# Chapter 15 - Fluid Mechanics
## Thursday, March 24th

- Fluids – Static properties
  - Density and pressure
  - Hydrostatic equilibrium
  - Archimedes principle and buoyancy
- Fluid Motion
  - The continuity equation
  - Bernoulli's effect
- Demonstration, *i*Clicker and example problems

**Reading: pages 243 to 255 in text book (Chapter 15)**

# Definitions: Density

Pressure, $\rho$, is defined as force per unit area:

$$\rho = \frac{\text{Mass}}{\text{Volume}} = \frac{M}{V} \qquad [\text{Units} - \text{kg.m}^{-3}]$$

Definition of mass – 1 kg is the mass of 1 liter ($10^{-3}$ $\text{m}^3$) of pure water.

Therefore, density of water given by:

$$\rho_{\text{H}_2\text{O}} = \frac{\text{Mass}}{\text{Volume}} = \frac{1\ \text{kg}}{10^{-3}\ \text{m}^3} = 10^3\ \text{kg} \cdot \text{m}^{-3}$$

# Definitions: Pressure ($p$)

Pressure, $p$, is defined as force per unit area:

$$p = \frac{\text{Force}}{\text{Area}} = \frac{F}{A}$$ [Units – $N.m^{-2}$, or Pascal (Pa)]

Atmospheric pressure (1 atm.) is equal to 101325 $N.m^{-2}$.

1 pound per square inch (1 psi) is equal to:

$$1 \text{ psi} = 6944 \text{ Pa} = 0.068 \text{ atm}$$

$$1\text{atm} = 14.7 \text{ psi}$$

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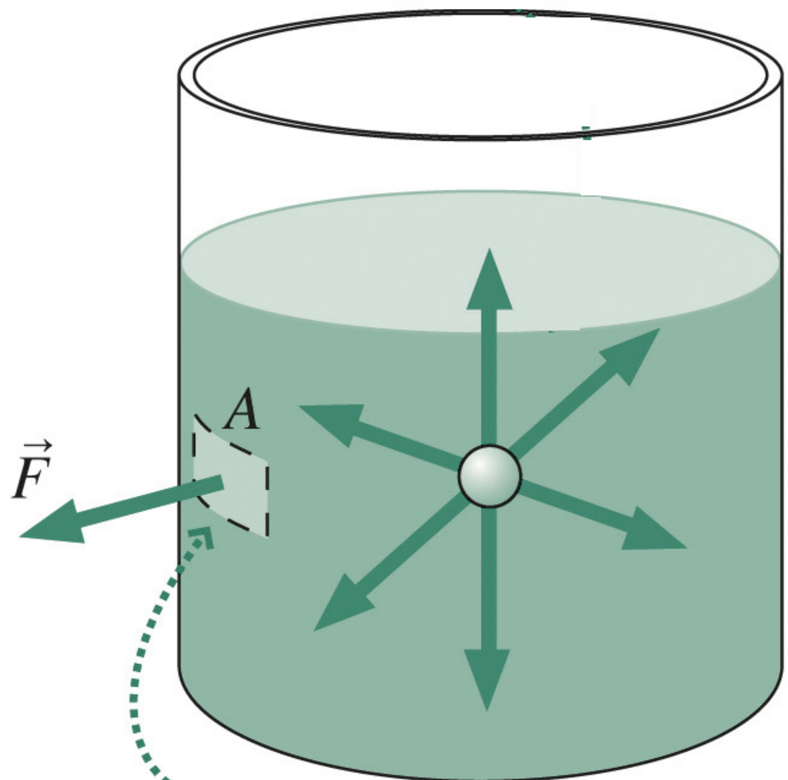
The fluid exerts pressure internally as well as on the container. The internal pressure is the same in all directions.




$\vec{F}$ is the force on the area $A$, so the pressure is $p = F/A$.

# Pressure in Fluids

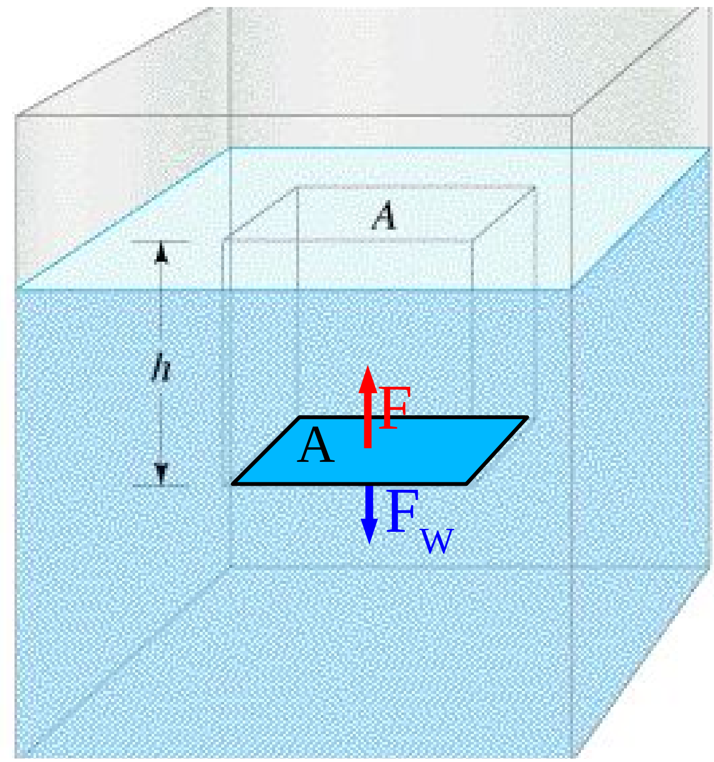
Pressure, $p$, is defined as force per unit area:

$$p = \frac{\text{Force}}{\text{Area}} = \frac{F}{A}$$ [Units – N.m$^{-2}$, or Pascal (Pa)]

In the presence of gravity, pressure in a static fluid increases with depth.

- This allows an upward pressure force to balance the downward gravitational force.
- This condition is hydrostatic equilibrium.
- **Incompressible fluids** like liquids have constant density; for them, pressure as a function of depth $h$ is

