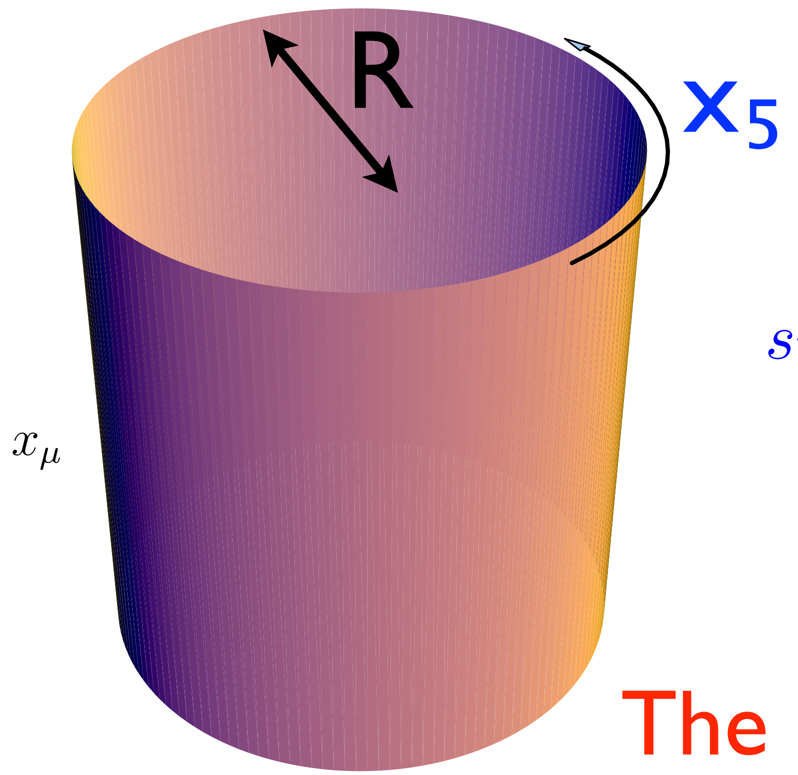
Extra dimensions

Origins...

Kaluza (1921) and Klein (1926) proposed an **extra spacial dimension** trying to unify EM with Gravity:



In 5 dimensions the gravitational field has more components:

$$h_{MN} = (h_{\mu\nu}, h_{5\mu}, h_{55})$$

$M = \mu, 5 = 1, 2, 3, 4, 5$

$symmetric\ under\ M \leftrightarrow N$

4 dimensional gravit. field

4 dimensional EM field

The extra spacial-dimension had to be compactified

R~1/MP

Einstein liked the idea:

"Long live the fifth dimension" Einstein's letter to Ehrenfest 21/1/1928

... but didn't work: Not possible to incorporate matter

The formalism developed however was useful:

For periodic extra dim., one can perform a Fourier expansion of 5D fields:

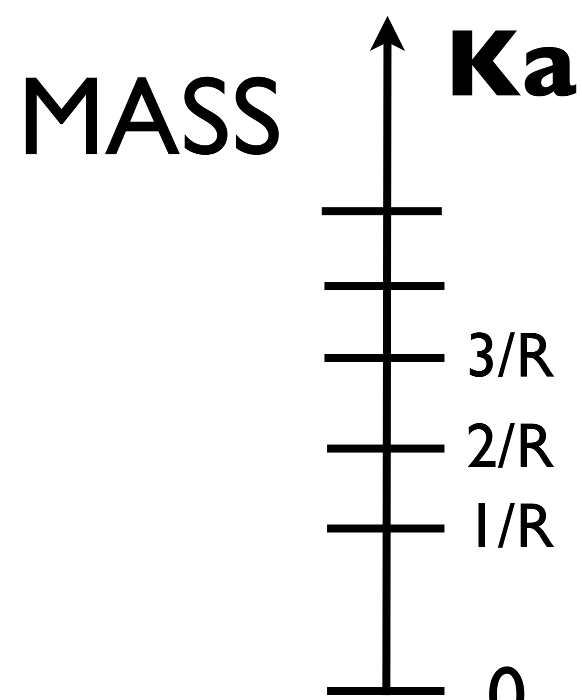
$$\Phi(x, x_5) = \sum_{n=-\infty}^{\infty} e^{inx_5/R}\,\Phi^{(n)}(x)$$

QM $\longrightarrow$ $e^{ip_5 x_5}$, $p_5 = n/R$

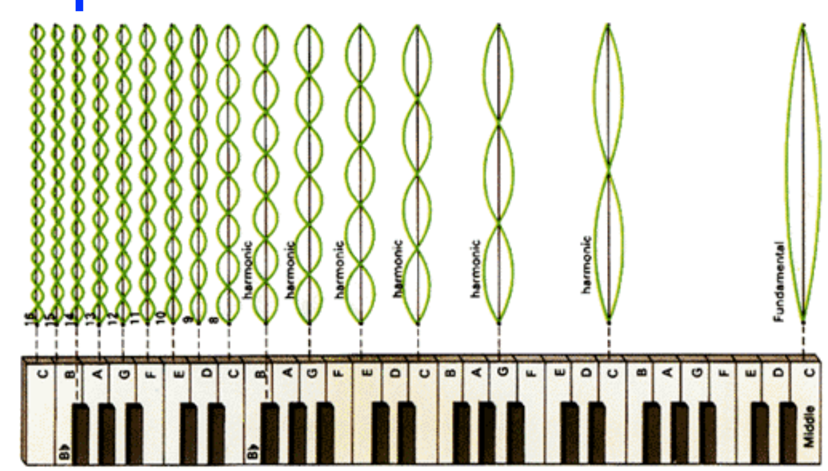
States with 5D momentum ~ mass in 4D: $p_\mu^2 = p_5^2$

5D field = 4D massless state (n=0)
+ infinity tower of 4D massive states

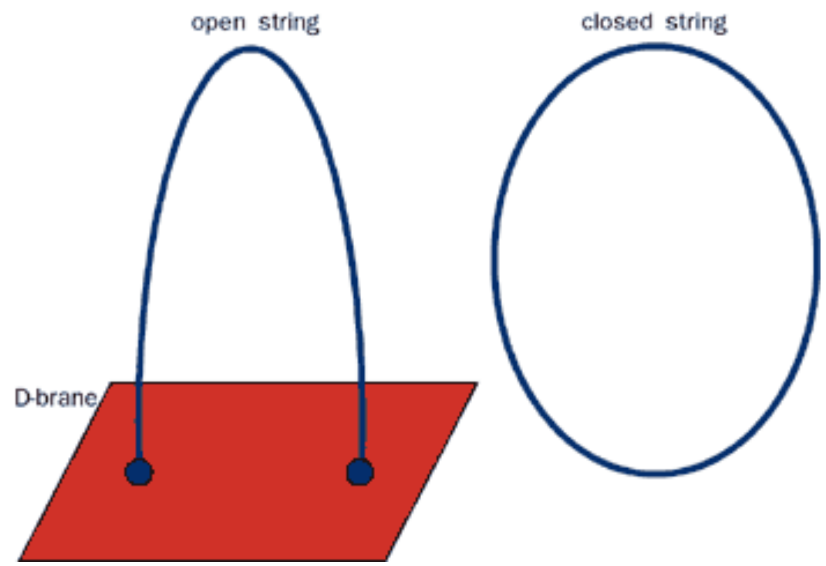
Kaluza-Klein states



...equivalent of harmonics

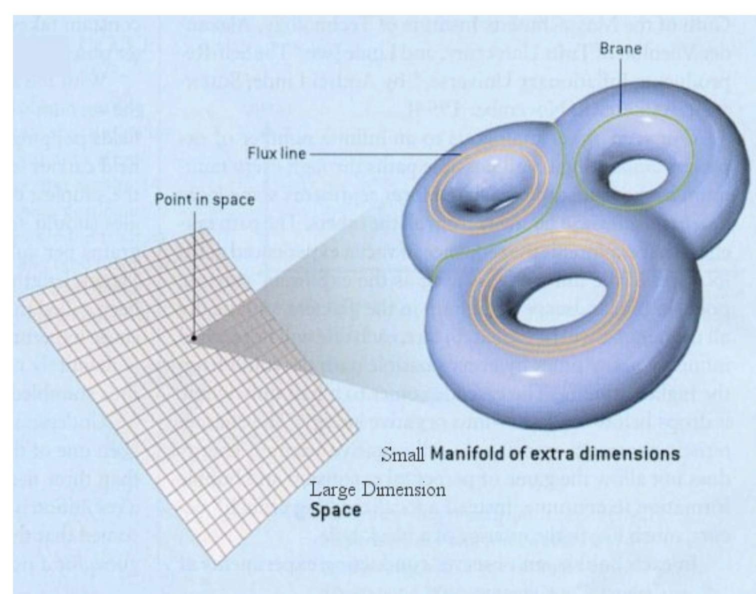


• The next incarnation of extra dimensions came around 1980, when **string theory** was developed as a theory of quantum gravity.



• It was shown that string theory could only be made consistent if the **spacetime dimensions were larger than four.**

The **extra dimensions** were supposed to be compactified close to the Planck scale:



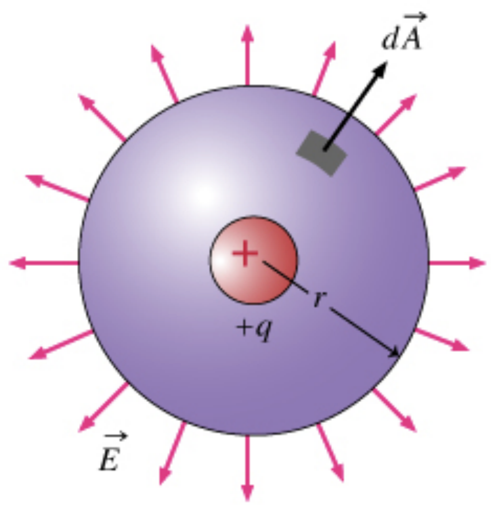
radius ~ $1/M_P$

Therefore not testable in near-future experiments

In 1998 Arkani-Hamed, Dimopoulos and Dvali (ADD) realize that extra dimensions could explain the weakness of gravity:

$G_N << G_F$

How? Gauss's law:



$$\oint_S d\Phi \sim Q_{int}$$

4 dim → $V \sim \frac{Q_{int}}{r}$

4+d dim → $V \sim \frac{Q_{int}}{r^{1+d}}$

$S \sim r^{2+d}$

d = number of extra dimensions

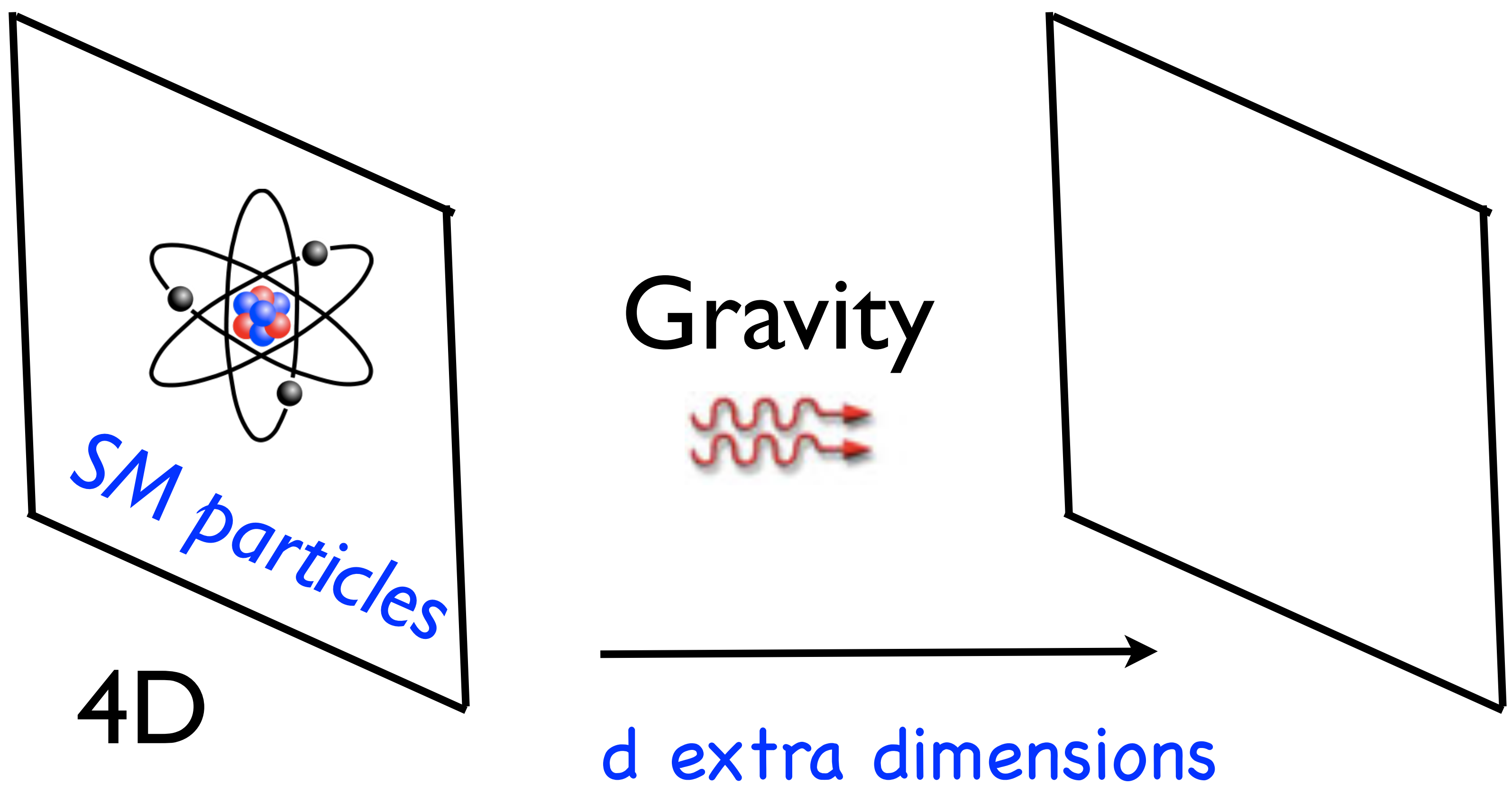
At large distances, the strength of a force becomes smaller in higher dimensions

BUT:

1) Only gravity could propagate in these extra dimensions

(otherwise all forces will be weak)

Possible in “Brane Worlds” (String constructions):



2) We see at large distances only 3 spacial dimensions, so these extra dimensions have to be compactified.

