Shai Simonson

The Mathematics of Levi ben Gershon

Much of the history of mathematics focuses on the Greeks (500 B.C.E.–400 C.E.), the founders of modern Western mathematical rigor. For a long time, the common notion was that little happened from the end of the Greek period until the solution of the general cubic equation in the Renaissance. However, mathematics in the period between the Greeks and the Renaissance, 400 C.E.– 1500 C.E., showed originality in both presentation and content.

One of the great mathematicians of this period, and perhaps the greatest of his generation, was Rabbi Levi ben Gershon (1288–1344), who until recently has remained in undeserved obscurity. His mathematics can be used at many different levels in the classroom. Levi—who was a rabbi, philosopher, astronomer, scientist, biblical commentator, and mathematician—was born in Provence in 1288 and lived there all his life. A prolific author, he wrote approximately one book a year from 1321 until his death in 1344. He wrote more than a dozen books of commentary on the Old Testament, a number of books on mathematics and logic, and a major philosophical work that included a groundbreaking chapter on astronomy.

Levi wrote in Hebrew, and most of his mathematical work has never been published in English. Scholars believe that he read only Hebrew and not Greek, Latin, or Arabic. The evidence comes from a surviving list of books from his personal library. All the books on the list are in Hebrew, including Hebrew translations of many major scientific and philosophical works available at the time. Besides recording his personal library, this list furnishes a rare sample of Levi's handwriting. The other extant manuscripts of his works are by later scribes. A complete bibliography appears in Kellner (1992).

The following list shows the diversity and scope of Levi's work.

- The earliest rigorous use of mathematical induction
- Pioneer work in the field of combinatorics
- Algorithms for square- and cube-root extraction, as well as solutions to a variety of simultaneous linear equations
- A commentary on Euclid, including an attempted proof of the fifth postulate
- A large work on trigonometry
- An elementary proof that the only pairs of harmonic numbers (numbers in the form 2^n3^m) that differ by 1 are the pairs (1, 2), (2, 3), (3, 4), and (8, 9)
- Important astronomical observations of the motions of the moon, earth, and sun and related theories that predate those of Copernicus and Kepler
- Invention of the Jacob's staff, a device that measures angles between heavenly bodies and that was for used for centuries by European sailors for navigation; **figure 1** depicts the use of the device but does not show the detailed and complex measurement markings that were etched on it.

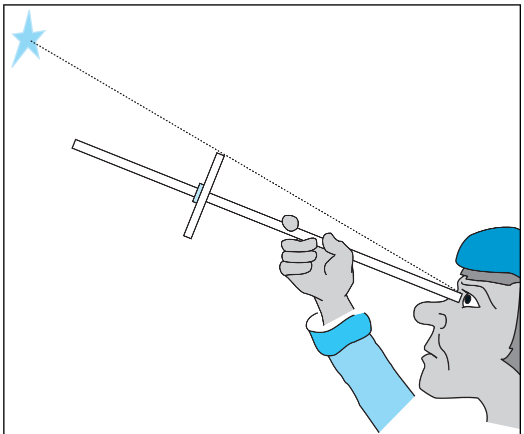

Fig. 1
The Jacob's staff

The mathematics of Levi ben Gershon can be used at many different levels in the classroom

- Description of the principles of the camera obscura (dark chamber), the forerunner of the modern camera

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We can only speculate on why Levi did not enjoy more recognition, but possible explanations include the following:

- His audience was not able to appreciate the material. Some of his mathematics and science was ahead of its time. In particular, the material on what we now call discrete mathematics and combinatorics was not likely to have been appreciated by his audience.
- His audience was restricted mainly to people who could read Hebrew. Although some of his works were translated by Christian scholars into Latin while he was still alive, most of his work was available only in Hebrew.
- Among those who read Hebrew, he was a controversial figure because of his rationalist philosophy, which foreshadowed the Renaissance by two centuries. His works were unpopular and even shunned by some. For example, his major philosophical work, *MilHamot Adonai* (Wars of God), was dubbed by critics *MilHamot Im Adonai* (Wars with God). Many Jews may have considered all his work, even the scientific material, to be off-limits.

