International Journal of Mechanical Engineering and Technology (IJMET)
Volume 8, Issue 8, August 2017, pp. 1109–1117, Article ID: IJMET_08_08_110
Available online at http://iaeme.com/Home/issue/IJMET?Volume=8&Issue=8
ISSN Print: 0976-6340 and ISSN Online: 0976-6359

KINETOSTATIC ANALYSIS OF SIX-BAR MECHANISM USING VECTOR LOOPS AND THE VERIFICATION OF RESULTS USING WORKING MODEL 2D

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ABSTRACT

In this paper is described use kinematics analysis is based on vectorial algebra known as vector loops for the six-bar mechanism. First is required to describe vectorial & scalar equations for each simple planar mechanisms, either closed-chain or open-chain. Dynamic equations of motions for kinetic analysis are used. Solution of vectorial/scalar equations is done using MathCAD software. Results are verified using Working Model 2D software.

Keywords: Mechanism, Kinematics, Kinetic, Dynamic, Vector loop

Cite this Article: Ahmet Shala and Mirlind Bruqi, Kinetostatic Analysis of Six-Bar Mechanism Using Vector Loops and the Verification of Results Using Working Model 2D, International Journal of Mechanical Engineering and Technology 8(8), 2017, pp. 1109–1117.
http://iaeme.com/Home/issue/IJMET?Volume=8&Issue=8

1. INTRODUCTION

Kinematics and dynamic study of mechanisms using plans and graphic methods is known that permissive error [1, 3]. To avert this error is required use of analytic methods [5], especially for dynamic study of smart mechanisms [3, 4], where error is not allowed. In despite of hard work during calculations which are required for analytic study, for smart mechanisms this method is required. Except that, using analytic method can be achieved required exactness, which can't be arrived using other two methods [1].

Usually, by using analytic study is determined relations of kinematics parameters of guided and guide links [6, 7].

Analytical methods for analysis of mechanisms are too many [5, 6], in view is presented application of vectors loops [1] method for kinematic analysis and use of dynamic equations of motions for kinetostatic analysis of mechanism.

2. KINEMATICS ANALYSIS OF MECHANISM USING VECTOR LOOPS

This represents one general study method, which is based on vectorial algebra. Based on this method, start & end connection points of each link represent the vectors with length same as link have, direction is determined according in the motion of mechanism.

In view is represented example of usage of this method for six-bar mechanism.

Basic data of links of mechanism (Figure 1):

Lengths

$L_1=O_2O_2=5\ cm$; $L_2=O_1A=2\ cm$; $L_3=AB=6\ cm$;

$L_{4a}=O_2B=4\ cm$; $L_{4b}=BC=1\ cm$; $L_5=CD=7\ cm$;

$y_D=1\ cm$

Masses

$m_2=2$ kg; $m_3=6$ kg; $m_4=5$ kg; $m_5=5$ kg; $m_6=1$ kg

Kinematic friction between slider D and basement μ= 0.1

Constant angular velocity of driver link 2, $\omega_2=1\ rad/s$

Driver motion: $\theta_2=\omega_2\cdot t\ rad$

Before starting any analysis it is important to check continues motion of given mechanism for minimum one circle of motion. Use of Grashof condition is very useful. In our example main basic mechanism is four-bar mechanism O_1ABO_2. The Grashof condition for a four-bar linkage states: If the sum of the shortest and longest link of a planar quadrilateral linkage is less than or equal to the sum of the remaining two links, then the shortest link can rotate fully with respect to a neighboring link. In other words, the condition is satisfied if $L_2+L_3\le L_1+L_4$ where L_2 is the shortest link, L_3 is the longest, and L_1 and L_4 are the other links.

Applying our data:

$$L_2+L_3\le L_1+L_4 \Rightarrow 2+6\le 5+4 \Rightarrow 8\le 9$$

Which is thru and our given mechanism is Grashof or time-continuously.

