Physics 457
Particles and Cosmology

Part 2

Standard Model of Particle Physics

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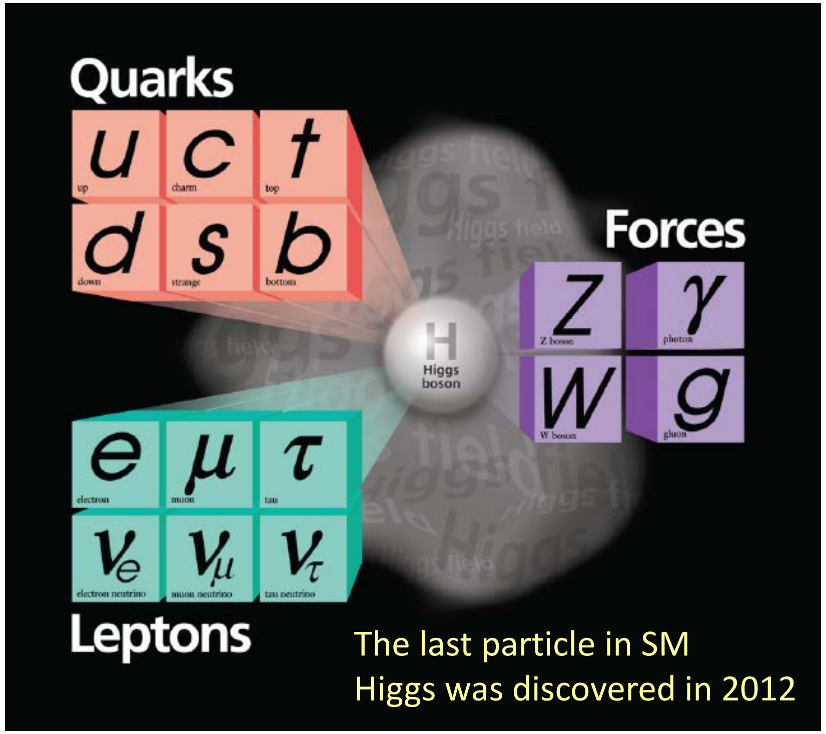
Long History of Particle Physics Development

1/23/2019

1900s:	e discovered (cathode ray tube)
	γ interpreted as a particle
1930s:	μ discovered (cosmic rays)
1950s:	ν_e observed (nuclear reactor)
	ν_μ discovered (BNL)
1960s:	1st evidence for quarks
	u and d observed (SLAC)
	s observed (BNL)
1970s:	***standard model is born***
	c discovered (SLAC, BNL)
	τ observed (SLAC)
	b observed (FNAL)
1980s:	W and Z observed (CERN)
1990s:	t quark observed (FNAL)
2000s:	ν_τ observed (FNAL)

Standard Model of Particle Physics



A theory of matter and forces.

Point-like matter particles (**quarks and leptons**), which interact by exchanging force carrying particles:

Photons, $W^{\pm}$ and Z, gluons

Particle masses are generated by Higgs Mechanism

Review: Schrodinger Equation

In nonrelativistic quantum mechanics, particles are described by Schrodinger equation

$$-\frac{\hbar^2}{2m}\nabla^2\psi + V\psi = i\hbar\frac{\partial\psi}{\partial t}$$

Ref. Griffiths Ch. 7

obtained by applying the quantum prescription

$$\vec{p} \Rightarrow -i\hbar\nabla, \qquad E \Rightarrow i\hbar\frac{\partial}{\partial t}$$

to the classical energy-momentum relation:

$$E = \frac{p^2}{2m} + V$$

The Schrodinger's equation is the 1st order in time-derivative and 2nd order in space-derivative.

Review: Klein-Gordon Equation

In relativistic quantum theory, spin-0 particle (1 d.o.f) is described by the Klein-Gordon wave equation

$$\left(\partial_\mu \partial^\mu + \frac{m^2 c^2}{\hbar^2}\right)\phi = 0 \quad \text{with} \quad \partial_\mu \partial^\mu \equiv \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2$$

Ref. Griffiths Ch. 7

ϕ is the scalar field. It is the result of quantum prescription

$$\vec{p} \Rightarrow -i\hbar\nabla, \qquad E \Rightarrow i\hbar\frac{\partial}{\partial t}$$

of the relativistic Energy-momentum relation:

$$E^2 = (pc)^2 + (mc^2)^2$$

$$\left(i\hbar\frac{\partial^2}{\partial t^2}\right)\phi = \left((-i\hbar\nabla)^2 + (mc^2)^2\right)\phi \quad \Rightarrow \quad \left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2\right)\phi = -\left(\frac{mc}{\hbar}\right)^2 \phi$$

Review: Dirac Equation (1)

Klein-Gordon equation is 2nd order derivative in time compared with the 1st order of Schrodinger's equation

- incompatible with probability conservation

Ref. Griffiths Ch. 7

Dirac set out to find out an equation that is consistent with the energy-momentum relation and yet 1st order in time. He noted that if the scalar equation

$$p_\mu p^\mu - (mc)^2 = 0$$

is turned into a 4x4 matrix equation, then the equation factorizes

$$\left(p_\mu p^\mu - m^2c^2\right) = \left(\gamma^\nu p_\nu + mc\right)\left(\gamma^\nu p_\nu - mc\right) = 0$$

and the energy-momentum equation becomes linear

$$\gamma^\nu p_\nu + mc = 0 \quad \text{or} \quad \gamma^\nu p_\nu - mc = 0$$

γ^ν =4x4 Dirac matrix
See next page

which leads to the Dirac equation with the quantum prescription

$$\vec{p} \Rightarrow -i\hbar\nabla, \qquad E \Rightarrow i\hbar\frac{\partial}{\partial t}$$

Review: Dirac Equation (2)

In relativistic quantum theory, spin-1/2 particle and its antiparticle (4 d.o.f) are described by the Dirac wave equation

$$i\hbar\gamma^{\mu}\partial_{\mu}\psi - mc\psi = 0$$

Ref. Griffiths Ch. 7

with ψ as a four-element column matrix called Dirac spinor

$$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \end{pmatrix}$$

And 4x4 γ-matrices have the following properties and can be represented by Pauli matrices (σ) as

$$\{\gamma^{\mu}, \gamma^{\nu}\} = 2g^{\mu\nu}$$

$$\gamma^0 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}, \qquad \gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}$$

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \qquad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Review: Classical Electrodynamics

In classic electrodynamics, E and B fields produced by a charge density ρ and current density j are determined by Maxwell equation.

$$\nabla . \mathrm{E} = \rho, \qquad \nabla \times \mathrm{E} + \frac{\partial \mathrm{B}}{\partial t} = 0$$

$$\nabla . \mathrm{B} = 0, \qquad \nabla \times \mathrm{B} - \frac{\partial E}{\partial t} = \mathrm{j}$$

Field strength tensor

$$\begin{pmatrix} 0 & -E_1 & -E_2 & -E_3 \\ E_1 & 0 & -B_3 & B_2 \\ E_2 & B_3 & 0 & -B_1 \\ E_3 & -B_2 & B_1 & 0 \end{pmatrix}$$

Lorentz invariant Lagrangian and eq.

$$\mathcal{L}^{EM} = -\tfrac{1}{4} F_{\mu\nu} F^{\mu\nu} - j^{\mu} A_{\mu}$$

$$F_{\mu\nu} = \partial_{\mu} A_{\nu} - \partial_{\nu} A_{\mu}$$

$$F_{\mu\nu} \to F_{\mu\nu}, \qquad A_{\mu} \to A_{\mu} + \partial_{\mu}\alpha$$

$$\frac{\partial \mathcal{L}}{\partial A^{\nu}} - \partial^{\mu} \frac{\partial \mathcal{L}}{\partial(\partial^{\mu} A^{\nu})} = 0$$

$$\partial_{\mu} F^{\mu\nu} = j^{\nu}$$

Review: EM Wave Equations

The electromagnetic wave equation (Can be derived from source less Maxwell Eqs)

$$\nabla^2 E = \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2}$$

admits plane wave solution

$$E \sim e^{i\left(\vec{k}\cdot\vec{r}-\omega t\right)} = e^{i\left(\vec{p}\cdot\vec{r}-Et\right)/\hbar} \quad \text{with } \vec{p} = \hbar\vec{k} \text{ and } E = \hbar\omega$$

Similarly both Klein-Gordon and Dirac's equations have plane wave solutions

$$\phi(x) \sim e^{i\left(\vec{k}\cdot\vec{r}-\omega t\right)} = e^{i\left(\vec{p}\cdot\vec{r}-Et\right)/\hbar} = e^{-ip\cdot x/\hbar}, \quad \psi(x) \sim u(p)e^{-ip\cdot x/\hbar}$$

with time-space and energy-momentum four vectors

$$x = \left(ct, \vec{r}\right), \quad p = \left(\frac{E}{c}, \vec{p}\right) \quad \text{note } p\cdot x = p^{\mu}x_{\mu} = Et - \vec{p}\cdot\vec{r}$$

These solutions describe streams of free particles (no interactions)

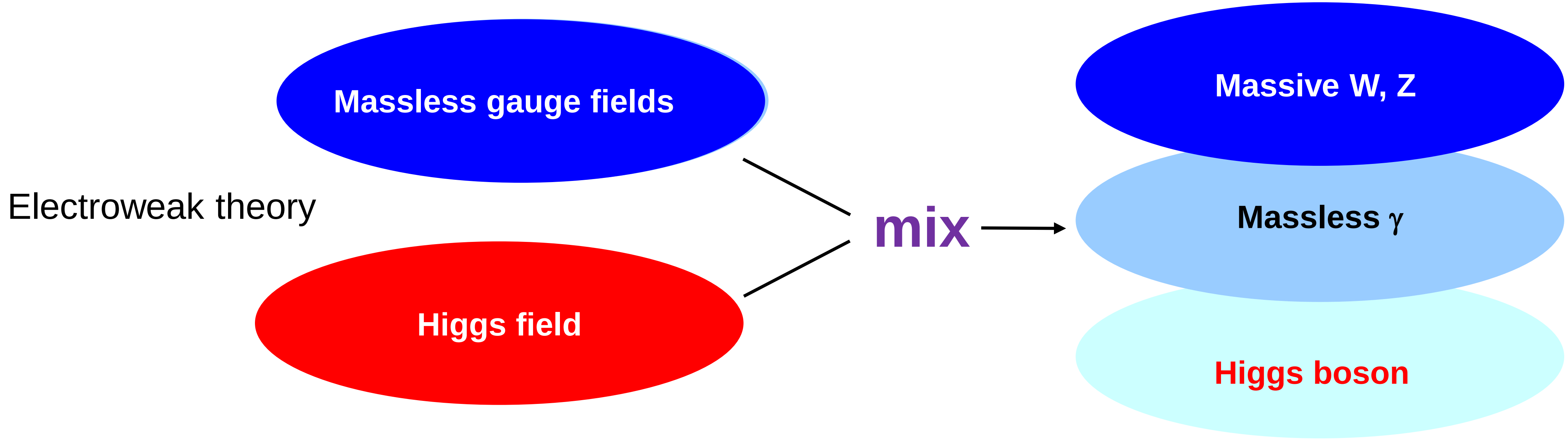
The Standard Model Framework

Theory is based on two principles:

- **Gauge invariance** (well tested with experiments over 50 years)

  Combines **Maxwell's theory** of electromagnetism, **Special Relativity** and **Quantum field theory**

- **Higgs Mechanism** (mystery in 50 years until Higgs discovery in 2012)



Symmetries and Gauge Invariance

- **Symmetries** $\longleftrightarrow$ **Conservation laws**
  Conservation laws observed in nature may be imposed as symmetries of the Lagrangian.

- **Gauge invariance** *In order to keep the laws of physics unchanged under symmetry transformations,* **Gauge fields must be introduced.** *Such symmetry is called gauge invariance.*

Examples

Symmetry	Conservation Law
Translation in time	Energy
Translation in space	Momentum
Rotation	Angular momentum
Gauge transformation	Charge

- **Often the symmetries are built in Lagragian density function, $L = T - V$**
- **Equation of motion (physics law) can be derived from Lagrangian**

Electron Free Motion Equation

- Example – equation of electron motion

  (*ref. Ch. 7, and 10, Griffiths book*)

  (*In nature unit*)

  $$L_e = \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi \implies (i\gamma^\mu \partial_\mu - m)\psi = 0$$

  $\psi \longrightarrow \psi' = e^{i\alpha}\psi \longrightarrow$ eq. of motion is unchanged. $\Rightarrow$ Q conservation.

γ^μ $- Dirac\ 4 \times 4\ matrices$

ψ $- electron\ wave\ function$ (4 componebts)

*Applying **Euler-Lagrange equation** to obtain the equation of motion:*

$$\partial^\mu \left(\frac{\partial L}{\partial(\partial_\mu \bar{\psi})} \right) - \frac{\partial L}{\partial \bar{\psi}} = 0$$

***L_e* here is an electron free motion Lagragian function (i.e. no interaction).**

The electron wave function phase transformation will not change the equation of motion of the electron –> symmetry transformation

The corresponding conservation law is that the electron charge is conserved

The Idea of Gauge Invariance

If, $\psi \to \psi' = e^{i\alpha(x)}\psi \to$ eq. of motion is changed!

Introduce a new field A_μ, and modify $\partial_\mu \to D_\mu = \partial_\mu - ieA_\mu$, let $A_\mu \to A_\mu - \partial_\mu \alpha(x)$, we obtain:

$(i\gamma^\mu D_\mu - m)\psi = 0 \Leftarrow L = L_e - eA_\mu \bar{\psi}\gamma^\mu \psi$

L is gauge invariant, A_μ is the gauge field, γ!

"Local" Gauge transformations of electron wave function and gauge field

e is the charge,
→EM interaction strength

Comment:

Gauge invariance naturally introduces the interaction into the Lagrangian function (theory)

Gauge Invariance Requires Massless Gauge Boson

- Remarks
  Gauge field must be massless, since the mass term in L should be
  $$Lm = \frac{1}{2}M^2 A^\mu A_\mu.$$
  But, under gauge transformation:
  $$A^\mu A_\mu \to (A^\mu - \partial^\mu \alpha)(A_\mu - \partial_\mu \alpha) \neq A^\mu A_\mu!$$
  Observations: weak gauge bosons are massive!

The First Gauge Theory - Quantum Electrodynamics

- Describes all interactions of light with matter, and those of charged particle with one another.
- Because the behavior of atoms and molecules is primarily electromagnetic in nature, all of atomic physics can be considered a test laboratory for the theory.
- **Precision tests** of QED have been performed in low-energy atomic physics, high energy collider experiments and condensed matter system.

QED: invariance under the local transformations of the abelian group U(1)

– transformation of electron field: $\Psi(x) \rightarrow \Psi'(x) = e^{ie\alpha(x)}\Psi(x)$

– transformation of photon field: $A_\mu(x) \rightarrow A'_\mu(x) = A_\mu(x) - \frac{1}{e}\partial_\mu \alpha(x)$

The Lagrangian density is invariant under above field transformations

$$\mathcal{L}_{QED} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + i\bar{\Psi}D_\mu\gamma^\mu\Psi \;\; - m_e\bar{\Psi}\Psi$$

with field strength $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ **and cov. derivative** $D_\mu = \partial_\mu - ieA_\mu$

coupling (→ e)

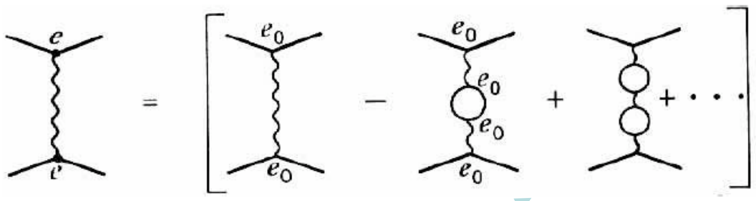
Running EM Interaction Coupling

However, due to charge screening, a depends on the energy-momentum transfer q:

$$\alpha(q^2) = \frac{\alpha(\mu^2)}{1 - \frac{\alpha(\mu^2)}{3\pi}\log\left(\frac{q^2}{\mu^2}\right)} \quad \text{or} \quad \frac{1}{\alpha(q^2)} = \frac{1}{\alpha(\mu^2)}\left\{1 - \frac{\alpha(\mu^2)}{3\pi}\log\left(\frac{q^2}{\mu^2}\right)\right\}$$

Vertex coupling constant α_{QED}:

$$\alpha_{QED} = \frac{e^2}{4\pi\epsilon_0\hbar c} = \frac{1}{137} = 0.0073$$

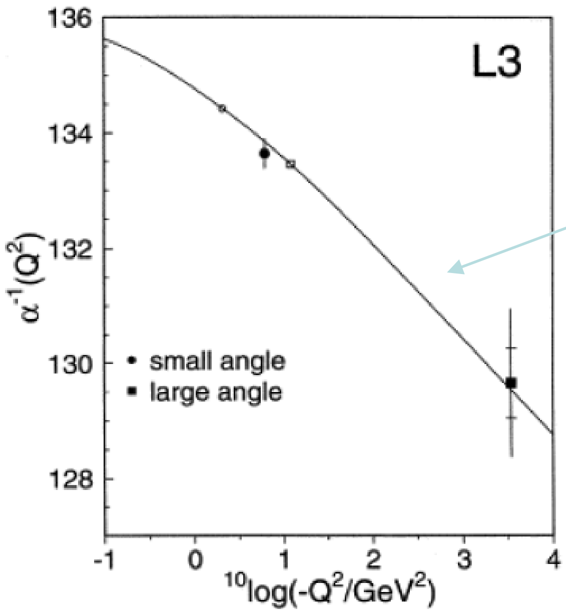


Higher-order diagrams

It increases at high energies, i.e., the interaction gets stronger. At mass of the Z-boson, the increase is about 7%:

$$\alpha(q^2 \sim 0) \approx \frac{1}{137} \quad \Rightarrow \quad \alpha(q^2 = M_Z^2) \approx \frac{1}{128}$$

