



Chapter 7

Forced Migration and World War II

In Section 1.4, I summarized the situation before 1914 like this: “In the last three decades before World War I, attention to national distinctions and feelings of national pride or imperial supremacy were extremely common, but by and large they *peacefully coexisted*—if I may put it like this—with increasing contact and collaboration among scientists from different empires or countries.”

As World War II was approaching, the situation was much more antagonistic. Everybody could either feel directly—or would find out—that

C'est un peu en se barbarisant qu'on se nationalise.

That is: Focussing on the nation renders us somewhat barbaric. This extraordinary statement was pronounced by the renowned Romanian historian Nicolae Iorga (1871–1940) at the 1933 International Congress of Historians in Warsaw, when he expounded the idea that “the nation, particularly in Southeastern Europe, was a late phenomenon. . . . Its appearance . . . marked the end of the Middle Ages, which was characterized essentially by universal ideas.”¹ His lucid comment linking nationalism and barbarism is all the more remarkable as Iorga himself not only enjoyed an international reputation as historian; he was also a right-wing, antisemitic nationalistic politician in his home country. He did criticize the Romanian fascist ‘Iron guard’, though. They assassinated him in November 1940.

The 1933 International Congress of Historians in Warsaw was actually surprisingly harmonious, in particular also between the Polish hosts and the German delegation.² One of its influential members was the Göttingen medievalist Karl Brandi (1868–1946), a personality who still managed to somehow combine a positive international spirit with strong nationalistic convictions all the way to a certain sympathy for the Nazi government. Back in Göttingen, though, Brandi was threatened by the ancient historian Ulrich Kahrstedt (1888–1962), an outright Nazi. In January 1934, Kahrstedt gave a public speech that implicitly called upon students to batter to death all members of the German delegation to the International Congress, and culminated in the declaration:

¹ See [Erdmann 2005], p. 173.

² Cf. [Erdmann 2005], Chapter 10, pp. 149–161.

We reject international science; we reject the international Republic of Letters; we reject research for the sake of research. In our country, medicine is taught, not in order to increase the number of known bacteria, but in order to keep the Germans healthy and strong. In our country, history is taught, not in order to say what has really happened, but in order to let the Germans learn from the past. In our country, the natural sciences are taught and learned, not in order to discover abstract laws, but in order to sharpen the toolkit of the Germans in their competition with other peoples.³

In the preceding chapters of Part II we have seen how selectively international nationalism was forced into Science International by World War I, and dictated the hapless episode of the IMU in the 1920s. By 1932, in beautiful neutral Zürich, it could perhaps still appear to participants of that sunny ICM as if those recent problems were now overcome. The way to a new buzzing international network of mathematicians seemed all the more open as many of the younger participants had already profited, for example, from Rockefeller grants. Transcontinental, open mathematics with an exciting new agenda was in the air.

However, as of May 1933, the Rockefeller Foundation had to reorient its activities towards emergency programs for refugees fleeing Europe.⁴ By the end of the thirties, an international reshuffling of mathematicians of unheard dimensions was under way. The new mobility was migration induced by politics. In the world of mathematics, this meant for example that lofty research hubs had to also function as employment agencies. The ensuing war had even stronger effects on the mathematical profession.

7.1 Global Redistribution of Scientists in the 1930s and 1940s

In Section 6.4, we have presented a triptych of international mathematical careers that originated in China. For each of the three mathematicians, it was the IAS Princeton that paved the way to a university position in the US. The IAS was rooted in the same setting that had also fuelled the Rockefeller Foundation: joining philanthropy with the idea of scientific excellence and the need for research centers beyond universities—see Section 5.2. In the thirties and forties, the IAS and the Rockefeller Foundation, along with many other institutions, had to face an increasing number of scientists in emigration.

It was in this context that Hermann Weyl—himself an emigré who had left Göttingen for the IAS Princeton in the Fall of 1933 because of the Nazis—was called upon to ponder the fate of the French *Bourbaki* group in a letter dated 22 March 1941 to the Rockefeller-sponsored New School for Social Research in New York. At the time, part of France was under German occupation, and André Weil and Claude Chevalley were already in the US. It seems that André Weil had taken the initiative to secure a bicontinental future for the Bourbaki project. This furnishes an extreme but instructive case where issues of migration, the interest of a small

³ See [Wegeler 1996], Section 3.2.2, pp. 147–162; Kahrstedt’s whole speech is reproduced on pp. 357–368; my translation of a passage on pp. 367–368.

⁴ See [Siegmond-Schultze 2001], Chapter VI.

but select group of mathematicians, and the fate of a major rewriting project of mathematics converge in Princeton, in Weyl's hands. In discussing his small list of French mathematicians, Weyl offers his personal reflections on the evolution of mathematics.

Dear Doctor Johnson:

André Weil told me that he had spoken to you about the *Bourbaki* enterprise. Under this collective pseudonym a set of young French mathematicians has started to publish a number of volumes concerning the basic disciplines of mathematics.

The accent in classical mathematics lay on calculus, and for everything related to calculus the great French *Cours d'Analyse* by Camille Jordan, [Édouard] Goursat (1858–1936) and others, have in the past played a very vital part in mathematical training all over the world. But in the last twenty-five years the emphasis has shifted to other fields like topology and algebra and it has become necessary to lay the foundations deeper. . . . [T]he time seems to have come when integration and a certain degree of standardization should and could be attempted with a fair hope of success. Sometimes such integration has been brought about by an individual work of genius of such paramount importance that nobody working in the field could evade its influence. Systematic efforts undertaken by a group expressly for this purpose are less sure of success; their achievement will perhaps do no more than solidify one school adhering to a special brand of abstract ideas without finding acceptance among other schools, or the foundations laid might soon prove too narrow, etc. I see these dangers and am therefore less enthusiastic about the enterprise than the entrepreneurs themselves. But there is an urgent need, and as far as I can see *Bourbaki* is trying very earnestly and intelligently to find the best and simplest way to arrange the fundamental ideas and to fix the nomenclature. Plan and execution of each volume are discussed in full detail by the whole group, and before the manuscript is finished, it will have been rewritten by three or four authors. It seems certain that no single member of the group could have accomplished what they have done by pooling their mathematical intelligence.

So far two small volumes, on abstract sets and topology, have appeared in French in the *Actualités Scientifiques et Industrielles*; preparation of the material for three more volumes is far advanced. But now the group has been broken up by the war; three of its leading members—André Weil, Chevalley, and Henri Cartan—are, or will soon be, in this country. A *conditio sine qua non* for the continuation of the work would be the bringing over of at least two more members, and this is the reason why I write to you about it.