$$p = p_0 + \rho g h$$




$p_0$ = pressure at surface

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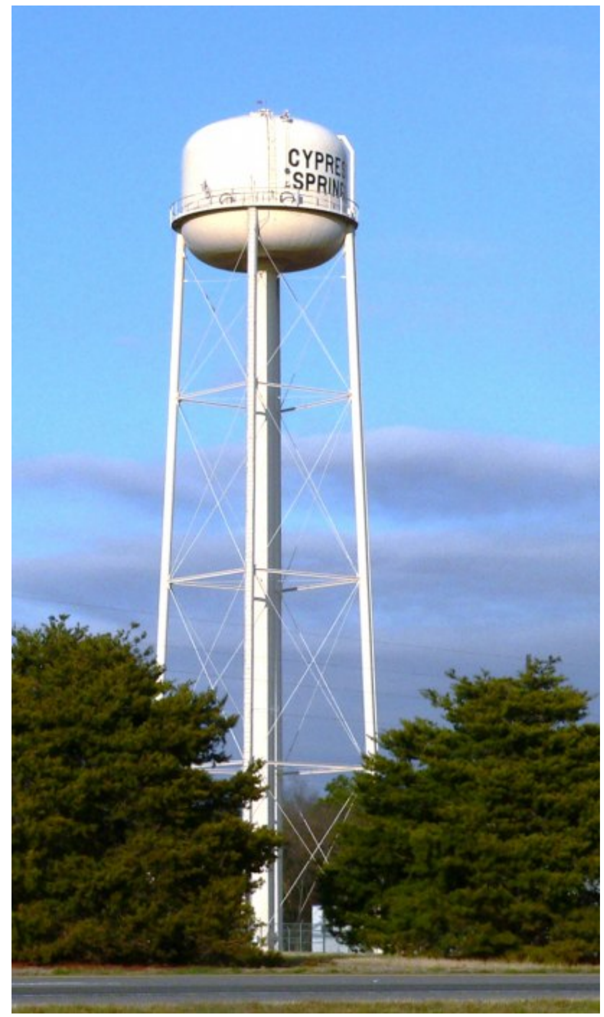


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# Pressure in Fluids

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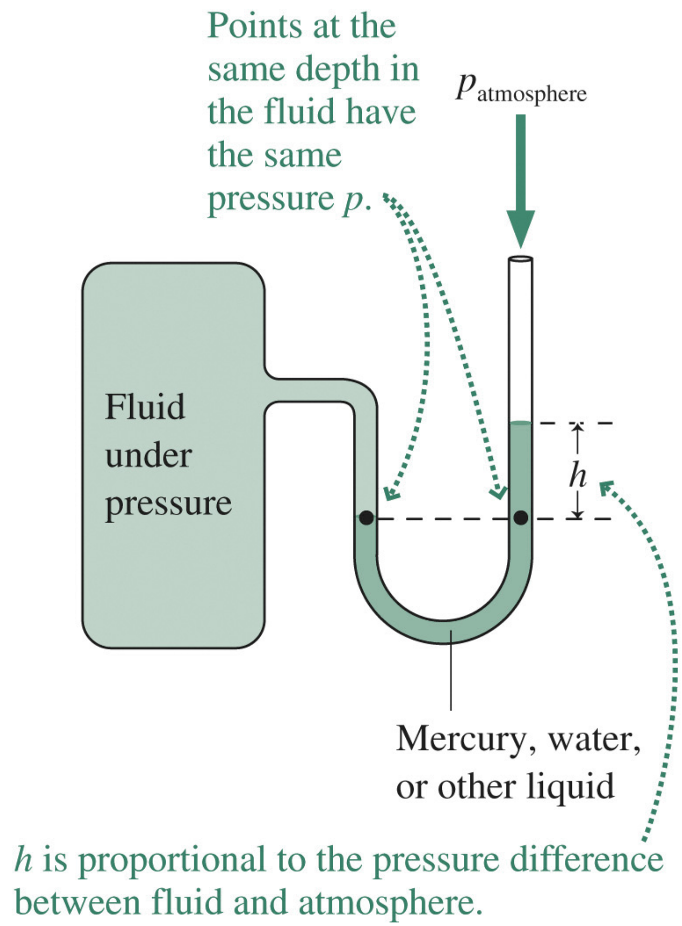
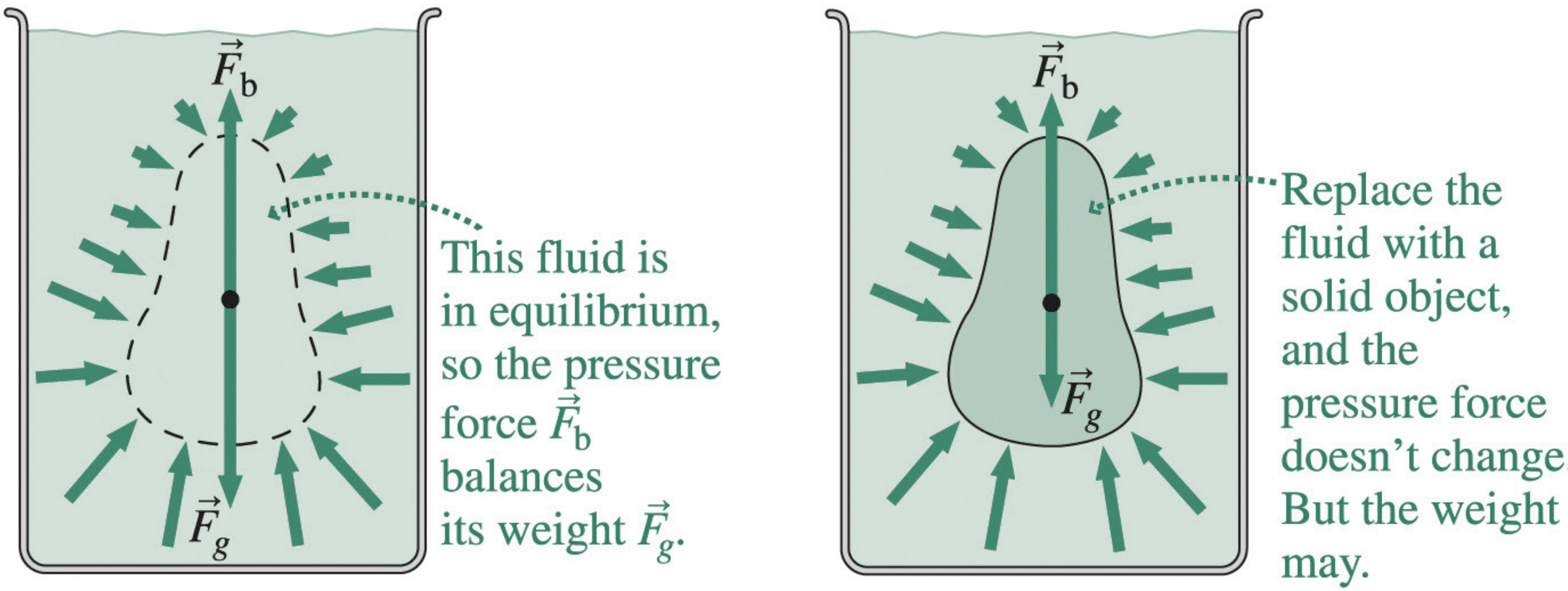
A manometer measures pressure differences.

