Two- and Three-dimensional Geometry in Tierceron Vaults: A Case Study of the Cloister at Norwich Cathedral

Nicholas Webb, James Hillson & Alexandrina Buchanan
University of Liverpool, Liverpool, UK

ABSTRACT: Located at the heart of Norfolk in East Anglia, the cloister at Norwich Cathedral has one of the most contested construction histories in English medieval architecture. Built 1297-1430 under a succession of patrons and master masons, the cloister's complex building sequence has invited a wide range of interpretations by architectural historians, including various theories regarding its design and construction process. However, these discussions have rarely taken account of the two-dimensional and three-dimensional geometry of the tierceron vaults above. A key exception to this is the work of Robert Willis (1800-75), who used what he termed the 'middle plan' to identify changes in the vault's three-dimensional form, identifying four designs in the east, south, west and north walks of the cloister. This paper uses a variety of digital surveying and analytical methods to re-examine the concept of the middle plan and its potential as a tool for comparing forms and geometries in medieval vaulting. Focusing on the east walk, it investigates the relationship between this comparative tool and the two- and three-dimensional geometries which it purportedly represents, outlining a series of design and constructional differences between the individual bays of the vault. By considering the potential implications of these observations for the building's construction sequence, the paper represents a comprehensive re-evaluation of the middle plan as a method for architectural study, suggesting new directions for research both for the cloister and construction history more generally.

1 INTRODUCTION

In his 1842 essay 'On the Construction of the Vaults of the Middle Ages,' Robert Willis introduced the concept of using a 'middle plan' to analyse the geometries of medieval vaulting. This method involved taking a horizontal section of the vault at the midpoint of its height, using the points of intersection with the rib intrados lines to plot a pattern corresponding to its specific three-dimensional form. The resulting middle plan was used as a comparative tool for identifying similarities and differences between the designs of individual bays, most notably in his study of the tierceron vaults of the cloister at Norwich Cathedral in East Anglia. Begun in 1297 and completed by 1430, this cloister was the product of a long and complex construction process including many changes in design distributed over several generations of patrons and master masons. Through detailed measurements taken using rods and plumb lines, Willis attempted to use the middle plan to quantify these changes graphically, identifying a sequence of four distinct forms corresponding to the cloister's east, south, west and north walks (Figure 1).

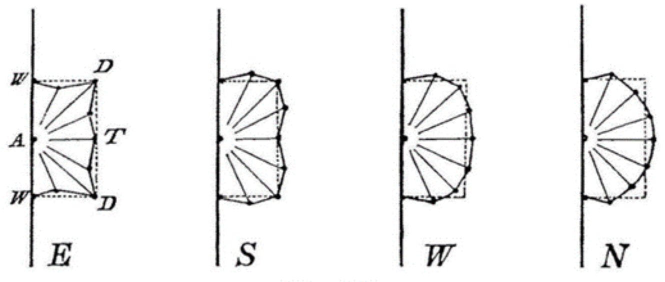

Fig. 33.

Figure 1. Norwich Cathedral Cloister, middle plans drawn by Robert Willis, published 1842.

The following discussion is an attempt to reassess the middle plan and its usefulness as a comparative method for analysing forms and geometries in medieval vaulting. The starting point for this is a digital survey of Norwich Cathedral Cloister conducted using a terrestrial laser scanner under the ‘Tracing the Past’ project at the University of Liverpool (www.tracingthepast.org.uk), a far wider series of investigations into the design and construction of vaults in England between the eleventh and sixteenth centuries. By converting our scans into 3D models, we were able to analyse the vaults at Norwich in unprecedented detail, extracting a wide range of geometrical data relating to their plans, dimensions and rib curvatures. This data was then used in conjunction with the middle plan to compare similarities and differences between individual bays, allowing us to identify at least eight distinct designs within the north, south, east and west walks (Figure 2). Focusing on the twelve regular bays within the cloister’s east walk (labelled E1-E12 from north to south), this paper examines how digital techniques can be used to investigate the relationship between the middle plan and the two-dimensional and three-dimensional geometries of medieval vaulting. In the process, it demonstrates how such a method can be used for identifying constructional differences between individual vault bays, as well as its implications for interpreting the wider building sequence and chronology of the cloister at Norwich.

