the forcing function to zero we find that the solution of the equation gives the first natural frequency:

$$\omega_n = \sqrt{\frac{k}{m}} \quad \left(m = \frac{W}{g} \right)$$

Note that damping is not included in this equation. We will add the effects of damping later when we consider fluid film bearings.

This solution is not directly applicable to all rotor systems as we shall see. In a simple example, shown in figure 2, consider a single mass rotor with a rigid shaft supported on identical bearings:

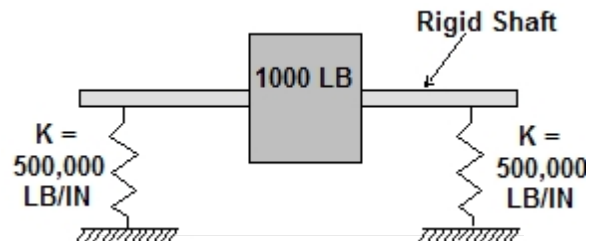


Figure 2 - Simple rigid shaft system

$$\begin{aligned} \omega_n &= \sqrt{\frac{k}{m}} \\ &= \sqrt{\left(\frac{1,000,000 \text{ LB}_f/\text{IN}}{1,000 \text{ LB}_m} \right) \left(\frac{386 \text{ IN-LB}_m}{\text{LB}_f\text{-SEC}^2} \right)} \\ &= \left(\frac{621.29 \text{ RAD}}{\text{SEC}} \right) \left(\frac{1 \text{ REV}}{2 \Pi \text{ RAD}} \right) \left(\frac{60 \text{ SEC}}{\text{MIN}} \right) \\ &= 5,933 \text{ RPM} \end{aligned}$$

However, a rigid shaft is not normally a reasonable assumption, and many shafts have more than one mass, either between bearings or overhung. Thus we must first find the shaft stiffness from beam theory.

For a circular shaft:

$$K_s = \frac{48 E I}{L^3}$$

$$I = \frac{\pi D^4}{64}$$

So, for example, let's envision a very simple case without external weights and without any overhung sections. I chose a 5 inch diameter shaft with a span of 80 inches. The stiffness of this shaft is:

$$K_s = \frac{48 (30,000,000) \left(\frac{\pi 5^4}{64} \right)}{80^3}$$

$$= 86,286 \text{ LB/IN}$$

Now, to include this in the previous example we must recalculate the system stiffness. The shaft is a spring supported on two other springs we call bearings. Springs in series add like resistors in parallel, that is inversely. So:

$$\frac{1}{K_{SYSTEM}} = \frac{1}{2 K_{BEARING}} + \frac{1}{K_{SHAFT}}$$

$$= \frac{1}{1,000,000} + \frac{1}{86,286}$$

$$= 79,432 \text{ LB/IN}$$

Notice that the system stiffness is always lower than any of the components. Thus, our natural frequency calculation is now (assuming that the total system mass is the same 1,000 LBm):

$$\omega_n = \sqrt{\frac{k}{m}}$$

$$= \sqrt{\left(\frac{79,432 \text{ LB}_f/\text{IN}}{1,000 \text{ LB}_m} \right) \left(\frac{386 \text{ IN-LB}_m}{\text{LB}_f\text{-SEC}^2} \right)}$$

$$= \left(\frac{175.1 \text{ RAD}}{\text{SEC}} \right) \left(\frac{1 \text{ REV}}{2 \pi \text{ RAD}} \right) \left(\frac{60 \text{ SEC}}{\text{MIN}} \right)$$

$$= 1,672 \text{ RPM}$$

This is a 255% decrease from the rigid case showing that, in this case, shaft stiffness is the dominant factor. This is usually true in most rotating equipment. This principal was originally developed in 1894 by Dunkerley, who stated that the principle of superposition applied to critical speeds in an inverse manner:

$$\frac{1}{\omega_{n_{system}}^2} = \frac{1}{\omega_{n_{shaft}}^2} + \frac{1}{\omega_{n_{mass1}}^2} + \frac{1}{\omega_{n_{mass2}}^2} + \dots$$

Additional information about the mathematical approach to rotor dynamics is available in abundance. A few of the best references are listed at the end of this paper.

To get a feel for a given rotor-bearing system, some general rules of thumb can be applied. The first of these is the concept of **modal mass**. Modal mass can be thought of as the effective mass "seen" by a particular mode at resonance. For a between bearing system at its first critical speed, the modal mass is equivalent to the mass that would yield an equivalent system if all the mass were lumped in the center of the span. Figure 3 is a plot of modal mass (modal mass/total mass) as a function of the stiffness ratio between bearing and shaft stiffness (2Kb/Ks). The stiffness ratio is very important and greatly affects the modal mass ratio among other things. As the support stiffnesses become much less than the shaft stiffness, the rotor begins to behave as a free body and the modal mass becomes equal to the total mass.

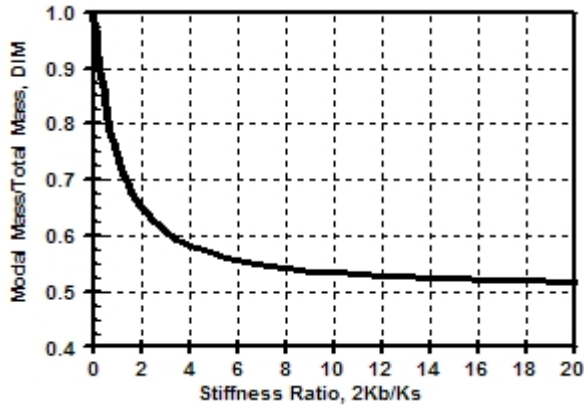


Figure 3 - Modal Mass Concept

As the support stiffnesses become much larger than the shaft stiffness, the modal mass ratio asymptotically approaches 0.5. For most actual cases that this author has encountered, modal mass ratios between 0.52 and 0.60 have been most common. When in doubt, use 0.55. As an example, see figure 4.

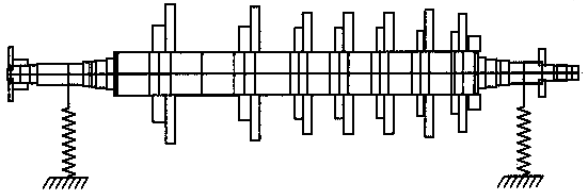


Figure 4 - Typical Compressor Rotor

This is an actual propylene refrigeration machine installed in an ethylene plant. Suppose we need a quick estimate of this machine's first critical speed so we don't accidentally run too close to it during a startup. We begin by calculating the shaft stiffness:

$$K_{shaft} = \frac{48 E I}{L^3}$$

$$= \frac{48 (30 \times 10^6) \Pi (14.25^4)}{64 \cdot 147^3}$$

$$K_{shaft} = 917,600 \frac{LB}{IN}$$

The bearings for this rotor are highly preloaded tilting shoe bearings and were calculated to have a stiffness of 3,500,000 LB/IN each. So to get the system stiffness we add the stiffnesses inversely:

$$\frac{1}{K_{system}} = \frac{1}{[2 K_{bearings}]} + \frac{1}{K_{shaft}}$$

$$= \frac{1}{7,000,000} + \frac{1}{917,600}$$

$$K_{system} = 811,200 \frac{LB}{IN}$$

If we were to use the total rotor mass to calculate the first critical speed:

$$\omega_n = \sqrt{\frac{K}{M}}$$

$$= \sqrt{\frac{(811,200 \cdot 386)}{11,959}}$$

$$= 161.8 \frac{RAD}{SEC} = 1,545 RPM$$

But this isn't even close to the actual critical speed. So, refer to figure 3 and calculate the stiffness ratio 2Kb/Ks = 7.63. This translates to a modal mass ratio of 0.56. So our first critical speed calculation becomes:

$$\omega_n = \sqrt{\frac{K}{M}}$$

$$= \sqrt{\frac{(811,200 \cdot 386)}{0.56 \cdot 11,959}}$$

$$= 216.2 \frac{RAD}{SEC} = 2,065 RPM$$

As can be seen in figure 5, the actual first critical speed of this machine was between 2000 and 2100 RPM. Thus we can see that this method works fairly well for between bearing rotors with evenly distributed external masses.