How large can they be?

Surprisingly, we have not measured very well gravity at distances smaller ~ 0.1 mm

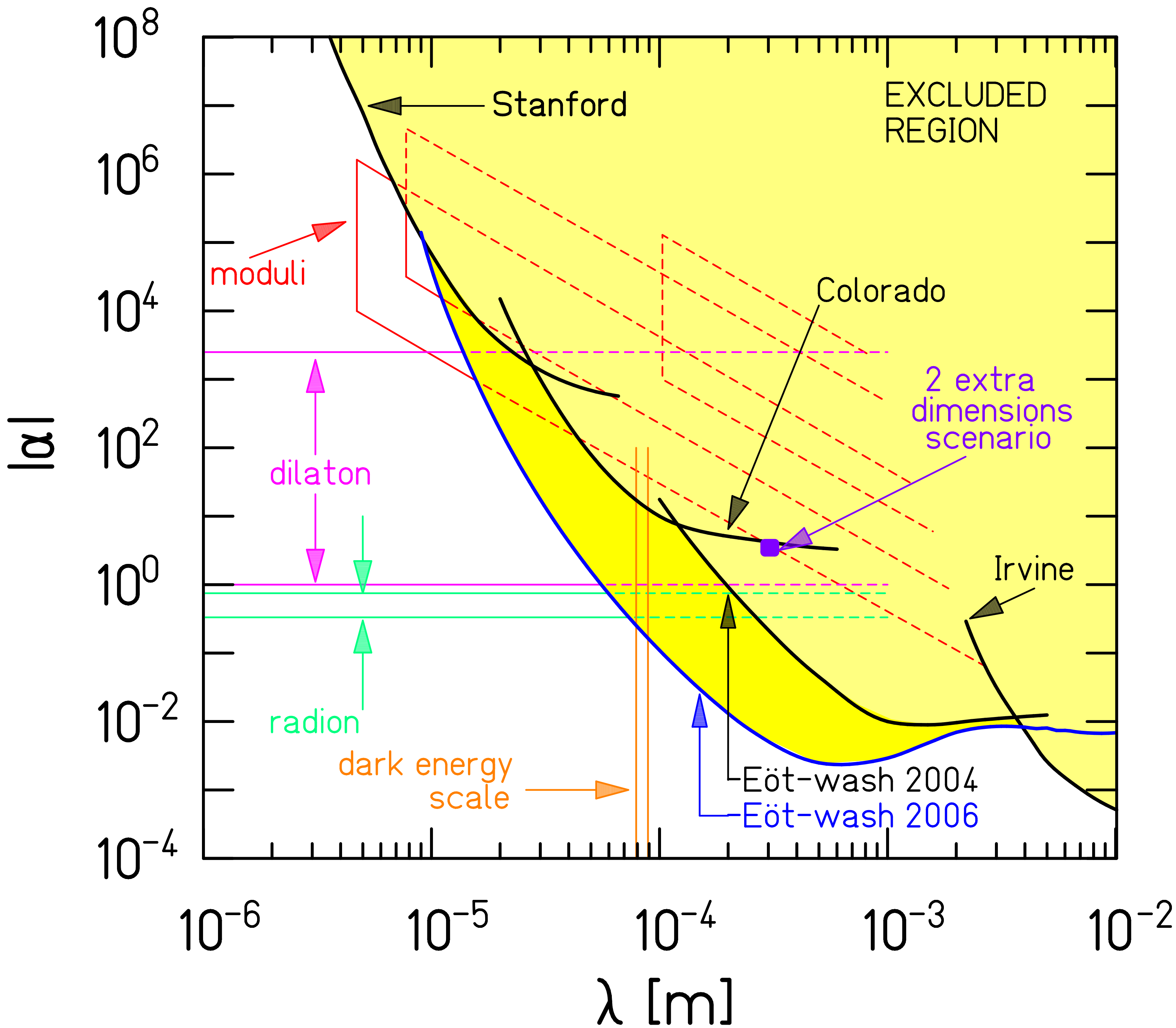
Constrains on extra forces:

$$F_{KK}(r) = -\alpha G_N \frac{m_1 m_2}{r} e^{-r/\lambda},$$

α = measures the strength of the interaction (α=1 → Gravit. strength)

λ = range of the interaction

Gravity coud be different at sub-mm scales: Extra dim of radius R~0.04 mm possible!



ADD proposed the following scenario:

1)

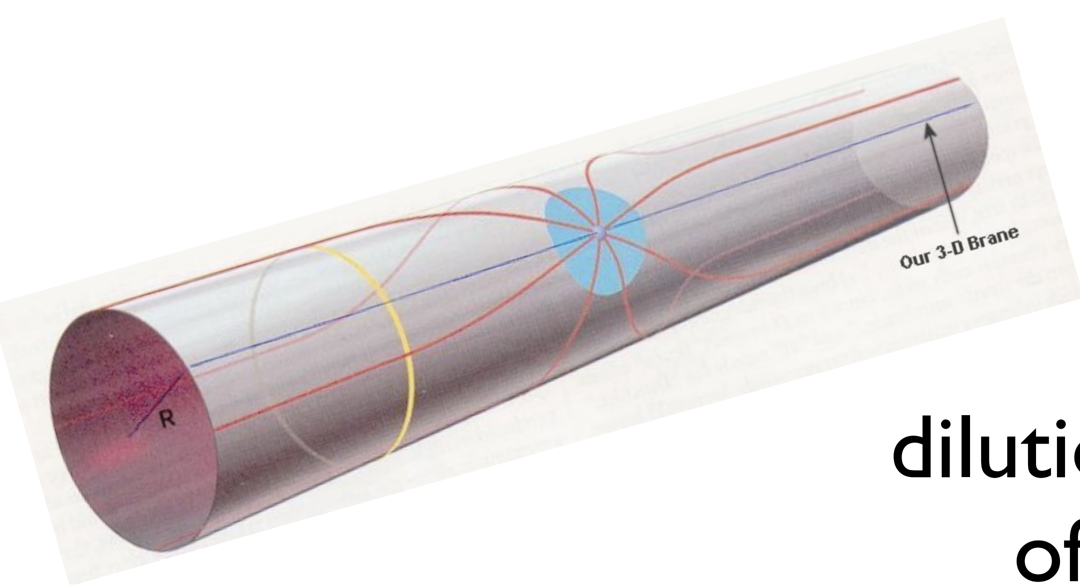
~ TeV — Cutoff of the SM ~ Λ ~ M_{string}: Scale at which gravity is strong

~ 100 GeV — EW scale

Since $M_W \sim \Lambda \sim M_{string}$, no big hierarchy problem!

2) Gravity propagating in *d* extra dimensions of size R:

Gravity strength :



$$G_N = \frac{1}{M_P^2} \sim \frac{1}{M_{string}^2} \frac{1}{(M_{string} 2\pi R)^d}$$

dilution factor due to the spreading of the gravitational field lines in *d* extra dimensions

$$G_N = \frac{1}{M_P^2} \sim \frac{1}{M_{string}^2} \frac{1}{(M_{string} 2\pi R)^d}$$

↓ M_{string} ~ TeV

d=1 → R ~10^8 Km Not possible

d=2→ R ~ 0.1 mm ~ at the verge of the exp. bounds

...

d=6 → R ~ 1/ MeV OK

Predictions:

1) For d=2, we expect deviations from Newtonian gravity at distances smaller than ~ 0.1mm

The 1st KK-graviton of mass ~1/(0.1mm) give a "new" interaction (a "new" force)

2) String theory at the reach of the LHC

But we already said...

"The only prediction of string theory is that there are no predictions"
Anonymous

The only generic ones:

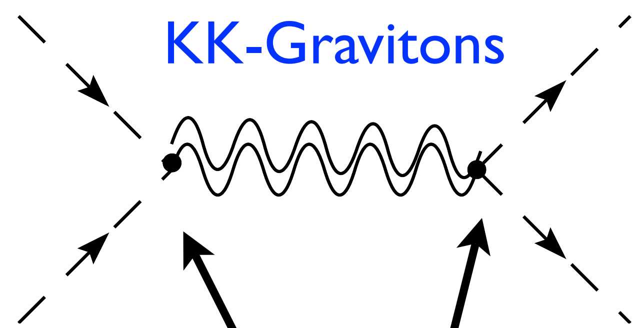
1) The space must be1+9 dimensional
2) There are string excitations of higher-energy

No clear predictions on what to expect!

Model-independent signals from gravity at ~TeV:

Gravity becomes strong at ~ TeV energies:

$$\sum_n$$

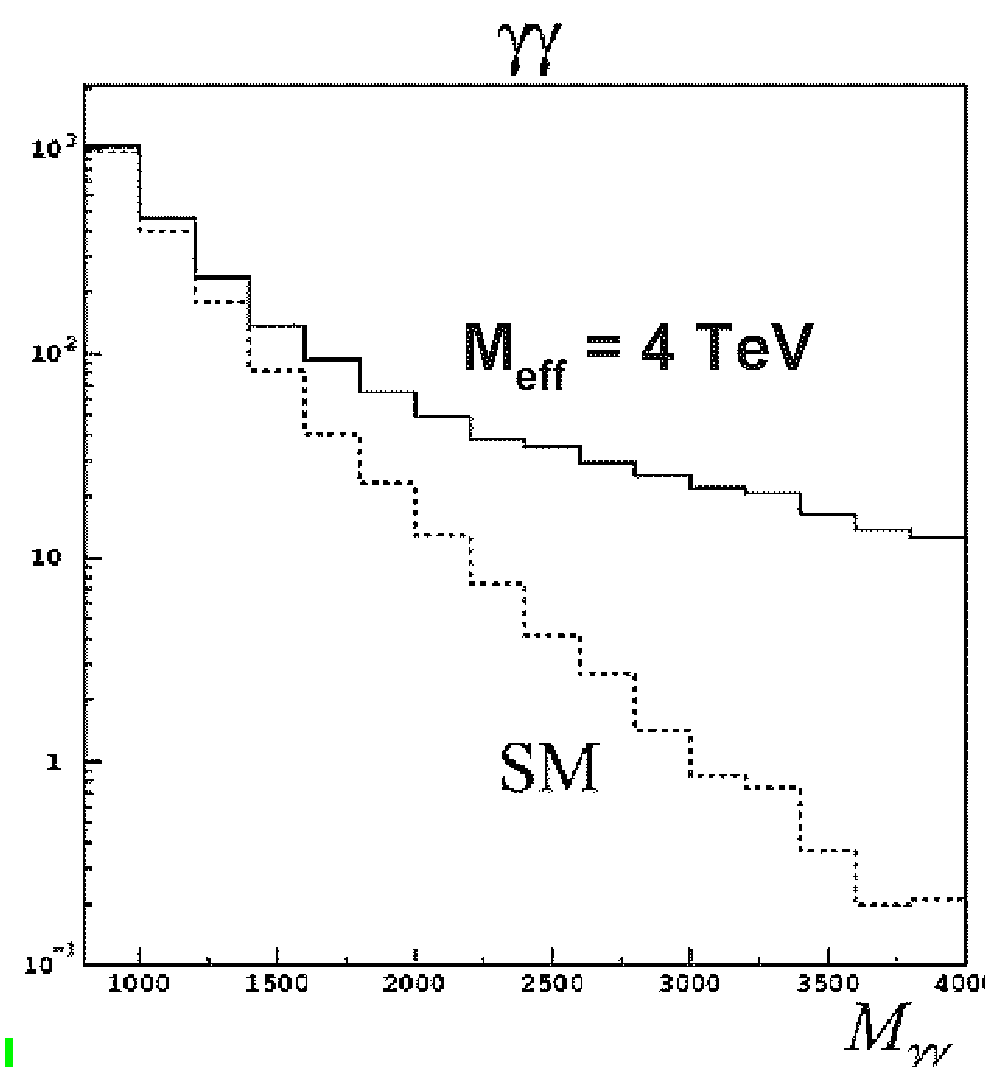


$$\sim \sum_n G_N Q^2 \sim \frac{1}{M_{string}^2} Q^2$$

Sum over all KK-gravitons

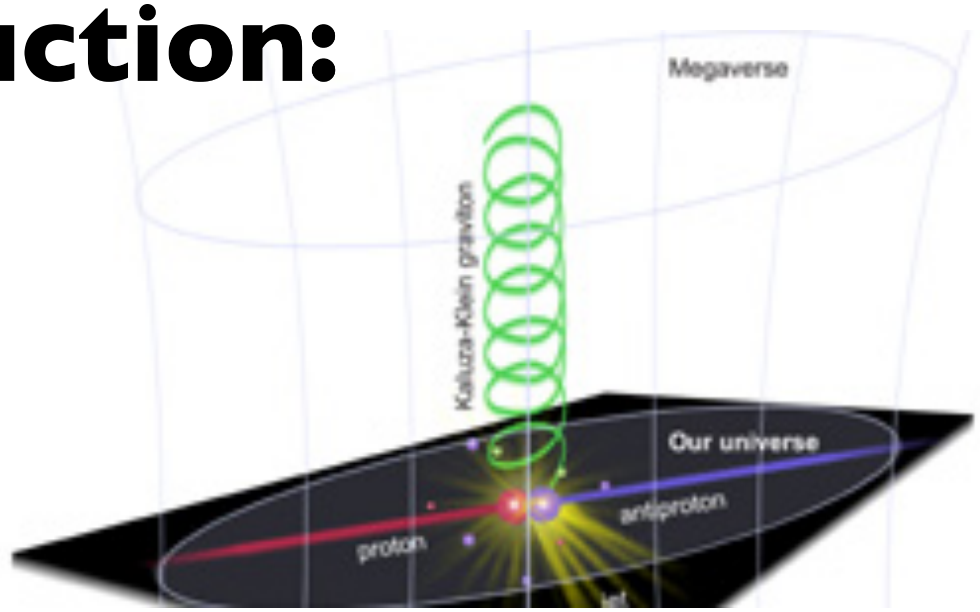
$$\mathrm{G_N} = \frac{1}{\mathrm{M_P^2}}$$