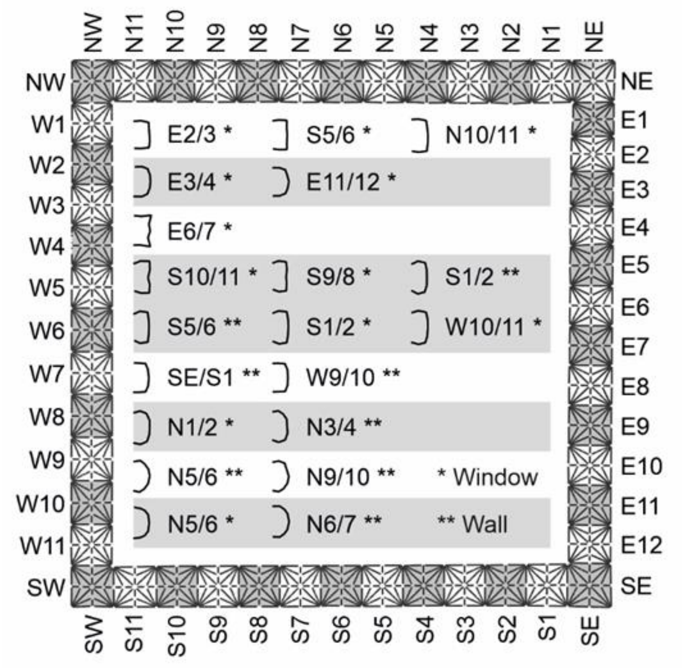

Figure 2. Norwich Cathedral Cloister, vault plan.

2 CHRONOLOGY

The chronology of Norwich Cathedral Cloister is one of the most contested in the history of English medieval architecture. This is in part because the evidence relating to its dating and construction sequence is not only extensive, but often inconclusive and at times can even appear contradictory. Surviving documentary records vary greatly in their level of detail, architectural and archaeological investigations have identified numerous points of significance in its material fabric and the stylistic developments in its window tracery and sculpture have invited diverse interpretations by architectural historians. The difficulty of interpreting this evidence is further compounded by the iterative nature of the cloister’s construction, with each bay consisting of a series of a separable elements that could have been designed, manufactured and installed at different stages of the building’s history (Figure 3).

The outer walls were provided by the previous Romanesque fabric of the cloister and the surrounding buildings, defined by the treasury, chapter house and dormitory on the east side, the refectory to the south, the guest house to the west and cathedral church to the north (Harris 2015). These were refaced in ashlar masonry and the floor level lowered, resulting in the installation of a seat and Purbeck marble columns. The forms of these were echoed on the window side of the cloister, which open out onto an approximately square garth (Figure 2). Above the capitals are the tas-de-charges for the vault above, which in the east walk appear to have been integrated into a single block along with the springers for the wall arch. This implies that both were planned and executed as a cohesive unit, presumably including the buttresses beyond. The window tracery, however, (as well as the numerous doorways on the wall side) may have been designed and installed separately, as might the wall above with its small window openings and the wooden roof. Lastly, the vaulting was not necessarily installed at the same time as the tas-de-charge stones. Consequently, though many aspects of the vault's design would necessarily have been set out at this earlier stage, others such as the style and iconography of the boss sculptures may not have been conceived until considerably later.

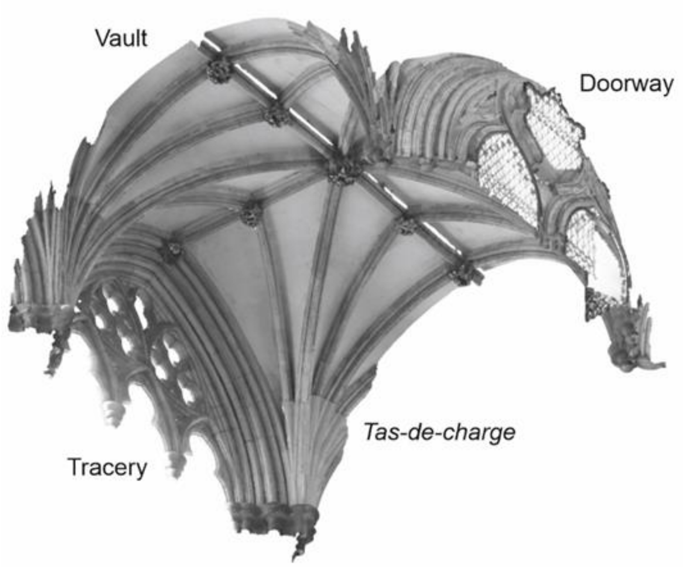

Figure 3. Norwich Cathedral Cloister, bay E7, mesh model, perspective view facing northwest.

