# ADDITIONAL IDENTITIES INVOLVING MERSENNE PRIMES

There are numerous point functions which can be used to generate prime numbers. Many of these go back hundreds of years and new ones are continually being discovered. We have been examining primes as a hobby for well over a decade and have found numerous new identities such as the fact that any number N is a prime p provided -

$$\sigma(N^2)=1+N+N^2$$

, where σ(N) is the sigma function of number theory.  Furthermore we have shown in earlier articles found on our Tech-Blog Web page that any prime five or greater must satisfy $N=6n\pm1$ and always have its number fraction-

$$f(N)=[\sigma(N)-N-1]/N$$

vanish.

Perhaps the best known and most studied of all primes have been and continue to be the Mersenne Primes first proposed by the French cleric, philosopher and mathematician Marin Mersenne (1588-1648). To date only 51 of these primes have been found so that they constitute only a very small fraction of possible primes in a given range. The definition of Mersenne Primes is-

$$M(p)=2^p-1$$

valid only for restricted primes p. Here is a list of the presently known 51 Mersenne Primes-

## List of known Mersenne primes

$M(p)=2^p-1$ with p equal to the primes

2, 3, 5, 7, 13, 17, 19, 31, 61, 89, 107, 127, 521, 607, 1279, 2203, 2281, 3217, 4253, 4423, 9689, 9941, 11213, 19937, 21701, 23209, 44497, 86243, 110503, 132049, 216091, 756839, 859433, 1257787, 1398269, 2976221, 3021377, 6972593, 13466917, 20996011, 24036583, 25964951, 30402457, 32582657, 37156667, 42643801, 43112609, 57885161, 74207281, 77232917, 82589933.

data source-Wikipedia

It is our purpose here to examine this list of primes in greater detail and thereby find some new identities. We begin by noticing that a mod(6) operation on any Mersenne Prime with p three or greater satisfies-

$$M(p)\mathrm{mod}(6)=1 .$$

Thus, for example p=19 where M(19)=534207, yields-

$$M(19)=6(87381)+1$$

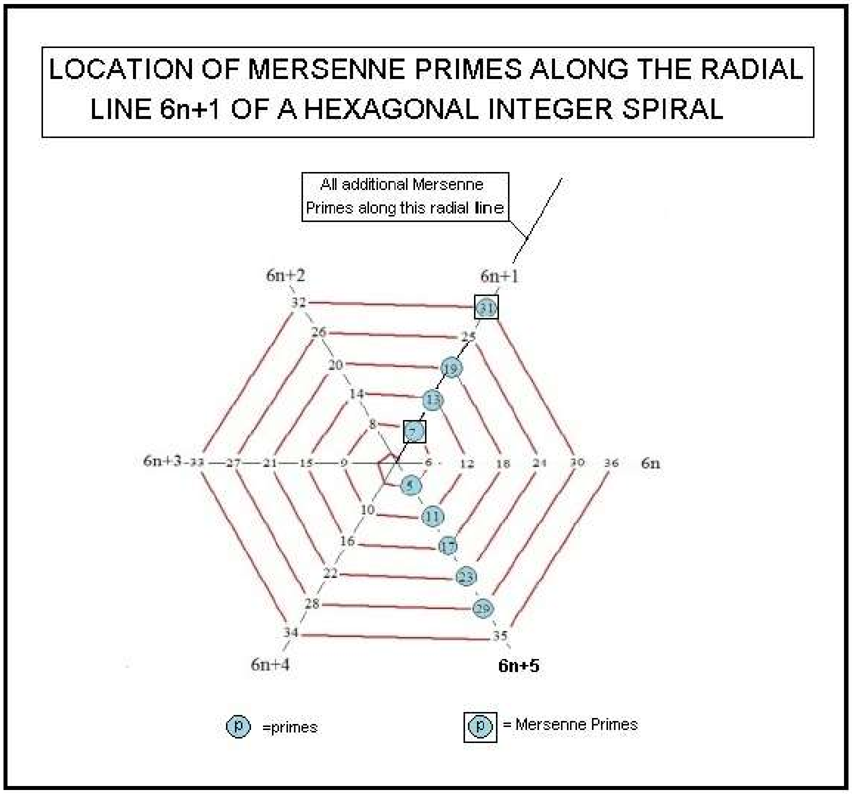
This means that M(19)-1 is a number divisible by six and hence that

M(19) has the form 6n+1 .

A convenient way to look at the Mersenne Primes geometrically is use of a hexagonal integer spiral first discovered by us about a decade ago while examining the Ulam Spiral. Here one has the point function-

$$[r,\theta] =[N,N\pi/3]$$

starting at N=5 and $\theta=-\pi/3$. Also one superimposes six radial lines which intersect the spiral at its vertexes. The resultant picture follows-




You will see there that all primes five or greater fall along the two radial lines 6n±1 and nowhere else. The more restrictive Mersenne Primes are found to lie only along the radial line in the first quadrant. Specifically the graph shows M(3)=7 and M(5)=31 in square boxes. All higher M(p)s also lie along this same radial line 6n+1.

Notice that one turn of the spiral raises the integer along the same radial line by six units. This must mean that-

$$M(p)-M(q)=6n$$

So, for instance,-

$$M(13)-M(5)=8191-31=8160=6(1360)$$

Application of this factor of six approach should make it possible to find the next Mersenne Prime number 52 (by the GIMPS approach) much faster since many numbers can be ignored in any prime number calculation.

A relation between M(n) and M(n+m)can be found as follows. We first note that-

$$M(n+1)=2M(n)+1$$

$$M(n+2)=2^2M(n)+3$$

$$M(n+3)=2^3M(n)+7$$

$$M(n+4)=2^4M(n)+15$$

On generalizing these equalities one finds-

$$M(n+m)=2^m[M(n)+1]-1$$

Thus-

$M(19)=2^{14}[M(5)+1]-1=524287$ using $M(5)=31$.

A table of the first ten Mersenne Primes generated by M(n+m) starting with M(2)=3 follow-

| integer-n | M(n) |
|---|---|
| 2 | 3 |
| 3 | 7 |
| 5 | 31 |
| 7 | 127 |
| 13 | 8191 |
| 17 | 131071 |
| 19 | 524287 |
| 31 | 2147483647 |
| 61 | 2305843009213693951 |
| 89 | 618970019642690137449562111 |

This same table can also be generated by the known Mersenne Primes insert given earlier. Note that the ns and M(n)s in this table are both primes as required. It is possible that n is prime but M(n) is not. In that case we are dealing with Mersenne composites of the form 6n+1. An example of such a number is the composite M(11)=2047=23 x 89.

One of the better known ancient Greek mathematician and geometers was Euclid of Alexandria. He published his famous book “The Elements” in about 300BC. In it he anticipated the Mersenne Primes by noting the following-

1+2=3

1+2+4=7

1+2+4+8=15

1+2+4+8+16=31

which in more modern language reduces to the general form-

$$\sum_{k=0}^{p-1} 2\hat{}k = 2^p - 1 = M(p)$$

That is, Mersenne Primes and composites can be written as finite length series in powers of 2. As an example we have-

1+2+4+8+16=2^5-1=31=M(5)

Note that if M(p) is a prime so will p be. However the reverse does not hold.

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