$\alpha(q^2)$ is called the running coupling constant



1/ α_{QCD} vs. Log(-Q^2) measurement at L3 experiment at CERN LEP

α_{QCD} increases as energy increases

Experimental Test of QED

On anomalous magnetic dipole moment a = (g-2) / 2

- **g-2 of the electron and fine structure constant**

  $g/2 = 1.001\ 159\ 652\ 180\ 85\ (76)$

  A precision better than one part in a trillion (ppt)

  $\alpha^{-1} = 137.035\ 999\ 070\ (98)$

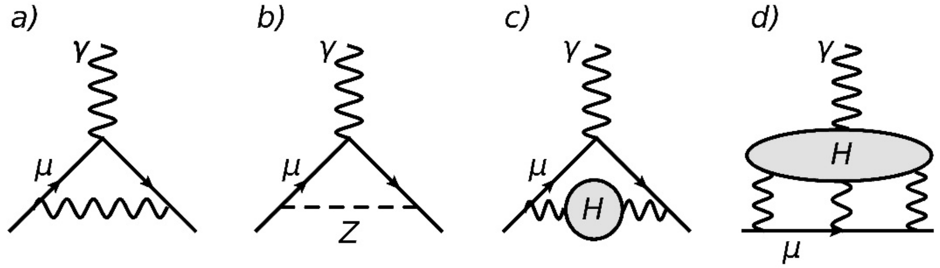
  A precision better then one part in a billion (ppb)

- **g-2 of the muon**

  $a_\mu(\mathrm{Exp}) = (g-2)/2 = 0.001\ 165\ 920\ 89(63)$ (ppm)

  $a_\mu(\mathrm{SM}) = (g-2)/2 = 0.001\ 165\ 918\ 28(49)$

Theoretical & Experimental Status of a_μ



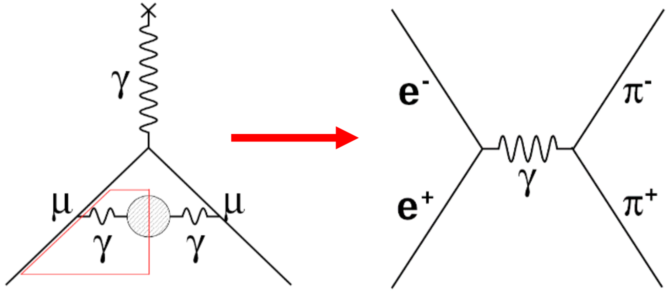
Source	Value (a_μ x 10^{-11})	Error
a) QED	116 584 718.95	0.08
b) EW	154	1
c) HVP	6850.6	43
d) HLBL	105	26

Summary	(a_μ x 10^{-11})
$a_\mu^{(EXP)}$	116 592 089(63)
$a_\mu^{(SM)}$	116 591 828(49)
$a_\mu^{(EXP)} - a_\mu^{(SM)}$	261(80) → 3.3σ

- **QED/EW uncertainties are tiny, e.g.**
  - Recent calculation to 5th order in α contributes 5×10^{-11} to a_μ
  - Known Higgs mass reduces error on EW from 2 to 1×10^{-11}
- **Error dominated by hadronic terms**
  - HVP can be determined from $e^+e^- \rightarrow$ hadrons data

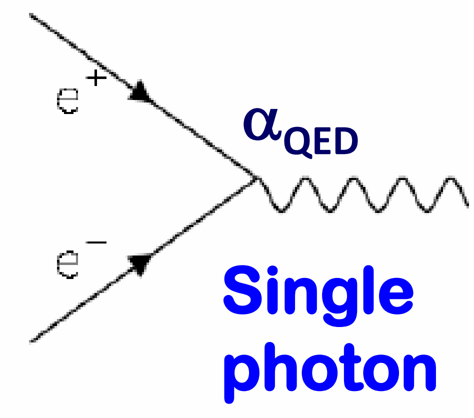
$$a_\mu^{had} = \frac{\alpha^2(0)}{3\pi^2} \int_{4m_\pi^2}^{\infty} ds \frac{K(s)}{s^2} R(s)$$

  - HLBL smaller overall error, but calculation model-dependent



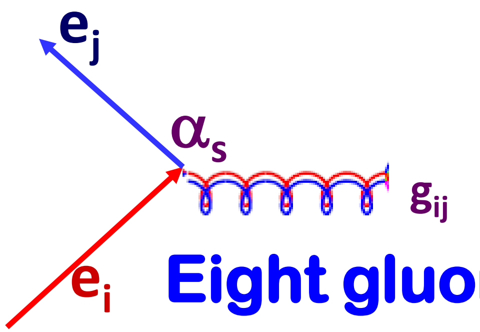
Gauge Theory: From QED to QCD

QED: scalar charge e



Non-Abelian extension of QED

QCD triplet color charge: $\begin{pmatrix} e_r \\ e_b \\ e_g \end{pmatrix}$



QED gauge transform

$$\vec{\nabla} \rightarrow \vec{\nabla} + i\, e\, \vec{A}$$

1 vector field (photon)

QCD gauge transform

$$\vec{\nabla} \rightarrow \vec{\nabla} + i\, \alpha\, \lambda_i\, \vec{G}_i$$

eight 3x3 SU(3) matrices

8 vector fields (gluons)

Gauge Theory of Strong Interactions

- The Lagrangian looks a lot like the one for QED: a field strength term representing the gluon field and a Dirac term for the quarks.
- However, it has one important difference: color.

$$\mathcal{L}_{QCD} = -\frac{1}{4} F^{a}{}_{\mu\nu} F_{a}^{\mu\nu} + \sum_{\text{flavors}} \bar{q}_i \left(i D_\mu \gamma^\mu - m \right)_{ij} q_j$$

- Within the quark model, the additional quantum number of color was initially introduced to accommodate the existence of the Δ^{++} baryon.
  - antisymmetry to satisfy Pauli exclusion principle carried by color
  - quarks and gluons carry color but observed hadrons are colorless
- The color degrees of freedom can also be directly probed in electron-positron collisions, by comparing the production of hadrons and muons.

$$R = \frac{\sigma\left(e^+e^- \to \text{hadrons}\right)}{\sigma\left(e^+e^- \to \mu^+\mu^-\right)} = N_c \sum_f Q_f^2$$

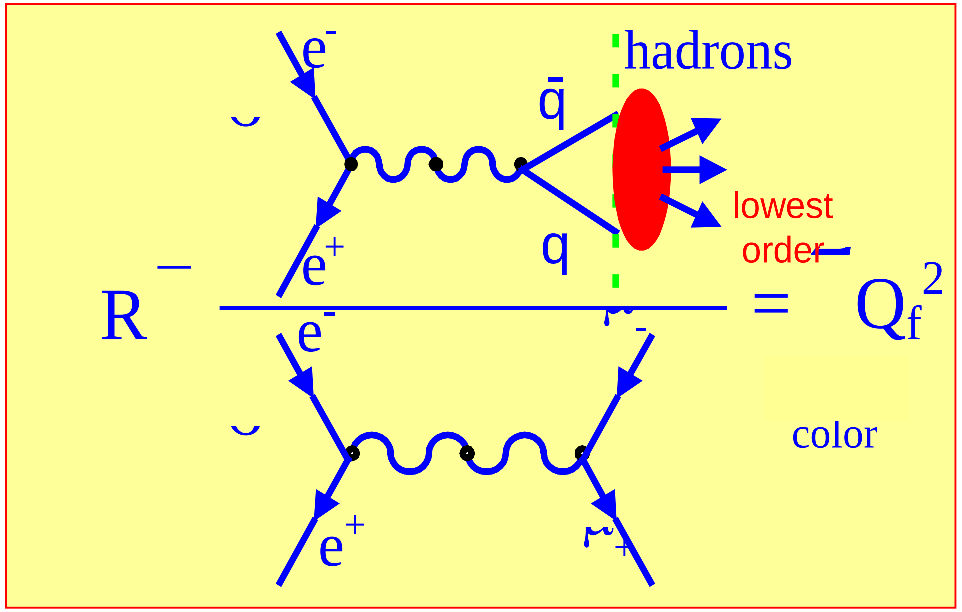
quark charge

assume N_c colors of quark

sum over active quarks

R-Value: A Prediction of the Quark Model

Only consider u, d, s quarks (experiment at e+e- colliders)



$$= \left(\tfrac{2}{3}\right)^2 + \left(\tfrac{1}{3}\right)^2 + \left(\tfrac{1}{3}\right)^2 = 2/3$$

Measurement shown that above R-value must be multiplied by 3 (number of color charge)

R-value Measurements in e+e- Collisions

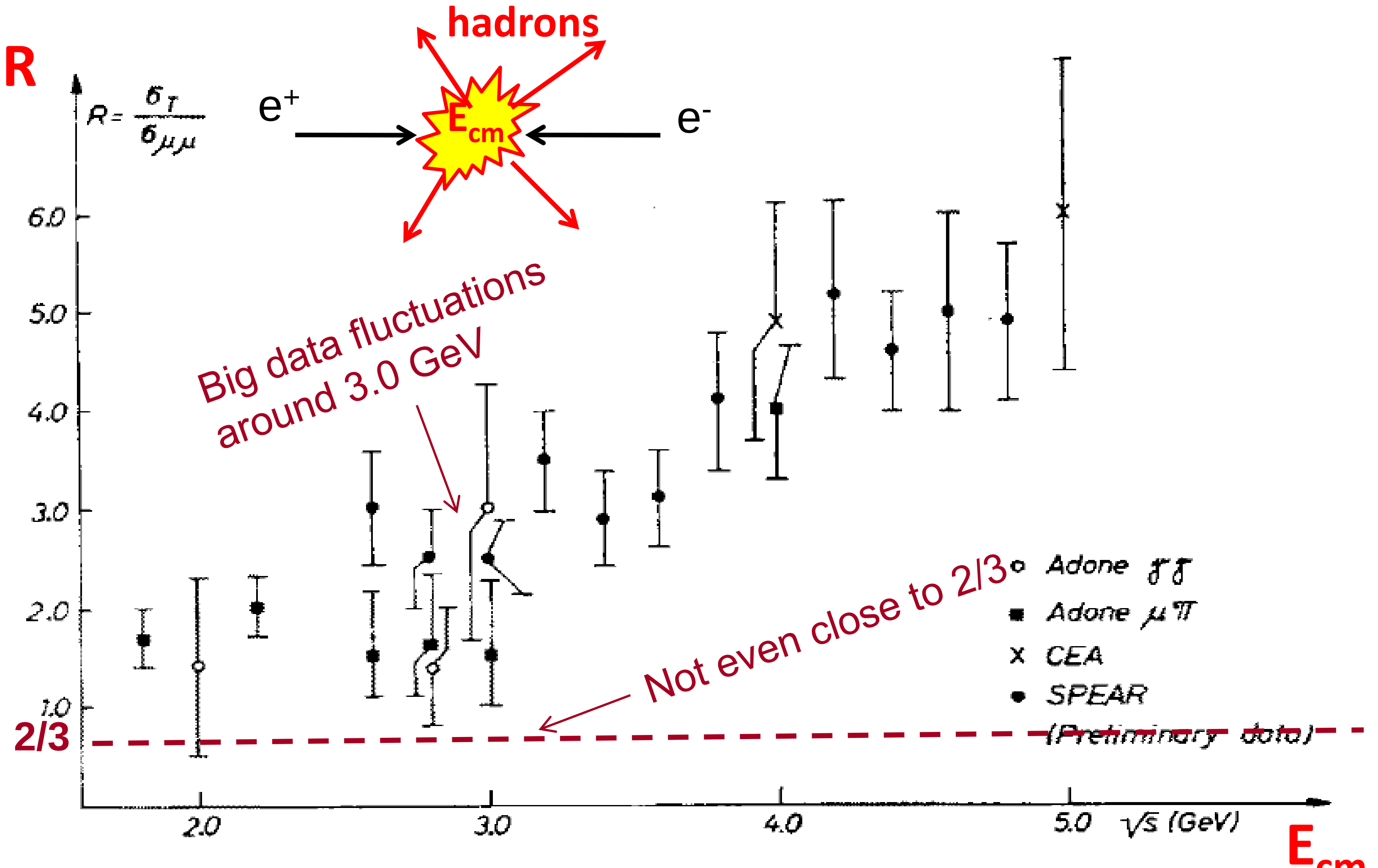
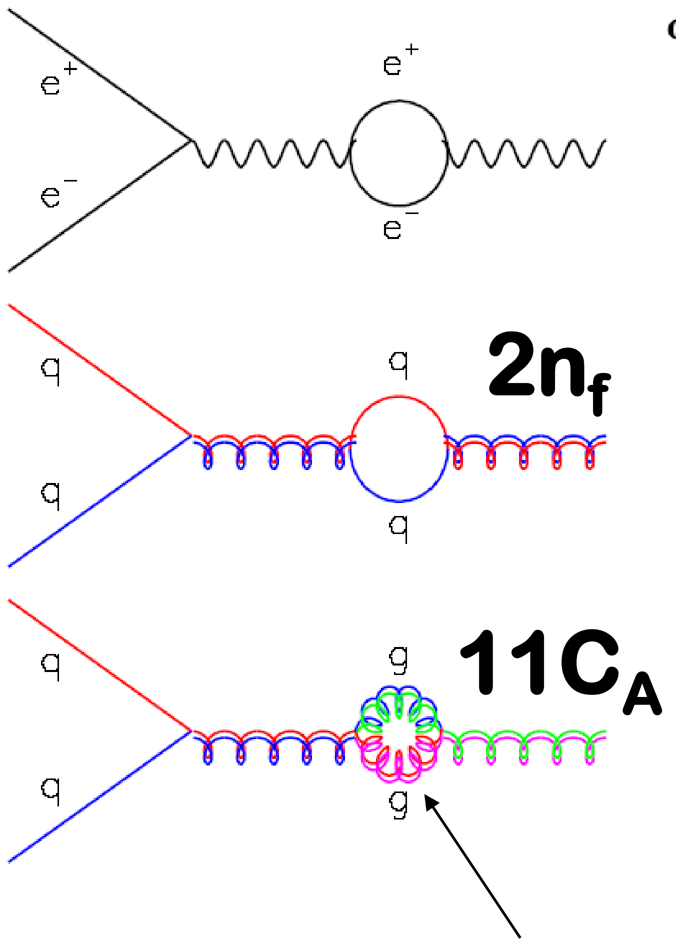
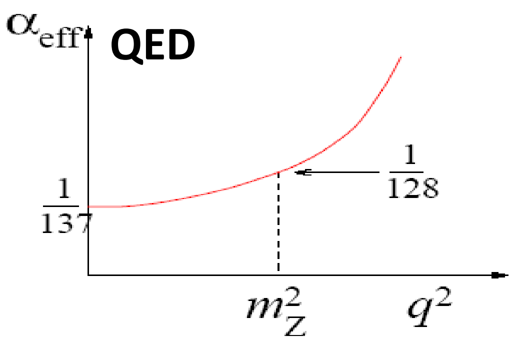


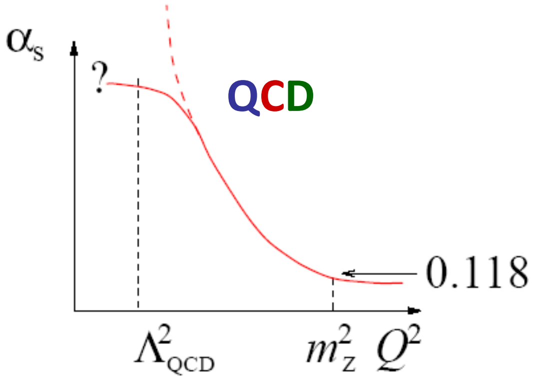
Fig. 35. $R = \sigma_{\text{hadrons, total}} / \sigma_{\mu\bar{\mu}}$ vs E

Compilation by: L. Paoluzi Acta Physica Polonica B5, 829 (1974)

Vacuum Polarization QED vs QCD







in QCD: C_A=3, & this dominates

α_s increases with distance

QED:

photons have no charge.