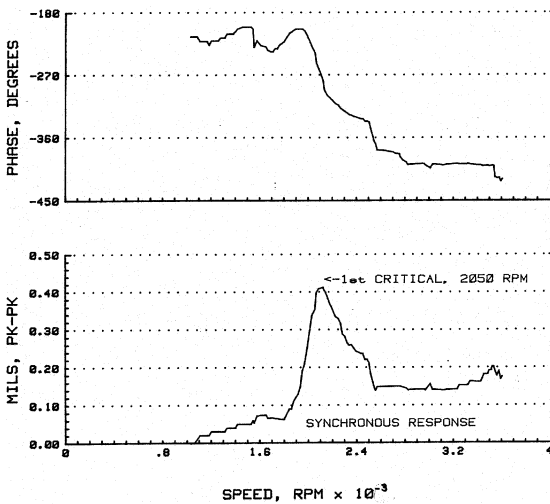


Figure 5 - Actual Compressor Startup

Mode Shapes

Associated with every resonance or critical speed is a **mode shape**. Mode shapes are only defined at resonance and are independent of the forcing function. At any other speed the rotor simply has a deflected shape that is due to the proximity of resonances and forcing function distribution.

Let's return to the simple 5-inch diameter shaft selected earlier where the overall length and bearing span are 80 inches. We'll examine the effects of changing these dimensions later. The mode shapes of any resonance of any rotor are only dependent on the shaft stiffness, the bearing stiffness, the shaft mass and the mass and distribution of the components mounted on the shaft. Sometimes, due to shrink fits, mounted components can increase the apparent shaft stiffness. Damping controls the severity of a resonance, not the mode shape.

The stiffness ratio, $2K_b/K_s$, can be used as a non-dimensionalizing factor to better understand the behavior of modes shapes. Let's look at the first three calculated mode shapes of our simple rotor as we vary the bearing stiffness as a function of shaft stiffness. Figure 6 shows how the mode shapes of the first three critical speeds change as the stiffness

ratio ranges from $2K_b=K_s$ to $2K_b=10K_s$.

Looking at the first critical speed, when the bearings are relatively soft ($2K_b=K_s$), the rotor has a translational "bouncing" motion up and down with little shaft bending. As the stiffness of the bearings increase, they clamp down on the shaft. More and more bending of the shaft appears until there is very little motion at the bearings.

The second critical speed has a rocking or pivotal motion with a point of no motion in the center. This is called a **node point**. At that resonant frequency, there is no motion at that point. At low bearing stiffnesses there is virtually no bending in the rotor but as the stiffness of the supports increases, the second critical speed takes on an "S" shape.

The third critical speed is different. At low (even zero) bearing stiffnesses, the rotor has a definite bent shape. This is a *free-free* or "bearingless" mode and is the lowest frequency at which the rotor will "ring" when hung and hit with a hammer. As the stiffness of the supports increases, this basic shape is retained with additional deflection at the ends near the bearings. The third critical speed mode shape has 2 node points regardless of the bearing stiffnesses. The number of nodal points for a simple shaft like this is always the critical speed number minus one.

It is an important point that the first two critical speeds of this type of simple rotor system are **inherently translational modes**. At zero support stiffness they do not exist. Bending is induced in the shaft as the supports increase in stiffness but only the third mode and higher are **inherently bending modes**. This way of referring to criticals speeds is commonly misused with the first critical speed often called a bending mode. It is not.

Some machines with very soft supports have critically damped first and second critical speeds: that is, you cannot detect them in normal operation. However, this does not make the third critical speed (which is almost always very excitable) suddenly become the

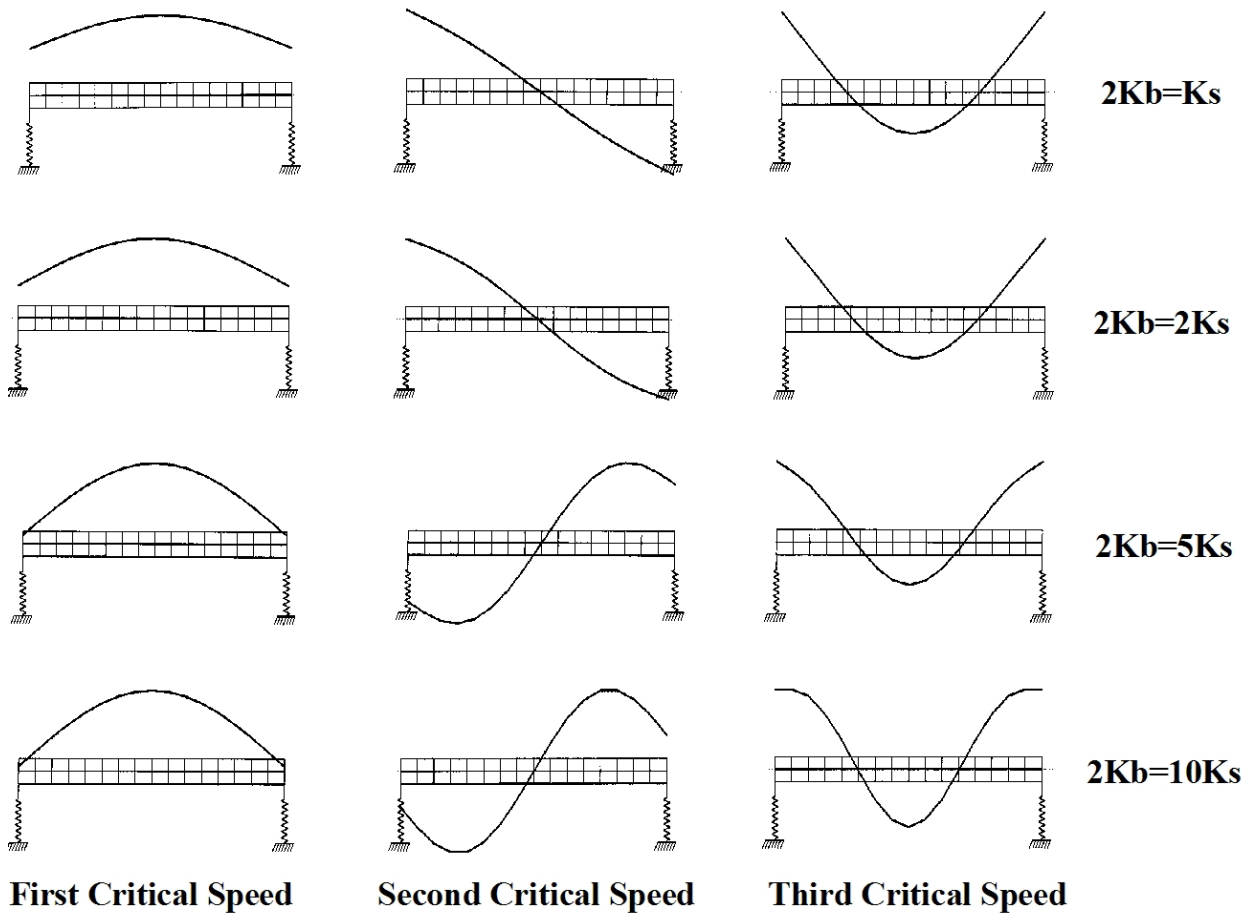


Figure 6 - Simple Rotor Mode Shapes at Different Bearing Stiffnesses

first critical speed. We need to keep our terminology consistent for better understanding.

Indeed, as we will see, reducing the amount of first mode shaft bending is a very important goal. It is easy to see that as shaft bending increases, the critical speed frequency will increase.

The Critical Speed Map

One of the most useful tools available to anyone evaluating the rotor dynamics of a rotor bearing system is the undamped critical speed map. This device allows us to completely quantify the possible critical speeds of our machine. It does not necessarily reveal anything about the true critical speed frequencies and it does not predict the severity

of the resonances. It does give a good guide that can be used to understand how various changes in the bearings or shafting will affect the critical speeds.

