O9e „Fringes of Equal Thickness"

Tasks

1 Determine the radius of a convex lens by measuring Newton´s rings using light of a given wavelength.

2 Using the experimental arrangement of *task 1* determine the wavelength of light passing a filter*.*

3 Determine the thickness of metal foils or wires using the interference at wedge-shaped layers.

Perform at least one of these additional experiments:

Z1 Determine the thickness or the diameter of an object chosen by yourself, e.g. a strand of hair.

Z2 Measure Newton's rings with water as medium between lens and glass plate.

Z3 Determine Poisson's number μ from the principal radii of curvature of a curved plate using fringes of equal thickness.

Literature

Physics, P. A. Tipler, 3rd Edition, Vol. 2, Chap. 33-1, 33-2
University Physics, H. Benson, Chap. 37.1, 37.5, 36.3
Physikalisches Praktikum, 13. Auflage, Hrsg. W. Schenk, F. Kremer, Optik, 2.0.1, 2.0.2, 2.1

Accessories

Abbe comparator for measuring the interference pattern, spectroscopic lamp, filters, lenses, glass plates, foils, wires, plate with bending device

Keywords for preparation

- Interference, coherence, coherence length
- Superposition of waves
- Interference (destructive, constructive), phase relationships, optical path difference
- Fringes of equal thickness, fringes of equal inclination, Newton´s rings
- Principle of light filters, e.g. interference filters

Remarks

At the beginning of the experiment the demonstrator will introduce you to the use of the Abbe comparator for the measurement of the interference pattern. This optical precision instrument consists of two microscopes to observe the object (interference pattern) as well as a precision length scale with a resolution of 0.2 µm. In order to minimize errors arising from the movement of the comparator table in different directions, the measurements should be performed while moving the table in one direction only.
The analysis of the data sets is done by linear regression. For each task about 10 measurements should be performed.

Newton´s Rings

Newton´s rings occur when monochromatic light interferes in the thin intermediate gap between a convex lens and a plane glass plate, see Fig. 1. In the Abbe comparator monochromatic light comes in horizontally, strikes the glass plate and is reflected downwards toward lens and plane glass plate. The light is partially reflected at each air/glass interface. Here we are interested in the reflections at the convex side of the lens and the upper side of the plane glass plate, since these waves interfere to generate Newton's rings. The contact between lens and glass plate must not be ideal. This is modeled by the introduction of a contact distance d_0; d_0 might be either positive, when e.g. dust particles are located between lens and glass plate, or negative, when the gravity pressure slightly indents the plane plate.
Ray 1 reflected at the top of the air film interferes with ray 2 reflected at the bottom of the air film, see Fig. 1. At a distance r_k from the contact point the air film has a thickness $d + d_0$, where d denotes the ideal thickness due to the convex curvature. The refractive index of air is taken to be unity. Ray 2 is reflected at an optically denser medium and acquires an additional phase shift of π.
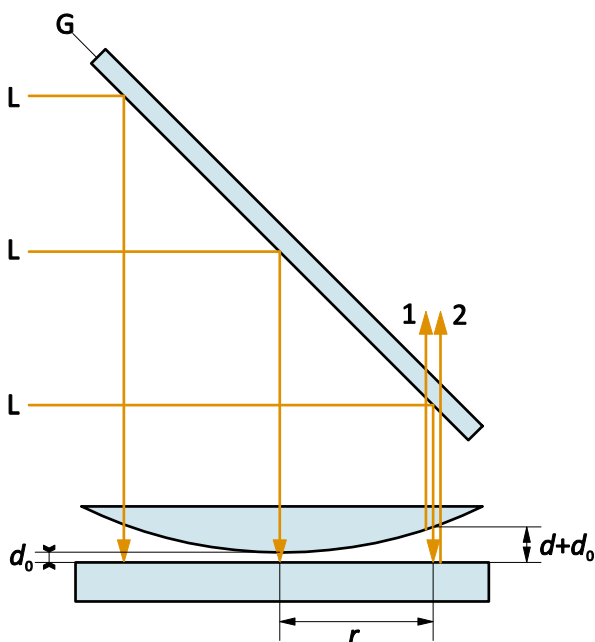

Fig. 1. Schematical setup for the measurement of Newton's rings showing the convex lens on the plane glass plate as well as the glass plate (G) used for deflection of the incoming light (L).

