# A Novel Approach for Designing Diversity Radar Waveforms that are Orthogonal on Both Transmit and Receive

Uttam K. Majumder, Mark R. Bell
School of Electrical and Computer Engineering
Purdue University, West Lafayette, Indiana 47907
Email: umajumde, mrb@purdue.edu

Muralidhar Rangaswamy
AFRL Sensors Directorate
WPAFB, Dayton, OH 45433

***Abstract*—In this paper, we present an approach to the design of orthogonal, Doppler tolerant waveforms for diversity waveform radar (e.g. MIMO radar). Previous work has given little consideration to the design of radar waveforms that remain orthogonal when they are received. Our research is focused on: (1) developing sets of waveforms that are orthogonal on both transmit and receive, and (2) ensuring that these waveforms are Doppler tolerant when properly processed.**

**Our proposed solution achieves the above mentioned goals by incorporating direct sequence spread spectrum (DSSS) coding techniques on linear frequency modulated (LFM) signals. We call it *Spread Spectrum Coded LFM (SSCL)* signaling. Our transmitted LFM waveforms are rendered orthogonal with a unique spread spectrum code. At the receiver, the echo signal will be decoded using it's spreading code. In this manner, transmitted orthogonal waveforms can be match filtered only with the intended received signals. From analytical expressions of the waveforms we have designed and from simulation results, we found that: (a) cross-ambiguity function of two LFM spread spectrum coded (orthogonal) waveforms is small for all delays and Dopplers (i.e. transmit and receive signals satisfy the orthogonality constraint), (b) The length and type of the spread spectrum code determines amount of suppression (i.e. complete orthogonal or near orthogonal of the received signal), (c) We can process the same received signal in two different ways; one method can provide LFM signal resolution and the other method can provide ultra high resolution.**

***Index Terms*—Linear Frequency Modulation (LFM), Multiple-input Multiple-output (MIMO), Direct Sequence Spread Spectrum (DSSS), Spread Spectrum Coded LFM (SSCL), Cross-Ambiguity Function (CAF), Orthogonal Waveform (OW).**

## I. Introduction

Waveform diverse radar (such as MIMO radar) promises improved performance (in terms of detection, resolution, etc) over conventional radar systems [10]. As a new paradigm for radar systems design, MIMO radar's novelty relies significantly on the waveform's structure. More specifically, it is assumed that, in a MIMO radar setting, both the transmitted and received waveforms will remain orthogonal under the Doppler shifts caused by targets' motion [14].

However, state-of-the-art MIMO radar research efforts do not fully address this key issue of how to maintain the orthogonality of the received waveforms. Researchers presented numerous MIMO radar signal processing concepts and exploitation algorithms based on the assumption that waveforms stay orthogonal. Hence, one might argue that enhanced performance of MIMO radar over traditional radar cannot be realized unless we fully address the issue of designing MIMO radar waveforms that will remain orthogonal on both transmit and receive.

## II. Previous Research on MIMO Radar Waveforms

To address the MIMO radar waveforms design issue, researchers attempted techniques such as employing polarization diverse waveforms, frequency diverse waveforms, coded waveforms, and combination of these methods [1], [2], [3], [4], [5], [6], [7]. Moran et.al. [9] presented polarization diverse waveforms on multiple channels for MIMO radar. Principal advantages claimed by this approach are: it enables detection of smaller radar cross section (RCS) targets and diversity gains. However, this research did not address whether waveforms will remain orthogonal on receive. Gladkova et.al. described a family of stepped frequency waveforms to attain high range resolution [11]. This paper demonstrated that a suitable choice of waveform's parameters leads to the essential suppression of its autocorrelation function (ACF) sidelobes. Similarly, Zoltowski et. al. [12], and Nehorai [13] also illustrated methods to exploit waveforms for MIMO radar applications.

Other approaches researchers attempted include separating the waveforms into separate frequency sub-bands that will not overlap even under the maximum expected Doppler shifts. In this case, the waveforms will remain orthogonal under arbitrary delays and Doppler shifts, but in general the Doppler shifts for a given target velocity and geometry will be different because of the different carrier (center) frequency of each sub-band.

## III. Our Approach For Orthogonal MIMO Radar Waveform Design

Mentioned earlier, our goal is designing a practial set of waveforms that should be Doppler tolerant and remain orthogonal (or near orthogonal) on receive. Hence, we choose LFM waveforms to satisfy good Doppler tolerance criteria. To fulfill the need for orthogonality on receive, we consider coding each of the transmited waveforms with a unique code, and each of these codes should be orthogonal to each other. This concept is familiar in communications and this is known as spread spectrum. Hence, we consider blending LFM waveforms with the spread spectrum technique for orthogonal (or near orthogonal) waveform design for MIMO radar. In communication, chirp modulated spread spectrum combined with antipodal signaling has been utilized to reduce bit error rate [8]. In radar literature, coded waveforms have been utilized to reduce sidelobes [2]. Our research provides a mathematical analysis for combining LFM waveforms with spread spectrum. Simulation results are used to validate the analysis. This technique solves a very fundamental and crucial problem (i.e. waveforms that should remain orthogonal on both transmit and receive).

