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Fish Harvesting Experienced by Depensation Growth Function

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Ref

1. Getachew Abiye Salilew, *Mathematical Bio-Economics of Fish Harvesting with Critical Depensation in Lake Tana,* TJPRC: JMCAR 3(2016) 1-14.

Fish Harvesting Experienced by Depensation Growth Function

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Abstract- Getachew [1] introduced there is overfishing in Lake Tana, Amhara, Ethiopia using critical depensation model. In this paper, we applied this technique for depensation model. We investigated that if we consider depensation model we found that there is overfishing.

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I. Introduction

The economics of renewable resource use is essentially a multi-disciplinary undertaking, integrating both biological and economic aspects. When modeling the dynamics of the resource, one has to choose a level of analysis. An intuitive entity is the organism itself (a fish, a tree, or a cow) that experiences growth and mortality. While growth and mortality determine the dynamics of the existing number of individuals, it is the potential to reproduce which characterizes renewable resources. Resources whose reproduction is completely outside of the control of the resource users can perhaps best be analyzed in the framework of "eating a cake of unknown size". We will meet resources whose reproduction can be completely controlled when discussing forestry issues, but aquaculture could be another example. For most resource management problems however, it will be useful to model a reproduction function which depends in some (possibly highly nonlinear, possibly very stochastic) way on the existing number of individuals, which in turn are influenced by the current exploitation regime. In the absence of regulation control over harvesting behavior, the resource stocks are subject to open access [1]. In addition to the viewpoint of an organism, one could also focus on the dynamics of the underlying processes. Most models will take an aggregated view means analyzing a fishery, a forest, an ecosystem as a whole.

In different renewable resource management, it is important to balance ecological and economic needs. For example if we consider one of the renewable resource (fish), the fishery management is the consideration of the ecological effects of harvesting. Fisherman work to provide fish for a growing human population but because of this some fish populations have been dangerously declining. A major focus in fishery management is how best to ensure harvesting sustainability [2, 4, 5, 6]. The object of the management is to devise harvesting strategies that will not drive species to extinction. Therefore, the notion of persistence, extinction times of the populations and precautionary harvesting policy, is always critical. A control variable of every fishery

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management is the fishing effort [3, 8], which is defined as a measure of the intensity of fishing operations. As fishery management is the balance between harvesting and its ecological implications, it is important to fish in such a way that a species is sustainable and not in danger of becoming extinct. Mathematical bio economics is the study of the management of renewable resources. It takes into consideration not only economic questions like revenue, cost, price, effort etc., but also the impact of this demand on the resource. The aim of fish harvesting management is to gain a sustainable development of activity so that, future generation can also benefit from the resources. In this paper we consider depensation (weak Alee effect) deterministic model with a constant harvest rate as well as time dependent. Optimization and numerical calculations were used to determine the harvest rate that produces maximum yield under different population density scenarios. The dynamic mathematical models set on the background of biology and economics knowledge. The integration of these seemingly different subjects namely mathematics, biology and economics creates the source of interesting results and give valuable applications for the peoples living with fishing activities and those policy makers who involved control of overfishing.

II. Mathematical Biology of Depensation Model

Deterministic models of fishery populations can be classified into three types namely compensation, depensation and critical depensation. Compensation model is a growth type where population declination is compensated by increased growth rate. Depensation model is the opposite case to composition growth model. The critical depensation modelis the generalized logistic model which is extremely in opposite of the depensation model. A population's dynamics are depensatory or depensation is said to occur if the per- capita rate of growth decreases as the density decreases to low levels. Component of the life-history such as fecundity or survival during a particular stage or the mechanisms that affect these components (such as group defense or mate-finding difficulty) are called depensatory if they decrease the per-capita growth rate as abundance declines to low levels. Depensation model is the label most often used in fisheries. The strong depensation model is called critical depensation model. By the work done [9] some populations experience reduced rates of survival and reproduction when reduced to very low densities. Mathematical biology expression of depensation is given by growth model as:

$$(dx/dt) = rx^{a}[1-(x/k)] \tag{1}$$

Here in the growth model (1), $x(t)$ represents fish biomass, r represents intrinsic growth rate of fish, k is ecological carrying capacity, $a \geq 0$ and $a \neq 1$, and (dx/dt) growth rate of fish without harvest. Model (1) has the property that for $a > 1$ there is, at low stock levels, depensation, which is a situation where the proportionate growth rate is an increasing function of the stock size, as opposed to being a decreasing function (compensation) in the simple logistic case where $a = 1$. The biological growth model (1) exhibiting depensation at the stock level below, y_0, and compensation thereafter as shown the figure below.