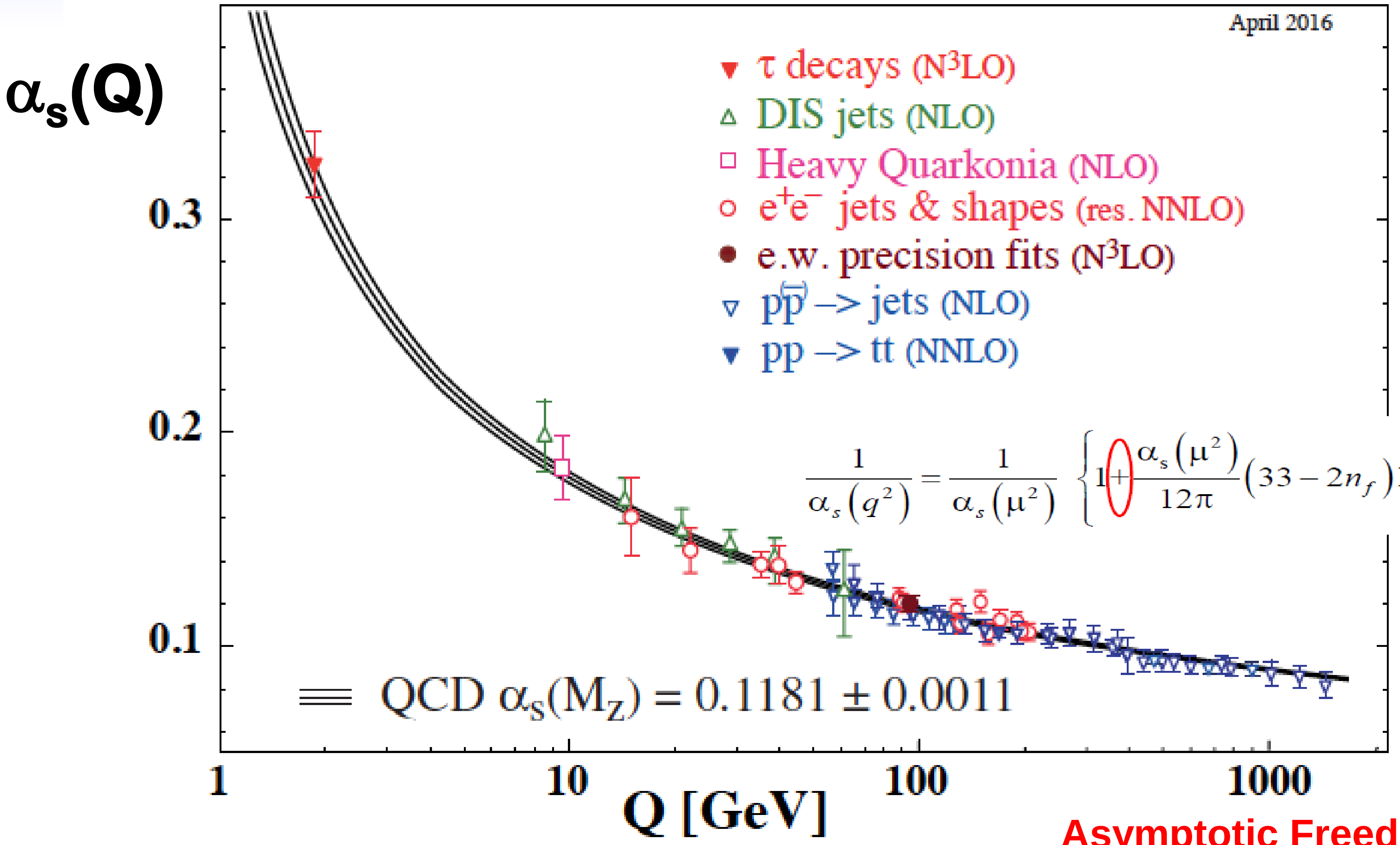
Coupling decreases at large distances (lower E).

QCD:

gluons carry color charges.

Gluons interact with each other. Coupling increases at large distances.

QCD Running Coupling Constant (α_s)



$$\frac{1}{\alpha_s(q^2)} = \frac{1}{\alpha_s(\mu^2)}\left\{1 + \frac{\alpha_s(\mu^2)}{12\pi}(33 - 2n_f)\log\left(\frac{q^2}{\mu^2}\right)\right\}$$

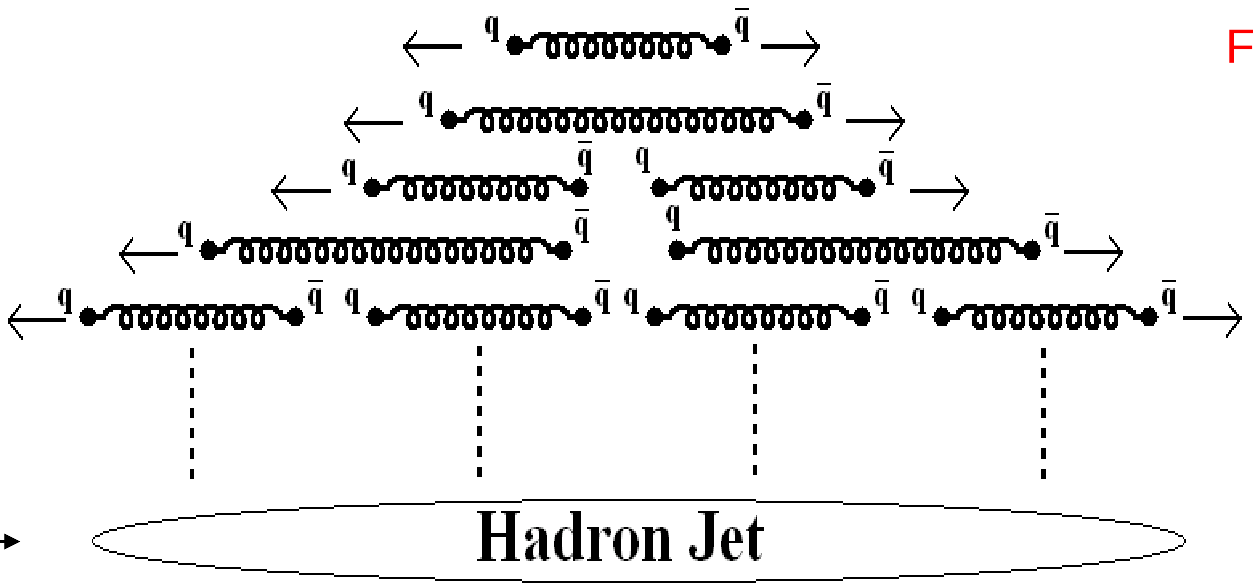
Asymptotic Freedom

Fragmentation - Physics with Jet

$$V(color) = -\frac{3}{4} \cdot \frac{\alpha_s}{r} + k \cdot r$$

As already discussed: Strong force is like a '**spring**' – at longer distance (lower energy) it becomes stronger)

From quark to jet

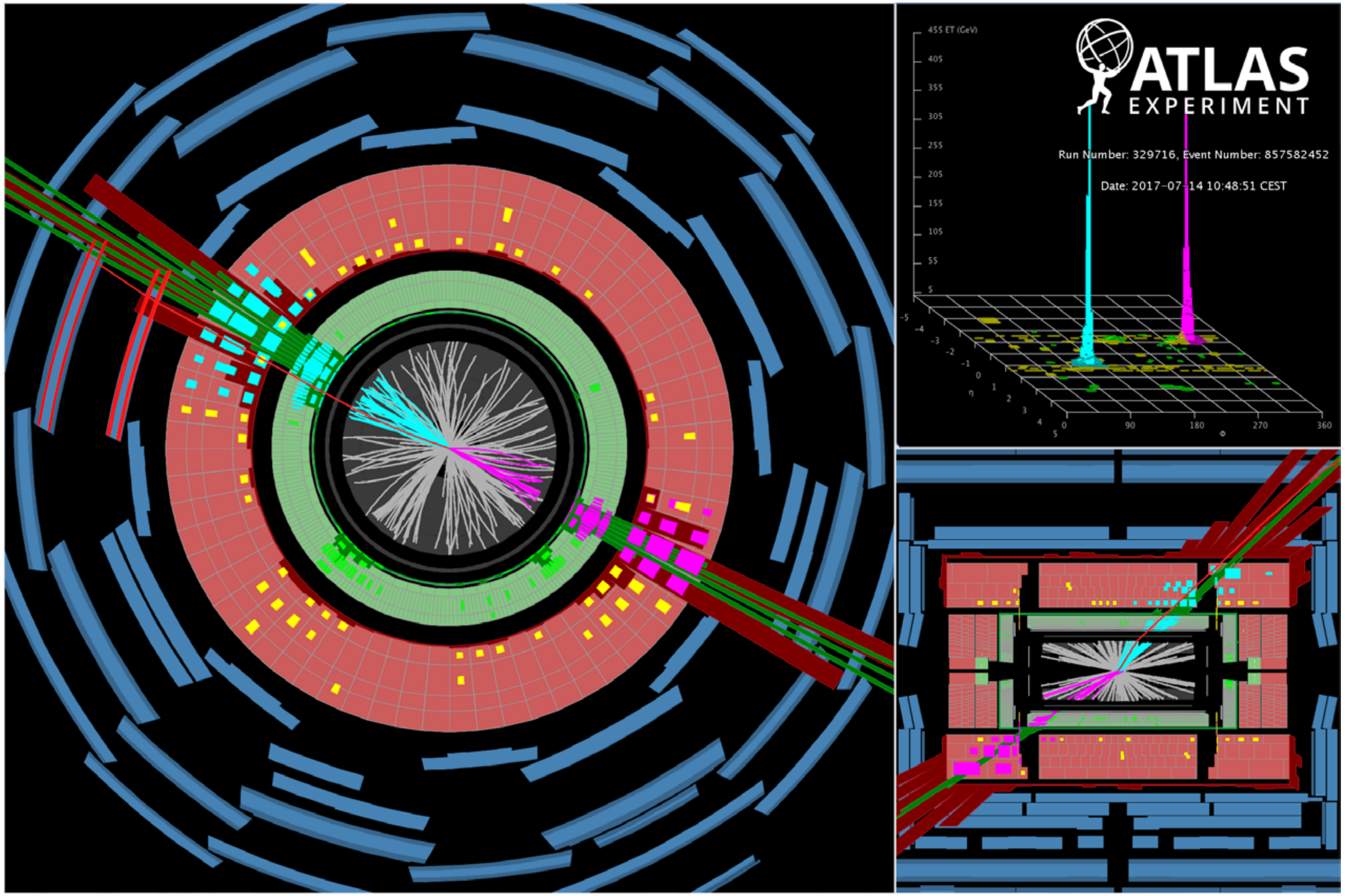


Observed in experiment →

- **When two quarks become separated, at some point it is more energetically favorable for a new quark/anti-quark pair to spontaneously appear out of the vacuum, than to allow the quarks to separate further.**
- **When quarks are produced by particle accelerators, we never see the individual quarks in detectors, but jets of many color-neutral particles (mesons and baryons) clustered together →fragmentation.**
- **Fragmentation is one of the least understood processes in particle physics.**

A Di-jet Event Recorded by ATLAS Detector

A di-jet event recorded during 2017, with m_{jj} = 9.3 TeV

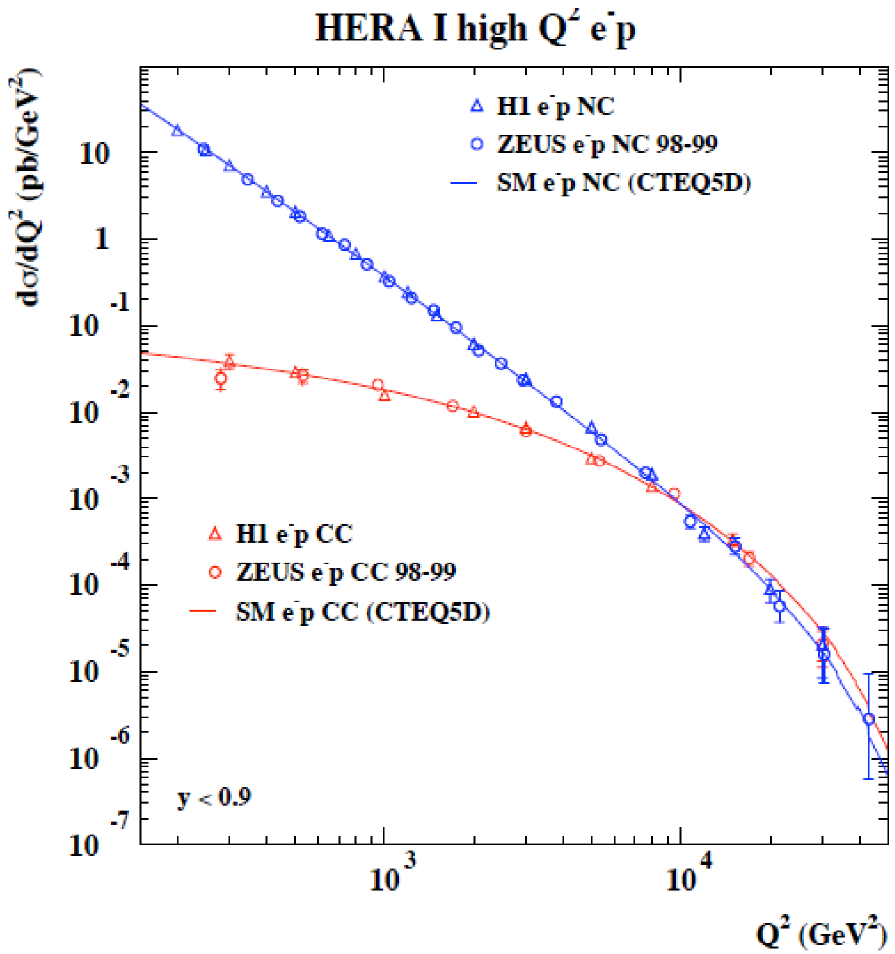


EM and Weak Force Unification

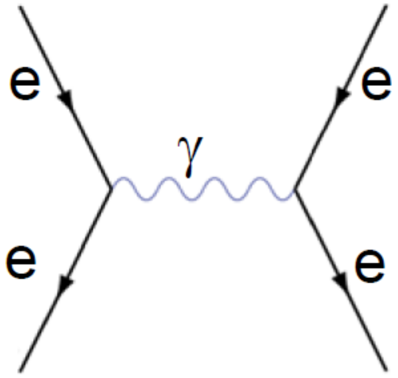
- EM and Weak forces become equally strong at short distances of order 10^{-15} cm

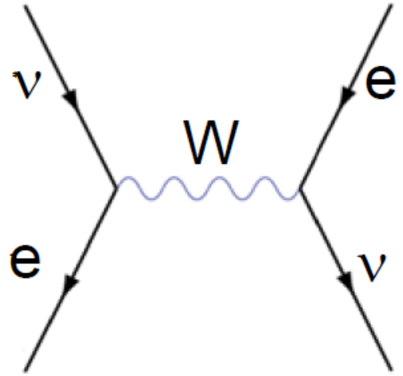
- Same theory describes both forces in a unified framework

[Nobel prize 1979: Glashow, Salam, Weinberg]



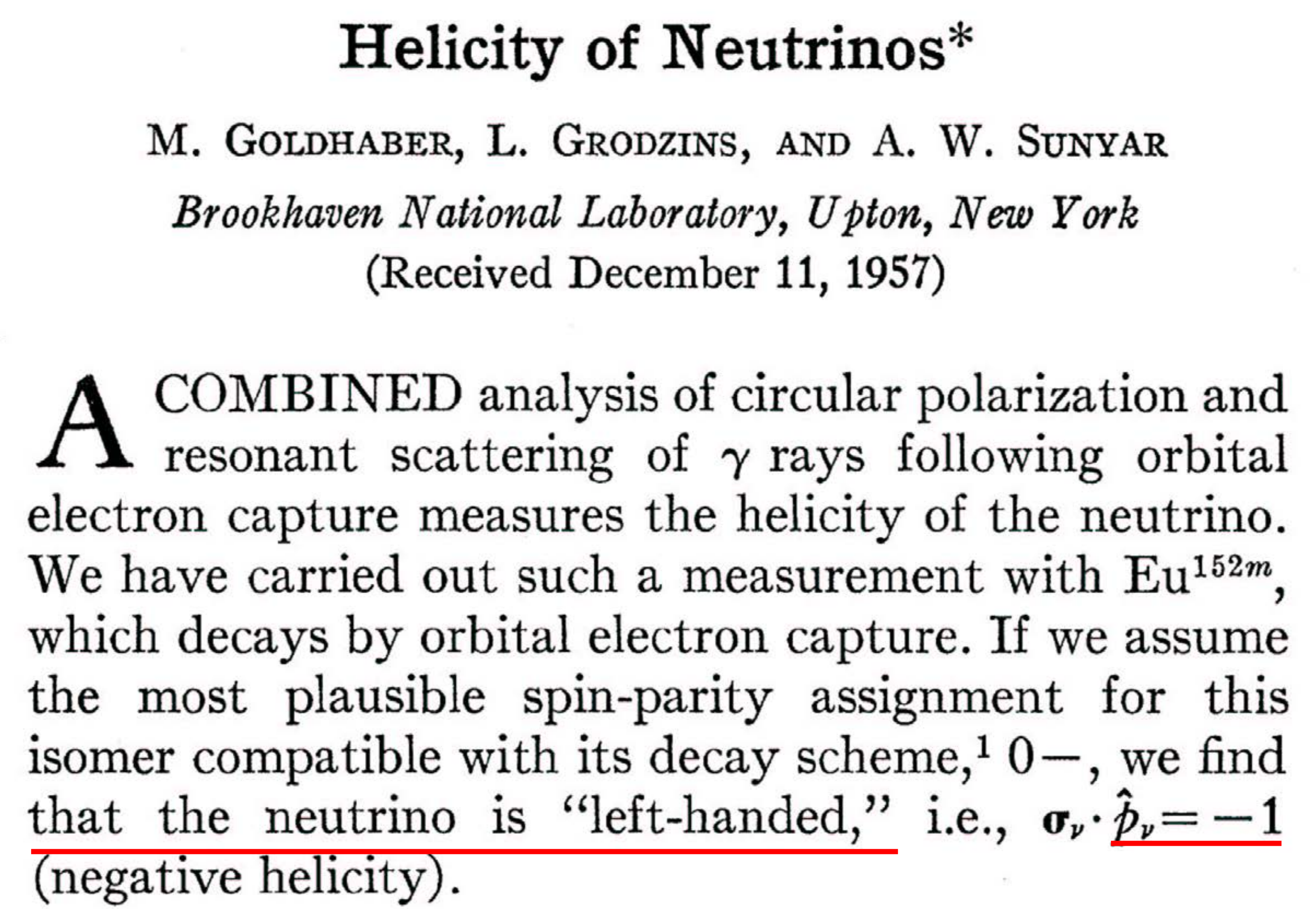
[blue=EM, red=weak]





Cannot describe the massive weak force carriers without breaking the symmetry !

Neutrinos are Left-handed



Helicity of Neutrinos*

M. Goldhaber, L. Grodzins, and A. W. Sunyar

Brookhaven National Laboratory, Upton, New York

(Received December 11, 1957)

A COMBINED analysis of circular polarization and resonant scattering of γ rays following orbital electron capture measures the helicity of the neutrino. We have carried out such a measurement with Eu^{152m}, which decays by orbital electron capture. If we assume the most plausible spin-parity assignment for this isomer compatible with its decay scheme,[1] 0−, we find that the neutrino is "left-handed," i.e., $\boldsymbol{\sigma}_\nu \cdot \hat{p}_\nu = -1$ (negative helicity).

