Fracture Mechanics

- It was shown that the theoretical cohesive stress is much greater than the observed fracture stress for metals
- This lead to the idea of defects or cracks which locally raise the stress to the level of the theoretical cohesive stress.
- The first successful theoretical approach for brittle fracture was introduced by **Griffith.**
- **Griffith's** equation shows a strong dependence of fracture strength on crack length.
- It is well established that even metals which fail in a completely brittle manner have undergone some plastic deformation prior to fracture.
- Therefore Griffith's theory was modified by **Orowan** to allow for the degree of plasticity always present in the brittle fracture of metals by the inclusion of a term γ_p which referred to plastic work required to extend the crack wall.

- **Orowan;**

$$\sigma_f = \left(\frac{2E(\gamma_s + \gamma_p)}{\pi a}\right)^{1/2} \approx \left(\frac{E\gamma_p}{a}\right)^{1/2} \qquad 2.2-1$$

where, *E is Young's modulus and γ_p is the plastic work required to extend the crack wall for a crack of length 2a.*

(note: The surface energy term can be neglected since estimates of the plastic work term are about 10^2 to 10^3 J/m^2 compared with values of γ_s of about 1 to 2 J/m^2.

- Eq 2.2-1 was modified by *Irwin* to replace the hard to measure γ_p with a term that was directly measurable.

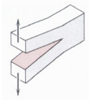
$$\sigma_f = \{(E\mathcal{G}_c)/(\pi a)\}^{1/2}$$

$\mathcal{G}_c$ corresponds to a critical value of crack-extension force and represent $2\gamma_p$.

- The critical value of $\mathcal{G}_c$ makes the crack propagate to fracture is called the toughness of the material.

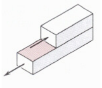
- There are three modes of fracture: Mode I, II and III.
  - Mode I.

Also known as the opening mode, which refers to the applied tensile loading. The most common fracture mode and used in the fracture toughness testing. And a critical value of stress intensity determined for this mode would be designated as K_{IC}.



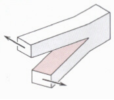
  - Mode II.

Also known as the shear mode, which refers to the applied shear stress in the in-plane direction. The shear stress applied normal to the leading edge of the crack but in the plane of the crack.



  - Mode III.

Also known as the tearing mode, which refers to the applied shear stress out of plane. Applied shear stress is parallel to the leading edge of the crack.



LINEAR ELASTIC FRACTURE MECHANICS (LEFM)

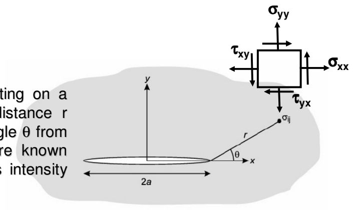
- Fracture mechanics is the discipline that allows one to assess the importance of cracks in components, irrespective of the mechanism by which the cracks grow.
- **LEFM** analysis is based on an analytical procedure that relates the stress-field magnitude and distribution in the vicinity of a crack tip to the nominal stress applied of the structural component, to the size, shape and orientation of the crack and to material properties.
- The fundamental principle of fracture mechanics is that the stress field ahead of a sharp crack in a structural member can be characterized in terms of a simple parameter K, *the stress intensity factor (was developed by Irwin in the 1950's).* This parameter, K, is related to both the nominal stress level (σ) and the size of the crack present (a).

LINEAR ELASTIC FRACTURE MECHANICS (LEFM)

- Fracture The crack tip stresses can be expressed as:

$$\sigma_{ij} = \frac{K_I}{\sqrt{2\pi r}} f_{ij}(\theta)$$

- Where σ_{ij} are the stresses acting on a material element dxdy at a distance r from the crack tip and at an angle θ from the crack plane, and f_{ij} (θ) are known functions of θ. KI is the stress intensity factor.



