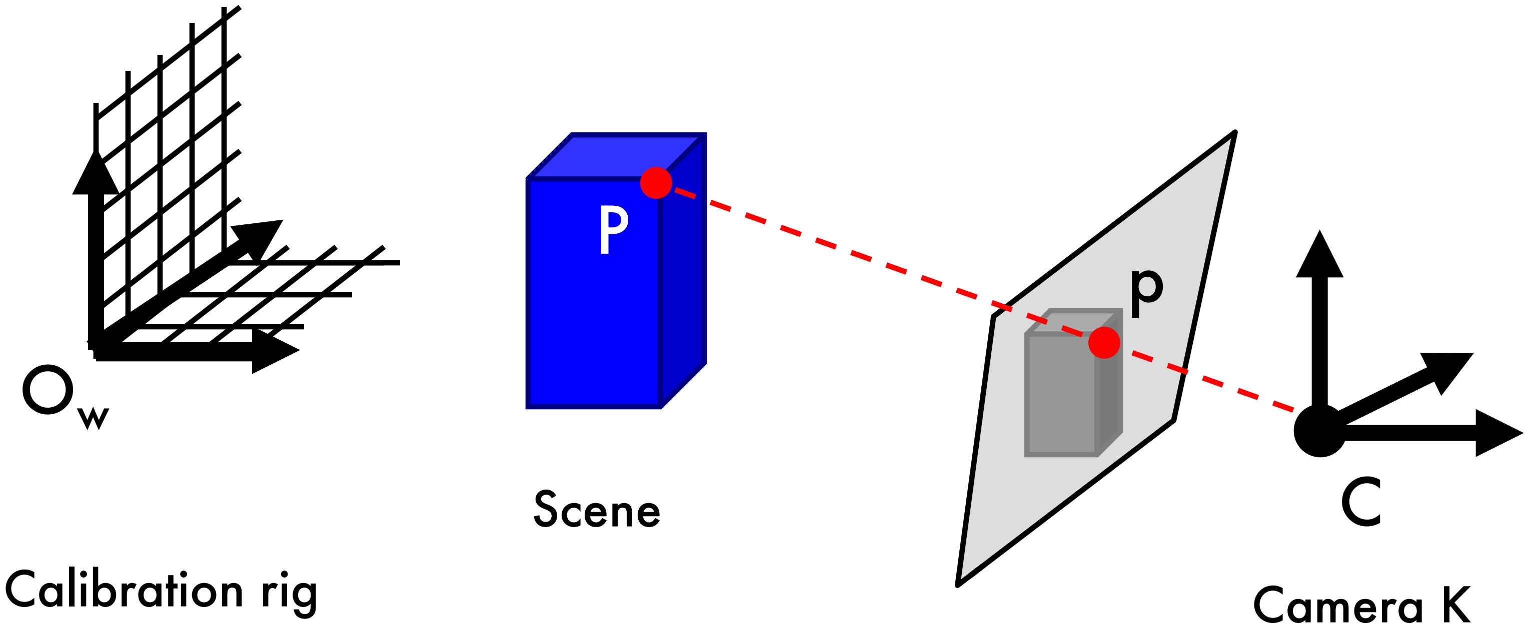
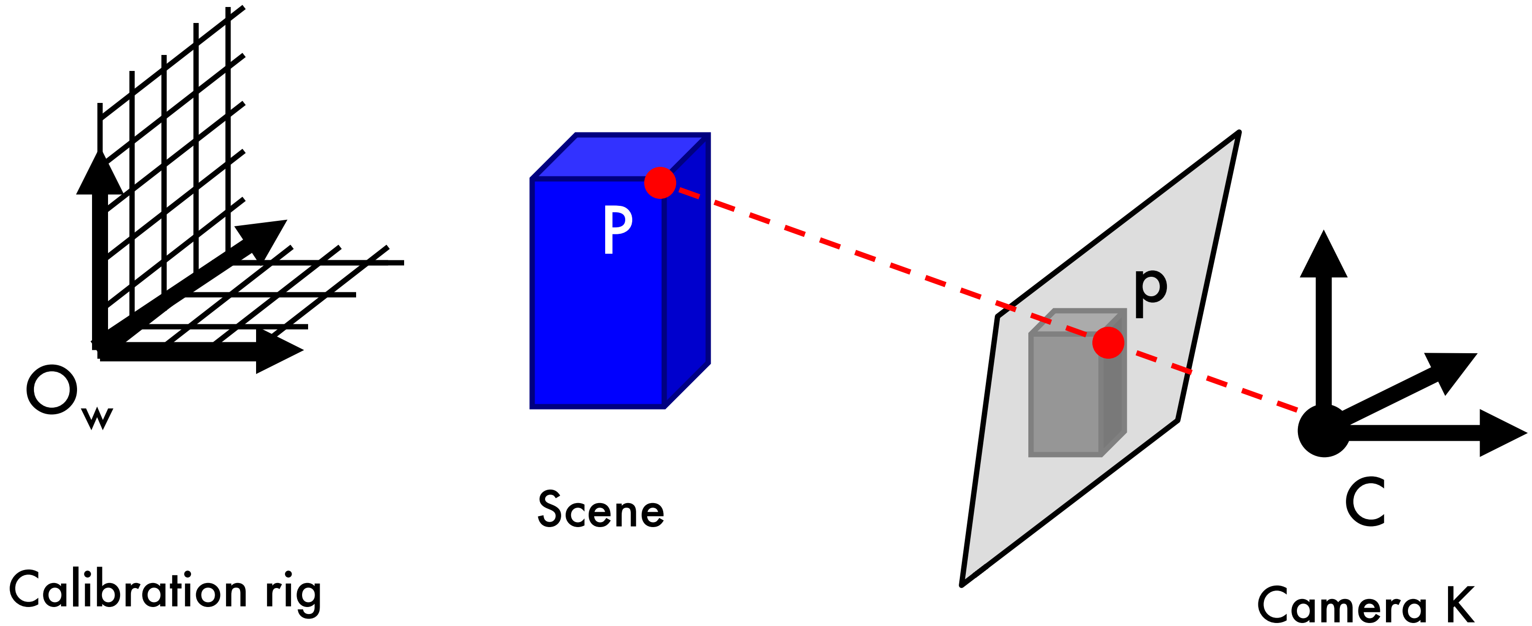
Recovering structure from a single view



From calibration rig	$\rightarrow$	location/pose of the rig, K
From points and lines at infinity + orthogonal lines and planes	$\rightarrow$	structure of the scene, K

Knowledge about scene (point correspondences, geometry of lines & planes, etc...

Recovering structure from a single view



Why is it so difficult?

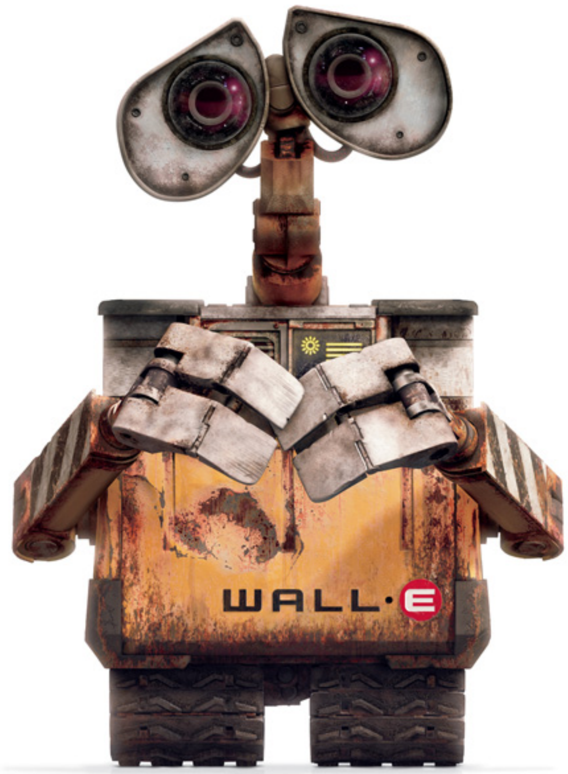
Intrinsic ambiguity of the mapping from 3D to image (2D)

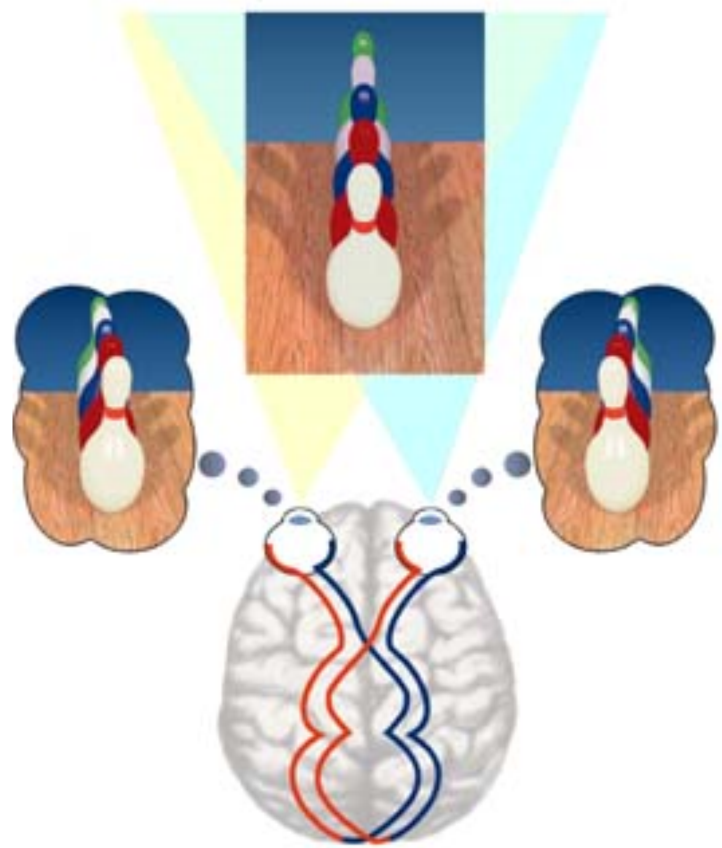
Recovering structure from a single view

Intrinsic ambiguity of the mapping from 3D to image (2D)

Courtesy slide S. Lazebnik

Two eyes help!

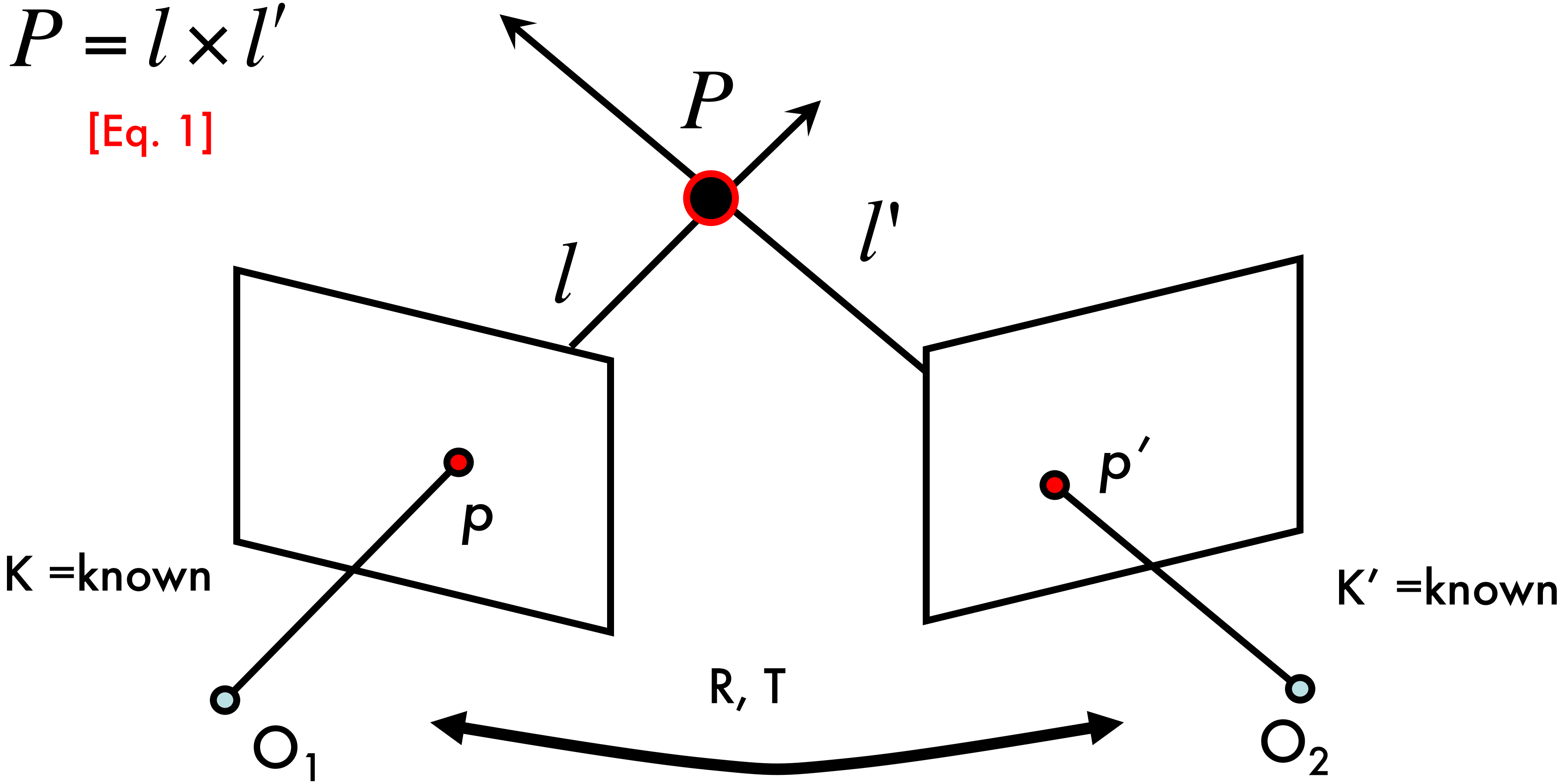




Two eyes help!

$$P = l \times l'$$

[Eq. 1]