## IV. LFM Signal and Corresponding Ambiguity Function

Consider an LFM signal be:

$$s(t) = \mathrm{rect}\left(\frac{t}{T}\right)\exp\left[i2\pi(f_0 t + \frac{1}{2}\alpha t^2)\right]$$
$$= \mathrm{rect}\left(\frac{t}{T}\right)\exp(i\pi\alpha t^2)\exp(i2\pi f_0 t)$$

Therefore,

$$s(t) = u(t)\exp(i2\pi f_0 t) \quad (1)$$

where

$f_0$ : is the carrier frequency
$\alpha$ : is the chirp rate
$T$ : is the pulse width
$u(t) = \mathrm{rect}(\frac{t}{T})\exp(i\pi\alpha t^2)$ : is the complex envelope

Now, the ambiguity function of the above LFM signal can be defined as:

$$\chi_s(\tau,\nu) = \int_{-\infty}^{\infty} s(t)s^*(t-\tau)e^{i2\pi\nu t}dt$$
$$= \int_{-\infty}^{\infty} \mathrm{rect}(\tfrac{t}{T})e^{i\pi\alpha t^2}\mathrm{rect}(\tfrac{t-\tau}{T})e^{-i\pi\alpha(t-\tau)^2}.e^{i2\pi\nu t}dt$$
$$= e^{-i\pi\alpha\tau^2}\int_{-\infty}^{\infty} \mathrm{rect}(\tfrac{t}{T})\mathrm{rect}(\tfrac{t-\tau}{T}).e^{i2\pi\alpha t\tau}.e^{i2\pi\nu t}dt$$
$$= e^{-i\pi\alpha\tau^2}\int_{-\infty}^{\infty} \mathrm{rect}(\tfrac{t}{T})\mathrm{rect}(\tfrac{t-\tau}{T}).e^{i2\pi(\nu+\alpha\tau)t}dt$$

After further simplification, closed form solution for ambiguity function of an LFM signal is given by:

$$\chi_s(\tau,\nu) = (T-|\tau|)\mathrm{sinc}[(\nu+\alpha\tau)(T-|\tau|)].$$
$$e^{i\pi\nu\tau}.e^{i\pi(\nu+\alpha\tau)T} \quad (2)$$

for $|\tau| \le T$, zero elsewhere.

In equation (2) above, $\tau$ and $\nu$ represent the delay and Doppler, respectively.
By adding two LFM signals (one with up-chirp and the other with down-chirp), we can create another LFM signal that could provide better interference suppression capability. Figure 1 shows ambiguity function plot of such a signal. In this paper, to plot ambiguity function $\chi(\tau,\nu)$, we used the convention $|\chi(\tau,\nu)|$.

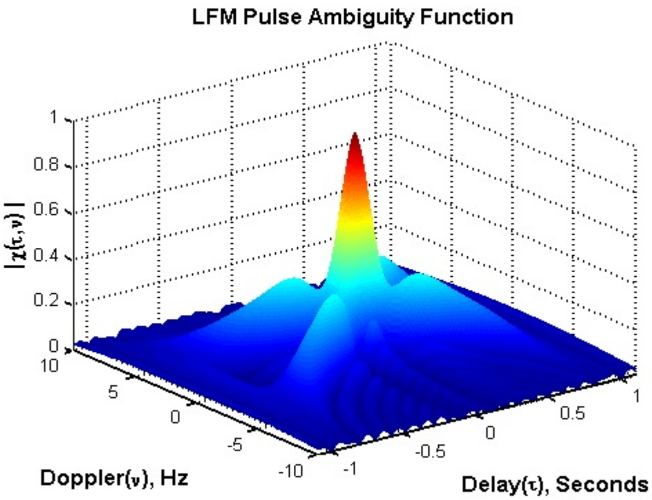


Fig. 1. Linear frequency modulated signal ambiguity function. The signal has been generated by combining an up-chirp and down-chirp signal.