Continuing on with the original example of a simple shaft, let's vary the support stiffnesses from 1,000 LB/IN to 1,000,000 LB/IN and calculate the first three critical speeds over this entire range. Figure 7 shows these calculations on a log-log plot. The first critical speed rises from 400 RPM when the bearing stiffness is 1,000 LB/IN to 1,200 RPM at 10,000 LB/IN to 2,800 RPM at 100,000 RPM and finally 3,600 RPM at 1 million LB/IN. At this point this curve becomes asymptotic and increasing the stiffness of the bearings will not significantly increase the first critical speed frequency. One

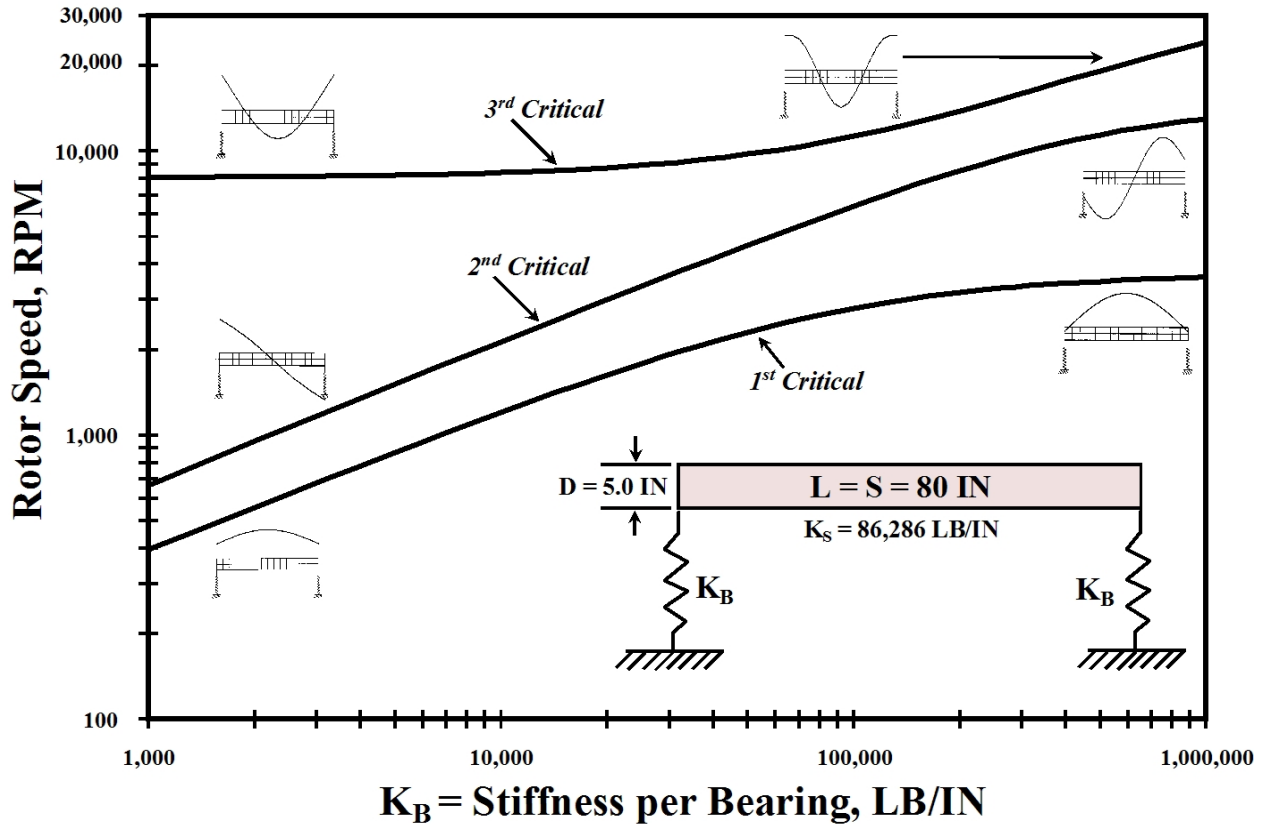


Figure 7 - Basic Undamped Critical Speed Map - The Effects of Bearings on Critical Speeds

can think of the regions on the map where changes in bearing stiffness affect the critical speeds as bearing dependent and those regions where the bearings have little effect as shaft geometry dependent.

The second critical speed behaves similarly while the third critical speed shows no significant variation for bearing stiffnesses up to 30,000 LB/IN. Then, it also increases in frequency. The corresponding mode shapes for each resonance are indicated for low and high support stiffnesses.

Now, having seen the effects of support stiffness on the first three critical speeds, let's look at how other parameters such as bearing span and shaft diameter affect the critical speeds.

The Effect of Bearing Span

Logically, one thinks that a long span between bearings supports will make the

system more flexible and thus lower the critical speeds. Indeed, since the shaft stiffness equation has the length term cubed and the first critical speed equation takes the square root of this, then the first critical speed should vary by the 3/2 power with bearing span. However, support stiffness must be taken into account as well. How then does bearing span affect the second and third critical speeds? The answer is not obvious. Let's look at what happens if we decrease the bearing span of our example rotor by 25% from 80 inches to 60 inches. The new shaft stiffness is:

$$K_s = \frac{48 (30,000,000) \left(\frac{\pi 5^4}{64} \right)}{60^3}$$

$$= 204,531 \text{ LB/IN}$$

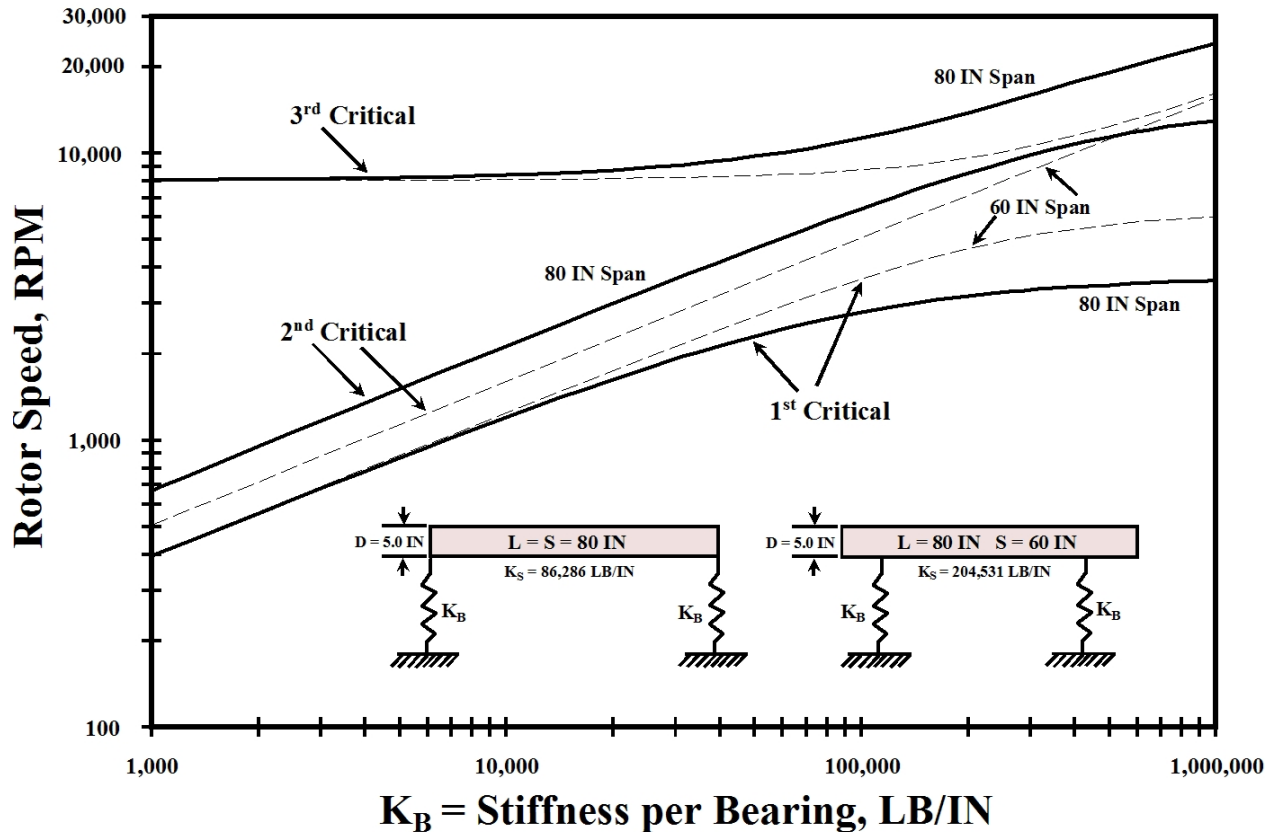


Figure 8 - Bearing Span Effect on Critical Speeds

Compared to the 80 inch span case stiffness of 86,286 LB/IN this is a remarkable increase of 137% in shaft stiffness. Figure 8 is an undamped critical speed map showing how this span change affects the first three critical speeds. The solid lines are the 80 inch span case and are identical to figure 7. The dashed lines are the critical speeds for the 60 inch span case. Looking at the first critical speed one can see that there is an overall increase in the first critical speed regardless of support stiffness, but since the mode is purely translational at very low support stiffnesses, the bearings exert less influence than at higher support stiffnesses when bending is introduced. The shorter span induces more bending at high support stiffnesses and thus drives up the first critical speed. Above 1,000,000 LB/IN the increase is a constant 70 percent.

The second critical speed behaves

somewhat differently. At low stiffnesses, the shortened bearing span causes a decrease in the second critical speed; the two curves cross at about 500,000 LB/IN support stiffness. Then, for higher stiffnesses, the second critical speed is higher for the shortened span case. At low stiffnesses this mode is pivotal with maximum amplitudes at the rotor ends and a node point in the center. Moving the bearings inward moves them away from the area of maximum amplitude and thus decreases their effectiveness. It is very important to note that the closer to a pivot point (node) the less effective a bearing will be. This is the reason for the decrease in the second critical speed for the shortened span case at low support stiffnesses. As with the first mode, as the bearings "clamp down" on the shaft, increased bending is induced and the shortened span increases the resistance to bending, driving up

the second critical speed.

The third critical speed experiences a similar phenomena. At very low support stiffnesses the span will have no effect whatsoever, but as the support stiffness begins to affect the third mode, the bearings are now closer to the 2 node points and thus have a reduced effect, lowering the third critical speed's frequency.

Thus, changes in bearing span are dependent upon the system's support stiffness. The first critical speed can be effectively raised by a decrease in span and this method is often used to raise the first critical speed out of an operating range. It will also raise the second critical if there is sufficient support stiffness.

critical speeds is shaft diameter. Designers of centrifugal compressors like small shaft diameters so that they can make the impeller eyes small, bearings small, seals small, keep the frame size down and the overall design compact and inexpensive. Since shaft diameter affects the moment of inertia, I , then the first critical should vary by the square root of I . Let's take our example rotor, hold the span constant at 80 inches and vary the shaft diameter by first decreasing it 25% to 3.75 inches and then increasing it 25% to 6.25 inches. Total rotor weight will vary as the diameter changes (total weight is a function of shaft radius squared) and you will see that the weight changes have an effect as well as the

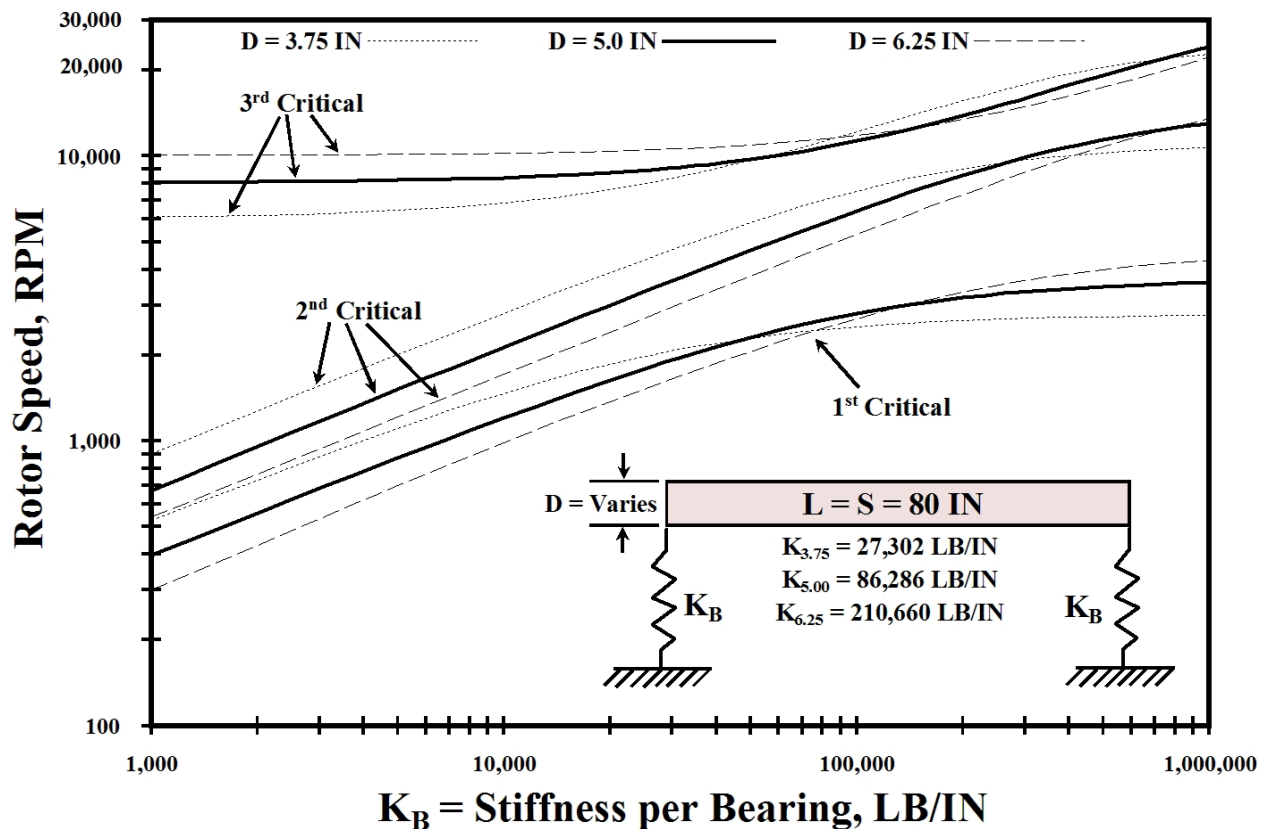


Figure 9 - Shaft Diameter Effect on Critical Speeds

The Effect of Shaft Diameter

The other major factor which controls

shaft diameter changes. Again we must look at the effect as a function of support stiffness. Figure 9 is the undamped critical speed map

which compares shaft diameters as a function of support stiffness. The solid line is the baseline case of 5.0 inch diameter shaft. The dotted line is the smaller 3.75 inch shaft and the dashed line is the larger 6.25 inch case.

The first critical speed lines show that at low support stiffness the smaller diameter shaft has a higher first critical speed and the larger shaft a lower first critical speed. This is because the dominant factor with very low support stiffness is the rotor weight. Since the shaft stiffness is not effective when no bending is taking place, the lighter rotor will raise the first critical speed and the heavier rotor will decrease it. As support stiffness rises, the logical change occurs and the smaller shaft has a much lower first critical speed. Since the smaller diameter shaft is more flexible, the bearing stiffness becomes dominant faster. Notice how this curve "flattens out" much faster and approaches an asymptotic value. This more flexible shaft has a smaller bearing dependent region on the critical speed map. The stiffer shaft increases the first critical speed at moderate and high support stiffnesses and also does not become asymptotic as fast. The stiffer shaft has a larger bearing dependent region on the map. Note that, in this case, when the bearing stiffness is about 80,000 LB/IN the shaft diameter has little effect on the first critical speed.