To be seen at LHC as deviations in Drell-Yan cross-sections for SM processes. Example: deviations in the γγ invariant mass distribution of pp→γγ:

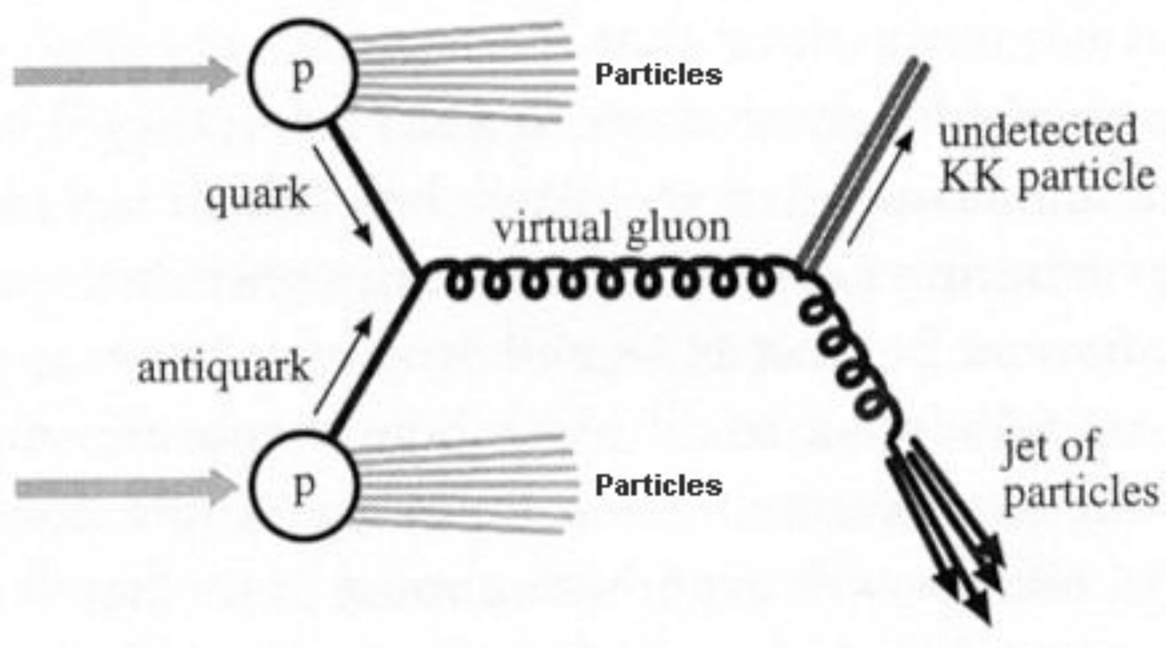


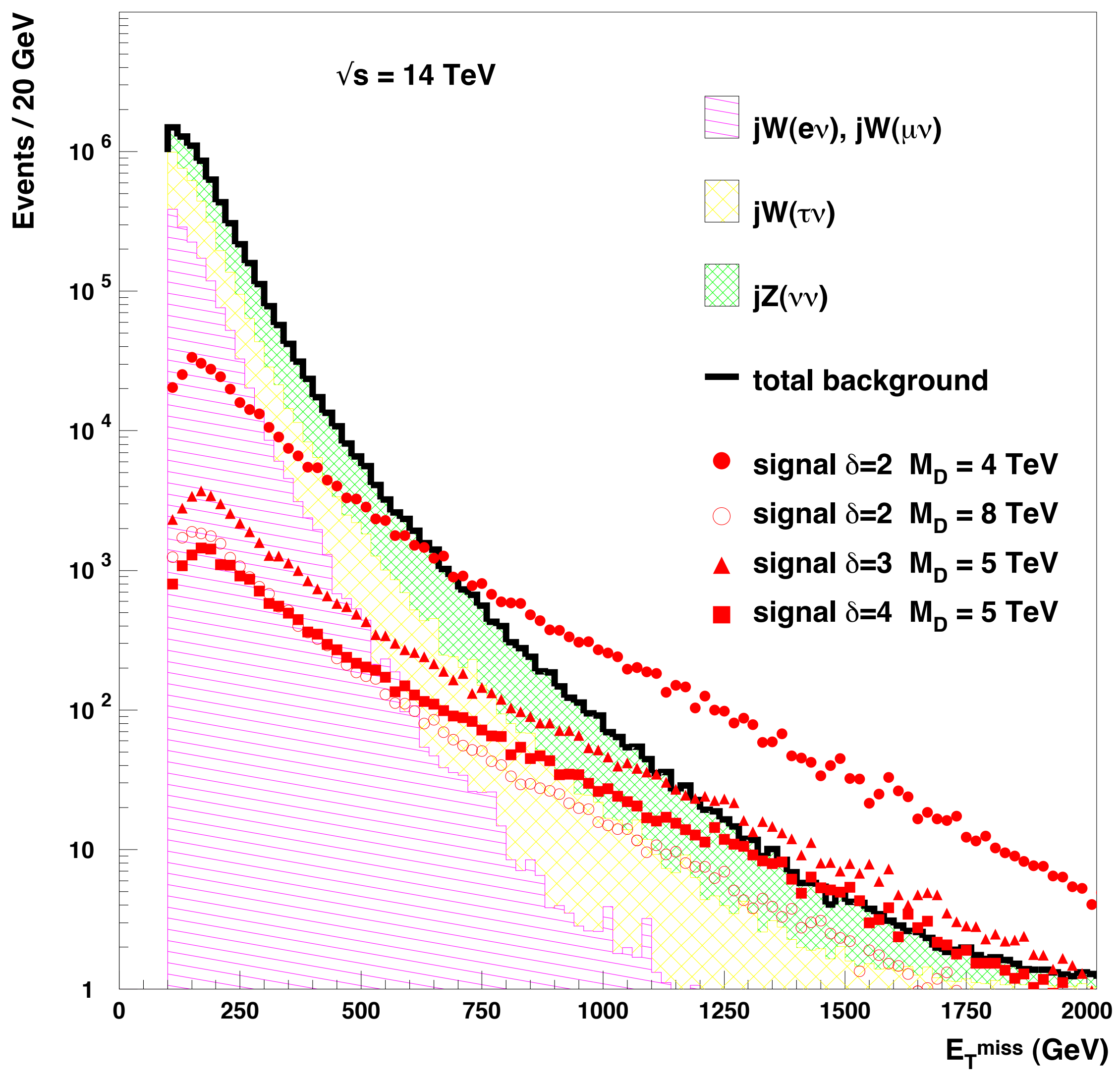
arXiv:hep-ex/0310020v1

KK-Graviton production:



Search for:
Mono-jet + Missing energy





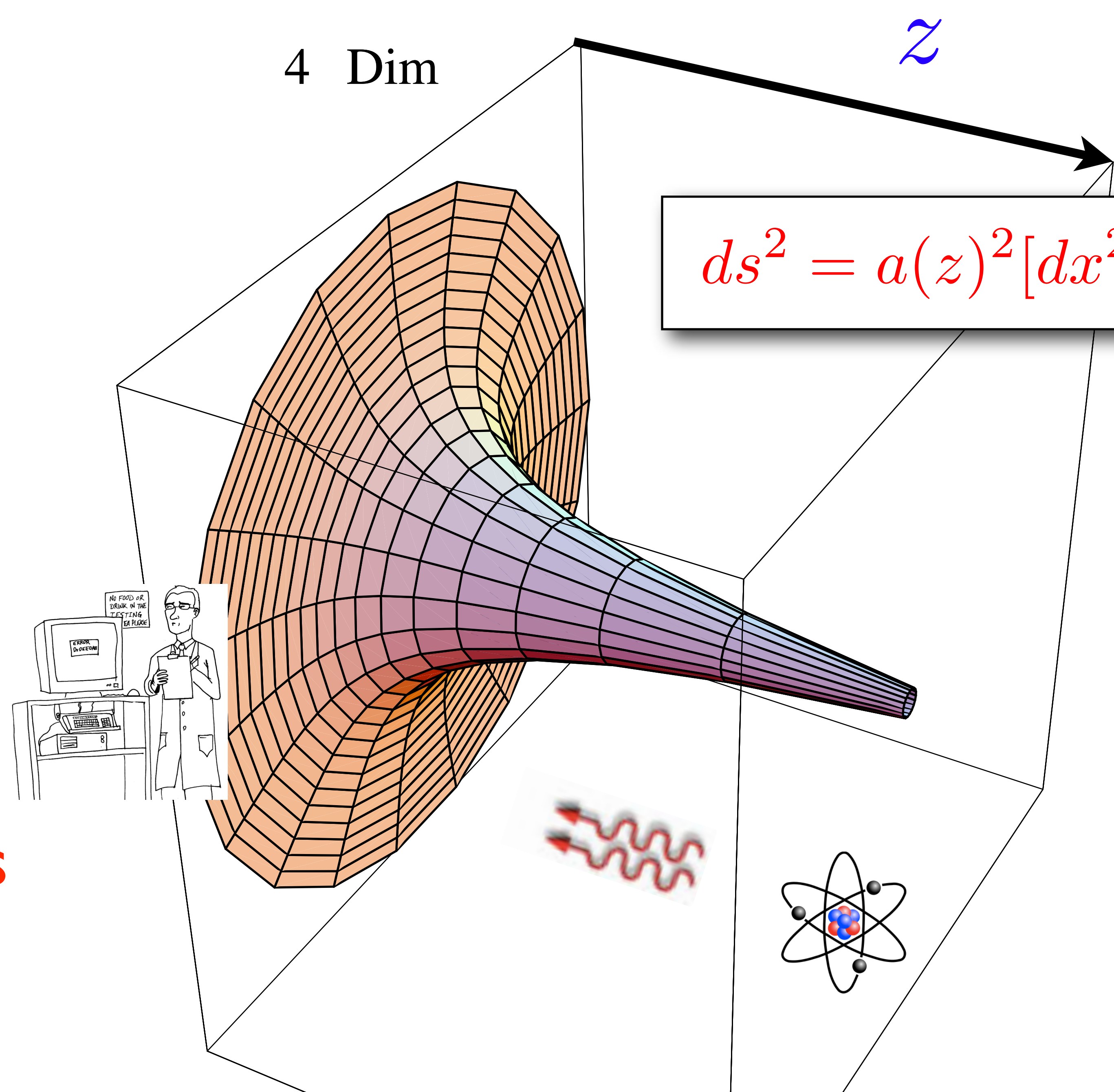
arXiv:hep-ex/0310020v1

In 1999 Randall and Sundrum had a different idea:

Use gravitational redshift factors to explain
the difference between M_P and the EW-scale

Assume that the extra-dimensional geometry is non-flat

Scales shrink as we move in the extra dimension



$$ds^2 = a(z)^2[dx^2 + dz^2]$$

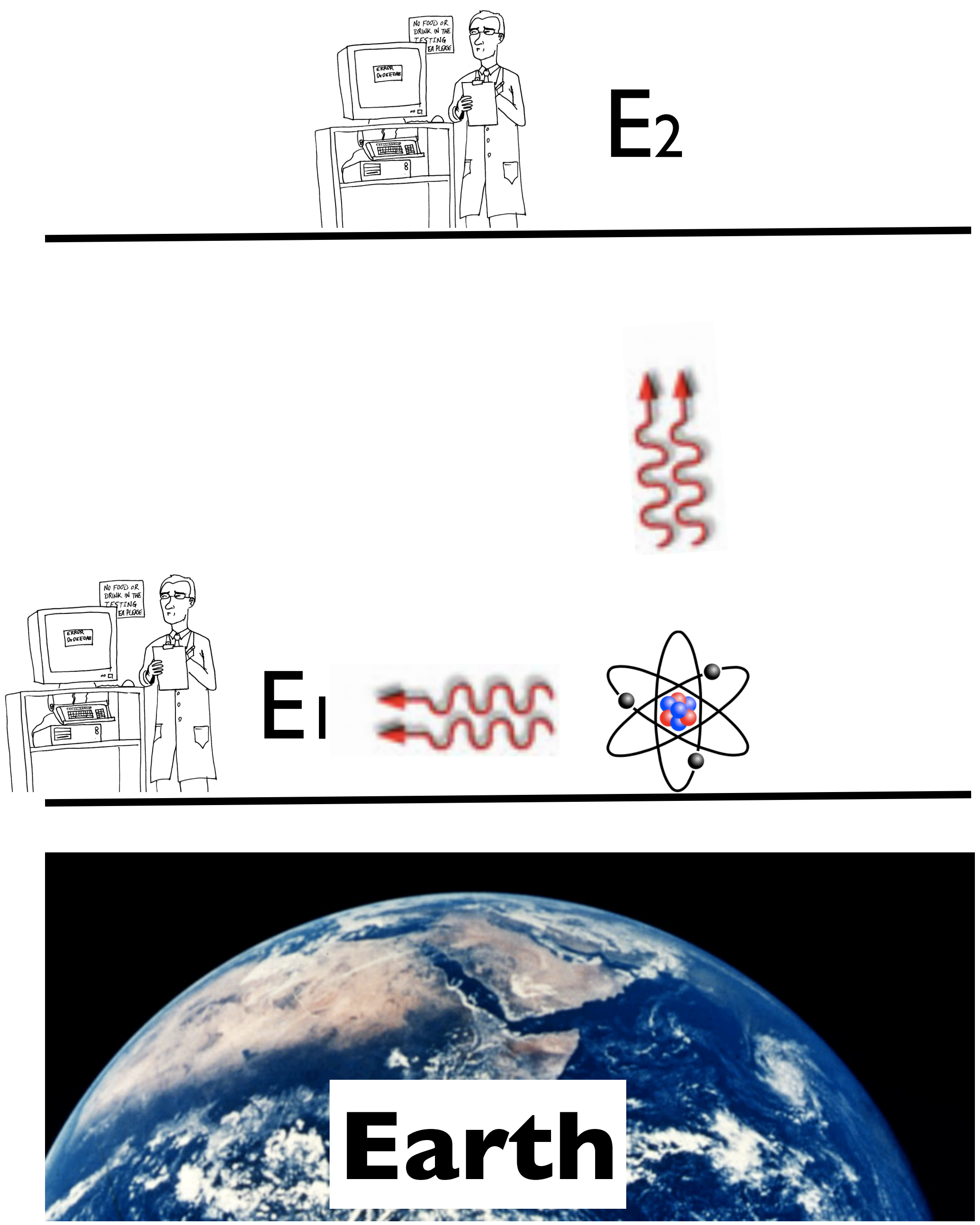
Anti-de-Sitter (AdS):