Stresses at a point ahead of the crack tip

LINEAR ELASTIC FRACTURE MECHANICS (LEFM)

- From the stress field solution it appears that the stresses on a material element as shown in Figure can be described by:

$$\sigma_x = \frac{K_I}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left(1 - \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)$$
$$\sigma_y = \frac{K_I}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left(1 + \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)$$
$$\sigma_Z = 0$$
$$\tau_{xy} = \frac{K_I}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\sin\frac{\theta}{2}\cos\frac{3\theta}{2}$$

For an orientation directly ahead of the crack, θ = 0, then $\tau_{xy} = 0$ and x = r.

Then the crack tip stresses in X and Y directions are:

$$\sigma_x = \frac{K_I}{\sqrt{2\pi x}}$$
$$\sigma_y = \frac{K_I}{\sqrt{2\pi x}}$$

- The crack tip stress must be proportional to the applied stress σ.

$$\sigma_y \propto \frac{\sigma}{\sqrt{2\pi x}}$$

- The crack tip stresses will also depend upon crack size. The stresses will be higher when "*a*" is longer. Hence the crack size "*a*" must appear in the numerator in equation

$$\sigma_y \propto \frac{\sigma\sqrt{a}}{\sqrt{2\pi x}}$$

- The crack tip stress must be proportional to the applied stress σ.

$$\sigma_y = C\frac{\sigma\sqrt{a}}{\sqrt{2\pi x}}$$

- With $C = (\pi)^{1/2}$, then

$$\sigma_y = \frac{\sigma\sqrt{\pi a}}{\sqrt{2\pi x}}$$

- Comparing the above equation with

$$\sigma_x = \frac{K_I}{\sqrt{2\pi x}}$$

- We get:

$$\boxed{K = \sigma\sqrt{\pi a}}$$

- K depends on the plate dimensions, for example width W
- As W decreases, the stress at the crack tip increases
- For any configuration the crack tip stresses will be:

$$\sigma_y = \frac{\sigma\sqrt{a}}{\sqrt{2\pi x}}C(a/L)$$

- Dividing $C/(\pi)^{1/2}$ and substitute $\sqrt{a}$ with $\sqrt{\pi a}$

$C(a/L)/(\pi)^{1/2} = \beta$.

$$\sigma_y = \frac{\beta\sigma\sqrt{\pi a}}{\sqrt{2\pi x}} = \frac{K_I}{\sqrt{2\pi x}}$$

- The stress intensity factor is always:

$$\boxed{K_I = \beta\sigma\sqrt{\pi a}}$$

(note: β is a dimensionless parameter or function that depends on both crack and specimen sizes and geometries, as well as the manner or load application and also can be written as Y or α).

- For an infinite plate, $\beta = 1$

(for planar specimens containing cracks that are much shorter than the specimen width)

e.g. for a plate of infinite width having a through-thickness crack.

$$K_I = \beta\sigma\sqrt{\pi a} = \sigma\sqrt{\pi a}$$

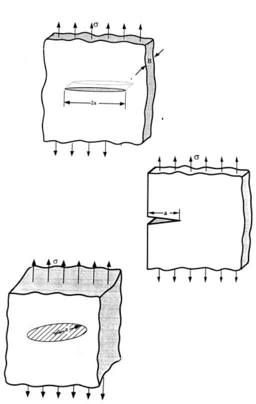
- For a semi-infinite plate, $\beta = 1.12$ and

(for a plate of semi-infinite containing an edge crack of length a)

$$K_I = 1.12\sigma\sqrt{\pi a}$$

- For a circular (penny-shaped) crack with radius (α), $\beta = 2/\pi$ and

$$K_I = \frac{2}{\pi}\sigma\sqrt{\pi a}$$



DESIGN USING FRACTURE MECHANIC (TOUGHNESS)

- A property that is a measure of a material's resistance to brittle fracture when a crack is present.
- From an engineering point of view, the stress intensity factor K_I can be used as stress to predict the *critical condition at fracture*

- Fracture occurs when:

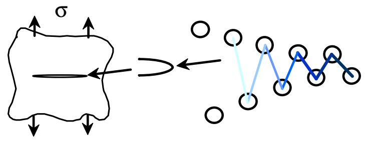
$$\boxed{K_I = K_{IC} = \sigma\sqrt{\pi a}} \Rightarrow \boxed{\sigma_{fr} = \frac{Toughness(K_{IC})}{\sqrt{\pi a}}}$$

Fracture Toughness: Material property which can be measured experimentally

Fracture stress

WHEN DOES A CRACK PROPAGATE?

