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Mixtilinear Incircles and Excircles

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Abstract: We use symbolic geometry to prove theorems about the radii of mixtilinear incircles and excircles

1. Radii of mixtilinear incircles and excircles

A mixtilinear incircle is tangent to 2 sides of a triangle and (internally) to the circumcircle. A mixtilinear excircle is tangent to 2 sides of a triangle and (externally) to the circumcircle. We show that the ratio of the radii of the mixtilinear excircle and the mixtilinear incircle is the same as the ratio of the corresponding excircle and the incircle of the triangle:
First the mixtilinear case:

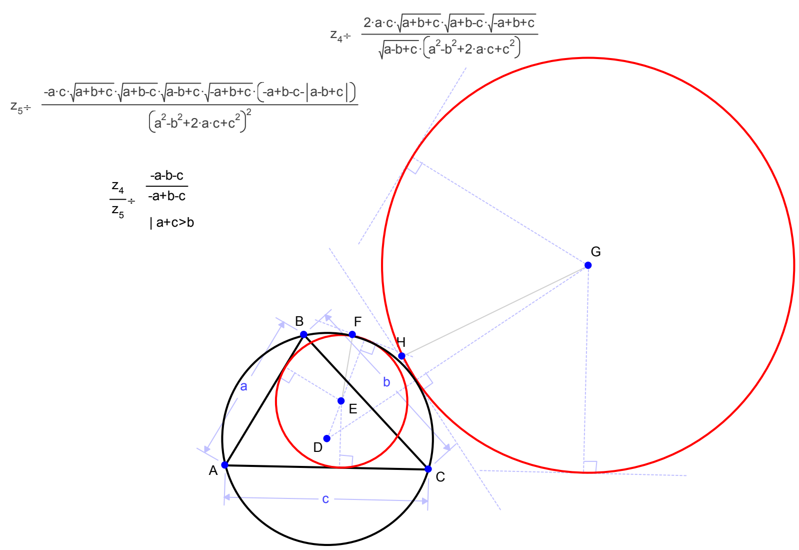

Figure 1: Radii of mixtilinear incircle and excircle and their ratio

Nor the ordinary incircle / excircle:

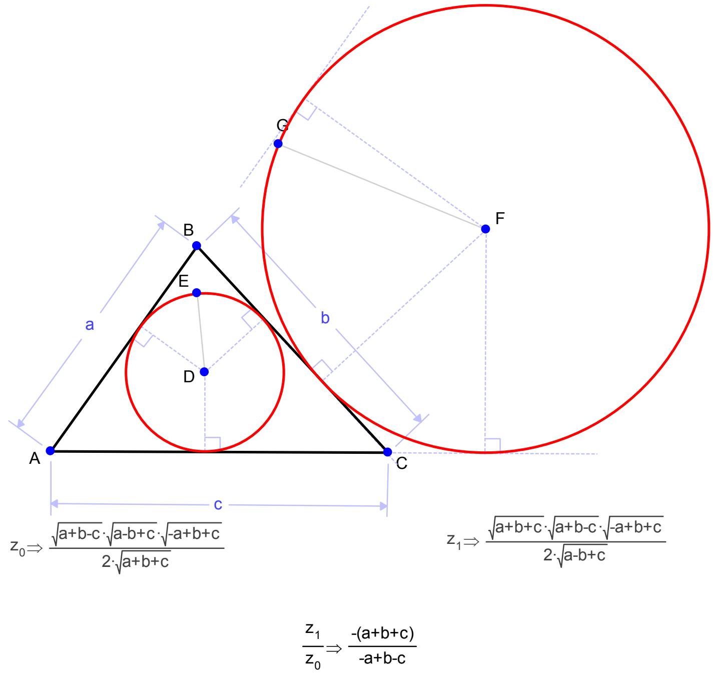

Figure 2: Radii of incircle and excircle and their ratio

If r is the radius of the incircle, and if s_1, s_2, s_3 are the radii of the excircles, we can easily see from result in fig. 2 that:

$$\frac{r}{s_0}+\frac{r}{s_1}+\frac{r}{s_2}=1$$

This result is more usually phrased:

$$\frac{1}{s_0}+\frac{1}{s_1}+\frac{1}{s_2}=\frac{1}{r}$$

If r_1, r_2, r_3 are the radii of the mixtilinear incircles, and if s_1, s_2, s_3 are the radii of the mixtilinear excircles

2. Distance between the centers of the mixtilinear incircle and excircle

The distance between these circles centers is shown in figure 3.