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Little information exists about Levi's personal life except that he came from a family of scholars and learned men. The literature contains many references to possible identities of his immediate family; however, most of these references are not completely established and some information is speculative or contradictory.

Life for Jews in most of fourteenth-century France was very difficult. France, and most of Europe at the time, was a collection of semi-independent regions—dukedoms, city-states, and papal states—which were often at war with one another. The king of France had loose control over all the provinces, but local control varied. In most of these regions, Jews were persecuted and blamed for economic hardship, plagues, and famines. Only in the region of Provence was life relatively good. In Provence, the Pope was self-exiled in Avignon, where he protected the Jews and allowed them to live with little or no oppression.

The total population of approximately two million people in fourteenth-century Provence included fifteen thousand Jews. The Jews worked as moneylenders, physicians, craftsmen, and merchants. Agriculture and farming were not possible occupations for them because of legal restrictions on who could control land. No established information indicates Levi's occupation, whom he influenced, who influenced him, or what he was like as an individual. Historians know, however, that he corresponded with important Christian scholars and that he probably had a group of students (Glasner 1995). He dedicated his book on trigonometry to Pope Clement VI, and he was commissioned by Phillip of Vitry, Bishop of Meaux, to write *De Numeris Harmonicis* (On harmonic numbers), a book that was immediately translated from Hebrew to Latin. His Christian colleagues called him *Leo Hebraus* or *Maestro Leon*. Jewish biblical scholars called him *RaLBaG* (the capital letters stand for Rabbi Levi Ben Gershon); and modern scholars call him *Gersonides*.

MANUSCRIPTS—PUBLISHING BEFORE THE INVENTION OF PRINTING

Publishing a book in the fourteenth century was different from publishing a book today. Printing presses, publishing companies, copyrights, royalties, and word processors did not exist. An author would sit on the floor and write out the manuscript by hand on costly paper. A student would often act as a scribe, using the task as an opportunity to study while proofreading and editing the author's work. Sometimes a professional scribe, who might have no understanding of the content, would be hired.

When the job was done, it was sent to someone or to a community of people who were interested in reading the manuscript. These people might make their own copies, and so on. The manuscripts might be kept in a public place, like a synagogue or church, or they could be the private property of wealthy patrons or scholars. Copies of a book might not be identical. The scribe might use the local dialect to change certain phrases. The placement of illustrations and examples, as well as the general formatting, would depend on the style of the scribe and the size of his paper. Errors would occur, especially in the work of a hired scribe who did not understand the material that he was paid to transcribe. Finally, a private owner of a manuscript would commonly write comments in the margins of the manuscript, correcting errors or adding commentary as he studied.

MAASEH HOSHEV

The topic of Levi's first book was logic. His second book, *Maaseh Hoshev* (The art of calculation), written in 1321, was a major mathematical work in two parts. The first part is a collection of sixty-eight theorems and proofs in Euclidean style about arithmetic, algebra, and combinatorics. The second part contains algorithms for calculation and is subdivided into six sections: "Addition and Subtraction"; "Multiplication"; "Sums"; "Combinatorics"; "Division, Square Roots and Cube Roots"; and "Ratios and Proportions."

The last section is followed by a long appendix of problems, which is about one-fifth of the whole book. Lange (1909) completed a critical edition of *Maaseh Hoshev* with a translation to German in

1909, but the last section of problems is completely missing, and other minor omissions were caused by Lange's lack of access to all the extant manuscripts. A critical edition of the problem section appears in Simonson (forthcoming). Other articles about *Maaseh Hoshev* include discussions of its content and its use of mathematical induction (Chemla and Pahaut 1992; Katz 1993; Langermann and Simonson forthcoming; Rabinovitch 1970; Simonson 2001).