The Electroweak Theory

- Left-handed weak-isospin doublets (only left-handed neutrinos are observed)

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \quad \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L \quad \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L$$

$$\begin{pmatrix} u \\ d' \end{pmatrix}_L \quad \begin{pmatrix} c \\ s' \end{pmatrix}_L \quad \begin{pmatrix} t \\ b' \end{pmatrix}_L$$

Note:
q' - weak interaction Eigen states
q - mass Eigen state

q and q' are connected by **CKM** matrix, will discuss late

Sometimes, people write q' as q

- **Universal strength** of the (charged-current) weak interactions
- Idealization that neutrinos are massless (no right-handed neutrino is observed)
- Based on $SU(2)_L \times U(1)_Y$ symmetry (Here, L: left-handed, Y: hypercharge)

Electroweak Interactions of Leptons

(Example: the first generation of lepton only)

- Left-handed fermions $SU(2)_L$ doublets, right-handed fermions $SU(2)_L$ singlets:

$$L \equiv \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \qquad R \equiv e_R$$

  - Weak Hypercharge $Y_L=-1$, $Y_R = -2$
  - Gell-Mann-Nishijima connection: $Q = I_3 + \frac{1}{2}Y$

- Gauge Fields introduced by $SU(2)_L$ x $U(1)_Y$ symmetry transformation invariance:
  - Weak isovector $\vec{B_\mu}$, *coupling g*: gauge transformation $B_\mu^l \rightarrow B_\mu^l - \varepsilon_{ikl}\, \alpha^j B_\mu^k - \left(\frac{1}{g}\right) \partial_\mu \alpha^l$,
  - Weak isoscalar A_μ, *coupling g'*: gauge transformation $A_\mu \rightarrow A_\mu - \partial_\mu \alpha$

- Gauge Field Tensor: $F_{\mu\nu}^l = \partial_\nu B_\mu^l - \partial_\mu B_\nu^l + g\varepsilon_{jkl} B_\mu^j B_\nu^k$, $SU(2)_L$

$$f_{\mu\nu} = \partial_\nu A_\mu - \partial_\mu A_\nu \qquad U(1)_Y$$

EW Interaction of Lepton Lagrangian

$$\mathcal{L} = \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{leptons}}$$

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4} F^{\ell}_{\mu\nu} F^{\ell\mu\nu} - \frac{1}{4} f_{\mu\nu} f^{\mu\nu},$$

←Massless gauge boson

$$\mathcal{L}_{\text{leptons}} = \overline{\mathsf{R}}\, i\gamma^{\mu} \left(\partial_{\mu} + i\frac{g'}{2} \mathcal{A}_{\mu} Y \right) \mathsf{R}$$

←Massless fermions

$$+ \ \overline{\mathsf{L}}\, i\gamma^{\mu} \left(\partial_{\mu} + i\frac{g'}{2} \mathcal{A}_{\mu} Y + i\frac{g}{2} \vec{\tau} \cdot \vec{b}_{\mu} \right) \mathsf{L}.$$

Note:

Dirac mass term forbidden by $SU(2)_L$ gauge invariance:

$$L = -m\overline{\psi}\psi = -m(\overline{\psi}_L \psi_R + \overline{\psi}_R \psi_L)$$

Theory: 4 massless gauge bosons ($\mathcal{A}_{\mu}$ b^1_{μ} b^2_{μ} b^3_{μ});

Nature: 1 (γ) **BUT, weak interaction force carries, W^+, W^- and Z are massive!**

Redefine →EM interaction and coupling

From SU(2)xU(1) → bosons $(b_\mu^1, b_\mu^2, b_\mu^3, A_\mu)$, *and couplings* (g, g')

Redefine to get physical gauge bosons

$$A_\mu = \frac{g \ A_\mu + g' b_\mu^3}{\sqrt{g^2 + g'^2}}$$

$$Z_\mu^0 = \frac{-g' A_\mu + g b_\mu^3}{\sqrt{g^2 + g'^2}}$$

$$W_\mu^\pm = \frac{b_\mu^1 \mp b_\mu^2}{\sqrt{2}}$$

EM Interaction → $\mathcal{L}_{A-l} = \frac{g g'}{\sqrt{g^2 + g'^2}} \bar{\psi} \gamma^\mu \psi A_\mu$

A_μ as γ, provided we identify $gg'/\sqrt{g^2 + g'^2} \equiv e$

Define: $g' = g \tan \theta_W$ θ_W: weak mixing angle

$$g = e/\sin \theta_W \geq e$$

$$g' = e/\cos \theta_W \geq e$$

Redefine → Weak Interaction and Coupling

$$Z_\mu = b^3_\mu \cos\theta_W - \mathcal{A}_\mu \sin\theta_W \qquad A_\mu = \mathcal{A}_\mu \cos\theta_W + b^3_\mu \sin\theta_W$$

$$\mathcal{L}_{Z-\nu} = \frac{-g}{4\cos\theta_W}\bar{\nu}\gamma^\mu(1-\gamma_5)\nu\, Z_\mu$$

Purely left-handed!

$$\mathcal{L}_{Z-e} = \frac{-g}{4\cos\theta_W}\bar{e}\,[L_e\gamma^\mu(1-\gamma_5) + R_e\gamma^\mu(1+\gamma_5)]\, e\, Z_\mu$$

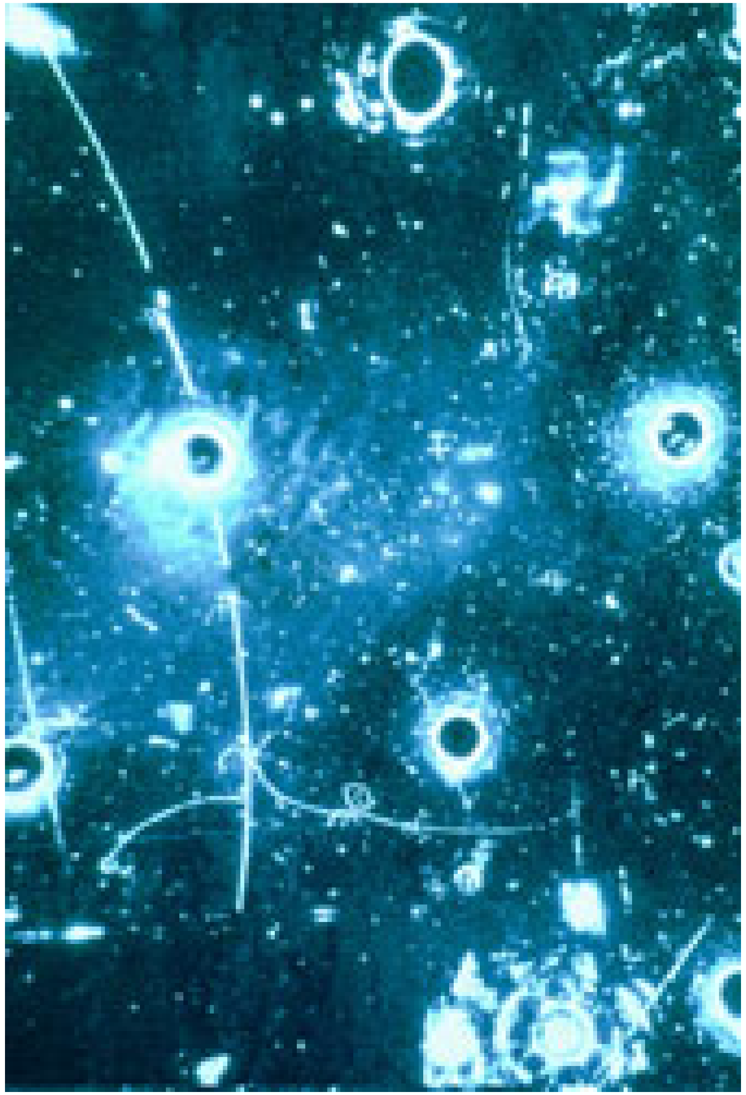
(Note, e, ν represent electron and neutrino wave functions in *L*)

$$\mathcal{L}_{W\text{-}\ell} = -\frac{g}{2\sqrt{2}}[\bar{\nu}\gamma^\mu(1-\gamma_5)eW^+_\mu + \bar{e}\gamma^\mu(1-\gamma_5)\nu W^-_\mu]$$

Re-define weak coupling $\sin^2\theta_W$

$$L_e = 2\sin^2\theta_W - 1$$
$$R_e = 2\sin^2\theta_W$$

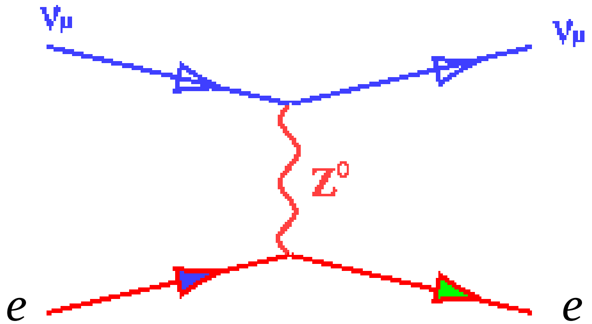
Discovery of the Neutral Current (1973)



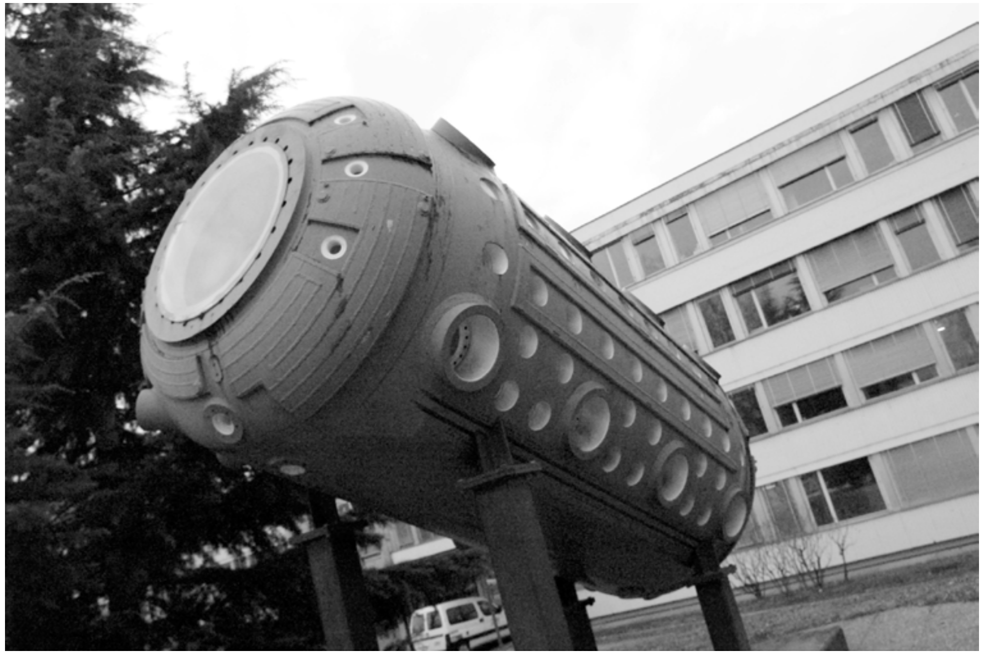
$\leftarrow \bar{\nu}_\mu$

In 1973, it came the observation of neutral current interactions as predicted by electroweak theory.

The huge Gargamelle bubble chamber photographed the tracks of a few electrons suddenly starting to move, seemingly of their own accord. This is interpreted as a neutrino interacting with the electron by the exchange of an unseen Z boson. The neutrino is otherwise undetectable, so the only observable effect is the momentum imparted to the electron by the interaction.



Gargamelle Bubble Chamber



On display at CERN

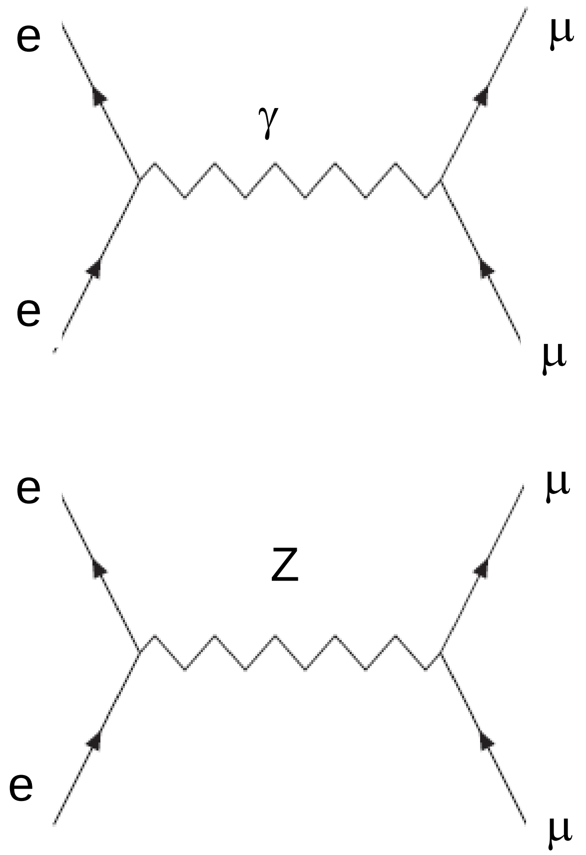
Gargamelle was the name of the particle detector used to make this discovery at the Proton Synchrotron accelerator. It was a large bubble chamber, a type of particle detector that uses a pressurized transparent liquid to detect electrically charged particles passing through it.

Named after the mother of Gargantua (the giant in the story by François Rabelais), Gargamelle measured 4 m long with a 2 m diameter, weighed 1000 tonnes, and contained 18 tonnes of liquid Freon. It was made especially for detecting neutrinos. These particles have no charge, and would leave no tracks in the detector, so the aim was to reveal any charged particles set in motion by the neutrinos and so reveal their interactions indirectly.

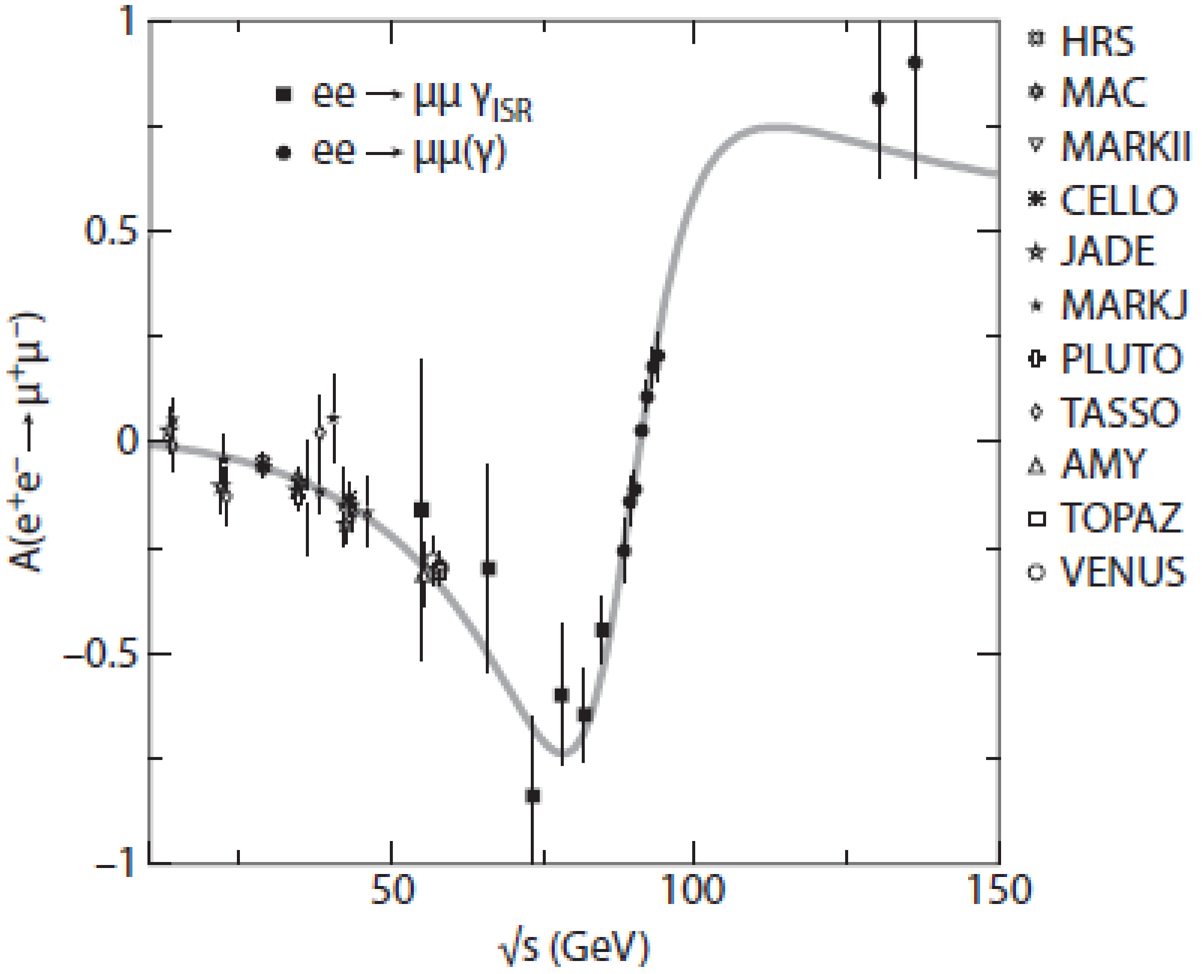
Indirect evidence of Z: Z-γ interference:

Before the Z boson discovery

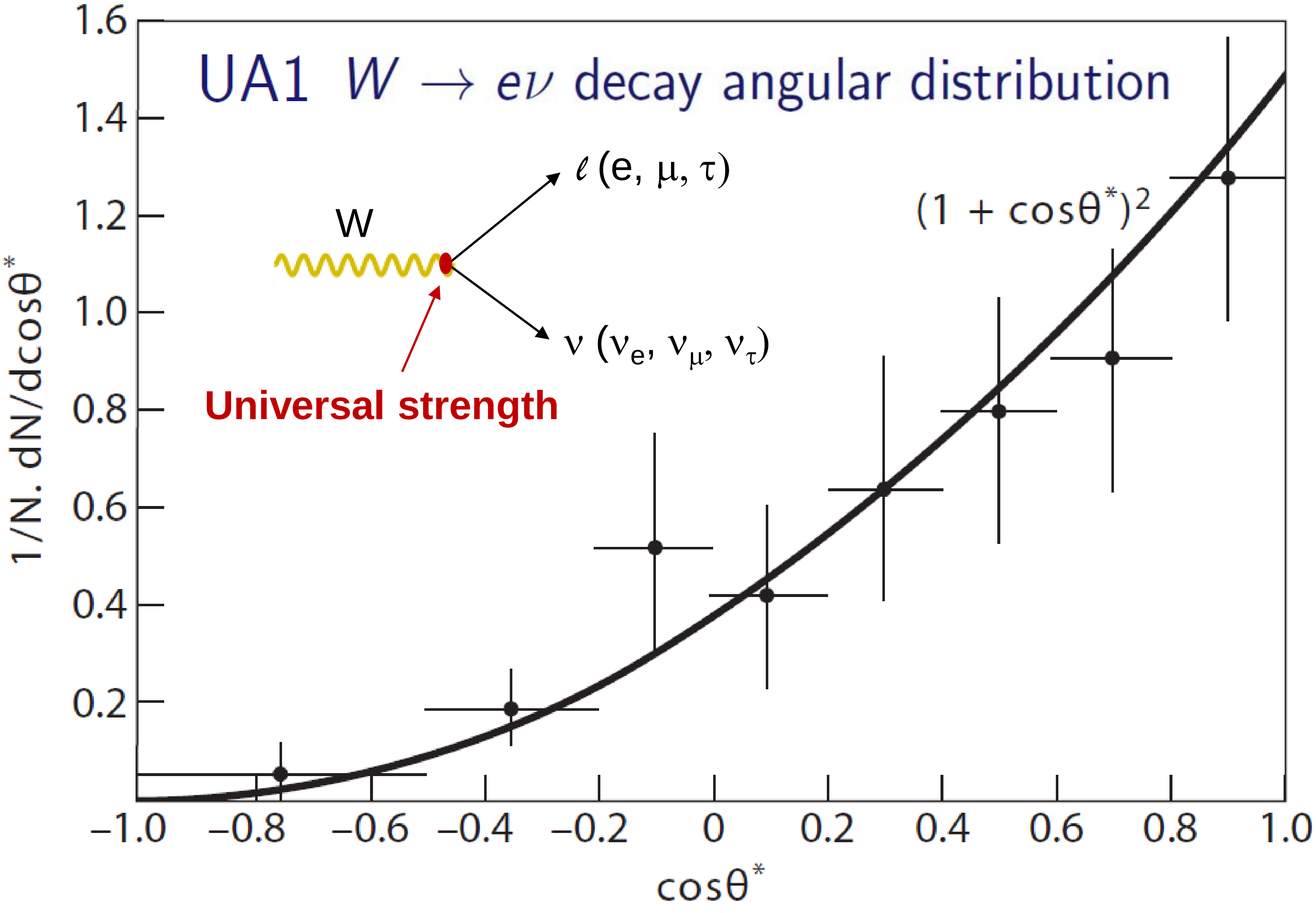
$e^+e^- \rightarrow \mu^+\mu^-$



Forward/backward asymmetry



Charged Current Interaction W→eν



Electroweak Interactions of Quarks

Use the first generation of quarks, the 2nd and the 3rd generation should be the same

- Left-handed doublet

	I_3	Q	$Y = 2(Q - I_3)$
$L_q = \begin{pmatrix} u \\ d \end{pmatrix}_L$	$\frac{1}{2}$	$+\frac{2}{3}$	$\frac{1}{3}$
	$-\frac{1}{2}$	$-\frac{1}{3}$	

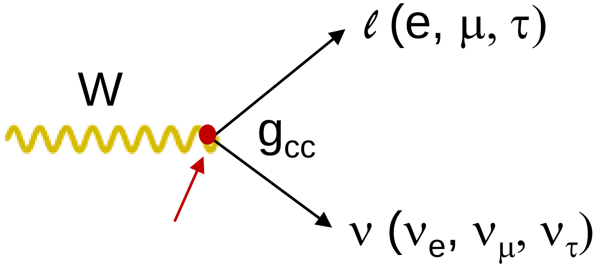
- two right-handed singlets

	I_3	Q	$Y = 2(Q - I_3)$
$R_u = u_R$	0	$+\frac{2}{3}$	$+\frac{4}{3}$
$R_d = d_R$	0	$-\frac{1}{3}$	$-\frac{2}{3}$

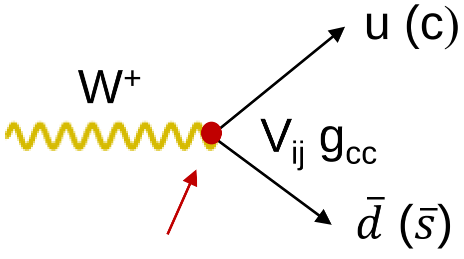
Is the Quark Interaction the Same as Leptons?

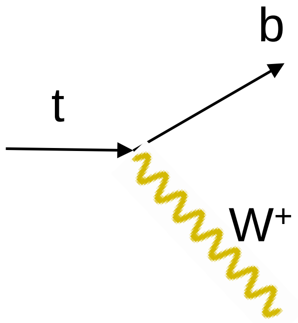
$\begin{pmatrix}\nu_e \\ e\end{pmatrix}_L \quad \begin{pmatrix}\nu_\mu \\ \mu\end{pmatrix}_L \quad \begin{pmatrix}\nu_\tau \\ \tau\end{pmatrix}_L$ *only* $\approx$ $\begin{pmatrix}u \\ d\end{pmatrix}_L \quad \begin{pmatrix}c \\ s\end{pmatrix}_L \quad \begin{pmatrix}t \\ b\end{pmatrix}_L$

Lepton charged current

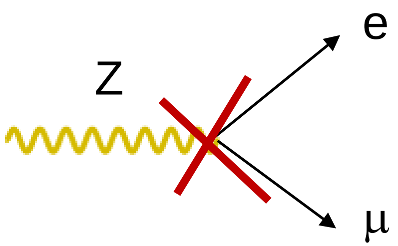


Quark charged current





No lepton and quark flavor change neutral currents have been observed!



Good $\begin{pmatrix}u \\ d\end{pmatrix}_L$ → Better $\begin{pmatrix}u \\ d'\end{pmatrix}_L$ Universe strength

d' = d $\cos\theta_c$ *+ s* $\sin\theta_c$, $\cos\theta_c$ *= 0.9736+-0.0010*

θ_c quark mixing angle, Cabibbo angle

Quark weak interaction and mass Eigen states are not the same → CMK matrix V_{ij}

CKM Matrix

Quark mass eigen-states are not the same as the weak eign-states. Weak eign-states are mixture of mass eigen-states. The mixing is described by a unitary Cabibbo-Kobayashi-Maskawa (CKM) matrix.

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

weak eigenstates

Cabibbo Kobayashi Maskawa (CKM) matrix

mass eigenstates

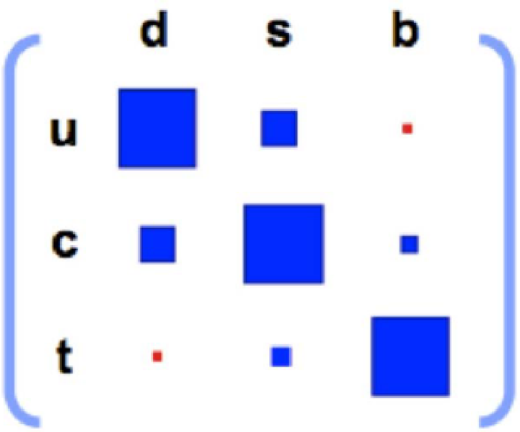
Mass eigen states - "physical" quarks, eigen states of strong interaction: $g^* \rightarrow u\bar{u}$ (mass eigen states)

Weak eigen states - admixtures of "physical" quarks, eigen states of weak interaction: $W^+ \rightarrow u\bar{d}$ (weak eigen states)

The CKM Matrix Parameters

The 3x3 unitary matrix can be parameterize by 4 independent parameters: 3 angles and 1 phase

The CKM matrix is mostly diagonal unlikely the PMNS neutrino mixing matrix.



$$V_{\mathrm{CKM}} = \begin{pmatrix} 0.97427 \pm 0.00014 & 0.22536 \pm 0.00061 & 0.00355 \pm 0.00015 \\ 0.22522 \pm 0.00061 & 0.97343 \pm 0.00015 & 0.0414 \pm 0.0012 \\ 0.00886^{+0.00033}_{-0.00032} & 0.0405^{+0.0011}_{-0.0012} & 0.99914 \pm 0.00005 \end{pmatrix}$$

The quarks arranged in 3 families/generators are weak eigen states, but the prime subscripts are often ignored.

$$\begin{pmatrix} u & c & t \\ d & s & b \end{pmatrix}$$

CKM Matrix and Weak Force

$$V_{\text{CKM}} = \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix}$$

- Connects u- and d- type quarks via the weak force
- Each element related to a transition probability, $|V_{ij}|^2$
- 3×3 unitary matrix is parameterised by three rotation angles and one complex phase
    - Phase changes sign under the CP operator
    - In SM, this phase is the single source of quark sector CP violation

Three Left-handed Doublets in SM

After discovery of bottom quark and tau-lepton, three generations of Fermions included in SM
Only massive particles have right-handed SU(2) singlets (experiments only observed left-handed ν)

$$Q_L = \begin{pmatrix} u \\ d' \end{pmatrix}_L \quad u_R, \quad d_R, \quad \begin{pmatrix} \nu \\ e \end{pmatrix}_L, \quad e_R$$

$$\begin{pmatrix} c \\ s' \end{pmatrix}_L \quad c_R, \quad s_R, \quad \begin{pmatrix} \nu \\ \mu \end{pmatrix}_L, \quad \mu_R$$

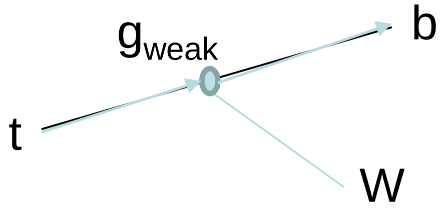
$$\begin{pmatrix} t \\ b' \end{pmatrix}_L \quad t_R, \quad b_R, \quad \begin{pmatrix} \nu \\ \tau \end{pmatrix}_L, \quad \tau_R$$

Except for masses, the generations are identical

Weak interaction Eigen states – not the same as quark mass Eigen states –> CKM matrix

Questions

Weak coupling



$$\begin{pmatrix} u \\ d \end{pmatrix}_L \quad \begin{pmatrix} c \\ s \end{pmatrix}_L \quad \begin{pmatrix} t \\ b \end{pmatrix}_L$$

The probability of top quark decay to b quark is $(g_{weak} x 0.99914)^2$ (where 0.99914 = V_{tb})

What is the probability of t→s +W, and t → d+W ?

What is the probability of c→s +W*, and c → s+W* ?

What is the probability of b→c +W*, and b → u+W* ?

……

$$V_{\mathrm{CKM}} = \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix}$$

Connects u- and d- type quarks via the weak force

Each element related to a transition probability, $|V_{ij}|^2$

Properties of W boson

Measured mass and total decay width

$$M_W = 80.399 \pm 0.023 \text{ GeV}, \;\; \Gamma_W = 2.085 \pm 0.042 \text{ GeV}$$

Possible decay final states are:

$$W \to \begin{pmatrix} \nu_e \\ e \end{pmatrix}, \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}, \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}; \; W \to \begin{pmatrix} u \\ d \end{pmatrix}, \begin{pmatrix} c \\ s \end{pmatrix}$$

$W \to \ell \bar{\nu}$ and $W \to q\bar{q}'$ decays would have the same branching ratios if it were not for that the fact that quarks have colors. Thus

$$Br(W \to \ell \nu) \approx \frac{1}{9} \approx 11\% \text{ (each flavor)}, \quad Br(W \to hadrons) \approx \frac{6}{9} \approx 67\%$$

The actual measured value is

$$Br(W \to \ell \nu) = (10.80 \pm 0.09)\%$$

Question: what's lifetime of the W boson?

Properties of Z boson

Measured mass and total decay width:

$$M_Z = 91.1876 \pm 0.0021 \text{ GeV}, \;\; \Gamma_Z = 2.4952 \pm 0.0023 \text{ GeV}$$

Possible decay modes are

$$Z \to e^+e^-, \mu^+\mu^-, \tau^+\tau^-; \quad \nu_e\bar{\nu}_e, \nu_\mu\bar{\nu}_\mu, \nu_\tau\bar{\nu}_\tau;$$

$$Z \to u\bar{u}, c\bar{c}; \qquad Z \to d\bar{d}, s\bar{s}, b\bar{b}$$

with branching ratios

$$Br\left(Z \to \ell^+\ell^-\right) = 3.363\% \;\; \left(\text{each flavor}\right);$$

$$Br\left(Z \to \nu\bar{\nu}\right) = 20.00\% \;\; \left(\text{all flavors}\right)$$

$$Br\left(Z \to u\bar{u}\right) = 11.6\% \;\; \left(\text{each up-quark}\right);$$

$$Br\left(Z \to d\bar{d}\right) = 15.6\% \;\; \left(\text{each down-quark}\right)$$

and

$$Br\left(Z \to hadron\right) \approx 70\%$$

Summary: the Standard Model

The SM of the electromagnetic, weak and strong interactions is:

- a relativistic quantum field theory,
- based on local gauge symmetry: invariance under symmetry group,
- more or less a carbon–copy of QED, the theory of electromagnetism.

The SM is based on the local gauge symmetry group

$$\mathrm{G_{SM}} \equiv \mathrm{SU(3)_C} \times \mathrm{SU(2)_L} \times \mathrm{U(1)_Y}$$

- The group $\mathrm{SU(3)_C}$ describes the strong force
- $\mathrm{SU(2)_L} \times \mathrm{U(1)_Y}$ describes the electroweak interaction

Test the Theory

- Discovery of **new particles** predicted by the theory
- Measure the production **cross sections (event rates)** for different process and to measure the predicted kinematic distributions – normally the differential cross sections (cross section will be discussed)

Successful Predictions of EW Theory

- neutral-current interactions
- necessity of charm
- existence and properties of $W^{\pm}$ and Z^{0}

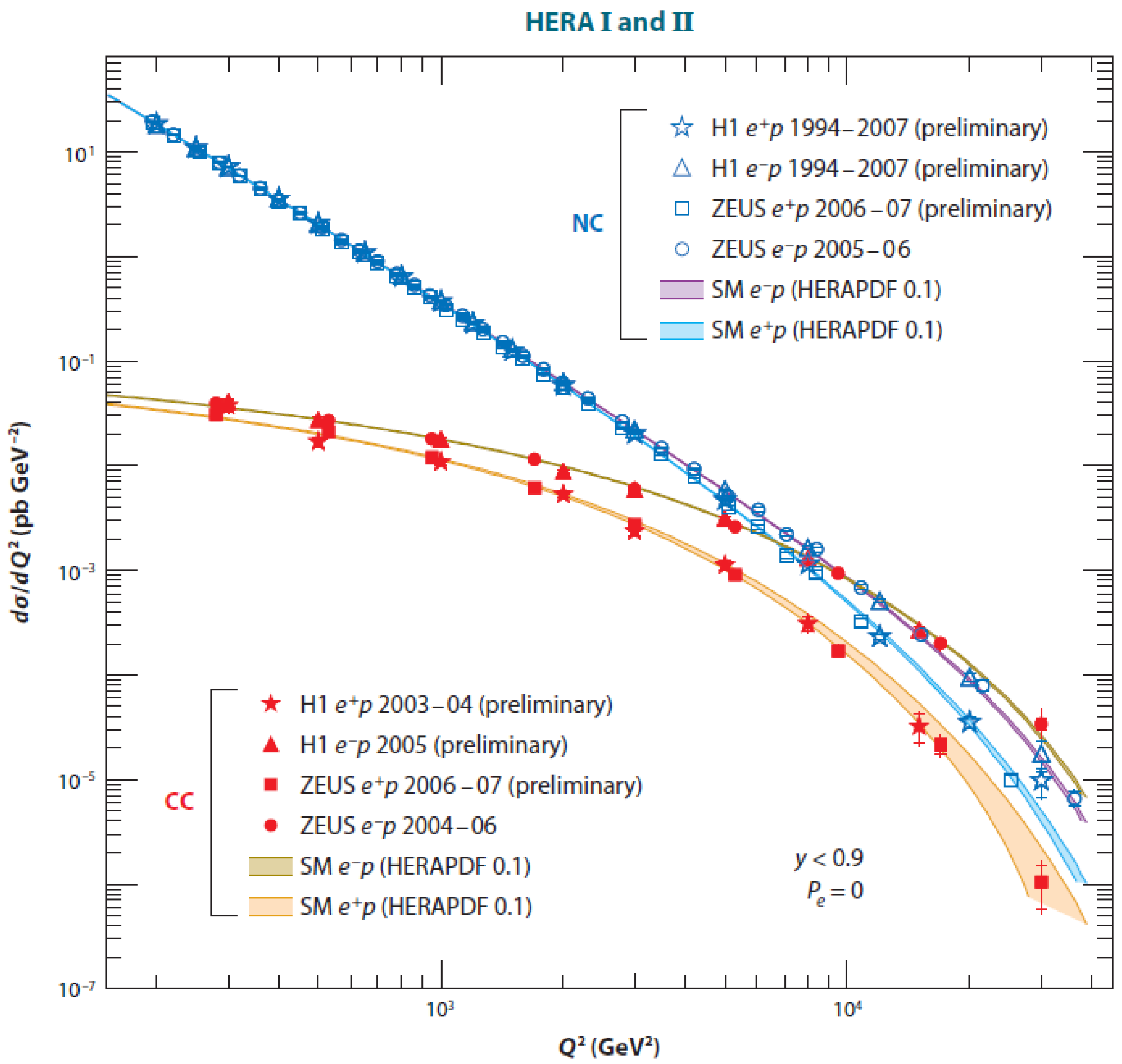
+ a decade of precision EW tests (one-per-mille)