## V. Direct Sequence Spread Spectrum Technique

Spread spectrum is a pioneering technique implemented in modern wireless communication CDMA (code division multiple access). Spread spectrum (SS) signal performs very well in high interference environment. In this section, we provide a brief description of Direct Sequence Spread Spectrum (DSSS) technique. The presentation provided here closely follows the texts by Proakis [15] and Simon [16]. Using standard spread spectrum signal notation, information signal can be expressed as:

$$A(t) = \sum_{n=-\infty}^{\infty} a_n P(t - nT_b) \quad (3)$$

where

$a_n$ : $\pm 1$
$P(t)$ : is the rectangular pulse of duration $T_b$

Now, $A(t)$ is multiplied by the coded signal

$$C(t) = \sum_{n=-\infty}^{\infty} c_n P(t - nT_c) \quad (4)$$

to produce the product or spreaded signal.

$$B(t) = A(t).C(t) \quad (5)$$

where

$c_n$ : is binary PN code of $\pm 1'$s
$P(t)$ : is the rectangular pulse of duration $T_c$

The product signal is then used to modulate the carrier signal and transmitted. So, the transmitted signal becomes:

$$T_x(t) = A(t)C(t).\cos(2\pi f_c t) \quad (6)$$

Received signal is the transmitted signal $T_x(t)$ and the interfering signal, $I(t)$ i.e.

$$R_x(t) = A(t)C(t)\cos(2\pi f_c t) + I(t) \quad (7)$$

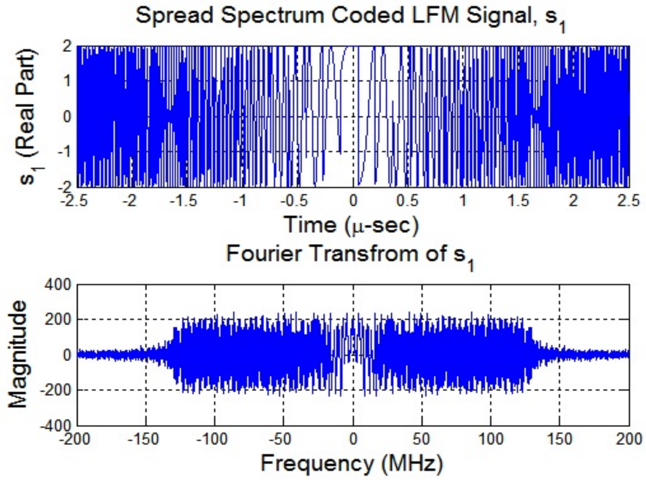


Fig. 2. Spread spectrum coded LFM signal $s_1$ and corresponding Fourier transform. Chirp rates used were, $\alpha_1 = (1 * B)/TP_1$ and $\alpha_2 = (-1 * B)/TP_1$, where $B$ is the bandwidth of the LFM signal after applying spread spectrum code, $TP_1$ is duration of the signal. Walsh-Hadamard code of length 64 has been used to spread the LFM signal.

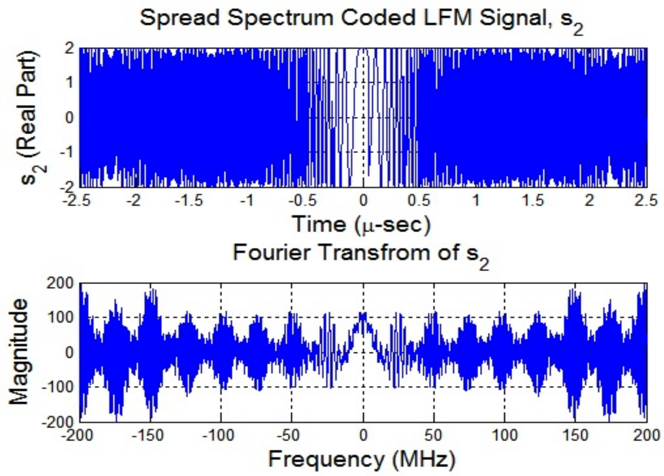


Fig. 3. Spread spectrum coded LFM signal $s_2$ and corresponding Fourier transform. Chirp rates used were, $\alpha_1 = (2 * B)/TP_2$ and $\alpha_2 = (-2 * B)/TP_2$, where $B$ is the bandwidth of the LFM signal after applying spread spectrum code, $TP_2$ is duration of the signal. Walsh-Hadamard code of length 64 has been used to spread the LFM signal.

In demodulation process, received signal $R_x(t)$ is multiplied by the coded waveform $C(t)$ i.e.

$$D_x(t) = R_x(t)C(t) \tag{8}$$

This process is also known as spectrum despreading. Output of the despreading process is the original information signal (after filtering process) i.e.

$$D_x(t) = A(t) \tag{9}$$

In equation (3) information rate is $\frac{1}{T_b}$ which is bandwidth $R$ of the information-bearing baseband signal. In equation (4), the rectangular pulse $p(t)$ and $T_c$ is known as *chip* and *chip interval* respectively. Also, $\frac{1}{T_C}$ is known as *chip rate* and this is approximately the bandwidth, $W$ of the transmitted signal (i.e. spreaded signal). Processing gain of a DSSS signal is defined as:

$$L_C = \frac{T_b}{T_c} = \frac{W}{R} \tag{10}$$

In DSSS signal, $L_C$ represents number of chips used in PN code. This is also known as bandwidth expansion factor and it represents reduction in power in the interfering signal.

