Math 6010 - Assignment 7

Due October 23, 2023

(1) Read Section 2.2.1-2.2.6 on the Recursion Theorem in [1].
(2) [1, Ex 2.2.9] Define the following set of minimal indices of partial computable functions

$$M := \{x \ : \ \neg(\exists y < x)[\varphi_x = \varphi_y]\}.$$

Prove that M is infinite but contains no infinite c.e. set.
Hint: Use the Recursion Theorem.

Disjoint sets A, B are **computably separable** if there exists a computable set C such that $A \subseteq C$ and $B \cap C = \emptyset$; else A, B are **computably inseparable**.

(3) Show that the Σ_1^0 sets

$$A := \{x \ : \ \varphi_x(x) = 0\} \text{ and } B := \{x \ : \ \varphi_x(x) = 1\}$$

are computably inseparable.
Hint: Show that no φ_e is the characteristic function of a separating set C.

(4) Show the Π_1^0-**Separation Principle**: If $A, B \subseteq \mathbb{N}$ are Π_1^0 and disjoint, then they are computably separable.
Hint: Use the Σ_1^0 Reduction Principle.

References

[1] Soare, Robert I. Turing computability : theory and applications. Springer, Berlin, 2016.