The only known copy of *Maaseh Hoshev* in the United States is in the rare book library of the Jewish Theological Seminary in New York. This copy is the source of most of the mathematical material in this article. The whole first half of the work and the last few pages of the second half are missing. Scattered comments in the margins are in at least two different handwritings, and a comment indicates that the manuscript was purchased in Hebron in 1909. The manuscript is bound into one book with a ten-page fragment of another medieval mathematics manuscript, *Sefer Hamispar* (The book of number), written by Abraham Ibn Ezra in the twelfth century. Presumably, some bookseller noticed the similar content and time period, bound the two fragments together, and sold them as medieval mathematics manuscripts. This practice was common and serves as an example of the exciting mysteries that accompany these manuscripts.

The twelve remaining manuscripts of *Maaseh Hoshev* are located all over the world. More copies may be forgotten in some old dusty basement, but any other copies were probably lost or destroyed over the years. Most of the twelve manuscripts that remain are incomplete. The first or last few pages have often fallen off. Wormholes have left some paragraphs unreadable. Some pages have turned yellow or brown and have deteriorated.

Handwriting, script styles, margin comments, watermarks, paper quality, and ink quality are all clues in determining the provenance of a manuscript and its history, but perhaps the clearest indication of the date of publication is the colophon. Although a medieval manuscript generally had no table of contents or index, it often had a colophon. A colophon is a short paragraph at the end of the manuscript that identifies the author and the date and gives a brief conclusion to the work. Unfortunately, these last pages are the ones that tend to fall off and get lost. Some manuscripts of *Maaseh Hoshev* bear a colophon that reads as follows:

> The author writes: The sixth section of this part is now finished, and with its completion, the book is complete. And praise is due only to God. And its completion was at the start of Nissan in the year 81 of the 6th millennium, as I reached the 33rd of my years. And bless the Helper.

The dates to which Levi refers are from the Hebrew lunar calendar. The fifteenth of Nissan is the start of Passover, and the year 5081 corresponds to 1321. Hence, *Maaseh Hoshev* was finished in spring 1321, when Levi was thirty-three years old. This evidence, incidentally, is how we know that his year of birth was 1288.

Other copies of *Maaseh Hoshev* bear colophons that indicate that the work was finished in fall 1322. See **figure 2.** Interestingly, these later colophons say nothing about Levi's age at the time of the manuscript's completion. These alternative colophons imply that Levi ben Gershon issued a second edition of *Maaseh Hoshev*. This evidence about a subsequent edition sheds light on his work habits and lifestyle (Simonson forthcoming).

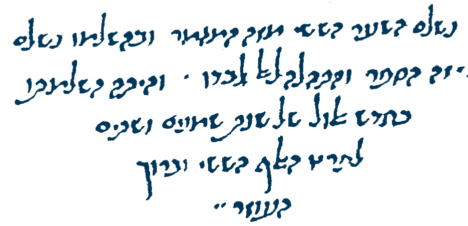

Fig. 2
Colophon from a manuscript of *Maaseh Hoshev*

A TASTE OF LEVI'S WORK

Levi's mathematics was diverse and brilliant. The small sample shown here gives a sense of his style. Ample material for the classroom can be found in his work, and the interested reader should consult the **bibliography.**

Word problems appear in the very earliest mathematical works, including those of the Babylonians and the Egyptians. This tradition of pedagogy and exposition has continued throughout history. Part 2, section f, of *Maaseh Hoshev* includes a large collection of word problems and solutions that deal with proportions and linear equations. These problems can be found in Simonson (forthcoming). The material is interesting, if only because it is old. Solving a word problem is perhaps more appealing if it comes from a 680-year-old manuscript than if it appears in the review exercises at the end of the chapter. Romance aside, seeing these problems in their historical context compels us to think of them as more human. Furthermore, the problems reflect on the day-to-day economic and sociological structure of fourteenth-century society and are therefore an interesting historical tool for a view of that culture and time. In any event, trying to solve the puzzles and looking at the solutions presented by Levi are fun.

Perhaps solving a word problem from a 680-year-old manuscript is more appealing

The author of this article translated the following examples from the Hebrew and added the punctuation for the sake of clarity.