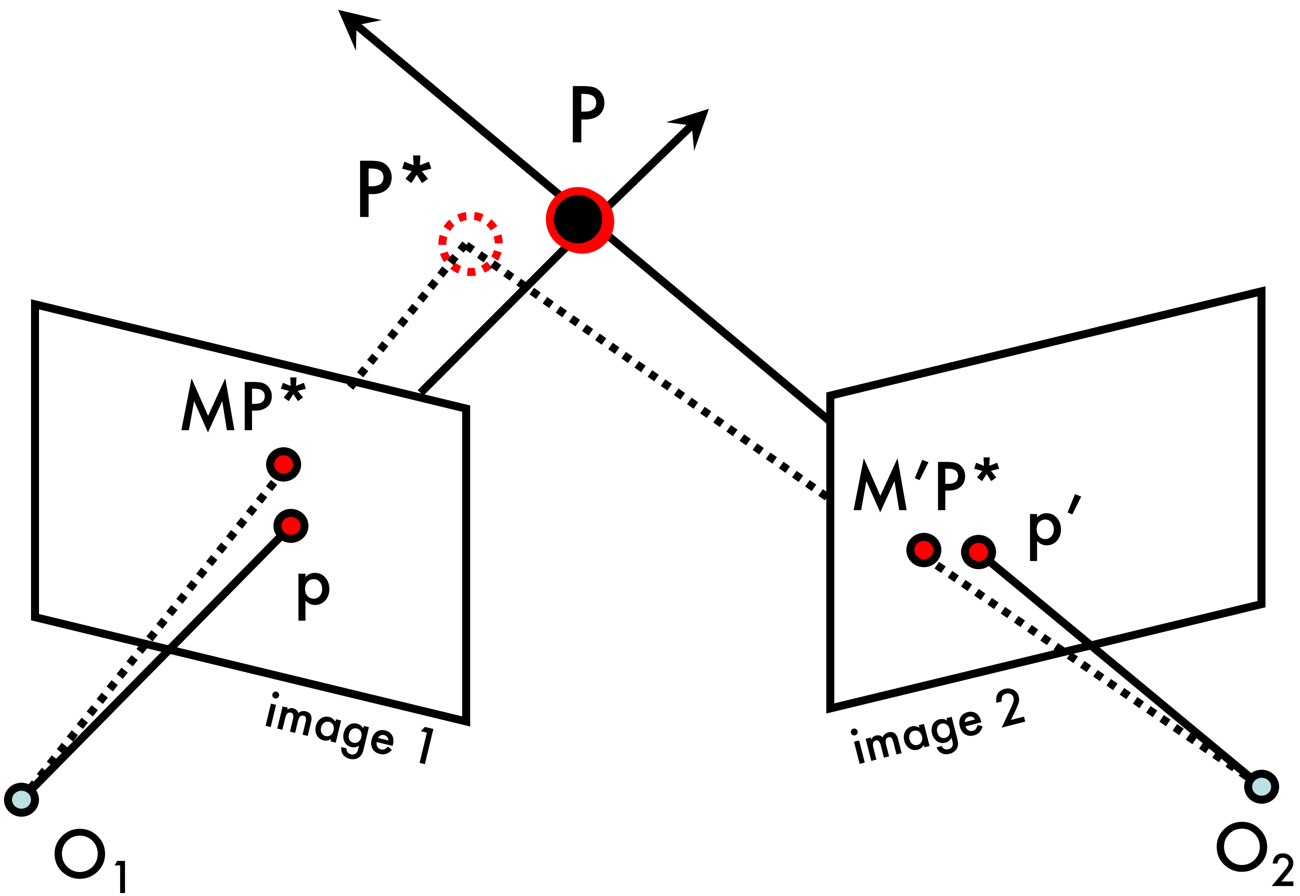


This is called triangulation

Triangulation

- Find P* that minimizes

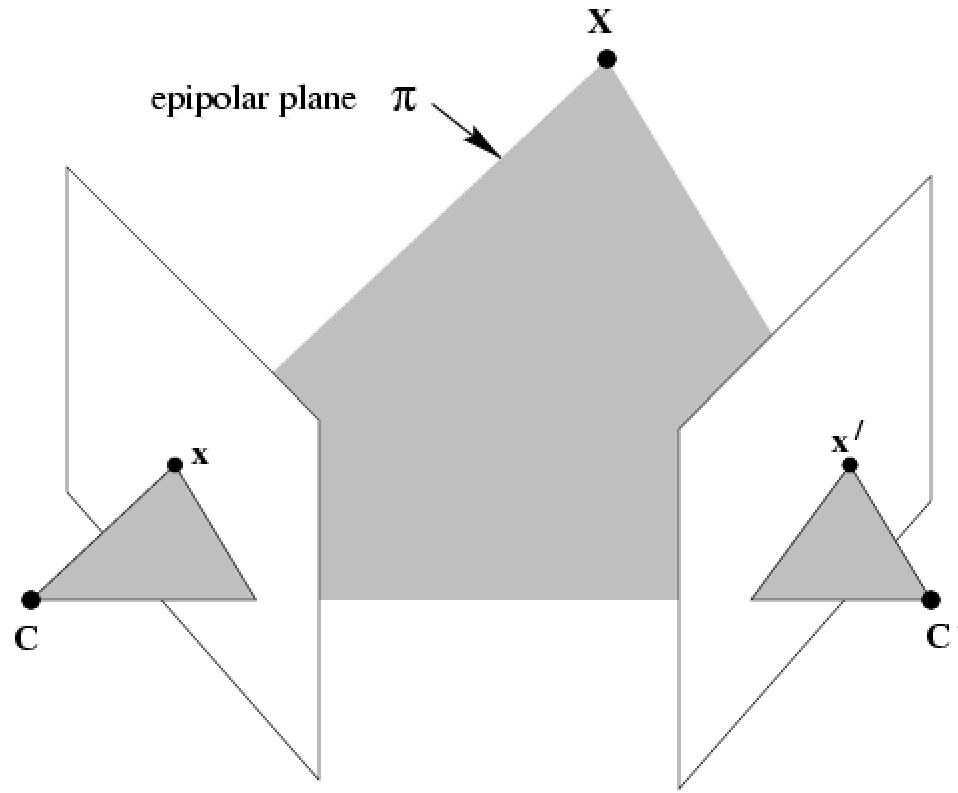
$$d(p, M\ P^*) + d(p', M' P^*)$$ [Eq. 2]



Multi (stereo)-view geometry

- **Camera geometry:** Given corresponding points in two images, find camera matrices, position and pose.
- **Scene geometry:** Find coordinates of 3D point from its projection into 2 or multiple images.
- **Correspondence:** Given a point **p** in one image, how can I find the corresponding point **p′** in another one?

The epipolar geometry



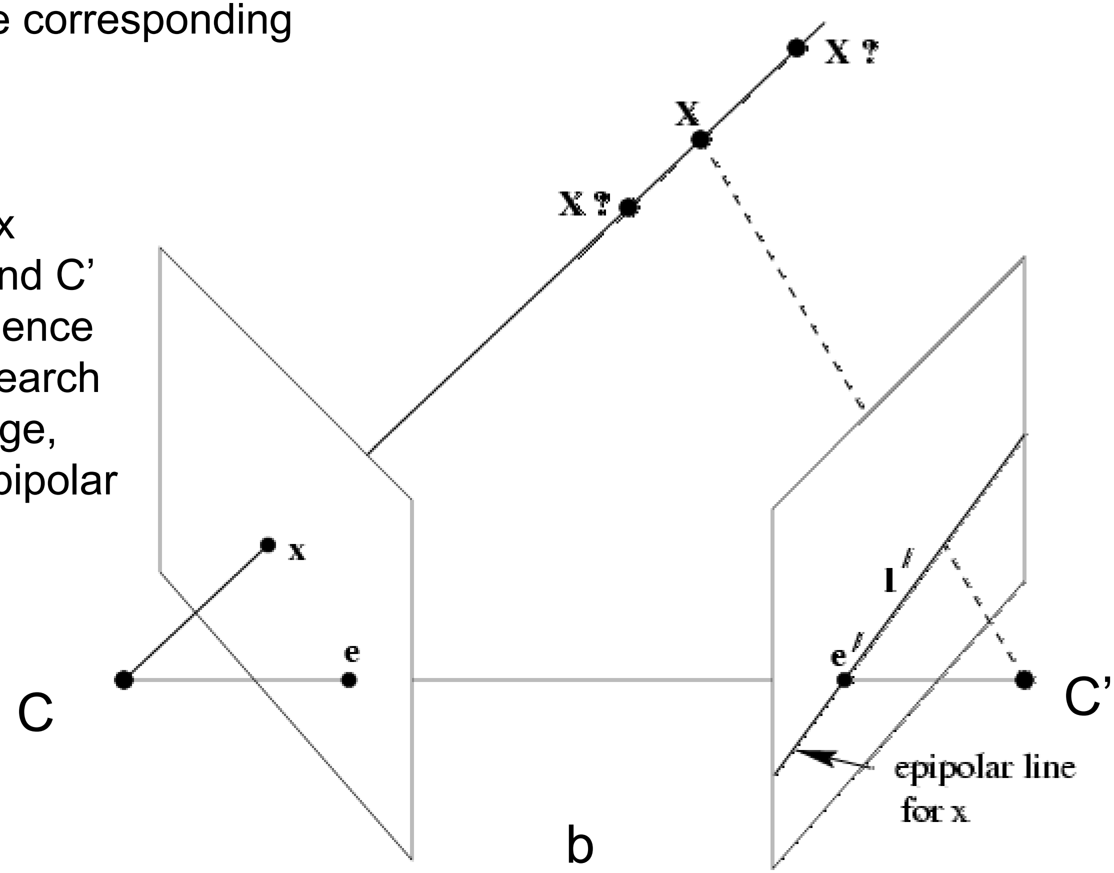
(a)

C,C',x,x' and X are coplanar

Fig. 9.1. **Point correspondence geometry.** *(a) The two cameras are indicated by their centres* **C** *and* **C**$'$ *and image planes. The camera centres, 3-space point* **X**, *and its images* **x** *and* **x**$'$ *lie in a common plane* π. *(b) An image point* **x** *back-projects to a ray in 3-space defined by the first camera centre,* **C**, *and* **x**. *This ray is imaged as a line* $\mathbf{l}'$ *in the second view. The 3-space point* **X** *which projects to* **x** *must lie on this ray, so the image of* **X** *in the second view must lie on* $\mathbf{l}'$.

The epipolar geometry

- If we know x, how is the corresponding point x’ constrained?
- l’ is the Epipolar line corresponding to point x
- Upshot: if we know C and C’ for a stereo correspondence algorithm, no need to search all over the second image, but just only over the epipolar line.

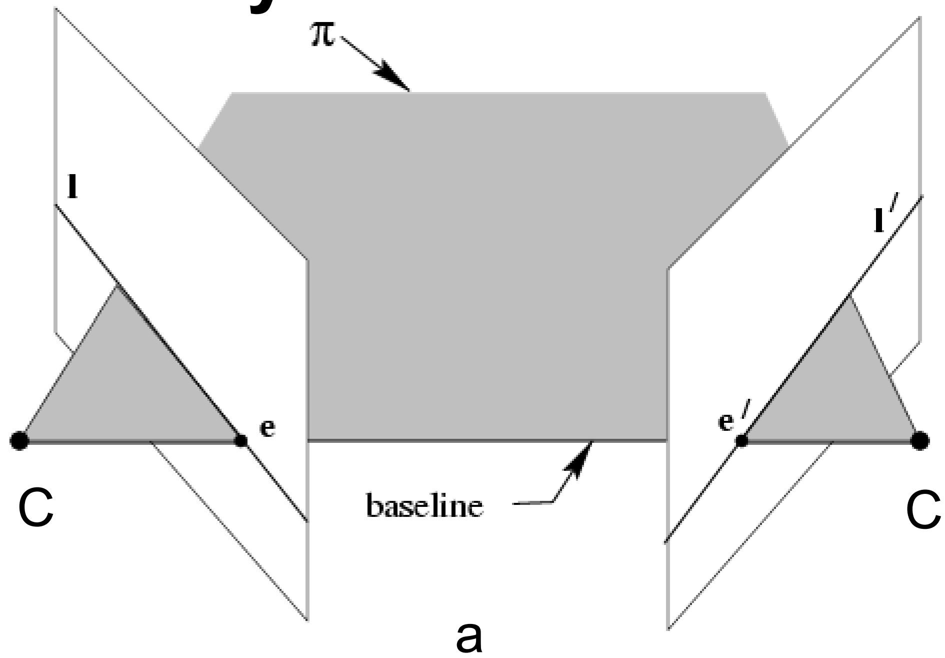


What if only C,C’,x are known?

Fig. 9.1. **Point correspondence geometry.** *(a) The two cameras are indicated by their centres* $\mathbf{C}$ *and* $\mathbf{C}'$ *and image planes. The camera centres, 3-space point* $\mathbf{X}$*, and its images* $\mathbf{x}$ *and* $\mathbf{x}'$ *lie in a common plane* π. *(b) An image point* $\mathbf{x}$ *back-projects to a ray in 3-space defined by the first camera centre,* $\mathbf{C}$*, and* $\mathbf{x}$. *This ray is imaged as a line* $\mathbf{l}'$ *in the second view. The 3-space point* $\mathbf{X}$ *which projects to* $\mathbf{x}$ *must lie on this ray, so the image of* $\mathbf{X}$ *in the second view must lie on* $\mathbf{l}'$.

The epipolar geometry

- Baseline: connects two camera centers
- Epipole: point of intersection of baseline with image plane
- Epipole: image in one view of the camera center of the other view.

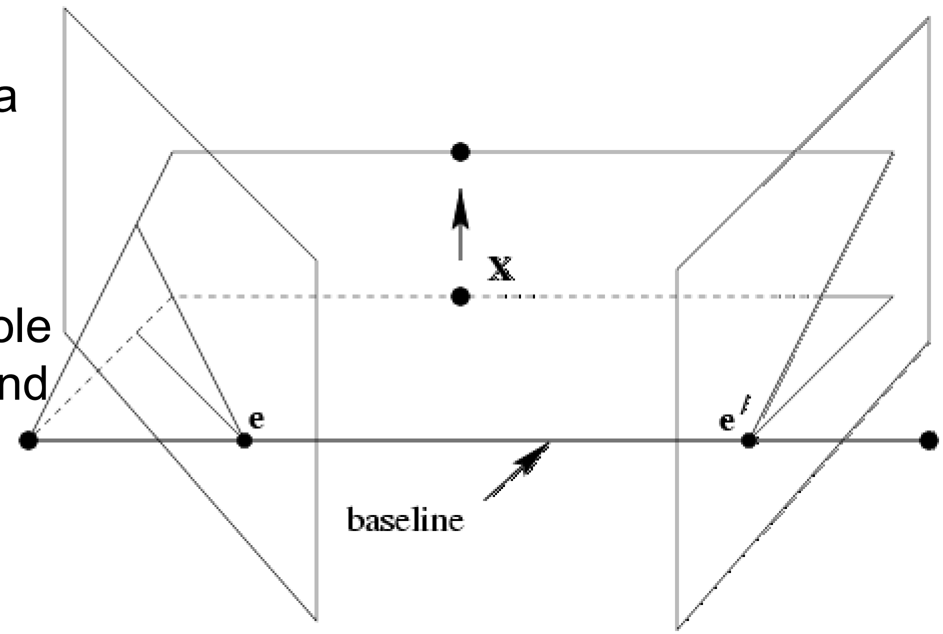


All points on π project on l and l'

Fig. 9.2. **Epipolar geometry.** *(a) The camera baseline intersects each image plane at the epipoles* $\mathbf{e}$ *and* $\mathbf{e}'$. *Any plane* π *containing the baseline is an epipolar plane, and intersects the image planes in corresponding epipolar lines* $\mathbf{l}$ *and* $\mathbf{l}'$. *(b) As the position of the 3D point* $\mathbf{X}$ *varies, the epipolar planes "rotate" about the baseline. This family of planes is known as an epipolar pencil. All epipolar lines intersect at the epipole.*

The epipolar geometry

- Epipolar plane: A plane containing the baseline.
- There is a one parameter family , or a pencil, of epipolar planes
- Epipolar line is the intersection of an epipolar plane with the image plane
- All epipolar lines intersect at the epipole
- An epipolar plane intersects the left and right image planes in epipolar lines, and defines the correspondence between the lines.