## VI. Cross-Ambiguity Function for Spread Spectrum Coded LFM Waveforms

In previous section, we derived closed form mathematical expression for an LFM signal's ambiguity function. We have also presented algorithmic steps for direct sequence spread spectrum concept. In this section, we develop fundamental mathematical equation integrating spread spectrum code into LFM signal.

We define indicator function as:

$$1_{[0\ T]}(t) = \begin{cases} 1, & 0 \le t \le T, \\ 0, & \text{otherwise.} \end{cases} \tag{11}$$

Let, an LFM signal be:

$$s(t) = e^{i\pi\alpha t^2}.1_{[0,T]}(t)$$

Now define two direct sequence spread-spectrum coded LFM signals as follows:

$$s_1(t) = \sum_{m}^{M-1} C_m P(t - mT_C)e^{i\pi\alpha_1 t^2} \tag{12}$$

$$s_2(t) = \sum_{n}^{M-1} D_n P(t - nT_C)e^{i\pi\alpha_2 t^2} \tag{13}$$

where

$C_m$ : first code sequence
$D_n$ : second code sequence (different from $C_m$)
$T_C$ : chip time
$P(t)$ : rectangular pulse
$\alpha_1,\ \alpha_2$ : different chirp rates

Then, cross-ambiguity function of $s_1(t)$ and $s_2(t)$ can be expressed as:

$$\chi_{s_1,s_2}(\tau,\nu) = \int_R s_1(t)s_2^*(t-\tau)e^{i2\pi\nu t}dt \tag{14}$$

$$= \int_R \left(\sum_{m=0}^{M-1} C_m P(t - mT_C).e^{i\pi\alpha_1 t^2}\right).$$
$$\left(\sum_{n=0}^{M-1} D_n P(t - nT_c - \tau).e^{i\pi\alpha_2 (t-\tau)^2}\right)^* e^{i2\pi\nu t}dt$$
$$= \sum_{m=0}^{M-1}\sum_{n=0}^{M-1} C_m D_n^*.$$
$$\int_R P(t - mT_C)P^*(t - nT_C - \tau).e^{i\pi[\alpha_1 t^2 - \alpha_2 (t-\tau)^2]}e^{i2\pi\nu t}dt$$

Let, $f(m,n,\tau,\nu) := \int_R P(t - mT_C)P^*(t - nT_C - \tau)$ $.e^{i\pi[\alpha_1 t^2 - \alpha_2 (t-\tau)^2]}e^{i2\pi\nu t}dt$

Therefore,

$$\chi_{s_1,s_2}(\tau,\nu) = \sum_{m=0}^{M-1}\sum_{n=0}^{M-1} C_m D_n^* f(m,n,\tau,\nu) \tag{15}$$

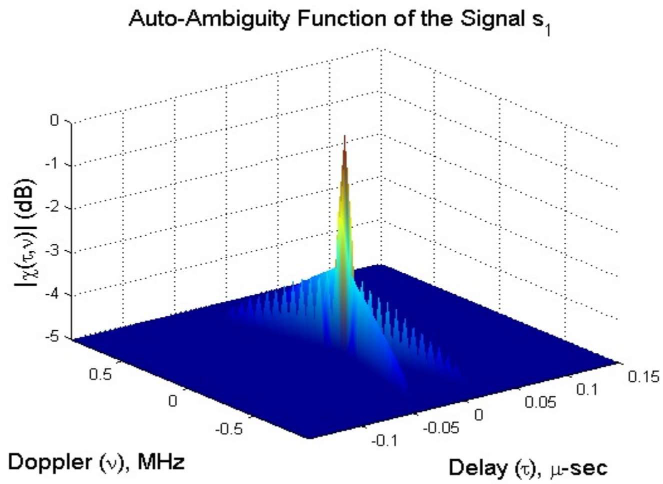


Fig. 4. Auto-ambiguity function (AAF) of the LFM signal $s_1$. First, we generated $s_1$ using an up-chirp rate, $\alpha_1 = (1 * B)/TP_1$ and down-chirp rate, $\alpha_2 = (-1 * B)/TP_1$. Then this signal was spreaded with Walsh-Hadamard code of length 64. The AAF has been evaluated on the spreaded signal.