- Gauge pressure is a measure of pressure relative to the ambient atmosphere:

$$p = p_{atm} + \rho g h$$

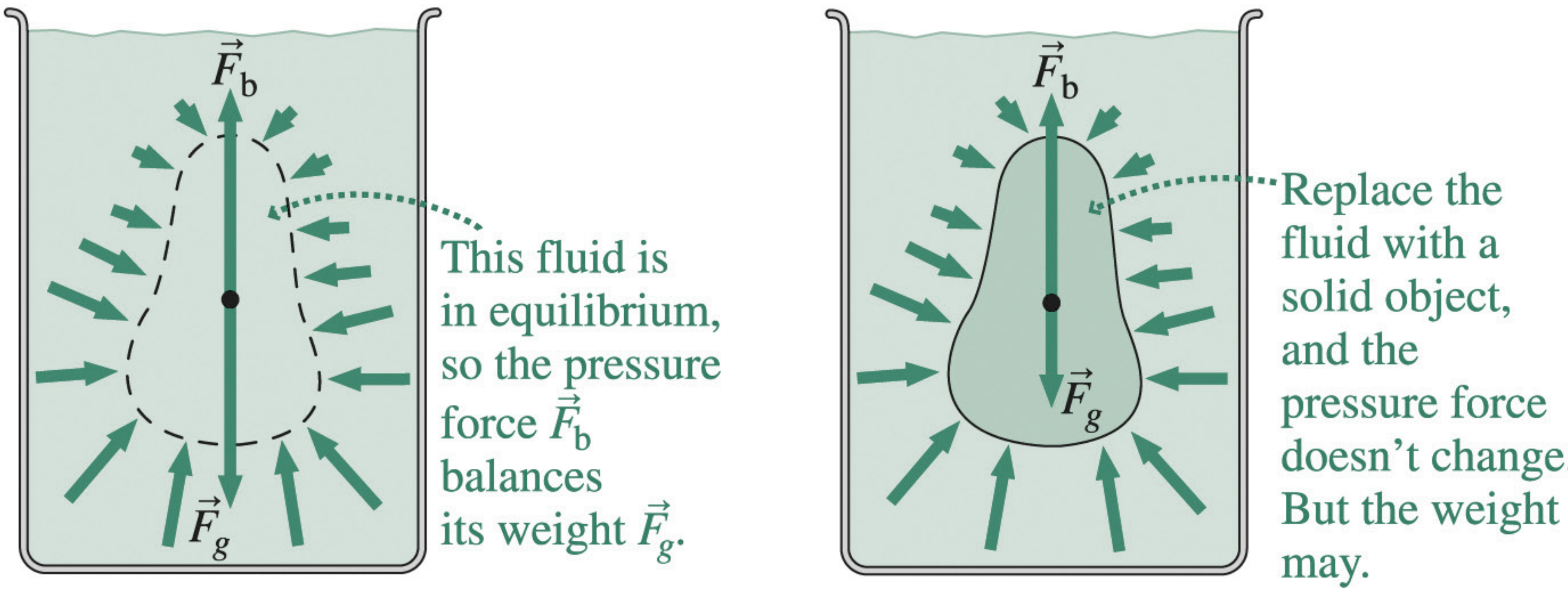


# Pressure in Fluids and Buoyancy




- When a fluid is in hydrostatic equilibrium, the force due to pressure differences on an arbitrary volume of fluid exactly balances the weight of the fluid.
- Replacing the fluid with an object of the same shape doesn't change the force due to the pressure differences.
  - Therefore the object experiences an upward force equal to the weight of the original fluid.

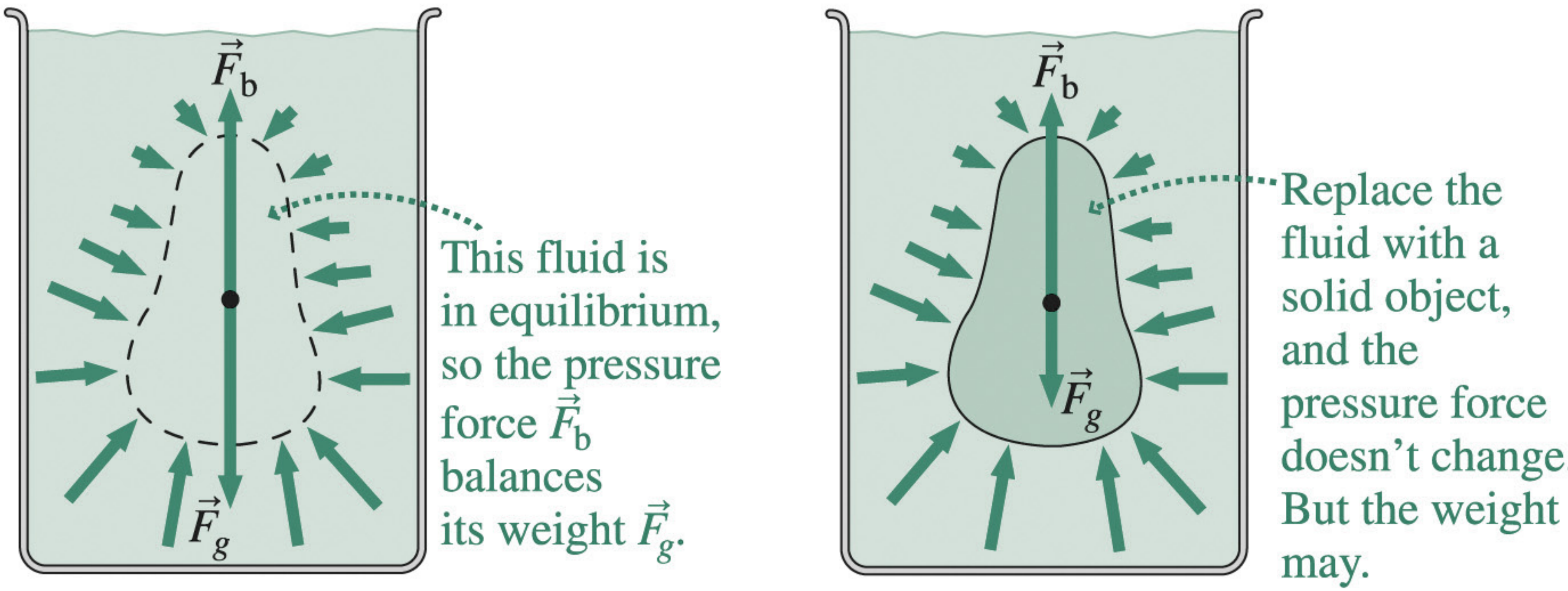
# Pressure in Fluids and Buoyancy




- When a fluid is in hydrostatic equilibrium, the force due to pressure differences on an arbitrary volume of fluid exactly balances the weight of the fluid.
- Replacing the fluid with an object of the same shape doesn't change the force due to the pressure differences.
  - This is **Archimedes' principle**, which states that the buoyancy force is equal to the weight of the displaced fluid:

$$F_b = Mg = \rho_f Vg$$

# Sink or float?




$$F_b = \rho_f V g \qquad F_g = \rho_o V g$$

$$F_b - F_g = \left(\rho_f - \rho_o\right) V g$$

$\rho_o < \rho_f$, float

$\rho_o > \rho_f$, sink

$\rho_o = \rho_f$, apparent weightlessness

Specific gravity:

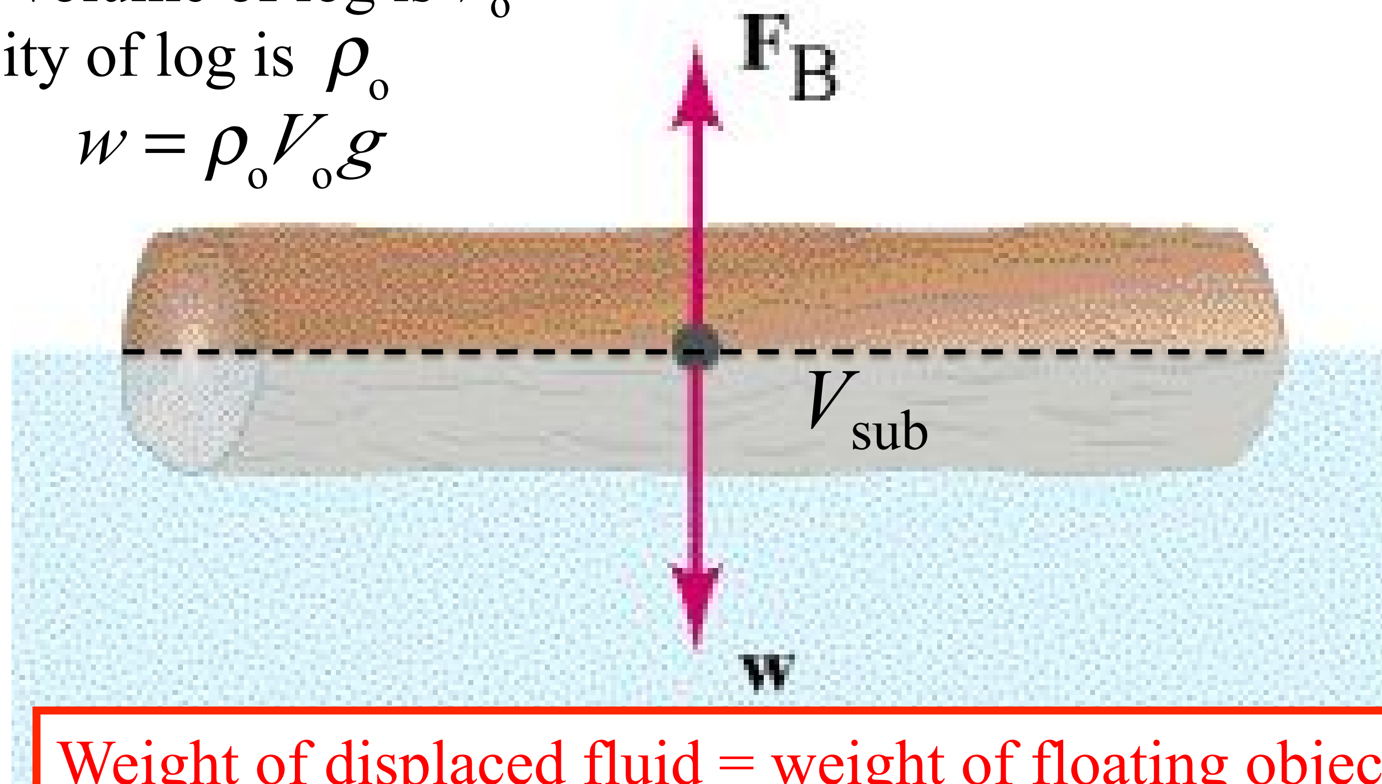
$$\text{S.G.} = \frac{\rho_o}{\rho_w}$$

# If it floats, how much is submerged?

Total volume of log is $V_o$

Density of log is $\rho_o$

$\Rightarrow \quad w = \rho_o V_o g$

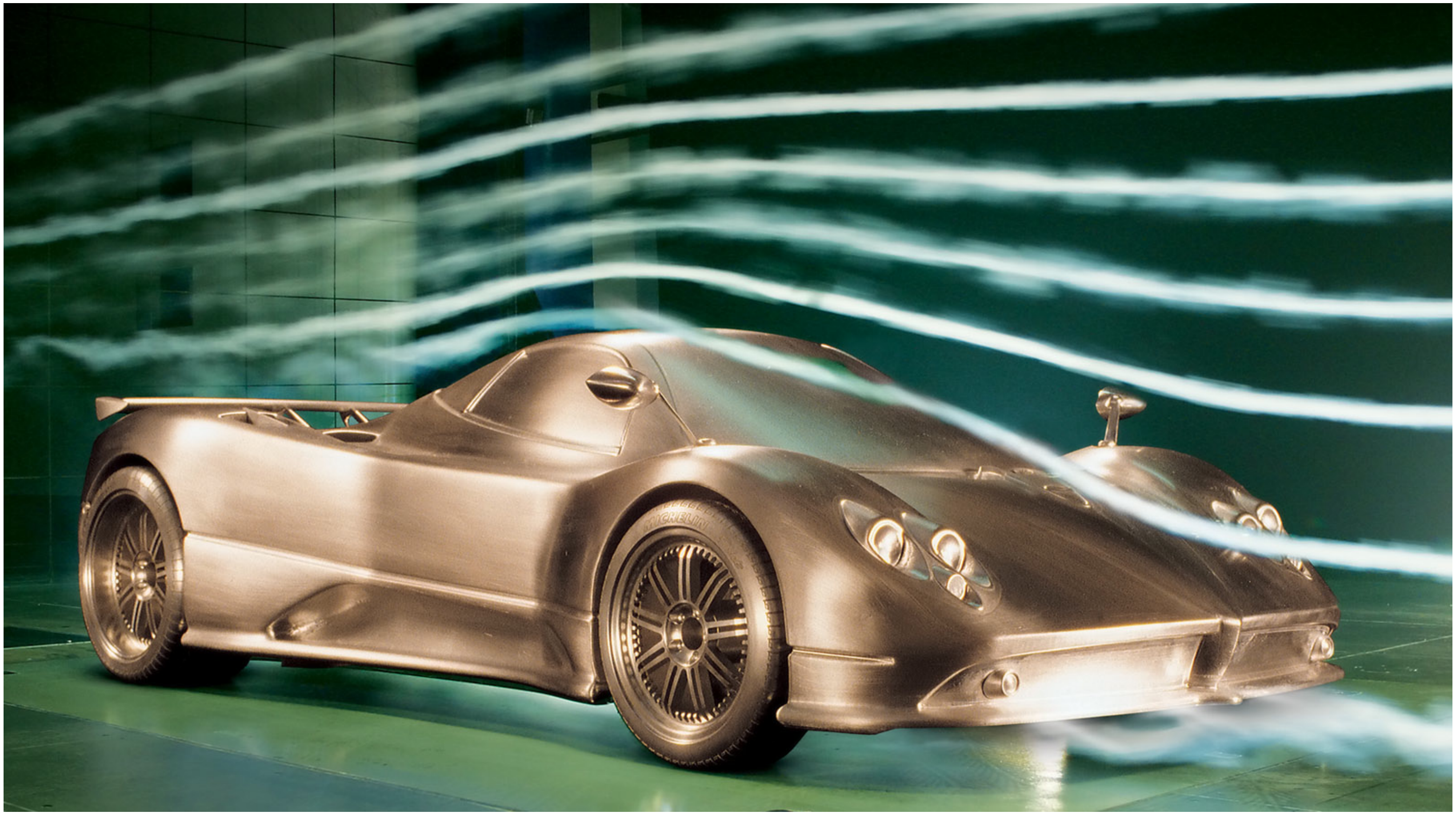



Weight of displaced fluid = weight of floating object

Displaced volume of fluid is $V_{sub}$ $\quad \Rightarrow \quad F_b = \rho_f V_{sub} g$