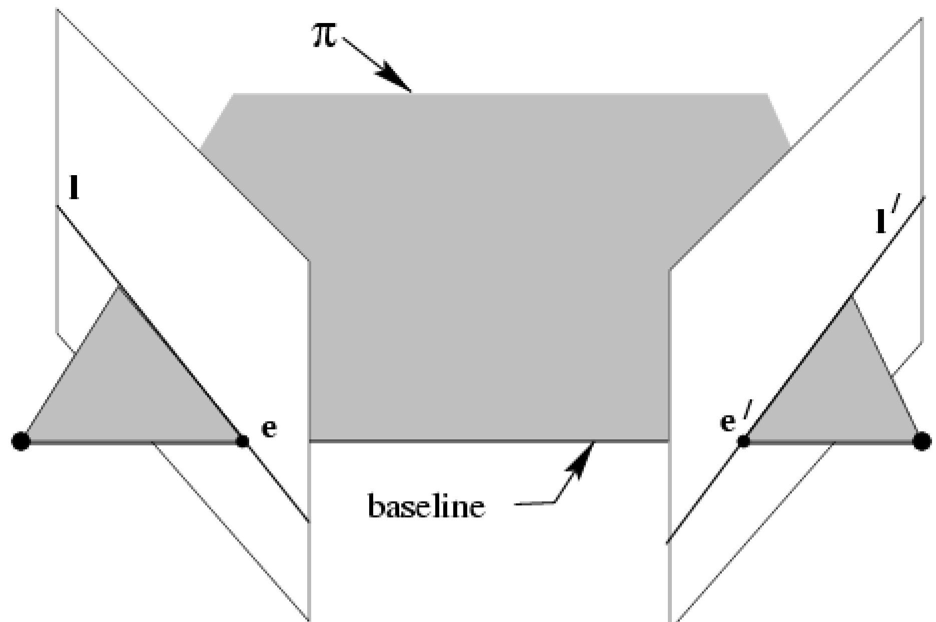
b

Family of planes π and lines l and l'
Intersection in e and e'

Fig. 9.2. **Epipolar geometry.** *(a) The camera baseline intersects each image plane at the epipoles* $\mathbf{e}$ *and* $\mathbf{e}'$. *Any plane* π *containing the baseline is an epipolar plane, and intersects the image planes in corresponding epipolar lines* $\mathbf{l}$ *and* $\mathbf{l}'$. *(b) As the position of the 3D point* $\mathbf{X}$ *varies, the epipolar planes "rotate" about the baseline. This family of planes is known as an epipolar pencil. All epipolar lines intersect at the epipole.*

The epipolar geometry

epipoles e,e’
= intersection of baseline with image plane
= projection of projection center in other image
= vanishing point of camera motion direction



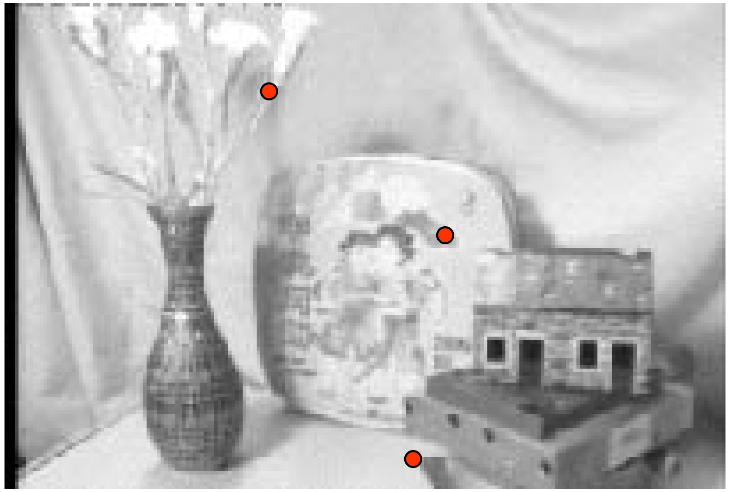
an epipolar plane = plane containing baseline (1-D family)

an epipolar line = intersection of epipolar plane with image
(always come in corresponding pairs)

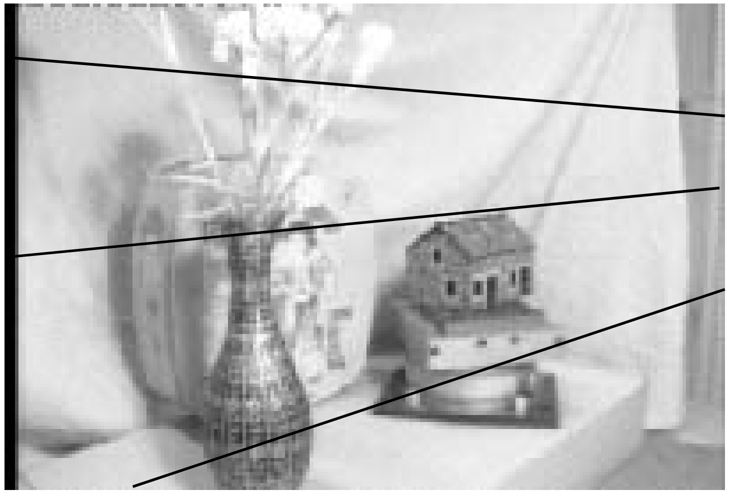
Two views of a collection of objects

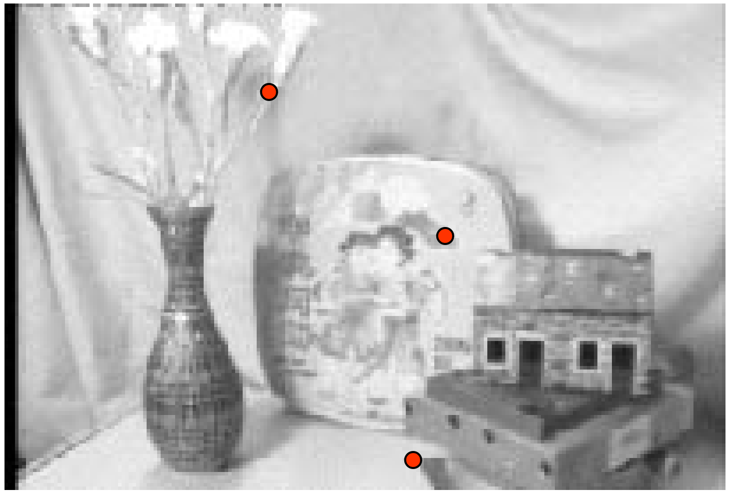
Assume that the full camera calibration is known. That is the extrinsic parameters of both cameras are known.

Some points have been identified in the right hand view

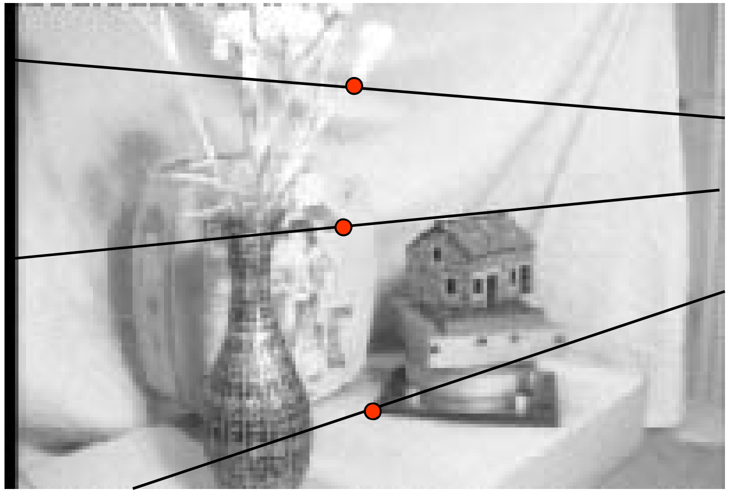


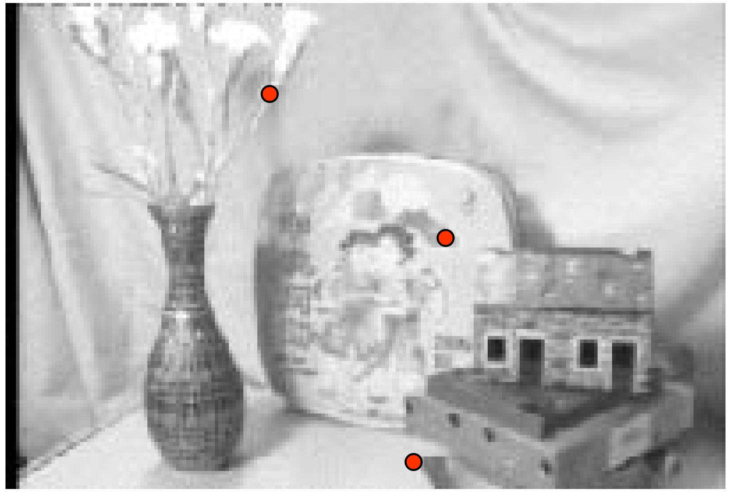
These points define epipolar lines in the left hand view



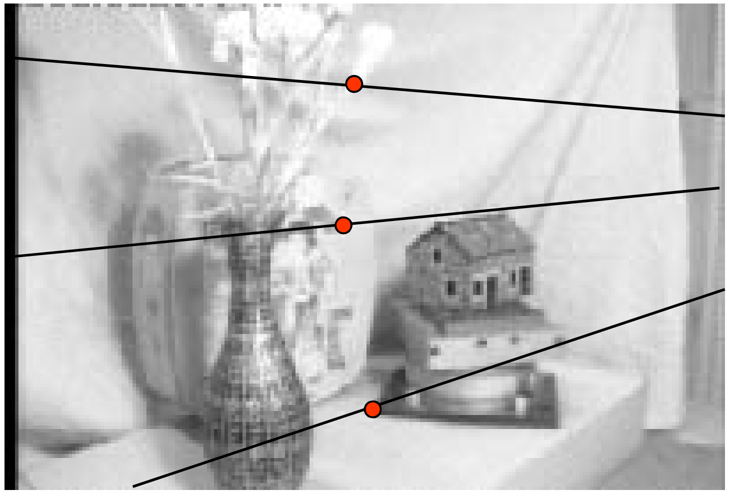


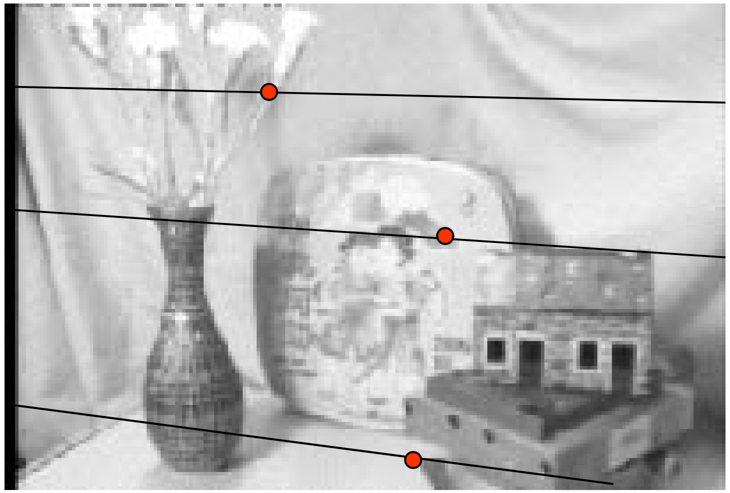
The corresponding points are located on the epipolar lines in the left hand view





The points in the left hand view in turn define epipolar lines in the right hand view, and these lines pass through the points in the right hand view.





Epipolar geometry is

- dependant only on the (internal and external) camera parameters.
- independent of the 3D structure of a scene.

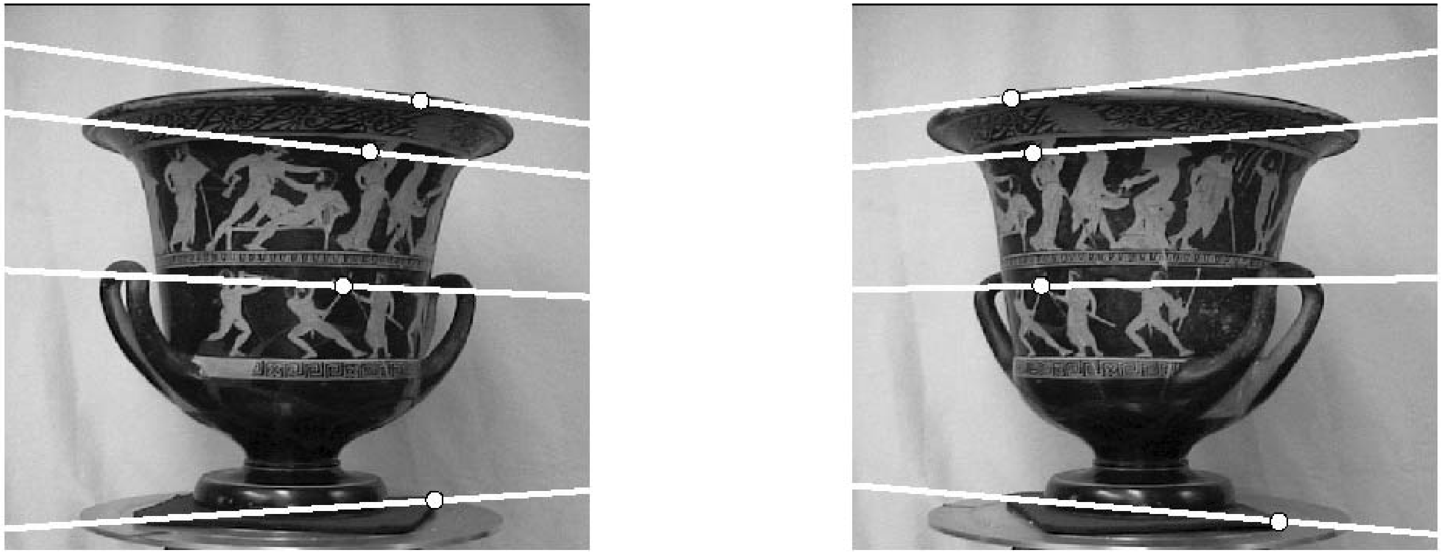
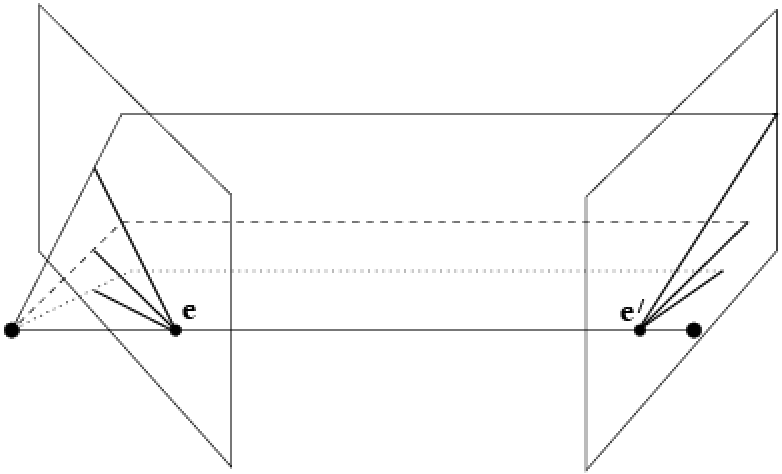


Figure courtesy of Richard Hartley, from *Multiple View Geometry in Computer Vision*, Hartley and Zisserman, Cambridge, 2000.