The second critical speed curves show a response similar to the first critical and for the same reasons. The mass influence is even greater and the support stiffness must be greater than about 750,000 LB/IN before the larger shaft's second critical increases over the baseline case.

The third critical is a bit different in that at low support stiffnesses, the system is inherently a bending mode and the stiffer shaft will be more resistant with the larger diameter and have a higher third critical speed. Likewise, the smaller diameter shaft will be less resistant to bending.

Shaft Diameter Effects on a Steam Turbine

The simple shaft example only illustrates basic ideas. In reality, most rotors have a significant amount of weight added as impellers or other disks. Let's consider an actual steam turbine and vary the shaft diameter. Figure 10 shows the original rotor.

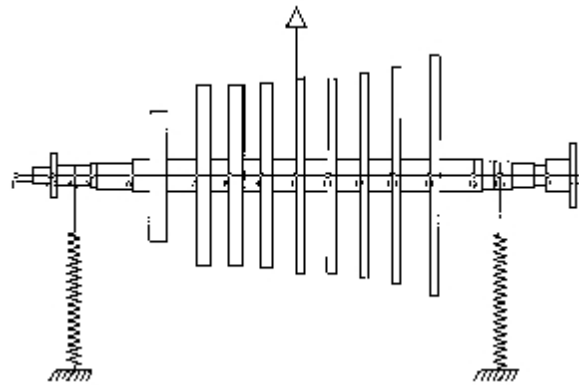


Figure 10 - Built-Up Turbine Rotor

This machine operated at 3,600 RPM driving a generator. The bearing span was 66.5 inches with a major shaft diameter of 6 inches. It was mounted on very stiff bearings and often rubbed while passing through the critical speed. If we take the bearing span and divide by the midspan diameter, the ratio is 11.1 to 1. This is an easy calculation to do and we know from experience that if this value exceeds 10 to 1 we probably have a problem machine. One way to tame this situation is to use a very soft bearing, but that often is not possible.

Instead, what about making the shaft bigger? Since the disks on this rotor make up a substantial percentage of the total rotor weight, this should mask the effect of the added shaft weight as we increase the diameter.

Figure 11 shows the effects of increasing the midspan diameter to 9 inches for a span/diameter value of 7.4 and to 12 inches for a span/diameter of 5.5. Note that the rotor weight increases by only 20 percent for the largest shaft. The bearing stiffness and damping values were held constant as was the

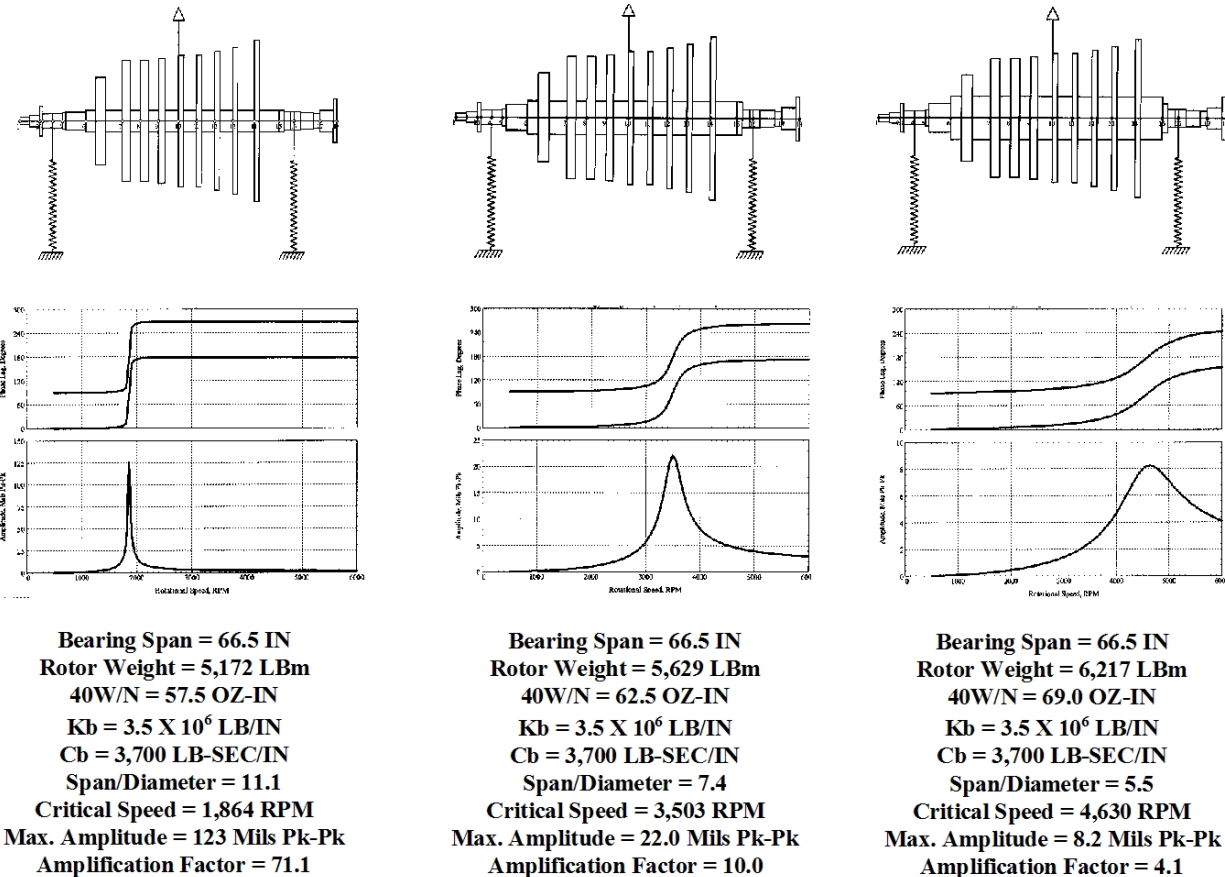


Figure 11 - Effects of Midspan Shaft Diameter on First Critical Speed Frequency, Amplitude, and Amplification Factor

imbalance, placed at the rotor center. This was adjusted to 40W/N for each case.

The amplification factor is defined by API as the “sharpness” of the resonance as shown in figure 12:

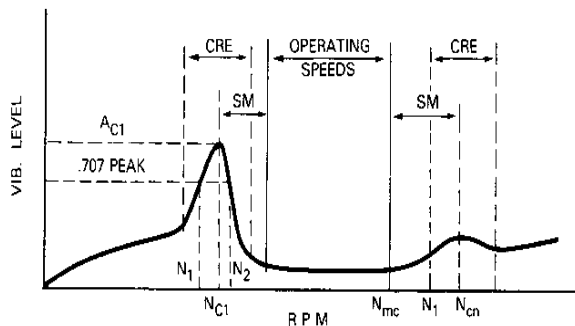


Figure 12 - API 617 Amplification Factor

$$AF = \frac{N_C}{N_2 - N_1}$$

The higher the amplification factor, the more violent the resonance becomes. In this case the original amplification factor was calculated to be over 70! Increasing the shaft diameter to 9 inches brings this down to a more reasonable amplification factor of 10 but this places the critical speed too close to the operating speed. By increasing the midspan diameter to 12 inches the amplification factor drops to a low 4.1 and moves the critical speed well above operating speed. Which turbine would you rather have?

The Effect of Mass Distribution

Since masses added to the shaft appear to have a significant effect, let’s look at some limiting cases. Taking our original 80 inch long rotor and attach 500 pounds in three ways. First of all would be the uniform distribution case, such as a multi-wheel compressor with

the stages equally spaced. Then there is the case of a single mass lumped in the center of the span such as a single wheel turbine. Finally consider the case where you have two-wheels with half the total mass lumped at the quarter-span points. Figure 13 shows the critical speed map for this as we take our example rotor and distribute 500 pounds as outlined above. The solid line is the uniform distribution case, the dotted line is the center-lumped case and the dashed line is the quarter-span lumped case. In all cases the total rotor mass is identical as is

center and thus the concentrated mass more easily affects the first critical. As the mass is distributed outward, the mass near the bearings contributes less to the modal mass and thus the first critical speed is raised.