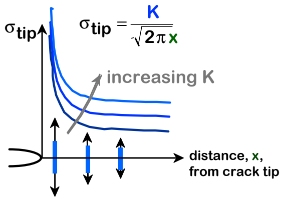
- ρ_t at a crack tip is very small!



- Result: **crack tip stress is very large.**

- Crack propagates when: **the tip stress is large enough to make:**

$K \geq K_c$



$$\sigma_{tip} = \frac{K}{\sqrt{2\pi x}}$$

GEOMETRY, LOAD, & MATERIAL

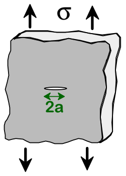
- Condition for crack propagation:

$K \geq K_c$

Stress Intensity Factor:
--Depends on load & geometry.

Fracture Toughness:
--Depends on the material, temperature, environment, & rate of loading.

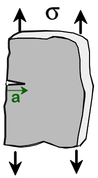
- Values of K for some standard loads & geometries:



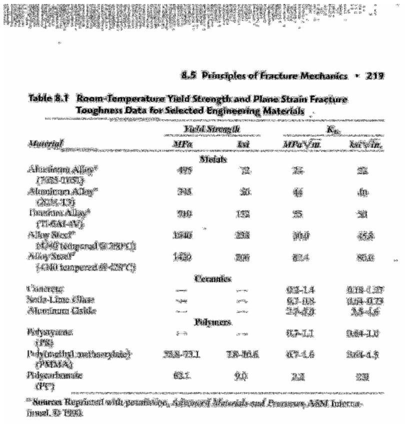
$$K = \sigma\sqrt{\pi a}$$

units of K :
$\mathrm{MPa}\sqrt{\mathrm{m}}$
or $\mathrm{ksi}\sqrt{\mathrm{in}}$

Adapted from Fig. 8.8, *Callister 6e.*



$$K = 1.1\sigma\sqrt{\pi a}$$

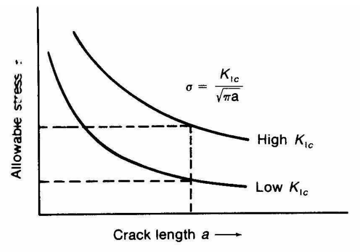


8.5 Principles of Fracture Mechanics • 219

Table 8.1 Room-Temperature Yield Strength and Plane Strain Fracture Toughness Data for Selected Engineering Materials

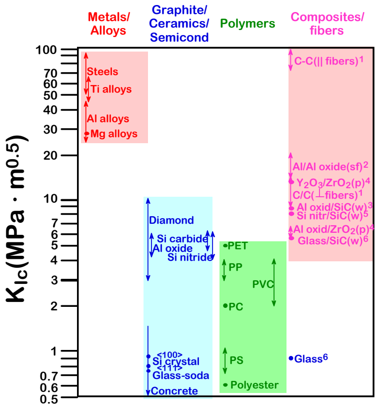
Material	*Yield Strength*		K_{Ic}	
	MPa	*ksi*	$MPa\sqrt{m}$	$ksi\sqrt{in.}$
	Metals			
Aluminum Alloy[a] (7075-T651)	495	72	24	22
Aluminum Alloy[a] (2024-T3)	345	50	44	40
Titanium Alloy[a] (Ti-6Al-4V)	910	132	55	50
Alloy Steel[a] (4340 tempered @ 260°C)	1640	238	50.0	45.8
Alloy Steel[a] (4340 tempered @ 425°C)	1420	206	87.4	80.0
	Ceramics			
Concrete	—	—	0.2–1.4	0.18–1.27
Soda-Lime Glass	—	—	0.7–0.8	0.64–0.73
Aluminum Oxide	—	—	2.7–5.0	2.5–4.6
	Polymers			
Polystyrene (PS)	—	—	0.7–1.1	0.64–1.0
Poly(methyl methacrylate) (PMMA)	53.8–73.1	7.8–10.6	0.7–1.6	0.64–1.5
Polycarbonate (PC)	62.1	9.0	2.2	2.0