Example: converging cameras



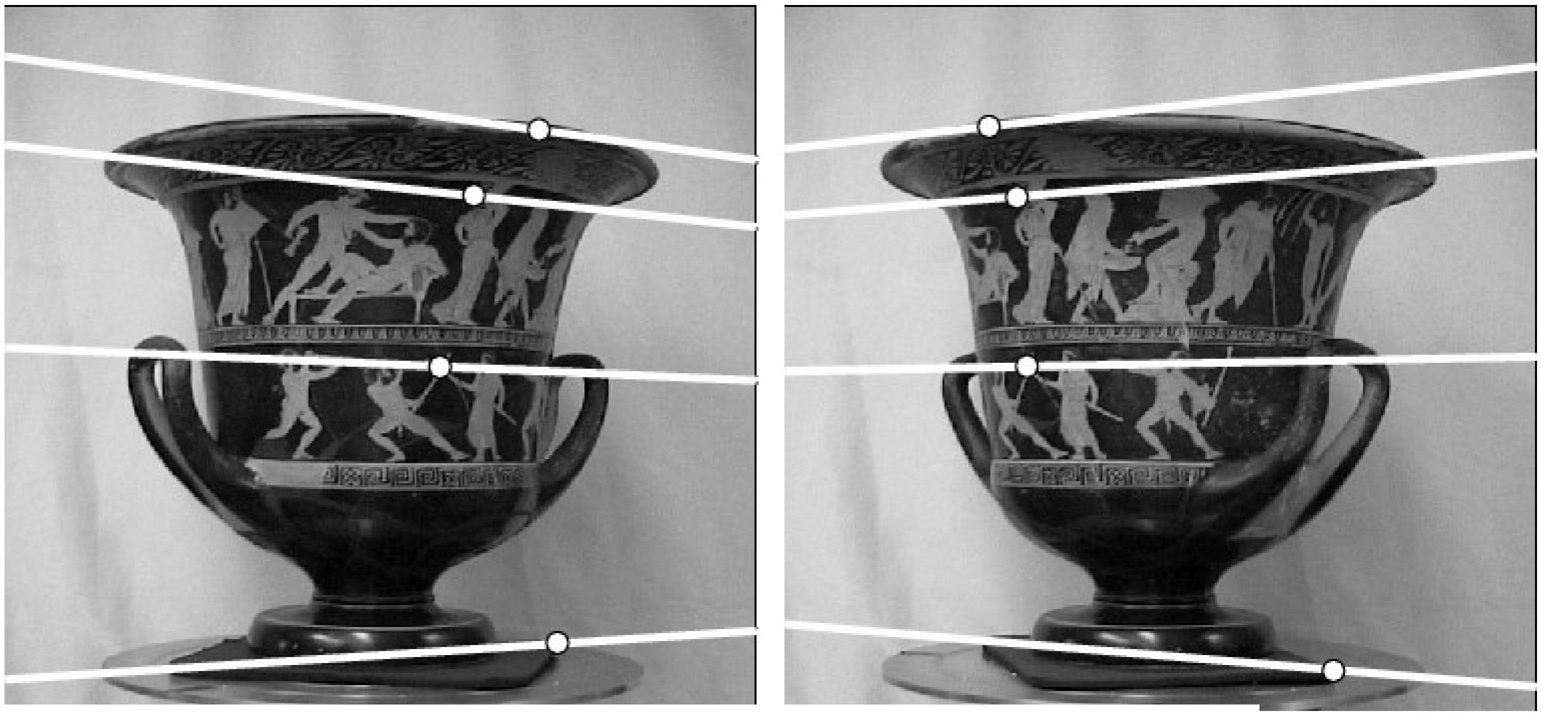
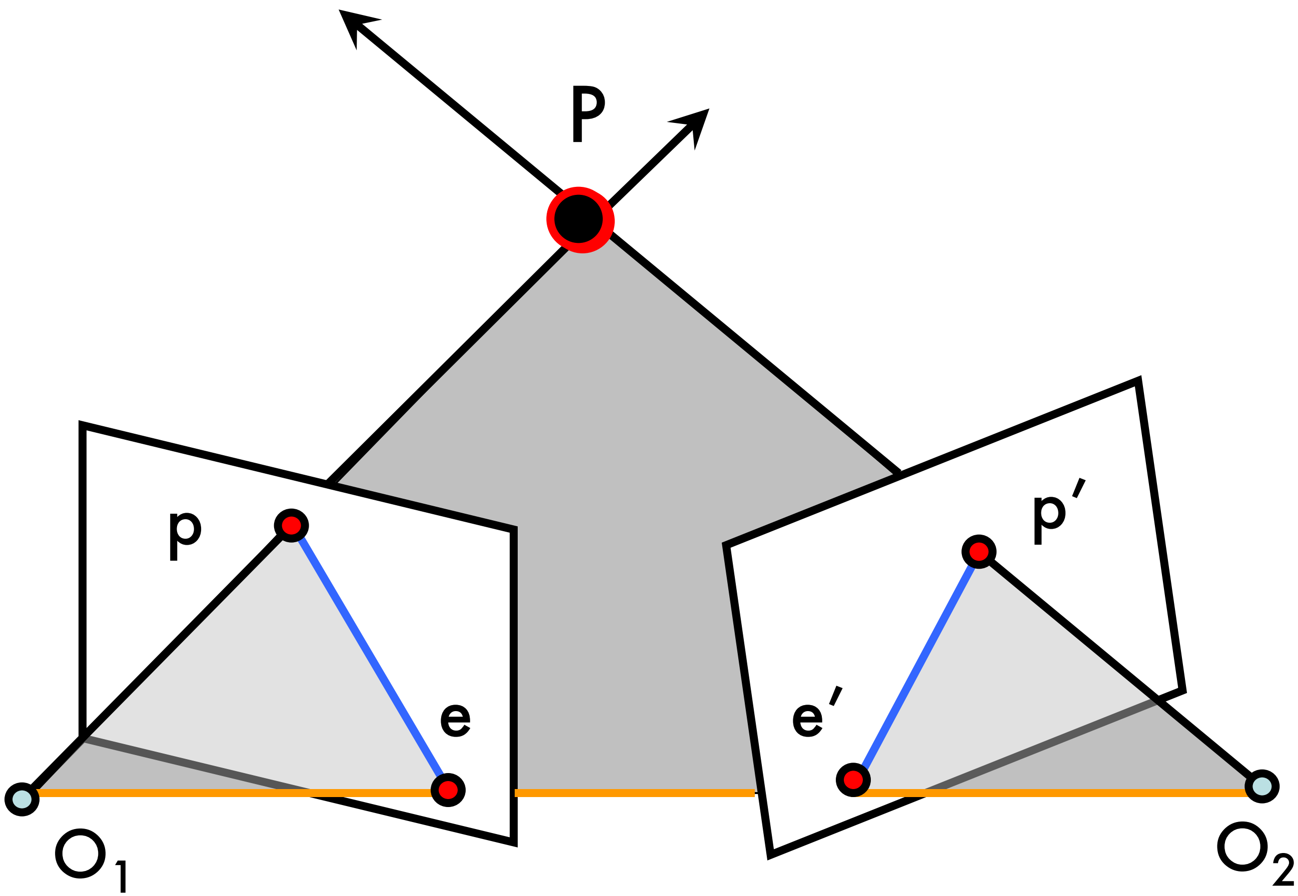


Fig. 9.3. **Converging cameras.** *(a) Epipolar geometry for converging cameras. (b) and (c) A pair of images with superimposed corresponding points and their epipolar lines (in white). The motion between the views is a translation and rotation. In each image, the direction of the other camera may be inferred from the intersection of the pencil of epipolar lines. In this case, both epipoles lie outside of the visible image.*

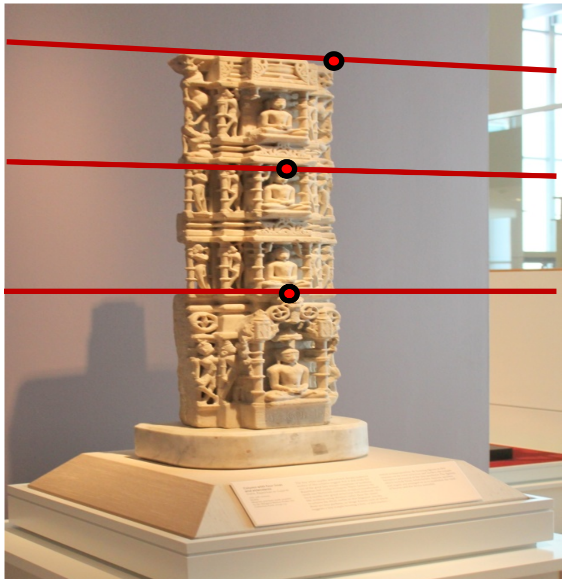
Epipolar geometry

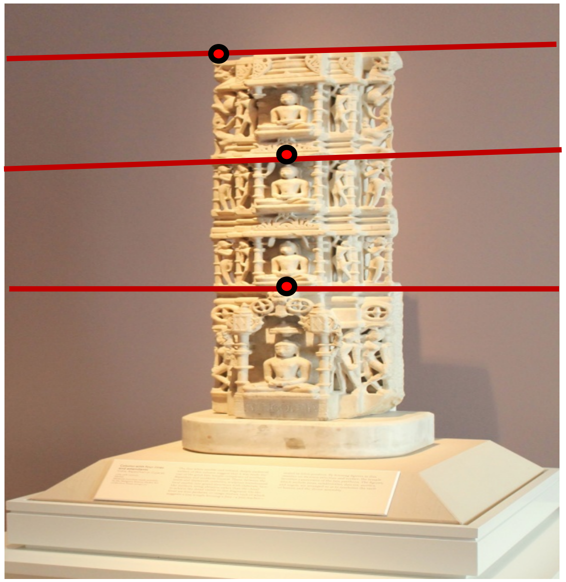


- Epipolar Plane
- Baseline
- Epipolar Lines
- Epipoles e, e′

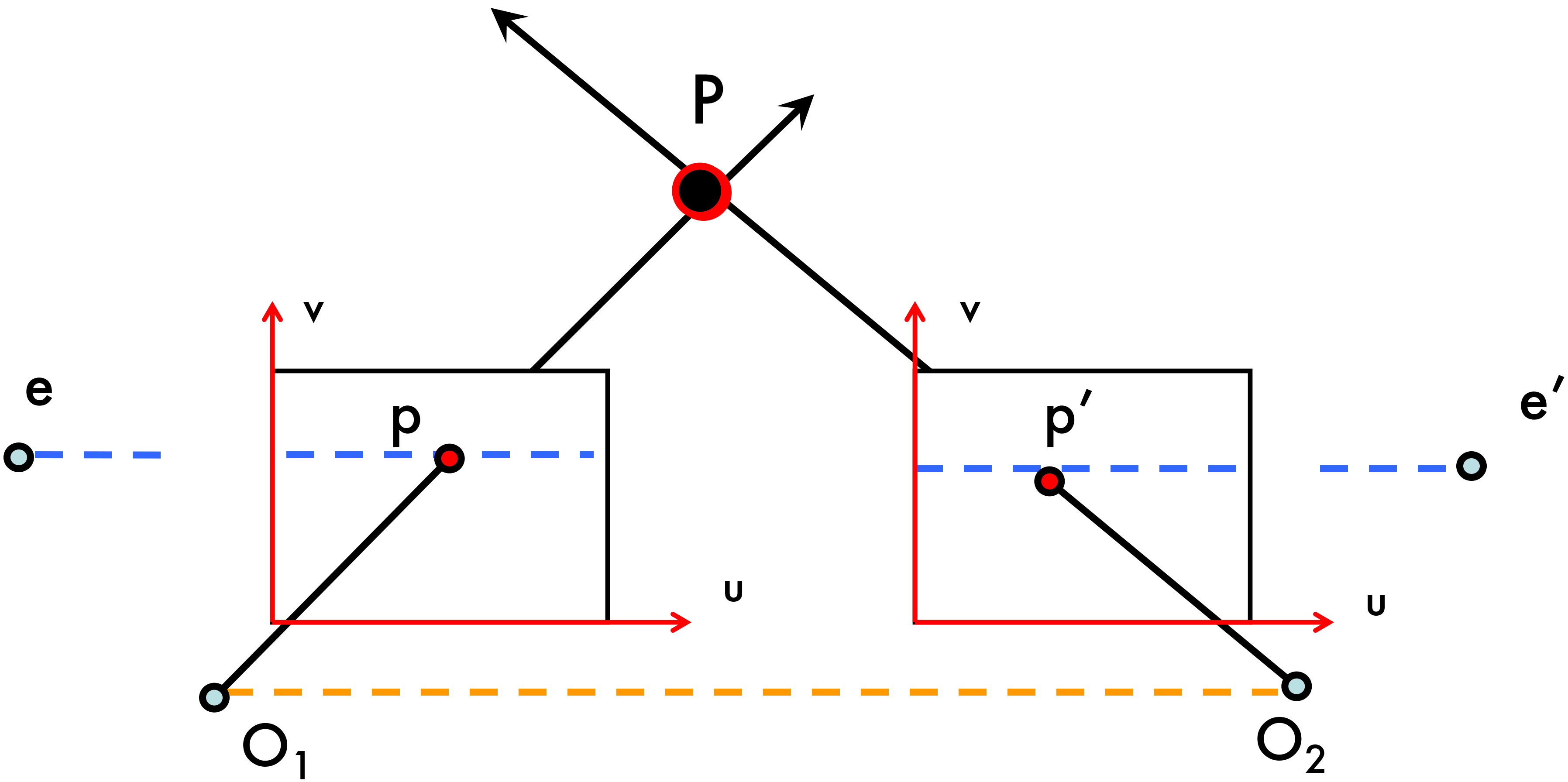
  = intersections of baseline with image planes
  = projections of the other camera center

Example of epipolar lines





Example: Parallel image planes



- Baseline intersects the image plane at infinity
- Epipoles are at infinity
- Epipolar lines are parallel to u axis