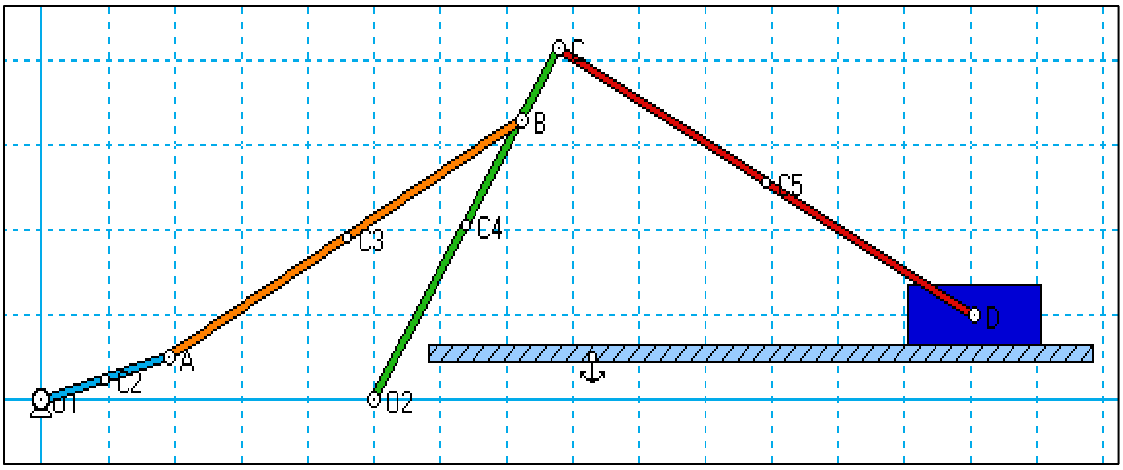


Figure 1 Six-bar planar mechanism

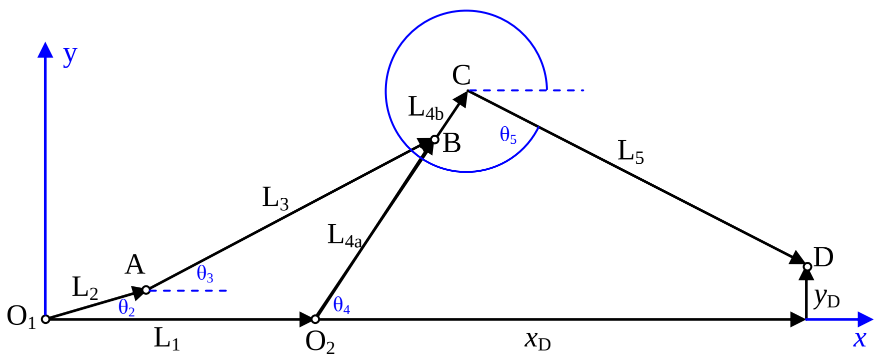


Figure 2 Scheme of vectors loops

From Figure 2 can be seen two vector loops which represent two vectorial equations of basic mechanisms, as in view:

$$L_2 + L_3 = L_1 + L_{4a}; \quad \text{(a)}$$

$$L_{4a} + L_{4b} + L_5 = x_D + y_D; \quad \text{(b)}$$

To find the solution of system of equations, time invariant, in Mathcad it is enough to call function "Given" in the beginning. To avoid inverse solutions it is required to declare initial position-values (approximately) of unknown parameters.