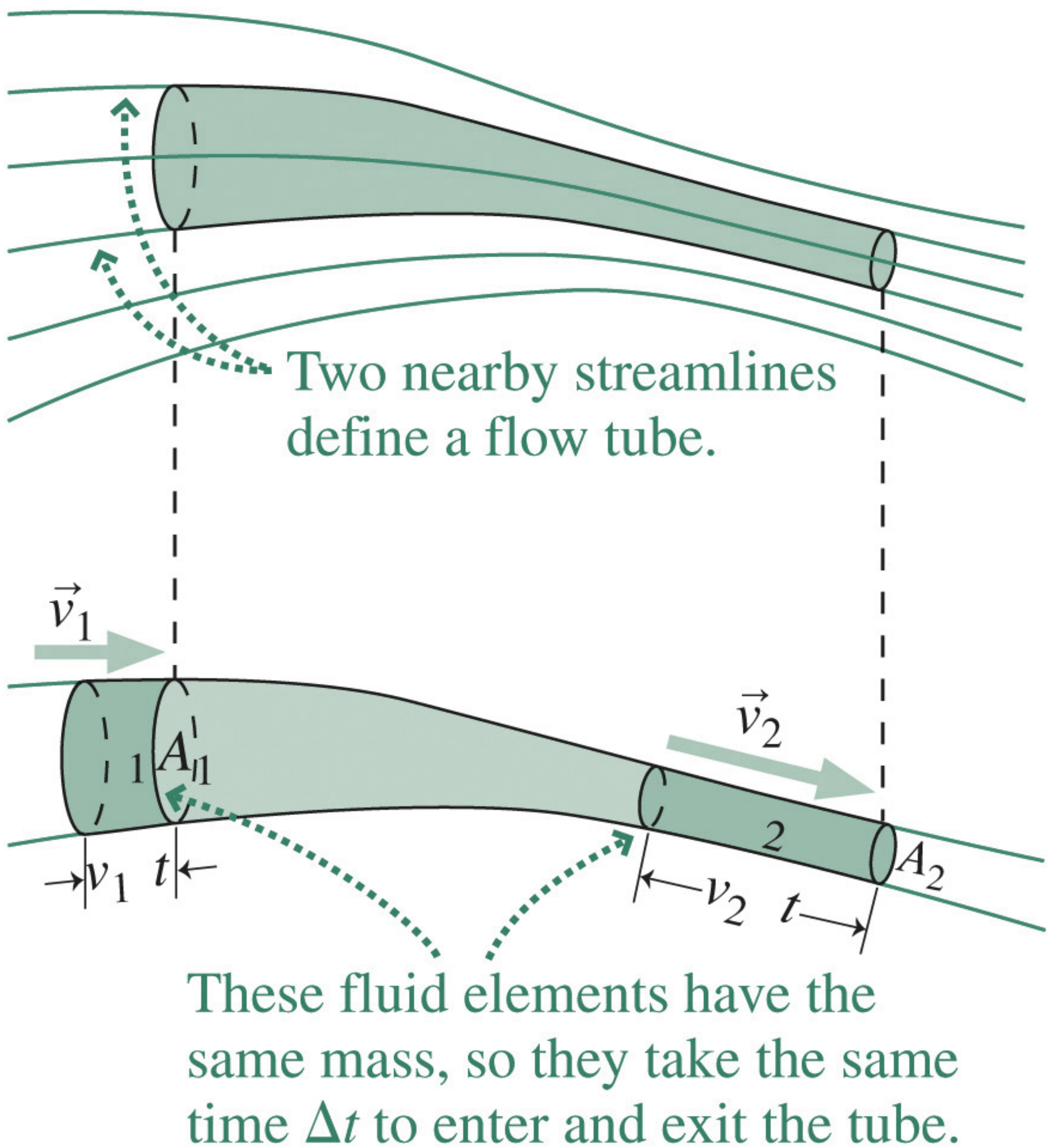
So, $\quad \rho_f V_{sub} \not{g} = \rho_o V_o \not{g} \quad \Rightarrow \quad \dfrac{V_{sub}}{V_o} = \dfrac{\rho_o}{\rho_f}$ [= specific gravity if fluid is water]

# The Continuity Equation



- The continuity equation derives from a fairly obvious conservation law: **conservation of mass**.
- The rate at which fluid (air) flows into a region has to be the same as the rate at which it flows out.

# The Continuity Equation




- Mass flow rate (kg/s) on the left must be equal to the mass flow rate on the right.
- Imaginary tubes bound the flow of the fluid.