Example: Parallel Image Planes

Example: motion parallel with image plane

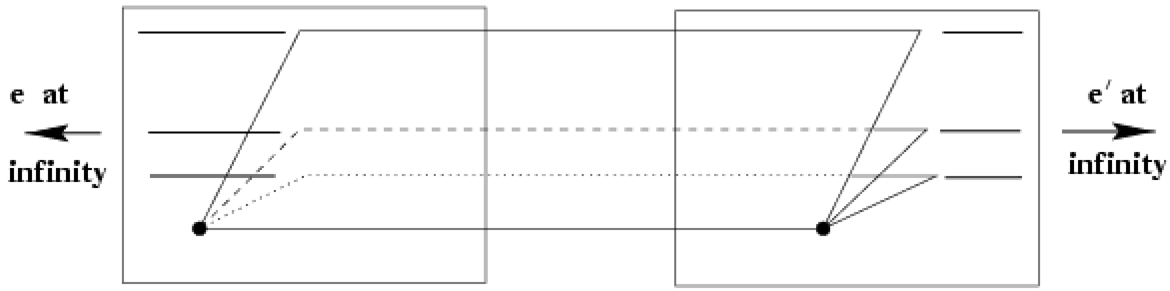
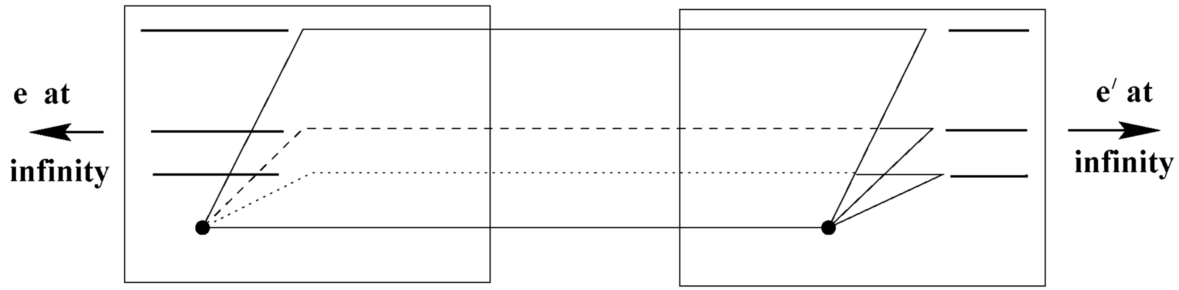


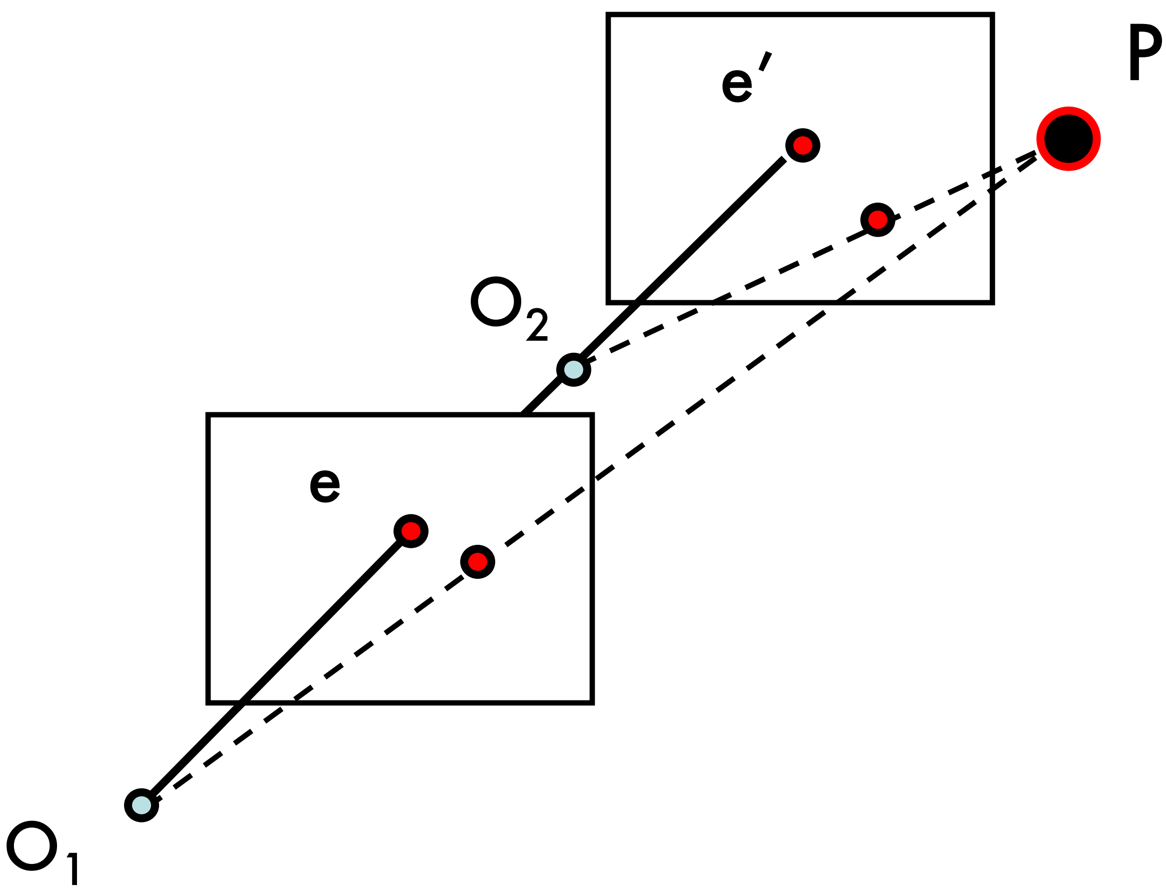
Fig. 9.4. **Motion parallel to the image plane.** *In the case of a special motion where the translation is parallel to the image plane, and the rotation axis is perpendicular to the image plane, the intersection of the baseline with the image plane is at infinity. Consequently the epipoles are at infinity, and epipolar lines are parallel. (a) Epipolar geometry for motion parallel to the image plane. (b) and (c) a pair of images for which the motion between views is (approximately) a translation parallel to the x-axis, with no rotation. Four corresponding epipolar lines are superimposed in white. Note that corresponding points lie on corresponding epipolar lines.*

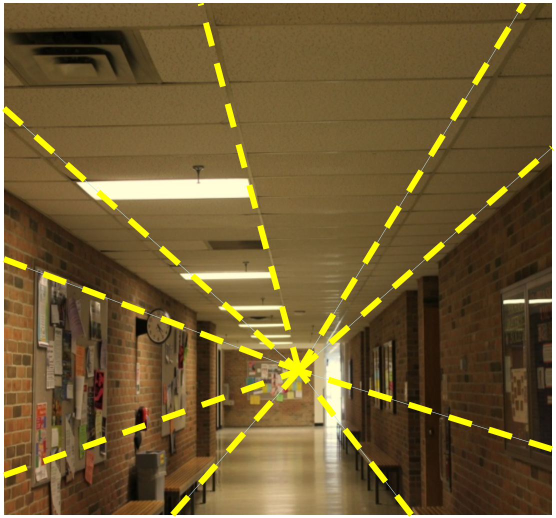
Epipolar geometry example

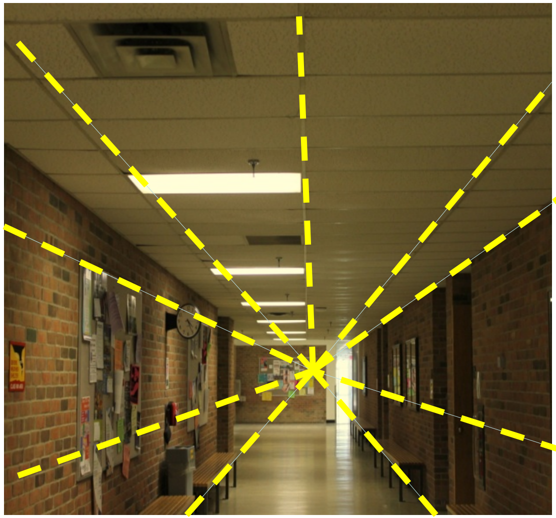


Epipolar geometry depends only on the relative pose (position and orientation) and internal parameters of the two cameras, i.e. the position of the camera centres and image planes. It does not depend on structure (3D points external to the camera).

Example: Forward translation





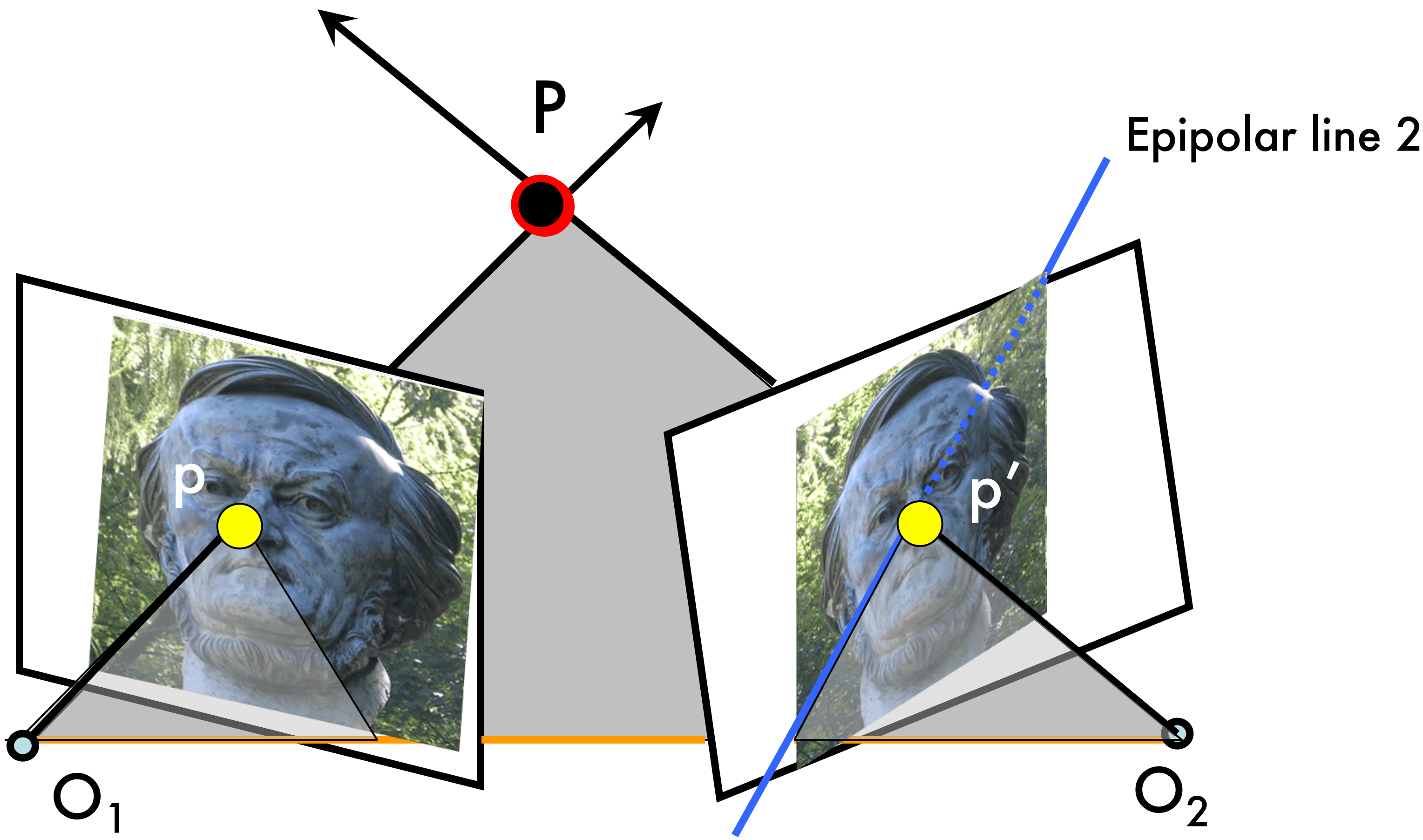


- The epipoles have same position in both images
- Epipole called FOE (focus of expansion)

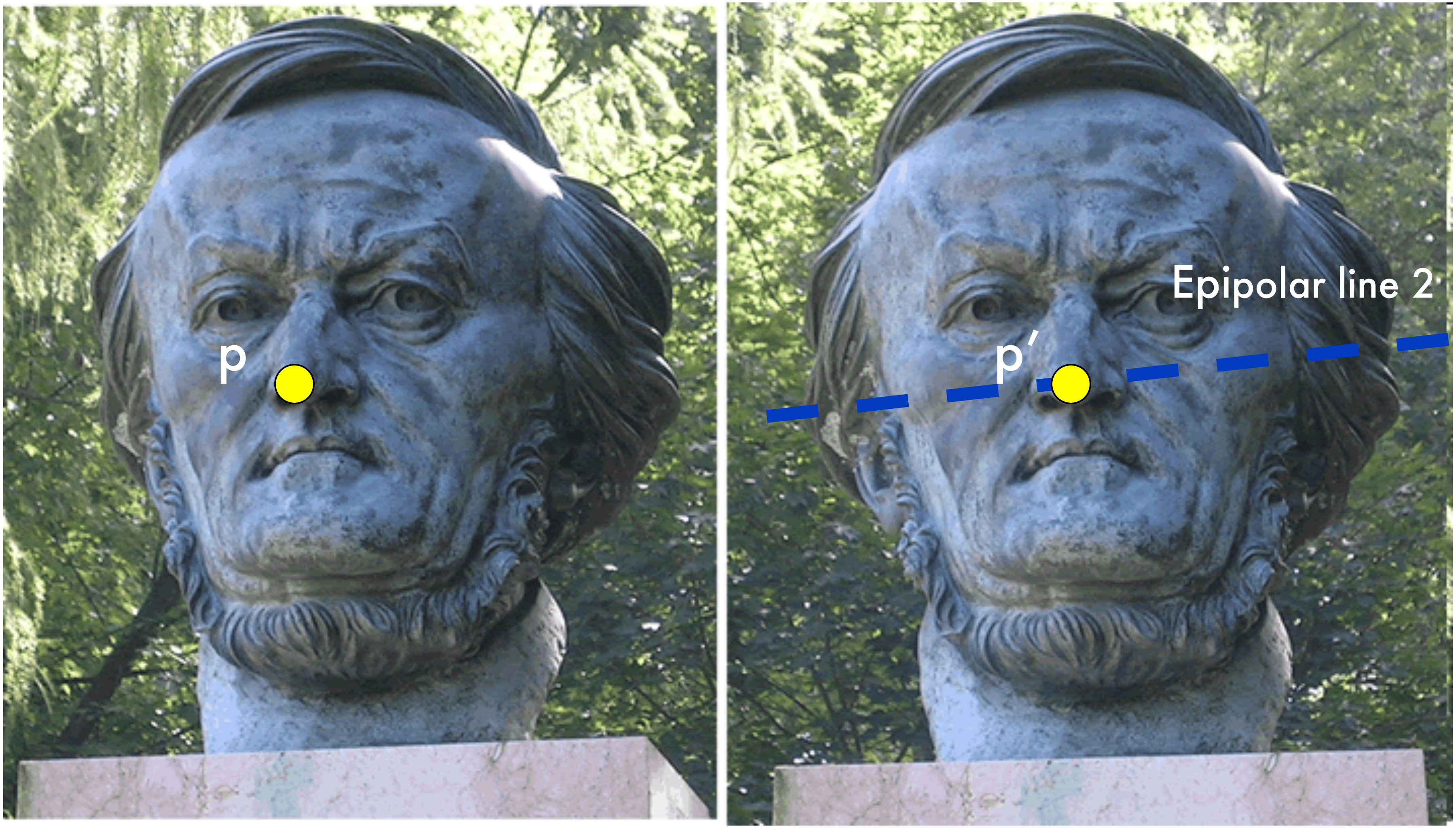
Epipolar Constraint

- Two views of the same object
- Given a point on left image, how can I find the corresponding point on right image?