Projections / components of equations (a) and (b) in horizontal (x) and vertical (y) direction are:

$$\left.\begin{aligned} L_2\cos\theta_2 + L_3\cos\theta_3 &= L_1 + L_{4a}\cos\theta_4 \\ L_2\sin\theta_2 + L_3\sin\theta_3 &= 0 + L_{4a}\sin\theta_4 \end{aligned}\right\} \quad \text{(c)}$$

$$\left.\begin{aligned} (L_{4a} + L_{4b})\cos\theta_4 + L_5\cos\theta_5 &= x_D \\ (L_{4a} + L_{4b})\sin\theta_4 + L_5\sin\theta_5 &= y_D \end{aligned}\right\} \quad \text{(d)}$$

From system of equations (c) and (d) can be found all required expressions for positional analysis of mechanism, usually unknown are angles: $\theta_3, \theta_4, \theta_5$ and distance-coordinate x_D:

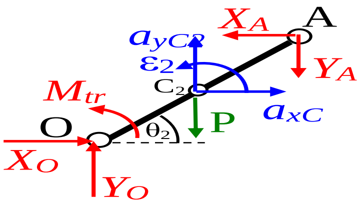
$$\text{kin\_sol}(t) := \text{Find}\left(\theta_3, \theta_4, \theta_5, x_D\right)$$

Using first and second derivative of equations (c) and (d) can be found required velocities and accelerations of characteristic points and links of mechanism.

3. DYNAMIC ANALYSIS OF MECHANISM

Dynamic analysis of mechanisms consists on writing of equations of motion based dynamics – kinetostatic laws e.g. dynamic equations of planar motion for each link of the mechanism. To find the solution of system of equations time invariant in Mathcad it is enough on top to call function "**Given**". Inverse solutions are avoided during kinematic analysis; here is enough to declare initial forces-values starting from zero (0) just to avoid imaginary solutions of unknown parameters.

Dynamic equations of motion for driven link 2:

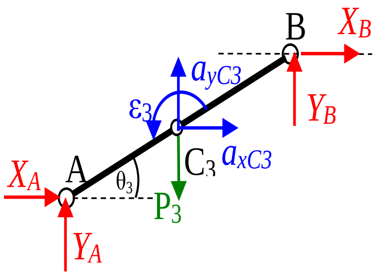


$$m_2 \cdot a_{xC2}(t) = X_{O1} - X_A \tag{1}$$

$$m_2 \cdot a_{yC2}(t) = Y_{O1} - Y_A - m_2 \cdot g \tag{2}$$

$$\begin{aligned} 0 = X_{O1}\frac{L_2}{2}\sin(\theta_2(t)) - Y_{O1}\frac{L_2}{2}\cos(\theta_2(t)) + X_A\frac{L_2}{2}\sin(\theta_2(t)) - \\ - Y_A\frac{L_2}{2}\cos(\theta_2(t)) + M_{tr} \end{aligned} \tag{3}$$

Dynamic equations of motion for link 3:

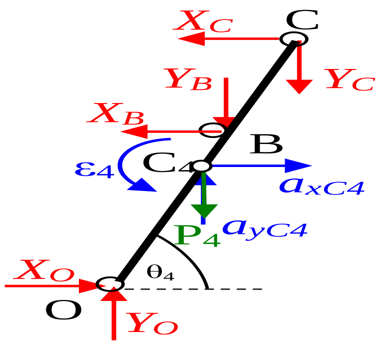


$$m_3 \cdot a_{xC3}(t) = X_A + X_B \tag{4}$$

$$m_3 \cdot a_{yC3}(t) = Y_A + Y_B - m_3 \cdot g \tag{5}$$

$$\begin{aligned} J_{C3} \cdot \varepsilon_3(t) = X_A\frac{L_3}{2}\sin(\theta_3(t)) - Y_A\frac{L_3}{2}\cos(\theta_3(t)) - \\ - X_B\frac{L_3}{2}\sin(\theta_3(t)) + Y_B\frac{L_3}{2}\cos(\theta_3(t)) \end{aligned} \tag{6}$$

Dynamic equations of motion for link 4:

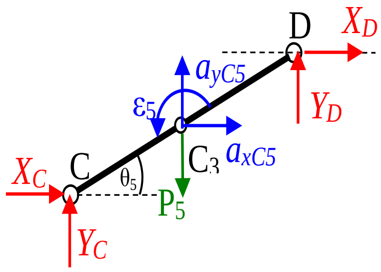


$$m_4 \cdot a_{xC4}(t) = X_{O2} - X_B - X_C \tag{7}$$

$$m_4 \cdot a_{yC4}(t) = Y_{O2} - Y_B - Y_C - m_4 \cdot g \tag{8}$$

$$J_{C4}\varepsilon_4(t)=\left(X_{O2}\frac{L_4}{2}+X_B\left(L_{4a}-\frac{L_4}{2}\right)+X_C\frac{L_4}{2}\right)\sin(\theta_4(t))-$$
$$-\left(Y_{O2}\frac{L_4}{2}+Y_B\left(L_{4a}-\frac{L_4}{2}\right)+Y_C\frac{L_4}{2}\right)\cos(\theta_4(t)) \quad (9)$$

Dynamic equations of motion for link 5:

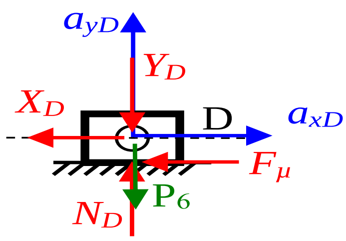


$$m_5\cdot a_{xC5}(t)=X_C+X_D \quad (10)$$

$$m_5\cdot a_{yC5}(t)=Y_C+Y_D-m_5\cdot g \quad (11)$$

$$J_{C5}\cdot\varepsilon_5(t)=(X_C-X_D)\frac{L_5}{2}\sin(\theta_5(t))-$$
$$-(Y_C-Y_D)\frac{L_5}{2}\cos(\theta_5(t)) \quad (12)$$

Dynamic equations of motion for link 6:



$$m_6\cdot a_D(t)=-X_D+F_\mu \quad (13)$$

$$0=-Y_D+N_D-m_6\cdot g \quad (14)$$

Coulomb's law for friction-normal force is:

$$F_\mu=-sign(v_D(t))\cdot\mu\cdot N_D \quad (15)$$

In previous equations are 15 unknown forces-moments:

$$\text{Dyn\_sol}(t) := \text{Find}(X_{O1},Y_{O1},X_A,Y_A,X_B,Y_B,X_{O2},Y_{O2},X_C,Y_C,X_D,Y_D,F_\mu,N_D,M_{tr})$$

Where: X_i and Y_i are components of joint forces, F_μ - friction force and M_{tr} - transmitted moment on driver link 2–joint O_1, or Torque of motor.

In the equation (15) is used signum function to take in consideration change of movement direction of slider D.

4. RESULTS OF KINEMATICS AND DYNAMICS ANALYSIS, USING SOFTWARE'S WORKING MODEL 2D AND MATHCAD FOR SIX-BAR MECHANISM

4.1. Graphical Results - Diagrams

After is obtained system which describe the motion of mechanism, required kinematics parameters are solved using MathCAD software, e.g. position, velocity, acceleration, angular velocity and acceleration, reaction forces on joints and Torque of motor (transmitted moment) placed on joint O_1.

Results are verified in Working Model 2D.

Based on Figure 2 position of point C_3 (centre of mass of link 3) is determined by coordinates:

$$x_{C3}(t) := L_2 \cdot \cos(\theta_2(t)) + \frac{L_3}{2} \cdot \cos(\theta_3(t)) \text{ and } \quad y_{C3}(t) := L_2 \cdot \sin(\theta_2(t)) + \frac{L_3}{2} \cdot \sin(\theta_3(t))$$

Where: $\theta_2(t) = \omega_2 \cdot t$ is given and $\theta_3(t)$ can be found from equations (a) and (b).