In October last year I sent Dr. Warren Weaver [1894–1978] a list of French mathematicians, mostly younger men, whom one could consider as candidates for the rescue action undertaken jointly by you and the Rockefeller Foundation.⁵

This list, which Weyl enclosed with the letter, runs as follows:

(1) Arnaud Denjoy, (2) Henri Cartan, (3) Jean Leray, (4) René de Possel, (5) Jean Delsarte, (6) Claude Chabauty (1910–1990), (7) Charles Ehresmann (1905–1979), (8) Charles Pisot (1910–1984), (9) Jean Dieudonné, and (10) Ervand Kogbetliantz (1888–1974).⁶ As to the last person of this list, let us mention in passing that the

⁵ See [Siegmond-Schultze 2001], pp. 284–285.

⁶ See [Siegmond-Schultze 2001], p. 285, footnote. To put this action into perspective, one should also bear in mind the activities launched in 1940 by Henri Laugier (1888–1973) and Louis Rapkine (1904–1948)—see [Dosso 2006].

Armenian, Moscow-trained mathematician Kogbetliantz was already a refugee in Paris since the early 1920s.⁷

The time for triage has come, and Weyl has to explain his choice:

It so happens that all the Bourbaki collaborators are on it. The two men whom Weil considers indispensable for continuation of the work are Jean Delsarte and Jean Dieudonné. Delsarte speaks no English and distrusts his linguistic abilities. It would be much better to place him in the French Catholic university in Montreal than anywhere in the United States. Things are different with Dieudonné who was a Proctor Fellow of Princeton University from 1927 to 1929.

In my opinion an invitation to this country to any democratic-minded foreign scholar who is threatened by the (let us hope short-lived) Nazification of the European continent should depend first of all on his scholastic standing, and then on its adaptability. The fact of his being indispensable for work like that of the Bourbaki group, however meritorious, should hardly play a decisive role in the selection. However, all the young French mathematicians (2)–(9), except (2) Henri Cartan and (3) Jean Leray, are of nearly equal rank. (2) is placed, (3) out of reach. Hence, if there is a possibility of bringing over to America one more young French mathematician, I should find it justifiable to concentrate on Dieudonné, and to try to establish Delsarte in Canada.⁸

The war situation reflected in this letter, and also the peculiar case of the Bourbaki group that Weyl was treating here, certainly make this document very special. Nonetheless the letter illustrates crucial aspects that any attempt to historically account for the scientific migration in the thirties and forties has to balance. This book is not the place to give such an account. All we do in this section is highlight the extent of the phenomenon by scattered thoughts and examples.

On the one hand, migration in general, and forced emigration in particular, involves both professional (in particular, scientific) and personal matters. What is more, the private aspects tend to be more pronounced than in ordinary career affairs, often dramatically so. Emigration is all about leaving a former life behind and letting yourself—and your family—in for a new cultural environment. An adequate account of emigration must therefore not restrict itself to extracting an ‘objective’ general map or measurement of the displacements, but give personal life stories their due share. This being said, integrating personal elements into a comprehensive study of migration phenomena is not only a stylistic challenge; it faces the well-known difficulties of any biographical endeavor (for short: one never knows enough about a person).⁹

⁷ Much more on what we do—and what we do not really—know about the eventful life of Kogbetliantz and his wife between the World Wars is summed up in a recent paper by Laurent Mazliak and Thomas Perfettini in [Mazliak & Tazzioli 2021], pp. 307–355. Their chapter also gives an overview of Russian refugee mathematicians in Paris in the 1920s and 1930s.

⁸ See [Siegmond-Schultze 2001], p. 285. As Siegmund-Schultze also duly notes, Henri Cartan would finally not come to the US during the war, and Leray remained in a German POW camp.

⁹ Cf. the standard reference about the history of mathematicians fleeing from Nazi Germany [Siegmond-Schultze 2009]. There the sequence of chapters follows the overall plan of the book, but the author adds ‘D’-sections, presenting documentary sources, and ‘S’-sections with individual case studies, to various chapters.

On the other hand, choosing the opposite approach, say, for the forced emigration of mathematicians instigated by the European fascist regimes before and during World War II, may suggest the rather cynical conclusion that the fascist pressure actually worked hand in hand with a global, genuinely international ‘consolidation and unification’ of mathematics. Indeed, taking applied mathematics in the US as an example, Richard Courant, himself an emigré mathematician, would joke about this later. When showing visiting colleagues the main building of the Courant Institute of Mathematical Sciences in New York—Warren Weaver Hall, which was built in the early sixties—he remarked that he principally owed this wonderful institute to two influential men: John Rockefeller who gave the money, and Adolf Hitler who provided the talent.¹⁰

The second point of view tends to only count migrations which can be considered scientifically successful; it passes over victims in silence. Otto Blumenthal for instance, the editor in chief of *Mathematische Annalen*, was attacked by Nazi circles on both political and racial grounds as early as 1932; he was dismissed from his chair in 1933. In 1939, aged 63, he did emigrate—but only as far as Holland, where he was arrested in 1943. He died in the concentration camp at Theresienstadt in 1944.¹¹

The scientific effect, or ‘success’, of emigration is a complex amalgam resulting from the encounter of the emigré with the host country. Taking Argentina as an example, the Spanish mathematician Julio Rey Pastor (1888–1962) had been present in Buenos Aires on a part-time basis since 1917, and permanently since 1927. He contributed immensely to the improvement of mathematics in Argentina, but his presence in Argentina may probably still be described best from the point of view of the Spanish metropolis interacting with the periphery.¹² The Catalan mathematician and engineer Esteve Terradas i Illa, however, is a case of emigration. He chose not to return to Barcelona after having participated in the Oslo ICM in 1936, because of the Spanish Civil War. He spent several years teaching in Buenos Aires and in La Plata, Argentina. But he eventually did return to Spain; his emigration was temporary. Terradas’s case shares with Rey Pastor’s a continuing exchange with Spain about returning to Europe.¹³ An emigrant to Argentina who was there to stay was the Italian mathematician Beppo Levi (1875–1961). He lost his chair in Bologna in 1938 due to the racial laws (*leggi razziali*), shortly before his retirement. With his wife and daughter he went to Rosario, Argentina, where he would play the central role in building up the Mathematics Department.

The founding of this institute at Rosario, upstream from Buenos Aires, took place at a time of cultural expansion of several provincial Argentinian cities, mainly Rosario, Córdoba, and Tucumán. A relative prosperity helped in the development of more substantial groups of professionals, mainly lawyers, medical doctors, and engineers, who promoted local cultural

¹⁰ Harold M. Edwards told me this anecdote during my first visit to Mercer Street. Cf. [Siegmund-Schultze 2001], p. 210.