Epipolar geometry



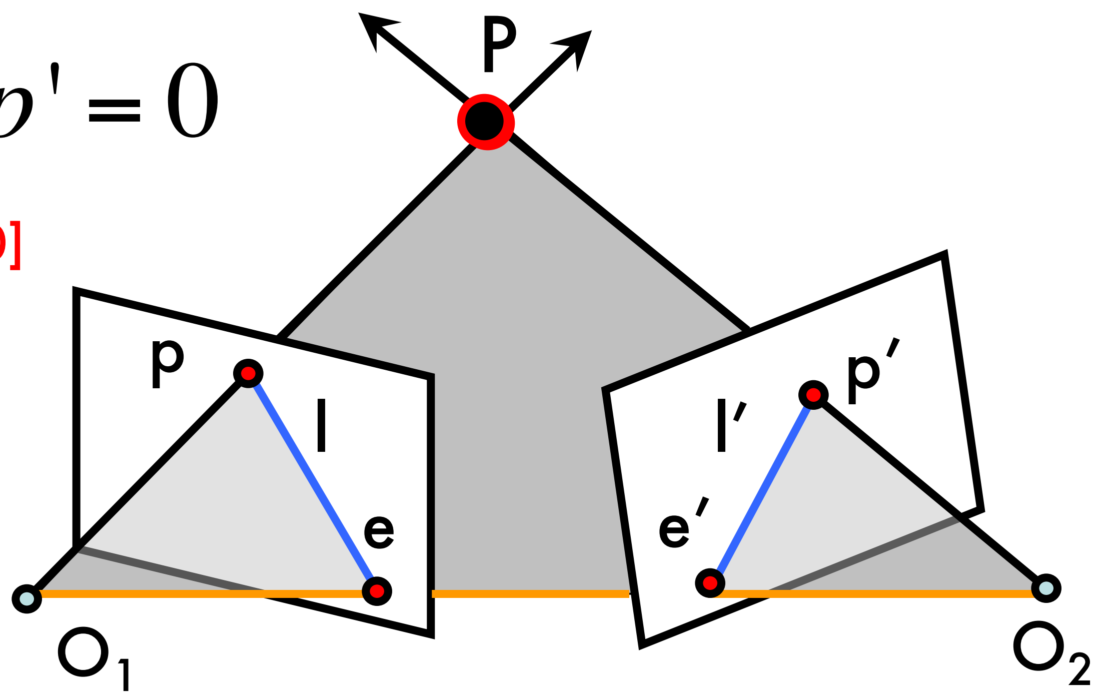
Epipolar Constraint



Epipolar Constraint

$$p^T \cdot E\, p' = 0$$

[Eq. 10]



- l = E p′ is the epipolar line associated with p′
- l′ = E^T p is the epipolar line associated with p
- E e′ = 0 and E^T e = 0
- E is 3x3 matrix; 5 DOF
- E is singular (rank two)

Fundamental Matrix

9.2 *The fundamental matrix* F 243

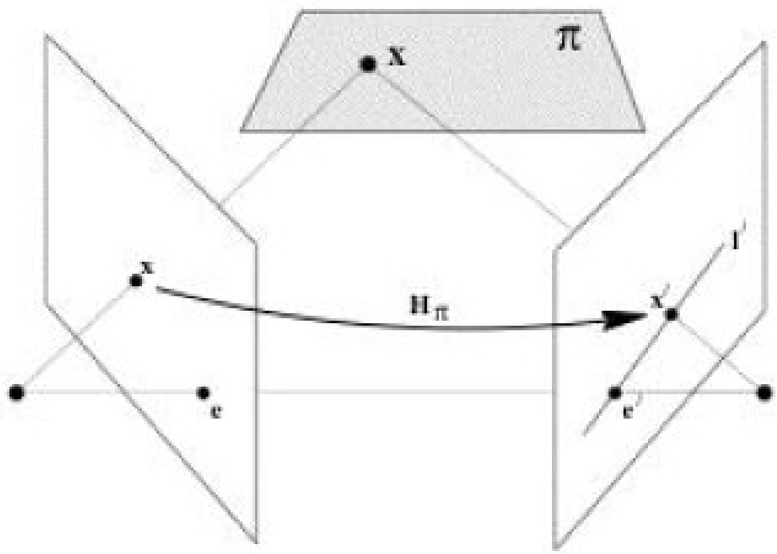


Fig. 9.5. *A point* $\mathbf{x}$ *in one image is transferred via the plane* π *to a matching point* $\mathbf{x}'$ *in the second image. The epipolar line through* $\mathbf{x}'$ *is obtained by joining* $\mathbf{x}'$ *to the epipole* $\mathbf{e}'$*. In symbols one may write* $\mathbf{x}' = \mathtt{H}_\pi \mathbf{x}$ *and* $\mathbf{l}' = [\mathbf{e}']_\times \mathbf{x}' = [\mathbf{e}']_\times \mathtt{H}_\pi \mathbf{x} = \mathtt{F}\mathbf{x}$ *where* $\mathtt{F} = [\mathbf{e}']_\times \mathtt{H}_\pi$ *is the fundamental matrix.*

F is a projective mapping x → l' from a point x in one image to its
Corresponding epipolar line in the other image

l' = Fx

The fundamental matrix F

algebraic representation of epipolar geometry

$$\mathrm{x} \mapsto \mathrm{l}'$$

we will see that mapping is (singular) correlation
(i.e. projective mapping from points to lines)
represented by the fundamental matrix F

Skew Symmetric Matrix for a vector a

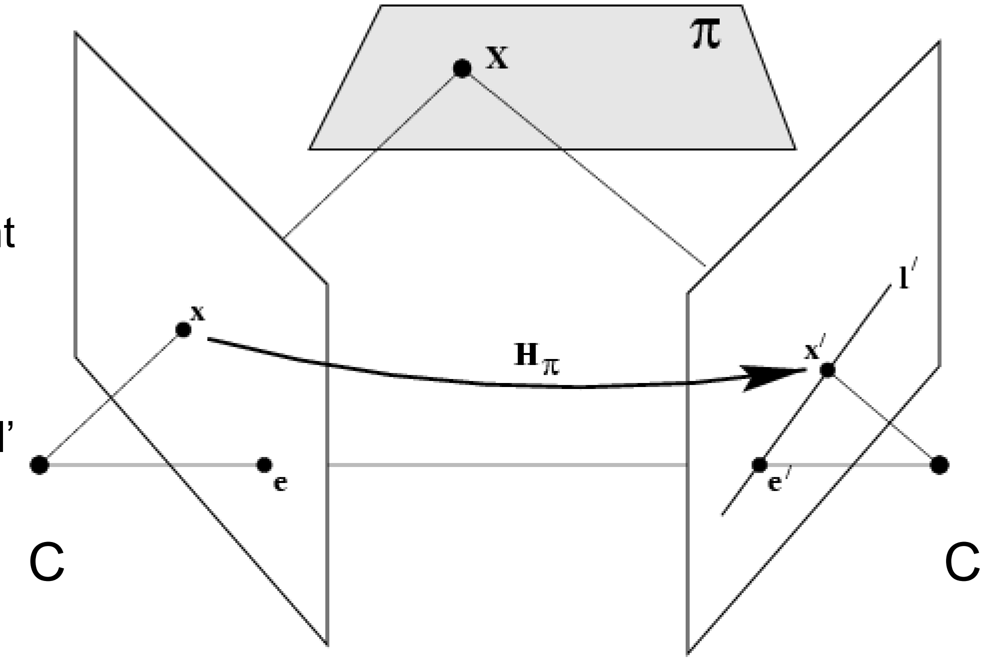
- $[a]_x$ is skew symmetric matrix for vector a
- If $a = (a_1 , a_2 , a_3)^T$ then,

$$[a]_x = \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix}$$

- Cross product between two vectors a and be can be written in terms of skew symmetric matrix for a:

$$a x b = [a]_x b$$

The fundamental matrix F
geometric derivation



- Plane π, not passing through either of the camera centers
- Ray through C corresponding to image point x, meets plane π in a point in 3D called X.
- Project X to a point x' in the second image
- "Transfer via the plane π".
- l' is the epipolar line for x ➔ x' must like on l'

$$\mathrm{l}' = \mathrm{e}' \times \mathrm{x}'$$

- x and x' are projectively equivalent to the planar point set X_i
- There is a 2D homography H_π mapping each x_i to x'_i

$$\mathrm{x}' = \mathrm{H}_\pi \mathrm{x}$$

$$\mathrm{l}' = \mathrm{e}' \times \mathrm{x}' = [\mathrm{e}']_\times \mathrm{H}_\pi \mathrm{x} = \mathrm{F}\mathrm{x}$$

mapping from 2-D to 1-D family (rank 2)

Result 9.1. *The fundamental matrix* F *may be written as* $\mathrm{F} = [\mathbf{e}']_\times \mathrm{H}_\pi$, *where* H_π *is the transfer mapping from one image to another via any plane* $\boldsymbol{\pi}$. *Furthermore, since* $[\mathbf{e}']_\times$ *has rank 2 and* H_π *rank 3,* F *is a matrix of rank 2.*

The fundamental matrix F

algebraic derivation

P^+ is pseudo inverse of P

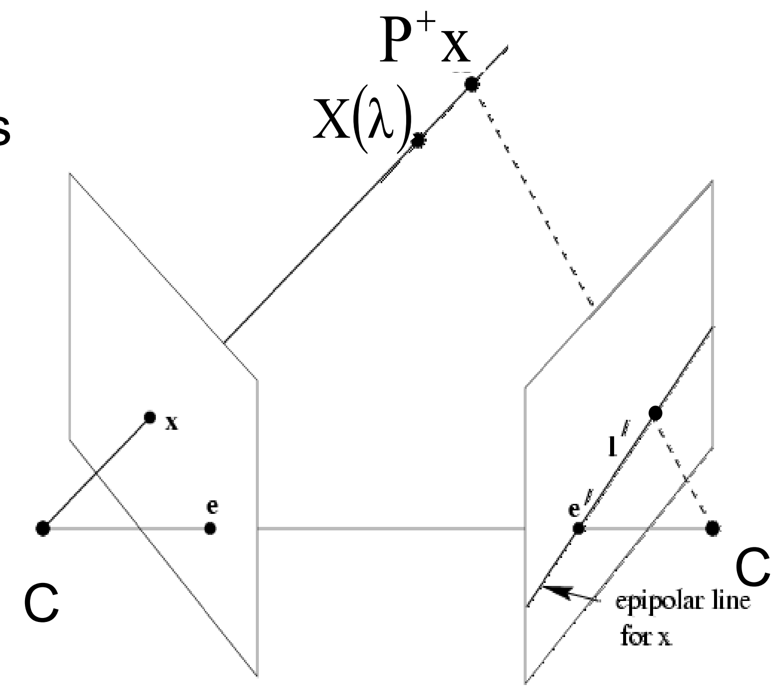
$$\left(P^+P = I\right)$$

$$l' = P'C \times P'P^+x$$

- Line l’ joints two points: can be written as cross product of those two points:
- First point is P’C which is e’
- Second point is projection P’ of P^+x onto second image plane

l’ = e’ cross product with (P’ P^+ x)

$$F = [e']_\times P'P^+$$



(note: doesn’t work for C=C’ $\Rightarrow$ F=0)

The fundamental matrix F

correspondence condition

The fundamental matrix satisfies the condition that for any pair of corresponding points $x \leftrightarrow x'$ in the two images

Combine these two: $\left(x'^T l' = 0\right)$ $l' = Fx$

$$x'^T F x = 0$$

Result 9.3. *The fundamental matrix satisfies the condition that for any pair of corresponding points* $\mathbf{x} \leftrightarrow \mathbf{x}'$ *in the two images*

$$\mathbf{x}'^{\mathsf{T}} F \mathbf{x} = 0.$$

- Upshot: A way of characterizing fundamental matrix without reference to camera matrices, i.e. only in terms of corresponding image points
- How many correspondences are needed find F? at least 7.

The fundamental matrix F

F is the unique 3x3 rank 2 matrix that satisfies $x'^T F x = 0$ for all $x \leftrightarrow x'$

(i) **Transpose:** if F is fundamental matrix for (P,P'), then F^T is fundamental matrix for (P',P)

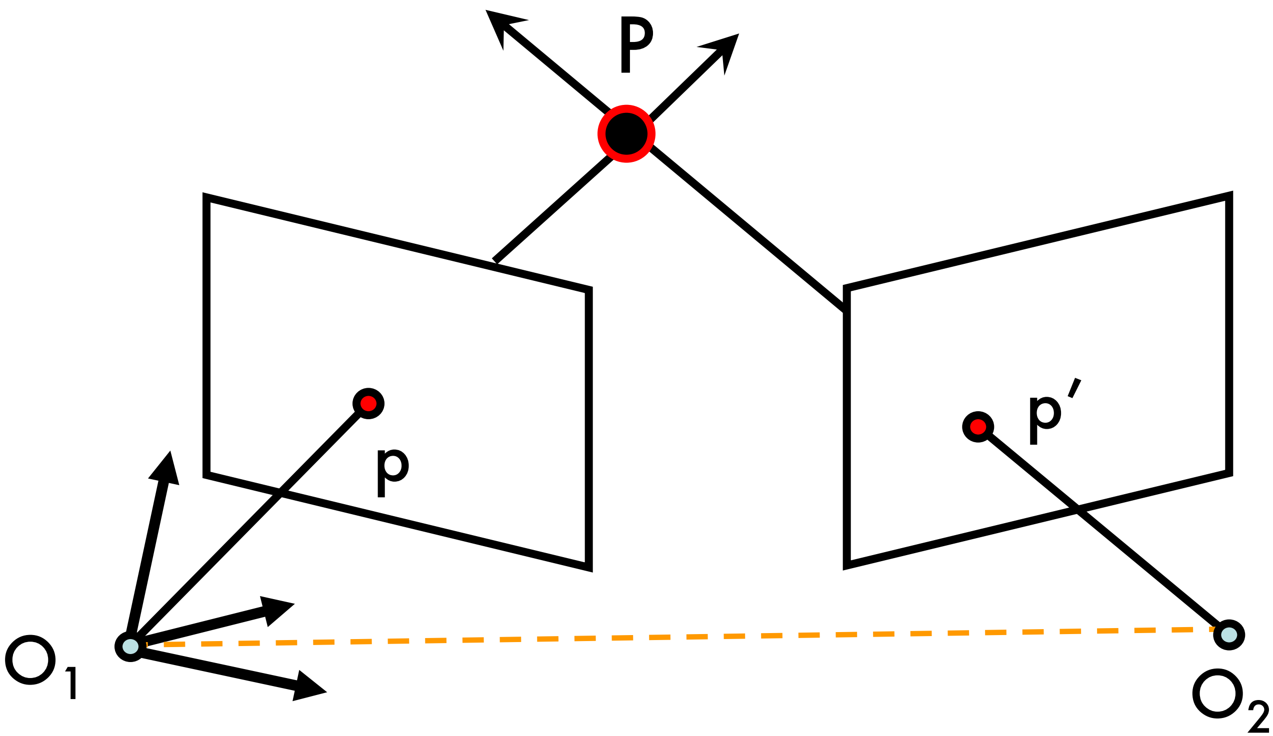
(ii) **Epipolar lines:** for any point x in the first image, the corresponding epipolar line is $l' = Fx$; same with converse: $l = F^T x'$ represents the epipolar line corresponding to x' in the second image