The mass distribution has an opposite effect on the second critical speed. Regardless of stiffness, the center lumped case has the highest second critical speed of the three cases. This is because of the node point in the center of the span and this lumped mass cannot exert much influence at all. At higher support

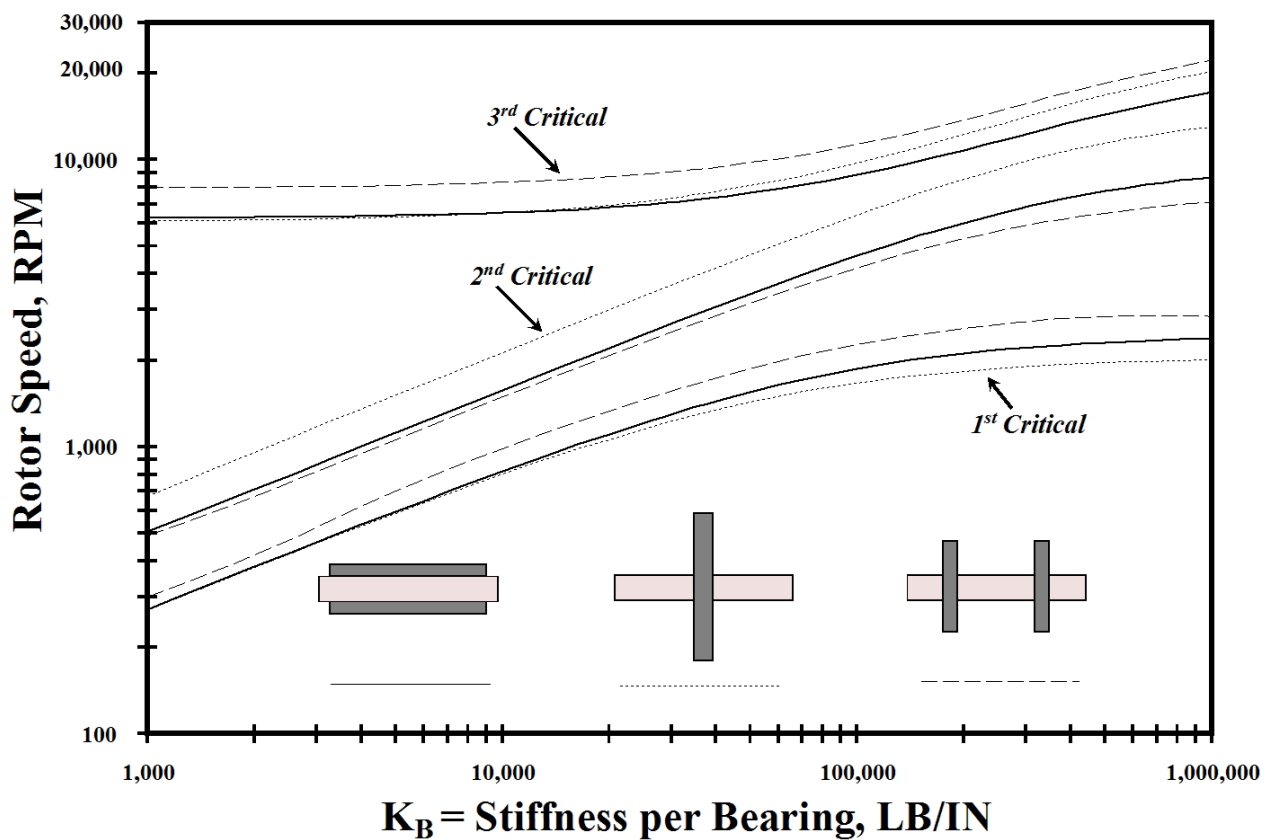


Figure 13 - Effect of Mass Distribution on Critical Speeds

the shaft diameter of 5 inches.

The first critical speed responds by having the highest critical speed for the quarter span lumped distribution and the lowest for the center-lumped case. This is logical when you think of the mode shape of the first critical speed. The maximum amplitude is at the rotor

stiffnesses the quarter span masses are nearest the high amplitude points and are effective in lowering the second critical speed.

The third critical speed is again different. The uniform case has the lowest third critical speed and the quarter span case has the highest. The mode shapes provide the

explanation. The quarter-span case has the masses close to the node points and thus are ineffective, causing an increase in the third critical speed. The center span case puts the mass at a point of maximum amplitude and is effective in lowering the third critical speed, but the uniform case puts mass at all three maximum amplitude points and is most effective in lowering the third critical speed.

To summarize, mass is effective in lowering critical speeds when it is near a maximum amplitude point. The modal mass for that mode is then increased causing a decrease in the resonant frequency of that mode. To drive up a critical speed, put the concentrated mass at a node point where it will not increase the effective modal mass.

Unbalance Response Analysis

Since imbalance is the most common excitation factor, one of the important things we are interested in when evaluating a rotor-bearing system is how that system will respond to this external forcing function. Many rotors suffer from the problem of buildup from a process residue or erosion and wear. Thus, the balance when installed may not maintain itself. The question is, what level of imbalance will result in unacceptable levels of vibration? It is necessary to introduce damping into the equations of motion to accurately model the unbalance response since damping is the only mechanism able to absorb energy at resonance. It is necessary to use a computer program to calculate accurate amplitudes on a rotor bearing system. Unbalance response calculation programs are very similar to critical speed programs with the addition of a known forcing function and realistic bearing stiffness and damping values for the supports. Once the physical model is constructed, the user specifies the location and the magnitude of the imbalance and the program calculates the steady-state response at fixed speed increments throughout a specified speed range. The results can be formatted as a Bodé or a polar plot.

Bodé plots are shown in figure 11 and an example of a polar plot is figure 14. Another type of output available from an unbalance response calculation is the operating speed deflection shape as shown in figure 15. This is very useful when setting vibration limits since it tells you the amplitude at any location compared to the vibration probe location.

The first question is how much imbalance to input into the program and where to place it on the rotor. The generally accepted balance tolerance in wide use today is $4W/N$ or 4 times the rotor weight in pounds divided by the maximum continuous speed in RPM. The units of imbalance are in ounce-inches or OZ-IN. If this level of imbalance is used, the predicted amplitudes will usually be unrealistically low. Since the calculations are linear, theoretically any amount could be used and the results scaled. I prefer to use $40W/N$ as a worst-case scenario.

To begin an analysis, mode shapes for a rotor are produced using the actual predicted bearing stiffnesses. See figure 6 for examples. Then the entire imbalance is placed at the maximum amplitude point indicated by the first undamped mode shape to excite the first critical speed. For the second critical speed, the $40W/N$ imbalance is split in two and half is placed at each end of the rotor 180° out-of-phase with each other and at the maximum amplitude points indicated by the second mode shape. Higher mode excitation will follow a similar procedure.

In summary, unbalance response calculations are not as straightforward as undamped critical speed calculations. You must already know something about the mode shapes before you can place the imbalances. Once the results of an unbalance response are obtained, the user can calculate an amplification factor as outlined earlier and begin to get a feel for the rotor's sensitivity.