$$\frac{\text{Mass}}{\text{time}} = \frac{\rho(vtA)}{t} = \rho vA$$

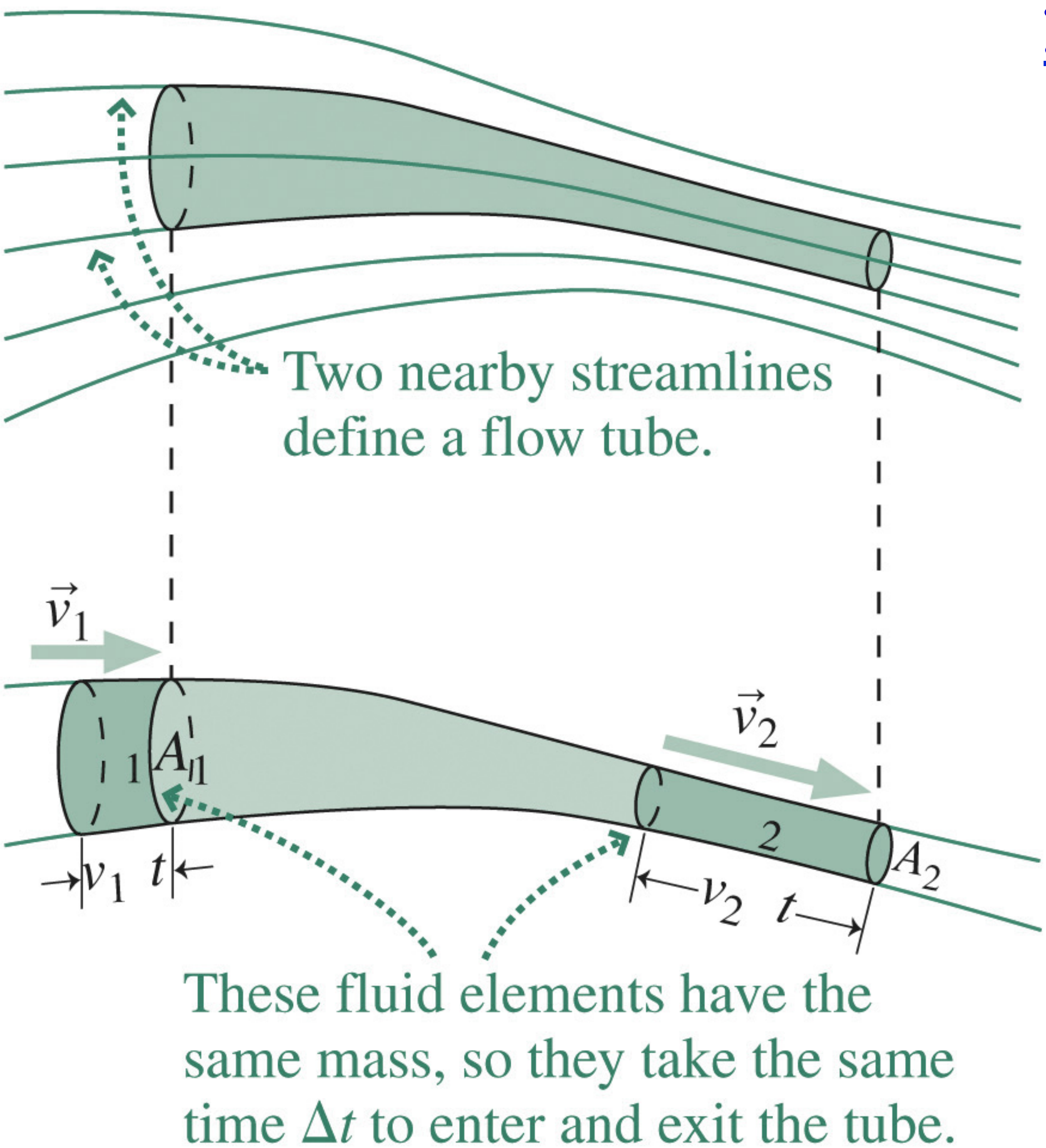
$v$ = velocity of fluid

**Continuity Equation:**

$$\rho vA = \text{constant}$$

$$\text{Or, } \rho_1 v_1 A_1 = \rho_2 v_2 A_2$$

# The Continuity Equation




**Incompressible fluids**

- To a good approximation, one may assume that most fluids are incompressible.
- This implies that their density does not vary when they flow.

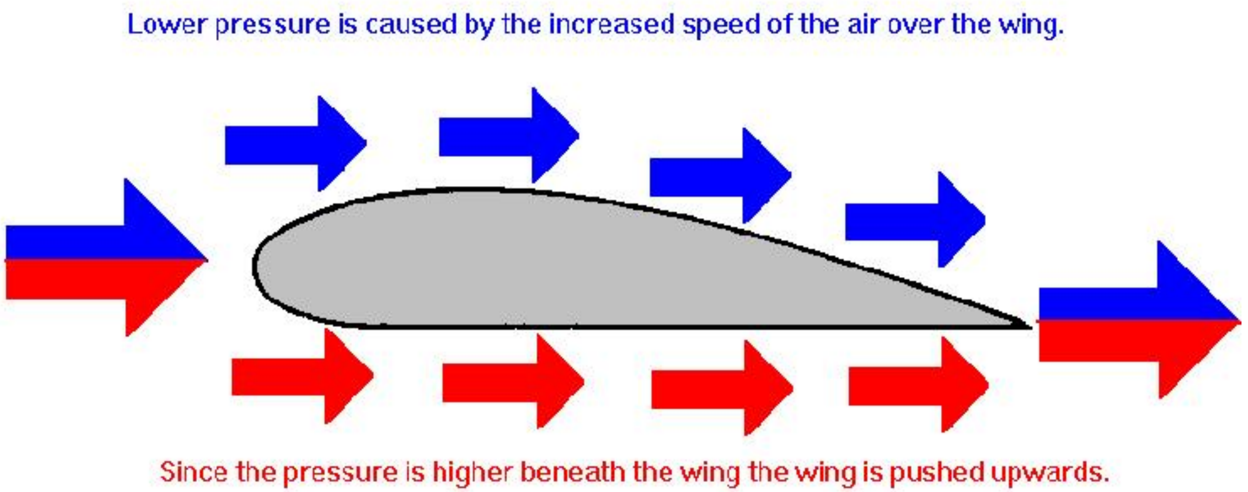
Thus:

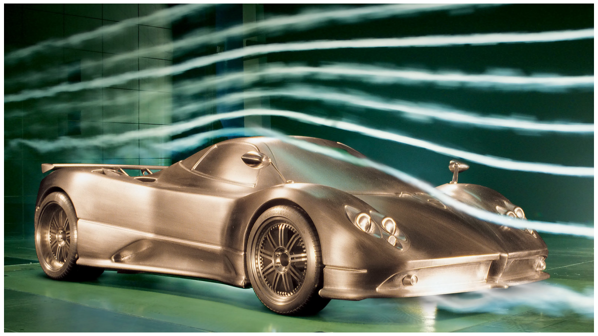
$$\rho = \text{constant}$$

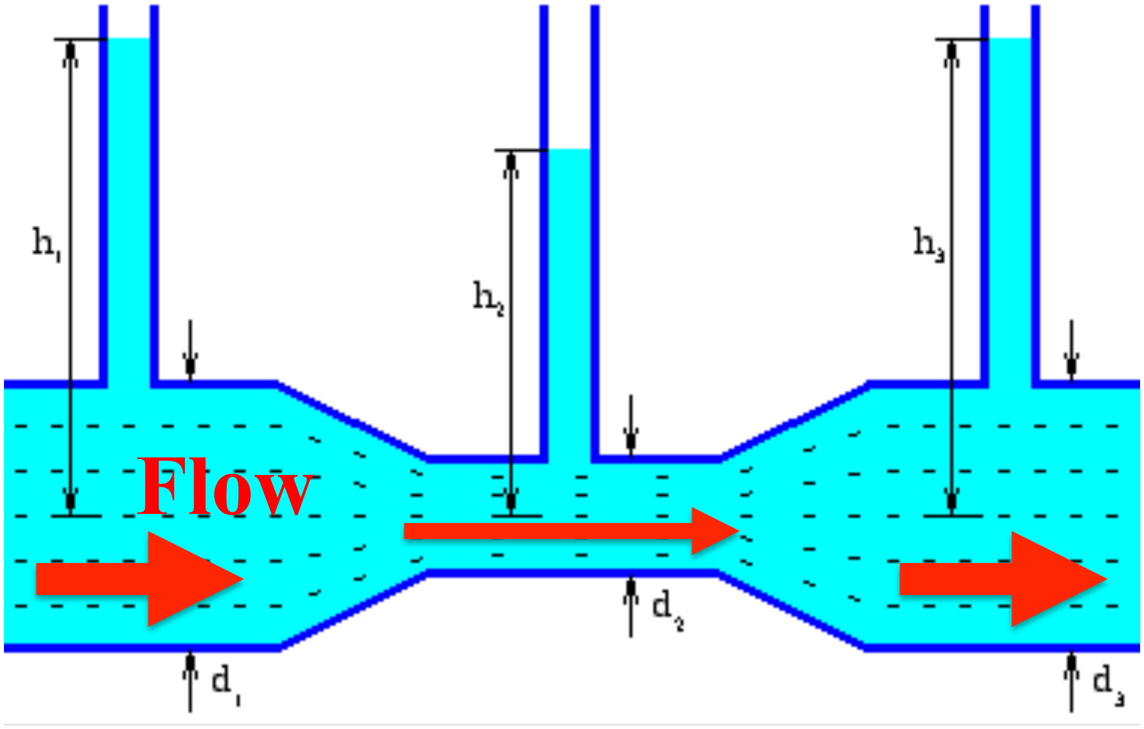
$$vA = \text{constant}$$

$$\Rightarrow \; v_1 A_1 = v_2 A_2$$

# Bernoulli's Principle





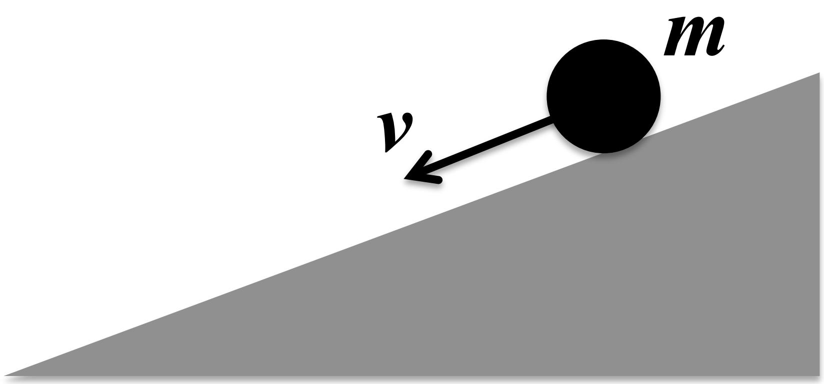

Recall:

$$\rho v A = \text{constant}$$

- Mass flow around an object affects the flow velocity.
- Apparently the flow velocity affects the pressure!!

# Bernoulli's Principle

- The Bernoulli equation derives from another conservation law that you already know: **conservation of energy**.




Energy conservation:

$$\Delta K + \Delta U = 0$$

$$\Rightarrow K + U = \text{constant}$$

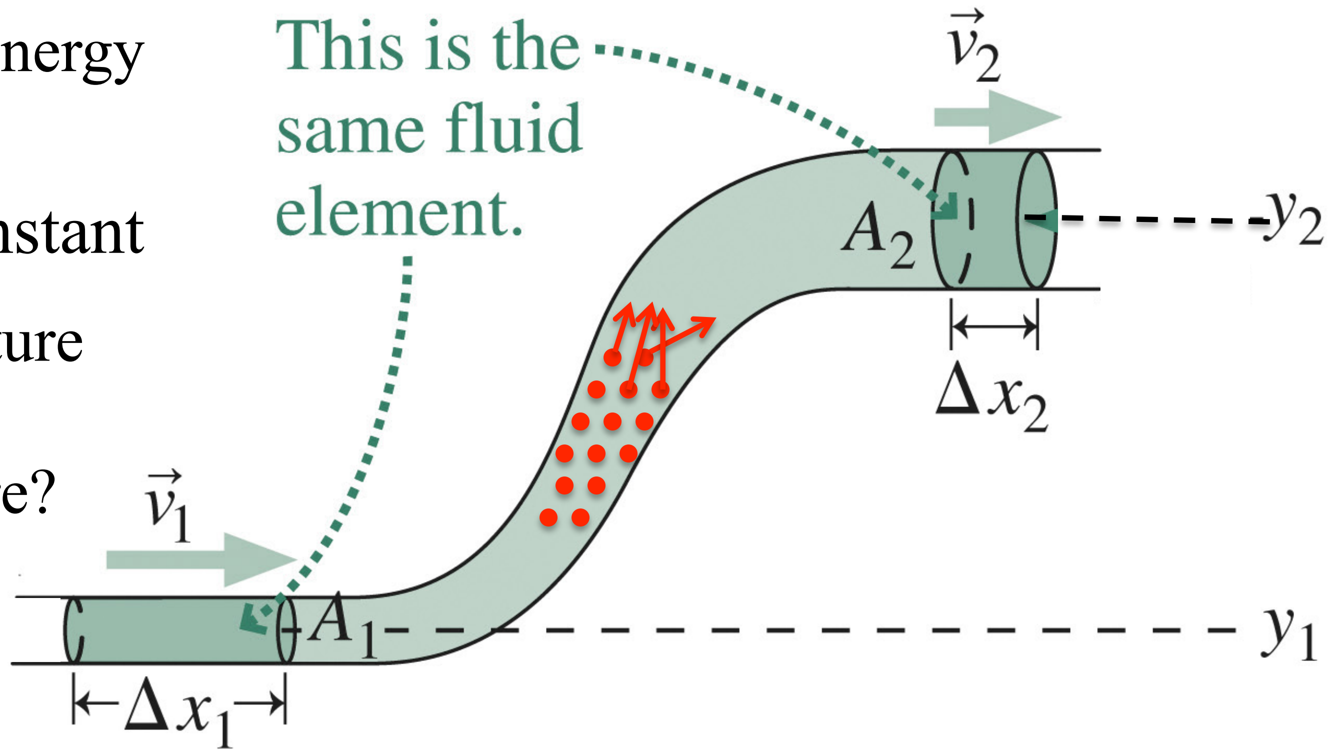
$$\text{Or } \tfrac{1}{2}mv^2 + mgh = \text{constant}$$

# Bernoulli's Principle

- The Bernoulli equation derives from another conservation law that you already know: **conservation of energy**.
  - How do we apply energy conservation?

$$\frac{1}{2}mv^2 + mgh = \text{constant}$$

  - Does this even capture all of the physics?
  - What about pressure?