Ref

3. F. Brauer and C. Castillo-Chavez, *Mathematical Models in Population Biology and Epidemiology*, Springer-Verlag, 2001.

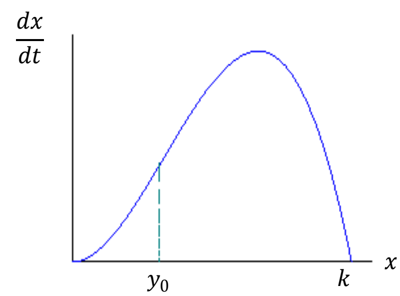

Figure 1: Growth curve of depensation model for $r = 1.4,\ a = 2k = 3.2,\ y_0 = 1$

Notes

In the figure 1 we have shown the rate change curve of the depensation model for some positive particular values of the parameters as shown. The curve is plotted for the population size function $x(t)$ versus the population rate change function $x' = (dx/dt)$. The maximum rate change of population, $Max(x')$, occurs when the population size be $x(t) = (ak/(a+1))$ for $a > 0$ and the corresponding maximum rate change of population is given by $Max(x') = \{r(ak/(a+1))^a(1-(a/(a+1)))\}$.

III. Solution of the Depensation Model

The solution of depensation model (1) with initial condition $x(0) = x_0$ is obtained as follows. Using techniques of separable variables the model (1) can be rewritten as $[\,dx/x^a(k-x)] = (r/k)dt$ and integrating both sides we get $\int[\,1/x^a(k-x)]dx = \int(r/k)dt$. To integrate the left hand side, we have to consider the following two cases by assuming that the initial value $x(0) = x_0$; a is an integer and $a > 1$ and then applying integration by partial fraction.

Case1: Let a is an even integer so that $a = 2n,\ n \in Z^+$.

$\int[1/x^a(k-x)]\,dx = \int[1/x^{2n}(k-x)]\,dx = \int[1/(x^2)^n\,(k-x)]\,dx = (r/k)\,t + \ell$, where $\ell \in \mathcal{R}$

If $n = 1$ then $a = 2$ and thus we do have $\int[1/x^a(k-x)]dx = \int[1/x^2(k-x)]dx$. The solution is obtained using integration by partial fraction and is $ln\left(\frac{x}{k-x}\right) - \frac{k}{x} = rkt + \ln\left(\frac{x_0}{k-x_0}\right) - \frac{k}{x_0}$. If $n = 2$ then $a = 4$ and thus we do have $\int[1/x^4(k-x)]dx = (r/k)t$. The solution is obtained using integration by partial fraction and is:

$$\ln\left(\frac{x}{k-x}\right) - \frac{k}{x} - \frac{k^2}{2x^2} - \frac{k^3}{3x^3} = rk^3t + \ln\left(\frac{x_0}{k-x_0}\right) - \frac{k}{x_0} - \frac{k^2}{2(x_0)^2} - \frac{k^3}{3(x_0)^3}$$

In general by mathematical induction for $a = 2n$ using partial fraction we get the solution:

$$\int \frac{dx}{(x^2)^n(k-x)} = \int (\frac{A_1x + B_1}{x^2} + \frac{A_2x + B_2}{(x^2)^2} + \cdots + \frac{A_nx + B_n}{(x^2)^n} + \frac{c}{k-x})dx = (r/k)t + \ell$$

This is the general solution. Here $A_1 = \frac{1}{k^{2n}},\ A_2 = \frac{1}{k^{2n-2}},\ \cdots,\ A_{n-1} = \frac{1}{k^4},\ A_n = \frac{1}{k^2}$; $B_1 = \frac{1}{k^{2n-1}},\ B_2 = \frac{1}{k^{2n-3}}, \ldots,\ B_{n-1} = \frac{1}{k^3},\ B_n = \frac{1}{k}$; $c = A_n = \frac{1}{k^2}$ and $\ell \in \mathcal{R}$.

Case 2: Let a is an odd integer so that $a = 2n + 1,\ n \in Z^+$.

