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Final Research Report for
Ministry of Fisheries Research Project SAM9701
Objectives 3

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Report Title: MIAEL estimators of virgin biomass for stocks where data are very limited

Authors: P.L. Cordue and S.L. Ballara

1. **Date:** 23 September, 1998
2. **Contractor:** NIWA
3. **Project Title:** Modelling of fish populations
4. **Project Code:** SAM9701
5. **Project Leader:** Chris Francis
6. **Duration of Project:**

   Start date: 1 October 1997
   Expected completion date: 30 September 1998

7. **Objective:**

This report covers work arising from Objective 3 of Project SAM9701:

> To develop optimal estimators of virgin biomass and MSY stocks where data are very limited, by inclusion of auxiliary information such as catch at age data.

8. Executive Summary

A simulation study was done to examine the performance of four alternative Minimum Integrated Average Expected Loss (MIAEL) estimators of virgin biomass for stock assessment scenarios where data are very limited. Two alternative data sets were considered: a reasonable CPUE time series with a one-off catch-at-age estimate; and a short trawl time series where numbers-at-age or biomass were available. In all, 32 stock assessment scenarios were considered, with the data sets being used with or without the age data, for two alternative theoretical stocks (with different biological parameters), each with two alternative bounds on virgin biomass, and a constant or increasing catch history.

The four estimators considered (for each of the 32 scenarios) were MIAEL estimators based on either a maximum likelihood or least squares estimator, each of which used the age data in two different ways. The first variation had the age data fitted directly within the model as catch-at-age or numbers-at-age. The second variation used a penalty function to fit estimates of the total mortality obtained (externally) from the age data. For selected stock assessment scenarios, the estimators which used the age

data internally were tested for robustness to errors in the assumed value of natural mortality and the assumption of average year class strengths.

The performance of the estimators varied enormously over the scenarios considered. In general, when age-data were used, either internally or externally to the model, the estimators performed far better than if the age data were not used. When catch-at-age data were used internally the maximum likelihood estimator was always better than the least squares estimator. However, when the (trawl) numbers-at-age data were used the maximum likelihood estimator generally performed worse than the least squares estimator. Internal use of the age data was normally better than external use, but not always.

In some circumstances, both estimators were very sensitive to an error in natural mortality; they were less sensitive to variation in year class strength, but still very sensitive for one particular scenario.

There are two conclusions from the study.

For any given stock, prior to the stock assessment, a variety of potential estimators should be tested, both in terms of their performance when the assumptions of the estimation model hold, and for their robustness to errors in those assumptions. The "best" estimator to use in the stock assessment will depend on the particular stock assessment scenario (biological parameters, catch history, scale of the bounds used, available data).

Also, when age data are used, the assumptions of known natural mortality and average year class strengths can lead to a totally unrealistic assessment of estimator performance (i.e., too optimistic). The current estimators need to be modified in some way to cope with the presence of unknown year class strengths and/or natural mortality.

9. Introduction

There are many New Zealand species for which stock assessment is problematic because of the sparseness of the available data. The difficulty of stock assessment for these species is made worse in that many stock assessment methods cannot use data of multiple types. For example, analysis of catch curves uses only age frequency data, and surplus production modelling uses abundance indices but ignores any age frequency data that may be available. In general, it is advantageous to use as much data as possible in a stock assessment; the difficulty is in using the data appropriately, especially if data of multiple types are combined.

The work described in this report (objective 3 of SAM9701) looks at several variations of an existing estimation technique which uses multiple data types. The variations of the technique are applied to stock assessment situations where data are very limited (e.g., several points in a relative abundance time series with a single catch-at-age estimate). Four Minimum Integrated Average Expected Loss (MIAEL) estimators of virgin biomass are compared for a variety of stock assessment scenarios. The MIAEL estimation technique was developed for use in hoki stock assessments (Cordue 1993) but has subsequently been applied to stock assessments for a variety of middle depths species (*see* Annala *et al.* 1998). Prior to this study, only one of the four estimators had been extensively used.

10. Methods

Objective 3 is "to develop optimal estimators of virgin biomass and MSY for stocks where data are very limited, by inclusion of auxiliary information such as catch-at-age data". To achieve this objective the estimation performance of several stock reduction MIAEL estimators of virgin biomass and MCY for a variety of generic stock models and data sets were examined. Three key activities were involved plus the production of this report.

1. <u>Create generic stock models and data sets</u>

A variety of theoretical stock assessment scenarios was created by combining a number of generic stock models and data sets. In each case the stock model was a deterministic single-stock two-sex age-structured model with a Beverton-Holt stock-recruit relationship (Cordue 1998a, and Appendix 1). Stock assessment scenarios were created as follows.

Biological parameters and available abundance data (Appendices 2 and 3) for a range of middle depths and inshore species were examined, and the final choices of parameters, catch histories, etc, were made after considering the actual stock assessment scenarios. The aim was to choose parameters, catch histories, etc, which are fairly typical of these stocks.

Two "generic stocks" with differences in various parameters, driven by different maximum ages of 15 years, or 30 years, were chosen (Table 1). Two alternative catch histories for 1975–1997 were chosen, with one case a constant catch for each year, and the other an increase in catches in recent years (Table 2), both common for a number of middle depths and inshore species.

Two alternative bounds on virgin biomass were generated for each particular stock model and catch history from assumptions about minimum and maximum exploitation levels (Cordue 1996) (Table 3). This was to give contrasting scales of uncertainty in the a priori knowledge about virgin biomass.

Two alternative data sets were chosen as follows: data set 1 had five recent relative abundance indices (log normal, with known c.v. of 40%), with a single recent catch-at-age estimate (multinomial, with known sample size). This mimicked a reasonable CPUE time series with a one-off catch-at-age estimate (Table 4). For data set 2, three recent relative abundance indices (log normal, with known c.v. of 30%), and corresponding estimates of number-at-age (multinomial, with known sample size) were used, which mimicked a reasonable trawl survey where numbers-at-age have been estimated.

2. From four alternative MIAEL estimators of virgin biomass determine which is best for each stock assessment setting

For each stock assessment setting (Table 5), four forms of stock reduction MIAEL estimation (Cordue 1993, 1995, 1998b, and Appendix 4) were used to determine MIAEL estimators of virgin biomass and their information indices. The information index of each estimator is a measure of its performance, and the indices are strictly comparable between estimators for a given stock assessment setting. The best estimator for each setting is that with the highest information index. Note, the information indices for the MIAEL estimators of MCY are equal to those of the MIAEL estimators of virgin biomass (as MCY is a given proportion of virgin biomass).

Each MIAEL estimator was a *best p* estimator with a proportional squared error loss function (Cordue 1995). The four estimators resulted from two alternative fitting procedures (maximum likelihood and least squares) and two alternative methods of using the age data (as estimates of Z, obtained outside the model, or as numbers or proportions at-age fitted with the abundance indices) (Appendix 5). Baseline information indices were calculated for each setting and each fitting procedure when the age data were not used in the estimation procedure. The increase in the information indices when the age data are used gives an indication of the worth of the age data. In data set 1, both CPUE, and catch-at-age data were used for the ageing run, and just CPUE data used in the non-ageing run. For data set 2, trawl survey numbers-at-age were used for the ageing run, and trawl survey biomass indices used in the non-ageing run (*see* Table 5). In all, 128 information indices were calculated (4 MIAEL estimators, and 32 stock assessment scenarios).

3. Investigate the robustness of the best MIAEL estimator(s) to departures from the model assumptions

Some of the MIAEL estimators considered in Activity 2 were tested for robustness to the assumptions of average year class strength and known natural mortality. The MIAEL estimators were tested by using a number of operating models (of reality) which had non-average year class strengths (Table 6), and different values of natural mortality (M±0.05) to that assumed in the estimation model. The testing was done for 8 different operating models and 4 different stock assessment scenarios, and using both maximum likelihood or least squares estimators using age data internally. In each case the true information index of the estimator of virgin biomass was calculated.

Stratified random sampling was used to obtain an approximate information index for each MIAEL estimator of B_0 (*see* Appendix 4 for the basic formulation of an information index). The bounds on B_0 in the operating model were split into eight equally sized intervals. Random values of B_0 were generated within the bounds using a uniform distribution, until there were exactly three values within each of the intervals (the same seed was used for all models in the generation of B_0s, so that models with the same bounds used the same B_0s). At each of the twenty four values of B_0 five hundred simulated point estimates were generated (using the given estimation model and the point estimator associated with the given MIAEL estimator. The simulated data were created using different seeds for each B_0, but the same seeds were used across all models.) In the cases where the operating and estimation models were the same, the approximate information index of the *best p* estimator was calculated directly from the simulated estimates. The *best p* estimator was determined by searching for the value of p which minimised, within the *best p* class of candidate estimators, the average proportional mean squared error (i.e., averaged over the twenty four points; this approximates the Integrated Average Expected Loss for a proportional squared error loss function—*see* Cordue 1995).

When the estimation model differed from the operating model, calculation of the information index required two steps. First, a value of p was noted: that corresponding to the *best p* estimator obtained when an operating model identical to the estimation model was assumed. The information index was then calculated (for the given value of p) from simulated data constructed using the correct operating model.

11. Results

The 128 information indices are presented in Table 7, and graphed in Figures 1, and 2. Note, when the data sets contain only a single biomass time series, then all four estimators are identical (maximum likelihood and least squares are the same because lognormal errors were used).

<u>Stock 1 (maximum age 15 years):</u>

Data set 1a (CPUE and catch at age - scenarios 1, 5, 9, and 13) showed similar trends for all 4 scenarios. The maximum likelihood fitting procedure using age data internally had the highest information index for all 4 scenarios, and the least squares fitting procedure using age internally had the lowest information index. Both the maximum likelihood and least squares fitting procedures using age data externally fell somewhere between the other two, and were at a similar level. Information indices ranged from about 20 to 90% for all 4 scenarios. Constant catch histories had higher information indices than increasing catch histories for all 4 scenarios. Information indices were higher for the smaller range in B_{min} and B_{max}, for each of constant catch, or increasing catch history scenarios.

For data set 1b (CPUE indices only - scenarios 2 , 6, 10, and 14), information indices ranged from about 7 to 20% for different scenarios, and were much lower than information indices in data set 1a. Increasing catch history scenarios had higher information indices than constant catch history scenarios. Information indices were higher for the smaller range in B_{min} and B_{max} for each of constant catch, or increasing catch history scenarios.

For data set 2a (Trawl survey numbers at age - scenarios 3, 7, 11, 15) the maximum likelihood fitting procedure using age data internally had the lowest information index for 3 of the 4 scenarios. The least squares fitting procedure using age data internally the highest information index, for the constant catch history scenarios, but fell in the increasing catch history scenarios. Both the maximum likelihood and least squares fitting procedures using age data externally were similar, and were close to the least squares procedure using age data internally for constant catch histories, but the same or higher for the increasing catch histories. Information indices ranged from about 18 to 90%. Constant catch histories had higher information indices than increasing catch histories for all 4 scenarios, and a wider range in information indices. Information indices were higher for the smaller range in B_{min} and B_{max} for each of constant catch, or increasing catch history scenarios.