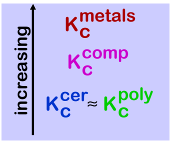
[a] Source: Reprinted with permission, *Advanced Materials and Processes,* ASM International, © 1990.



Relation between fracture toughness and allowable stress and crack size

FRACTURE TOUGHNESS





Based on data in Table B5, *Callister 6e*.

Composite reinforcement geometry is: f = fibers; sf = short fibers; w = whiskers; p = particles. Addition data as noted (vol. fraction of reinforcement):

1. (55vol%) *ASM Handbook*, Vol. 21, ASM Int., Materials Park, OH (2001) p. 606.
2. (55 vol%) Courtesy J. Cornie, MMC, Inc., Waltham, MA.
3. (30 vol%) P.F. Becher et al., *Fracture Mechanics of Ceramics*, Vol. 7, Plenum Press (1986). pp. 61-73.
4. Courtesy CoorsTek, Golden, CO.
5. (30 vol%) S.T. Buljan et al., "Development of Ceramic Matrix Composites for Application in Technology for Advanced Engines Program", ORNL/Sub/85-22011/2, ORNL, 1992.
6. (20vol%) F.D. Gace et al., *Ceram. Eng. Sci. Proc.*, Vol. 7 (1986) pp. 978-82.

DESIGN AGAINST CRACK GROWTH

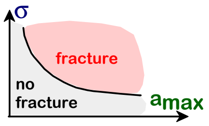
- Crack growth condition:

$$Y\sigma\sqrt{\pi a} \nearrow K \geq K_c$$

- **Largest**, most **stressed** cracks grow first!

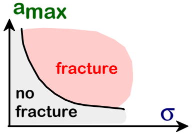
--Result 1: Max flaw size dictates design stress.

$$\sigma_{design} < \frac{K_c}{Y\sqrt{\pi a_{max}}}$$



--Result 2: Design stress dictates max. flaw size.

$$a_{max} < \frac{1}{\pi}\left(\frac{K_c}{Y\sigma_{design}}\right)^2$$



DESIGN EX: AIRCRAFT WING

- **Material has K_c = 26 MPa-m$^{0.5}$**
- **Two designs to consider...**

Design A	Design B
--largest flaw is 9 mm	--use same material
--failure stress = 112 MPa	--largest flaw is 4 mm
	--failure stress = ?

- **Use...** $\sigma_c = \frac{K_c}{Y\sqrt{\pi a_{max}}}$
  - **Key point: Y and K_c are the same in both designs.**

--Result:

$$\left(\overset{112\text{ MPa}}{\sigma_c}\sqrt{\overset{9\text{ mm}}{a_{max}}}\right)_A = \left(\sigma_c\sqrt{\overset{4\text{ mm}}{a_{max}}}\right)_B$$

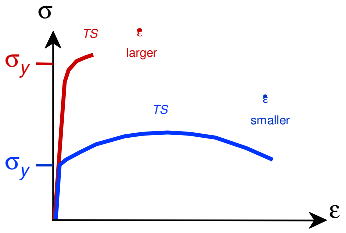
Answer: $(\sigma_c)_B = 168\,\text{MPa}$

- **Reducing flaw size pays off!**

Loading Rate

- Increased loading rate...
  -- increases σ_y and *TS*
  -- decreases %*EL*

- Why? An increased rate gives less time for dislocations to move past obstacles.