$$a = \frac{L}{z}$$

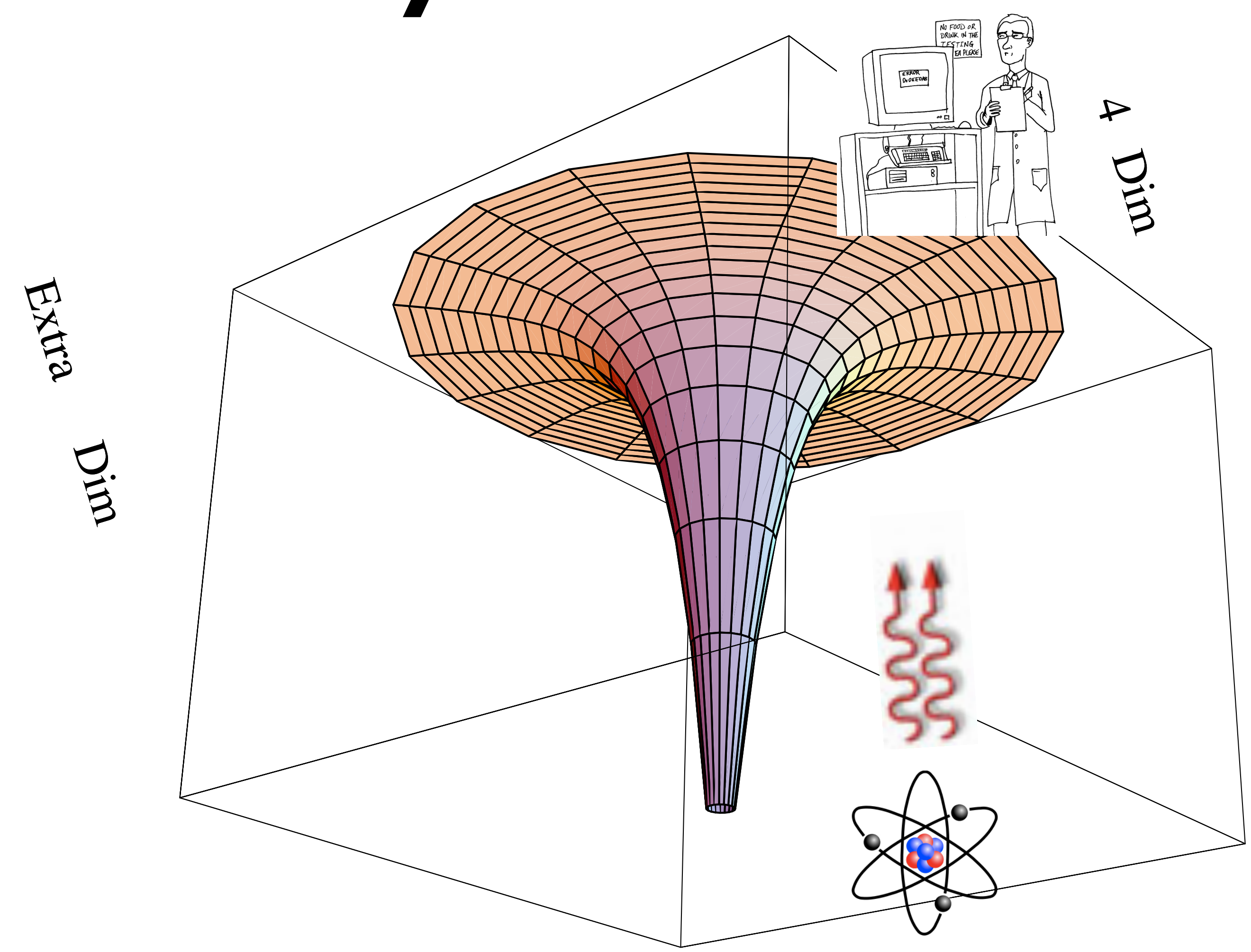
Here the observer sees that the atomic transition is less energetic

Qualitatively can be understood as the photon loosing kinetic energy as it climbs up the gravitational potential well:

$$E_2 < E_1$$

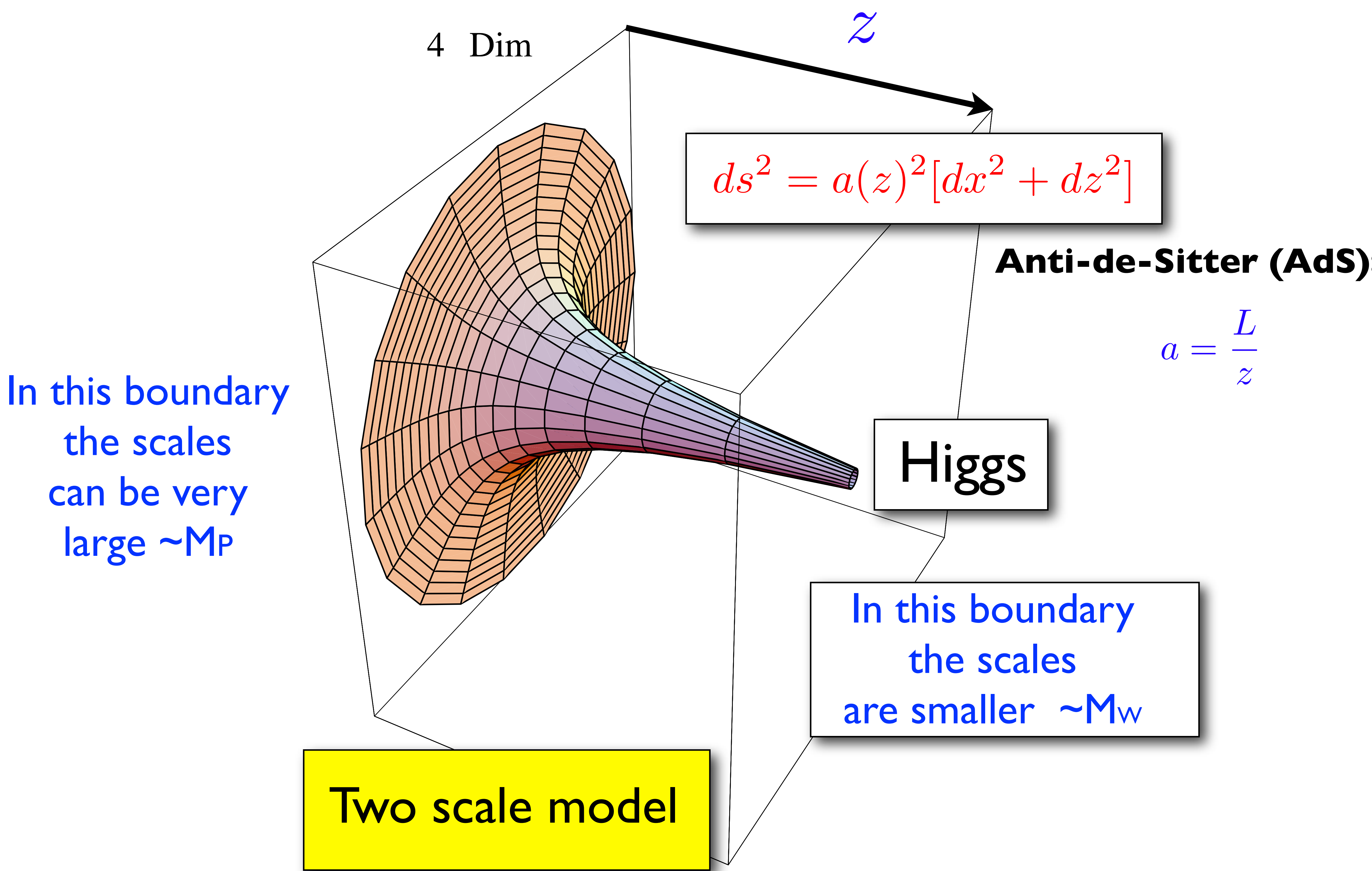


Similarly...

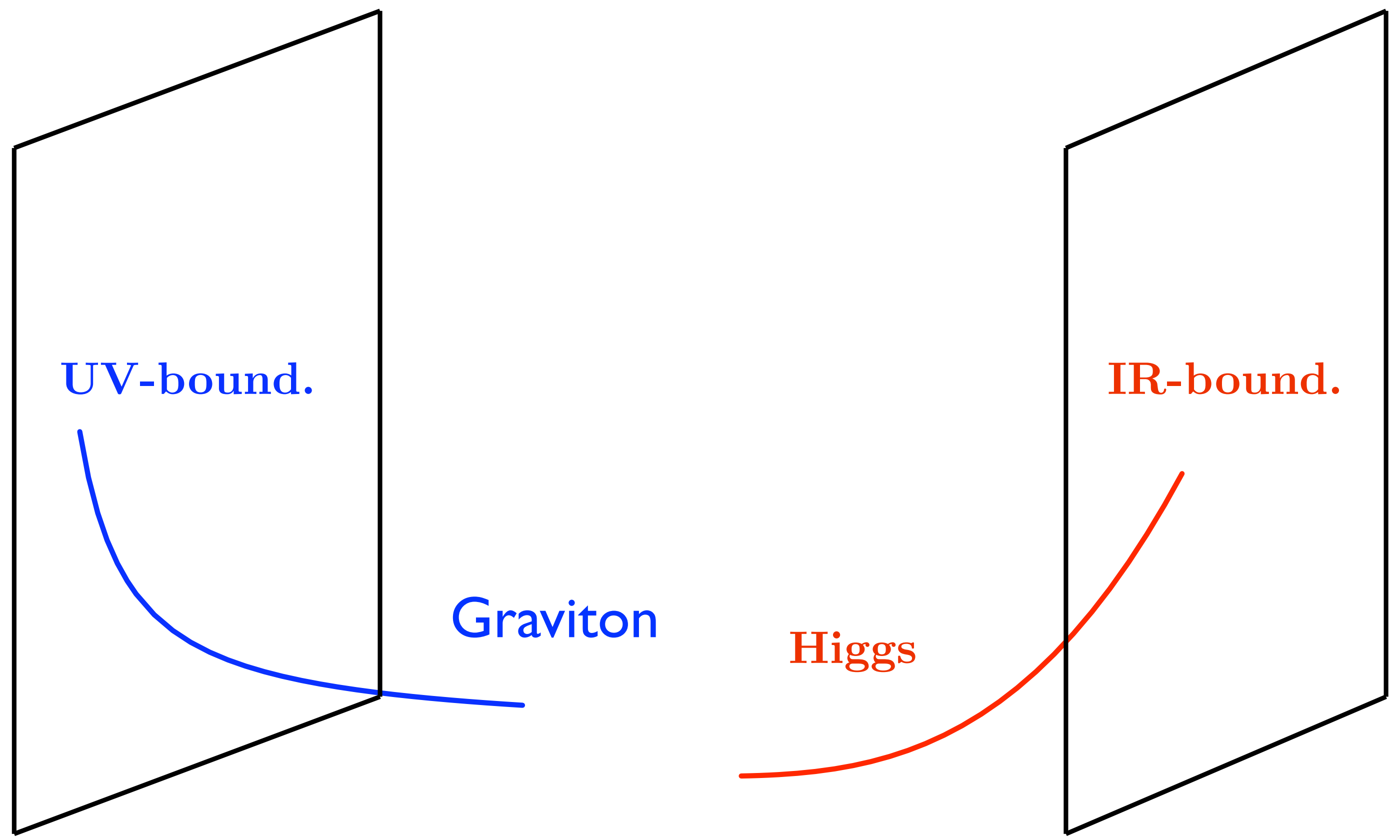


Randall-Sundrum Idea

Placing the Higgs in the interior of the extra dimension could explain why its mass-term is smaller than M_P:



Alternative understanding by looking at the wave-function of a graviton in a AdS-space



As in QM: Small overlapping of wave-functions = small couplings

➥ gravity is weak for the Higgs!!