- The particles in the fluid interact (PHY2049).
- This gives rise to an additional potential energy term that depends on the pressure of the fluid

# Bernoulli's Principle

- The Bernoulli equation derives from another conservation law that you already know: **conservation of energy**.
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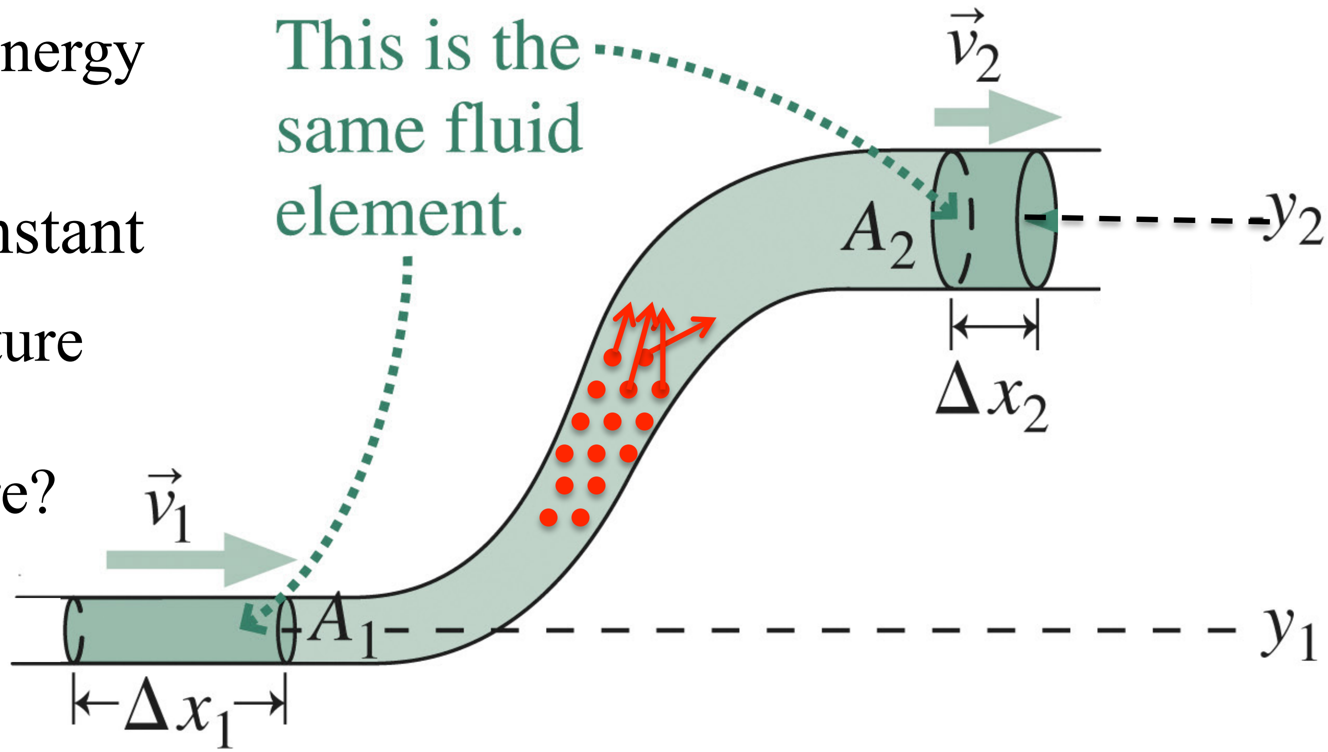



Note:

$$\text{Pressure} \times \text{Volume} = \frac{\text{Force}}{\text{Area}} \times \text{Volume} = \text{Force} \times \text{Length} \equiv \text{Joules}$$

$$\Rightarrow \quad pV + \tfrac{1}{2}mv^2 + mgy = \text{constant}$$

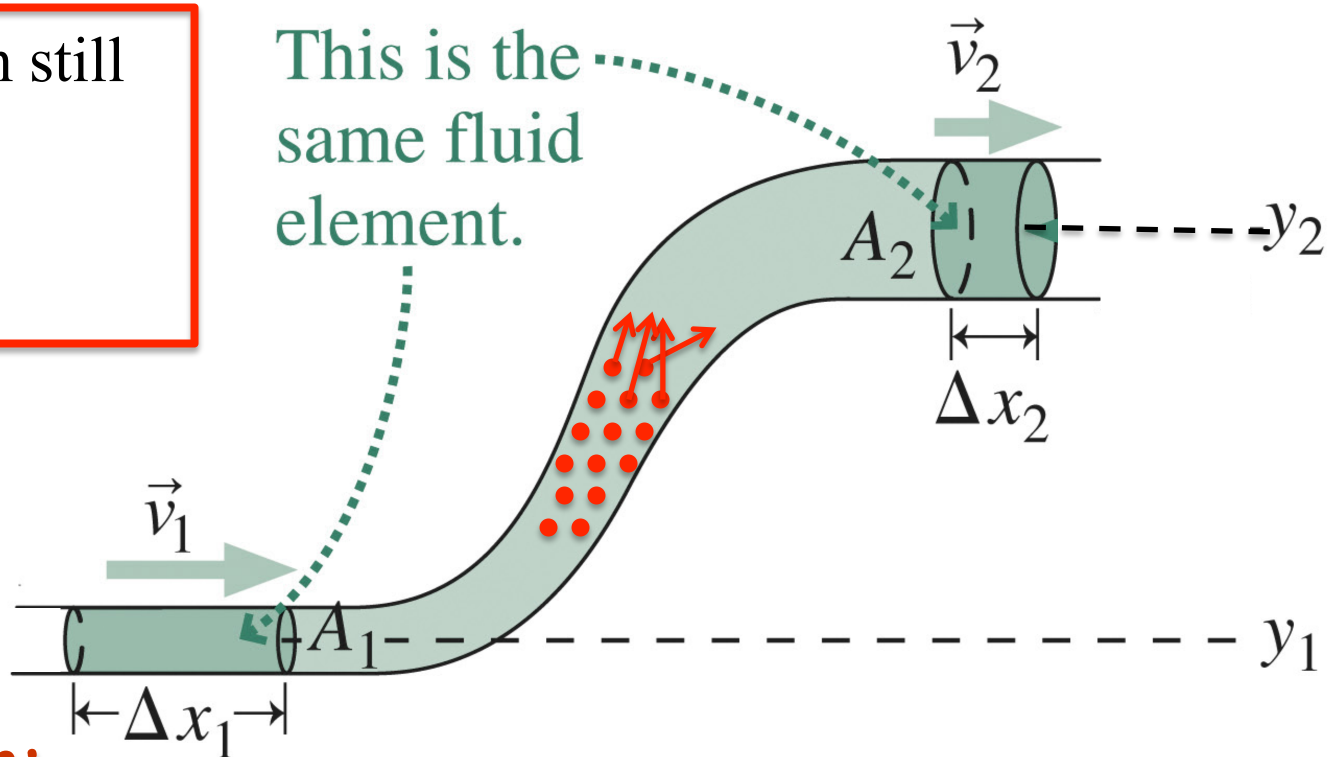
P.E. K.E. P.E.

# Bernoulli's Principle

- The Bernoulli equation derives from another conservation law that you already know: **conservation of energy**.

- Continuity equation still applies:

$$v_1 A_1 = v_2 A_2$$




## Bernoulli's Equation:

$$p + \tfrac{1}{2}\frac{m}{V}v^2 + \frac{m}{V}gy = p + \tfrac{1}{2}\rho v^2 + \rho g y = \text{constant}$$

$$\Rightarrow \qquad p_1 + \tfrac{1}{2}\rho_1 v_1^2 + \rho_1 g y_1 = p_2 + \tfrac{1}{2}\rho_2 v_2^2 + \rho_2 g y_2$$