For data set 2b (trawl survey abundance indices - scenarios 4, 8, 12, 16) the information indices were close to 0%, and were much lower than information indices in data set 2a. Information indices were higher for the smaller range in B_{min} and B_{max} for each of constant catch, or increasing catch history scenarios.

Stock 2 maximum age 30 years:

In general, the trend in all 4 data sets was similar for both stocks, except for data set 2a (*see* Figure 1, and 2), and all corresponding scenarios for both stocks were at slightly different information index percentages.

Data set 1a (CPUE and catch at age - scenarios 17, 21, 25, and 29) showed a similar trend to stock1, except that for the constant catch scenarios, the least squares fitting procedure using age data internally was lower for the wider range in B_{min} and B_{max}.

Data set 2a (trawl survey numbers-at-age - scenarios 19, 23, 27, and 31): Scenarios 19, and 23 (constant catch scenarios) had similar information indices for all estimators, with information indices higher than in stock1. Scenarios 27 and 31 (increasing catch scenarios) had much lower information indices, with the least squares fitting procedure using age data internally performing better than the other estimators. For run 31, the trend in estimators fitted that of stock 1, for age data fitted internally, had the lower information index for the maximum likelihood fitting procedure, and higher information index for the least squares fitting procedure.

Data set 1b (CPUE indices only), and data set 2b (Trawl survey abundance indices only) had similar trends to stock 1, and had much lower information indices than stock 2 data sets 1a and 2a respectively.

Robustness study

The MIAEL estimators for scenarios 17, 19, 29, and 31 were tested by using a number of operating models which had different values of natural mortality ($M \pm 0.05$), or different (lower and higher) year class strengths, to that assumed in the estimation model. The results of the robustness study are presented in Table 8, and Figures 3 and 4.

Estimator performance was not robust to errors in natural mortality. In general, the true information index of the estimator of virgin biomass was smaller than the information index calculated assuming average year class strengths and known natural mortality. The true information index for a higher mortality was always worse than the assumed mortality (Figure 3), and for a lower mortality was usually worse, except in run 17 (both maximum likelihood and least squares fitting procedures), and run 29 for the least squares fitting procedure. In some cases the assumed information index was very different to the true information index (scenarios 29, and 31). Errors in year class strength also created problems, as the true information index for below and above average year class strengths were generally worse (Figure 4), except for run 31 (with the below average year class strength scenarios). Note, a negative information index indicates that the estimation performance is worse than that achieved by ignoring the data and just choosing the point in the range which minimises the integrated average expected loss (i.e., using the *best k* estimator, *see* Appendix 4).

12. Conclusions

There are two conclusions from the study.

For any given stock, prior to the stock assessment, a variety of potential estimators should be tested, both in terms of their performance when the assumptions of the estimation model hold, and for their robustness to errors in those assumptions. The "best" estimator to use in the stock assessment will depend on the particular stock assessment scenario (biological parameters, catch history, scale of the bounds used, and available data).

Also, when age data are used, the assumptions of known natural mortality and average year class strengths can lead to a totally unrealistic assessment of estimator performance (i.e., too optimistic). The current estimators need to be modified in some way to cope with the presence of unknown year class strengths and/or natural mortality.

13. Publications

Nil.

14. Data Storage

Not applicable.

15. References

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Table 1: Parameters used in each "stock"

(a) Biological parameters

	Stock1		Stock2	
	Male	Female	Male	Female
Maximum age (years)	15	15	30	30
von Bertalanffy growth parameters				
L_∞	81.1	86.1	119.0	160.1
K	0.308	0.308	0.108	0.076
t_0	-0.627	-0.384	-1.24	-1.05
Length weight				
a	.00963	.00963	.00126	3.296
b	3.173	3.173	.00126	3.296
Natural mortality	0.3	0.3	0.18	0.18

Nb: For stock1, a "BNS2" stock was used, and for stock2, a "LIN3&4" stock

(b) Maturity ogives:

	Stock1		Stock2	
Age	male	female	male	female
4	0.01	0.01	0.01	0.01
5	0.05	0.05	0.10	0.10
6	0.25	0.25	0.30	0.30
7	0.50	0.60	0.70	0.80
8	0.75	0.80	1.00	1.00
9	1.00	1.00	-	-

Nb: for stock 1, maturity ogive data "made up", and for stock 2 maturity ogive taken from LIN 3&4

(c) Selectivities

Ground	Age	Stock1	Stock2
Spawning	3	1.0	-
	4	1.0	-
	5	1.0	1.0
	6	1.0	1.0
	7	1.0	1.0
	8	1.0	1.0
	9	1.0	1.0
	10	1.0	1.0
Home	1	0.01	0.01
	2	0.05	0.05
	3	0.1	0.1
	4	0.2	0.2
	5	0.4	0.4
	6	0.7	0.6
	7	0.9	0.8
	8	1.0	1.0
	9	1.0	1.0
	10	1.0	1.0

Nb: For each stock, the same for males and females

(d) Other parameters

Corridor ogive	1.00 at age 1
Proportion of male larvae	0.50
Steepness	0.75
Length of spawning season	0.3333 years
Proportion spawning	1.00
Proportion available to fleet (spawning season)	1.00

Table 2: Catches histories used in both "stocks"

	Constant catch		Increasing catch		
Year	prespawning	spawning	prespawning	spawning	total
1975	1995	3000	20	40	60
1976	1995	3000	20	50	70
1977	1995	3000	80	125	205
1978	1995	3000	200	600	800
1979	1995	3000	150	430	580
1980	1995	3000	300	450	750
1981	1995	3000	120	180	200
1982	1995	3000	137	210	247
1983	1995	3000	150	220	370
1984	1995	3000	150	490	640
1985	1995	3000	230	340	570
1986	1995	3000	310	470	780
1987	1995	3000	370	560	930
1988	1995	3000	540	806	1346
1989	1995	3000	1030	1554	2584
1990	1995	3000	1180	1770	2950
1991	1995	3000	1400	2119	3519
1992	1995	3000	1610	2413	4023
1993	1995	3000	1770	2655	4425
1994	1995	3000	1705	2550	4255
1995	1995	3000	2100	3160	5260
1996	1995	3000	2040	3055	5095
1997	1995	3000	2240	3355	5595

Table 3: Alternative bounds on virgin biomass for each stock and catch history constructed from different maximum and minimax exploitation rates

Bound		Catch	Maximum rate		Minimax rate			
Set	Stock	history	home	spawning	home	spawning	B_{min}	B_{max}
1	1	constant	0.5	0.7	0.01	0.05	24499	67767
2	1	constant	0.5	0.7	0.005	0.01	24499	127259
3	1	increasing	0.5	0.7	0.01	0.05	17810	72650
4	1	increasing	0.5	0.7	0.005	0.01	17810	139777
5	2	constant	0.6	0.8	0.01	0.05	58273	105924
6	2	constant	0.6	0.8	0.01	0.01	58273	189751
7	2	increasing	0.6	0.8	0.01	0.05	31791	92320
8	2	increasing	0.6	0.8	0.01	0.01	31791	191611

Table 4: Data sets used

Data set	Data	Description
1a, 1b *	CPUE	5 observations (1992-1996), *c.v.* of 40% each year spawning ground, mid-spawning season relative indices, mature biomass lognormal errors
1a *	Catch-at-age	1 observation (1995), multinomial, sample size of 100 spawning ground, end of spawning season ageing from 2 to 10 years (stock1), and 2 to 18 (stock2), no plus group ageing error of 15% (1 year either side) *c.v.* 40% (for least squares estimator)
2a	trawl survey numbers-at-age	3 observations (1993–1995), *c.v.* of 30% each year (and each age) home ground, mid pre-spawning season ageing from 2 to 10 years (stock1), and 2 to 18 (stock2), plus group ageing error of 15% (1 year either side) lognormal errors
2b	trawl survey abundance	3 observations (1993–1995) *c.v.* of 30% each year home ground, mid pre-spawning season relative indices, biomass lognormal errors

* Data set 1a includes both CPUE and catch-at-age data (with equal "source" weights; see Appendix 5), and data set 1b is CPUE data only.

Table 5: Stock assessment scenarios (Note: odd numbered scenarios use age data, and even numbered scenarios do not; add 16 to the run number for stock 1 to get the analogous run for stock 2; add 4 to the run number to get the analogous run for a different bound set)

Run no.	Stock[1]	Catch history[2]	Bound set[3]	Data set[4]
1	1	constant	1	1a CPUE and catch-at-age
2	1	constant	1	1b CPUE
3	1	constant	1	2a trawl survey numbers-at-age
4	1	constant	1	2b trawl survey abundance indices
5	1	constant	2	1a CPUE and catch-at-age
6	1	constant	2	1b CPUE
7	1	constant	2	2a trawl survey numbers-at-age
8	1	constant	2	2b trawl survey abundance indices
9	1	increasing	3	1a CPUE and catch-at-age
10	1	increasing	3	1b CPUE
11	1	increasing	3	2a trawl survey numbers-at-age
12	1	increasing	3	2b trawl survey abundance indices
13	1	increasing	4	1a CPUE and catch-at-age
14	1	increasing	4	1b CPUE
15	1	increasing	4	2a trawl survey numbers-at-age
16	1	increasing	4	2b trawl survey abundance indices
17	2	constant	5	1a CPUE and catch-at-age
18	2	constant	5	1b CPUE
19	2	constant	5	2a trawl survey numbers-at-age
20	2	constant	5	2b trawl survey abundance indices
21	2	constant	6	1a CPUE and catch-at-age
22	2	constant	6	1b CPUE
23	2	constant	6	2a trawl survey numbers-at-age
24	2	constant	6	2b trawl survey abundance indices
25	2	increasing	7	1a CPUE and catch-at-age
26	2	increasing	7	1b CPUE
27	2	increasing	7	2a trawl survey numbers-at-age
28	2	increasing	7	2b trawl survey abundance indices
29	2	increasing	8	1a CPUE and catch-at-age
30	2	increasing	8	1b CPUE
31	2	increasing	8	2a trawl survey numbers-at-age
32	2	increasing	8	2b trawl survey abundance indices

1, 2, 3, 4: for details see Tables 1, 2, 3, 4 respectively

Table 6: Year class strengths used in the robustness study: (a) below average, (b) above average