MEASUREMENT OF FRACTURE TOUGHNESS

- Fracture toughness is a material property which characterise the crack resistance and the value of K_{IC} can be found by testing of the same material with different geometries and with combinations of crack size and fracture stress.
- Knowledge of K_{IC} under standard conditions can be used to predict failure

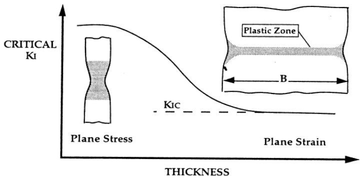
---

MEASUREMENT OF FRACTURE TOUGHNESS

- K_{IC} is usually measured under plane strain conditions, and the minimum thickness, *B* of the test specimen to achieve plane strain is:

$$\boxed{B \geq 2 \cdot 5\left(\frac{K_{IC}}{\sigma_{ys}}\right)^2}$$

σ_{ys} is the 0.2 % offset yield strength



Effect of specimen thickness on K_{IC}

FRACTURE MECHANICS SPECIMENS

Double Cantilevered Beam

Charpy

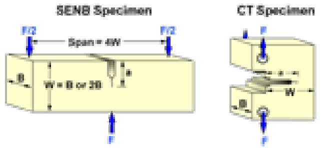
Wedge Open Load

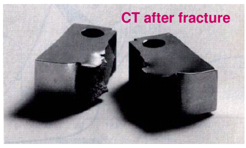
Compact Tension

All images adapted from www.alspi.com

FRACTURE TOUGHNESS: Specimen Configurations

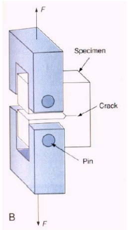
- Several types of specimens are permitted in ASTM standards
- Compact tension (CT) specimens and single edge notched bend (SENB) are the most common.

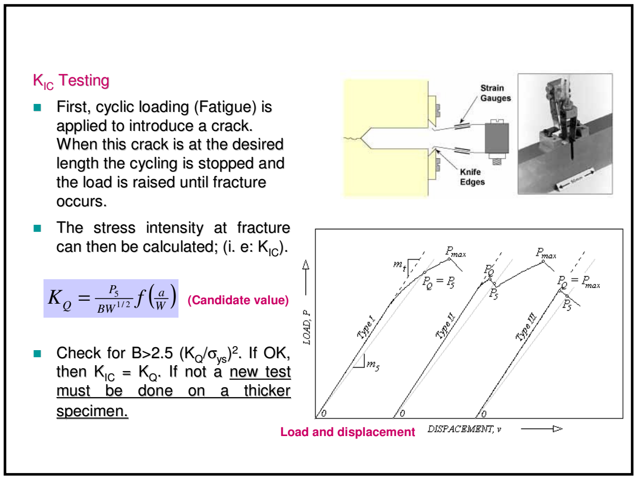




CT after fracture

CT specimen





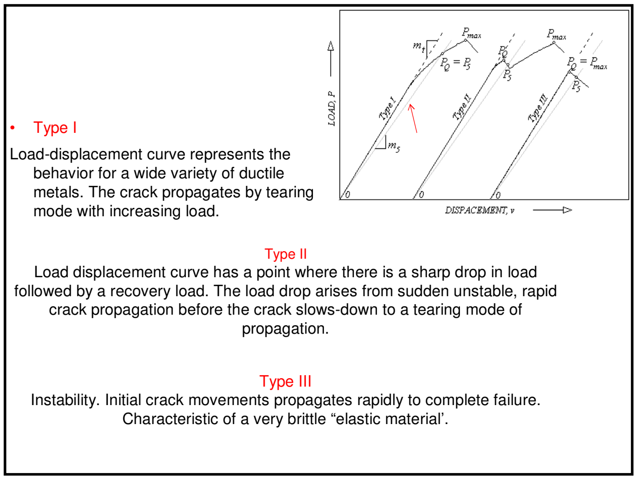
K_{IC} Testing

- First, cyclic loading (Fatigue) is applied to introduce a crack. When this crack is at the desired length the cycling is stopped and the load is raised until fracture occurs.
- The stress intensity at fracture can then be calculated; (i. e: K_{IC}).