Velocity and acceleration is calculated using firs and second derivatives of position:

$$v_{C3}(t) = \sqrt{\left(\frac{d}{dt} x_{C3}(t)\right)^2 + \left(\frac{d}{dt} y_{C3}(t)\right)^2} \text{ , } a_{C3}(t) = \sqrt{\left(\frac{d^2}{dt^2} x_{C3}(t)\right)^2 + \left(\frac{d^2}{dt^2} y_{C3}(t)\right)^2}$$

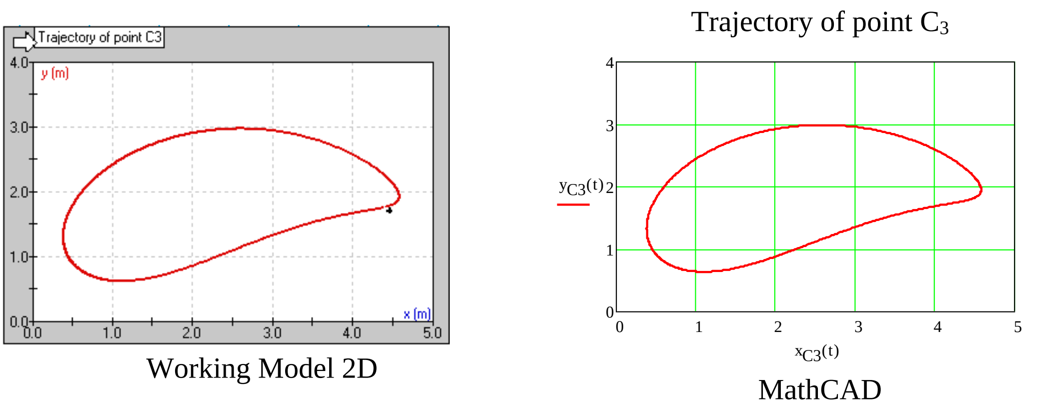


Figure 3 Trajectory of point C_3

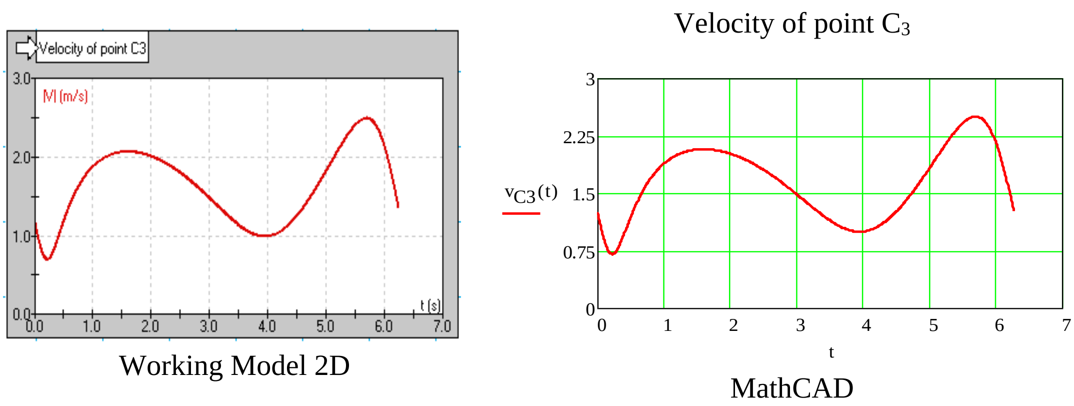


Figure 4 Velocity of point C_3

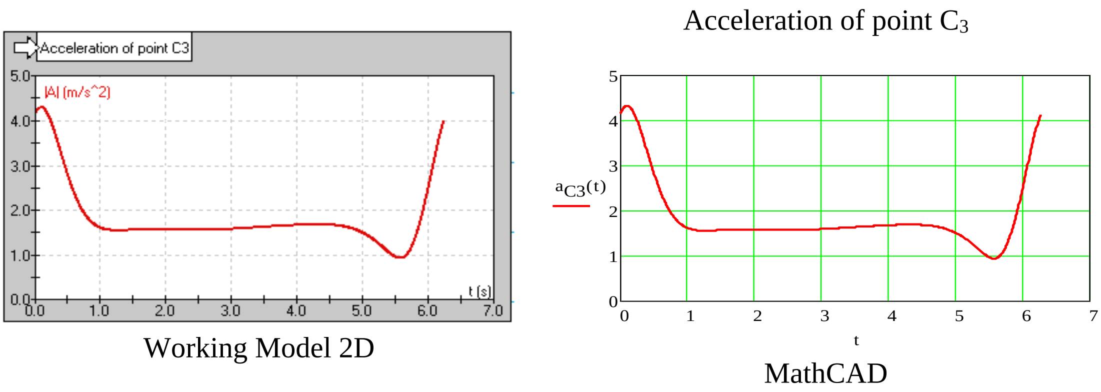


Figure 5 Acceleration of point C_3

From Figure 2 position of point C (joint between link 4 and link 5) is determined by coordinates:

$$x_C(t) := L_1 + L_4 \cdot \cos(\theta_4(t)) \text{ and } y_C(t) := L_4 \cdot \sin(\theta_4(t))$$

Where; $\theta_4(t)$ can be found from equations (a) and (b).

Velocity and acceleration is calculated using first and second derivatives of position:

$$v_C(t) = \sqrt{\left(\frac{d}{dt} x_C(t)\right)^2 + \left(\frac{d}{dt} y_C(t)\right)^2}, \; a_C(t) = \sqrt{\left(\frac{d^2}{dt^2} x_C(t)\right)^2 + \left(\frac{d^2}{dt^2} y_C(t)\right)^2}$$

From solution of dynamic equations of motion (1) to (15) is represented diagram of reaction force on joint C and Transmitted moment – Torque of motor on joint O_1.

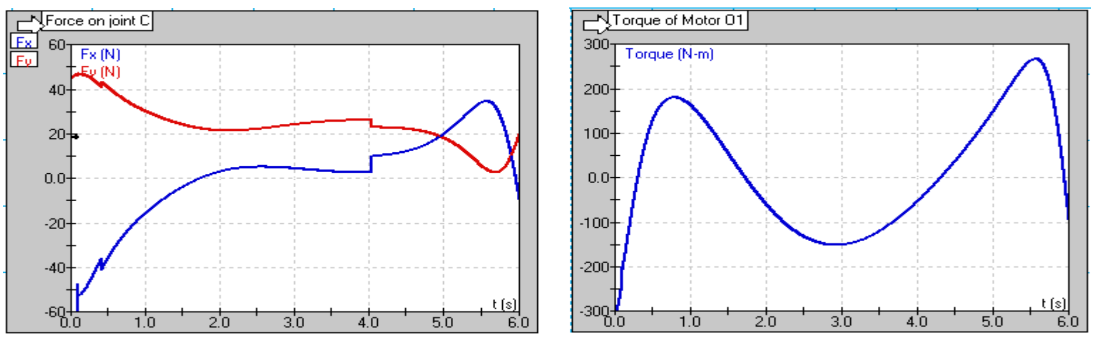


Figure 6 Reaction force on joint C and Transmitted moment – Torque of motor on joint O_1.

4.2. Numerical Results

For comparisons of used software's in Table 1 are represented numerical results for e.g. time 0.5 seconds.

Table 1 Results of kinematics parameters in MathCAD and Working Model 2D for time t=0.5 s

	Time	***x* position of sliderD**	**Velocity of point C_2**		**Velocity of point C_3**		**Velocity of point C_4**		**Velocity of point C_5**		**Velocity of slider D**
	t	x	v_x	v_y	v_x	v_y	v_x	v_y	v_x	v_y	v
WM2D	0.5	14.135	-0.479	0.878	-0.636	0.987	-0.196	0.137	-0.460	0.137	-0.527
Mathcad	0.5	14.135	-0.479	0.878	-0.636	0.987	-0.196	0.137	-0.460	0.137	-0.527