Year	YCS (a)	YCS (b)
1975	1	1
1976	1	1
1977	1	1
1978	0.5	1.2
1979	0.5	1.2
1980	1.2	0.8
1981	1.2	0.8
1982	0.8	1.8
1983	0.8	1.8
1984	0.5	1.2
1985	0.5	1.2
1986	1.2	0.8
1987	1.2	0.8
1988	0.8	1.8
1989	0.8	1.8
1990	0.5	1.2
1991	0.5	1.2
1992	1.2	0.8
1993	1.2	0.8
1994	0.8	1.8
1995	0.8	1.8
1996	1	1
1997	1	1

Table 7: Information indices (%) from MIAEL estimation of B_0 for different stock assessment scenarios using least squares and maximum likelihood fitting procedures

	Maximum likelihood		Least Squares	
Run no	external*	internal*	external*	internal*
1	64	88	64	48
2	8	8	8	8
3	88	61	88	90
4	2	2	2	2
5	49	82	49	39
6	7	7	7	7
7	80	18	80	85
8	0.3	0.3	0.3	0.3
9	44	76	44	31
10	19	19	19	19
11	52	52	52	46
12	1	1	1	1
13	35	67	37	19
14	11	11	11	11
15	49	25	49	32
16	0.3	0.3	0.3	0.3
17	58	80	70	23
18	10	10	10	10
19	91	93	91	92
20	1	1	1	1
21	53	76	64	32
22	5	5	5	5
23	89	88	89	89
24	0.5	0.5	0.5	0.5
25	24	60	36	29
26	17	17	17	17
27	55	55	55	68
28	2	2	2	2
29	19	54	32	25
30	11	11	11	11
31	53	38	53	62
32	1	1	1	1

* external = estimates of total mortality fitted externally using the age data; internal = age data fitted internally within the model as catch-at-age or numbers-at-age

Table 8: Results of robustness study using maximum likelihood[1], and least squares[2] fitting procedures using scenarios where age data were fitted internally within the model

(a) error in mortality

Run no	ML[1] information indices (%)			LS[2] information indices (%)		
	M	M-0.05	M+0.05	M	M-0.05	M+0.05
17	80	74	29	23	45	-4
19	93	38	5	92	41	5
29	54	-98	-2	25	32	17
31	38	-36	6	62	-151	-17

(b) error in YCS

Run no	ML[1] information indices (%)			LS[2] information indices (%)		
	average	below average	above average	average	below average	above average
17	80	53	68	23	11	14
19	93	90	61	92	90	59
29	54	44	48	25	31	17
31	38	66	7	62	74	24

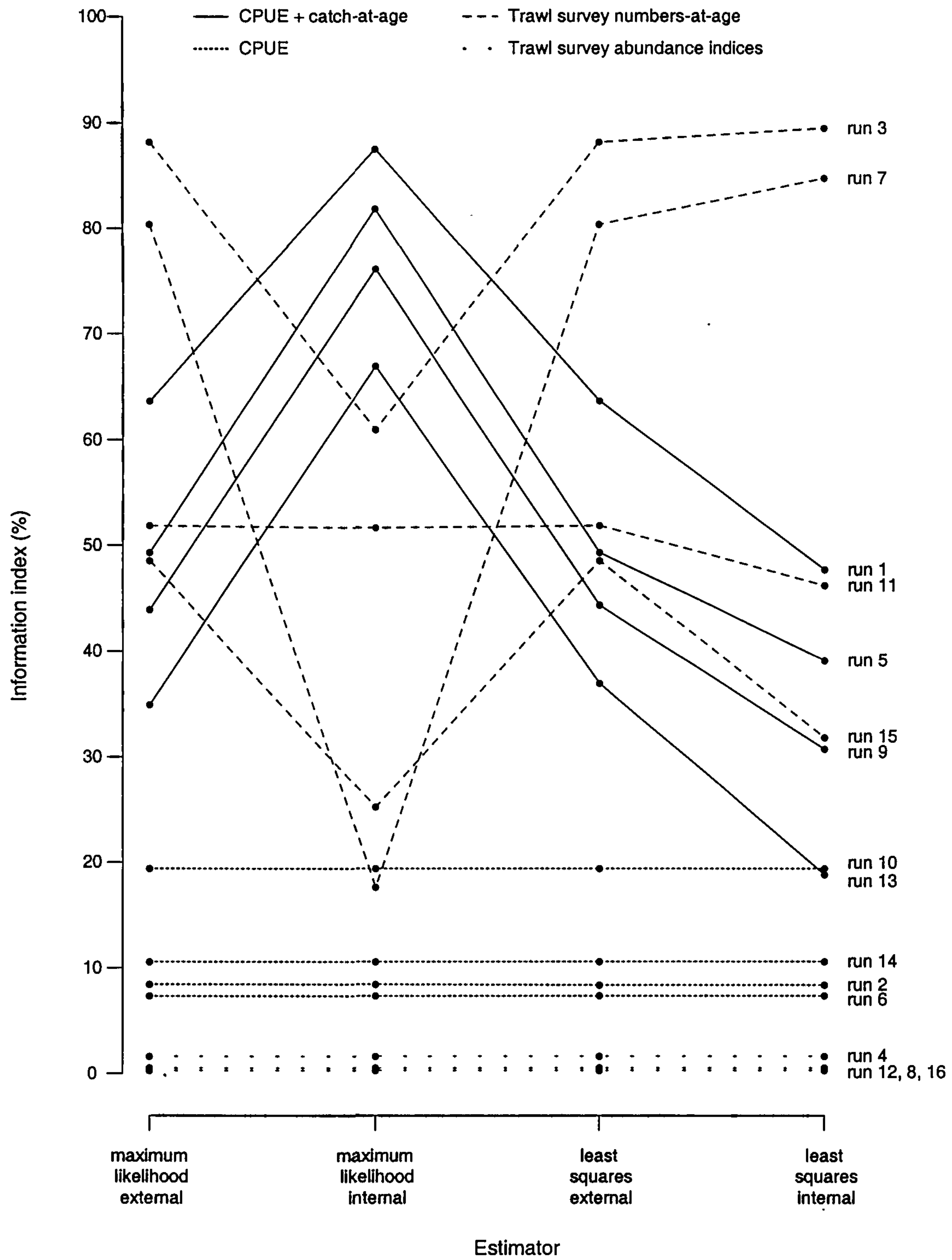

Figure 1a: Results of MIAEL estimation for stock 1 stock assessment settings. (external = estimates of total mortality fitted externally from the age data, internal = age data fitted directly within the model; *see* Table 5 for a description of each run).

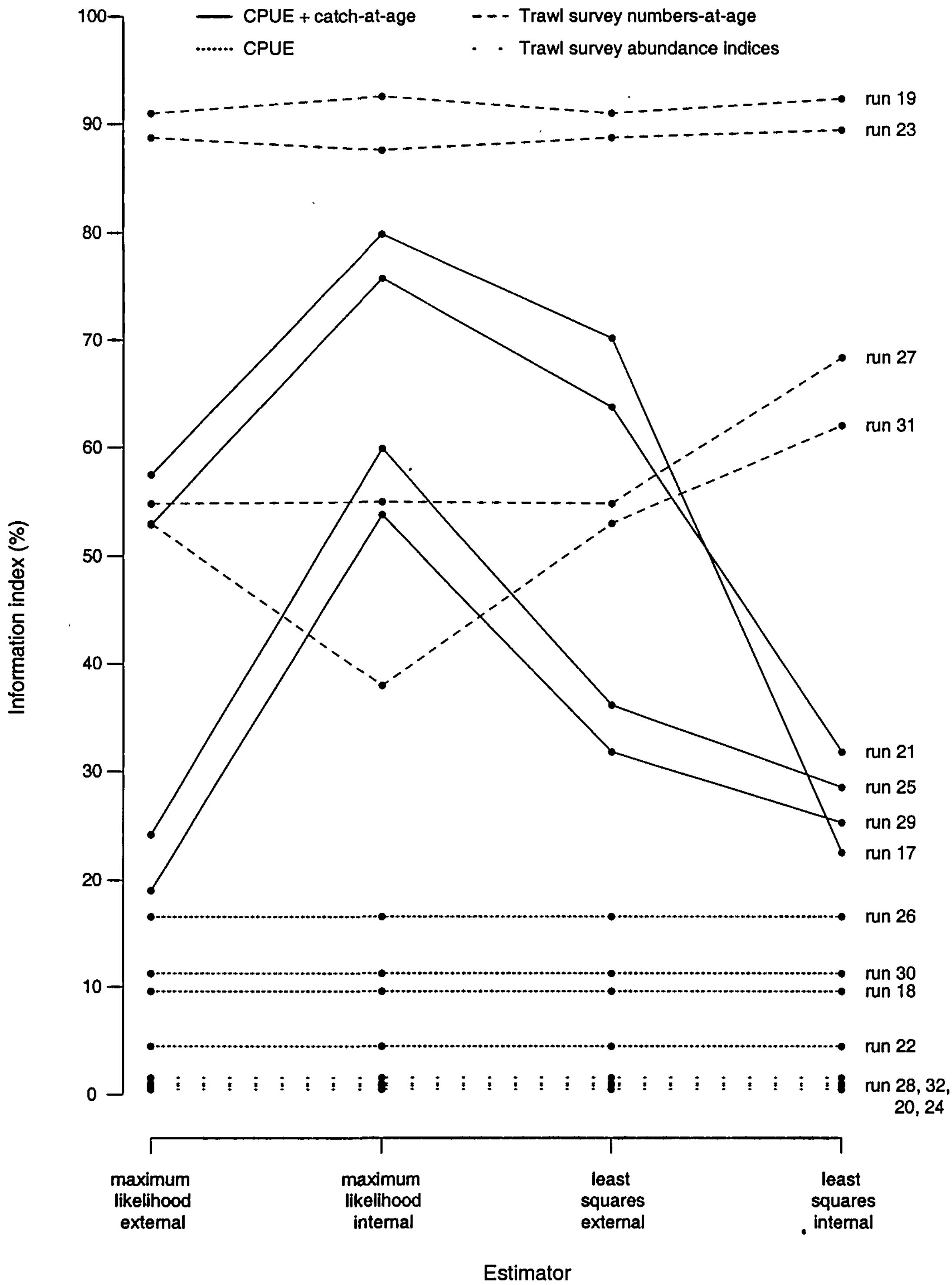

Figure 1b: Results of MIAEL estimation for stock 2 stock assessment settings. (external = estimates of total mortality fitted externally from the age data, internal = age data fitted directly within the model; *see* Table 5 for a description of each run).