$$K_Q = \frac{P_5}{BW^{1/2}} f\left(\frac{a}{W}\right)$$ **(Candidate value)**

- Check for $B > 2.5 (K_Q/\sigma_{ys})^2$. If OK, then $K_{IC} = K_Q$. If not a <u>new test must be done on a thicker specimen.</u>

Load and displacement



- Type I

Load-displacement curve represents the behavior for a wide variety of ductile metals. The crack propagates by tearing mode with increasing load.

Type II

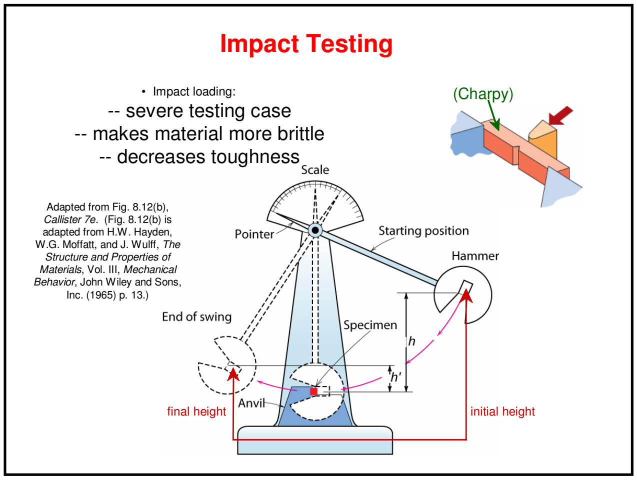
Load displacement curve has a point where there is a sharp drop in load followed by a recovery load. The load drop arises from sudden unstable, rapid crack propagation before the crack slows-down to a tearing mode of propagation.

Type III

Instability. Initial crack movements propagates rapidly to complete failure. Characteristic of a very brittle “elastic material’.

Impact testing

- **Standard laboratory tensile test could not extrapolated to predict fracture behavior e.g. under some circumstances ductile metal can fracture abruptly and with very little plastic deformation.**
- **Type of materials to be tested: which have**
  1. **Deformation at a relatively low temperature**
  2. **A high strain rate (e.g. rate of deformation)**
  3. **A triaxial stress state (which may be introduced by the presence of a notch.**



Impact Testing

- Impact loading:

-- severe testing case
-- makes material more brittle
-- decreases toughness

Adapted from Fig. 8.12(b), *Callister 7e*. (Fig. 8.12(b) is adapted from H.W. Hayden, W.G. Moffatt, and J. Wulff, *The Structure and Properties of Materials*, Vol. III, *Mechanical Behavior*, John Wiley and Sons, Inc. (1965) p. 13.)

Charpy-Izod Impact test

one of the primary function : to determine whether or not a material experiences a ductile-to-brittle transition.

ductile-to-brittle transition

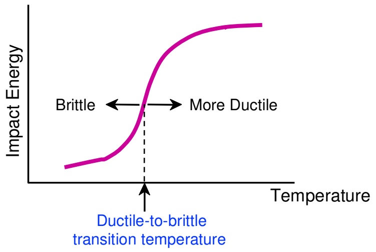
Is related to the temperature dependence of the measured impact energy absorption.