In 1999 when Sundrum was explaining this idea in a conference in Santa Barbara, E. Witten stood up and *more or less* said:

"This is as having a composite Higgs made of strongly-coupled fields of a conformal field theory (CFT)"

What did he have in mind?

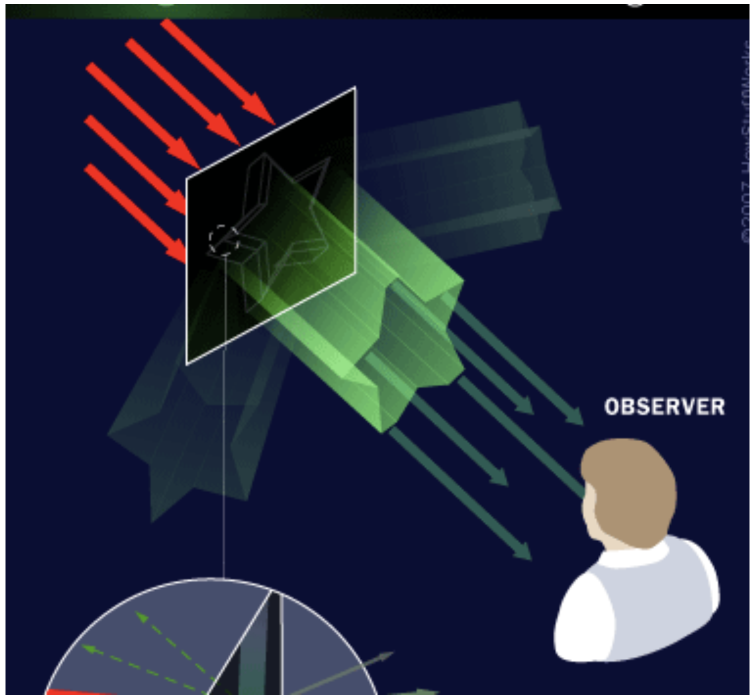
The AdS/CFT correspondence:

Maldacena 97

Strongly coupled 4D theories in certain limits ⟵ duality ⟶ Weakly coupled gravity theories in higher-dimensions

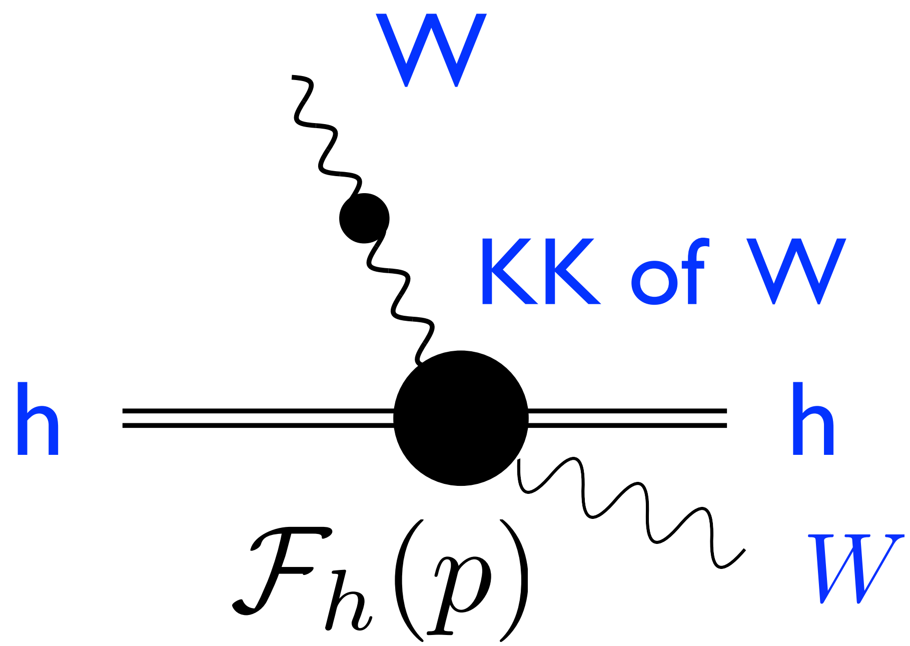
Composite states have similar dynamics as particles in a curved extra dimension

➡ **Holography:** "4D composite states encode 5D information"

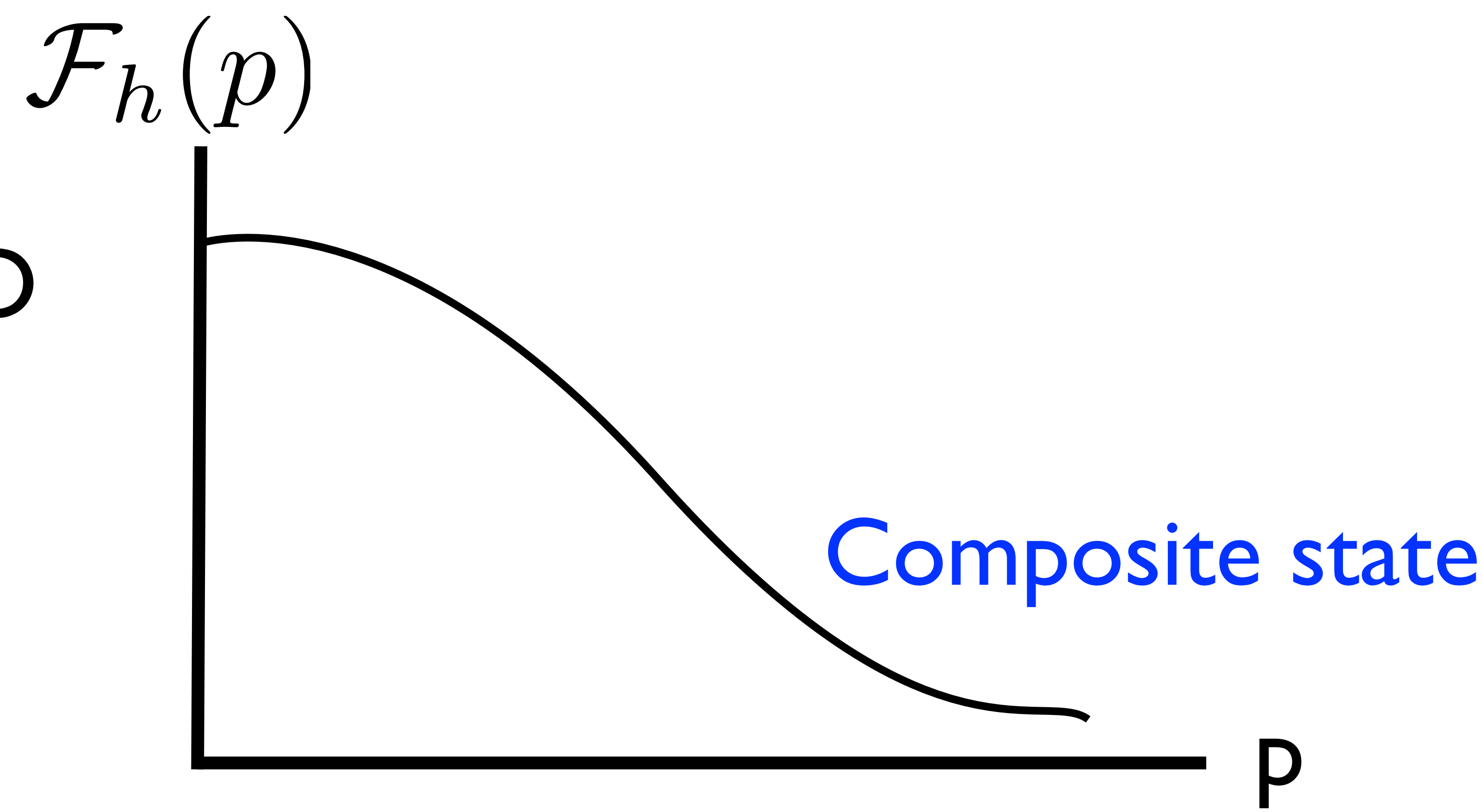


One can check certain things for a Higgs in an AdS-extra dimension:

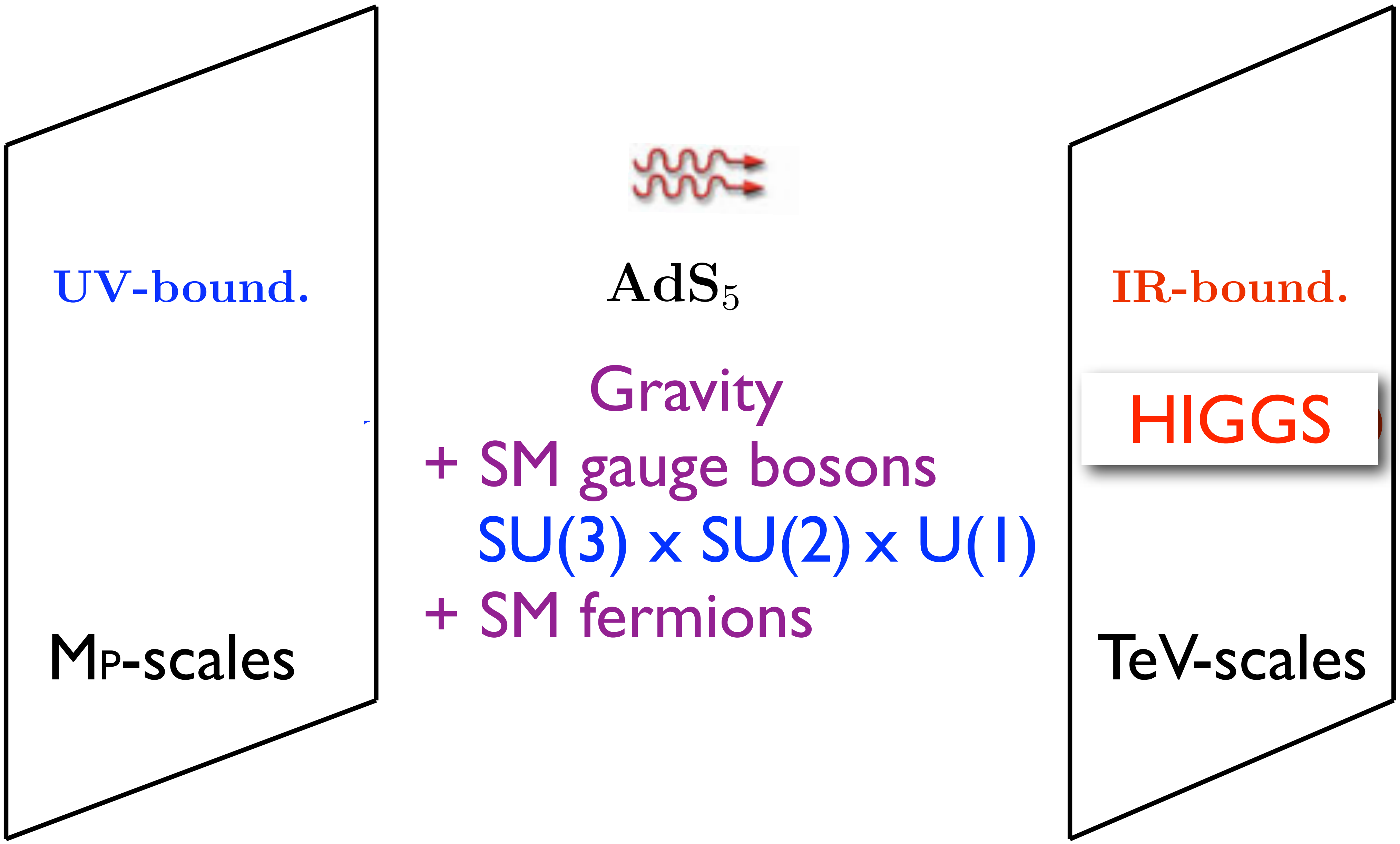
Example:



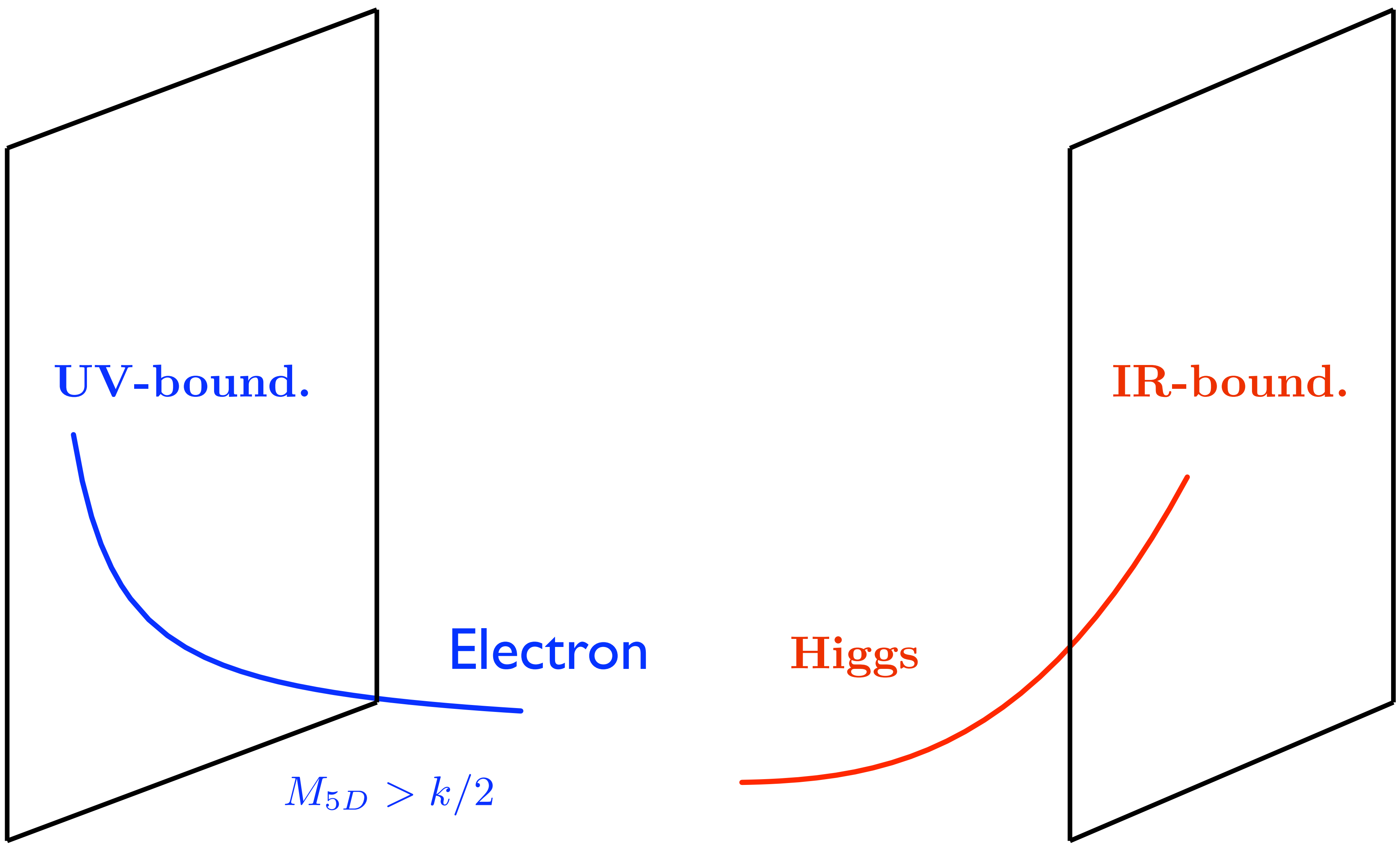
The form factor of a 5D Higgs follows the expectation for a composite state