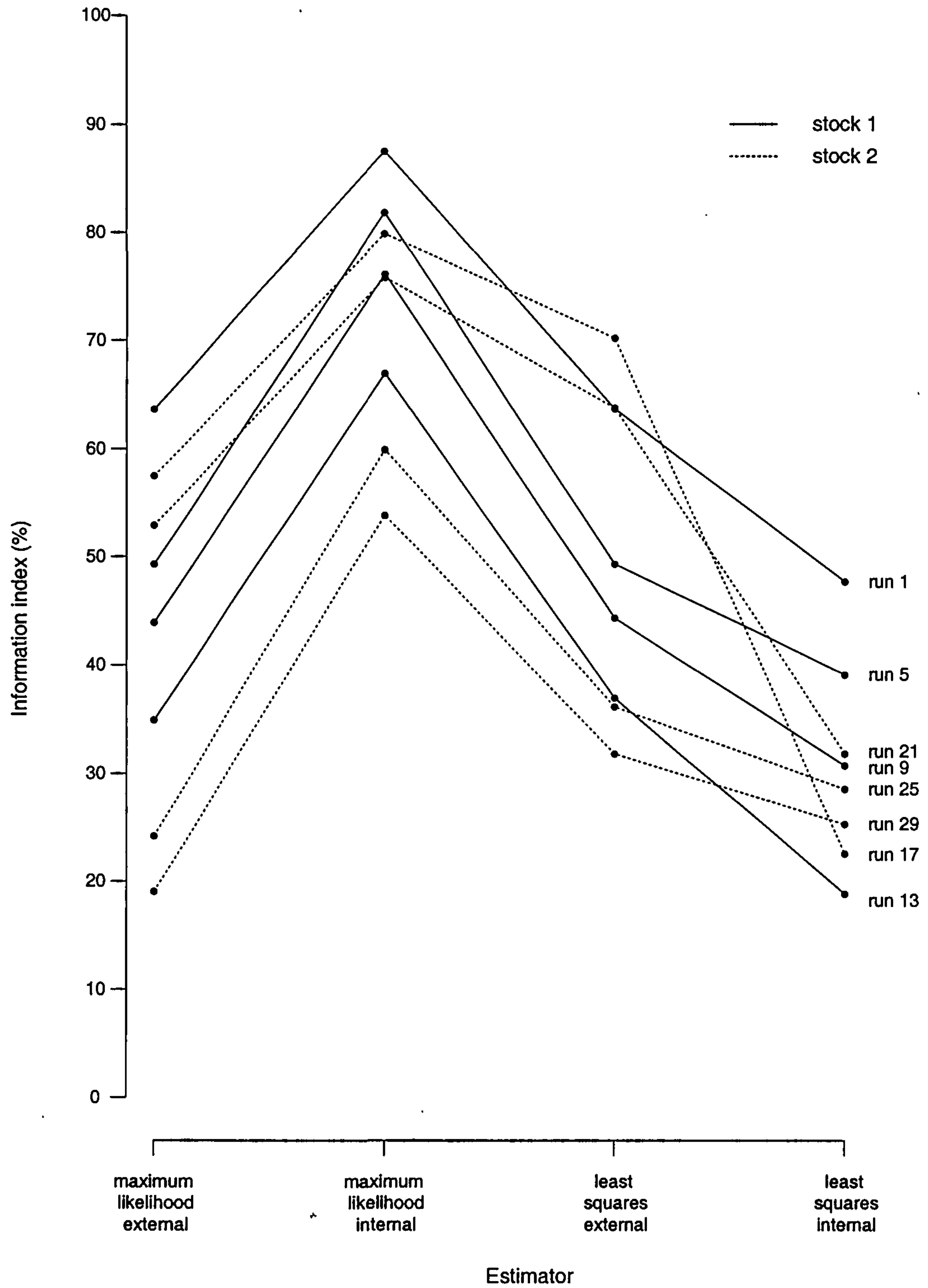

Figure 2a: Comparison of stock 1 and 2 CPUE and catch-at-age stock assessment settings. (external = estimates of total mortality fitted externally from the age data, internal = age data fitted directly within the model; *see* Table 5 for a description of each run).

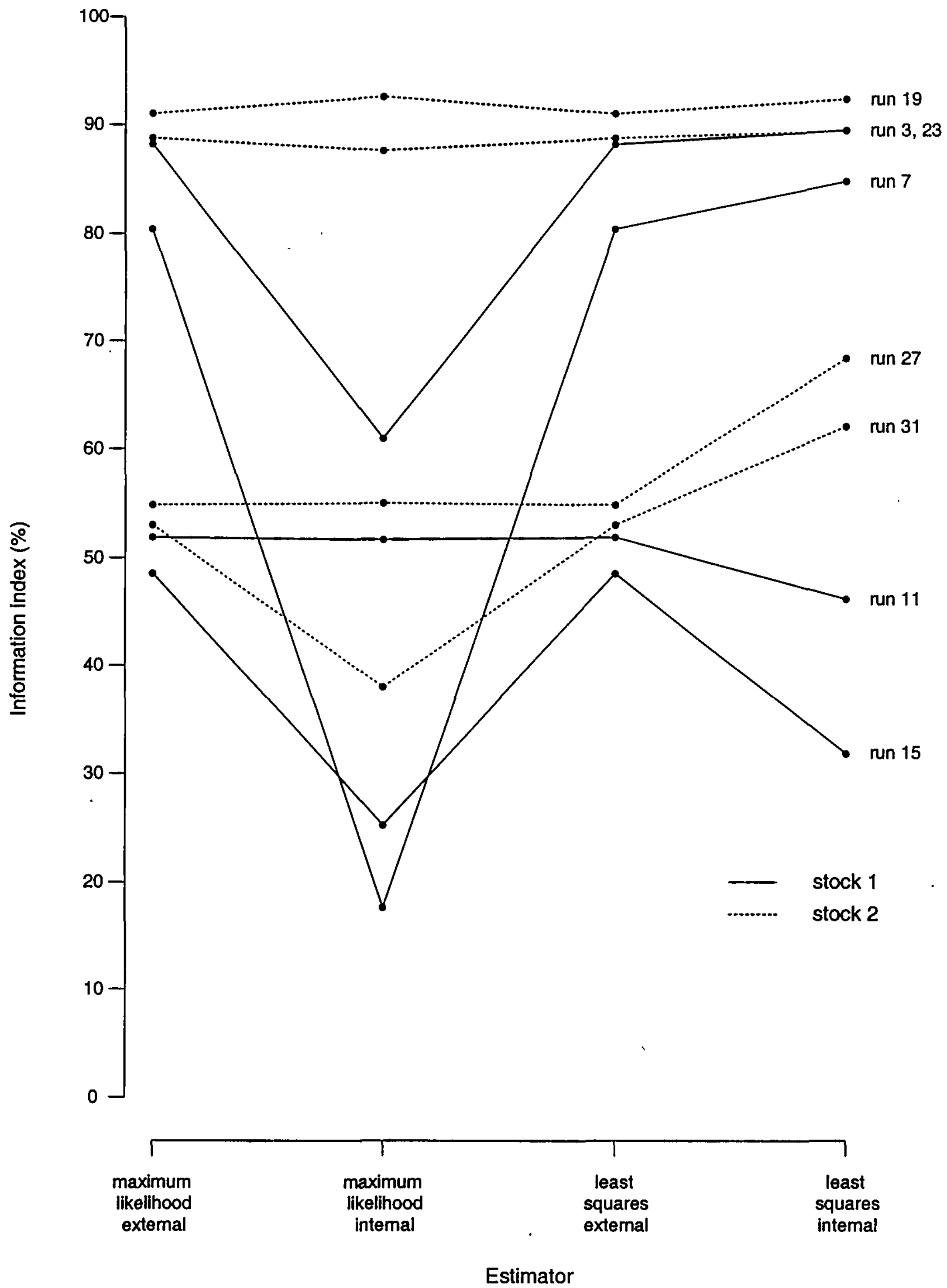

Figure 2b: Comparison of stock 1 and 2 trawl survey numbers-at-age stock assessment settings. (external = estimates of total mortality fitted externally from the age data, internal = age data fitted directly within the model; *see* Table 5 for a description of each run).

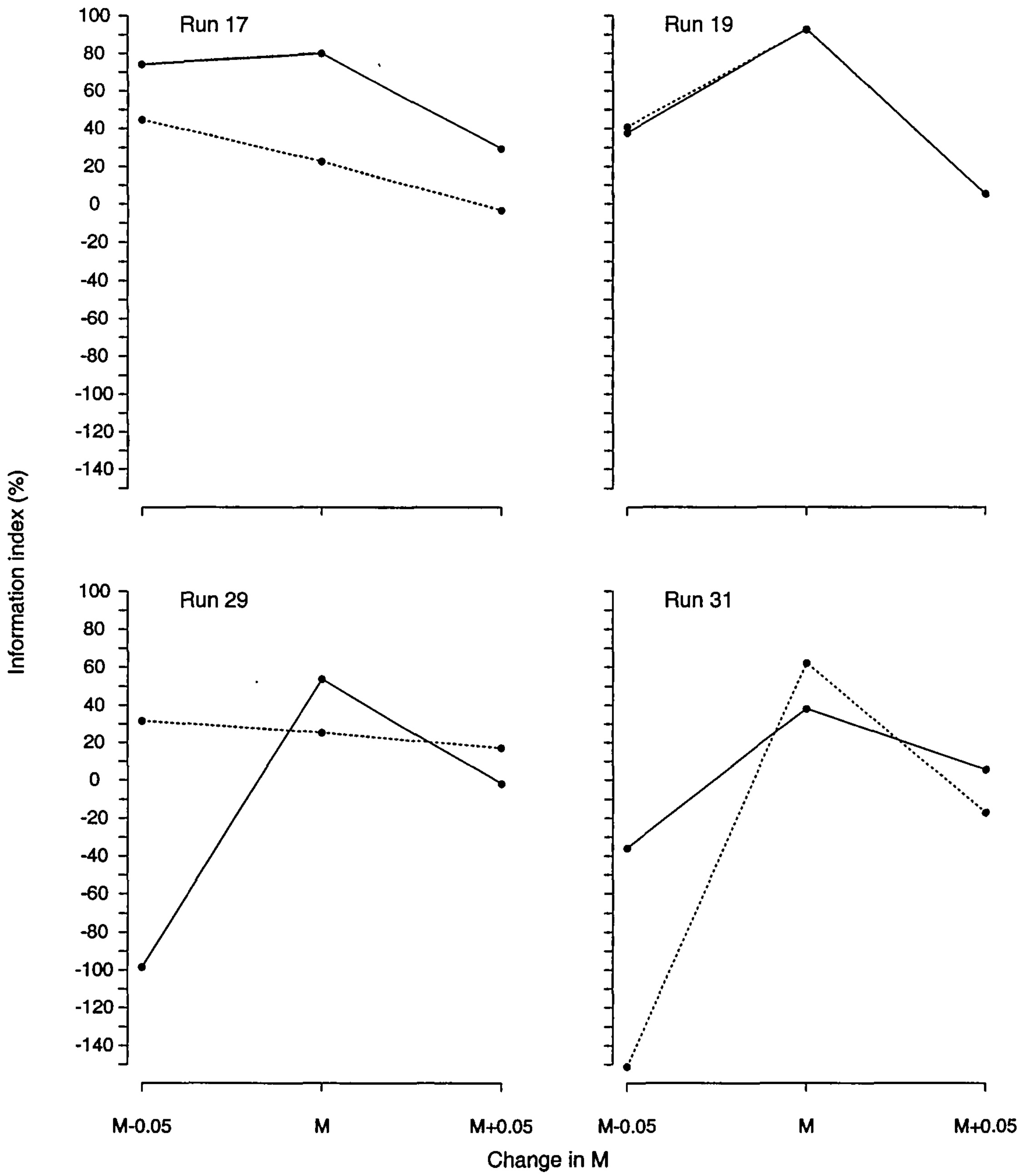

Figure 3: Robustness to the assumptions of known mortality for selected stock assessment settings. (The solid line indicates the maximum likelihood fitting proceedure, and the dotted line indicates the least squares fitting proceedure).