Given the variability of the evidence in the material fabric and the frequent ambiguity of the documentary records, it is unsurprising that a wide variety of interpretations of the building's chronology have been proposed. The earliest dates from the Middle Ages, consisting of an account in the cathedral's Primum Registrum written at some point between 1430 and 1558 (Fernie 1993, 166-67). This provides a date range of 1297-1430 for the cloister, stating that work began with the entrance to the chapter house followed by the adjoining three bays (E6-E8), with the rest of the east walk (NE-E5, E9-SE) and the first ten bays of the south being built under Bishop John Ely (John Salmon) (S1-S10). The account was based on a variety of documentary sources some of which are still extant, the most notable being the apologia of John Worstead which gives a date range of 1313/14-1329/30 for the south walk. This evidence was picked up by the historians Eric Fernie and Arthur Whittingham (1972, 1993), who suggested that the whole east walk was completed 1297-1314, starting with E6-E8 (1297-99), before moving south (E9-SE) and concluding with the northeast corner (E5-NE) (1299-1314). Francis Woodman (1996), by contrast, proposed a separation between the execution of the vaulting and the walls below, with the walls being completed c. 1297-1309 and the vaults constructed after a significant gap in the works, advancing sequentially from E7-E9 (c. 1316-17) to E10-SE (c. 1317-19) to E5-NE (c. 1327-29). Veronica Sekules (2006) concurred with Fernie and Whittingham regarding the walls, but suggested a revised date of 1320-30 for the vaults based on the sculptural style of its bosses. This dating was questioned by Paul Binski (2015), who proposed an earlier date of 1297-1315 for the vaults E6-SE and 1315-20 for E5-NE. These positions have been further challenged by Robert Hawkins (2019), who has identified a slower, more iterative set of changes in sculptural style with the bosses, dating the vaults of E6-E8 to 1297-1305, E9-SE to 1305-10 and E5-NE to 1310-14. However, none of these approaches have attempted to take account of the variations in underlying geometry between the vault's bays, something which can only be accomplished through detailed measurement and analysis.

3 VAULT PLANS

The starting point for any geometrical survey of a medieval vault is its plan. By taking digital laser scans at strategic points throughout the cloister, we created detailed point cloud models made up from hundreds of thousands of individual measurements for each of the vault bays. These were then converted into mesh models and imported into Rhinoceros, an advanced 3D modelling software platform, where the intrados lines of the ribs could be re-traced in three-dimensions, producing wireframe models that were then converted into best fit arcs to quantify their curvatures. The resulting point clouds, mesh models and wireframes could then be exported as orthographic representations, producing a set of detailed and highly accurate plans for the purpose of comparison between bays.

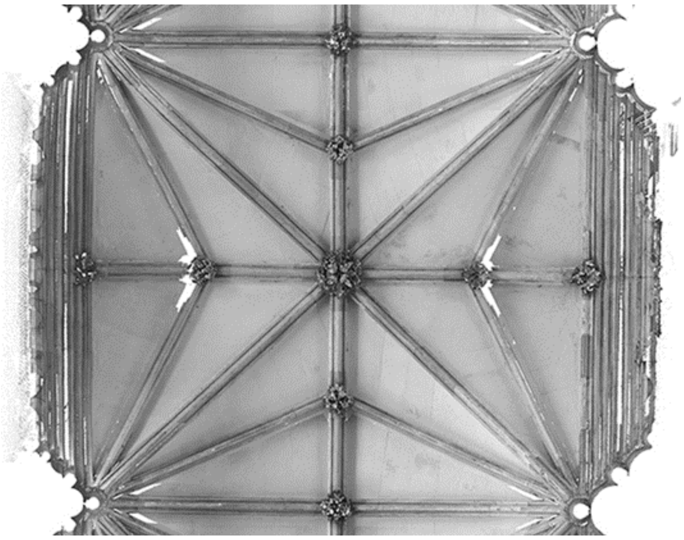

Figure 4. Norwich Cathedral Cloister, bay N7, orthophoto.