$\int[1/x^a(k-x)]dx = \int[1/x^{(2n+1)}(k-x)]dx = (r/k)t + \ell$, where $\ell \in \mathcal{R}$

If $n = 1$ then $a = 3$ and thus we have, $\int[1/x^3(k-x)]dx = (r/k)t$. The solution is obtained using integration by partial fraction and is:

$$\ln\left(\frac{x}{k-x}\right) - \frac{k}{x} - \frac{k^2}{2x^2} = rk^2t + \ln\left(\frac{x_0}{k-x_0}\right) - \frac{k}{x_0} - \frac{k^2}{2(x_0)^2}$$

If $n = 2$ then $a = 5$ and thus we do have $\int[1/x^5(k-x)]dx = (r/k)t$. The solution is obtained using integration by partial fraction and is:

$$\ln\left(\frac{x}{k-x}\right) - \frac{k}{x} - \frac{k^2}{2x^2} - \frac{k^3}{3x^3} - \frac{k^4}{4x^4} = rk^4t + \ln\left(\frac{x_0}{k-x_0}\right) - \frac{k}{x_0} - \frac{k^2}{2(x_0)^2} - \frac{k^3}{3(x_0)^3} - \frac{k^4}{4(x_0)^4}$$

In general by mathematical induction for $a = 2n + 1$ and using partial fraction we get the solution:

$$\int \frac{dx}{x^{2n+1}(k-x)} = \int \left(\frac{A_1}{x} + \frac{A_2}{x^2} + \frac{A_3}{x^3} + \cdots + \frac{A_{2n}}{x^{2n}} + \frac{A_{2n+1}}{x^{2n+1}} + \frac{B}{k-x}\right)dx = (r/k)t + \ell$$

This is the general solution. Where $A_1 = \frac{1}{k^{2n+1}}$, $A_2 = \frac{1}{k^{2n}}$, $A_3 = \frac{1}{k^{2n-1}}$, $A_4 = \frac{1}{k^{2n-2}}$, $\cdots$, $A_{2n} = \frac{1}{k^2}$, $A_{2n+1} = \frac{1}{k}$, $B = \frac{1}{k^{2n+1}}$ and $\ell \in \mathcal{R}$.

When we combine the above two cases we found that $\forall a \in Zl^+$ and $a \geq 2$ the general implicit solution of the depensation model is:

$$\int \frac{dx}{x^a(k-x)} = ln\left(\frac{x}{k-x}\right) - \sum_{n=1}^{a-1} \frac{1}{n}\left(\frac{k}{x}\right)^n = r(k^{a-1})t + \ell \tag{2}$$

Result (3) is the required particular implicit solution of the depensation model (1).

$$ln\left(\frac{x}{k-x}\right) - \sum_{n=1}^{a-1} \frac{1}{n}\left(\frac{k}{x}\right)^n = r(k^{a-1})t + ln\left(\frac{x_0}{k-x_0}\right) - \sum_{n=1}^{a-1} \frac{1}{n}\left(\frac{k}{x_0}\right)^n \tag{3}$$

The following graph represents the stock level of the depensation model.

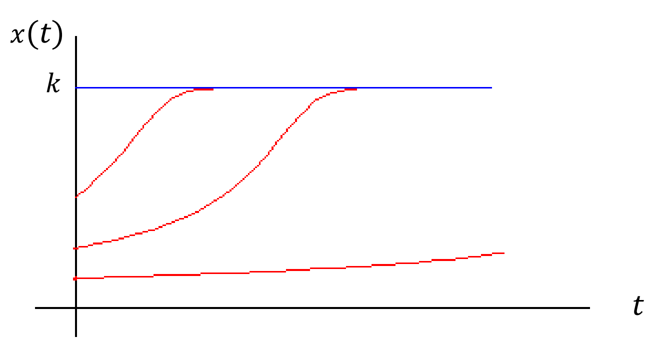

Figure 2: Typical solution curve for depensation model for $r = 1.6$, $k = 2.0$, $a = 3.0$

Notes

In figure 2 we have time series plot for depensation model which verifying local stability of the three equilibrium point in model (1). The depensation model has two equilibrium points namely the trivial and non-trivial equilibrium points $x = 0$ or $x = k$ respectively which are obtained by making $\frac{dx}{dt} = rx^a\left(1 - \frac{x}{k}\right) = 0$. The equilibrium point $x = 0$ is semi stable since $f'(0) = 0$. The non-trivial equilibrium point $x = k$ is stable for $r > 0$ and unstable for $r < 0$[7, 11].