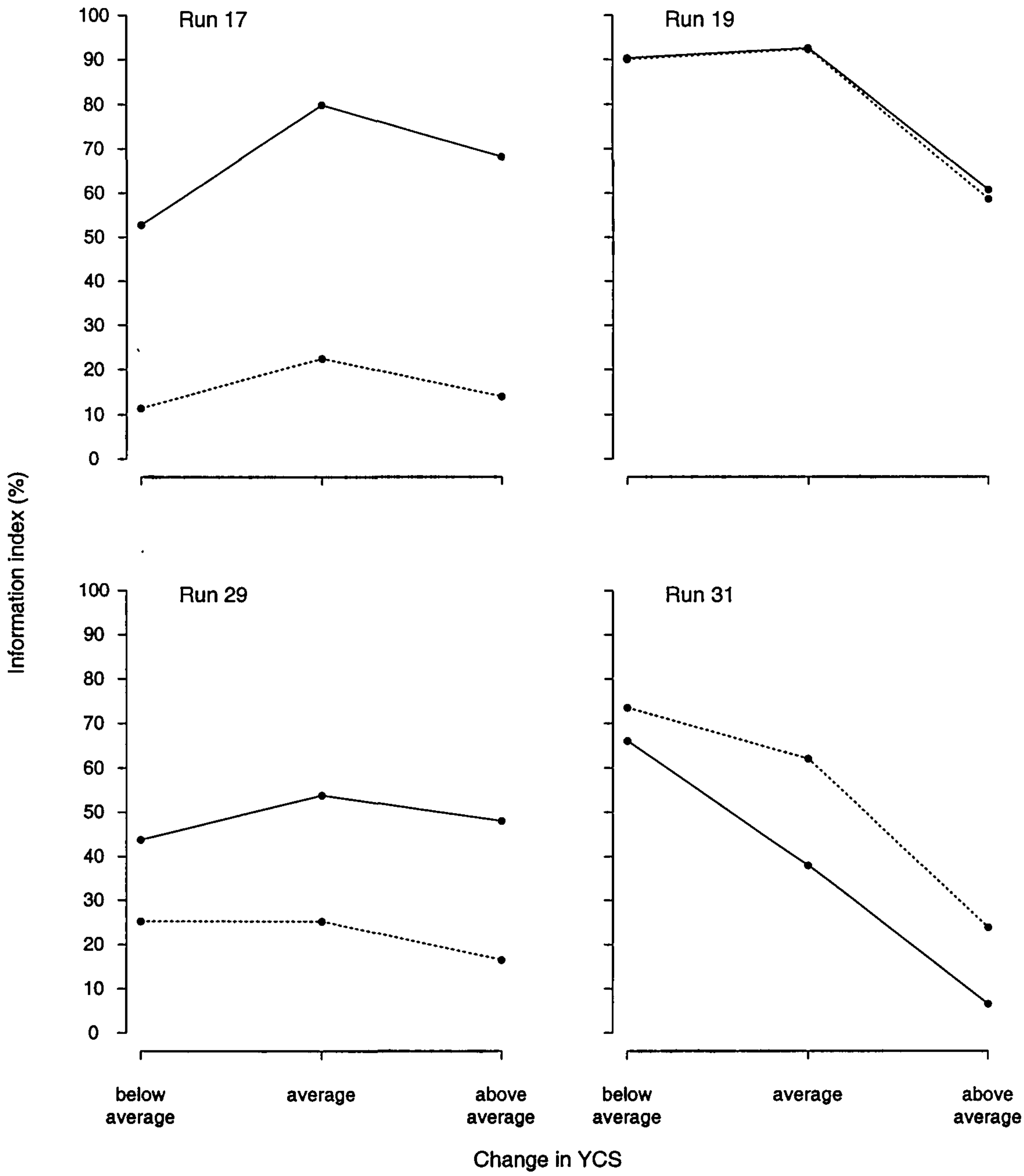

Figure 4: Robustness to the assumptions of average year class strength for selected stock assessment settings. (The solid line indicates the maximum likelihood fitting proceedure, and the dotted line indicates the least squares fitting proceedure).

Appendix 1: The single stock population model

In the population dynamics model, fish are categorised by ground, sex, age, and maturity. Given the level of complexity of the categorisation it is best to present the mathematical equations in a very descriptive form using categorical variables. Categorical variables are given in italics and specific members of a category (except ages) are given in bold italics.

The following abbreviations are used in the category member names:

"spawning" = ***sp***, "home" = ***hm***, "corridor" = ***cor***, "female" = ***fem***,
"immature" = ***imm***, "mature" = ***mat***, "maximum age" = ***amax***.

The categorical variables and their associated categories are:

ground	{ ***sp***, ***hm***, ***cor*** }
sex	{ ***male***, ***fem*** }
age	{ 0, 1, ... , ***amax*** }
maturity	{ ***imm***, ***mat*** }.

The fishing year is divided into eight stages with associated "cycle points". The numbers of fish in each category in year i and cycle point j are denoted by $N_{i,j}$ (*ground, sex, age, maturity*). Unless otherwise stated, an equation involving one or more categorical variables is valid for each member of the associated category or categories (where the particular combination of values is valid; an example of an invalid combination is "mature fish aged 0 years"). Equations are applied consecutively. Note, equations of the form "A += B" are shorthand for "$A = A + B$". Similarly for "A –= B" and "A *= B".

The notation for various population parameters used in the equations is as follows:

nurs(*age*)	Corridor migration ogive: proportion that migrate from the corridor to the nursery.
m_og(*sex, age*)	Maturity ogive: proportion of immature fish that mature.
sel_hm(*sex, age*)	Fishing selectivities in the home ground.
sp_length	Length of the spawning season as a proportion of the year.
spawn_p	Proportion of mature fish that migrate to the spawning ground.
sel_sp(*sex, age*)	Fishing selectivities in the spawning ground.
R_i	The year class strengths: multipliers of the recruitment obtained from the Beverton-Holt stock-recruit relationship.
p_male	The proportion of male larvae.

Stage 1: **Beginning of fishing year**

$N_{i,1}$ (*ground, sex, age, maturity*) = $N_{i-1,8}$(*ground, sex, age, maturity*)

Stage 2: **Corridor migrations and maturity**

$N_{i,2}$ (*ground, sex, age, maturity*) = $N_{i,1}$ (*ground, sex, age, maturity*)

(a) Larvae move from the spawning ground to the corridor.

$N_{i,2}$(***cor***, *sex*, 0, ***imm***) = $N_{i,2}$ (***sp***, *sex*, 0, ***imm***)

$N_{i,2}$(***sp***, *sex*, 0, ***imm***) = 0

(b) Some juveniles move from the corridor to the nursery (home).

$N_{i,2}$(***hm***, *sex, age*, ***imm***) += *nurs*(*age*) * $N_{i,2}$(***cor***, *sex, age*, ***imm***)

$N_{i,2}$(***cor***, *sex, age*, ***imm***) –= *nurs*(*age*) * $N_{i,2}$(***cor***, *sex, age*, ***imm***)

(c) Some juveniles mature (and become adults).

For $age \geq 1$

$N_{i,2}$ (***hm***, *sex, age*, ***mat***) += *m_og*(*sex, age*) * $N_{i,2}$(***hm***, *sex, age*, ***imm***)

$N_{i,2}$ (***hm***, *sex, age*, ***imm***) –= *m_og*(*sex, age*) * $N_{i,2}$(***hm***, *sex, age*, ***imm***)

Stage 3: **Pre-spawning season: first half**

$N_{i,3}$ (*ground, sex, age, maturity*) = $N_{i,2}$ (*ground, sex, age, maturity*)

(a) Natural and fishing mortality are applied to fish in the home ground.

For $age \geq 1$

$N_{i,3}$ (***hm***, *sex, age, maturity*) *= exp[–*t* * (*F*(*sex, age*) + *M*(*sex*))]

where the fishing mortalities *F*(*sex, age*) are calculated from the Baranov catch equation using the selectivities *sel_hm*(*sex, age*) and the pre-spawning season catch. The time period *t* for stage 3 is 0.5 * (1 – *sp_length*).

(b) Natural mortality is applied to fish in the corridor.

For $age \geq 1$

$N_{i,3}$ (***cor***, *sex, age, maturity*) *= exp[$-t$ * $M(sex)$]

Stage 4: Pre-spawning season: second half

$N_{i,4}$ (*ground, sex, age, maturity*) = $N_{i,3}$ (*ground, sex, age, maturity*)

(a) Natural and fishing mortality are applied to fish in the home ground.

For $age \geq 1$

$N_{i,4}$ (***hm***, *sex, age, maturity*) *= exp[$-t$ * ($F(sex, age) + M(sex)$)]

where the $F(sex, age)$ and t are as in Stage 3.

(b) Natural mortality is applied to fish in the corridor.

For $age \geq 1$

$N_{i,4}$ (***cor***, *sex, age, maturity*) *= exp[$-t$ * $M(sex)$]

Stage 5: Ageing and spawning migration

$N_{i,5}$ (*ground, sex, age, maturity*) = $N_{i,4}$ (*ground, sex, age, maturity*)

(a) Fish and larvae age 1 year.

$N_{i,5}$ (*ground, sex, amax, maturity*) += $N_{i,5}$ (*ground, sex, amax* – 1, *maturity*)

For *age* = (*amax* – 1) down to *age* = 1

$N_{i,5}$ (*ground, sex, age, maturity*) = $N_{i,5}$ (*ground, sex, age* – 1, *maturity*).

Also,

$N_{i,5}$ (*ground, sex,* 0, ***imm***) = 0

(b) Some adults move from the home ground to the spawning ground.

$N_{i,5}$ (***sp***, *sex, age,* ***mat***) = *spawn_p* * $N_{i,5}$ (***hm***, *sex, age,* ***mat***)

$N_{i,5}$ (***hm***, *sex, age,* ***mat***) –= *spawn_p* * $N_{i,5}$ (***hm***, *sex, age,* ***mat***)

Stage 6: Spawning season: first half

$N_{i,6}$ *(ground, sex, age, maturity)* = $N_{i,5}$ *(ground, sex, age, maturity)*

(a) Natural and fishing mortality are applied to fish in the spawning ground.