All four walks of the cloister at Norwich are covered by a relatively simple tierceron vault (Figure 4). Wall ribs and transverse arches provide the boundaries of the bay, with the remaining vault surface being divided up by diagonals and tiercerons that rise to meet the longitudinal and transverse ridge ribs. The design is remarkably consistent throughout the cloister, with only minor differences occurring from bay to bay. The principal point of variation is in the orientation of the tiercerons. Though our analyses of tierceron vault plans at other sites has often revealed the use of proportional systems for laying out rib patterns, this does not appear to be the case at Norwich. The point of intersection for the tiercerons along the ridge rib does not correspond to any known proportional division of the vault's plan, and the extended intrados lines of do not converge on any other defined point within its boundaries (Figure 5). Instead, the intrados lines converge on points just beyond the apexes of the transverse and longitudinal arches. Whilst similar methods of construction have been observed elsewhere in English medieval vaults, these largely made use of circular geometries, a method which does not seem to have been used at Norwich. Furthermore, the degree of deviation from the vault plan in these external points of convergence varies considerably from section to section, leaving it unclear exactly how the orientations of the tiercerons were derived. One possibility is that they were determined by the radial cuts flanking the boss stones rather than a single point, with the changes in angle corresponding to variations in masonry. Another is that it represents a more ad hoc optical correction to the vault's form, analogous to the minor geometrical manipulations which we have previously identified in the choir and nave vaults at Exeter cathedral (Buchanan and Webb 2019).

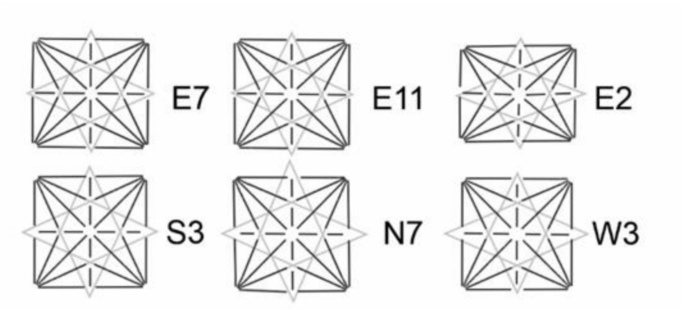

Figure 5. Norwich Cathedral Cloister, vault plans with best fit arcs (black) and tierceron intrados lines (grey).

Throughout the vaulting of the east walk the layout of the tiercerons is relatively consistent, with the main differences being in the proportions of the individual bays. Bay sizes are variable throughout the cloister, with differences appearing both between walks and internally within them. The south walk is perhaps the most consistent, with approximately square bays averaging 4.05m on the longitudinal axis (east-west) and 3.97m on the transverse axis (north-south). By contrast, the vaults in the east walk are slightly wider in the transverse direction (averaging 4.15m) and the distances spanned by the longitudinal axis vary considerably from bay to bay. The longitudinal dimensions of bays E5-E11 sit at a consistent average of 3.93m, but those of bays E4 and E12 (average 3.75m) and E1-E3 (average 3.39m) are significantly smaller. This has long been ascribed to the location of the chapter house entrance, as this is slightly offset from the position which would be necessary for an even distribution of the bays. If E6-E8 are the earliest part of the walk to be constructed as has been suggested, then it appears that the initial set of dimensions were continued southward for as long as possible, with some compression being required to span the uneven gaps left at the north and south ends. Yet whilst the dimensions of the vault plan might vary from bay to bay, the fundamental geometry on which it was based appears to have remained fairly consistent through its 133 years of construction, especially within the east walk.