Notes

IV. Mathematical Bio-Economics of Depensation Model

A fishery is an area with an associated fish or aquatic population which is harvested for its commercial or recreational value. Population dynamics describes the ways in which a given population grows and shrinks over time, as controlled by birth, death, and emigration or immigration. It is the basis for understanding changing fishery patterns and issues such as habitat destruction, predation and optimal harvesting rates. With the natural positive population growth the population size can be brought down whenever harvesting is introduced. Schaefer catch equation is a bilinear short-term harvest function and it assumes that effort always removes a constant proportion of the stock. Depensation Mathematical Bio-Economics model is given by $\frac{dx}{dt} = f(x) - h(E,x)$ where $f(x) = rx^a[1 - (x/k)]$ is the growth function of fish and $h(E,x) = qEx$ is the harvest function of fish. And thus we do have

$$(dx/dt) = rx^a[1 - (x/k)] - qEx \tag{4}$$

Here in the model (4), $x(t)$ represents fish biomass, r represents intrinsic growth rate of fish, k is ecological carrying capacity, $a \geq 0$ and $a \neq 1$, t is time, (dx/dt) is growth rate of the fish with harvest function $h(E,x)$.

a) Equilibrium Points of Bio-Economics of Depensation Model

The equilibrium points of model (4) are obtained by making $(dx/dt) = 0 \Leftrightarrow rx^a[1 - (x/k)] - qEx = 0$. This implies that the trivial equilibrium point $x = 0$ or the non-trivial equilibrium point: $rx^{a-1}\left(1 - (x/k)\right) - qE = 0$ for $a > 1$ are the equilibrium points. If we take $a = 2$, we get, $rx^2 - rkx + qkE = 0$. So that the non-trivial equilibrium points are

$$x_1 = \frac{rk + \sqrt{(rk)^2 - 4rqkE}}{2r} \quad or \quad x_2 = \frac{rk - \sqrt{(rk)^2 - 4rqkE}}{2r} \tag{5}$$

Provided that $rk > 4qE$ and since $rk > \sqrt{(rk)^2 - 4rkqE}$ both equilibrium points are positive for positive parameters r, q, E and k.

The stability analysis of the equilibrium points is obtained by identifying the algebraic sign of the first derivative of the function at each equilibrium points. That is, $(dx/dt) = g(x) = rx^a[1 - (x/k)] - qEx$. So, its first derivative for $a = 2$ is, $g'(x) = 2rx - \frac{3r}{k}x^2 - qE$. Since $g'(0) = -qE < 0$ implies equilibrium point $x = 0$ is stable. Next we have, $g'(x_1) = -(rk/2) + 2qE - \frac{1}{2}\sqrt{(rk)^2 - 4rkqE} < 0$ this implies that x_1 in (5) is stable. Further we have, $g'(x_2) = -\frac{rk}{2} + 2qE + \frac{1}{2}\sqrt{(rk)^2 - 4rkqE}$ this implies that, $x_2 = \frac{rk - \sqrt{(rk)^2 - 4rqkE}}{2r}$ is stable if $\sqrt{(rk)^2 - 4rkqE} < rk - 4qE$ and unstable otherwise.

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b) Maximum Sustainable Yield (MSY) Of The Depensation Model

Schaefer catch equation is a bilinear short-term harvest function and it assumes that effort always removes a constant proportion of the stock.

$$H(E,x) = qEx \tag{6}$$

Where H =catch measured in terms of biomass; E fishing effort and q is a constant catchability of coefficient. And substituting the non-trivial bio-economic equilibrium points of (5) in (6) gives the harvesting function as a function of effort E. Let $H(x_1, E) = H_1(E)$ and $H(x_2, E) = H_2(E)$ then we got

$$H_1(E) = (qE/2r)\left[kr + \sqrt{(kr)^2 - 4krqE}\right] \tag{7}$$

$$H_2(E) = (qE/2r)\left[kr - \sqrt{(kr)^2 - 4krqE}\right] \tag{8}$$

The effort at the maximum sustainable yield denoted by E_{MSY} is obtained by making the first derivative of H with respect to the effort E equal to zero. That is $\frac{d(H_1)}{dE} = 0$, gives $E = 0$ or $E = \frac{2kr}{9q}$. Thus, $E_{MSY} = \frac{2kr}{9q}$. We have the same result for, $\frac{d(H_2)}{dE} = 0$.