Five Dimensional composite Higgs model

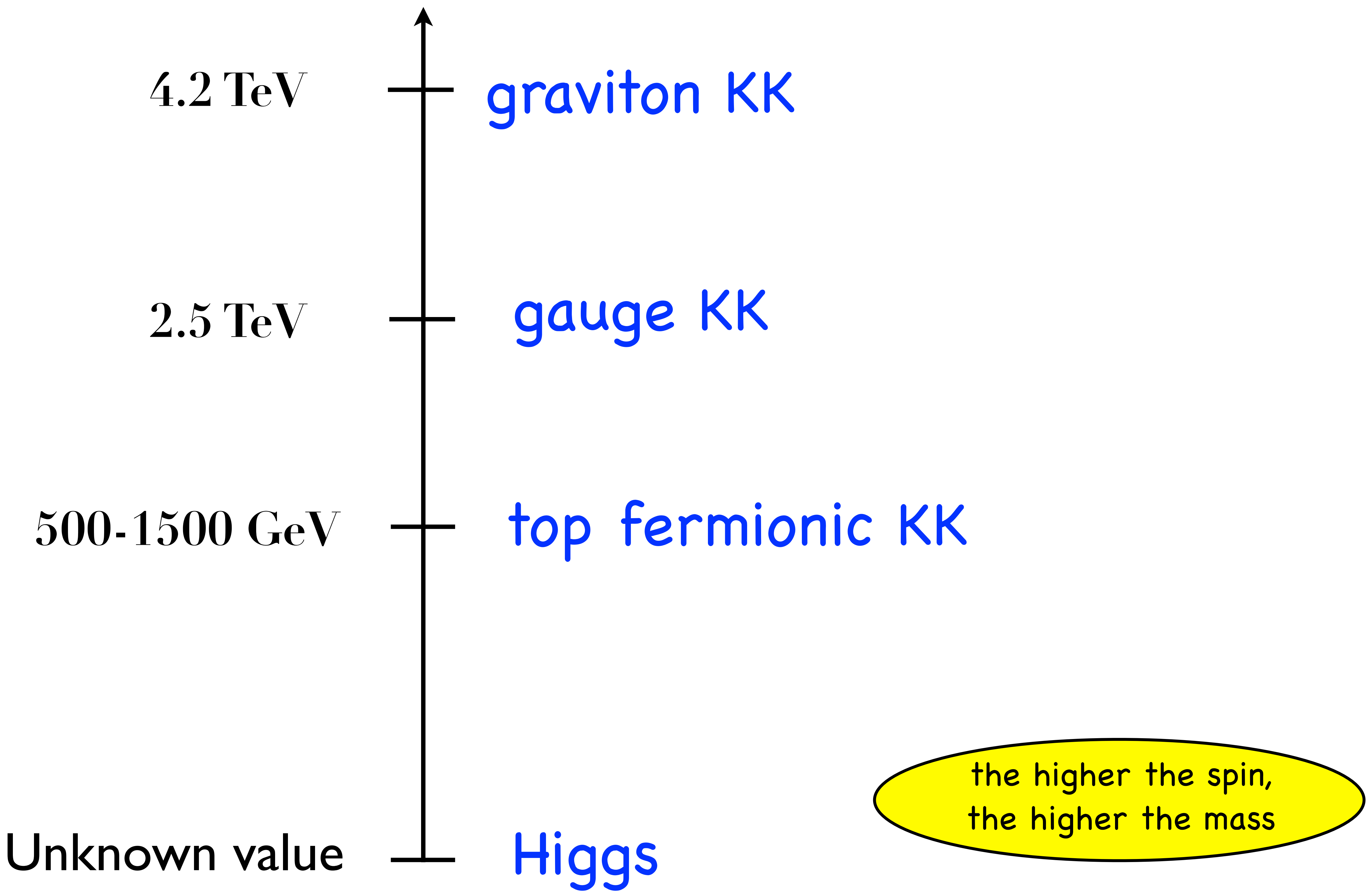


Small masses for fermions (e.g. electron) easy to generate by having the wave-functions picked towards the opposite boundary to the Higgs



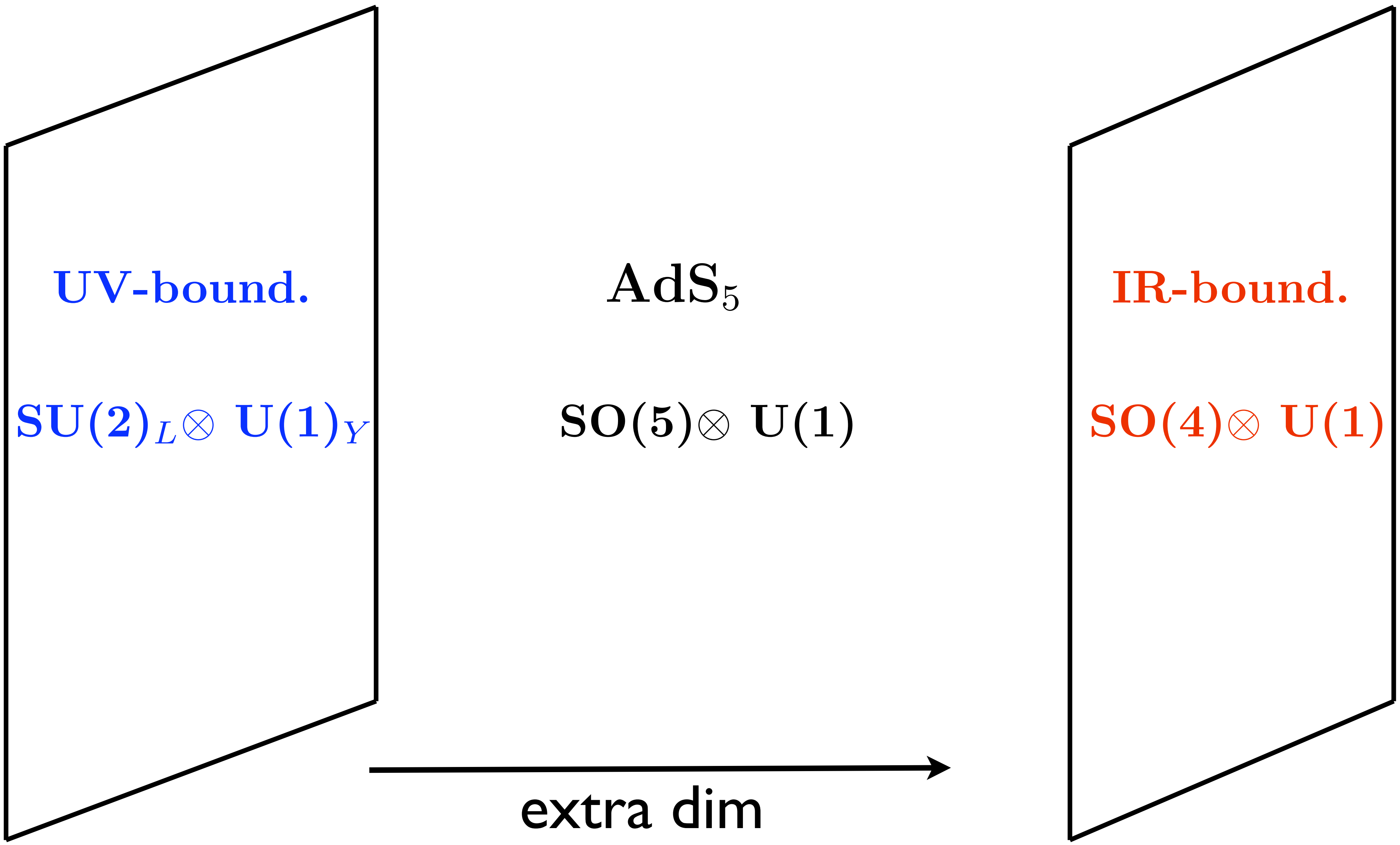
⇒ **Nice "geometrical" explanation of the smallness of some of the SM fermion masses**

Spectrum


4.2 TeV
graviton KK
2.5 TeV
gauge KK
500-1500 GeV
top fermionic KK
Unknown value
Higgs
the higher the spin,
the higher the mass

Five Dimensional composite (PGB) Higgs model

The bosonic sector:



Why this symmetry breaking pattern?

We are in 5D: $\mathbf{A_M} = (\mathbf{A}_\mu, \mathbf{A_5})$

Massless boson spectrum:

- $\mathbf{A}_\mu$ **of** $\mathbf{SU(2)}_L \otimes \mathbf{U(1)}_Y$ = **SM Gauge bosons**
- $\mathbf{A_5}$ **of SO(5)/SO(4) = 2 of** $\mathbf{SU(2)}_L$ = **SM Higgs**

$\hookrightarrow$ **Higgs-gauge unification**

Hosotani mechanism

Higgs mass protected by 5D gauge invariance!

$$A_5 \to A_5 + \partial_5 \theta$$

→ shifts as a PGB

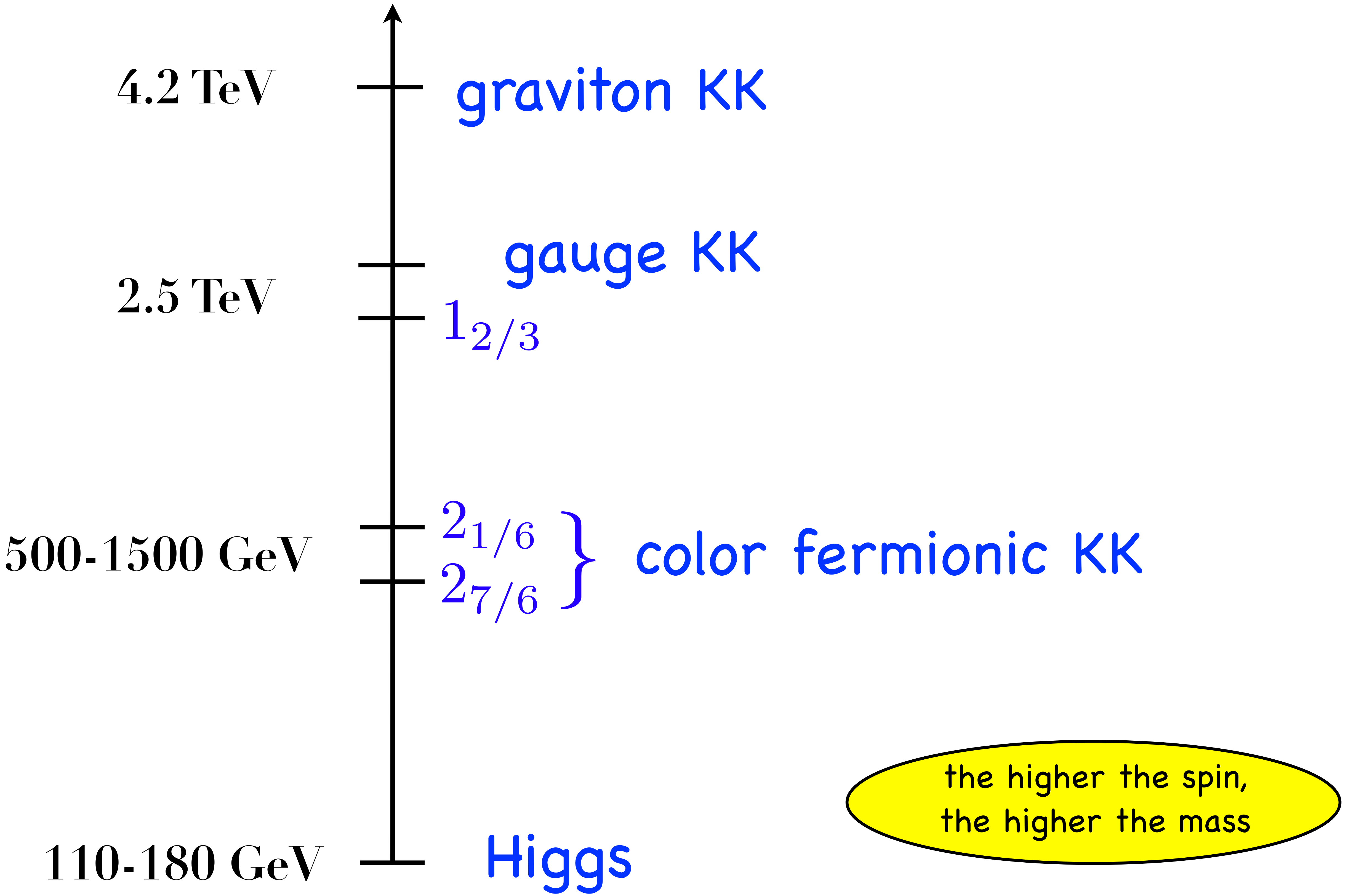
Predictions

Light Higgs $+$ **KK resonances for each SM field in complete reps of the bulk group SO(5)**