The optical path difference between ray 1 and ray 2 is

$$\Delta x = 2(d + d_0) + \lambda / 2 \qquad \qquad (1a)$$

and the corresponding phase shift is

$$\delta = 2\pi \frac{\Delta x}{\lambda} = \frac{4\pi}{\lambda}(d + d_0) + \pi \quad . \qquad \qquad (1b)$$

The relationship between the radius of the kth interference ring r_k and the radius R of the lens follows from Fig. 2:
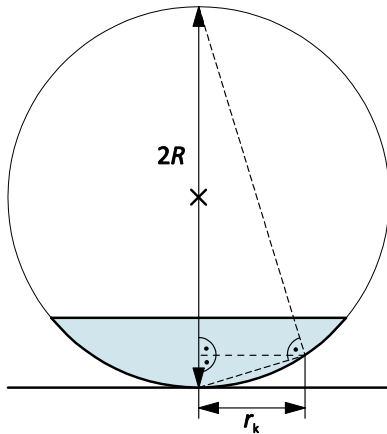
$$d(2R - d) = r_k^2 \quad . \qquad \qquad (2)$$

Fig. 2. Using the third Pythagorean theorem to relate d and r_k.

Using Eqs. (1) and (2) as well as the condition for constructive interference (bright rings) one obtains in the limit $d \ll R$

$$r_k^2 = \left(k - \frac{1}{2}\right)R\lambda - 2d_0 R \quad . \qquad \qquad (3)$$

Derive the equation for destructive interference (dark rings).

For the analysis of the data in *tasks 1* and *2* plot r_k^2 vs. k and determine R resp. λ from the slope. In order to estimate the uncertainty use

$$R\lambda = \frac{r_{k_2}^2 - r_{k_1}^2}{k_2 - k_1} , \qquad \qquad (4)$$

where k_1 and k_2 denote the first and highest measured diffraction order. Compare this uncertainty with that determined from the slope.

Interference in wedge-shaped layers

A wedge-shaped air gap between two flat glass plates can be created by placing a film, metal foil, thin wire, strand of hair etc. at one end of the glass plates as shown in Fig. 3. As in the case of Newton's rings monochromatic light is incident normally on the glass plates and the interference fringes are observed in reflection.

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Fig. 3.   Interference at wedge-shaped air films.
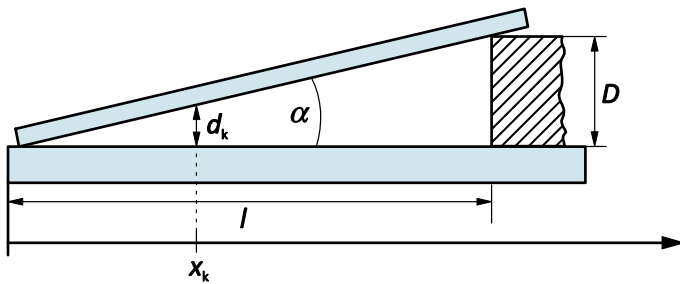
Let x_k be the horizontal position of the kth bright fringe, corresponding to a plate separation d_k as shown in the Fig. 3. Then the phase shift is $\delta_k = 2\pi \Delta x_k / \lambda = 4\pi d_k / \lambda + \pi$ and with the condition for constructive interference and the relation $\tan\alpha = d_k/x_k = D/l$ one obtains

$$x_k = \frac{l\lambda}{2D}k - \frac{l\lambda}{4D}.$$ (5)

Derive the equation for dark fringes.