¹¹ See [Bergmann et al. 2012], pp. 88–89 and *passim*.

¹² See Eduardo Ortiz’s account of mathematical relations with the “Iberian periphery” in the nineteenth century, in [Goldstein, Gray, Ritter 1996], Chapter 15, pp. 323–343.

¹³ See [González Redondo 2002].



Fig. 7.1 Beppo Levi, about 1930. (Courtesy Laura Levi.)

activity in these cities and invited leading intellectuals and artists from Buenos Aires to lecture or visit there. These professionals were financially better off, and their clients were richer yet. Societies, orchestras, art galleries, and publishing houses began to emerge in this period in Rosario.¹⁴

Beppo Levi was formally reinstated in his Bologna post in July 1945, but decided against returning to Italy because he was approaching the Italian age limit of 75 years for retiring, and he cared a lot for what he had built up in Rosario.¹⁵

As far as German mathematicians looking for a country of refuge are concerned, Siegmund-Schultze provides a truly global survey:

Examples from various host countries show how widespread economic problems and political resentment, such as anti-Semitism, made acculturation difficult. Some countries, such as Austria and Poland, had to be ruled out as host countries from the outset, since they offered

¹⁴ See [Schappacher & Schoof 1996], p. 67, based on information from Eduardo Ortiz. Cf. [Ortiz 1988].

¹⁵ See [Levi 2000], pp. 75–77. Laura Levi also stresses her father's interest and contact with the physics community, and corrects accordingly the caption of the group picture reproduced on p. 67 of [Schappacher & Schoof 1996], which in fact shows the 1948 meeting of the *Asociación Física Argentina* (AFA).

similar, if not quite as extreme, political conditions as Germany. Others, such as Italy and the Soviet Union, also ruled by dictatorial regimes, served nevertheless and somewhat surprisingly as temporary host countries. Hopes harbored by Turkey to profit from the German immigration for its own science system failed due to Hitler's expansion policies and the death of Kemal Atatürk in 1938; both circumstances forced the refugees to go on to safer places. Australia was a rather less attractive option for emigrants because of the rudimentary state of mathematics there at that time. Although some authorities involved in emigration tried to use Australia to ease the situation in other host countries, only two mathematicians finally ended up there before the end of the war.¹⁶

These general indications are then detailed according to countries, or continents in the particularly rich corresponding 'D' section of his book.¹⁷

Forced emigration was an important factor, if not the initial source for putting what would later become the state of Israel on the global map of mathematics. The Zionist movement had inspired the founding of the Hebrew University (HU) and its Mathematical institute—today called the Einstein Institute of Mathematics (EIM)—at Jerusalem in 1925. Edmund Landau gave a talk at the opening of the HU which actually reflected “the way the Zionist cause was inextricably linked to, and determined by, European political agendas” of the 1920s, in particular regarding Science International.¹⁸

During the fourth decade of the twentieth century, with the voluntary emigration and enforced expulsion of scientists and scholars from Nazi Germany, new centers of mathematical research were created. The great nineteenth-century German scientific heritage, which had hitherto slowly pervaded Europe and abroad, now dispersed to new intellectual havens. Former students of the German academic system carried their heritage to new harbors to anchor their scientific expertise, and implement their intellectual traditions from Istanbul to New York and Buenos Aires. Displaced mathematicians were part of this migration. Although it took place at roughly the same time, the founding of the EIM at HU belongs to a different kind of phenomena. The EIM was less the outcome of the push of anti-Semitism and Nazism, and more a result of the pull exerted by the Jewish national movement.¹⁹

The quasi economic push-and-pull model is one of the lenses through which migration phenomena have been investigated.²⁰ However, multiple methods and questions should always be kept in mind, for instance, if a loss or gain of people also meant a loss or gain for science, and so forth.²¹ Let us return to Shaul Katz's account:

It was the Zionist vision that drove a few dozen scholars and scientists, most of them European, to prefer the new university in Jerusalem opened in 1925, over their mainly European *alma mater*. There is no other overwhelming explanation for Landau's coming for a short period to Jerusalem in 1928, followed by the arrival of [Adolf Abraham Halevi] Fraenkel in 1929. And it was a sort of mathematical idiosyncrasy of Landau, coupled with a certain variety of European national movement, Zionism, that embraced wholeheartedly

¹⁶ See [Siegmond-Schultze 2009], p. 103.

¹⁷ See [Siegmond-Schultze 2009], pp. 104–148.

¹⁸ See [Corry & Schappacher 2010], p. 427; the claim quoted is elaborated in this article.

¹⁹ See [Katz 2004], pp. 226–227.

²⁰ See [Lee 1966].

²¹ See [Ash 2011].

pure science and its promotion as one of its exalted cultural ideals (a kind, so they tended to believe, of national transformation of the biblical “From Out of Zion Goes Forth Torah”) that begot the pure-mathematics trajectory of EIM. Concomitantly, Landau, Fraenkel, and [Mihály-Michael] Fekete [1886–1957] were proud intellectual inheritors of this variety of the Berlin tradition that not only conceived pure mathematics as a sublime neo-humanistic ideal, but also in parallel also disdained applied mathematics. Therefore EIM maintained the cultivation of pure mathematics only. Since the framework of European migration of the 1930s does not suggest itself as a proper comparative historical one for EIM case, a more general family of phenomena with more historical depth and geographical width invites attention. It is the comparative perspective of the process of implementation of Western science outside Europe.²²

It was not my intention to confuse the reader with scattered examples—to which one may also add the three men discussed in Section 6.4—, approaches, and remarks. But the far-reaching global reshuffling of mathematicians and mathematical centers of the 1930s and 1940s was as dramatic as it is complicated to sort out. Mapping out the whole migrational reshaping of the terrain of Mathematics International in the thirties and forties would require yet another book.

This exodus was, to be sure, a source of a tremendous upsurge in the internationalization of mathematics, especially in the sense of new and unexpected personal encounters and oral communication. Still, this type of internationalization was shaped in a peculiar way by emigration patterns. It was not necessarily healthy or natural when compared to the secular, long-term internationalization of mathematics that had been well under way in the decades before. Without entering into the foggy field of counterfactual history, it is important to focus on the losses for the various national cultures in mathematics in Europe that were brought about by the expulsions not just in Germany but also in other countries such as Poland, Hungary, and Austria. These losses were more than the sufferings of the refugees and the deaths of the victims.²³

7.2 What World War II Meant for Mathematics

The Second World War was of “a far greater magnitude than the preceding world war, it was to engulf a larger area, bringing with it the horror of systematic genocide exemplified by the Holocaust. Over and above territorial considerations, the very future of civilization was at stake.” Its theaters included Western Europe from April to June 1940, the German invasion of Russia as of June 1941, and Japan’s overrunning of the whole of South-East Asia.