top: $5 = 2_{7/6} + 2_{1/6} + 1_{2/3}$

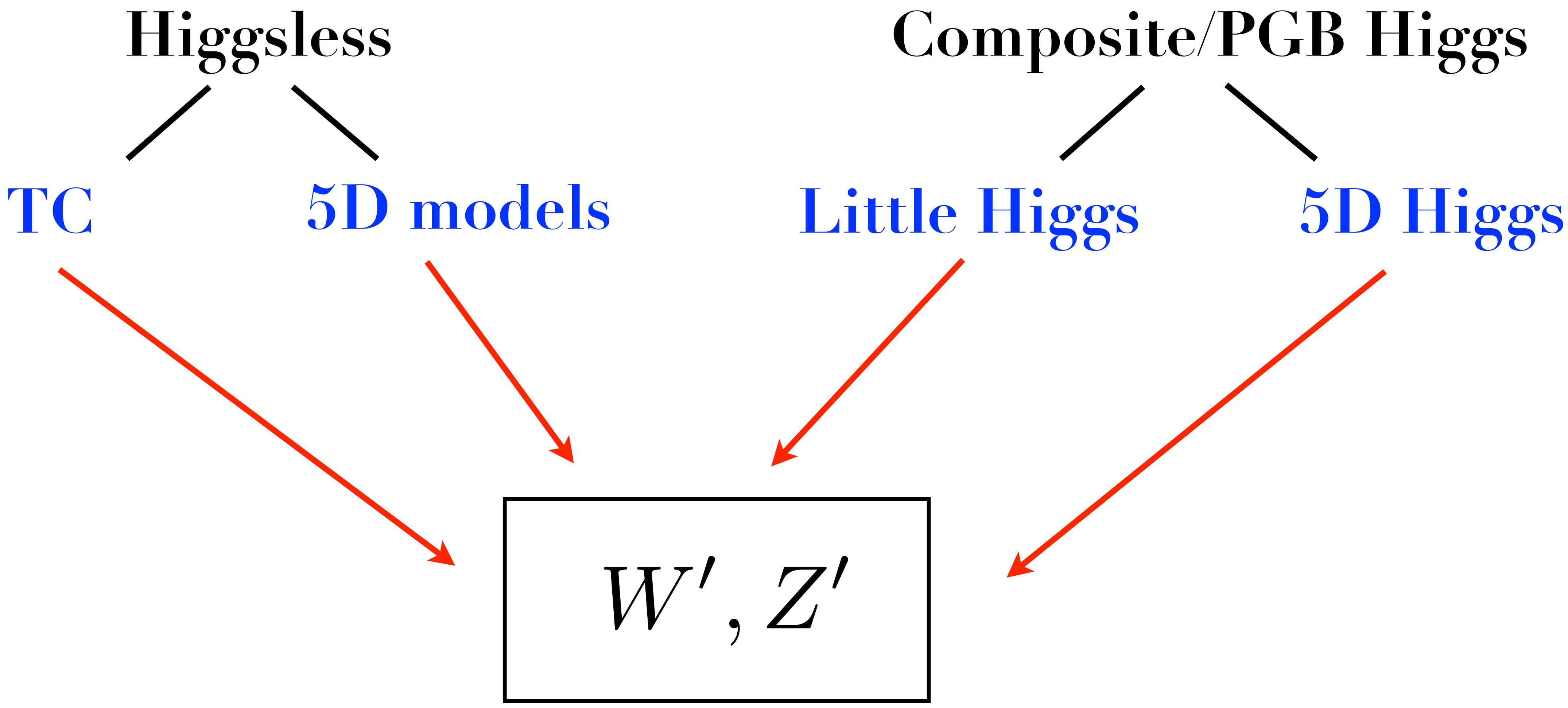
→ **exotic states of Q=5/3**

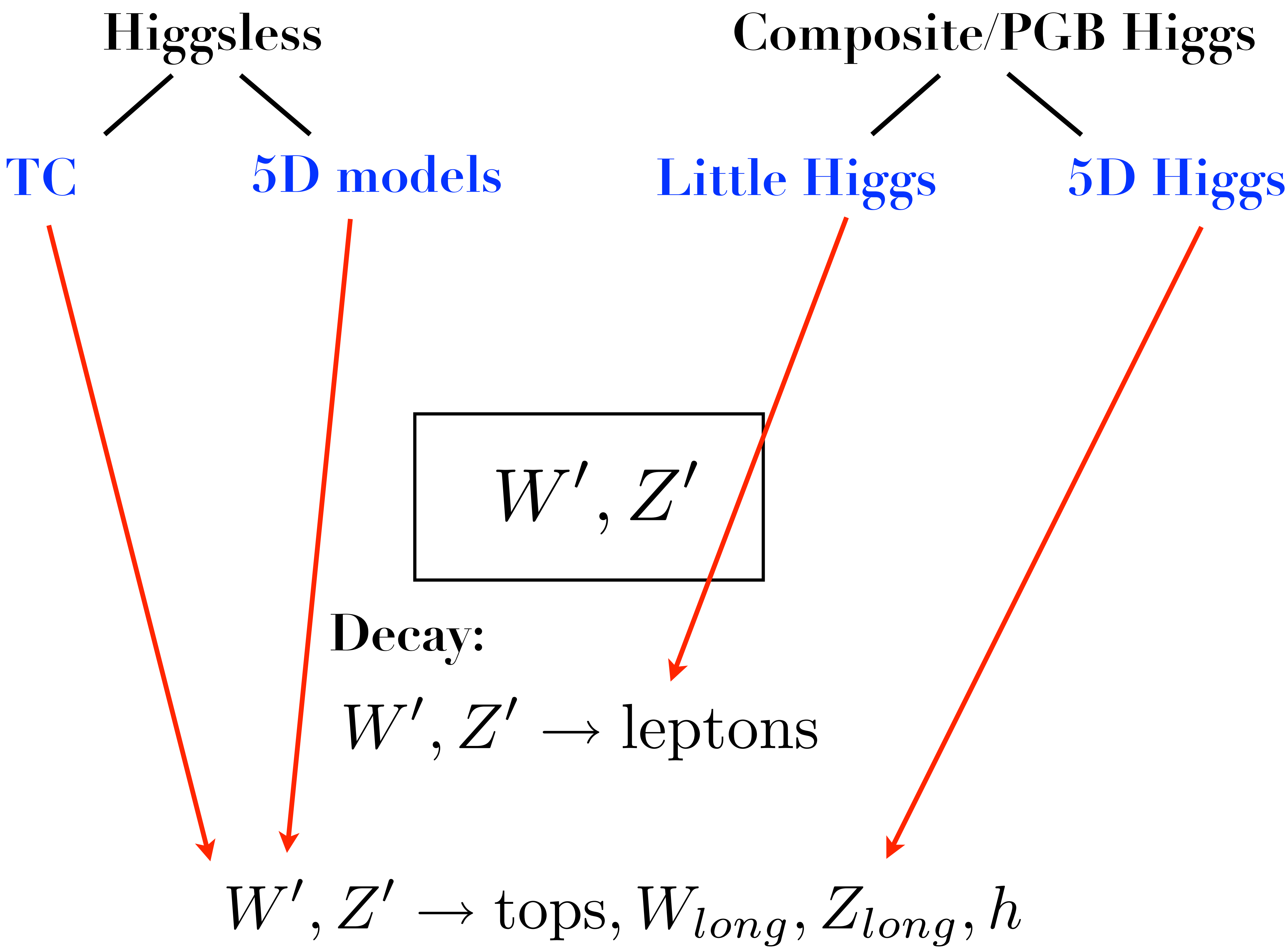
Spectrum


4.2 TeV
graviton KK
gauge KK
2.5 TeV
$1_{2/3}$
$2_{1/6}$
$2_{7/6}$
500-1500 GeV
color fermionic KK
110-180 GeV
Higgs
the higher the spin,
the higher the mass

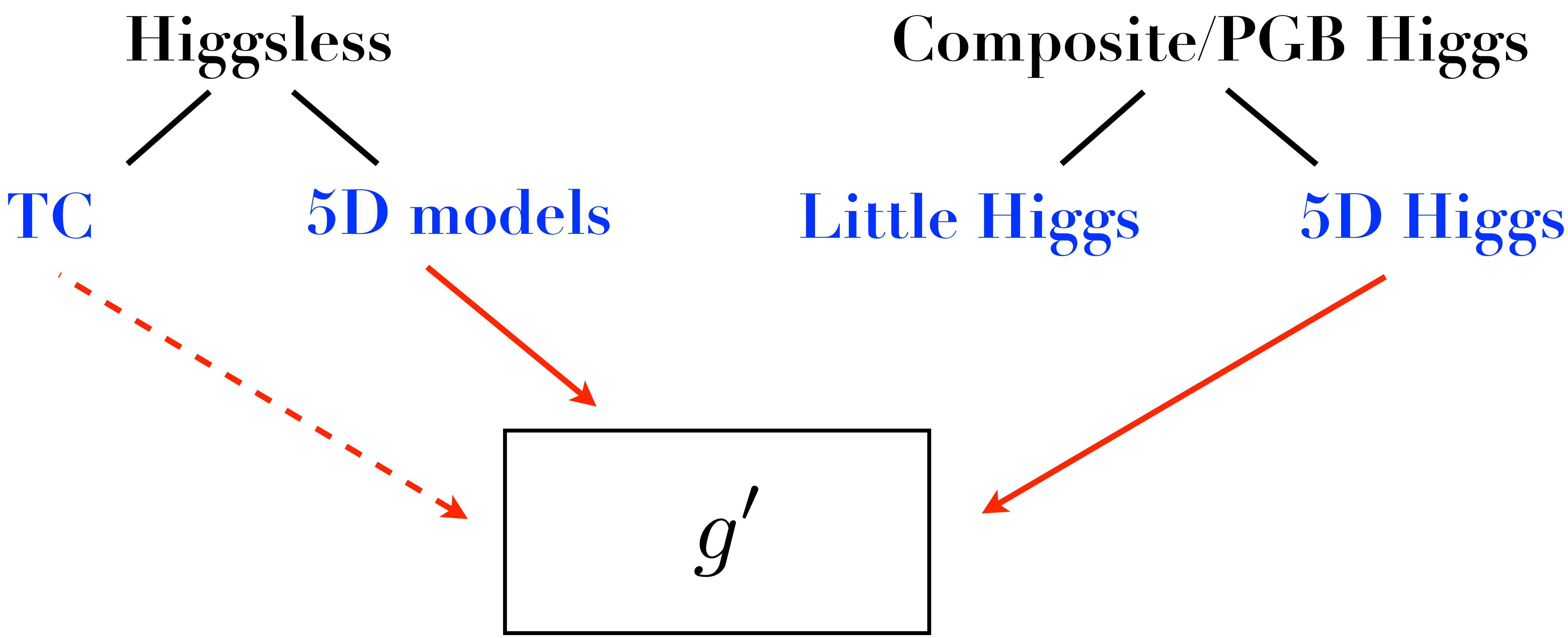
How to see the KK at Hadron Colliders?