M_Z	$91\,187.6 \pm 2.1$ MeV
Γ_Z	2495.2 ± 2.3 MeV
$\sigma^0_{\text{hadronic}}$	41.540 ± 0.037 nb
Γ_{hadronic}	1744.4 ± 2.0 MeV
Γ_{leptonic}	83.984 ± 0.086 MeV
$\Gamma_{\text{invisible}}$	499.0 ± 1.5 MeV

$$\Gamma_{\text{invisible}} \equiv \Gamma_Z - \Gamma_{\text{hadronic}} - 3\Gamma_{\text{leptonic}}$$

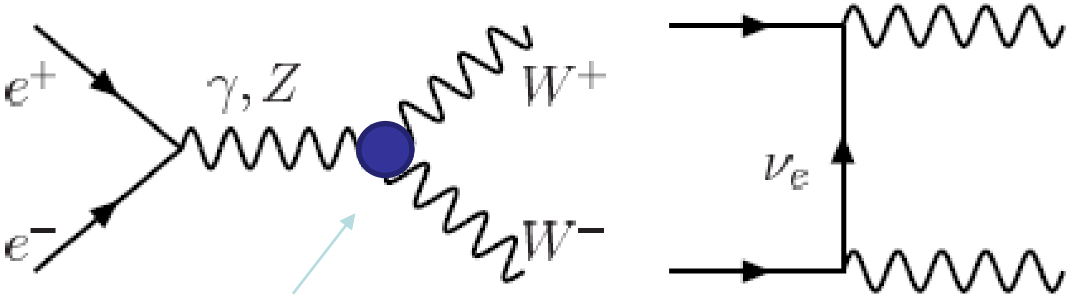
light ν : $N_\nu = \Gamma_{\text{invisible}}/\Gamma^{\text{SM}}(Z \to \nu_i\bar{\nu}_i) = 2.92 \pm 0.05 \quad (\nu_e, \nu_\mu, \nu_\tau)$

Electroweak Theory Tests



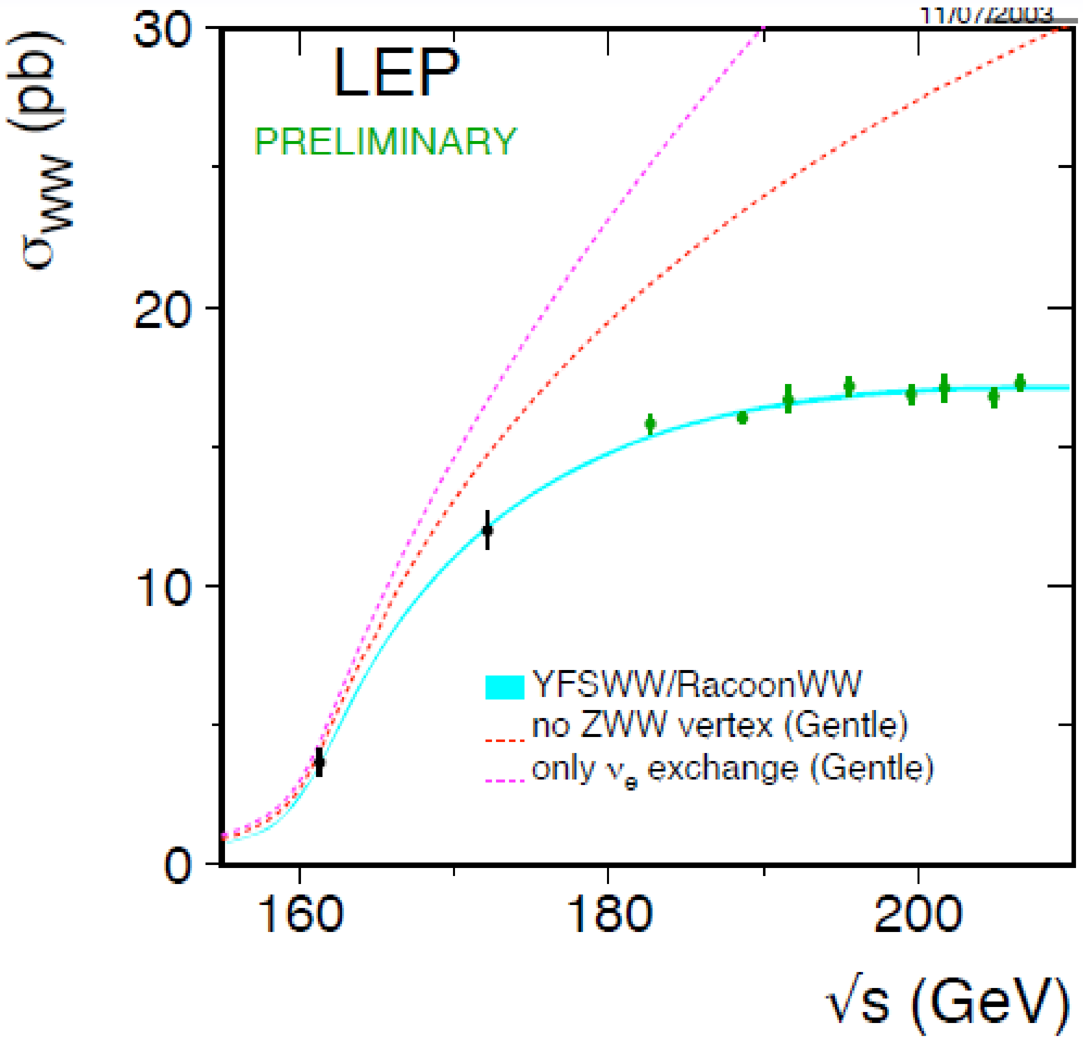
Test of $SU(2)_L \times U(1)_Y$ Structure at LEP

WW production at LEP2:



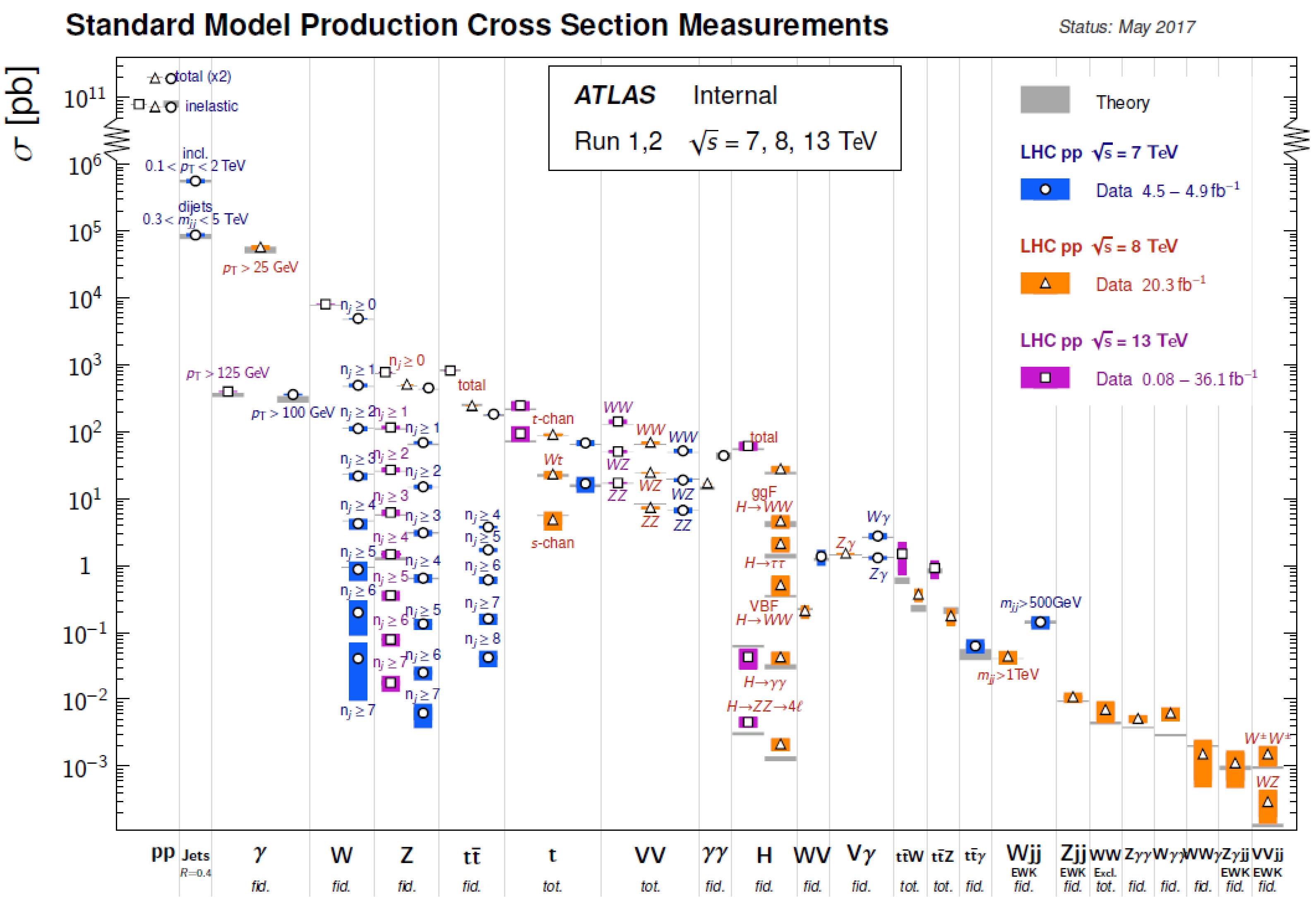
Non-Abelian triple-gauge boson coupling (WWZ)

$SU(2)_L \times U(1)_Y$ gauge structure checked rather precisely at LEP2



sum is well-behaved; gauge symmetry!

Precision Test of EW Theory at LHC



Generate Masses by Symmetry Breaking

UM Professor Veltman (Nobel Prize 1999)

Described History at a Higgs Lecture at the Physics Dept. of Michigan

The use of a field in the vacuum to generate masses was first published by **Schwinger** in 1957.

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Anderson (1958) discussed massive quantum electrodynamics as perceived in superconductivity.

This led **Higgs-Brout-Englert** (1964-1967) to their work, which is the use of a field in the vacuum to give mass to vector bosons.

In 1968 **Kibble** worked this out in a non-abelian model of vector boson with a mass due to the Higgs system. **Weinberg** (1968), knowing the work of Kibble, used the correct group as proposed by **Glashow** (1961) in a theory of weak interactions of leptons.

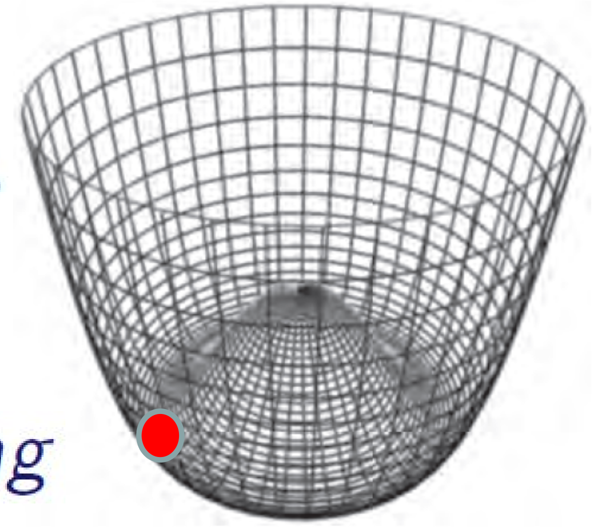
Renormalizability for gauge theories with vector boson masses generated through a vacuum field was proven in 1971 (**Veltman+'t Hooft**).

Symmetry Law Need Not Imply Symmetric Outcome

Unique
vacuum state

degenerate
vacuum states

spontaneous
symmetry breaking

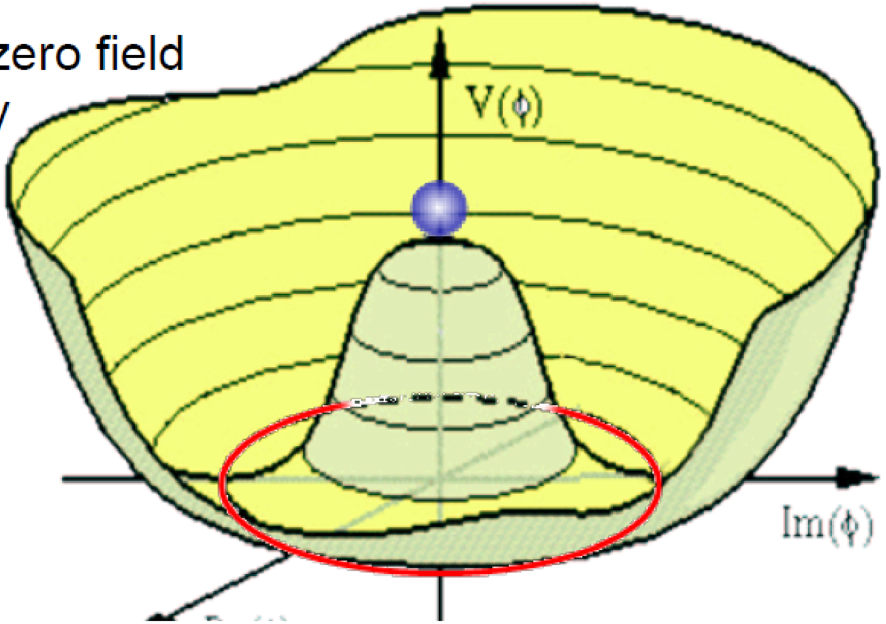


Spontaneous Symmetry Breaking

1964 - 1968

Higgs Guralnik Hagen Kibble Brout Englert

- introduce a field with a special potential
- continuously degenerate ground state at non-zero field
- field equations have same symmetry as theory
- every ground state breaks the symmetry
- can describe massive particles by coupling them with the Higgs field
- the quantum of this field is the Higgs boson



Higgs mechanism

Spontaneous Symmetry Breaking

❖ Introduced a field with ***a special potential***; Field equations have same symmetry as gauge theory

$$\mathcal{L} = (D_\mu\phi)^\dagger(D^\mu\phi) - V(\phi)$$

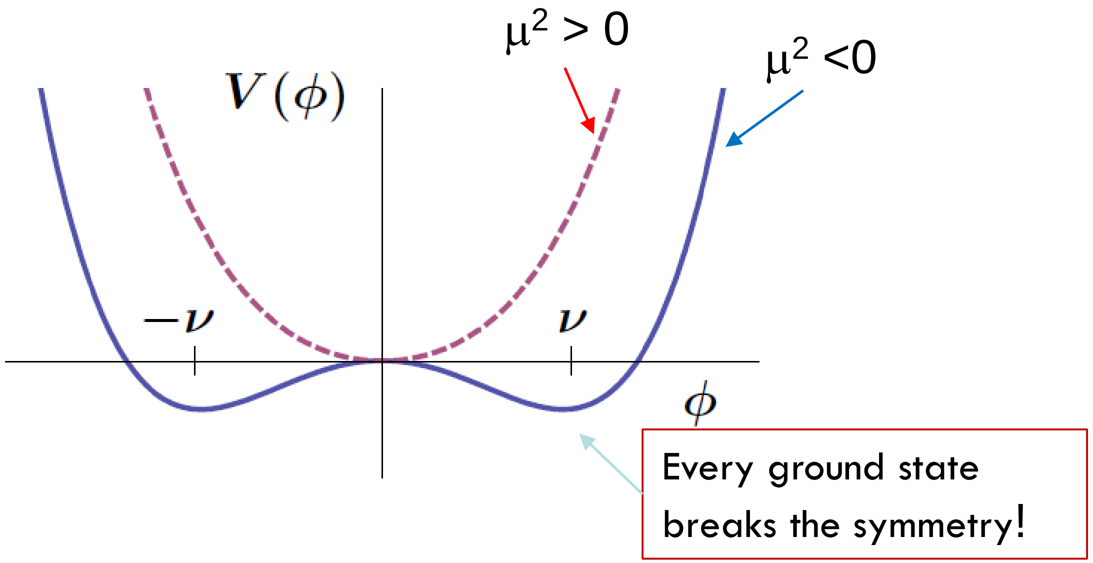
$$D_\mu = \partial_\mu - ig\vec{W}_\mu.\vec{\sigma} - ig'\frac{Y}{2}B_\mu$$

$$V = \mu^2\Phi^+\Phi + \lambda(\Phi^+\Phi)^2$$

$$\Phi = \frac{1}{\sqrt{2}}\begin{pmatrix}\phi_1 + i\phi_2 \\ \phi_3 + i\phi_4\end{pmatrix} = \begin{pmatrix}\varphi^+ \\ \varphi^0\end{pmatrix}$$

❖ **If $\mu^2 < 0$, then spontaneous symmetry breaking --Continuously degenerate ground state at non-zero field. The minimum of potential at**

$$\langle\Phi\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}0 \\ v\end{pmatrix}$$



Higgs Mechanism

expanding ϕ field around the vacuum

$$\langle\Phi\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix}$$

$$\mathcal{L}_\phi = (D^\mu\phi)^\dagger D_\mu\phi - V(\phi)$$

$$\phi(x) = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ \nu + H(x) \end{pmatrix}$$

Insert to →

$$V(\phi) = \mu^2\phi^\dagger\phi + \lambda(\phi^\dagger\phi)^2$$

→ **Higgs boson massive; self-interaction**

$$V(\phi) = -\frac{\mu^4}{4\lambda} - \mu^2 H^2 + \lambda\nu H^3 + \frac{\lambda}{4}H^4$$

The second term in $V(\phi)$ represents the mass (at the tree-level) of the physical Higgs boson:

$$M_H = \sqrt{-2\mu^2} = \sqrt{2\lambda}\,\nu,$$

where λ is a parameter of the SM which is not specified in the model. The third and the 4th terms are the Higgs boson self-interaction terms.