(i) **Epipoles:** for any point x, the epipolar line $l' = Fx$ contains the epipole e'. Thus $e'^T F x = 0, \forall x \Rightarrow e'^T F = 0$; similarly $Fe = 0$
e' is the left null vector of F; e is the right null vector of F

(i) **F** has 7 d.o.f. , i.e. 3x3-1(homogeneous)-1(rank2)

(ii) **F** is a correlation, projective mapping from a point x to a line $l' = Fx$ (not a proper correlation, i.e. not invertible)
If l and l' are corresponding epipolar lines, then any point x on l is mapped to the same line l' ➔ no inverse mapping➔ F not proper correlation

Epipolar Constraint



[Eq. 13] $$\boxed{\mathrm{p}^{T}\,\mathrm{F}\,\mathrm{p}' = 0}$$

$$F = K^{-T} \cdot \left[T_{\times}\right] \cdot R\,K'^{-1}$$

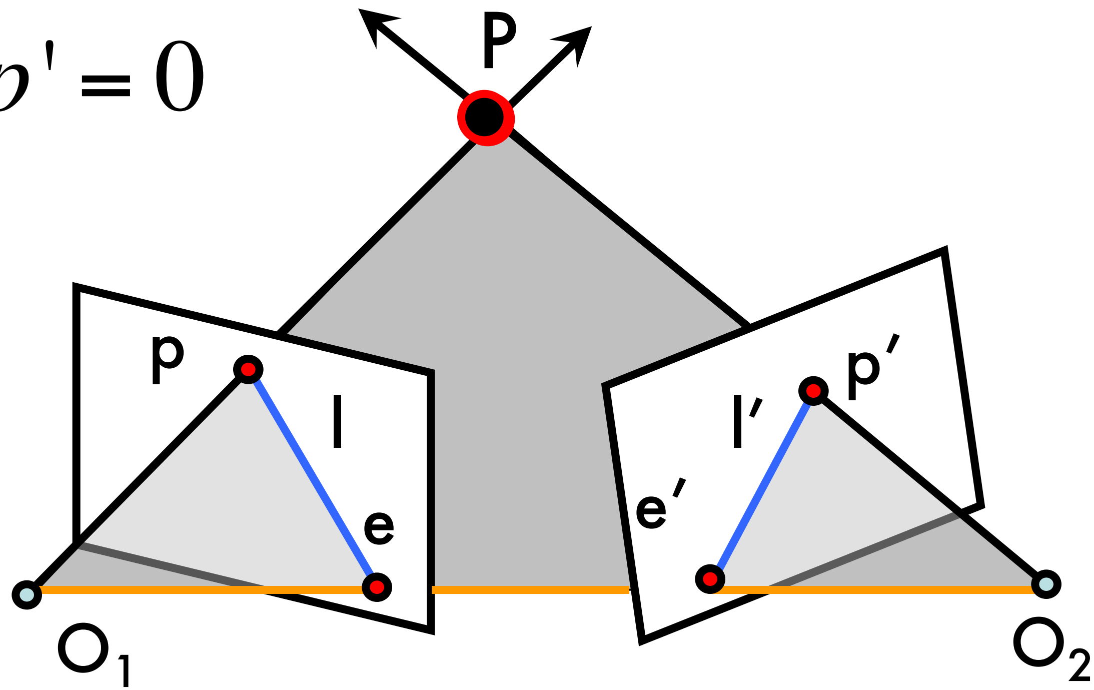
[Eq. 14]

F = Fundamental Matrix

(Faugeras and Luong, 1992)

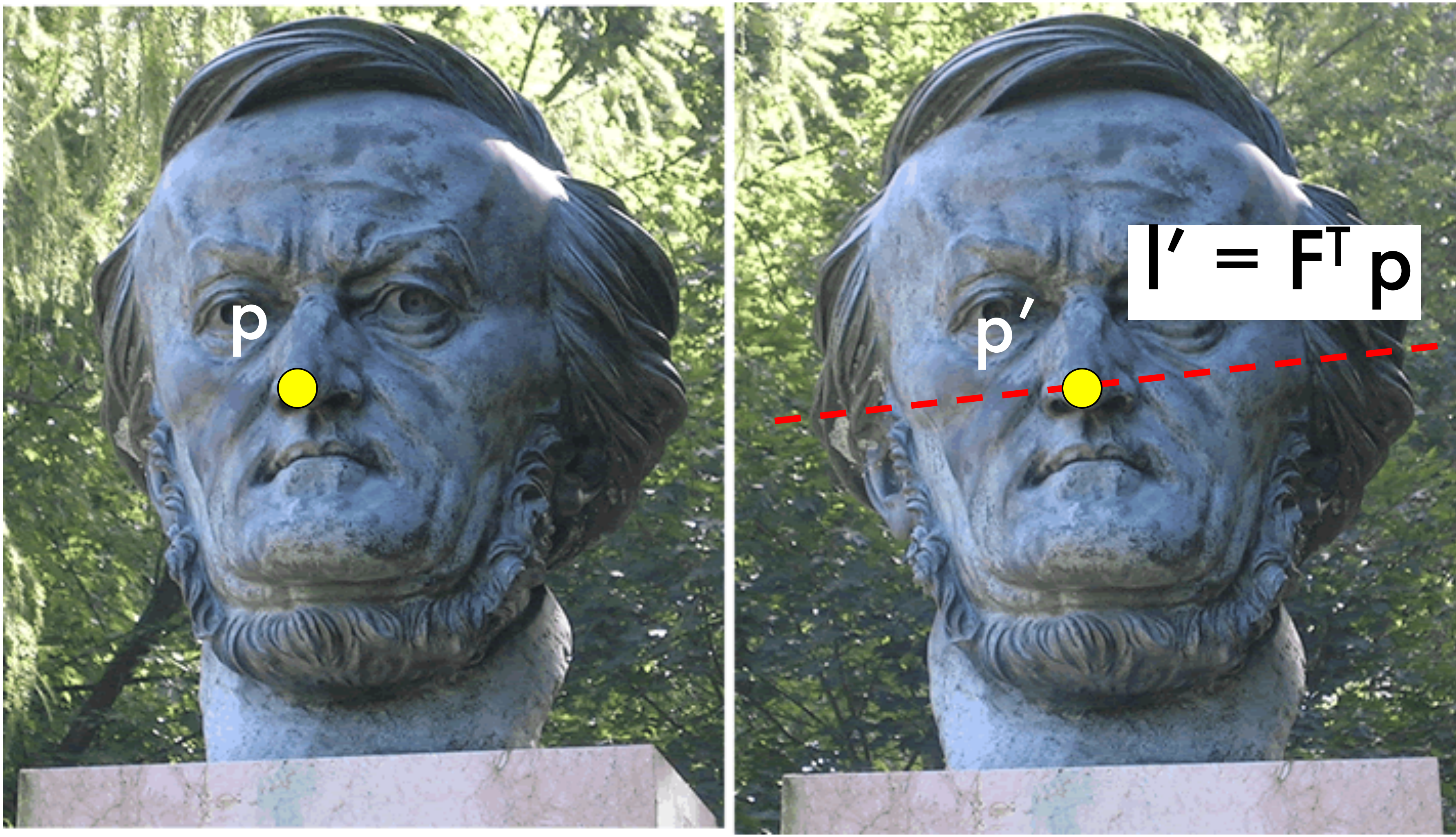
Epipolar Constraint

$$p^T \cdot F\, p' = 0$$



- l = F p′ is the epipolar line associated with p′
- l′= F^T p is the epipolar line associated with p
- F e′ = 0 and F^T e = 0
- F is 3x3 matrix; 7 DOF
- F is singular (rank two)

Why F is useful?



- Suppose F is known
- No additional information about the scene and camera is given
- Given a point on left image, we can compute the corresponding epipolar line in the second imag

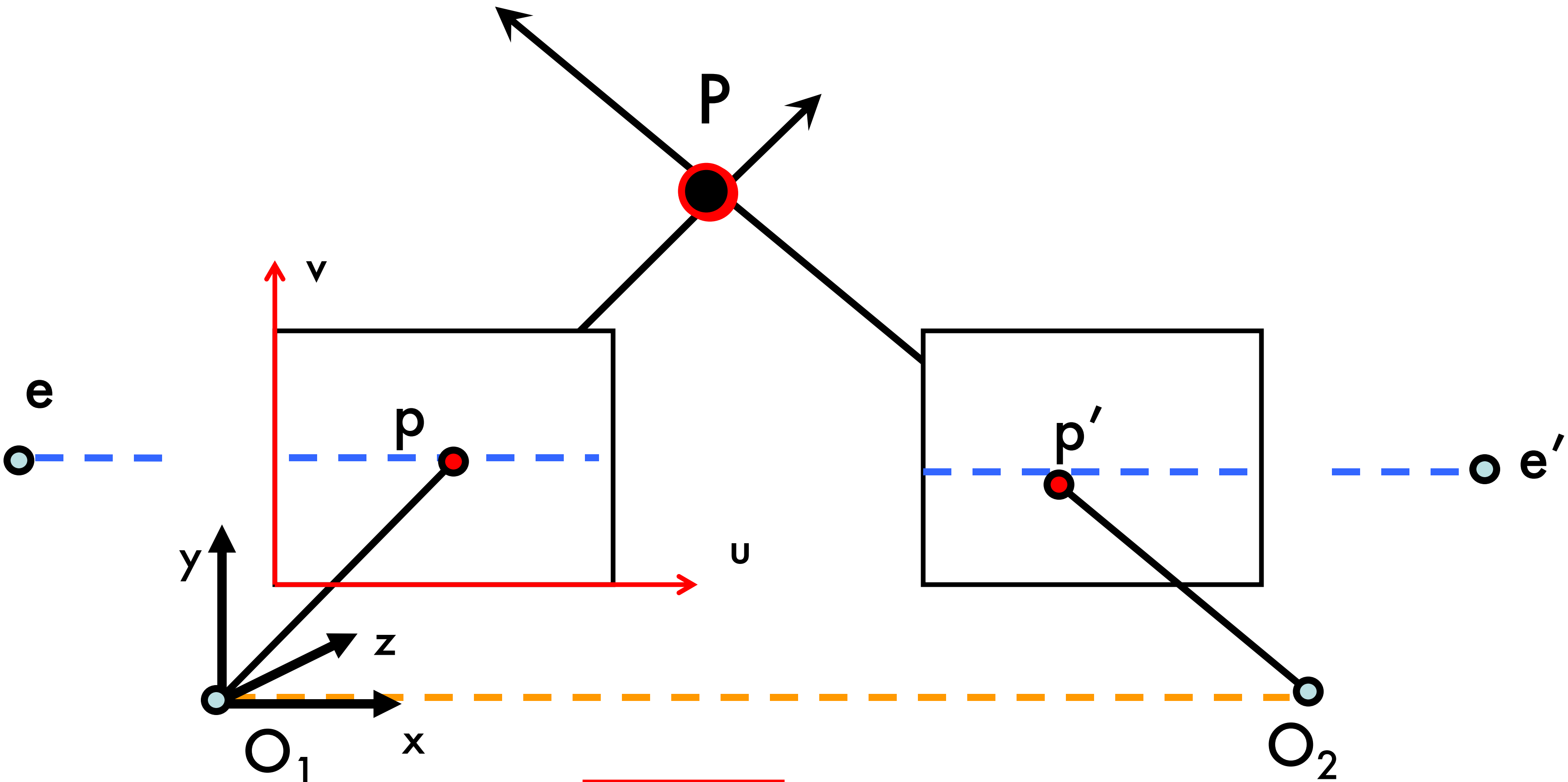
Why F is useful?

- F captures information about the epipolar geometry of 2 views + camera parameters

- MORE IMPORTANTLY: F gives constraints on how the scene changes under view point transformation (without reconstructing the scene!)

- Powerful tool in:
  - 3D reconstruction
  - Multi-view object/scene matching

Example: Parallel image planes



$K_1=K_2$ = known

x parallel to O_1O_2

$\boxed{E=?}$

Hint :