The Japanese bombing of the American naval base at Pearl Harbor on 7 December 1941 enabled President Roosevelt to surmount the pacifism widely supported by the American public and lead the United States into the war. Until then, the American participation was limited to providing equipment to Great Britain and Russia under the Lend Lease Programme. Despite the Americans’ superior weaponry and their contribution to the defeat of Germany in May 1945, the conflict with Japan appeared likely to endure. To curtail it, the United States resorted to atomic weapons in August 1945. The resistance movements in occupied

²² See [Katz 2004], p. 227.

²³ From Siegmund-Schultze’s chapter in [Parshall & Rice 2002], p. 339.

France, Belgium, Norway, Greece, Yugoslavia, Poland and Russia and in South-East Asia were the protagonists of a conflict, which was a key feature of the war despite its lower profile.

[Another theater] could be added: the battles in North Africa, which continued with the landings in Italy and the collapse of the Fascist dictatorship.

The Second World War also differed from the preceding war by doing away with the dividing line between civilians and combatants. The bombing of Warsaw, Leningrad, Rotterdam, London and Coventry by the Germans, the Allied bombing of Berlin, Hamburg and Dresden, and finally the atomic bombs dropped on Hiroshima and Nagasaki, all targeted civilian populations. If those who died of hunger are included, the civilian death toll probably numbered approximately fifty million.²⁴

In Chapter 3 above, in order to capture the impact of World War I on Mathematics International, we had to address not only the new role of mathematics and mathematicians in warfare, but also the violent nationalism fired by the war, because this spirit stood at the cradle of the IRC and the IMU in 1919–1920. Different but analogous observations apply to World War II. The role played by science was even more pronounced than during the First World War, and there was a greater variety of mathematical applications, many of which would flourish over the following decades. World War II also prepared a new global political landscape: The Cold War, which would reshape the professional structures for mathematical research and determine the first decades of the IMU after its renewed birth in 1950–1951.

Before going into this, let us start with a peculiar episode from the German occupation of France in 1940.

7.2.1 Searching for the Hiding Place of the IMU

During World War II, Harald Geppert (1902–1945)—the elder brother of Maria-Pia Geppert (Section 6.2.2)—was in charge of both German review journals, the *Jahrbuch* and the *Zentralblatt* (Section 6.3.3). The *Jahrbuch* would not survive the war; but in the first war years it was still trying to squeeze the delay between the publication of the papers and their reviews. The *Zentralblatt*, on the other hand, had just lost a number of its referees in the fight that had precipitated the foundation of *Mathematical Reviews*. Thus Geppert was trying to fill those gaps with mathematicians recruited in the large part of Europe that had come under German control by the end of 1940. Irrelevant and piecemeal as it may seem at first, this endeavor would be an important element of the attempt to re-order Europe under German domination, as far as mathematics was concerned. Individuals who were invited to write some of these much needed reviews for the *Zentralblatt* would not only get paid, but would get access to recent literature in their field, which was otherwise hard to obtain in times of war. The extent to which, say, a French mathematician had

²⁴ All quotes in this preamble to the present section are from [Gopal et al. 2008], p. 6.

accepted, or not, to write reviews could therefore become a key issue after the war when it came to judge if he behaved like a *résistant* against the Germans, or rather like a *collaborateur* during the occupation.²⁵

Against the double background of his responsibility for the *Zentralblatt* and ongoing political discussion about the would-be German re-ordering (*Neuordnung*) of science in Europe, Geppert was sent on an official mission to Paris in December 1940. His explicit agenda, however, was to search for hidden signs of life of the IMU. Now that the Germans controlled Paris, they wanted to make sure to extinguish whatever might still smolder of that anti-German international construct. Thus on 3 December 1940, the minister confidentially ordered Geppert to travel to Paris in order to investigate what Geppert himself had alerted the ministry to in the first place:

While preparing for a re-ordering of international scientific cooperation in the international unions, associations, etc., I was led to examine the International Mathematical Congresses and the former *Union internationale de mathématique*. It has come to my attention that there exists in Paris an *Institut Poincaré*, which also organizes international meetings in the domain of mathematics, which are different from the International Mathematical Congresses that take place regularly. It seems that this *Institut* perpetuates on its own account the *Union Mathématique*.

I herewith order you to undertake before long an official journey to Paris in order to assess directly on the spot the importance that has to be attributed to the activity of the *Institut Poincaré*. I point out that the extent of your findings may be of fundamental importance for my future decisions.²⁶

After his return from Paris to Berlin Geppert, in an attempt to respond to the object of his mission, submitted a survey of the ICMs that had taken place since World War I, based on the various ICM Proceedings. We quote starting with the Zürich ICM:

The following International Congress took place in 1932 in Zürich. The IMU is mentioned neither in the invitation nor during the Congress. But at the final session an international commission is formed—its only German member was the Jew²⁷ Hermann Weyl—“in order to re-study the question of the international collaboration in the sphere of mathematics and to make propositions with regard to its reorganization at the next congress.”²⁸ Obviously, this commission was to ensure a future substitute for the IMU, which was still in existence.

The next ICM took place in 1936 at Oslo. It was again called without any intervention by the IMU. However, the Union suddenly appears in the minutes of the final session of the Congress, where Prof. Gaston Julia reports on the activity of the international commission mentioned before. After several meetings over the years the commission has determined that the creation of a truly international organization of mathematicians encounters unsurmountable difficulties and must therefore be delayed. Whether this means that the activity of the union has to be considered terminated, or whether it continues to be alive because of the lack of a truly international organization, is not clear from the minutes. From the

²⁵ See the detailed analysis in [Eckes 2018], which also connects the review issue to Geppert's and Hasse's vain attempts to free certain French POWs.

²⁶ Quoted from [Siegmond-Schultze 1993], p. 179; my translation.

²⁷ In fact, not even the Nazi administration claimed that Weyl was Jewish.

²⁸ See Section 6.2.2 above.

German side, professor Blaschke, Hamburg, has participated in these meetings. The next International Congress of Mathematicians was planned to take place in 1940 at Princeton, USA, but has been adjourned because of the war.

Two questions thus remain to be settled: that of the creation of an international organization of mathematicians, whose need is documented by the events described; and the organization of the next International Congress of Mathematicians, which will be called by the American Mathematical Society.²⁹

About five years after Harald Geppert's suicide (Berlin, 4 May 1945) both questions were settled: in the US and in particular thanks to Marshall Stone.