For $age \geq 1$

$N_{i,6}$ *(**sp**, sex, age, maturity)* *= exp[−*t* * (*F(sex, age)* + *M(sex)*)]

where the fishing mortalities *F(sex, age)* are calculated from the Baranov catch equation using the selectivities *sel_sp(sex, age)* and the spawning season catch. The time period *t* for stage 6 is 0.5 * *sp_length* .

(b) Natural mortality is applied to fish in the corridor and the home ground.

For $age \geq 1$ and *ground* ε { ***cor***, ***hm*** }

$N_{i,6}$ *(ground, sex, age, maturity)* *= exp[−*t* * *M(sex)*]

(c) Larvae are created in the spawning grounds.

$larvae_i = R_i$ * *virginR* * *fbio* / [*alpha* + *beta* * *fbio*]

where *fbio* is the biomass of the females present in the spawning ground, *virginR* is the number of larvae needed to maintain deterministic equilibrium prior to fishing, and *alpha*, *beta* are the parameters of the Beverton-Holt stock-recruit relationship given by *steep*.

(d) Larvae are split by sex.

$larvae_i$(**male**) = *p_male* * $larvae_i$

$larvae_i$(***fem***) = (1 − *p_male*) * $larvae_i$

$N_{i,6}$ *(**sp**, sex, 0, **imm**)* = $larvae_i$*(sex)*

Stage 7: Spawning season: second half

$N_{i,7}$ *(ground, sex, age, maturity)* = $N_{i,6}$ *(ground, sex, age, maturity)*

(a) Natural and fishing mortality are applied to fish in the spawning ground.

For $age \geq 1$

$N_{i,7}$ *(**sp**, sex, age, maturity)* *= exp[−*t* * (*F(sex, age)* + *M(sex)*)]

where the *F(sex, age)* and *t* are as in Stage 6.

(b) Natural mortality is applied to fish in the corridor and the home ground.

For $age \geq 1$ and $ground\ \varepsilon\ \{\ \boldsymbol{cor},\ \boldsymbol{hm}\ \}$

$N_{i,7}\ (ground,\ sex,\ age,\ maturity)\ {*}{=}\ \exp[\ -t * M(sex)\ ]$

Stage 8: End of fishing year

$N_{i,8}\ (ground,\ sex,\ age,\ maturity) = N_{i,7}\ (ground,\ sex,\ age,\ maturity)$

(a) Adults return from the spawning ground to their home ground.

$N_{i,8}\ (\boldsymbol{hm},\ sex,\ age,\ \boldsymbol{mat}) \mathrel{+}= N_{i,8}\ (\boldsymbol{sp},\ sex,\ age,\ \boldsymbol{mat})$

$N_{i,8}\ (\boldsymbol{sp},\ sex,\ age,\ \boldsymbol{mat}) = 0$

Appendix 2: Biological Parameters

Stock	Natural mortality		Von Bertalanffy growth parameters						Length-weight				Maximum	Ageing
	male	female	male			female			male		female		Age (m,f	Data
			K	t_0	L_∞	K	t_0	L_∞	a	b	a	b	or both)	Validated
HAK1	0.22	0.20	0.259	-0.60	92.5	0.185	-0.18	115.5	0.0016	3.36	0.0015	3.37	30	Y
HAK4	0.22	0.20	0.294	-0.01	88.8	0.181	-0.18	116.1	0.0051	3.11	0.0045	3.11	30	Y
HAK7	0.22	0.20	0.308	0.00	111.1	0.194	0.00	111.1	0.00275	3.23	0.00113	3.41	30	Y
LIN3, 4	0.18	0.18	0.108	-1.24	119.0	0.076	-1.05	160.1	0.00126	3.296	0.00126	3.296	30	Y
LIN5, 6	0.18	0.18	0.194	0.16	95.1	0.113	-0.67	125.7	0.00139	3.278	0.00139	3.278	30	Y
LIN7	0.18	0.18	0.087	-0.13	146.1	0.090	0.22	165.9	0.00126	3.290	0.0126	3.290	30	Y
SKI1,2	0.25	0.25	0.266	-0.35	87.4	0.194	-0.55	105.0	0.0008	3.55	0.0034	3.22	17?	Y
SKI3,7	0.23	0.23	0.242	-0.66	88.5	0.178	-0.88	104.2	0.0033	3.19	0.0018	3.32	17?	Y
WAR3	0.21	0.21	0.241	-0.46	63.8	0.209	-0.79	66.3	0.0015	3.09	0.0016	3.07	17	Y
SBW	0.20	0.20	0.350	-0.93	47.6	0.320	-1.03	51.5	0.00515	3.092	0.00407	3.152	22	Y
GUR1	-	-	0.569	-0.552	28.8	0.641	0.189	36.4	0.00988	2.99	0.00988	2.99	16	Y
GUR3	0.35	0.29	0.49	-0.26	42.2	0.44	0.1	48.2	-	-	-	-	16,13	Y
GUR7	0.31	0.31	0.37	-0.96	40.3	0.40	-0.36	45.7	-	-	-	-	15	Y
RCO3	0.76	0.76	0.47	0.06	68.5	0.41	-0.03	76.5	0.0145	2.892	0.0074	3.059	6?	Y
STA3	0.23	0.23	0.19	-1.19	59.12	0.18	-0.22	73.92	0.015	3.01	0.015	3.01	20	N
STA5	0.23	0.23	0.19	-1.19	59.12	0.18	-0.22	73.92	0.024	2.92	0.024	2.92	20	N
BCO5	0.27-0.38	0.27-0.38	-	-	-	-	-	-	0.00001	3.10	0.00002	2.95	12–17	N

Biological Parameters continued:

Stock	Natural mortality		Von Bertalanffy growth parameters						Length-weight				Maximum	Ageing
	male	female	male			female			male		female		Age (m,f	Data
			K	t_0	L_∞	K	t_0	L_∞	a	b	a	b	or both)	Validated
BNS2	0.3	0.3	0.308	-0.627	81.1	0.308	-0.384	86.1	0.00963	3.173	0.00963	3.173	15	Y
BYX2	0.23	0.23	0.11	-3.56	51.1	0.08	-4.10	57.5	0.0226	3.018	0.0226	3.018	20	Y
TAR3	0.1	0.1	0.2085	-1.397	42.1	0.2009	-1.103	44.6	0.0433	2.77	0.0400	2.79	40+	Y
TAR4	0.1	0.1	0.1666	-2.479	44.7	0.2205	-1.026	44.6	0.017	3.02	0.023	2.94	40+	Y
TRE1	0.03	0.03*	0.29	-1.40	44.9	0.30	-1.40	44.45	0.016	3.064	0.016	3.064	45	Y
TRE1	0.30	0.30+	0.29	-1.40	44.9	0.30	-1.40	44.45	0.016	3.064	0.016	3.064	45	Y
	* <35 years, conservative, + >35 years													
BAS1	0.1	0.1	-	-	-	-	-	-	0.2734	2.382	0.2734	2.382	40+	N
HPB1	0.1	0.1	-	-	-	-	-	-	0.0142	3.003	0.0142	3.003	40+	N
HPB2	0.1	0.1	-	-	-	-	-	-	0.0242	2.867	0.0242	2.867	40+	N
HPB7,8	0.1	0.1	-	-	-	-	-	-	0.01423	2.998	0.01423	2.998	40+	N
BAR4	0.3	0.3	-	-	-	-	-	-	0.0117	2.82	0.0074	2.94	10	N
BAR5	0.3	0.3	-	-	-	-	-	-	0.0075	2.90	0.0075	2.90	10	N
MOK1,3,4,5	0.14	0.14	0.208	-0.029	66.95	0.208	-0.029	66.95	0.055	2.713	0.055	2.713	33	N

Biological Parameters continued:

Stock	Natural mortality		Von Bertalanffy growth parameters						Length-weight				Maximum	Ageing
	male	female	male			female			male		female		Age (m,f	Data
			K	t_0	L_∞	K	t_0	L_∞	a	b	a	b	or both)	Validated
ELE3	0.35	0.35	*1 0.231	-0.78	74.7	0.096	-0.87	156.9	0.0091	3.02	0.0091	3.02	13-15	?
			*2 0.473	-0.24	66.9	0.195	-0.53	113.9						
			*3 0.089	-0.96	141.5	0.060	-1.06	203.6						
			*4 0.466	-0.38	62.7	0.224	-0.69	94.1						
	*1 Pegasus Bay 1966–68, *2 Pegasus Bay 1983–84, *3 Canterbury Bite 1966–68 , *4 Canterbury Bite 1988													
GMU1	0.33	0.33	0.47	0.73	35.84	0.45	0.72	40.10	0.036	2.7537	0.036	2.7537	15	N
JDO1	0.38	0.38	0.480	-0.251	36.40	0.425	-0.223	41.13	0.048	2.70	0.048	2.70	12	N
SWA4	0.27	0.24	0.41	-0.71	51.8	0.33	-1.04	54.5	0.00848	3.214	0.00848	3.214 *1	23	Y
SWA4	0.27	0.24	0.41	-0.71	51.8	0.33	-1.04	54.5	0.00473	3.380	0.00473	3.380 *2	23	Y
										L-W: *1 from Ch Rise, *2 from Southland				
WWA	0.23-0.31	0.23-0.31	-	-	-	-	-	-	-	-	-	-	15-20	N

Appendix 3: Available data

Stock	Catches	Abundance index	Detail
HAK1	1975–96/97	Trawl survey	Nov/Dec 1992, 1993, 1994; biomass + numbers-at-age
		Trawl survey	Apr/May 1992, 1993, 1996, (1998?); biomass + numbers-at-age
HAK4	1975–96/97	Trawl survey	Dec/Jan 1991–1998; biomass + numbers-at-age
		CPUE	Trawl;1991–1996
HAK7	1975–96/97	Observer data	1990–1997, prop. catch-at-age
		CPUE	index1: 1989–1997; index2:1992–1996 (use only one index, which?)
LIN3, 4	1972–96/97	Trawl survey	Dec/Jan 1991–1998; biomass + numbers-at-age
		CPUE	index1: 1992–1995; index2:1990–1995 (index 1 preferred)
LIN5, 6	1972–96/97	Trawl survey	Nov/Dec 1992, 1993, 1994; biomass + numbers-at-age
		Trawl survey	Apr/May 1992, 1993, 1996, (96/97?); biomass + numbers-at-age
		CPUE	Puysegur trawl: 1989–1995 (index no good, probably shouldn't use)
LIN7	1972–96/97	CPUE	Longline 1990–1997
		Observer data	ageing data
SKI1, 2		CPUE	Index1 SKI1E 1989–1997
		CPUE	Index2 SKI1W 1994–1997
		CPUE	Index3 SKI2 1990–1997
SKI3,7		Trawl survey	Shinkai Feb-Apr 1981–1983; biomass
		Trawl survey	Tangaroa Feb/Mar 1993–1996; biomass + prop catch-at-age