$\boldsymbol{R} = I \qquad \boldsymbol{T} = (T, 0, 0)$

Essential matrix for parallel images

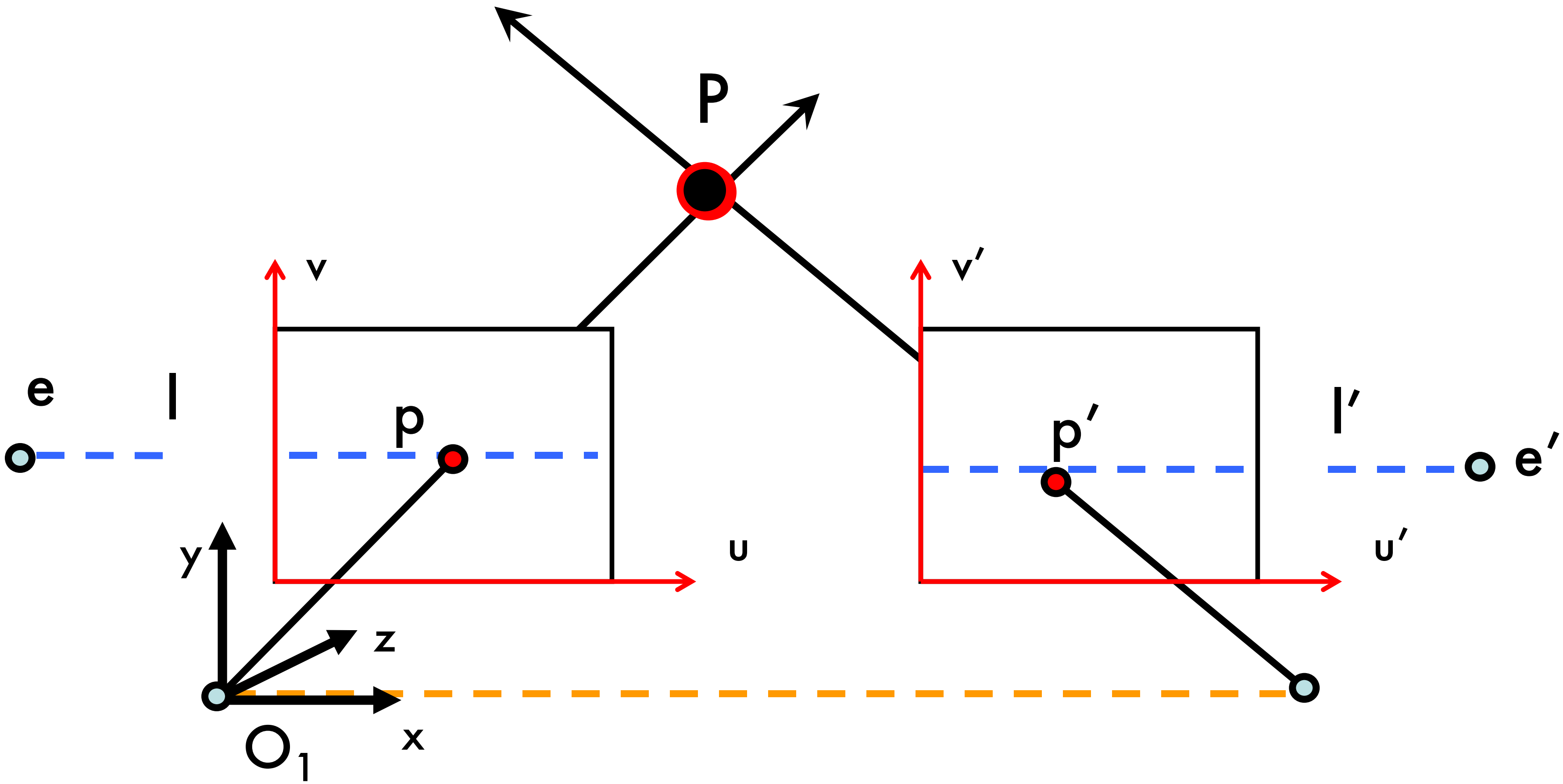
$$\mathbf{E} = [\mathbf{T}_\times] \cdot \mathbf{R}$$

$$\mathbf{E} = \begin{bmatrix} 0 & -T_z & T_y \\ T_z & 0 & -T_x \\ -T_y & T_x & 0 \end{bmatrix} \mathbf{R} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -T \\ 0 & T & 0 \end{bmatrix}$$

[Eq. 20]

$$\mathbf{T} = [\ \mathrm{T}\ \ 0\ \ 0\ ]$$
$$\mathbf{R} = \mathbf{I}$$

Example: Parallel image planes

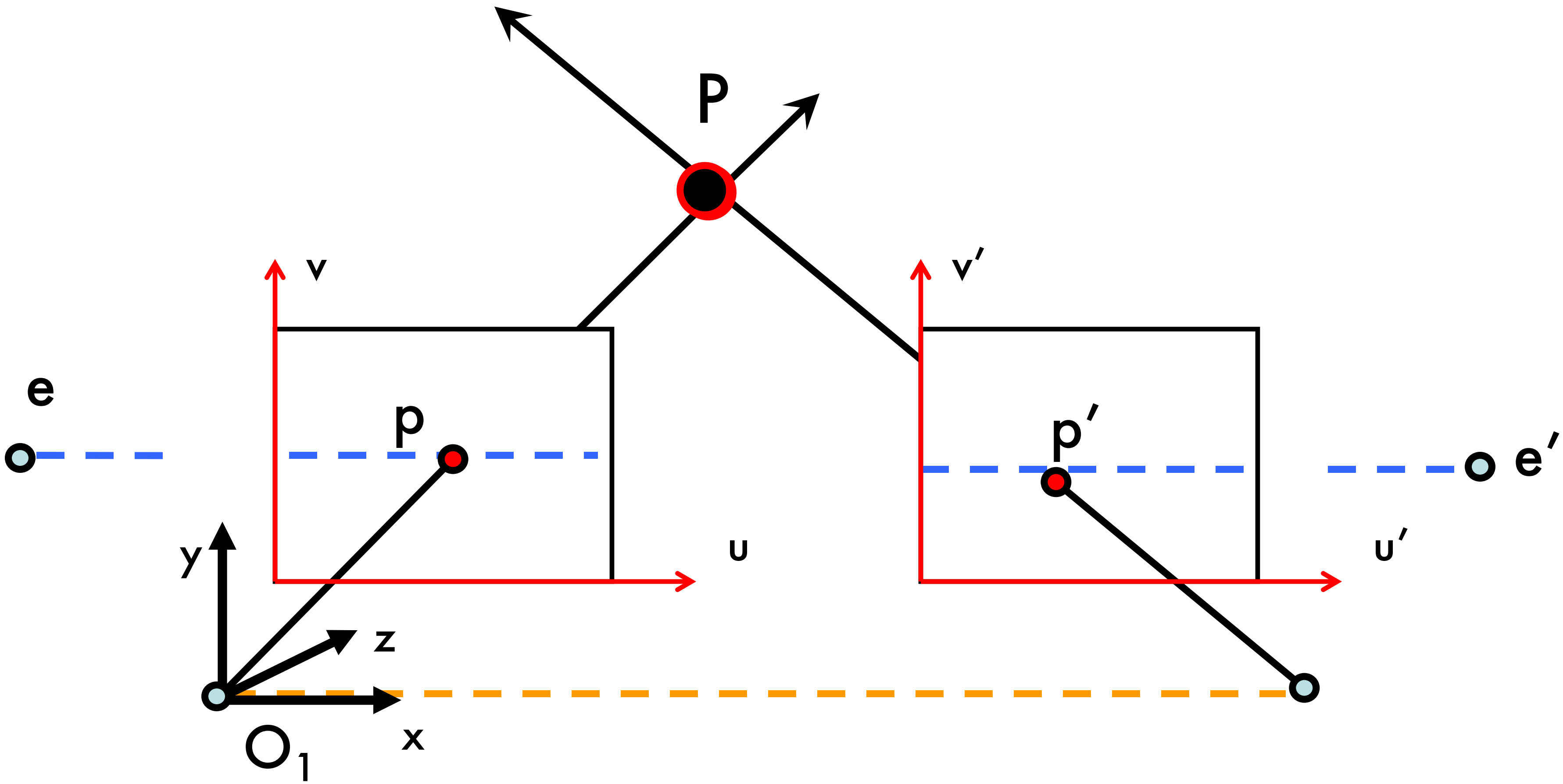


What are the directions of epipolar lines?

$$l = \mathrm{E}\, p' = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -T \\ 0 & T & 0 \end{bmatrix} \begin{bmatrix} u' \\ v' \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -T \\ T v' \end{bmatrix}$$

horizontal!

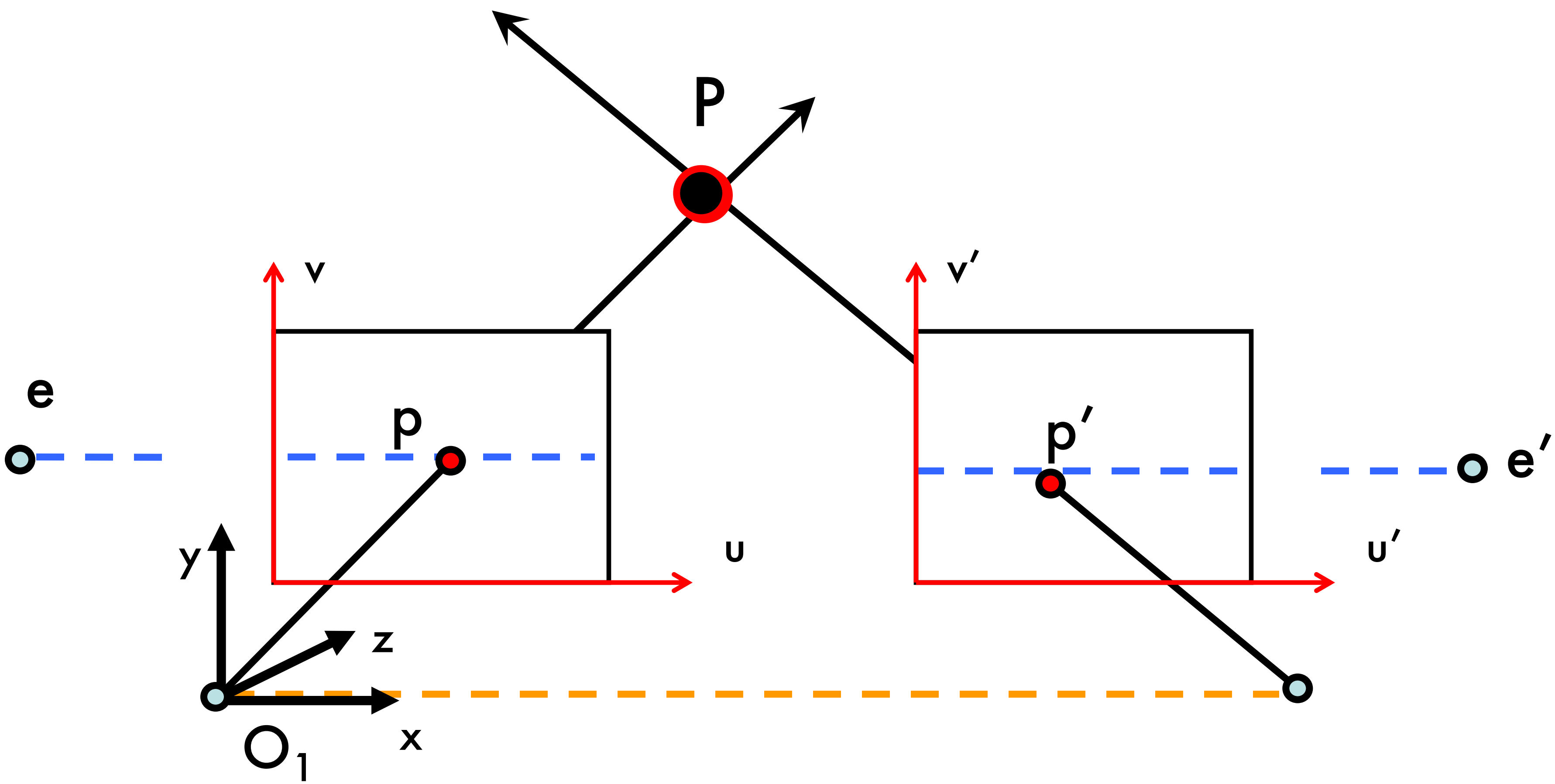
Example: Parallel image planes



How are p and p' related?

$$p^T \cdot E\, p' = 0$$

Example: Parallel image planes

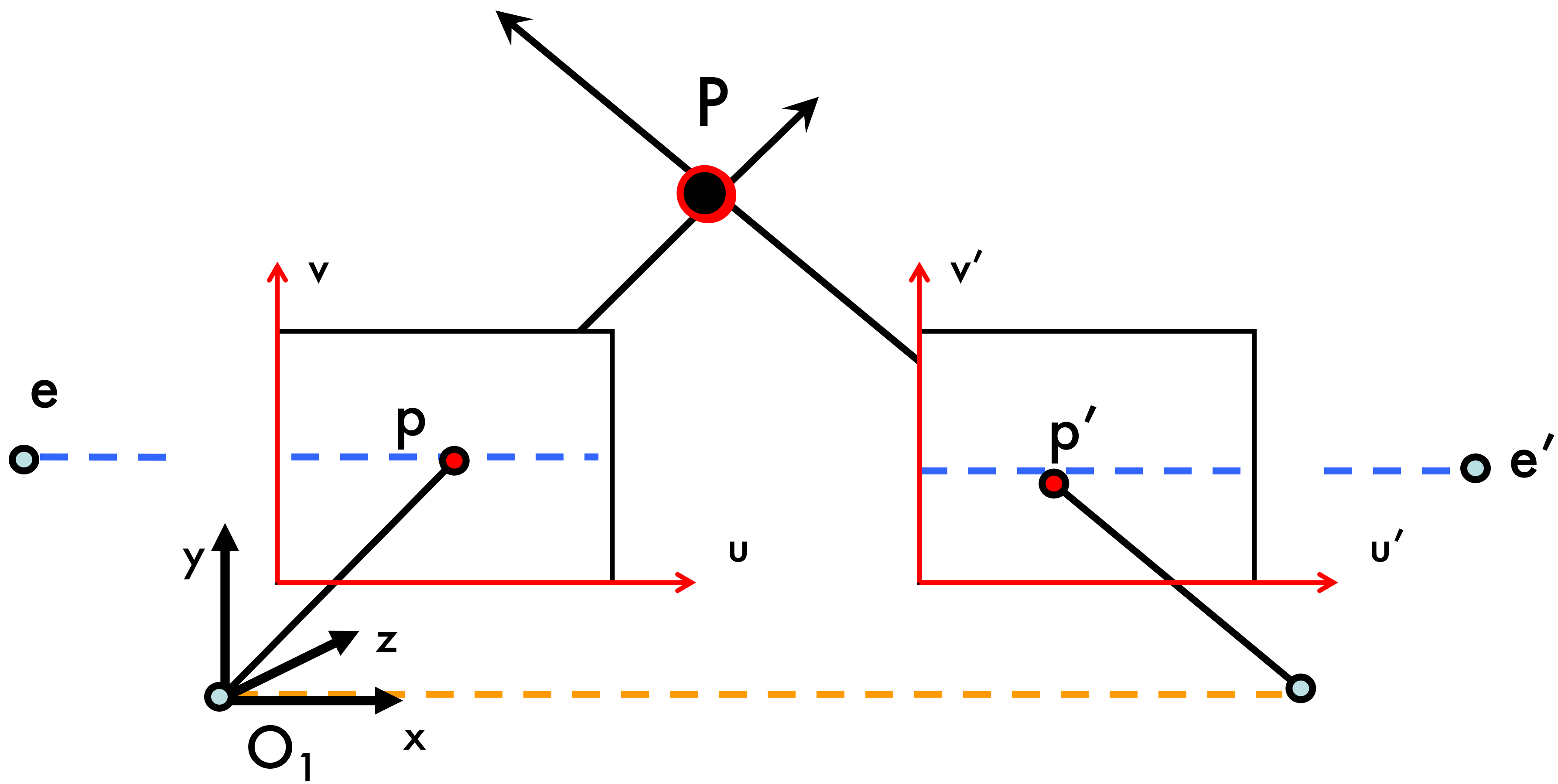


How are p and p′ related?

$$\Rightarrow \begin{pmatrix} u & v & 1 \end{pmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -T \\ 0 & T & 0 \end{bmatrix} \begin{pmatrix} u' \\ v' \\ 1 \end{pmatrix} = 0 \Rightarrow \begin{pmatrix} u & v & 1 \end{pmatrix} \begin{pmatrix} 0 \\ -T \\ Tv' \end{pmatrix} = 0 \Rightarrow Tv = Tv'$$

$$\Rightarrow v = v'$$

Example: Parallel image planes



Rectification: making two images "parallel"

Why it is useful?

- Epipolar constraint → v = v′
  - New views can be synthesized by linear interpolation

Application: view morphing

S. M. Seitz and C. R. Dyer, *Proc. SIGGRAPH 96*, 1996, 21-30