One of the LHC goal

Higgs Mechanism

expanding ϕ field around the vacuum $\langle\Phi\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v\end{pmatrix}$

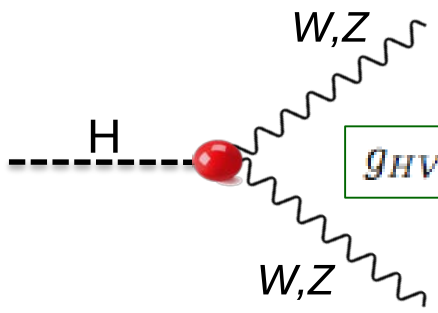
$$\phi(x) = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ \nu + H(x)\end{pmatrix}$$

Generate masses for weak interaction bosons → $\mathcal{L}_\phi = (D^\mu\phi)^\dagger D_\mu\phi - V(\phi)$

$$T(\phi) = (D_\mu\phi)^\dagger(D^\mu\phi) = \frac{1}{2}(\partial_\mu H)^2 + \frac{g_1^2}{8}(W_\mu^1 - iW_\mu^2)(W^{1\mu} + iW^{2\mu})(\nu + H)^2$$
$$+\frac{1}{8}(g_1 W_\mu^3 - g_2 B_\mu)(W^{3\mu} - g_2 B^\mu)(\nu + H)^2,$$

W, and Z become massive; & interacting with H

$$(D^\mu\phi)^\dagger D_\mu\phi = \frac{1}{2}(\partial_\mu H)^2 + M_W^2 W^{\mu+}W_\mu^-\left(1+\frac{H}{\nu}\right)^2 + \frac{1}{2}M_Z^2 Z^\mu Z_\mu\left(1+\frac{H}{\nu}\right)^2$$
$$= \frac{1}{2}(\partial_\mu H)^2 + M_W{}^2 W^{\mu+}W_\mu^- + \frac{2M_W{}^2}{\nu}W^{\mu+}W_\mu^- H + \frac{M_W{}^2}{\nu^2}W^{\mu+}W_\mu^- H^2$$
$$+\frac{1}{2}M_Z{}^2 Z^\mu Z_\mu + \frac{M_Z{}^2}{\nu}Z^\mu Z_\mu H + \frac{M_Z{}^2}{2\nu^2}Z^\mu Z_\mu H^2$$



$g_{HVV} = 2M_V^2/v$

Generate Fermion Mass

$$L_e = \begin{pmatrix} \nu_e \\ e_L \end{pmatrix}$$

$$R_e = (e_R)$$

$$e_L = \frac{1-\gamma^5}{2} e$$

$$e_R = \frac{1+\gamma^5}{2} e$$

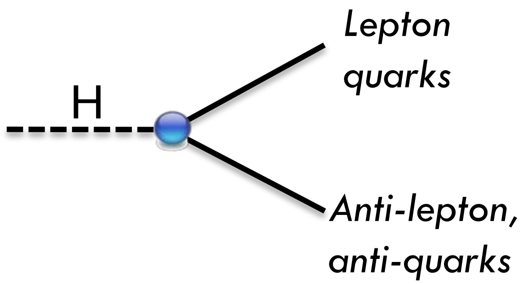
γ^5 is a 4×4 matrix $\begin{pmatrix} 0 & I_{2\times2} \\ I_{2\times2} & 0 \end{pmatrix}$

$$L^e_{Yukawa} = g_e(\overline{L_e}\phi e_R + \phi^\dagger \overline{e_R} L_e)$$

expanding ϕ field around the vacuum $\phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \nu + H(x) \end{pmatrix}$

$$L^e_{Yukawa} = \frac{g_e \nu}{\sqrt{2}}(\overline{e_L} e_R + \overline{e_R} e_L) + \frac{g_e}{\sqrt{2}}(\overline{e_L} e_R + \overline{e_R} e_L)H$$

- The first term is the mass term: $m_e = \frac{g_e \nu}{\sqrt{2}}$ $\quad \boldsymbol{g_e \propto m_e}$
- The second term represents the interaction between electron and the Higgs boson with the interaction coupling proportional to the electron mass. Such coupling is called Yukawa coupling.



What is the difference between gauge couplings and Yukawa couplings?

$$\mathcal{L}_{SU(2)_L \times U(1)_Y} = \mathcal{L}_{gauge} + \mathcal{L}_{fermion} + \mathcal{L}_{\phi} + \mathcal{L}_{Yukawa}$$

$$\mathcal{L}_{gauge} = -\frac{1}{4} W^i_{\mu\nu} W^{\mu\nu i} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}$$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

No self-interactions

$$W^i_{\mu\nu} = \partial_\mu W^i_\nu - \partial_\nu W^i_\mu - g_2 \epsilon_{ijk} W^j_\mu W^k_\nu$$

Self-interactions

$$\mathcal{L}_{fermion} = \sum_{\psi = L_L, L_R, Q_L, u_R, d_R} \bar{\psi} i \gamma^\mu D_\mu \psi$$

$$D^\mu = \partial^\mu - i g_1 \frac{Y}{2} B_\mu - i g_2 \frac{\tau^i}{2} W^i_\mu$$

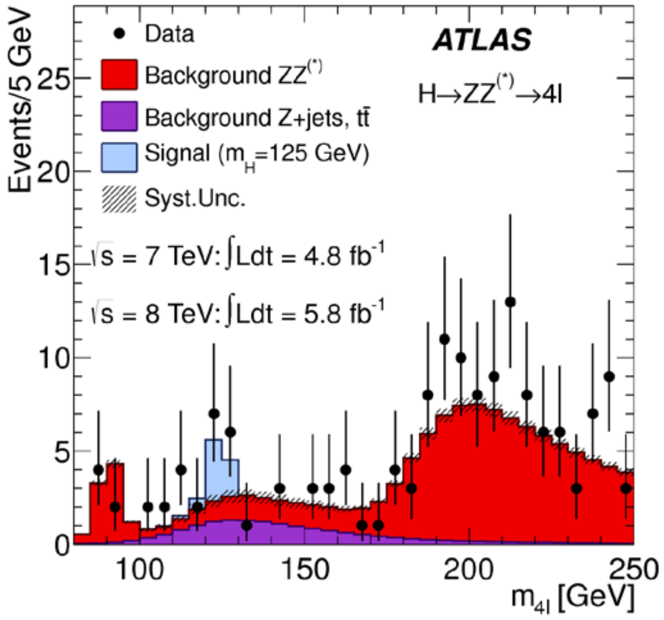
Interaction with gauge fields

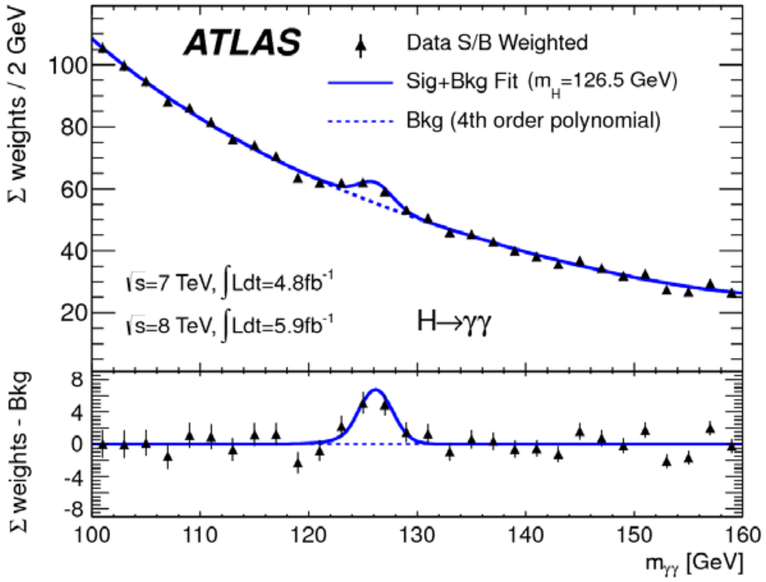
$$\mathcal{L}^f_{Yukawa} = g_f (\overline{L_f} \phi R_f + \overline{R_f} \phi^\dagger L_f)$$

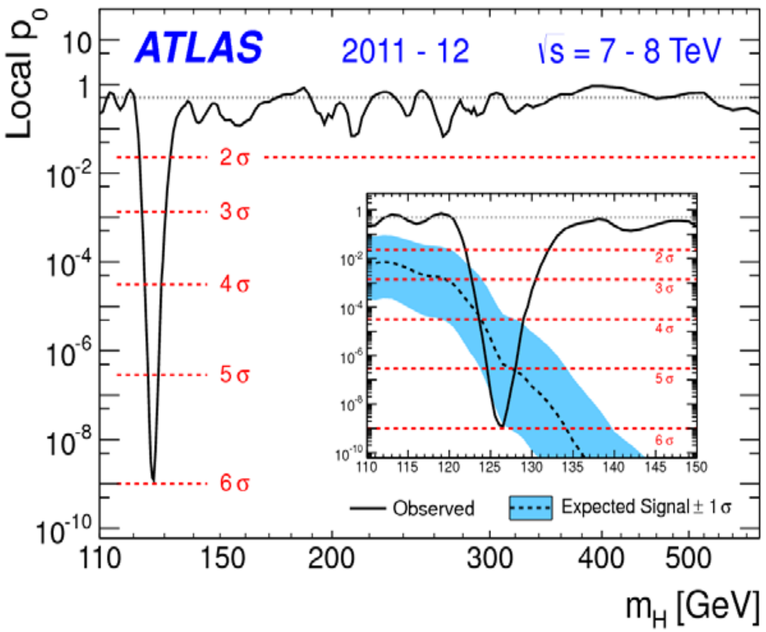
What are the forms of gauge transformations? Which leads no mass terms can be included in Lagarangion

Question: what is the gauge couplings? Are they universal or non-universal? If Yukawa couplings universal?

Higgs Discovery with 4l and γγ Inv. Mass







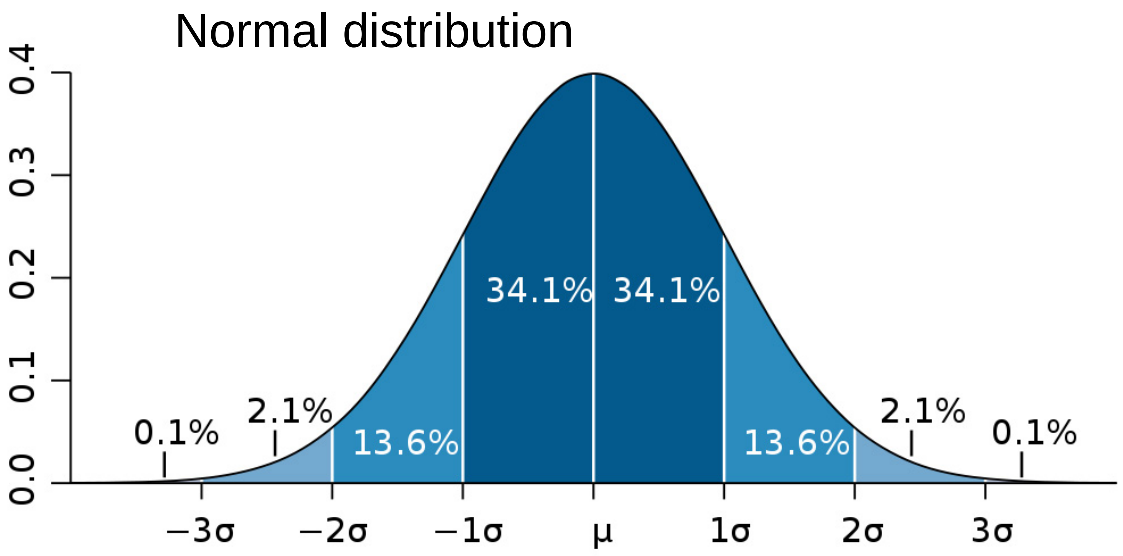
Invariant mass calculation using measured energies and momenta of the lepton (or photon); we plot the inv. Mass spectra.

$$m_{4l} = \sqrt{(E_1^l + E_2^l + E_3^l + E_4^l)^2 - (\vec{p_1} + \vec{p_2} + \vec{p_3} + \vec{p_4})^2}$$

$$m_{2\gamma} = \sqrt{(E_1^\gamma + E_2^\gamma)^2 - (\vec{p_1} + \vec{p_2})^2}$$

Statistic test to see if data is consistent with background only model.
-- **Higgs boson signal made > 5σ deviations from the background only model**

Statistical Significance in Terms of σ



It's a question that arises with every major new finding in science: **What makes a result reliable enough to be taken seriously?** The answer has to do with statistical significance.

The unit of measurement usually given when talking about statistical significance is the standard deviation (σ). The term refers to the amount of variability in a given set of data: whether the data points are all clustered together, or very spread out.

If a data point is a few standard deviations away from the model being tested, this is strong evidence that the data point is not consistent with that model.

Criteria in particle discovery is 5σ deviations from the background only model

Free parameters in SM

Parameters of the SM: 18 free parameters

- **9 fermions masses, 4 CKM parameters** (See before two pages for detail)
- **3 coupling** g_s, g_2, g_1 **and 2 parameters from EWSB scalar potential.**

More precise inputs, $\alpha_s, \alpha(M_Z^2), G_F, M_Z$ **and** M_H **(unknown)**

Fermion masses, not explained in SM

$$\begin{pmatrix} u & c & t \\ d & s & b \end{pmatrix} \text{ with mass } =\sim \begin{pmatrix} 5 & 1{,}500 & 172{,}000 \\ 5 & 100 & 4{,}500 \end{pmatrix} \text{ MeV}$$

$$\begin{pmatrix} \nu_e & \nu_\mu & \nu_\tau \\ e^- & \mu^- & \tau^- \end{pmatrix} \quad m_e = 0.511 \text{ MeV} \quad m_\mu = 105.6 \text{ MeV} \quad m_\tau = 1.777 \text{ GeV}$$

In SM, $m_\nu = 0$.
But in recent years experiment, we discovered ν oscillation → $m_\nu \neq 0$!

SM Shortcomings

- No explanation of Higgs potential
- No prediction for M_H
- Doesn't predict fermion masses & mixings
- M_H unstable to quantum corrections
- No explanation of charge quantization
- Doesn't account for three generations
- Vacuum energy problem
- Beyond scope: dark matter, matter asymmetry, etc.

Introduction to Cross Section

- **The most important quantity to describe a scattering (also called "collision" or "reaction") process is the "cross section" (σ).**
- **It is a yardstick of the probability of a reaction between the two colliding particles**

Consider a generic $2 \to 2$ process with beam particle a and target particle b: $a + b \to a' + b'$

The number of scatterings per unit time $\dot{N} \propto \Phi_a$, $\dot{N} \propto N_b$ with
Φ_a: the beam particle flux (particles per unit area per unit time);
N_b: the number of target particles

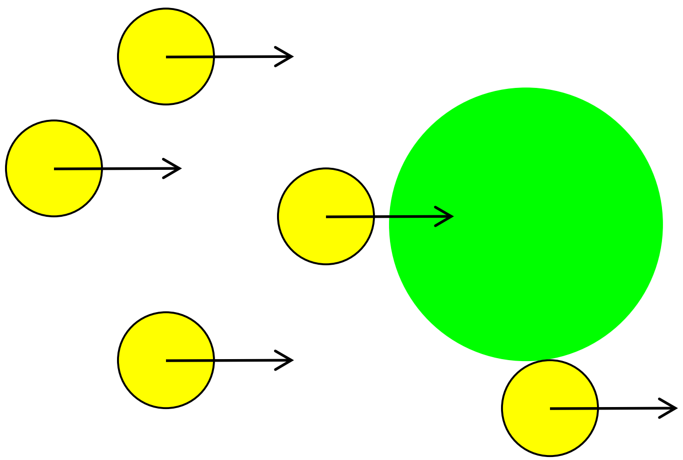
Cross Section

The proportional constant is the cross section

$$\sigma = \frac{\dot{N}}{\Phi_a \cdot N_b}$$

$$= \frac{\text{number of scatterings per unit time}}{\text{beam particles per unit time per unit area} \times \text{target particles}} \quad \text{(uniform beam)}$$

$$= \frac{\text{number of scattering per unit time}}{\text{beam particles per unit time} \times \text{target particles per unit area}} \quad \text{(uniform target)}$$



For the collisions of two hard balls, it is the geometric cross sectional area

$$\sigma = \pi\left(R_1 + R_2\right)^2$$

where R_1 and R_2 are radii of the beam and target balls.