And thus the corresponding Maximum Sustainable Yield in (7) to be

$$MSY_1 = H_1(E_{MSY}) = \frac{4k^2r}{27}$$

Again the corresponding Maximum Sustainable Yield in (8) to be

$$MSY_2 = H_2(E_{MSY}) = \frac{2k^2r}{27}$$

From this we conclude that, $MSY_1 = 2MSY_2$.

c) The Open Access Yield (OAY) For The Depensation Model

A work done in [10] shows that economic models of fishery are underlined by biological models and it is impossible to formulate any useful economic model of fishery without specifying the underlining biological dynamics of the fishery. Based on constant price and unit cost of effort the total revenue denoted by TR will be calculated using the formula $TR(E) = p.H(E)$, where p is the average price per kilogram of fish. The relationship between cost and effort is assumed to be linear and then the total cost of fishing effort denoted by TC is defined as $TC(E) = c.E$, where c is the unit cost of effort that includes cost of labor and capital and E is the unit of effort and thus the total economic rent of fishery denoted by TER defined as

$$TER(E) = TR(E) - TC(E) \tag{9}$$

At the open access point, total fishing costs are equal to total revenues from the fishery. Then the open access effort is obtained by equating $TR(E) = TC(E)$. Where $TR(E) = pqxE$ and $TC(E) = cE$ which yields $pqxE = cE$. To calculate the effort for the Open Access Yield we used two non-trivial equilibria in (5). And thus we have two

Ref

10. G. Munro, *Fisheries, Extended Jurisdiction and the Economics of Common Property Resources.* The Canadian Journal of Economics 15(1982).

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equations namely $pqx_1E = cE$ and $pqx_2E = cE$. And substituting their corresponding values respectively gives

$$\frac{pq}{2r}E\left(kr + \sqrt{(kr)^2 - 4krqE}\right) = cE \quad (10)$$

$$\frac{pq}{2r}E\left(kr - \sqrt{(kr)^2 - 4krqE}\right) = cE \quad (11)$$

From (10), we have $E = 0$ $E = \frac{cr}{pq^2}\left(1 - \frac{c}{pqk}\right)$ that is $E_{OAY} = \frac{cr}{pq^2}\left(1 - \frac{c}{pqk}\right)$ provided, $c < pqk$.

And thus the corresponding Open Access Yield to be

$$OAY_1 = H_1(E_{OAY}) = \frac{q}{2r}.E_{OAY}\left(kr + \sqrt{(kr)^2 - 4krqE_{OAY}}\right)$$

$$OAY_1 = \frac{c}{2pq}(1 - \frac{c}{pqk})\left(kr + \sqrt{(kr)^2 - \frac{4kr^2c}{pq}(1 - \frac{c}{pqk})}\right)$$

Equation (11) gives $E = 0$ or $E = \frac{cr}{pq^2}\left(1 - \frac{c}{pqk}\right)$ that is $E_{OAY} = \frac{cr}{pq^2}\left(1 - \frac{c}{pqk}\right)$ provided, $c < pqk$.

And thus the corresponding Open Access Yield to be

$$OAY_2 = H_2(E_{OAY}) = \frac{q}{2r}E_{OAY}\left(kr - \sqrt{(kr)^2 - 4krqE_{OAY}}\right)$$

$$OAY_2 = \frac{c}{2pq}\left(1 - \frac{c}{pqk}\right)\left(kr - \sqrt{(kr)^2 - \frac{4kr^2c}{pq}(1 - \frac{c}{pqk})}\right)$$

d) The Maximum Economic Yield (MEY) of Depensation Model

The maximum economic yield is attained at the profit maximizing level of effort which is obtained using equation (9). So, $[d(TER(E))/dE] = 0$ implies $[d(TR(E))/dE] = [d(TC(E))/dE]$. To calculate the effort for the Maximum Economic Yield we used two non-trivial equilibria x_1 and x_2 in (5). And thus we have two equations namely $[d(pqx_1E)/dE] = [d(cE)/dE]$ and $[d(pqx_2E)/dE] = [d(cE)/dE]$. And substituting the corresponding values of x_1 and x_2 in these equations give respectively

$$\frac{d}{dE}\left(\frac{pq}{2r}E\left(kr + \sqrt{(kr)^2 - 4krqE}\right)\right) = \frac{d(cE)}{dE} \quad (12)$$

$$\frac{d}{dE}\left(\frac{pq}{2r}E\left(kr - \sqrt{(kr)^2 - 4krqE}\right)\right) = \frac{d(cE)}{dE} \quad (13)$$

From equation (12), we do have the following

$$E^2 - \frac{2r}{9kq^2}\left[k\left(kq + \frac{2c}{p}\right) - \frac{2c^2}{p^2q}\right]E + \frac{r^2ck}{9pq^3}\left(1 - \frac{c}{pqk}\right) = 0$$

Setting $A = k\left(kq + \frac{2c}{p}\right) - \frac{2c^2}{p^2q}$ and $B = 1 - \frac{c}{pqk}$, we obtain

$$E=\frac{r}{3q}\left[\frac{A}{3kq}\pm\sqrt{\left(\frac{A}{3kq}\right)^2-\frac{ckB}{pq}}\right],\ \text{provided that}\ (A/3kq)^2\geq(ckB/pq)\ \text{and,}\ A>0.$$