7.2.2 Mathematics for the War

In Section 3.3 we briefly described the effects of the First World War on mathematicians and on mathematics. We have seen in particular that applied research topics imposed by the needs of the battlefields would modify the appearance, the context, and thus finally the substance of mathematics. And we have seen in the Italian example (Section 3.3.1) how the organization of military research during the First World War would create structures of scientific policy that outlived the war.

Thus prepared what to look for, we now turn to World War II. Scientific man and woman³⁰ power was mobilized for the new war effort on a considerably larger scale than during World War I.³¹ Note that the enrolment in scientific work for the war could save the life of a young man who would otherwise be sent to the front; leaving a relatively safe place in a decoding unit in Berlin to volunteer for frontline fighting could amount to suicide, as in the case of the fanatic Nazi Oswald Teichmüller (1913–1943).³²

The domains for which mathematicians were in high demand during World War II cover a substantially broader spectrum than in the previous war, and include a few recent, budding subdisciplines of mathematics. Here is a rough overview of the main areas:³³

²⁹ Geppert to Ministry, 29 December 1940; my translation. For the German and French archival sources of copies of this report, see [Eckes 2018], pp. 299 and 305.

³⁰ The presence of women in science for World War II was not limited to the numerous women computers; see for instance Kathleen Williams's chapter "Improbable Warriors: Mathematicians Grace Hopper and Mina Rees in World War II" in [Booß-Bavnbeck & Høystrup 2003], pp. 108–125.

³¹ This seems obvious, for instance if one looks at the whole spectrum of mathematical domains that were pushed during WW II. However, I have not been able to find reliable estimates from the various countries of, say, mathematicians enrolled in war research in the forties.

³² Cf. the reflections about the adjacencies between Teichmüller's work on (quasi-)conformal maps and ongoing aerodynamic research in [Epple & Remmert 2000], pp. 291–293.

³³ See Siegmund-Schultze's schematic overview, with references, of mathematical war work in Germany, the US, USSR, UK, Italy, France, and Japan in [Booß-Bavnbeck & Høystrup 2003], pp. 63–74.

- Aerodynamics/hydrodynamics, especially problems near super-sonic speed and air foil design.
- Ballistics of torpedoes, anti-aircraft gunnery, and rockets.
- Cryptography. Among the countless cryptography units in all countries at war,³⁴ Turing's work at Bletchley Park has received the greatest attention in the literature on World War II.
- Development of early electronic computers.
- Operations research.
- Game theory.
- Cybernetics.³⁵

Some of these domains really took off only after the war; game theory for instance. In the United States, the Manhattan Project working on the Atomic bomb, and continuing later with the H-bomb, naturally enlisted mathematicians. It required heavy numerical calculations. Mathematical problems arose in this context from

- Gas dynamics, and from
- Statistical approaches of various kinds, in particular the Monte Carlo method introduced (after the war) by Stanisław Ulam (1909–1984) and Nicholas Metropolis (1915–1999).

In the US, John von Neumann was the central figure, almost the incarnation of mathematical war research. The organizational setup of mathematical war research in the US and its consequences will be discussed in the next Section 7.2.3. The broad panorama of mathematical fields and the great number of mathematicians enlisted for war-related research, and its continuation after 1945, make it impossible to present an overall account. We visit a few examples instead.

For the period of World War II itself, an interesting contrast between Germany and the UK transpires from the report on *Applied Mathematical Research in Germany, with Particular Reference to Naval Applications* by the *British Intelligence Objectives Sub-committee* (BIOS), based on investigations made in June–August 1945 by John Todd (1911–2007), G.E.H. [Gerd Edzard Harry] Reuter (1921–1992), Friedrich G. Friedlander (1917–2001), Donald Harry Sadler (1908–1987), A. Baxter (?) and Fred Hoyle (1915–2001). We quote from the general observations at the beginning of the report:

2. There is no possibility of 'controlling' mathematical research, i.e. preventing work being carried out on 'war' subjects. It is abundantly clear from our observations in Germany and from information obtained from U.S.A. (and, to a much less extent, from our experience in U.K.) that almost any top-class mathematician practising in the most abstract fields can very quickly make substantial contributions in the mathematics of technology.

...

³⁴ See for instance [Weierud & Zabell 2019] for the German case.

³⁵ This term for the new science born out of his war research was coined by Norbert Wiener only in 1947. See [Galison 1994].

4. Nevertheless we feel that the mathematicians in U.K. made a bigger contribution to the war effort than those in Germany. On the one hand a considerable number of younger mathematicians in Germany were actually put in the fighting services, on the other, those in Government Departments and in Industry did not appear to work as conscientiously as the majority of those similarly placed in U.K. As evidence of this may be mentioned the fact that members of this party were continuously being asked to take with them, manuscripts prepared in 'Sparetime', for submission to editors of mathematical journals here or in U.S.A. Very few of the English mathematicians had energy left for such activities.³⁶

And on the war work of the Number Theorist Helmut Hasse, who during the war was in charge of a research group at the High Command of the German Marine Forces (OKM), the committee notes:

H[asse] seemed to have an exaggerated opinion of the value of his trajectory work, which, in our opinion, though elegant, is of little practical value. He stated he had forgotten all the details of his work but said they could be extracted from the OKM documents which he understood to be in our possession—he asked that we should send him copies of his own reports! It was considered unnecessary to encourage him to remember details of the work, as it appeared that in his position as administrative head of FEP III he was content to leave all technical matters to Prof. Karl Willy Wagner [1883–1953], and devote his energies to rather unpractical matters.³⁷

Kolmogorov's work on the probability theory of firing techniques provides another, different example of a well-known mathematician's work occasioned by the war. It would fill a special volume of the *Proceedings of the Steklov Mathematical Institute* published in 1945.³⁸

More historical research, in particular also comparative research, on the nature of mathematics for the War in various countries is still a desideratum for the history of mathematics in the twentieth century. Indeed,

during the war(s) a lot of at least potentially applicable theoretical work was done in various countries—whether they were involved in the war effort or not—that escaped attention of men such as Norbert Wiener abroad and was likewise not noticed due to the communication blackout during much of the war(s) and even later in the Cold War. Mathematical work or mathematics-related engineering work that was potentially war-important, such as done in France by É[mile] Borel on game theory and émigrés W[olfgang/Vincent] Döblin [1915–1940] and F[elix] Pollaczek [1892–1981] on Markov chains and queuing problems, or in Germany by K[onrad] Zuse [1919–1995] on digital computers, was not, for various historical reasons, actually . . . transferred into the war effort and therefore partly or temporarily ignored in the countries that would write the history of the war and set the norms for the scientific enterprise after 1945, especially the United States.³⁹

³⁶ See BIOS Report 79 (1945), pp. 2–3.