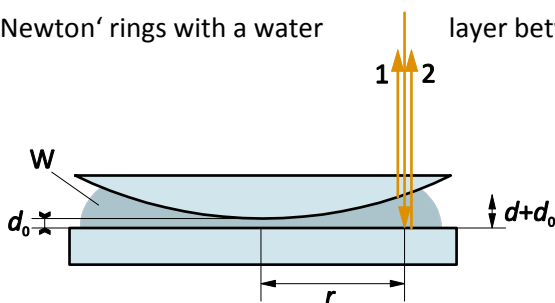For adjacent equidistant dark or bright fringes separated by Δ one has

$$\Delta = \frac{l\lambda}{2D} \ .$$ (6)

Measure 10 different values x_k in dependence on k and plot x_k vs. k. Determine D from the slope, the measured value l and the given wavelength λ.

Additional task: Newton's rings with a water layer

Fig. 4. Newton' rings with a water   layer between lens and glass plate.

The analysis follows the reasoning above for the case of Newton's rings at an air gap. Here the optical path difference is modified by the refractive index n that differs significantly from unity

$$\Delta = 2n(d+d_0) + \frac{\lambda}{2} .$$ (7)

This leads to a modified equation for the bright fringes:

$$r_k^2 = (k-1)\frac{\lambda R}{n} - 2d_0 R .$$ (8)

Derive the corresponding equation for the dark fringes.

Additional task: Determination of Poisson's number

Consider a thin plate (length l, width b and thickness h) suspended on two blades S, S', see Fig. 5. Via two additional blades S_1, S_1', see Fig. 5, a force along the z-direction is exerted on the plate by some fastening device. Under this load the plate bends in the yz-plane (primary bending). In this bending the upper layers of the plate are stretched, experiencing at the same time a lateral contraction, whereas the lower layers are compressed and simultaneously laterally expanded. The lateral contraction and expansion leads also to a bending in the xz-plane (secondary bending).
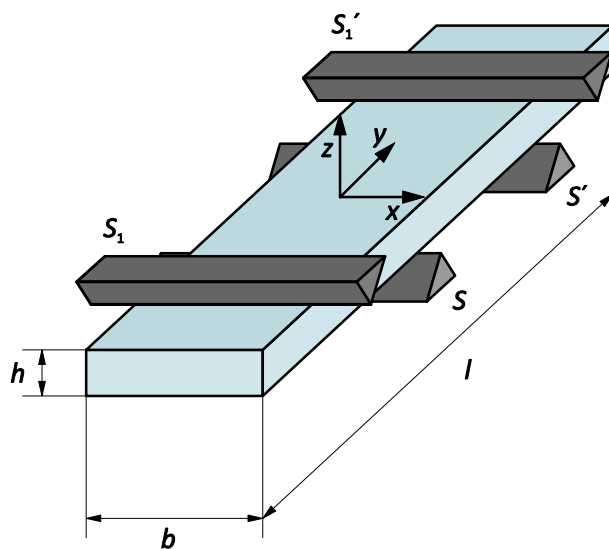

Fig. 5. Transparent plate under pressure in a fastening device.

The surface profile of the bent plate is shown, albeit rather exaggerated, in Fig. 6. Both in the yz- and the xz-plane the surface profile can be characterized in a good approximation by its radius of curvature, yielding a saddle-point profile
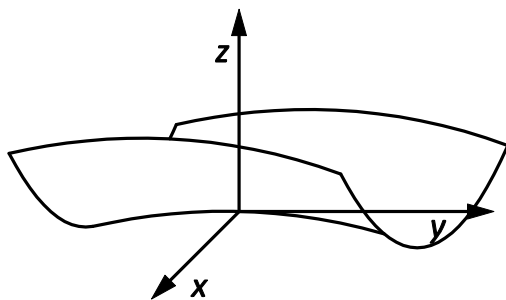