4 MIDDLE PLANS

Such consistency in two-dimensional geometry does not seem to have extended into the vault's three-dimensional form, which can differ significantly from bay to bay. The most straightforward means of visualising these changes is provided by the middle plan, which could be drawn digitally using Rhinoceros. Starting with the wireframe models of intrados lines traced from the cloister vaults, we identified a horizontal plane at the midpoint between the apex height and impost level given by the abaci of the capitals below. The 'section' command was then used on the wireframe, identifying the points of intersection between the plane and the rib intrados lines. These points were then connected together to form a two-dimensional plan, which could then be exported as a more accurately measured digital equivalent of the middle plans produced by Willis. Whilst the height of the ridge rib does fluctuate slightly from bay to bay, the resulting sections nevertheless provide a reasonable method for identifying formal changes within a run of vaulting, their particular forms corresponding directly to the differences in their geometry and proportions.

Detailed study reveals a range of different types of middle plan within the confines of the east walk alone (Figure 6). Bays E5-E9 share a common pattern, with the diagonals taking the distinctive form of prominent spikes. It was this which was the middle plan identified with the east walk by Willis (1842), suggesting that the bay which he selected lay somewhere within this run (Figures 1-2). The tas-de-charges connecting E9 to E10 and E4 to E5 are both transitional, one half reproducing the plan of E5-E9 and the other the more rounded form found in bays E11-E12. This rounded type can also be found at the intersection between E3 and E4, but for E2 to E3 the middle plan changes again, taking on a new, more rectangular form. Sections of the ribs taken at the level of the abacus reveal that all of the tas-de-charges in the east walk had a similar pattern at their springing point, yet as the ribs rose their curvatures began to diverge. As the underlying geometry of the vault plan was relatively similar throughout the east walk, there were therefore only two possible causes for these changes in three-dimensional form: differences in bay sizes or differences in the curvatures of the ribs.

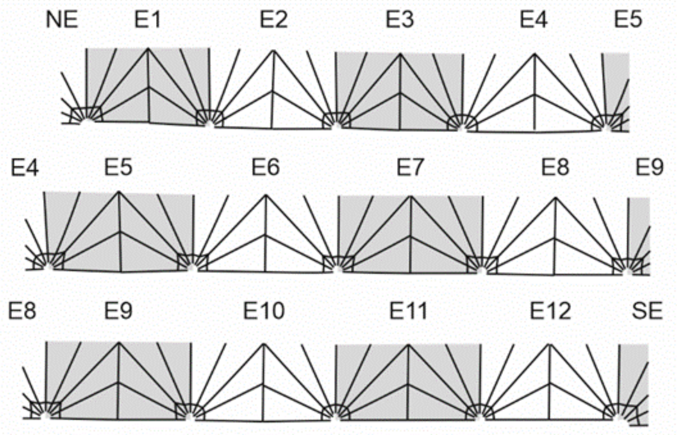

Figure 6. Norwich Cathedral Cloister, east walk, window side, middle plans.

5 RIB CURVATURES

As our far wider study of medieval English vaults has demonstrated, the majority of ribs in medieval vaults were laid out in two dimensions as curves constructed through simple geometrical operations. Using the best fit arcs derived from the tracing process described above, we were able to extract geometrical data for each of the vault ribs in Norwich Cathedral Cloister, specifically their springing points, spans, notional apex heights, radii and the vertical and horizontal positions of their centres. Through fixing some of these variables in advance, vault designers were able to provide a set of parameters within which a rib's curvature could be defined, deriving the remaining variables by selecting an appropriate geometrical method. Through studying the patterns presented by these geometrical data, we were able to propose hypotheses regarding the design process for each rib within the cloister. Each vault's hypothetical design process was then modelled digitally and the resulting wireframe compared directly to the traced intrados lines, allowing us to assess its accuracy both in terms of geometrical data and visual congruity. Through a rigorous process of trial and error, we were able to discover at least three different geometrical designs within the east walk alone, as well as number of minor variations from bay to bay.