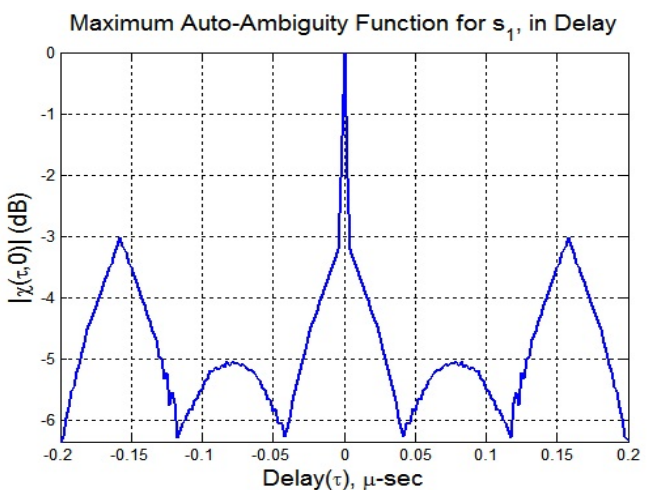


Fig. 5. Zero-Doppler cut (i.e. when $\nu = 0$) of the auto-ambiguity function (AAF) presented in Fig. 4 for signal $s_1$.

Equation (15) above represents the fundamental equation of a ***Spread Spectrum Coded LFM (SSCL)*** signaling. Some important properties of the SSCL signaling have been listed below. The proofs for these properties are straightforward:

1) ***Cross-Ambiguity property of SSCL signaling:*** When the codes $C_m$ and $D_n^*$ are orthogonal, $\chi_{s_1,s_2}(\tau,\nu) \cong 0$. This property implies that matched filter response of a transmitted signal $s_1$ with a received signal $s_2$ will be small if $s_2$ does not have the same code as the $s_1$ (i.e. cross-ambiguity function will be almost zero when $C_m$ and $D_n^*$ are orthogonal).
2) ***Auto-Ambiguity property of SSCL signaling:*** When codes $C_m$ and $D_n^*$ are the same, $\chi_{s_1,s_2}(\tau,\nu)$ will provide the highest return. This property implies that matched filter response of a transmitted signal $s_1$ with a received signal $s_2$ will be the highest if $s_2$ has the same code as the $s_1$ (this also implies that $s_1 = s_2$).

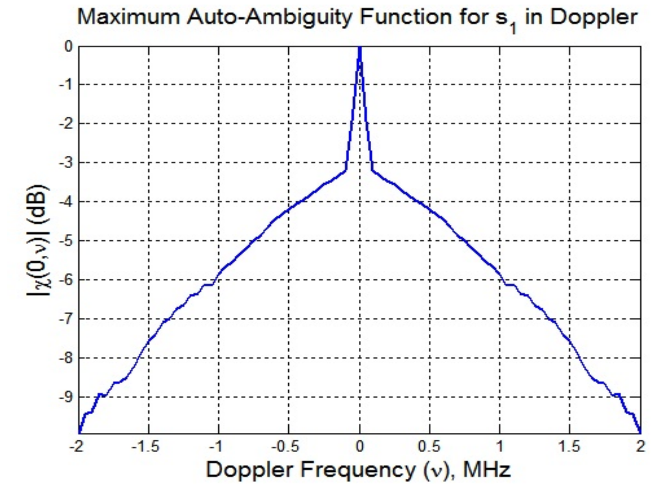


Fig. 6. Zero-Delay cut (i.e. when $\tau = 0$) of the auto-ambiguity function (AAF) presented in Fig. 4 for signal $s_1$.

3) ***Code property of SSCL signaling:*** The type of code such as Walsh-Hadamard code, Gold code, Kasami code etc. will influence the cross-ambiguity or auto-ambiguity response (i.e. degree of orthogonality of the received signal).
4) ***Code length property of SSCL signaling:*** The length of code such as 8, 16, 32 or 512 will also determine degree of orthogonality of the received signal. Furthermore, the length of code will also determine bandwidth expansion of the SSCL signaling and hence increased resolution.
5) ***Time bandwidth property of SSCL signaling:*** Increased time bandwidth product can be achieved by SSCL signaling. Bandwidth expansion provides the unique capability of this SSCL signaling. First of all, after dispreading the code, we can get our original LFM signal back and get our usual LFM signal resolution (Doppler tolerant). Secondly, by processing the coded signal we can get ultra-high resolution to separate closely spaced targets.
6) ***Bandwidth reduction property of SSCL signaling:*** We can use biorthogonal codes to reduce the bandwidth (by a factor of half) requirement of SSCL signaling. This does not affect the performance of SSCL signaling significantly.