- At higher temperatures the charpy v-notch (CVN) is relatively large (refer to ductile mode of fracture)
- Low temperature: the impact energy drops suddenly over a relatively narrow temperature range (small energy , brittle fracture)

- **Increasing temperature...**

--increases %*EL* and K_c

- **Ductile-to-Brittle Transition Temperature (DBTT)...**



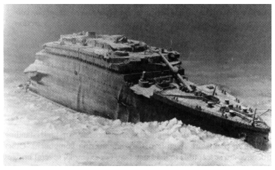
Adapted from Fig. 8.15, *Callister 7e.*

- For material that exhibit ductile-brittle behavior should be used only above this transition temperature , to avoid brittle and catastrophic failure.
  - ❖ Example: Titanic and Liberty ship
    - Vessel material : steel alloy which has adequate ductility at room temperature tensile test.
    - accident occurred at ~ 4°C
    - Each fracture crack originated at some point of stress concentration e.g. at sharp corner or fabrication defect, then propagated around the entire girth of the ship

---

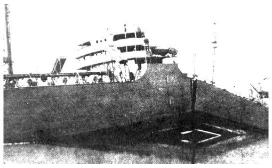
DESIGN STRATEGY:

- **Pre-WWII: The Titanic**



Reprinted w/ permission from R.W. Hertzberg, "Deformation and Fracture Mechanics of Engineering Materials", (4th ed.) Fig. 7.1(a), p. 262, John Wiley and Sons, Inc., 1996. (Orig. source: Dr. Robert D. Ballard, *The Discovery of the Titanic*.)

- **WWII: Liberty ships**



Reprinted w/ permission from R.W. Hertzberg, "Deformation and Fracture Mechanics of Engineering Materials", (4th ed.) Fig. 7.1(b), p. 262, John Wiley and Sons, Inc., 1996. (Orig. source: Earl R. Parker, "Behavior of Engineering Structures", Nat. Acad. Sci., Nat. Res. Council, John Wiley and Sons, Inc., NY, 1957.)

- **Problem: Used a type of steel with a DBTT ~ Room temp.**

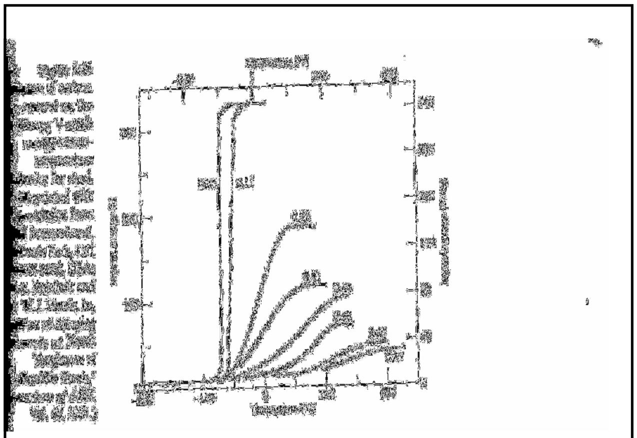
Materials which experienced a ductile-to-brittle transition

- **Low strength steel , BCC crystal structure**

The transition temperature is sensitive to both alloy composition and microstructure.

e.g.

- **decreasing the average grain size results in a lowering of the transition temperature (strengthen and toughen the steels)**
- **Increasing the carbon content: strength increase, raises the CVN transition temperature**



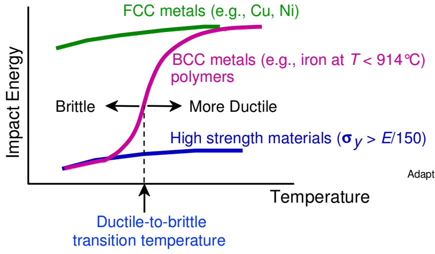
Materials which do not experienced a ductile-to-brittle transition

- Low strength FCC metals (some Al and Cu alloys) and most HCP metals, and always retain high impact energies or remain ductile with decreasing temperature.
- High strength materials e.g. high strength steel and titanium alloys. Impact energy is relatively insensitive to temperature but these materials are very brittle, reflected by their low impact energy values.

