Department of Mathematics

Math 6720 (CRN 78217): Topics in Logic

Classical and Computable Model Theory

Spring 2020

MW 3:45–5:00pm

Phillips Hall, Room 736

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Course Description

We will cover a variety of important topics in model theory, and their counterparts in computable model theory. Model theory, which emerged as a distinct field in the 1940's through the works of Gödel, Löwenheim, Malcev, Skolem, and Tarski, provides a rigorous framework for the notions of formal language, meaning, and truth. In the 1930's, Church, Gödel, Kleene, Post, and Turing, developed computability theory, the mathematical theory of algorithms. Interaction of computability theory with model theory, as well as other areas of mathematics, has resulted in computable model theory and, more generally, in computable mathematics.

While some mathematical constructions are algorithmic, or can be replaced by algorithmic ones yielding the same results, others are intrinsically nonalgorithmic. For example, every decidable theory has a decidable model. While the standard model of arithmetic is computable, there are no computable nonstandard models (that is, ones with infinite numbers). Other examples of negative results include the undecidability of Hilbert's Tenth Problem (for integers), and the undecidability of the word problem in combinatorial group theory. To this day, Hilbert's Tenth Problem for rationals remain as one of the main open problems in mathematics.

While free groups on different finite numbers of generators greater than one have the same first-order theory (as Sela established solving Tarski's famous problem), we can use infinitary language to describe different free groups, and can use computable infinitary language to describe different computable free groups. In general, we use sophisticated methods of computability theory to calibrate the complexity of undecidable problems.

Reading material, which will be provided in class, will be selected from:

D. Marker, *Model Theory*, 2000, Springer.

C.C. Chang and H.J. Keisler, Model Theory, 1990, North-Holland.

C.J. Ash and J.F. Knight, *Computable Structures and the Hyperarithmetical Hierarchy*, 2000, Elsevier.

V. Harizanov, “Pure computable model theory,” chapter in: *Handbook of Recursive Mathematics*, vol. 1, 1998, North-Holland, pp. 3–114.

E. Fokina, V. Harizanov, and A. Melnikov, “Computable model theory,” chapter in: *Turing's Legacy*, 2014, Cambridge University Press, pp. 124–194.

Required background

Math 2971 or an equivalent, and familiarity with the notion of an algorithm. Math 6720 can be taken for credit repeatedly. Advanced **undergraduate** students may also take this course for credit.

Grading

Based on take-home assignments and projects, class participation, and in-class presentations.