## VII. Experiments on SSCL Signaling for Auto and Cross-Ambiguity Response

We set up an experiment for SSCL signaling presented in equation (15) and examined the auto and cross-ambiguity responses. Table I presents key parameters choosen to evaluate the auto and cross-ambiguity function of the SSCL signaling-ing. In terms of transmit orthogonal code selection, we used Walsh-Hadamard code.

We used different bandwidths for the LFM (chirp) signals that correspond to the length of the codes. Our initial bandwidth of 4 MHz has been used for the code length of 1. This is the scenario of just using a LFM signal without any spread spectrum coding. From this signal we expect to get

| Parameters | Values |
|---|---|
| Bandwidth $(B)$ | 4,36,130,260,1000 MHz |
| First Pulse Duration$(TP_1)$ | 10 $\mu$sec |
| Second Pulse Duration $(TP_2)$ | 10 $\mu$sec |
| First Pulse Chirp Rate$(\alpha_1)$ | $(1 * B)/TP_1$ |
| Second Pulse Chirp Rate $(\alpha_2)$ | $(-2 * B)/TP_2$ |
| First Pulse Code Length $(NC_1)$ | 1,8,32,64,256 |
| Second Pulse Code Length $(NC_2)$ | 1,8,32,64,256 |

TABLE I
EXPERIMENTAL PARAMETERS USED TO EXAMINE SSCL WAVEFORM PRESENTED IN EQUATION (15).

| Code Type | Code Length | Bandwidth (MHz) | Max. AAF (dB) | Max. CAF (dB) |
|---|---|---|---|---|
| - | 1 | 4 | 0 | -5 |
| Walsh-Hadamard | 8 | 36 | 0 | -9 |
| Walsh-Hadamard | 64 | 260 | 0 | -14 |
| Walsh-Hadamard | 256 | 1000 | 0 | -16 |
| Biorthogonal | 32 | 130 | 0 | -13.5 |

TABLE II
SSCL SIGNALING PERFORMANCE ANALYSIS. IN THIS TABLE, B IS THE BANDWIDTH, MAX. AAF IS THE MAXIMUM AUTO-AMBIGUITY FUNCTION, MAX. CAF IS MAXIMUM CROSS-AMBIGUITY FUNCTION.

an ambiguity response as shown in Figure 1. Then we used a bandwidth of 36 MHz that corresponds to code length 8. The new bandwidth has been calculated using the following formula (as a result of spread spectrum coding) and multiplied by a factor of 4:

$$BW = \frac{1}{TC_1} + \frac{1}{TP_1} \quad (16)$$

where, $TC_1 = \frac{TP_1}{NC_1}$ and assume that $NC_1 = NC_2$.

Similarly, we have calculated and used bandwidths of 260 MHz and 1000 MHz that corresponds to the code lengths of 64 and 256 respectively. In addition, we used a bandwidth of 130 MHz which corresponds to a code length of 32 to experiment with the biorthogonal code. The length 32 biorthogonal code has been generated using a length 64 Walsh-Hadamard code.

## VIII. RESULTS AND ANALYSIS

To reiterate our research problem, we wanted to design waveforms for diversity radar that should remain orthogonal both on transmit and receive and should be Doppler tolerant. The chief benefit of the waveforms that remain orthogonal on receive is that we can separate them with certainty to be matched filter with their corresponding transmitted waveforms.

Two signals' (with code) cross-ambiguity response is a measure to determine degree of orthogonality (i.e. whether near orthogonal or completely orthogonal). The lower the value of maximum cross-ambiguity response, the more the signals approach to become exactly orthogonal. One key attribute of our waveform is that it reveals the non-orthogonal waveforms with the maximum value of cross-ambiguity response. This implies that both signals must be the same and has been spreaded with the same code. In this case, cross-ambiguity response is just the auto-ambiguity response of two signals. On the other hand, any two signals that have cross-ambiguity response that is smaller than the maximum cross-ambiguity response must be orthogonal to each other. In this manner, we can separate all receive waveforms without cross-ambiguity response being exactly zero i.e. exactly orthogonal.

Table II presents key results obtained from our waveform (i.e. SSCL signaling) presented in equation (15). Intuitively, for a given code, the longer it is, the better cross-ambiguity response we should expect. From Table II, we can see that by increasing the code length, we obtained better cross-ambiguity response (i.e. lower CAF value). With code length of 1, maximum cross-ambiguity response observed was about -5 dB. This case is just using LFM signal with chirp diversity; no influence from spread spectrum code. With code length of 8, maximum cross-ambiguity response observed was about -9 dB. In this case, we started to see the influence of spread spectrum code. Similarly, using the code length of 64 and 256, we have observed improved cross-ambiguity response.