Available data continued:

Stock	Catches	Abundance index	Detail
WAR3	1970–96/97	Trawl survey	Shinkai 1981–1983; biomass
		Trawl survey	Tangaroa Feb/Mar 1993–1996; biomass + prop catch-at-age
SBW	1978–96/97	Campbell	catch-at-age 1979–1996
			acoustic data 1993–1995
			CPUE 1986–1996 (not useful)
SBW	1978–96/97	Bounty	catch-at-age 1990–1997
			acoustics 1993–1995, 1996
SBW	1978–96/97	Pukaki	catch-at-age 1989–1997
			acoustics 1993–1995, 1996
GUR1		Trawl survey	WCNI: 1986, 1987, 1991, 1994; biomass (Survey for GUR1 & GUR8 ?)
			Hauraki Gulf: 1984–1990, 1992–1994; biomass
			ECNI: 1993–1995: biomass (for GUR1 & GUR2 ?)
GUR2		Trawl survey	Bay of plenty: 1983, 1985, 1987, 1990, 1992, 1996; biomass
			ECNI: 1993–1995: biomass (for GUR1 & GUR2 ?)
GUR3		Trawl survey	ECSI: 1991–1994, 1996: biomass
		CPUE	Index A: 1989/90–1995/96
			Index B: 1982/83–1995/96
GUR7		Trawl survey	WCSI & Tasman/Golden Bay: 1993–1995
		CPUE	1991/92–1995/96 (may not be useful)
GUR8		Trawl survey	WCNI: 1986, 1987, 1991, 1994; biomass (Survey for GUR1 & GUR8

Available data continued:

Stock	**Catches**	**Abundance index**	**Detail**
RCO2		Trawl survey	ECNI: Feb/Mar 1993–1995; biomass, ageing data
RCO3		Trawl survey	ECSI: May/Jun 1991–1994, 1996; biomass, ageing data
			Southland: Feb/Mar 1993–1996; biomass, ageing data
			ECSI: Dec/Jan 1996–97; biomass, ageing data
RCO7		Trawl survey	WCSI: Mar/Apr 1992, 1994, 1995: biomass, ageing data
STA2		Trawl survey	ECNI inshore: 1993–1996; biomass estimates; numbers-at-age
			ECNI scampi: 1995–1995; biomass estimates; numbers-at-age
		CPUE	1991/92–1995/96 (Linear & combined indices) (not useful)
STA3	1983–96/97	Trawl survey	ECSI: 1991–1994, 1995; biomass estimates; numbers-at-age
		Trawl survey	Chatham Rise (western side): 1992–1998; numbers-at-age
		CPUE	1991/92–1995/96 (Linear & combined indices) (not useful)
STA4		Trawl survey	Chatham Rise: 1992–1998; biomass estimates; numbers-at-age
STA5		Trawl survey	Stewart-Snares: 1993–1996; biomass estimates; numbers-at-age
			Stewart-Snares: BAZ5: 1993–1996 (Banded stargazer) biomass estimates; numbers-at-age
		CPUE	1991/92–1995/96 (Linear & combined indices)
STA7	1983–96/97	Trawl survey	WCSI: 1992, 1994, 1995, 1997; biomass estimates; numbers-at-age
		CPUE	1991/92–1995/96 (Linear & combined indices) (not useful)

Available data continued:

Stock	Catches	Abundance index	Detail
BNS1		CPUE	1990–1996
BNS3	1981–96/97	CPUE	1991/92–1995/96 2 series: target bluenose, target ling (neither useful)
BNS7, 8	1981–1988	CPUE	FMA7: 1991/92–1996/97 (not useful) FMA8: 1991/92–1996/97 (not useful)
TAR2		Trawl survey	Cape Runaway to Cook Strait: 1993–1996; biomass, num-at-age? (not useful)
TAR3		Trawl survey	Pegasus Bay to Banks Peninsula: 1991–1994, 1996; biomass, num-at-age? (not useful)
TAR7		Trawl survey	Tasman Bay to Haast: 1992, 1994, 1995; biomass, num-at-age? (not useful)
TRE		Trawl survey	these exist, but are not considered useful due to partial pelagic nature of fish
BAR1		Trawl survey	ECSI: May/Jun 1991–1994, 1996; biomass, num-at-age? ECNI: 1993–1996; biomass, num-at-age?
BAR5		Trawl survey	Southland: Feb/Mar 1993–1996; biomass, num-at-age?
BAR7		Trawl survey	WCSI: Mar/Apr 1992, 1994, 1995; biomass, num-at-age?

Available data continued:

Stock	Catches	Abundance index	Detail
ELE3		Trawl survey	ECSI: 1991–1994, 1996; biomass, num-at-age?
ELE5		Trawl survey	Stewart-Snares: 1992–1995; biomass, num-at-age?
ELE7		Trawl survey	WCSI:1992, 1994, 1995; biomass, num-at-age?
JDO1		Trawl survey	Bay of Plenty: 1983, 1985, 1987, 1990, 1992, 1996; biomass, num-at-age?
		Trawl survey	WCNI: 1986–1989, 1991, 1994; biomass, num-at-age?
		Trawl survey	Hauraki Gulf: 1984–1990, 1992–1994; biomass, num-at-age?
		Trawl survey	ECNI: 1993–1995; biomass, num-at-age?
SWA		Trawl survey	unusable
		CPUE	WCSI: 1985/86–1995/96, not very useful

Appendix 4: MIAEL estimation, the information index, and *best p* estimators

This appendix gives the reader who is unfamiliar with MIAEL estimation a detailed introduction to the motivation and definitions of the method. For further details on MIAEL estimation see Cordue (1998b) and for *best p* estimators see Cordue (1995).

Decision theory and point estimation

Point estimation can be considered as a special case of decision theory (Wald 1950; Fergusson 1967; Berger 1985). In the general decision problem there is an unknown "state of nature" and a decision maker. The decision maker has to choose between a number of alternative actions, each of which will result in a "loss" depending on the true state of nature. The decision maker may conduct an experiment (i.e., observe some random variable whose distribution (hopefully) depends on the state of nature) in order to help them decide on the "best" action.

For example, a classic statistical problem is estimating the probability of getting "heads" from the single toss of a given coin (a special case of a Bernoulli experiment). The unknown "state of nature" is the probability of getting "heads". The "actions" available to the decision maker are their possible choices for the estimate: any real number from 0 to 1 inclusive. The "loss" in this case is estimation loss; presumably the further away that the estimate is from the true value, the greater the loss. The usual experiment conducted by the decision maker is to toss the coin n times, and record the total number of times that "heads" occurs. On the basis of this observation, they choose their estimate (action).

In more precise terms, for the general decision problem, there is an unknown state of nature θ contained in a parameter space Θ. The decision maker can observe a random variable X which has observable values in Obs(X) (with a generic observation denoted by x), and probability density function $p(x \mid \theta)$. An action $a \in A$ must be chosen, and this will result in a non-negative loss given by the function $L : \Theta \times A \to \mathrm{R}$. The solution to the decision problem is to find a decision rule $d : \mathrm{Obs}(X) \to A$, which minimises (in some sense) the expected loss $\mathrm{E}_{X|\Theta}[\, L(\theta,d(X))\,]$. ($\mathrm{E}_{X|\Theta}$ denotes the expectation with respect to X assuming that θ is the true state of nature.) The expected loss $\mathrm{E}_{X|\Theta}[\, L(\theta,d(X))\,]$ is called the risk function of d, and will be denoted here by $R(\theta,d)$.

In the general point estimation problem, estimating $g(\theta)$ for some given function g, the actions consist of the possible choices for the estimate, so that $d(X)$ is simply an estimator (and for $x \in \mathrm{Obs}(X)$, $d(x)$ is an estimate). The loss function will then be a function of $g(\theta)$ and $d(X)$ and should in some sense measure the "distance" between them, with increasing loss as the "distance" increases. An estimator $d(X)$ which in some sense minimises $R(\theta,d)$, is then minimising the expected "distance" between $g(\theta)$ and $d(X)$. For example, a commonly used loss function is squared error $[\, g(\theta) - d(X)]^2$, which results in mean squared error as a risk function. An optimal estimator in this case, then, minimises (in some sense) mean squared error.

Continuing with the coin tossing example, rather than estimating θ, the probability of getting "heads", it may be desired to estimate a function of θ, say θ^2. Also, the decision rule d, might be "divide the total number of heads observed by the number of trials, and square the result". That is, $d(X) = (X/n)^2$ where X is the total number of heads observed in

n trials. If the loss function L is squared error, then $L(\theta,d(X)) = [\ \theta^2 - X^2/n^2\ ]^2$. The risk function of the estimator $(X/n)^2$ is $E_{X|\theta}[\ \theta^2 - X^2/n^2\ ]^2$.

The general formulation is intuitively appealing, but there is the difficulty of deciding in what sense the risk function is to be minimised. In general there will not be an estimator with minimum risk for all values of θ. (Consider for example, estimating $\theta \in [0,1]$ with a squared error loss function. For any constant $k \in [0,1]$, $d(X) = k$ has zero risk when $\theta = k$, hence there cannot be an estimator of θ with uniformly minimum mean squared error.) Three main approaches have been used: imposition of a special property to form a "class" of estimators within which uniformly minimum risk is sought (e.g., considering only unbiased estimators); minimising the maximum risk (minimax estimation); and minimising a weighted average risk (e.g., Bayes estimation, where the weighting is given by the prior distribution of Θ—although, of course, Bayes estimation can be developed more simply and independently of the approach described here). MIAEL estimation is related to Bayes estimation, but its formulation differs because the averaging is done over $g(\Theta)$ rather than Θ (note, $g(\Theta) = \{\ g(\theta)\ |\ \theta \in \Theta\}$).

MIAEL estimation

The main idea behind MIAEL estimation is that since $g(\theta)$ is the object of interest, the minimisation of estimation risk should be done in the $g(\theta)$ domain (i.e., within $g(\Theta)$ rather than Θ). Also, a uniform weighting is used in the integration of risk (across $g(\Theta)$) because, inasmuch as $g(\theta)$ is unknown, there is little reason to require preferential estimator performance in any particular region of $g(\Theta)$. The aim is to minimise the "average" risk, given the estimation losses encapsulated in the specified loss function.