Thus efforts at maximum economic yield are:

$$E_{MEY_1}=\frac{r}{3q}\left[\frac{\left[k\left(kq+\frac{2c}{p}\right)-\frac{2c^2}{p^2q}\right]}{3kq}+\sqrt{\left(\frac{1}{3kq}\right)^2\left[k\left(kq+\frac{2c}{p}\right)-\frac{2c^2}{p^2q}\right]^2-\frac{ck}{pq}\left[1-\frac{c}{pqk}\right]}\right]$$

$$E_{MEY_2}=\frac{r}{3q}\left[\frac{\left[k\left(kq+\frac{2c}{p}\right)-\frac{2c^2}{p^2q}\right]}{3kq}-\sqrt{\left(\frac{1}{3kq}\right)^2\left[k\left(kq+\frac{2c}{p}\right)-\frac{2c^2}{p^2q}\right]^2-\frac{ck}{pq}\left[1-\frac{c}{pqk}\right]}\right]$$

And thus the corresponding Maximum Economic Yields in (7) to be

$$MEY_1=H_1\left(E_{MEY_1}\right)$$

$$MEY_1=\frac{1}{6}\left[\frac{\left[k\left(kq+\frac{2c}{p}\right)-\frac{2c^2}{p^2q}\right]}{3kq}+\sqrt{\left(\frac{1}{3kq}\right)^2\left[k\left(kq+\frac{2c}{p}\right)-\frac{2c^2}{p^2q}\right]^2-\frac{ck}{pq}\left[1-\frac{c}{pqk}\right]}\right]\times$$

$$\times\left(kr+\sqrt{(kr)^2-\frac{4kr^2}{3}\left[\frac{\left[k\left(kq+\frac{2c}{p}\right)-\frac{2c^2}{p^2q}\right]}{3kq}+\sqrt{\left(\frac{1}{3kq}\right)^2\left[k\left(kq+\frac{2c}{p}\right)-\frac{2c^2}{p^2q}\right]^2-\frac{ck}{pq}\left[1-\frac{c}{pqk}\right]}\right]}\right)$$

$$MEY_2=H_1\left(E_{MEY_2}\right)$$

$$MEY_2=\frac{1}{6}\left[\frac{\left[k\left(kq+\frac{2c}{p}\right)-\frac{2c^2}{p^2q}\right]}{3kq}-\sqrt{\left(\frac{1}{3kq}\right)^2\left[k\left(kq+\frac{2c}{p}\right)-\frac{2c^2}{p^2q}\right]^2-\frac{ck}{pq}\left[1-\frac{c}{pqk}\right]}\right]\times$$

$$\times\left(kr+\sqrt{(kr)^2-\frac{4kr^2}{3}\left[\frac{\left[k\left(kq+\frac{2c}{p}\right)-\frac{2c^2}{p^2q}\right]}{3kq}-\sqrt{\left(\frac{1}{3kq}\right)^2\left[k\left(kq+\frac{2c}{p}\right)-\frac{2c^2}{p^2q}\right]^2-\frac{ck}{pq}\left[1-\frac{c}{pqk}\right]}\right]}\right)$$

Similarly from equation (13), we have the same effort as the above. And thus the corresponding Maximum Economic Yields in (8) to be

$$MEY_3=\frac{1}{6}\left[\frac{\left[k\left(kq+\frac{2c}{p}\right)-\frac{2c^2}{p^2q}\right]}{3kq}+\sqrt{\left(\frac{1}{3kq}\right)^2\left[k\left(kq+\frac{2c}{p}\right)-\frac{2c^2}{p^2q}\right]^2-\frac{ck}{pq}\left[1-\frac{c}{pqk}\right]}\right]\times$$