Figure 4 shows auto-ambiguity response of the signal $s_1$. The code used was Walsh-Hadamard of length 64. This is the case, where $s_1 = s_2$ and $C_m = D_n^*$. As expected, the auto-ambiguity function takes the shape of the LFM signal presented in Figure 1.

Figure 7 shows cross-ambiguity response of the signals $s_1$ and $s_2$. Two orthogonal codes used were Walsh-Hadamard of length 64. We also applied two different chirp rates for each signals. The key attribute of Figure 7 is that we don't see the shape of LFM signal ambiguity response anymore. This is due to the fact that signals $s_1$ and $s_2$ were multiplied by orthogonal codes. We observe that, cross-ambiguity response approaches to -14 dB.

From Table II, we can see that by increasing the code length, we can achieve lower cross-ambiguity and hence better orthogonality of the received signals. However, the longer the code, the greater the bandwidth expansion. In practical application, we know that bandwidth is a scarce resource. In particular, using bandwidth more than 1 GHz could be very expensive for various reasons. In such a scenario, we can use biorthogonal coding to reduce the bandwidth (by a factor of half) requirement of the SSCL signaling. In Table II, we see that by using a biorthogonal code of length 32 our waveform achieves a cross-ambiguity response of about - 13.5 dB, which almost comparable (-14 dB) to using Walsh-Hadamard code of length 64. However, biorthogonal code reduces the bandwidth by a factor of half.

## IX. CONCLUSION

We have designed a novel Doppler tolerant and orthogonal waveform for waveform diverse radar applications. We called this waveform spread spectrum coded LFM (SSCL) signaling and it's cross-ambiguity function has been presented in equation (15). The contributions of this research to MIMO radar applications are the followings: (1) Designing waveform that will remain orthogonal on receive has not been accomplished previously; our research provides a solution to this critical problem, (2) This waveform inherits Doppler tolerant property of the LFM waveform when properly processed, (3) This

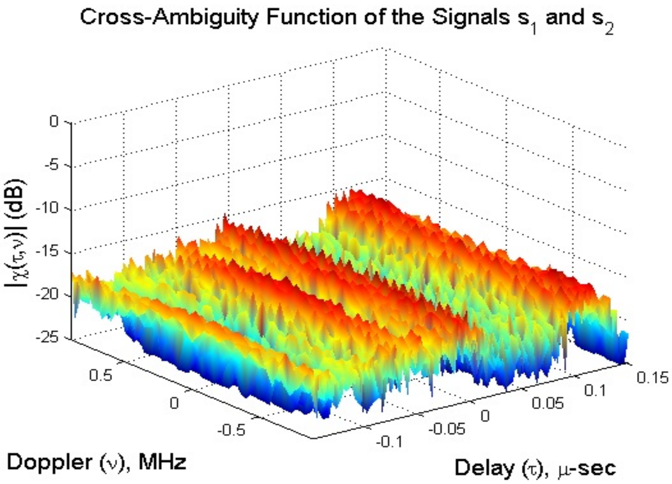


Fig. 7. Cross-ambiguity function (CAF) of two LFM signals $s_1$ and $s_2$. First, we generated $s_1$ using an up-chirp rate, $\alpha_1 = (1 * B)/TP_1$ and down-chirp rate, $\alpha_2 = (-1 * B)/TP_1$. Second, we generated $S_2$ using an up-chirp rate, $\alpha_1 = (2 * B)/TP_1$ and down-chirp rate, $\alpha_2 = (-2 * B)/TP_1$. Then this signals were spreaded with Walsh-Hadamard code of length 64. The key attribute of this figure is that maximum cross-ambiguity becomes about -14dB.

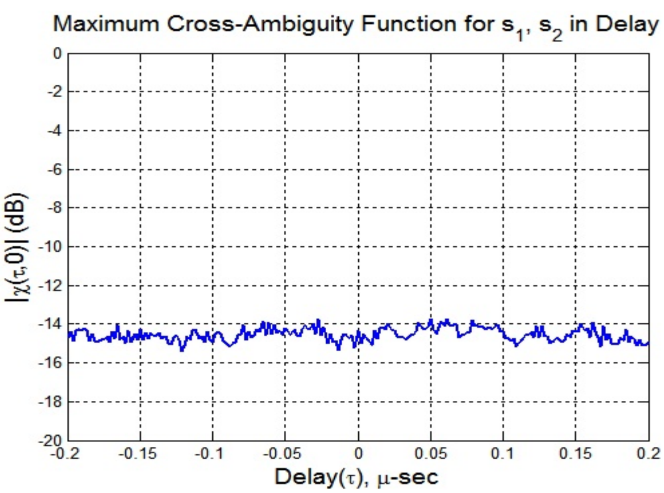


Fig. 8. Zero-Doppler cut (i.e. when $\nu = 0$) of the cross-ambiguity function (CAF) presented in Fig. 7 for signals $s_1$ and $s_2$.

waveform allows processing of received signal in two different ways. First, if we despread the received signal, we will get back our original LFM signal and hence get resolution capability of simple LFM signal. Second, if we process the received signal with the spread spectrum code in it, we will get ultra high resolution capability to separate closely spaced targets. This is a unique capability of our proposed waveform.

