

Math 6010 - Assignment 8

Due March 15, 2021

Disjoint sets A, B are **computably separable** if there exists a computable set C such that $A \subseteq C$ and $B \cap C = \emptyset$; else A, B are **computably inseparable**.

- (1) Show that the Σ_1^0 sets

$$A := \{x : \varphi_x(x) = 0\} \text{ and } B := \{x : \varphi_x(x) = 1\}$$

are computably inseparable.

Hint: Show that no φ_e is the characteristic function of a separating set C .

- (2) Show the Π_1^0 -**Separation Principle**: If $A, B \subseteq \mathbb{N}$ are Π_1^0 and disjoint, then they are computably separable.

Hint: Use the Σ_1^0 Reduction Principle.

- (3) Show that there exists $e \in \mathbb{N}$ such that

$$W_e = \{e\}.$$

Hint: Argue that there exists a computable $f(x, y)$ such that $W_{f(x, y)} = \{x\}$. Then apply the Uniform Recursion Theorem to find a fixed point of f .

- (4) Show that a set A is productive iff $\bar{K} \leq_m A$.

Hint for \Leftarrow : Use that \bar{K} is productive.