Cross Section

- The cross-section is a physical quantity with the dimension of area;
- It is determined by physics (the interaction that causes the scattering) and is independent of specific experimental design;
- The commonly used unit is barn (b) defined as

$$1\text{ b} = 10^{-24}\text{ cm}^2;\ \ 1\text{ mb} = 10^{-3}\text{ b};\ \ 1\ \mu\text{b} = 10^{-6}\text{ b}, ...$$

In reality, only a faction of all the scatters are detected since most of the detectors do not cover the entire solid angle. In this case, we measure the differential cross-sections such as

$$\sigma(\theta) = \frac{d\sigma}{d\Omega} = \frac{1}{\Phi_a \cdot N_b}\frac{d\dot{N}}{d\Omega},\quad \sigma(\theta, E) = \frac{d^2\sigma}{dE\ d\Omega},\ \ldots$$

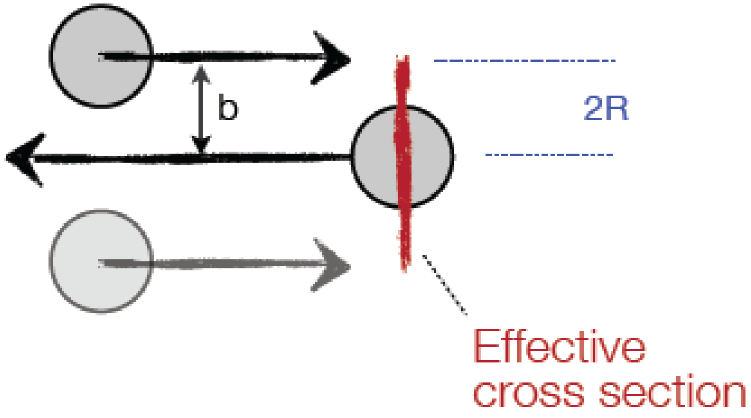
Cross Section

Standard cross section unit: [σ] = mb, nb, pb, fb, …

In natural unit: [σ] = GeV^{-2} with the conversion

$$1\ \mathrm{GeV}^{-2} = 0.389\ \mathrm{mb} \quad \text{or} \quad 1\ \mathrm{mb} = 2.57\ \mathrm{GeV}^{-2}$$

Estimating the
proton-proton cross section:

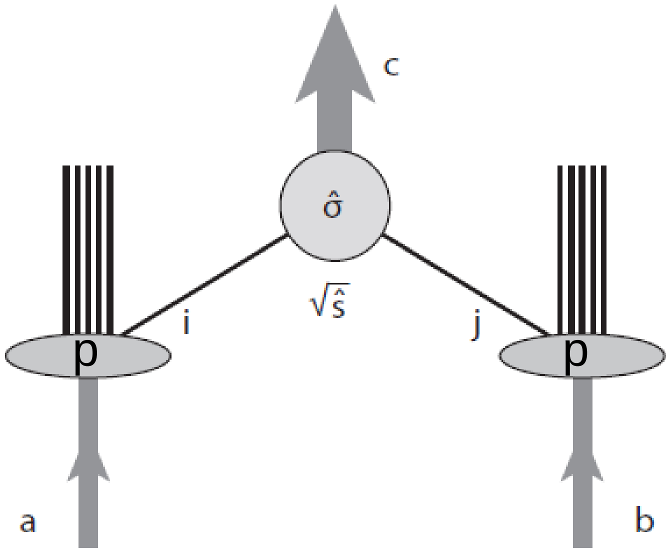


Proton radius: R = 0.8 fm

Strong interactions happens up to b = 2R

$$\sigma = \pi(2R)^2 = \pi \cdot 1.6^2\ \mathrm{fm}^2$$
$$= \pi \cdot 1.6^2\ 10^{-26}\ \mathrm{cm}^2$$
$$= \pi \cdot 1.6^2\ 10\ \mathrm{mb}$$
$$= 80\ \mathrm{mb}$$

Hard scattering Cross Sections



$$d\sigma(a+b\to c+X) = \sum_{ij}\int dx_a dx_b\, \delta(\tau - x_a x_b)\cdot \qquad (\tau = \hat{s}/s)$$

$$f_i^{(a)}(x_a,Q^2) f_j^{(b)}(x_b,Q^2) d\hat{\sigma}(i+j\to c+X),$$

$d\hat{\sigma}$: elementary cross section at energy $\sqrt{\hat{s}} = \sqrt{x_a x_b s}$

High-energy p: broadband unseparated beam of quark, anti-quark, gluons

(For hard scattering) a broad-band, unseparated beam of quarks, antiquarks, gluons, & perhaps other constituents, characterized by parton densities

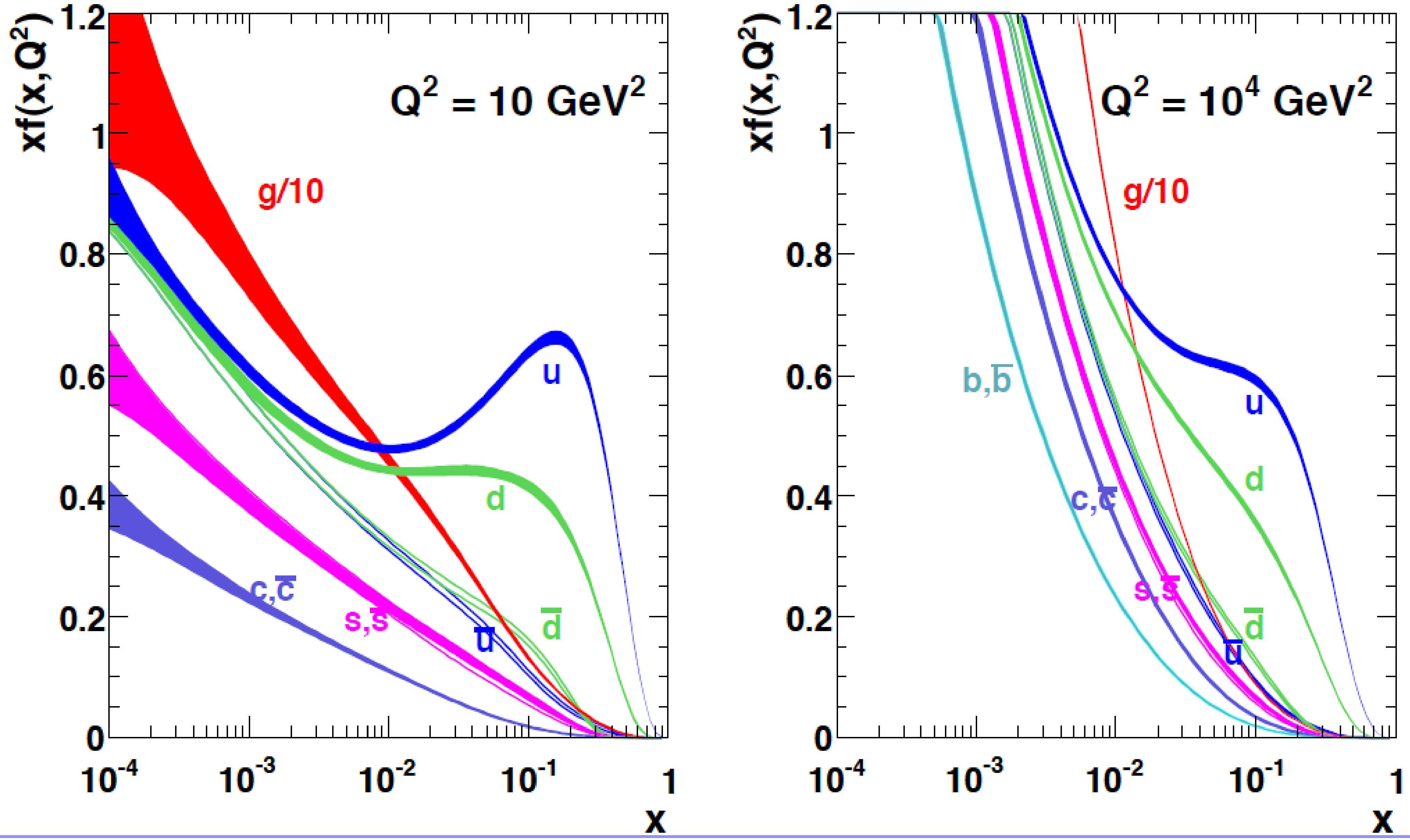
$$f_i^{(a)}(x_a, Q^2),$$

. . . number density of species i with momentum fraction x_a of hadron a seen by probe with resolving power Q^2.

Q^2 evolution given by QCD perturbation theory

$f_i^{(a)}(x_a, Q_0^2)$: nonperturbative

MSTW 2008 NLO PDFs (68% C.L.)



Cross Section Measurement

- *Select events based on underlying physics process final state*

$$N_i(selected\ number\ events) = \sigma_i(predicted) \times BR \times \int L\,dt \times Acceptance$$

The sources of the same final state need to be estimated by MC, particularly you need to understand the expected signal size before you jumping in analysis details

- *Connection between experiment and theory: cross section σ*

Prediction from Modeling/simulation
To be compared with data
subtract background

Estimation from data/MC

$$\sigma = \frac{N_{signal} = N_{data} - N_B}{BR \times \int L\,dt \times Acceptance}$$

Theory/Previous measurement

Accelerator

MC simulation/ SF from data

Cross Section Measurement Uncertainties

- Assume Lumi (L), Acc (A) and estimated background (B) are independent. Error propagation gives the relative systematical uncertainty expression:

$$\frac{\Delta\sigma}{\sigma} = \sqrt{\left(\frac{\Delta L}{L}\right)^2 + \left(\frac{\Delta A}{A}\right)^2 + \left(\frac{\Delta N_B}{N_{data} - N_B}\right)^2}$$

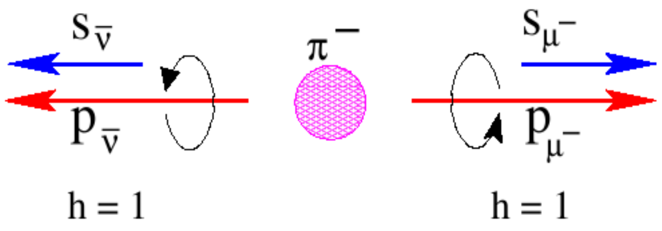
- The static uncertainty can be estimated with the Poisson distribution; for large number of observed events, N, the related uncertainty should be just $\frac{1}{\sqrt{N_{data} - N_B}}$
- *Likelihood fit method is used in final cross section extraction*

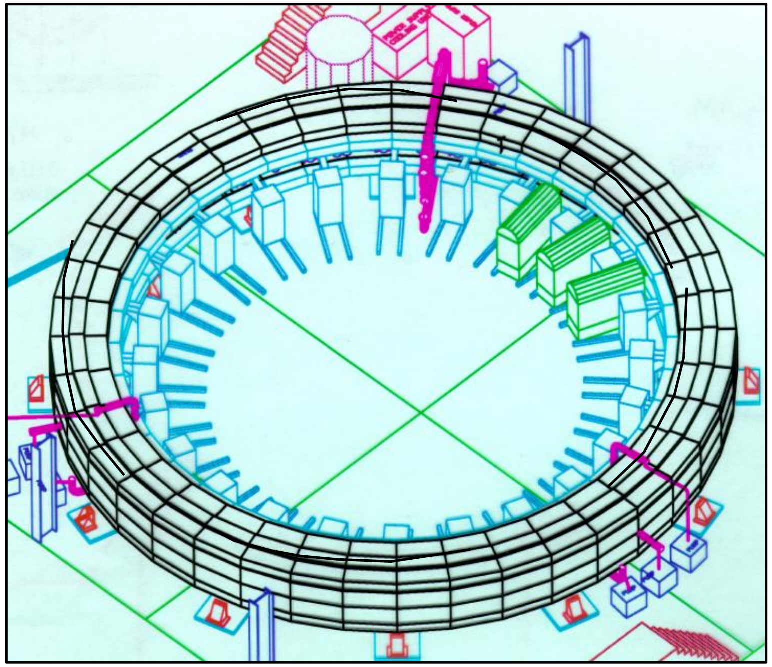
Questions

- What are the gauge symmetry groups under which the theory is invariant under symmetry transformation? How many force carrier particles in the SM? What are their masses?
- What is the charged current? Can you provide an example? What is the neutral current? Can you provide an example?
- Can quark change the flavor in charged current interactions? How about in neutral current interactions?
- What is the W and Z decay width? From which, can you derive the life-time of the W and Z bosons?
- Can you estimate what is the branching fraction of $W \rightarrow \mu\nu$? If the fractions for W decays to electron+ν or tau+ν are the same?
- Why the CKM matrix needs to be introduced to the SM? Do we know its origin?
- How the masses are generated in the SM? Are the interaction strength of the Higgs boson to all the massive particles universal?
- What are the mass values of neutrinos in the SM? Can Higgs mechanism generate neutrino mass?
- Do we fully understand the electroweak symmetry breaking mechanism? Why or why not?
- What is the Higgs field potential shape ($V(\Phi)$ vs. Φ)? How to test it experimentally?
- What is the most interesting particle physics question that could be addressed in next 10 - 20 years?

G-2 Experiment (at BNL and Fermilab)

1. Inject polarized muon source

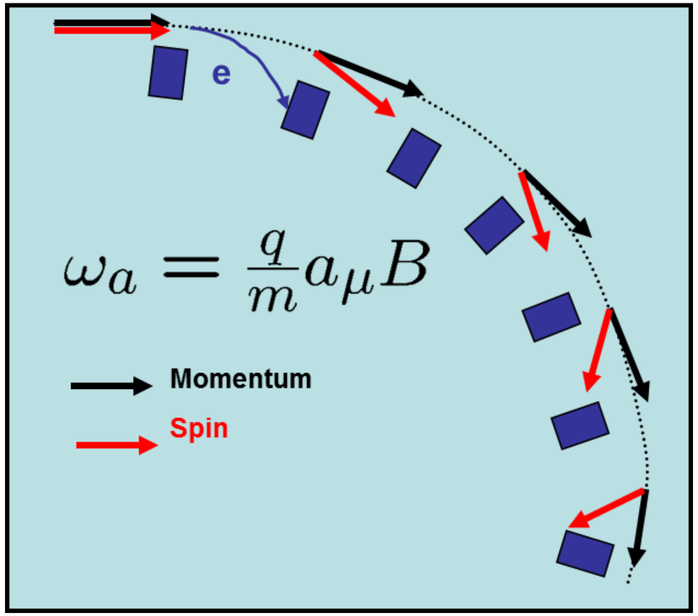




2. γ=29.3 μ allow E-field vert. focusing

$$\vec{\omega}_a = -\frac{e}{m}\left[a_\mu \vec{B} - \left(a_\mu - \frac{1}{\gamma^2 - 1}\right)\frac{\vec{\beta}\times\vec{E}}{c}\right]$$

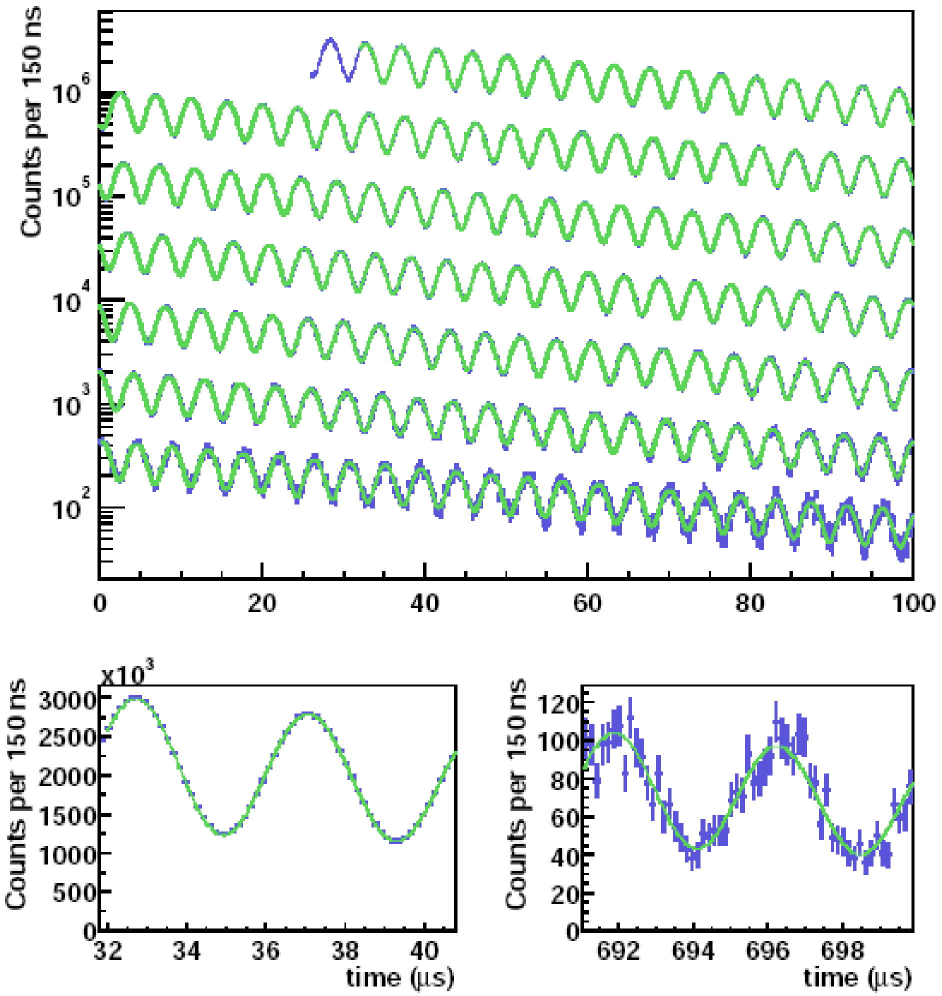
3. μ spin precession relative to momentum in cyclotron is directly proportional to a_μ



$$\omega_a = \omega_S - \omega_C = \left(\frac{g-2}{2}\right)\frac{eB}{mc} = a\frac{eB}{mc}$$

G-2 Experiment (at BNL and Fermilab)

4. Highest energy decay electrons emitted when spin and momentum vectors parallel



$$\omega_a = \omega_S - \omega_C$$

$$= \left(\frac{g-2}{2}\right)\frac{eB}{mc} = a\frac{eB}{mc}$$

ω_a and eB are the two quantities to be measured

Parton Distribution Functions Literature

The state of the art is reviewed in A. De Roeck & R. S. Thorne, *Prog. Part. Nucl. Phys.* **66**, 727 (2011).

Recommendations and assessments of uncertainties are given by the PDF4LHC Working Group.

Convenient access to many sets of parton distributions is available through the Durham HEPData Project Online.

A common interface to many modern sets of PDFs is M. R. Whalley & A. Buckley, "LHAPDF: the Les Houches Accord Parton Distribution Function Interface."

Deeply Inelastic Scattering $\rightsquigarrow f_i^{(a)}(x_a, Q_0^2)$