## X. Future Research

Auto and cross-ambiguity function plots presented in this paper based on SSCL signaling in (15) illustrate matched filter output in a single element of an antenna array. We will extend this to a large number of antenna elements and develop a matched filter receiver structure. We then analyze performance (probability of detection vs. false alarm) of this waveform in a noisy/clutter environment. We also examine performance of other codes.

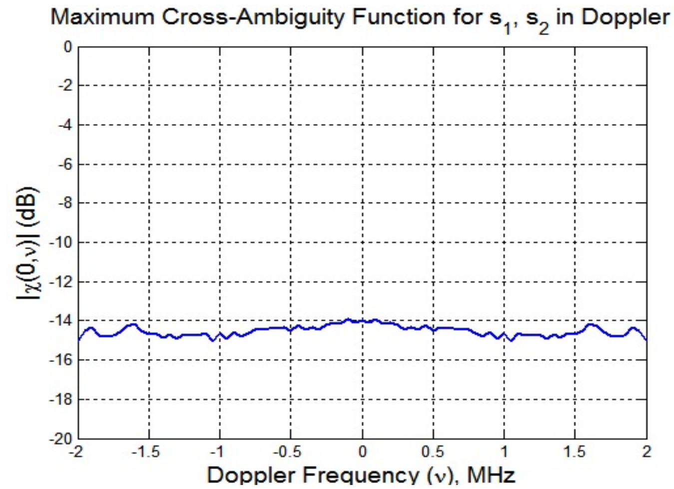


Fig. 9. Zero-Delay cut (i.e. when $\tau = 0$) of the cross-ambiguity function (CAF) presented in Fig. 7 for signals $s_1$ and $s_2$.

## References


[1] P. M. Woodward, *Probability and Information Theory, With Applications to Radar* McGraw-Hill Book Co. Inc, New York, NY 1953.
[2] N. Levanon and E. Mozeson, *Radar Signals*, 1st ed. Harlow, England: Wiley-IEEE Press,2004.
[3] C.E. Cook and M. Bernfeld, *Radar Signals: An Introduction to Theory and Application*, 1st ed. New York, Academic Press,1967.
[4] R. Calderbank, S. Howard, and B. Moran, Waveform Diversity in Radar Signal Processing, IEEE Signal Processing Magazine, 2009
[5] J.J.M. de Wit, W. L. van Rossum, and A.J. de Jong, Orthogonal Waveforms for FMCW MIMO Radar, IEEE Radar 2011
[6] S. R.J. Axelsson, Suppressed Ambiguity in Range by Phase-Coded Waveforms, IEEE Radar 2001
[7] S. Searle,S. Howard Waveform Design and Processing for Multichannel MIMO Radar, IEEE Radar 2010
[8] S. Hengstler, D. Kasilingam A Novel Chirp Modulation Spread Spectrum Technique for Multiple Access, Proceedings of the IEEE International Symposium on Spread Spectrum Techniques and Applications (ISSSTA 2002), vol. 1, pp. 73-77, September 2002
[9] S. Howard, A. Calderbank, and W. Moran A Simple Signal Processing Architecture for Instantaneous Radar Polarimetry, IEEE Transactions on Information Theory, VOL. 53, NO. 4, APRIL 2007
[10] J. Li and P. Stoica, *MIMO Radar Signal Processing*, 1st ed. John Wiley, and Sons, New York, NY 2009.
[11] I. Gladkova Analysis of Stepped-Frequency Pulse Train Design, IEEE Transactions on Aerospace and Electronic Systems, VOL. 45, Issue 4, October 2009
[12] M. Zoltowski, Moran, W., et. al. Unitary Design of Radar Waveform Diversity Sets, Asilomar Conference, 2008
[13] S. Sen., A. Nehorai OFDM MIMO Radar for Low-grazing Angle Tracking, Asilomar Conference, 2009
[14] B. Friedlander On the Relationship Between MIMO and SIMO Radars, IEEE Transactions on Signal Processing, VOL. 57, No 1, January 2009
[15] J. Proakis and M. Salehi, *Communication Systems Engineering*, Prentice-Hall, Inc, New Jersey, NJ 1994.
[16] M. Simon and J. Omura, et.al, *Spread Spectrum Communications* Computer Science Press, Maryland, MD 1985.