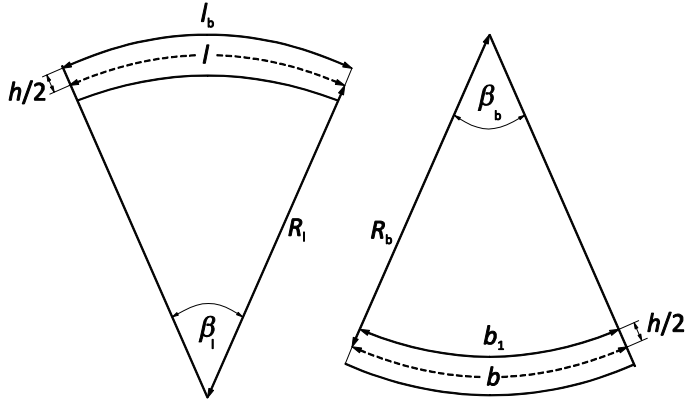Fig. 6. Saddle-point surface-profile of the bent plate with strongly exaggerated curvature.

Fig. 7. Cross-section through the bent plate in the *yz*- (left) and *xz*-plane (right).

From Fig. 7 one reads off the relative elongation

$$\frac{l_1 - l}{l} = \frac{\beta_l h}{2l} = \frac{h}{2R_l} \ , \qquad \text{(y-direction)} \tag{9}$$

and the relative contraction

$$\frac{b - b_1}{b} = \frac{\beta_b h}{2b} = \frac{h}{2R_b} . \qquad \text{(x-direction)} \tag{10}$$

Therefore, for Poisson's number μ one obtains in good approximation

$$\mu = \frac{R_l}{R_b} \ . \tag{11}$$

The saddle-point surface-profile shown in Fig. 6 is described by

$$z = R_b \left\{ 1 - \sqrt{1 - \left(\frac{x}{R_b}\right)^2} \right\} + R_l \left\{ \sqrt{1 - \left(\frac{y}{R_l}\right)^2} - 1 \right\} \ . \tag{12}$$

Experimentally, $x \ll R_b$ and $y \ll R_l$, such that the Taylor expansion of the square roots can be truncated after the terms $(x/R_b)^2$ and $(y/R_l)^2$ to yield

$$z = \frac{x^2}{2R_b} - \frac{y^2}{2R_l} \ . \tag{13}$$

Sections through the saddle-point profile parallel to the *xy*-plane (*z* = const) yield hyperbolas. For *z* = 0 one obtains the pair of lines, see Fig. 8,

$$y = +\sqrt{\frac{R_l}{R_b}} \, x \quad \text{and} \quad y = -\sqrt{\frac{R_l}{R_b}} \, x \ . \tag{14}$$

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The smaller of the two angles between these lines be denoted by 2α, see Fig. 8. Then one obtains for Poisson's ratio

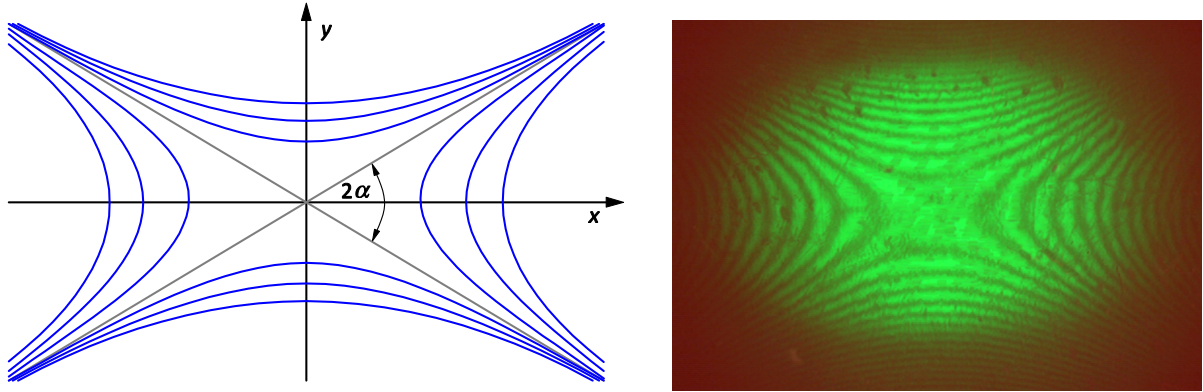$$\mu = \frac{R_l}{R_b} = \tan^2 \alpha \quad . \tag{15}$$

Fig. 8. Interference pattern.