³⁷ See BIOS Report 79 (1945), pp. 48–49.

³⁸ See A.N. Shiryaev's account of it in [Booß-Bavnbeck & Høyrup 2003], pp. 103–107.

³⁹ From Siegmund-Schultze's chapter in [Booß-Bavnbeck & Høyrup 2003], p. 28.

7.2.3 How World War II Reshaped the World – the Case of Mathematics

World War II brought a tremendous impetus to mathematics. Indeed the military interest in all the areas of mathematics we have listed above led to the creation of applied research groups and of new specialized research institutes, in all countries at war. These institutes, and the whole organization of war research would reconfigure the professional setup of scientific disciplines. Let us start in the European countries under fascist rule:

Aerodynamics, the scientific basis of aviation, represents one of the most significant successes in the mathematization of the technological sciences in the twentieth century. At the same time, ballistic problems concerning projectiles and missiles, in the air and under water, were tackled with the help of mathematical methods on an increasingly larger scale. During World War II, new coding and decoding projects required mathematical support. In some countries, this process had started already in or right after World War I.

In all these areas, the traditional university system proved insufficient for the organization of specific mathematical research extensive enough to meet both armament and warfare interests. As in many other scientific and technological fields, research institutes outside the university system were founded with state, military, and industrial participation. Mathematicians either significantly shaped, or even entirely supported, these institutes.⁴⁰

What happened after the war to those newly created structures, and to the whole war administration of mathematical research, would of course depend on the country and on individual circumstances. To mention a peculiar example known to many mathematicians, the “Mathematical Research Institute” at Oberwolfach, Germany, is today a conference center of international reputation. But it started out in November 1944—very late in the day as far as World War II was concerned—as a *Reichsinstitut* with the mission to coordinate mathematical war research in Germany.⁴¹

What was the long term effect of the war for mathematics? Looking at individuals, there were surely a number of mathematicians who had been enlisted in military research during World War II, but who would later look back on this period as a passing spell in their professional life, after which they took up (as soon as this was materially possible) their previous work more or less where they had left it. Looking at nations, the strongest and most influential long time repercussion of the war effort on the development of the mathematical profession seems to have taken place in the USA. There a certain divide opened up after the war, between those who returned to pure mathematics the way they had practiced it before—typically, in the axiomatizing spirit of ‘Consolidation and Unification’ inherited from the thirties—and those who followed up the type of applied problems they had worked on for the nation at war and ended up establishing more than one new mathematical speciality. To do this the latter could avail themselves of new employment patterns inherited from the

⁴⁰ See [Epple et al. 1995], p. 132. This paper then goes on to compare various research structures for aerodynamics and mathematics in the two fascist states Italy and Germany.

⁴¹ See [Remmert 2020] for the history of the Oberwolfach institute in the first years after World War II.

administration of war research. However, pure mathematics could also profit from these rich new funding facilities. There was nonetheless a parting of the ways in the US mathematical community about how to position oneself with respect to the Cold War and the corresponding advent of *Big Science*.⁴² Since the US would become the leading nation for Mathematics International after 1945, what happened there would also affect mathematical communities in other countries as well as international organizations. It was principally through the development in the US that World War II influenced the kind of mathematics showcased by the IMU and at the ICMs.

The Second World War has brought about in the United States important changes in mathematical practice, in the scientific, intellectual, and social networks of mathematicians, bringing them into closer contact with physicists, engineers, economists, and specialists of the social sciences, as well as with military officers and politicians. The mathematicians were confronted with various concrete and pressing problems for which solutions, or rational, formal approaches were urgently wanted. At the end of the war an important part of the mathematicians returns to their traditional academic universe, taking up the research they had briefly interrupted. In the mathematical community at the universities and its international institutions a certain ideology of pure mathematics develops and seems to become dominant at the end of the 1950s. This 'purism' in which part of the community tries to shelter is in part a reaction against the American tradition of utilitarianism. It also has to be linked with the political context of the Cold War and the climax of McCarthyism. The mathematicians which represent this tendency consider having already paid their due to the global conflict; they now want to be able to dedicate themselves to the most abstract fundamental research, far from all preoccupations with politics or applications. However, there are also other mathematicians, other groups which have emerged during the war and whose interests as well as social and professional networks continue to hold their own, independently of the purist mathematicians.⁴³

One could have imagined that World War II would create a sort of transparency between pure and applied mathematics, which would then likewise reshape the professional situation of mathematical research in society and politics. But this did not happen, neither during the war nor afterwards.

During the war, Warren Weaver directed the *Applied Mathematics Panel* that was created

to coordinate the services of mathematicians and to serve as a clearinghouse for mathematical information pertinent to the war. . . . Weaver's panel supervised an effort that employed close to three hundred people, including such mathematicians as John von Neumann, Richard Courant, Jerzy Neyman, Garrett Birkhoff, Harold Hotelling [1895–1973], and Oswald Veblen; wrote several hundred technical reports; and spent nearly three million dollars. The panel encouraged new developments in statistics, numerical analysis and computation, the theory of shock waves, and operational research, and served as a training ground for mathematically-minded workers in fields like economics, one of the more famous being the eventual Nobel Prize winner Milton Friedman [1912–2006]. The panel also promoted the institutionalization of applied mathematics through its support, e.g., of Brown University's Program in Applied Mechanics, Jerzy Neyman's Statistical Laboratory

⁴² *Don't forget your mittens!* Laurie Anderson.

⁴³ See [Dahan 2004], p. 50; my translation.

at Berkeley, and Richard Courant's group in applied mathematics at NYU. When ground was broken for the Courant Institute at New York University in 1962, Warren Weaver was there to wield a shovel for the building that would bear his name.⁴⁴

And yet,

judged in terms of its larger ambitions—the central coordination of wartime mathematics—the panel failed. Furthermore, the success it did achieve split the nation's mathematicians into angry factions. . . . [The panel's] forgotten trials and tribulations illuminate both the uneven development of American mathematics at the outbreak of World War II as well as the imperial ambitions of those who, like Vannevar Bush [1890–1974], James [Bryant] Conant [1893–1978], and Warren Weaver, took the lead in the mobilization of wartime science.⁴⁵

Adding to the places just mentioned Los Alamos, Aberdeen Proving Ground and CalTech, and also Princeton, we are looking at a list of the main centers of applied mathematics launched in the US during the war where mathematicians, physicists and engineers rubbed shoulders.