The integrated average expected loss of $d(X)$ when estimating $g(\theta)$ with risk function R is

$$I[\ d(X)\ ] = \int_{z \in g(\Theta)} \left[\frac{\int_{\phi \in g^{-1}(z)} R(\phi, d)\, d\phi}{\int_{\phi \in g^{-1}(z)} d\phi} \right] dz$$

where

$$\forall\ \theta \in \Theta \quad g^{-1}\ (g(\theta)) \ = \ \{\ \phi \mid \phi \in \Theta \text{ and } g(\phi) \ = \ g(\theta)\}$$

and if $g^{-1}(z)$ is finite, then integration over $g^{-1}(z)$ is interpreted as simple summation. If $d \in D$ is such that for every $d' \in D$, $I\,[d'(X)] \geq I\,[d(X)]$ then $d(X)$ is a MIAEL estimator within the class D.

This definition requires some clarification. In the MIAEL acronym, "EL" denotes Expected Loss (expectation over X of the loss function). The "A" is Averaging of Expected Loss for each point in $g(\Theta)$. For each $z \in g(\Theta)$, the Average Expected Loss is given by the ratio of the integrals in the definition of $I[d(X)]$. Since $z \in g(\Theta)$ and

$\phi \in g^{-1}(z)$, there exists $\theta \in \Theta : g(\phi) = g(\theta)$. The denominator in the ratio is a "count" of the number of points in $g^{-1}(z)$ (explicitly if $g^{-1}(z)$ is finite) and the numerator is the "sum" of the expected losses. Note, that if g is 1 to 1 then $g^{-1}(z)$ contains only a single point, and the "A" is redundant.

In the coin tossing example, if θ or θ^2 were being estimated, then g is 1 to 1 (as $\theta \in [0,1]$) and no averaging of expected losses occurs. However, if $g(\theta) = \theta(1 - \theta)$ was being estimated, then $g(\Theta) = [0,1/4]$ and for any $z \in [0,1/4]$, $g^{-1}(z) = \{ t_0, 1 - t_0 \}$ where t_0 is a solution to $z = \theta(1 - \theta)$. Hence, for every point in $g(\Theta)$, the ratio of the integrals is an average of exactly two expected losses. Note, that there is no guarantee that $R(t_0,d) = R(1 - t_0,d)$. In general, the risk of a decision rule is a function of θ, not $g(\theta)$.

Returning to the general case, note that $I[d(X)]$ does not necessarily exist (it may be infinite) and hence for some classes a MIAEL estimator may not exist. If a MIAEL estimator does exist it may not be unique. However, in almost every practical fisheries application there will be sufficient ancillary information available to allow θ and $g(\theta)$ to be bounded. In that case, MIAEL estimators within many general classes will exist and be unique within the class. Also, in some circumstances, a global MIAEL estimator will exist (*see* Theorems 1–3 in Cordue 1998b).

An information index

Point estimates by themselves are sometimes not particularly useful to fishery managers. It is generally desirable to include some measure of the uncertainty of an estimate. This is traditionally done by providing a confidence interval at some high level of confidence (traditionally 95%, more recently 90%). The confidence interval approach is of limited value in some fisheries applications, particularly in "risk" analysis, where confidence intervals on "risk" (if they were ever calculated) would often include the interval [0,1]. If MIAEL estimation is used then a natural measure of estimator uncertainty can be provided by comparing the relative performance of the MIAEL estimator which uses the observations and the MIAEL estimator which does not.

Let D be a class of estimators (based on X, estimating $g(\theta)$, with loss function L), and let the information index of $d \in D$ be defined as

$$Info\,[\,d\,] = 1 - \frac{\mathrm{I}[\,\mathrm{d(X)}\,]}{\mathrm{I}[\,\mathrm{K}\,]}$$

where K (called the *best k* estimator) is the MIAEL estimator of $g(\theta)$ (under loss function L) before the experiment is observed (i.e., when no observations are available). If D contains a MIAEL estimator $M(X)$, then for every $d \in D$, $Info(M) \geq Info(d)$.

If X has a distribution which does not depend on θ then $Info[d] \leq 0$ (since $I[d(X)] \geq I[K]$). Also, as estimation losses cannot be negative, for every $d \in D$ $I[d(X)] \geq 0$. Hence, an information index (as defined) is always less than 1, and equals 1 if and only if $I[d(X)] = 0$. Note that provided $K \in D$, $Info[M]$ is always in the interval [0,1]. The MIAEL estimator K can easily be found for a squared error loss function (and other simple loss functions). In the case when $g(\Theta) = [a,b]$ with a squared error loss function, $K =$

$(a + b)/2$. (For K under proportional squared error, see Cordue 1995.) Under fairly general conditions, it is always the case that $K = k$ for some $k \in g(\Theta)$ (*see* Theorem 1 in Cordue 1998b).

The *best p* estimator

Finding a MIAEL estimator from the class of all estimators is often difficult or impossible. To find a MIAEL estimator for a particular problem it is often necessary to construct a restricted class of estimators and determine the MIAEL estimator within the class. One way to construct a class of estimators is to build it around a standard estimator, derived from a method such as least squares or maximum likelihood. This is how "*best p*" estimators are constructed; they are MIAEL estimators within particular classes of estimators built from a "base" estimator.

Continuing with the notation above, let

$$P = \{\, p\, d(X) + (1-p)\, K \;\mid\; p \in \mathrm{R} \,\}$$

for some estimator $d(X)$ where K is the *best k* estimator. The MIAEL estimator in the class P is called a *best p* estimator; it is derived from the base estimator $d(X)$. Note, both $d(X)$ and K are in the class P, and that because $K \in P$ it follows that the information index of the MIAEL estimator is between 0 and 1.

Appendix 5: The least squares and maximum likelihood estimators

Least squares

In general, a least squares estimate is a function of a vector of parameter values which minimises a weighted sum of squared differences between the observations and the predicted values (as given by the vector of parameters when input into the model). The form of the sum of squares used for the least squares estimator in this paper is:

$$\sum_{k \varepsilon K} w_k \left[\ln(X_k) - \ln(P_k)\right]^2$$

where K indexes all observed values (individual biomass indices or individual proportions or numbers at age and sex), and for $k \varepsilon K$, X_k is the kth observation, P_k is the kth predicted value, and w_k is the kth weight. The weights for each observation were calculated using the method described below.

Each observation has a "source code": observations with the same source code are theoretically derived from the same "source" (e.g., a series of trawl surveys—the source—giving as observations a time series of biomass indices, or a time series of biomass indices and a corresponding set of estimated numbers at age and sex). Let S be a subset of K which indexes observations with a particular source code, then for $s \varepsilon S$,

$$w_s = \frac{uy}{Wc_s^2}$$

where u is a specified source weight, y is the number of years for which there are observations from the source, c_s is a specified *c.v.*, and

$$W = \sum_{s \varepsilon S} \frac{1}{c_s^2}$$

Each observation also has a "q code": observations with the same q code are assumed to belong to a relative time series. Let Q be a subset of K which indexes observations with a particular q code, then for each $j \varepsilon Q$,

$$P_j = qT_j$$

where q is a proportionality constant and T_j is the predicted value before scaling. The value of q which minimises the sum of squares can be found analytically and is equal to:

$$\exp\left[\left(\frac{1}{\sum_{j \varepsilon Q} w_j}\right) \sum_{j \varepsilon Q} w_j \log\left(\frac{X_j}{T_j}\right)\right]$$

Maximum likelihood

In general, a maximum likelihood estimate is a function of a vector of parameter values which gives the highest "probability" (or likelihood) of observing the actual observations. Maximum likelihood estimators are usually obtained by minimising the negative log likelihood (which is equivalent to maximising the likelihood). To derive the log likelihood of the observations it is necessary to specify the statistical distribution of the observations. For the estimator in this document observations of three types were used: relative biomass, estimated numbers at age, and estimated catch at age (as proportions).

For a time series of relative biomass indices or estimated numbers at age and sex, let K index the observations in the time series, then for $k \varepsilon K$ it is assumed for observation X_k that $X_k = qT_k\varepsilon_k$ where $\varepsilon_k \sim N(1, c_k^2)$, q is a proportionality constant, T_k is the true value, and c_k is a given *c.v.* In the case of estimated numbers at age and sex, the T_k are actual numbers at age and sex after application of an age and sex specific selectivity, and ageing error (in this study ageing error was assumed from age 3 with 70% of fish aged correctly and a 15% error one year either side).

For a time series of estimated proportion at age and sex in the catch, let I index years and J index age and sex, and let $Y_i = \langle Y_{ij} \rangle_{j \varepsilon J}$. It is assumed that $Y_i \sim \mathrm{Mult}(n_i, p_i)$ where "Mult" denotes the multinomial distribution, n_i is the given sample size in year i, and $p_i = \langle p_{ij} \rangle_{j \varepsilon J}$ is the vector of true proportions (after ageing error transformation, if specified).

All of the time series are considered to be mutually independent, so that the combined log likelihood is the sum of the individual log likelihoods. For the time series of estimated proportion at age and sex in the catch, the non-constant portion of the negative log likelihood is:

$$-\sum_{i \varepsilon I} n_i \sum_{j \varepsilon J} O_{ij} \log(p_{ij})$$

where $O_{ij} = Y_{ij} / n_i$ is the observed proportion at age and sex in year i.

For a time series of relative biomass indices or estimated numbers at age and sex (indexed by K as above), the non-constant portion of the negative log likelihood is:

$$\sum_{k \varepsilon K}\left[\log(qT_k c_k) + \frac{1}{2c_k^2}\left(\frac{X_k}{qT_k} - 1\right)^2\right]$$

The value of q which minimises the above equation can be found analytically and it is equal to:

$$\frac{\sqrt{m_1^2 + 4m_2} - m_1}{2}$$

where

$$m_1 = \frac{1}{n}\sum_{k \varepsilon K}\frac{X_k}{T_k c_k^2} \quad \text{and} \quad m_2 = \frac{1}{n}\sum_{k \varepsilon K}\frac{X_k^2}{T_k^2 c_k^2}$$

and n is the cardinality of K.

Penalty function fitting of Z

Variations of the maximum likelihood and least squares estimators were formed by incorporating a penalty function based on the ratio of an "external" (i.e., outside the model) estimate of Z and the predicted estimate of Z. The observed and predicted survival rates were estimated using the maximum likelihood estimator (Chapman & Robson 1960):

$$s = \frac{\overline{a}}{1+\overline{a}}$$

where $\overline{a}$ is the average age beyond a user specified minimum age (usually the age of full recruitment or vulnerability).

A squared log ratio was used as a penalty function using a high multiplier so that for the estimate the predicted survival rate is approximately equal to the observed survival rate. For a single observation of catch at age or estimated numbers at age, let s_o be the observed survival rate and s_p be a predicted survival rate, then the penalty P added to the total sum of squares or the negative log likelihood is

$$P = 100\left[\log(\frac{s_o}{s_p})\right]^2$$

If there are multiple observations of catch at age or estimated numbers at age, then a penalty is added for each observation.