Perhaps the earliest of design of vaulting can be found in E6-E8, the three bays which directly front the entrance to the chapter house (Figure 7). The apex height of the longitudinal ridge appears to have been defined in advance (average 2.07m), though it is not at present clear exactly how it was derived. The springing points and spans for each rib would have been defined on the vault plan, perhaps using 1:1 scale drawings conducted on a tracing floor or another large two-dimensional surface (Pacey 2007). For the transverse ribs (AF, BG, CF and DG), the centre appears to have been positioned on the plane of the impost, suggesting that they used the 'chord method'. This involved constructing the perpendicular bisector of a chord drawn between the apex and springing point of the rib, defining the centre using the line's point of intersection with the impost level. The same method appears to have been applied to the longitudinal ribs (AE, BE, CH and DH), though the apex heights of these are slightly higher than the ridge in order to accommodate the window and wall arches below (average 2.17m). A similar radius to the transverse ribs can also be found in the tiercerons (AJ, AK, BJ, CK etc.), but the level of the centre also approximates that of the impost. Either the chord method was used again, or the 'three circles' method was used to transfer the radius from rib to rib. This involved drawing two circles of a defined radius centred on the apex and springing point of the rib, with the lower point of intersection between them giving the missing centre. For the diagonal ribs (AZ, BZ, CZ and DZ), the best correlation which we found involved using the ‘two chord’ method. This involved defining a third point along the circumference of an adjacent rib’s curvature, in this case the transverse rib. Using the vault plan, the horizontal position of this third point could then be projected orthogonally, allowing it to be relocated along the diagonal rib’s span. By constructing the perpendicular bisectors of the two chords formed by springing point, third point and apex, it was possible to define the centre using the intersection between the two perpendicular lines. When viewed in section along the vault’s tunnel, the result would be a similar apparent curvature for the two ribs, although not an identical one. This hypothesis is further supported by the middle plan, in which the position of the diagonals and transverse ribs can be located fairly closely along the same line perpendicular to the wall, strongly suggesting that the third point was located roughly at the midpoint of the vault’s height.

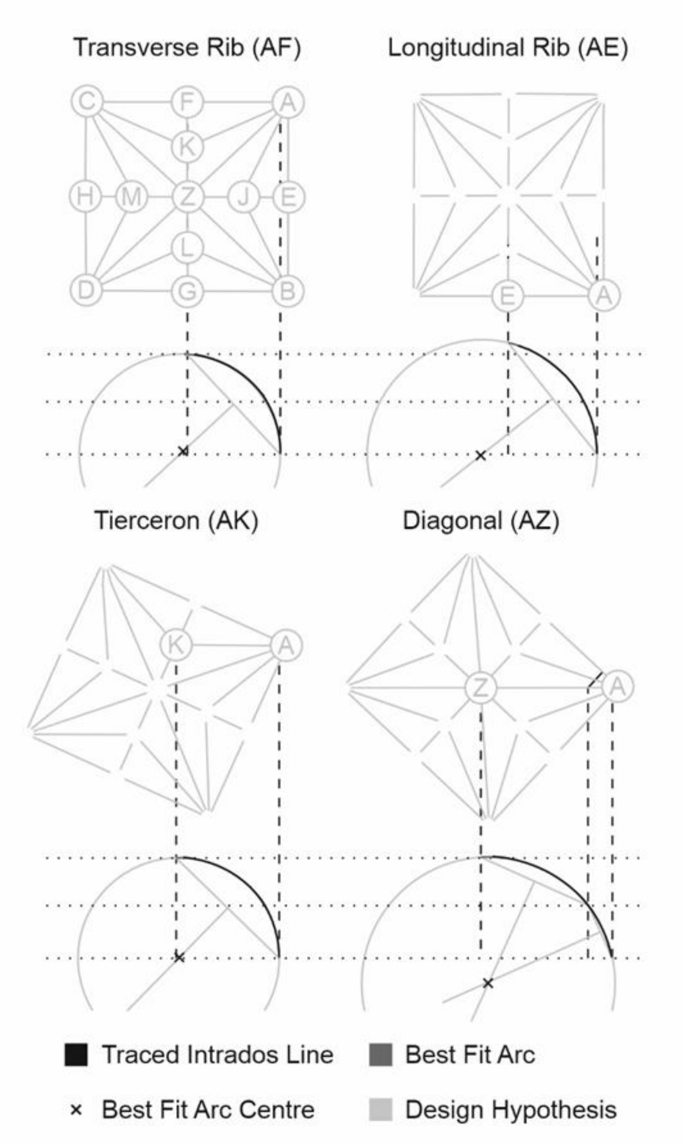

Figure 7. Norwich Cathedral Cloister, bay E7, rib geometry.