The most significant reconfiguration which emerges from these works, both on supersonic flow and on nuclear questions, concerns hydrodynamics, computers and numerical analysis. This reconfiguration shatters the established hierarchies between 'pure' and 'applied'. It blurs the borderline between what clearly belongs to mathematics and what does not belong to mathematics and would normally have been classified in the domain of engineering science or physics. Von Neumann emerges as someone who has realized this recomposition of interests for himself early on. From the beginning of the 1940s he convinces himself of the importance of hydrodynamics for all the physical sciences and for mathematics and of the fact that it requires a radically new development of methods and of computational capacity. When the project of an electronic computer gets under way, von Neumann, [Herman H.] Goldstine [1913–2004] and their collaborators explain that the economy of the machines absolutely calls for a profound remodelling of numerical analysis and for the elaboration of new algorithms. Also the program of digital meteorology chosen as a priority full scale application for the Princeton computer is an example of this reconfiguration of interests and practices.⁴⁶

After the war, there was widespread

concern that the vitality and flourishing of wartime research would dissolve in the postwar period. The scientists would go back to the kind of work they did before the war with the consequence that the research cooperation within the military-university-industry complex, which had proved itself so productive during the war, would simply disappear. Not surprisingly there was a shared belief that the USA had to be strong scientifically in order to be strong militarily. . . .

The National Science Foundation was not established until 1950 and in the meantime the military services initiated different channels for supporting scientific research. There were two primary places where the new mathematical techniques that emerged during the war became the subject of military funded basic research, Project RAND and the Office of Naval Research (ONR).⁴⁷

⁴⁴ See [Owens 1989], pp. 287–288.

⁴⁵ See [Owens 1989], p. 289. See also [Parshall 2015], pp. 295–302.

⁴⁶ See [Dahan 2004], pp. 54–55; my translation.

⁴⁷ See [Kjeldsen 2003], pp. 133–134.

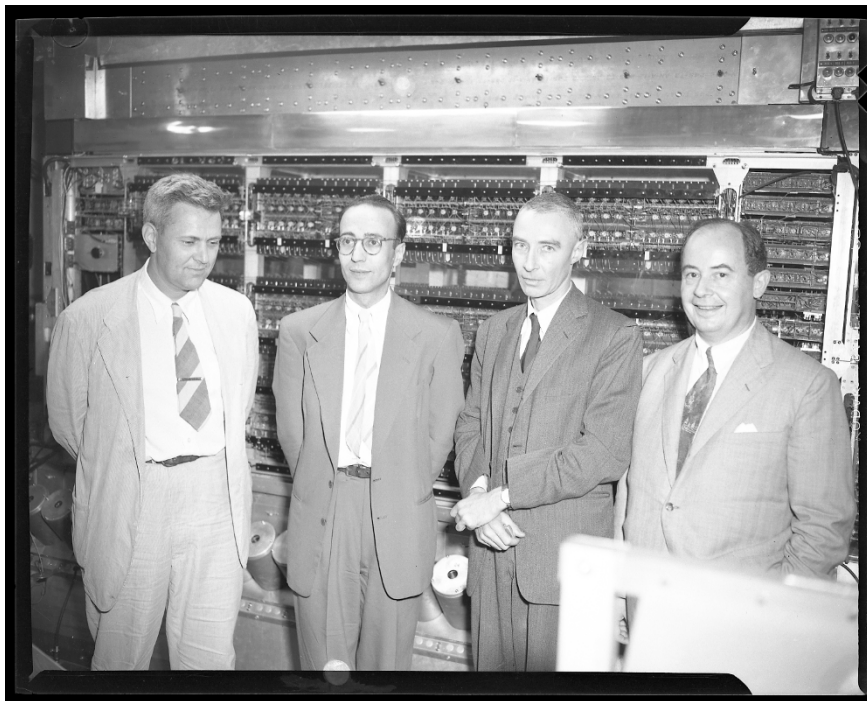


Fig. 7.2 Julian Bigelow, Herman Goldstine, J. Robert Oppenheimer, and John von Neumann in front of MANIAC, the Institute for Advanced Study computer, 1952. Credit: [Arch. IAS].

Several new mathematical disciplines grew from this peculiar constellation in the wake of World War II. We mention Operations Research—in particular Nonlinear Programming—and Game Theory.

The ONR was established within the US Navy in 1946 to ensure the continuation of the vitality and thriving of scientific research done during the Second World War. During the first four years of its existence it was the main sponsor for government supported research in the USA. It continued the practise of the war organisation Office of Scientific Research and Development (OSRD) that had been the vehicle for the mobilisation of civilian scientists during the war. Like OSRD, ONR supported scientific projects through contracts with scientists working in the universities, projects of which many were proposed by the investigators.

The logistics programme of ONR originated in 1948 as a result of the mathematician George B. Dantzig's [1914–2005] work with so-called programming planning methods in the US Air Force during and after WW II. An Air Force programme was a huge logistics schedule for Air Force activities. During the war Dantzig had worked on these programmes and taught Air Force staff how to calculate the programmes. The methods they used were slow and inefficient. It took more than 7 months to set up such a programme. After the war Dantzig went back to work for the US Air Force Headquarters where he functioned as mathematical advisor. Together with a group of Air Force people he worked on programming planning problems. In October 1947 the Princeton people became aware of this work because Dantzig visited John von Neumann, in von Neumann's capacity as a consultant for the Air Force, to

discuss the possibility of solving such an Air Force programme. At this point Dantzig and his group at the Air Force had built a mathematical model for the programming problem, a model they first called programming in a linear structure and soon after became known as a linear programming problem. John von Neumann had just completed the first book on game theory with Oskar Morgenstern [1902–1977] and he suggested that Dantzig’s programming problem was equivalent to a so-called finite zero-sum two-person game. This connection to game theory provided the linear programming problem with a mathematical foundation in the theory of systems of linear inequalities and the theory of convexity.⁴⁸

It is remarkable how seamlessly the history of Operations Research slides from World War II to the Cold War. This is illustrated by the US airlift operation Vittles during the Berlin Blockade in 1948–1949.⁴⁹

As another illustration of the same general process let us quote from Kjeldsen’s summary of how “game theory became the main subject of mathematical research at the RAND Corporation”:

According to the historian [and economist] Philip Mirowski [b. 1951], the disregard shown by economists brought von Neumann to search for another ‘home’ for game theory. Given the time, the place, and the concept of optimal strategies for winning a game, which fitted perfectly into the war context, and given von Neumann’s multiple connections, reputation, and influence within the military-science complex during the war, the military context was an obvious choice. Project RAND in Santa Monica, California became the most important home for game theory. This project originated in March 1946 by the initiative of Army Air Force Chief of Staff Henry H. ‘Hap’ Arnold and Donald Douglas, the president of Douglas Aircraft. In the beginning the project functioned as a subsidiary of Douglas Aircraft but in 1948 Project RAND became a free-standing nonprofit corporation, a so-called ‘thinktank’.