Often, based on experience with a given rotor configuration, an analyst will place a complex distribution of imbalances on a rotor

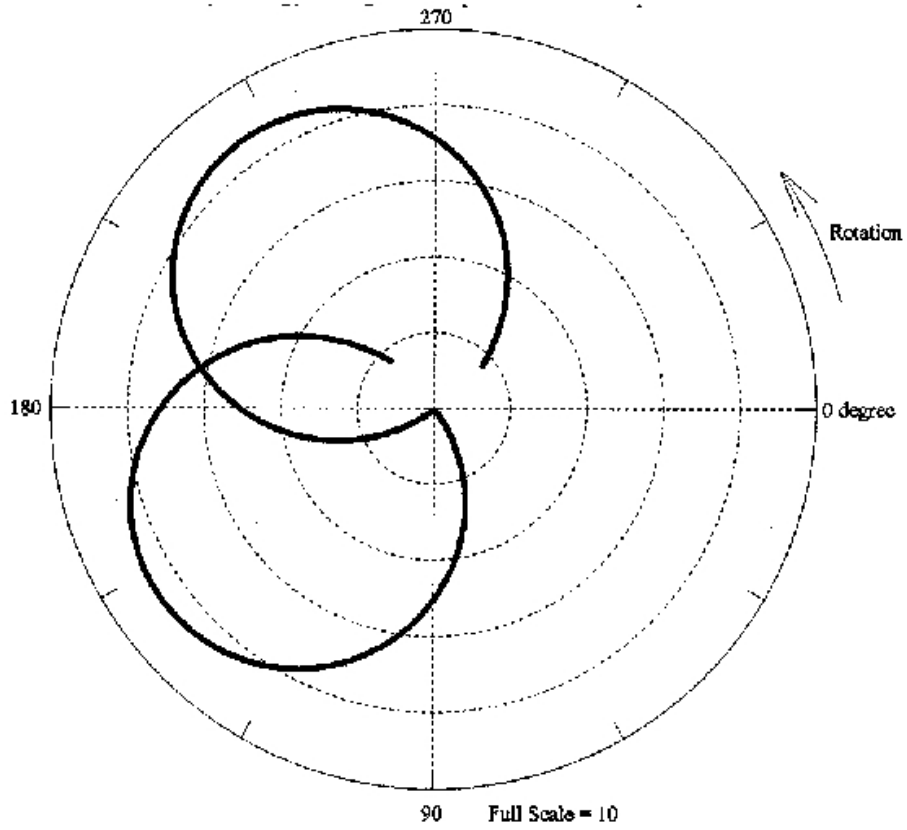


Figure 14 - Typical Polar Plot of Unbalance Response Calculations

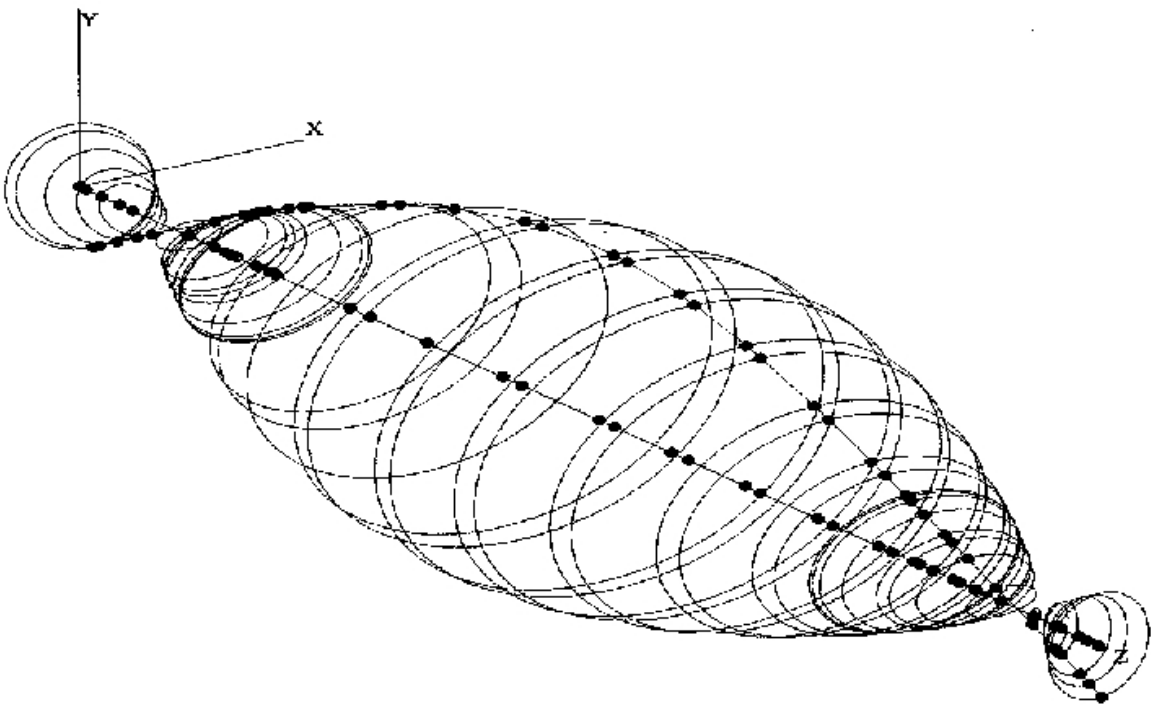


Figure 15 - Three Dimensional View of Compressor Operating Deflection Shape

(for example at wheel locations, overhangs and couplings, etc.) in order to excite all pertinent modes simultaneously.

Stability Analysis

Stability analysis is the most difficult aspect of rotor dynamics. It is not nearly as well understood as the previously covered subjects and the computer programs that are available require that the user estimate the amount of destabilizing influences in the system. The destabilizing forces are commonly designated *cross-coupling* forces. These cross-coupling forces cause an orthogonal forward displacement of the rotor for each normal rotor displacement. If the force is sufficient in magnitude to overcome the damping in the system, an instability occurs. Stability is a function of rotor geometry, bearing-to-shaft stiffness ratio, bearings, seals, and fluid dynamics.

The best measure of rotor stability is the logarithmic decrement. Often called the log dec for short, this factor is related to amplification factor and is defined as the natural log of the ratio of two successive resonant amplitudes. When the log dec is positive, the systems vibrations die out with time and the system is stable. However, if the log dec is less than zero, the system's vibrations grow with time and the system is unstable. The relationship of the log dec, abbreviated "δ" to amplification factor is this:

$$AF = \frac{\Pi}{\delta}$$

The log dec can be experimentally determined by momentarily exciting a running rotor at one of its natural frequencies either by a forcing function or an impulse. By recording the resultant "ring-down" the log dec can be calculated. Compressor surge is an excellent exciter (and wrecker) of machinery, but if well controlled this may be used to impulse the rotor

and excite the first critical while running at a higher speed. This is shown in figure 16 which is the time trace of an axial compressor's displacement probe signal during a surge test. A log dec of 0.35 can be calculated from this trace.

System instability is almost always manifested by the presence of forward subsynchronous circular whirling of the shaft at a frequency equal to the first critical speed. In most cases the driving mechanism is the fluid film in the journal bearings. Two distinct phenomena can occur that are related but different:

Oil Whirl: This is a circular forward subsynchronous vibration that nominally occurs near 50% of synchronous rotor speed and tracks with rotor speed. To visualize this, consider the oil wedge in the bearing. The oil at the surface of the bearing is at zero speed. The oil at the journal surface is at the shaft surface speed and the average oil speed is then 50% of running speed. This is not entirely true however, due to surface roughness differences, the shaft is usually smoother than the bearing and this is why oil whirl often occurs at 42% to 48% of rotor speed. If the journal area is very rough, say due to a rusty shaft, oil whirl can be observed at frequencies greater than 50% of synchronous speed. Oil whirl can be induced by bushings and seals as well as bearings. Many bearing designs are "anti-whirl" bearings such as tilting pad bearings.

Oil Whip: This phenomena is similar to oil whirl, but occurs at a constant frequency. This frequency is almost always the first critical speed. The shaft must also be turning at a speed greater than twice the first critical speed. Many systems will run very well until exactly twice the first critical speed is reached and the rotor will go into violent oil whip. Oil whip can also be induced by seals and bushings and it can be controlled in bearings by using a more stable design Whirl and whip can be very violent and can destroy a machine in minutes

so it is very important to prevent it at the design stage.

One of the first things the design auditor should check is the bearing to shaft stiffness ratio, $2K_b/K_s$. As this ratio exceeds a value of 10, the rotor is starting to become a "noodle" and will be susceptible to instabilities. The use of plain journal bearings in a rotor which exceeds twice its first critical speed should be a warning sign. Floating oil bushings and high pressure labyrinth seals like balance piston drums should also be looked at as potential cross-coupling sources.