Notes

$$\times\left(kr-\sqrt{(kr)^2-\frac{4kr^2}{3}\left[\frac{\left[k\left(kq+\frac{2c}{p}\right)-\frac{2c^2}{p^2q}\right]}{3kq}+\sqrt{\left(\frac{1}{3kq}\right)^2\left[k\left(kq+\frac{2c}{p}\right)-\frac{2c^2}{p^2q}\right]^2-\frac{ck}{pq}\left[1-\frac{c}{pqk}\right]}\right]}\right)$$

$$MEY_4=\frac{1}{6}\left[\frac{\left[k\left(kq+\frac{2c}{p}\right)-\frac{2c^2}{p^2q}\right]}{3kq}-\sqrt{\left(\frac{1}{3kq}\right)^2\left[k\left(kq+\frac{2c}{p}\right)-\frac{2c^2}{p^2q}\right]^2-\frac{ck}{pq}\left[1-\frac{c}{pqk}\right]}\right]\times$$

$$\times\left(kr-\sqrt{(kr)^2-\frac{4kr^2}{3}\left[\frac{\left[k\left(kq+\frac{2c}{p}\right)-\frac{2c^2}{p^2q}\right]}{3kq}-\sqrt{\left(\frac{1}{3kq}\right)^2\left[k\left(kq+\frac{2c}{p}\right)-\frac{2c^2}{p^2q}\right]^2-\frac{ck}{pq}\left[1-\frac{c}{pqk}\right]}\right]}\right)$$

V. Parameter Estimation

a) *Basic Parameters Estimation*

Using the time series data [1], we have the following parameters estimation.

Table 1

Parameters	Symbol	Value
Carrying capacity	k	$2.57\times10^{13}\,kg\ of\ fish$
Catch ability constant	q	$2.197\times10^{-11}\ perday$
Cost of effort	c	$182.50\ birr/kg$
Price of effort	p	$11 birr/kg$
Intrinsic growth rate	r	0.5

Table 2: Parameter estimation for depensation model

Description	Formula	Value [kg per day]
E_{MSY}	$2rk/9q$	1.3×10^{23}
E_{OAY}	$cr(1-(c/pqk))/pq^2$	1.7×10^{22}
E_{MEY}	$r(A+\sqrt{A^2-B})/3q$	1.29×10^{23}
MSY	$4k^2r/27$	4.9×10^{25}
OAY	$c(1-(c/pqk))\left(kr+\sqrt{(kr)^2-4r^2B}\right)/2pq$	91.4×10^{23}
MEY	$MEY_1=\varphi\left(kr+\sqrt{(kr)^2-(4kr^2\varphi/3)}\right)/6$	48.9×10^{24}

Where; $A=\left[\frac{k\left(kq+\frac{2c}{p}\right)-\frac{2c^2}{p^2q}}{3kq}\right]$, $B=\frac{ck}{pq}\left[1-\frac{c}{pqk}\right]$, and $\varphi=A+\sqrt{A^2-B}$

Ref

1. Getachew Abiye Salilew, *Mathematical Bio-Economics of Fish Harvesting with Critical Depensation in Lake Tana,* TJPRC: JMCAR 3(2016) 1-14.

Table 3: Parameter estimation to depensation model for different values of r.

r=0.5	R=1	R=2
$E_{MSY} = 1.3 \times 10^{23}$	$E_{MSY} = 2.6 \times 10^{23}$	$E_{MSY} = 5.20 \times 10^{23}$
$MSY_1 = 4.89 \times 10^{25}$	$MSY_1 = 9.795 \times 10^{25}$	$MSY_1 = 19.59 \times 10^{25}$
$MSY_2 = 2.45 \times 10^{25}$	$MSY_2 = 4.898 \times 10^{25}$	$MSY_2 = 9.795 \times 10^{25}$
$E_{OAY} = 1.67 \times 10^{22}$	$E_{OAY} = 3.336 \times 10^{22}$	$E_{OAY} = 6.67 \times 10^{22}$
$OAY_1 = 91.47 \times 10^{23}$	$OAY_1 = 182.94 \times 10^{23}$	$OAY_1 = 365.885 \times 10^{23}$
$OAY_2 = 2.77 \times 10^{23}$	$OAY_2 = 5.53 \times 10^{23}$	$OAY_2 = 11.07 \times 10^{23}$
$E_{MEY_1} = 1.29 \times 10^{23}$	$E_{MEY_1} = 2.58 \times 10^{23}$	$E_{MEY_1} = 5.162 \times 10^{23}$
$E_{MEY_2} = 8.4 \times 10^{21}$	$E_{MEY_2} = 16.81 \times 10^{21}$	$E_{MEY_2} = 33.62 \times 10^{21}$
$MEY_1 = 48.97 \times 10^{24}$	$MEY_1 = 97.94 \times 10^{24}$	$MEY_1 = 195.885 \times 10^{24}$
$MEY_2 = 46.79 \times 10^{23}$	$MEY_2 = 93.58 \times 10^{23}$	$MEY_2 = 187.156 \times 10^{23}$
$MEY_3 = 23.94 \times 10^{24}$	$MEY_3 = 47.88 \times 10^{24}$	$MEY_3 = 95.75 \times 10^{24}$
$MEY_4 = 6.92 \times 10^{22}$	$MEY_4 = 13.84 \times 10^{22}$	$MEY_4 = 27.68 \times 10^{22}$