In the first decade after the war RAND was the center for mathematical research in game theory. The first mathematicians working there were recruited mainly from the Applied Mathematics Panel. . . . This group at RAND was the first established group of game theorists and they all either came from the war work or had connections to mathematicians who had been involved with OSRD. The group at RAND held lengthy summer sessions in game theory and collaborated with another military financed project—the logistic project—in Princeton.⁵⁰

Another, analogous example of continuity from war work to fundamental scientific reorientations of the 1950s and 1960s is Norbert Wiener’s conception of cybernetics as analyzed in Peter Galison’s penetrating study.

What we have seen in Wiener’s cybernetics is the establishment of a field of meanings grounded not through zeitgeist but explicitly in the experiences of war. For however far telephone relaying technology or A.N. Kolmogoroff’s statistics had come before the war, it was the mass development and deployment of guided missiles, torpedoes, and anti-aircraft fire that centralized the technology to scientists and engineers. To the thousands of servicemen who used and faced this new generation of weapons, the ‘human’ character of self-regulating machines seemed all too human. After all, trying to shoot down a Junkers JU

⁴⁸ See [Kjeldsen 2006], pp. 34–35. Cf. [Kjeldsen 2019], pp. 147–155.

⁴⁹ See Chapter 2 of the inspiring book [Erickson et al. 2013], pp. 51–80, which first focusses on the same scientific development as the last quote, complementing it at the end by a look at developments in the USSR.

⁵⁰ See [Kjeldsen 2003], pp. 135–136; see also pp. 146–14 for a discussion of how this institutional fixation may have influenced the development of the young theory.

88 heading for London or a V-1 buzz bomb doing the same thing was not all that different. A skipper trying to dodge a self-guided torpedo could be excused for referring to the device as ‘trying’ to kill him, as could the pilot ascribing airfoil self-adjustment to the work of ‘gremlins.’ And in the specific case of Wiener, [Julian] Bigelow [1913–2003], Weaver, and their colleagues, it is perhaps understandable that the pilot of an enemy plane could be said to ‘behave like a servo-mechanism.’ While prewar behaviorists might have cautioned against the ascription of internal states, war made it impossible; reading the hidden enemy meant reading his actions. In the mechanized battlefield, in those life-and-death confrontations with an enshrouded enemy, the identity of intention and self-correction was sustainable, reasonable, even ‘obvious.’⁵¹

The mathematical landscape that resulted from the new actors and attitudes had repercussions on the way mathematicians would approach classical fields such as analysis, which is after all

one of the oldest branch[es] of mathematics, especially linked to the study of nature, physics, and engineering science. Various conceptions of analysis and what its teaching should be strongly opposed those of pure and applied mathematicians. In the 1940s and 1950s, the emphasis put by the former on functional analysis was enormous. For Bourbaki, this was justified by the general state of confusion in mathematics at the time. In fact, except for Laurent Schwartz [1915–2002], none of its members was really an analyst. Bourbaki labored towards a conception in which algebra, analysis, and topology would form a single unified domain giving rise to vast syntheses at increasing levels of abstraction. Traditional branches of analysis were considered bleak and limited in their ambitions. When he tackled nonlinear oscillations, Solomon Lefschetz noticed that differential equation theory was deemed the most boring topic possible. L[ennart] Carleson [b. 1928] has described the reigning state of mind regarding classical analysis: ‘There was a period, in the 1940s and 1950s when classical analysis was considered dead and the hope for the future of analysis was considered to be in the abstract branches, specializing in generalization.’ Writing in 1978, he went on: ‘As is now apparent, the rumor of the death of classical analysis was greatly exaggerated and during the 1960s and 1970s the field has been one of the most successful in all mathematics.’⁵²

We leave this chapter with an example from the other side of the Cold War, of a long-term development of a war-related mathematical problem, whose solution would provide a central result of the theory of optimization.

In 1970, at the World Congress in Nice, Prof. [Lev Semenovich] Pontryagin [1908–1988] gave a plenary talk on differential games, which was motivated by pursuit-evasion strategies of aircrafts for a very simplified model of behavior. During the after-talk discussions, A. Grothendieck put a rhetorical question to Pontryagin. He said that though the listeners witnessed a beautiful piece of mathematics, still he would like to know whether the speaker feels himself morally responsible for supporting military trends in the society. Pontryagin’s answer was quite definite and blunt. He was convinced, he said, that, on an intellectual level, any intellectual problems could be discussed openly in a developed society, and if we would follow to the logical end Prof. Grothendieck’s recommendation, we should be prohibited from speaking openly about some topics of abstract Algebra, since Cryptography, which has much deeper correlations with military problems than the differential game considerations he spoke about, is completely based on the theory of finite fields.

⁵¹ See [Galison 1994], p. 263.

⁵² See [Dahan 2001], p. 242; the author goes through further milestones of the story in her text, which we do not follow up here.

Lev Semenovich Pontryagin was one of the leading figures in 20th century algebraic topology and topological algebra, but in mid-1950s he abandoned topology, never to return to it, and completely devoted himself to purely engineering problems of mathematics. He organized at the Steklov Mathematical Institute a seminar on applied problems of mathematics, often inviting theoretical engineers as speakers, since he considered a professional command over the engineering part of the problem under investigation to be mandatory for an adequate mathematical development. . . .

Pontryagin was led to the formulation of the general time-optimal problem by an attempt to solve a concrete fifth-order system of ordinary differential equations with three control parameters related to optimal maneuvers of an aircraft, which was proposed to him by two Air Force colonels during their visit to the Steklov Institute in the early spring of 1955. Two of the control parameters entered the equations linearly and were bounded, hence from the beginning it was clear that they could not be found by classical methods, as solutions of the Euler equations. The problem was highly specific, and very soon Pontryagin realized that some general guidelines were needed in order to tackle the problem. I remember he even said half-jokingly, ‘we must invent a new calculus of variations.’ As a result, [a] general time-optimal problem was formulated. . . .⁵³

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⁵³ See Gamkrelidze’s Chapter in [Booß-Bavnbek & Høystrup 2003], pp. 160–161. The chapter goes on to explore the meaning of Pontryagin’s Maximum Principle all the way to its geometric bearing.