The same design appears to have been used for bay E9, but from E10 southwards the geometry of the diagonals changes. When tracing their curvatures, it quickly became apparent that the rib intradoses could not be reproduced using a single centred arc. Instead, it seems more likely that they were designed using a two-centred arc. However, this possibility introduced the problem of how to identify the point of transition between the two curvatures. Focusing on bay E11, we adopted a parametric approach to testing, creating best fit arcs by dividing the traced data incrementally at 5%

intervals of the vault’s apex height. The closest results were found markedly above the level of the middle plan, situated firmly within the 55-65% margin (~1.14-1.35m). Yet the problem with this approach was that it was based on a theoretical point defined mathematically and did not take account of the physical structure of the masonry. By using orthophotos, photographs and other survey data, we were able to identify the horizontal and radial cuts of the component tas-de-charge stones and voussoirs for each of the diagonal ribs. By transferring the positions of these cuts onto the traced intrados lines within Rhinoceros, we were able to test the effects of constructing best fit lines divided at different joints. The closest match which we could find between model and reality was produced by defining the point of transition at the first radial cut of the tas-de-charge. This varied considerably from bay to bay, but was usually within the 55-65% margin that had previously been observed. It therefore seems likely that the lower curvature of the diagonal was applied to the tas-de-charge, with the upper belonging to the voussoirs above.

As the lower radius of the diagonals in E10-E12 is close to that of the transverse arches and tiercerons and the centre appears to be on the impost level, it is likely that it was set out using the ‘two circles’ method (Figure 8). This involved drawing a circle of the previously defined radius centred on the springing point, using its point of intersection with the impost to define the centre. The point of transition would then be set according to the tas-de-charge, providing an effective springing point for the upper curvature of the rib. However, the derivation of the upper radius is more difficult to define. Whilst the best fit arc does generally fit the traced data closely, it consistently pulls away slightly as it approaches the point of transition to the lower arc (Figure 9). This raises the possibility that there may have been some kind of transitional block or voussoir between the two arcs. However, such a disparity would also be consistent with many of the best fit lines we have constructed for single-centred arcs, and could therefore be either an inherent flaw in the retracing process or the result of some form of settlement. The closest result which we found for reproducing the upper curvature involved constructing a segmental arch, using a variation of the chord method where the centre was defined by the intersection between the bisector and a vertical line descending from the apex of the rib.

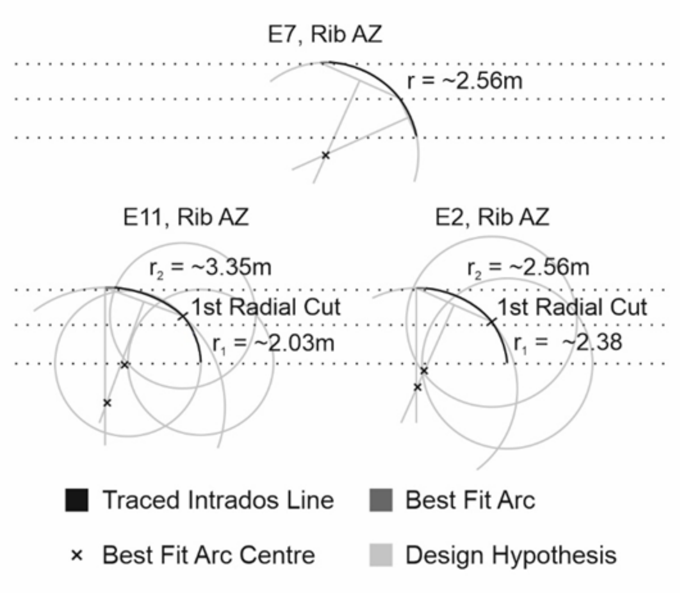

Figure 8. Norwich Cathedral Cloister, diagonal rib geometries.

The process described above marks a significant departure from the methods proposed by Willis (1842), who set out two more complex geometrical processes for ensuring a smooth transition between the two curvatures. However, when these were tested using parametric modelling. the results for the east walk were very different from the traced intrados lines. Closer inspection of the diagonals reveals that there is a sharp, visible disjunction in curvature between the upper and lower arcs.