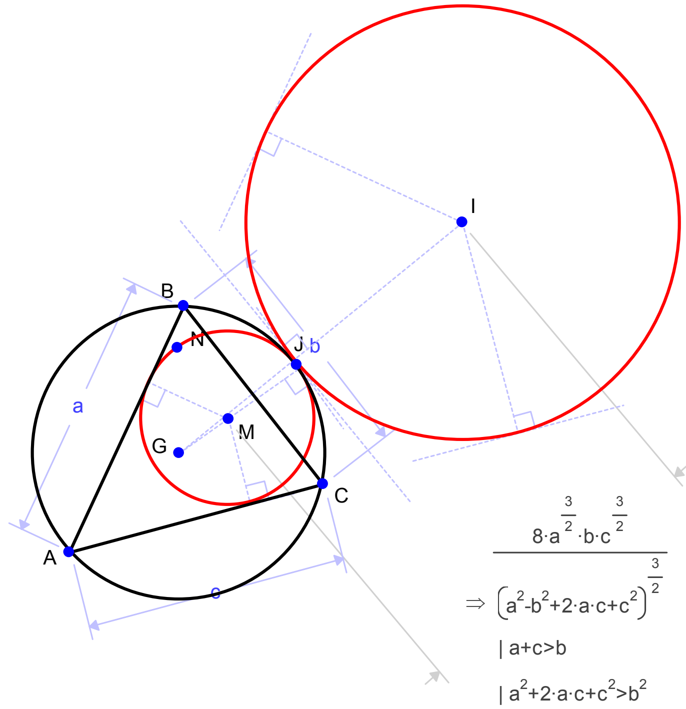

Figure 3: Distance between mixtilinear incircle and excircle centers

The distance between the centers of the incircle and excircle are shown in figure 4.

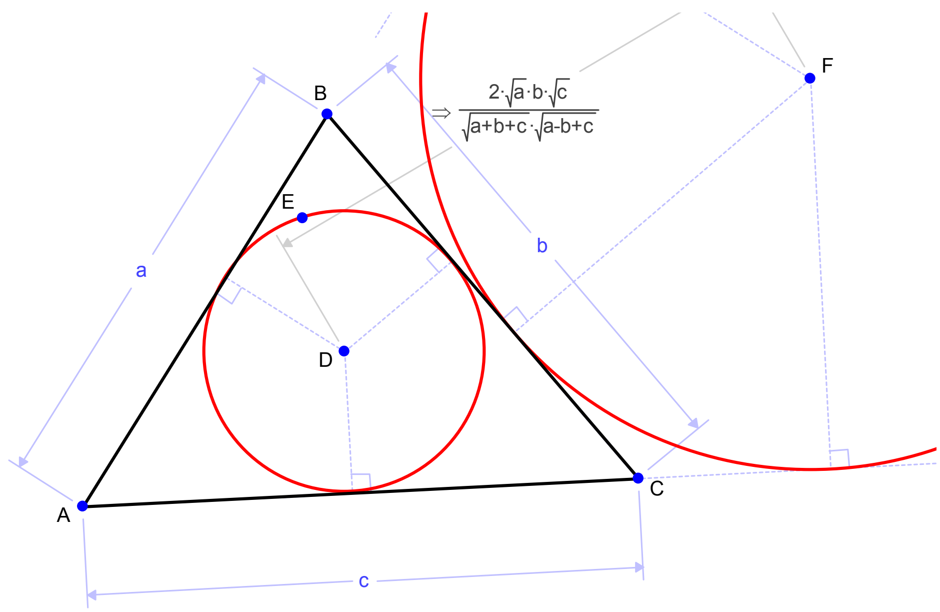

Figure 4: Distance between incircle and excircle centers

Let D be the distance between the mixilinear incircle and excircle, and let d be the distance between the regular incircle and excircle, then:

$$D = \frac{d^3}{b^2}$$