Question. A merchant sells products of varied prices, and a buyer wants to purchase an equal measure of each product using a given quantity of his money.

Solution. The method is to add up the prices per single measure of each product and remember the result. And he should take from each product the fraction of a measure equal to the ratio of the total money in his hand to the remembered result. And this is what was requested.

Example. An example of this is when a merchant sells four medications. The cost of the first medication is 7 peshutim per litra; the cost of the second medication is 8 peshutim per litra; the cost of the third medication is 10 peshutim per litra; the cost of the fourth medication is 15 peshutim per litra. And the buyer comes to buy an equal quantity of each with 3 dinars. And with 3 dinars and 4 peshutim he can take a litra of each one. And the ratio of 3 dinars to 3 dinars and 4 peshutim is 9 tenths, and so he should take from each medication. I mean to say 9 tenths of a litra. And the total price is 3 dinars.

The problem is not difficult, but the solution is presented differently from the way that it would be presented today. Certainly we would set up an equation with the total price equal to some unknown times each of the given prices per litra, and we would solve for that unknown. Perhaps Levi's method offers a fresh look at the problem without wielding the unnecessary heavy tool of algebra.

More interesting perhaps is that the problem incidentally teaches us the exchange rate between peshutim and dinars. Deducing that twelve peshutim are in one dinar is a simple exercise. *Peshut* is from the Arabic *pashit* and is used in the Talmud to mean a small coin probably of copper, whereas *dinar* is used to denote larger coins of silver or gold. In fourteenth-century Provence, coins included a small, thin copper-and-silver coin called the *denier* and a large silver coin called the *gros tournois,* which was worth twelve deniers. See **figure 3.** Levi seems to have used the anachronistic Talmudic terms to refer to the actual local coinage of his area.

The study of original sources is an adventure in the sciences and the humanities

Fig. 3
Gros tournois, a heavy silver coin that was one-half inch in diameter and that was equivalent to twelve deniers. Note the twelve distinct markings around the outside edge.

The following problem is more famous; variations of it have appeared as recreational mathematics puzzles for centuries (Singmaster 1995).

Question. A certain container has various holes in it, and one of the holes lets all the contents drain out in a given time. And so on for each of the holes. How much time will it take to empty the container when all the holes are opened?

Solution. First, calculate how much drains from each hole in an hour, add them all up, and note the ratio to the full container. This ratio is equal to the ratio of one hour to the time it will take to empty the container when all holes are open.

Example. For example, when a barrel has some holes, and the first hole empties the full barrel in 3 days, and the second hole empties the full barrel in 5 days, and from another hole the full barrel empties in 20 hours, and from another hole the full barrel empties in 12 hours. And so the first hole empties 1 of 72 parts of the barrel in an hour, the second hole 1 of 120 parts, the third hole 1 of 20 parts, and the fourth hole 1 of 12. And when we add them all up, the total that empties from all the holes in an hour is 56 of 360 parts of the full barrel. Divide this into 1 full barrel and we divide 360 by 56 to get 6 whole and 25 first parts and 43 second parts, and so the time to empty the barrel is approximately 6 hours and 25 minutes and 43 seconds.

When Levi uses "first parts" and "second parts," he is referring to 1/60ths and 1/3600ths. In his time, people commonly used mixed bases, base ten for the integer part and base sixty for the fractional part. In base sixty, more fractions have nonrepeating representations.

More than twenty problems, mostly longer and more complex than the ones presented here, are included in part 2, section f, of *Maaseh Hoshev*. Levi's problems can supply a large amount of classroom material.

Using historical original sources is a great way to interest students and teach mathematics. The mathematics of the Middle Ages is a particularly underused source. Levi's work furnishes a variety of good examples, and the material presented in this article is a tiny sampling of this untapped reservoir. For more problems and material by Levi ben Gershon than would fit in this article, contact the author by e-mail at shai@stonehill.edu, or send a self-addressed, stamped envelope.

The study of original sources is an adventure in the sciences and the humanities for the mutual benefit of teaching and research.

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