The fastening device is positioned under the microscope of the Abbe comparator. On the bent transparent plate a flat glass plate is placed; the whole setup is illuminated by monochromatic light as described above and an interference pattern of the type shown in Fig. 8 is observed. For the distances x_k and y_k of the apexes of the hyberbolas from the origin one obtains $x_k = \pm\sqrt{2R_b z_k}$ and $y_k = \pm\sqrt{2R_l |z_k|}$, where the z_k now denote optical path differences. Measure the lengths $2x_k$ and $2y_k$ for several orders of the interference pattern, plot $x_k^2 = f_1(k)$ and $y_k^2 = f_2(k)$ and determine the slopes S_b and S_l, respectively. Then Poisson's ratio is obtained from

$$\mu = \frac{S_l}{S_b} = \frac{R_l}{R_b} \quad . \tag{16}$$

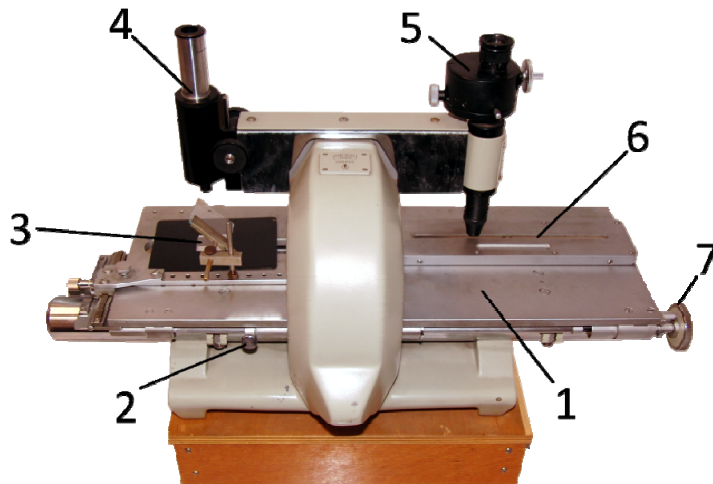**Measurements with the 'Abbe-Komparator'**

Fig. 9. Abbe-Komparator

1 table, 2 fixation screw for movable table, 3 object, 4 measuring microscope (enlarged image of the object), 5 microscope to measure the enlarged glass scale (1 mm, 0.1 mm, 0.01 mm, 0.001 mm), 6 glass scale, 7 knob for the fine-adjustment of the movable table
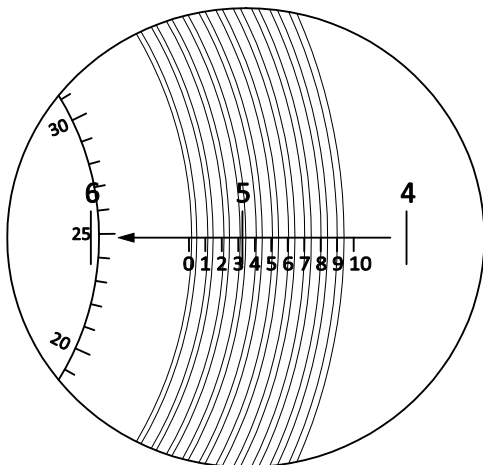

Fig. 10. Glass scale as seen in microscope (12).

reading = 5 mm + 0.3 mm +0.024 mm + 0.0008 mm

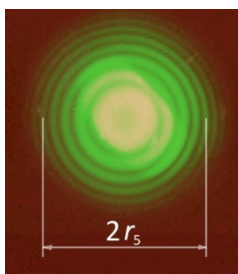= 5.3248 mm

with an uncertainty of 0.0002 mm

Fig. 11. Newton's rings. The fringes are not equally spaced. Measure from the left highest order to the right highest order or vice versa.
Calculate the diameter from the difference of the readings:

$$\left| le_5 - ri_5 \right| = 2r_5.$$