(and similarities/distinction with other models)

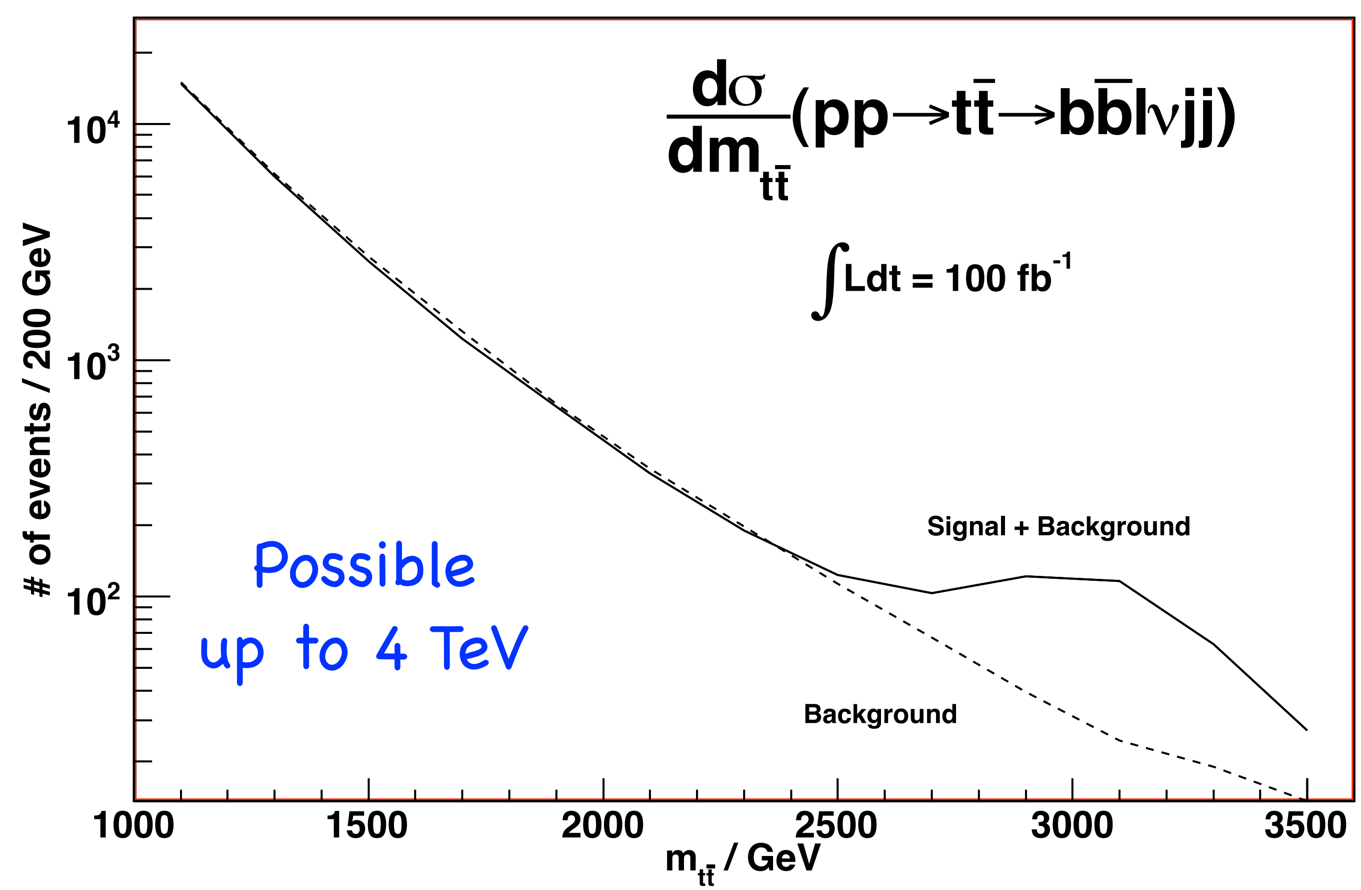

Higgsless
Composite/PGB Higgs
TC
5D models
Little Higgs
5D Higgs
W', Z'



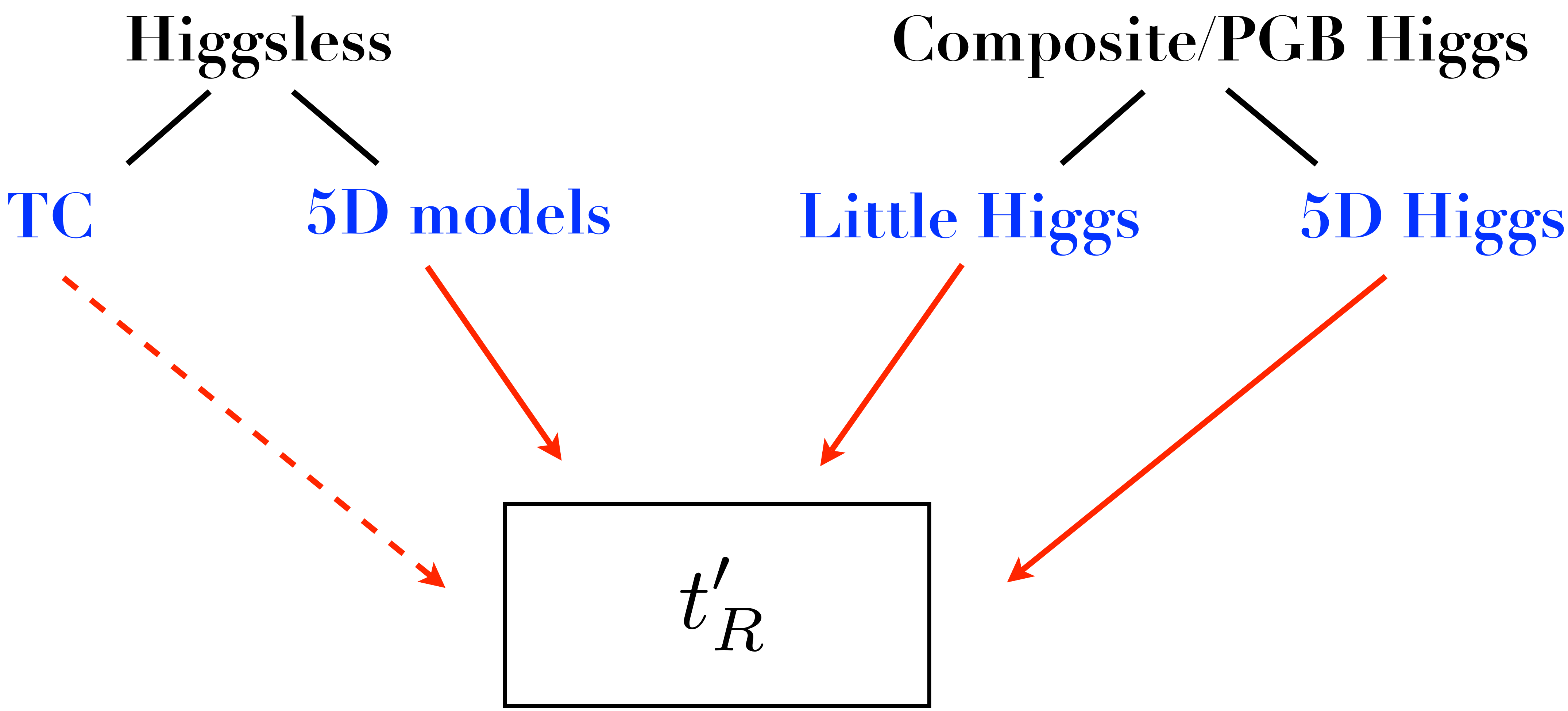
Possible to see up to 2 TeV



Decay: $g' \to t\bar{t}$

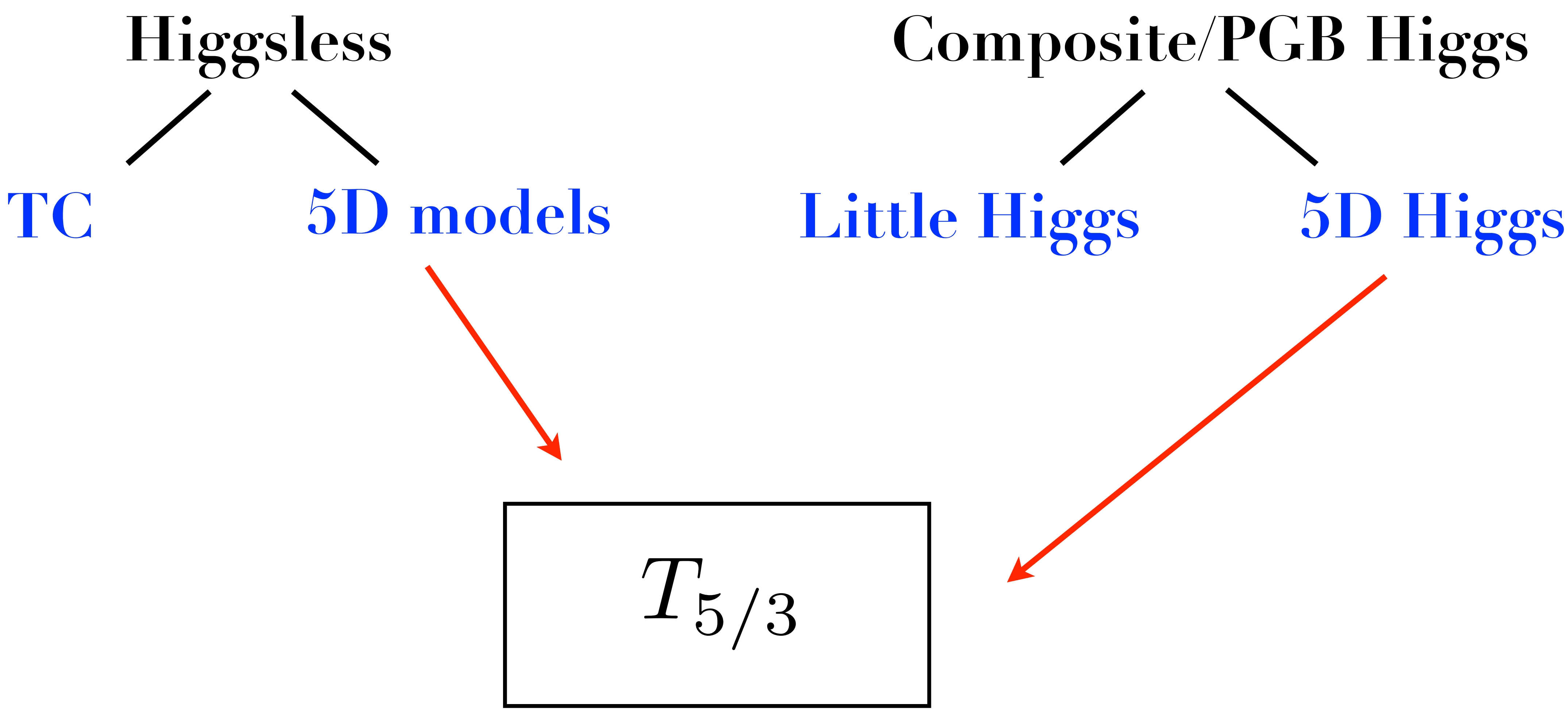


Agashe et al



Decay: $t'_R \rightarrow W_{long} b$

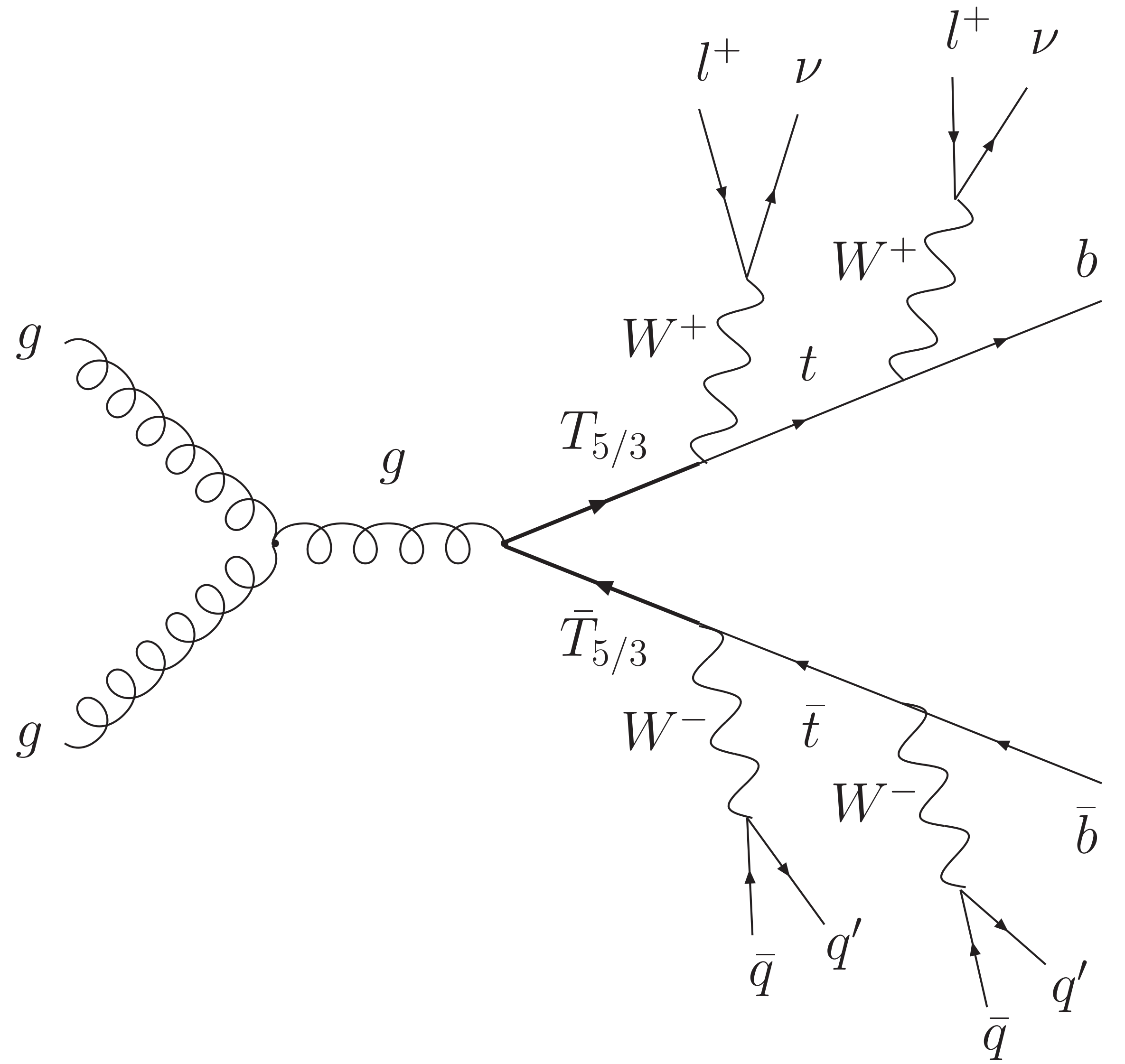
feasible to see up to 1-2 TeV



Decay: $T_{5/3} \longrightarrow W_{long} t$

feasible to see up to 1-2 TeV

If this fermion is light, it can be double produced:



two like-sign leptons

masses up to 1 TeV reached with an integrated luminosity of 20/fb

Contino,Servant,
see also Saavedra, Wulzer, Disertori