	Time	**Angular velocity of link 3**	**Angular velocity of link 4**	**Angular velocity of link 5**	**Acceleration of point C_3**		**Acceleration of point C_4**		**Acceleration of point C_5**		**Acceleration of point D**
	t	ω_3 [s^{-1}]	ω_3 [s^{-1}]	ω_3 [s^{-1}]	a_x	a_y	a_x	a_y	a_x	a_y	*Ax*
WM 2D	0.5	-0.278	0.096	-0.044	-2.744	0.80	-2.334	1.599	-5.465	1.598	-6.263
Mathcad	0.5	-0.278	0.096	-0.044	-2.744	0.80	-2.333	1.599	-5.465	1.599	-6.262

	Angular acceleration link 3	Angular acceleration link 4	Angular acceleration link 5
	ε_3 [s^{-2}]	ε_4 [s^{-2}]	ε_5 [s^{-2}]
WM2D	0.668	1.131	-0.510
Mathcad	0.668	1.131	-0.510

Table 2 Results of dynamic parameters in MathCAD and Working Model 2D for time t=0.5 s

	Time	**Force A**		**Force B**		**Force O_2**		**Force C**		**Force D**	
	t	X_A	Y_A	X_B	Y_B	X_{O2}	Y_{O2}	X_C	Y_C	X_D	Y_D
WM2D	0.5	-218.617	-58.680	202.151	122.318	154.341	220.630	-36.142	41.284	8.82	15.741
Mathcad	0.5	-218.605	-58.674	202.139	122.313	154.332	220.628	-36.140	41.286	8.82	15.743

	Time	**Force O_1**		**Friction Force D**	**Reaction Force D**	**Torque of Motor O_1**
	t	X_{O1}	Y_{O1}	F_μ	N_D	M_{tr}
WM2D	0.5	-220.372	-40.026	2.555	25.546	123.837
Mathcad	0.5	-220.360	-40.019	2.555	25.550	123.840

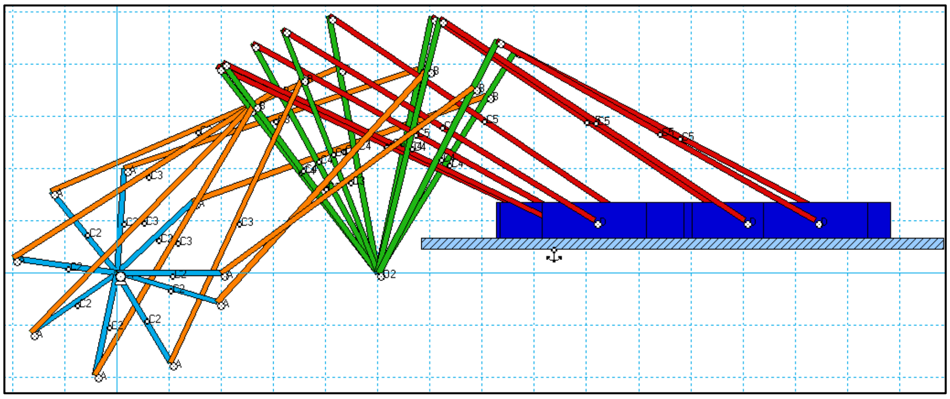


Figure 7 Different positions (9 trucking frames) of mechanisms during the motion

4. CONCLUSIONS

Based on results can be concluded that Working Model in comparison with MathCAD need less time and less theoretical knowledge's to have exact results of any engineering models generally, especially any 2D or 3D model of mechanisms.

From Tables 1 and 2, can be see small difference on results which result of use of derivative and integral mathematical operations. Difference shows advantage of use of Working Model, because results from this software are more realistic in comparison with results from MathCAD which are theoretical – ideal, received by strict calculations.

In second circle, motion of mechanism is stabilized. Results from both software's are identically.

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