- **Increasing temperature...**

--increases %*EL* and K_c

- **Ductile-to-Brittle Transition Temperature (DBTT)...**

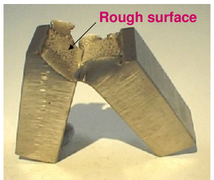


Adapted from Fig. 8.15, *Callister 7e.*

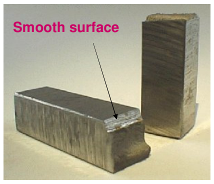
TYPES OF FRACTURE

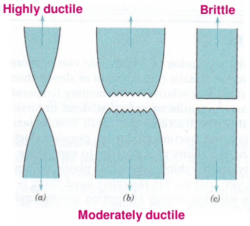
- In general, materials can be broadly classified into two groups depending on their mechanical behaviour.
  1. Materials that behave in a **ductile** manner
  2. materials that behave in a **brittle** (**cleavage**) manner.

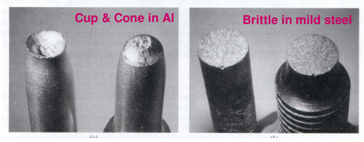
Ductile fracture is high energy fracture and occurs with large plastic deformation. Characterised by stable crack growth



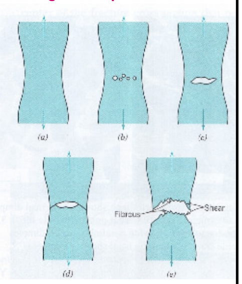
Brittle fracture is low energy fracture and occurs with no or little plastic deformation. Characterised by unstable crack growth





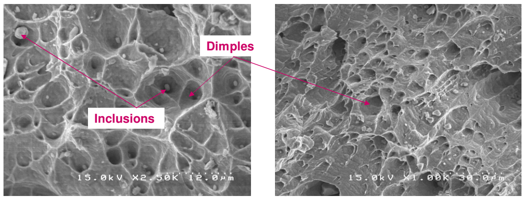


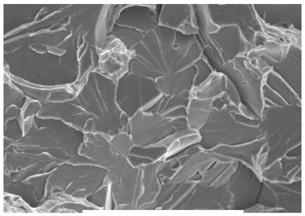
Stages in Cup & Cone fracture



MECHANISM OF DUCTILE FRACTURE

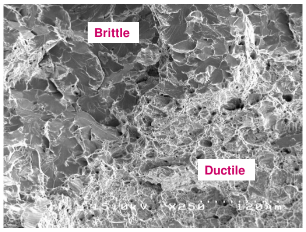
- The observed stages in ductile fracture are as follows:
  1. Formation of voids around an inclusion or second phase particles
  2. Growth of the void around the particles
  3. Coalescence of the growing void with adjacent voids.





Brittle (cleavage) fracture

Mixed mode fracture



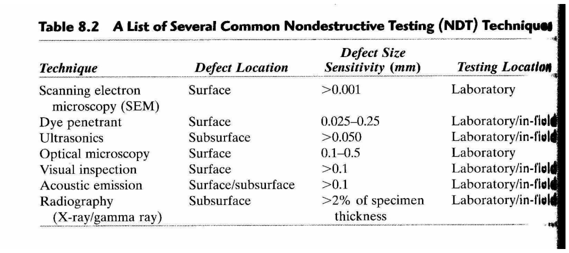


Table 8.2 A List of Several Common Nondestructive Testing (NDT) Techniques

Technique	*Defect Location*	*Defect Size Sensitivity (mm)*	*Testing Location*
Scanning electron microscopy (SEM)	Surface	>0.001	Laboratory
Dye penetrant	Surface	0.025–0.25	Laboratory/in-field
Ultrasonics	Subsurface	>0.050	Laboratory/in-field
Optical microscopy	Surface	0.1–0.5	Laboratory
Visual inspection	Surface	>0.1	Laboratory/in-field
Acoustic emission	Surface/subsurface	>0.1	Laboratory/in-field
Radiography (X-ray/gamma ray)	Subsurface	>2% of specimen thickness	Laboratory/in-field

---

SUMMARY

- **Engineering materials don't reach theoretical strength.**
- **Flaws produce stress concentrations that cause premature failure.**
- **Sharp corners produce large stress concentrations and premature failure.**