b) *The Economic Model Estimation*

In this section we calculated the profit for depensation models by using real data [1]. In a commercial fishery, the appropriate measure of gross benefits is the total revenue that accrues to firms. Assuming that fish are sold in a competitive market, each firm takes the market price p as given and so the revenue obtained from a harvest H is given by $TR(E) = pH(E)$. And finally the economic rent or profit denoted by P is defined in terms of total cost TC and total revenue TR by $P = TR - TC$.

c) *The depensation economic model parameter estimations*

In this case we do have the following parameter estimation with the harvest at the given type of Effort is obtained by: $H(E) = \frac{qE}{2r}\left(kr + \sqrt{(kr)^2 - 4krqE}\right)$.

Table 4: The depensation economic model parameter estimations

r	c	p	q	k	E_{MSY}	E_{OAY}	E_{MEY}	$H(E_{MSY})$	$H(E_{OAY})$	$H(E_{MEY})$
0.5	182.5	11	2.1×10^{-11}	2.5×10^{13}	1.3×10^{23}	1.6×10^{22}	1.2×10^{23}	4.8×10^{25}	91.4×10^{23}	48.9×10^{24}

And thus the profit with different type of harvest function is given by:
The depensation economic model for Maximum Sustainable Yield (MSY)

$$P(E_{MSY}) = TR(E_{MSY}) - TC(E_{MSY}) = p.H(E_{MSY}) - c.E_{MSY}$$

$$P(E_{MSY}) = 53.877549 \times 10^{25} - 237.330848 \times 10^{23} = 51.50424 \times 10^{25}\ birr$$

The depensation economic model for Open Access Yield (OAY)

$$P(E_{OAY}) = TR(E_{OAY}) - TC(E_{OAY}) = p.H(E_{OAY}) - c.E_{OAY}$$

$$P(E_{OAY}) = 1006.1903 \times 10^{23} - 30.44 \times 10^{23} = 975.7503 \times 10^{23}\ birr$$

The depensation economic model for Maximum Economic Yield (MEY)

$$P(E_{MEY}) = TR(E_{MEY}) - TC(E_{MEY}) = p.H(E_{MEY}) - c.E_{MEY}$$

$$P(E_{MEY}) = 538.684 \times 10^{24} - 235.5227436 \times 10^{23} = 515.13173 \times 10^{24}\ birr$$

VI. Results and Conclusions

Biologically overfishing occurs when fish species are caught at a rate faster than they can reproduce. A continuous increase of effort might result in an increase catch

Ref

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but at a decreasing rate or more effort may result in proportionality a smaller harvest, which means the additional effort will have less return.

Using data [1] there is over fishing for different cases of the natural growth rates $r = 0.5$, $r = 1$ and $r = 2$ of fish as in table 3. Without loss of any generality we prefer to analyze the tabular approximate value for $r = 0.5$ as our choice of the parameter is similar to that of $r = 1$ and $r = 2$. When the natural growth rate $r = 0.5$, carrying capacity $k = 2.57 \times 10^{13} kg\ of\ fish$, effort for maximum sustainable yield, $E_{MSY} = 1.300443 \times 10^{23}\ kg\ of\ fish$, effort for open access yield $E_{OAY} = 0.1668069 \times 10^{23}\ kg\ of\ fish$, effort for maximum economic yield $E_{MEY} = 1.2905355813 \times 10^{23}\ kg\ of\ fish$. And thus we observed from these values that all efforts are greater than the carrying capacity therefore there is overfishing if we consider the depensation model.

In the economic point of view we have the approximate price of total population of fish in [1] is, $28.2 \times 10^{13}\ birr$. In the depensation economic model parameter estimations we have: $P(E_{MSY}) = 51.50424 \times 10^{25} birr$, $P(E_{OAY}) = 975.7503 \times 10^{23}\ birr$ and $P(E_{MEY}) = 515.13173 \times 10^{24}\ birr$. And thus in the depensation model the economic rent or the profit obtained by all kinds of effort are greater than the price of the total population of fish and therefore there is overfishing. To keep the sustainability of fish we must reduce the effort levels.

Ref

1. Getachew Abiye Salilew, *Mathematical Bio-Economics of Fish Harvesting with Critical Depensation in Lake Tana,* TJPRC: JMCAR 3(2016) 1-14.

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