The bearing analysis should look carefully at the predicted operating speed eccentricity ratio. If this ratio becomes less than 0.2 the likelihood of bearing instabilities exist. The reason for this is that a centered bearing has equal clearance all around and is easily perturbed from this position. Many bearings counteract this centering phenomena by inducing an external load on the shaft by preloading the bearing, or in the case of the pressure dam bearing, the oil itself creates a pressure wedge in the upper half of the bearing that pushes down on the journal, increasing the eccentricity, and the stability. Often you will see a groove cut in the bottom half of a bearing. This groove reduces the bearing's load carrying capacity and at the same time increases the eccentricity and the stability. Sometimes a quick fix for an unstable machine is to mill out a pressure dam and/or load groove in the plain journal bearing. This needs to be done with extreme care as the critical speeds and unbalance response will also be affected by such a change.

There are other destabilizing forces called aerodynamic cross-coupling. This is a fluid induced force around the periphery of centrifugal wheels or other rotating components.

There are semi-empirical equations for estimating the amount of cross coupling. One of these was developed by Mr. J. C. Wachel

now retired.

For compressible flow:

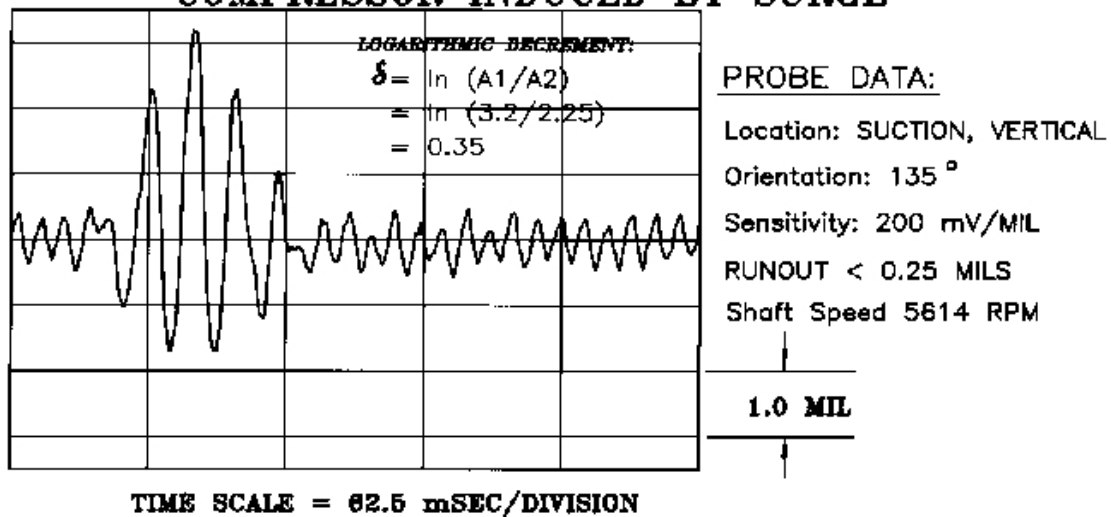
$$K_{xy} = \frac{[6300 \text{ HP (mol wt)} \rho_D]}{[N D h \rho_S]}$$

HP = stage horsepower
mol wt = fluid molecular weight
 ρ_D = discharge density
 ρ_S = suction density
N = speed in RPM
D = impeller pitch diameter
h = restrictive dimension in flow path

This equation should be used carefully, particularly in determination of the factor "h".

Figure 17 shows the results of a typical stability study on a compressor. The log dec is calculated for several speeds and plotted. The cross-over point in this case is 7,240 RPM which means this is the speed where the machine begins to show signs of subsynchronous vibration.

RESONANT EXCITATION OF AXIAL AIR COMPRESSOR INDUCED BY SURGE



THIS DATA WAS CAPTURED DURING A SURGE TEST
 WITH VARIABLE INLET GUIDE VANES 100% OPEN.
 FLOW OF 75,000 SCFM @ 30 PSIG DISCHARGE.

Figure 16 - Determining the Log Dec from Compressor Vibration Trace

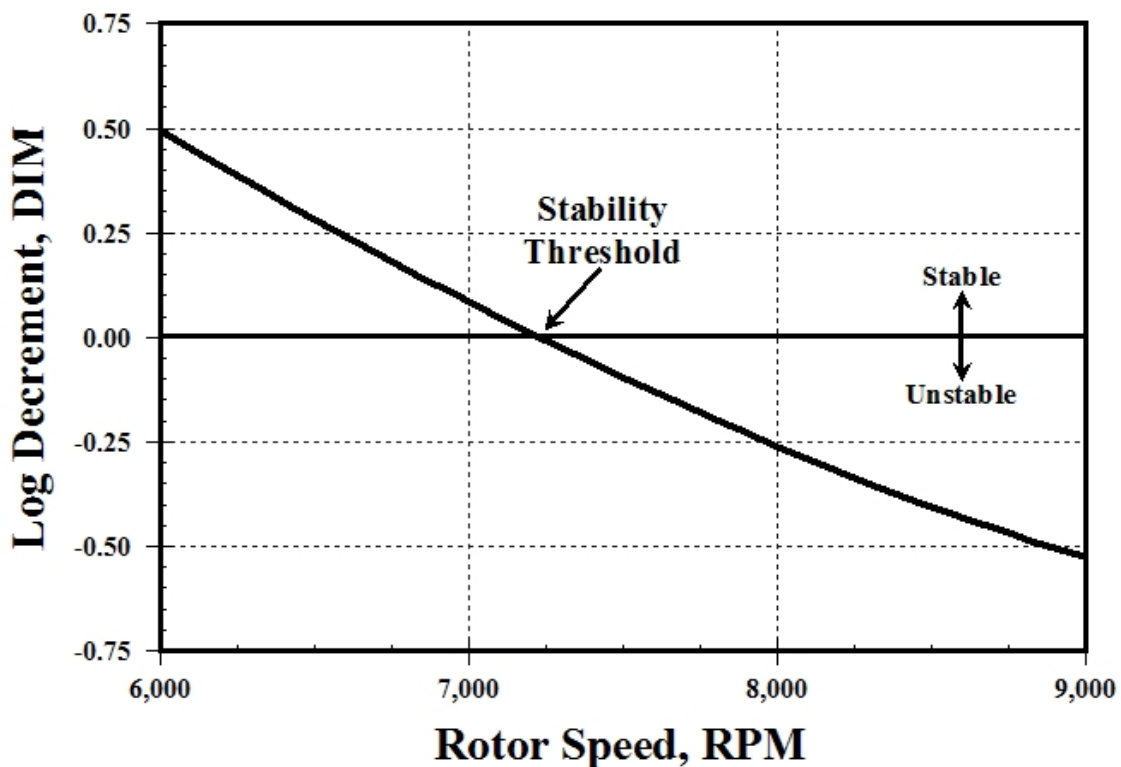


Figure 17 - Stability as a function of Speed for a Propylene Compressor

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Some of the computer programs used by the author have been developed by the ROMAC Laboratory at the University of Virginia. The primary tool used was the DyRoBeS suite of analysis programs developed by Dr. Wen Jeng Chen of Eigen Technologies, Inc. and distributed by RODYN Vibration Analysis, Inc.

<u>Nomenclature</u>	
C	- damping, Lbf-SEC/IN
C_c	- critical damping, Lbf-SEC/IN
E	- Young's modulus, PSI
g	- Gravitational Constant, 386.088 LBm-IN/LBf-Sec ²
I	- area moment of inertia, IN ⁴
K,k	- spring stiffness, LBf/IN
K_B	- bearing stiffness, LBf/IN
K_s	- shaft stiffness, LBf/IN
K_{yx}	- Cross-Coupling, LB/IN
M,m	- Mass, LBf (mass = weight/g)
N	- rotational speed, RPM
N_{cx}	- critical speed number x, RPM
π	- 3.14159...
S	- Sommerfeld number

ω	- frequency, RAD/SEC
ω_d	- damped natural frequency, RAD/SEC
ω_n	- undamped natural frequency, RAD/SEC
x	- displacement, inches
x'	- velocity, inches/second
x''	- acceleration, inches/second ²
δ	- logarithmic decrement, dimensionless
ζ	- damping ratio - C/C _c , dimensionless

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