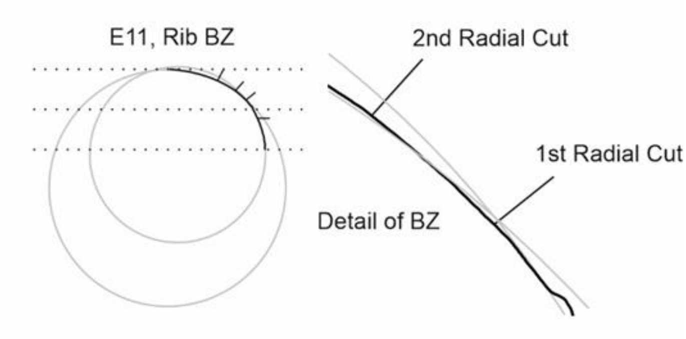

Figure 9. Norwich Cathedral Cloister, diagonal rib geometries.

A more complex situation can be seen at the north end of the east walk. As in the case of bays E9-E12, the vault in bay E5 follows the same design as E6-E8, and the diagonals in E4 are modified to reproduce the same geometry as E10-12. In bay E3, however, the setting out method for the diagonals appears to have changed again, with the southern half corresponding to those in E4 and the northern half employing an entirely different two-centred curvature (Figure 8). The lower radius is significantly larger than those on the south, approximating the radius of the longitudinal rib, and the level of the centre is significantly below that of the impost. This suggests that the three circles method may have been used, with the radius being transferred from the longitudinal rather than transverse arches. It is not clear how the apex height of this lower curve may have been derived in this instance, but the upper radius appears to have been set out using the same segmental arch method as that used in bays E10-E12.

The differences in geometry described above correspond precisely to the differences in middle plan between the vault's bays. However, geometry is not the only feature to change between these sections, as the longitudinal dimensions of the bays in E1-E3 are radically different to those in E4-E12. By using parametric modelling, we able to investigate the relative effects of changes in geometry and changes in bay size on the form of the middle plan. The resulting models demonstrated that the effect of the bay dimensions on the middle plan was relatively negligible, suggesting that it was the geometry which was the principal difference in three-dimensional form between the different sections of the walk.

6 CONCLUSIONS

As the preceding discussion has demonstrated, the middle plan remains an effective means of identifying changes in the design process and constructional technique between individual vaults. Digital surveying and analytical techniques allow for fast, accurate comparison between vault bays, allowing a far more detailed comparison of sections and three-dimensional geometries than was possible using analogue methods. Our study of the cloister at Norwich has revealed a far more graduated series of design changes than Willis' original investigation had identified, further complicating the contested design sequence of this building. Whilst the evidence does support the traditional sequence starting with E6-E8 then advancing southwards before continuing north, the disparity in middle plan and geometry between E4-E3 and E3-E1 suggests a more convoluted design and construction process than has hitherto been suspected, perhaps taking the form of an additional interruption in building at tas-de-charge level within bay E3. Whereas it has often been supposed that architectural designs were particular to individual master masons, the transitional forms of the tas-de-charges at E9/E10 and E5/E4 suggest that they were instead quite capable of reusing old designs even whilst in the process of adopting their new replacements.

Though the reasons behind these design changes remain inscrutable, some possibilities are suggested by details in the material fabric. If there was a significant gap between the installation of the tas-de-charge and the vaulting, then a division between two curvatures at this level for diagonals could be advantageous as it would allow for more flexible adjustments at a later stage. Such a change may have been prompted by a flaw within the original bays, as the window side tas-de-charge stones for bay E7 include evidence of later modifications to increase their height (Woodman 1996). However, as the transitional voussoirs in bays E10-E12 demonstrate this was not always successful, an observation which may well have prompted the further changes found in bays E3-E1. Alternatively, it may simply have been an attempt to modulate the vault three-dimensional form through geometrical manipulation, perhaps in order to obtain a specific optical effect.

A final point arising from this study is that the middle plan is not solely a modern analytical tool, but may also have had a degree of real design value for medieval masons. Though it remains unlikely that middle plans were actually drawn out during the design process, the use of the two-chord method in bays E6-E8 suggests that the midpoint of a vault's height could at times be a significant part of the design process. Something similar could also be argued for the two-centred arcs, though the point of transition which defined them was more variable and usually significantly above the level of the middle plan. It is possible that further study of the cloister and other sites may reveal the involvement of additional types of level-based geometry in setting out rib curvatures, with the middle plan serving as a critical means for identifying the methods involved.

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