Lecture #4

Warmups: 1) Simplify the expression in such a manner that there are no radials / fractional exponents in the denominator

$$\frac{x}{(x^{1/4} - \sqrt[4]{3})} - \frac{3}{(x^{1/4} + \sqrt[4]{3})}$$

$$\frac{a}{c} + \frac{b}{d} = \frac{ad}{cd} + \frac{bc}{dc}$$

$$\frac{x(x^{1/4} + \sqrt[4]{3})}{(x^{1/4} - \sqrt[4]{3})(x^{1/4} + \sqrt[4]{3})} - \frac{3(x^{1/4} - \sqrt[4]{3})}{(x^{1/4} - \sqrt[4]{3})(x^{1/4} + \sqrt[4]{3})}$$

$$(\sqrt{a} - \sqrt{b}) \cdot (\sqrt{a} + \sqrt{b}) = a - b$$

$$= \frac{x(x^{1/4} + \sqrt[4]{3}) - 3(x^{1/4} - \sqrt[4]{3})}{x^{1/2} + \cancel{x^{1/4}3^{1/4}} - \cancel{x^{1/4}3^{1/4}} - 3^{1/2}}$$

$$= \frac{x(x^{1/4} + 3^{1/4}) - 3(x^{1/4} - 3^{1/4})}{\sqrt{x} - \sqrt{3}} \cdot \frac{\sqrt{x} + \sqrt{3}}{\sqrt{x} + \sqrt{3}}$$

$$= \frac{\left[x(x^{1/4} + 3^{1/4}) - 3(x^{1/4} - 3^{1/4})\right] \cdot (\sqrt{x} + \sqrt{3})}{x - 3}$$

2) Find all solutions to the equation

$$(x^2 - 1)^{1/4} \pm 2(x^2 - 1)^{1/8} + 1 = 0$$

Try to reduce to solving a quadratic.

Substitute $y = (x^2 - 1)$

$$y^{1/4} \pm 2y^{1/8} + 1 = 0$$

Solve for y first then go back and solve for x

Sub $z = y^{1/8}$,

$$z^2 \pm 2z + 1 = 0$$

" " z " " " " " y

$$(z \pm 1)(z \pm 1) = 0$$

$\Rightarrow z = -1$, $z = 1$

$\Rightarrow \sqrt[8]{y} = -1$, $1 = y^{1/8} \Rightarrow y = 1$

$\Rightarrow$ No such y exists

$\Rightarrow$ there are no solutions.

$\Rightarrow \quad 1 = x^2 - 1$

$\Rightarrow \quad 2 = x^2$

$\Rightarrow \quad x = \pm\sqrt{2}$

3) Find all solutions by completing the square

$$x^2 - 82x + 7 = 0$$

Dream: Find real # a st

$$x^2 - 82x + 7 + a = (x+b)^2 \qquad (\star)$$

If the dream is real, then we can easily solve

Add a to both sides

$$x^2 - 82x + 7 + a = a$$

We use $(\star)$, LHS is $(x+b)^2$

$$\Rightarrow (x+b)^2 = a$$

$\sqrt{\ }$ both sides

$$x + b = \pm\sqrt{a}$$

$$\Rightarrow x = -b \pm \sqrt{a}$$

Solve for a and b.

First expand RHS,

$$x^2 - 82x + 7 + a = x^2 + 2bx + b^2$$

$$\Rightarrow -82 = 2b, \quad 7 + a = b^2$$

$$\Rightarrow b = -41, \quad a = (41)^2 - 7$$

Solutions are $x = +42 \pm \sqrt{41^2 - 7}$

Ex: $x^2 - 2x = -1$

Want $(x+b)^2 = x^2 - 2x + a = -1 + a$

Expand $x^2 + 2bx + b^2 = x^2 - 2x + a = -1 + a$

$\Rightarrow$ $b = -1$, $a = 1$

Plug in: $(x-1)^2 = x^2 - 2x + 1 = 0$

$\Rightarrow$ $x = 1$

Thm: (Quadratic Eqn): The solutions of the quad. eqn

$$ax^2 + bx + c = 0 \qquad (*)$$

are

$x^2 = 1$

$x = \pm 1$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$\hookrightarrow$ if we plug this into $(*)$, we get zero.

Ex: $x^2 - 2x + 1 = 0$

$a = 1$, $b = -2$, $c = 1$

Plug into eqn,

$$\frac{2 \pm \sqrt{\cancel{4 - 4\cdot 1 \cdot 1}}^{\,0}}{2\cdot 1} = \frac{2}{2} = 1$$

Ex: $0 = 81x^2 + 7x - 31$

Defn: The discriminant D of the $ax^2 + bx + c = 0$ is

$$D = b^2 - 4ac$$

↳ $D > 0 \Rightarrow \sqrt{b^2-4ac} = \sqrt{D}$ makes sense and

So there exists 2 unique real solutions.

↳ $x^2 + 4x + 3 = 0$
$\quad = (x+3)(x+1)$

→ Soln $x=-3$, $x=-1$ ⇒ 2 unique soln.

$D = 16 - 12 = 4 > 0$ ✓ out

↳ $D = 0 \Rightarrow \sqrt{b^2-4ac} = \sqrt{D} = 0$

So quad formula ⇒ there is one soln.

↳ $x^2 - 2x + 1 = 0$
$\quad = (x-1)(x-1)$

→ $x = 1$ is only soln

$D = 4 - 4 = 0$ ✓ out

↳ $D < 0 \Rightarrow \sqrt{b^2-4ac} = \sqrt{\text{neg \#}}$ ⇒ undefined

So there are no soln

↳ $x^2 + 1 = 0$

$\Rightarrow x^2 = -1$

Squares are pos ⇒ no such x exists, ie, no soln.

$D = 0^2 - 4\cdot 1\cdot 1 = -4$ ✓ out.

Rmk: Quad eqn have at most 2 real solutions.

Rmk: Solve fractional eqns

↳ solutions when plugged in need to make sense

↳ solutions to eqn must be in the dom. of the eqn

Ex: $$\frac{1}{x} - \frac{22}{x-3} = \frac{-12}{x+3}$$

$$d \cdot \frac{a}{c} = \frac{b}{d} \cdot d$$
$$\Rightarrow \frac{ad}{c} = b$$

Step I: make common denom.

$$\frac{a}{c} = \frac{b}{c} \Rightarrow a = b$$

$$\frac{(x-3)(x+3) - 22x(x+3)}{\cancel{x}\cancel{(x-3)}\cancel{(x+3)}} = \frac{-12x(x-3)}{\cancel{x}\cancel{(x-3)}\cancel{(x+3)}}$$

$ac = bc$

Step 2: Simplify

$$(x-3)(x+3) - 22x(x+3) = -12x(x-3)$$

Cancel like denominator terms that appear on opp. sides of the "=" sign.

Step 3: Multiply out

$$x^2 - 9 - 22x^2 - 66x = -12x^2 + 36x$$
$$\Rightarrow 11x^2 - 102x - 9 = 0$$

Step 4: Quad form

$$x = \frac{102 \pm \sqrt{102^2 + 4 \cdot 99}}{22}$$

Step 5: Check the soln lie in the dom.

↳ values of x st when we plug them in we get something defined

What is domain of

$$\frac{1}{x} - \frac{22}{x-3} + \frac{12}{x+3} = 0$$

$$\text{dom} = \{x \mid x \neq 0,\ x \neq 3,\ x \neq -3\}$$

Ex: $$\frac{x-1}{x-1} = x \text{ ; What are solutions.}$$

Simplify, $1 = x$

But is 1 a soln

A: NO! 1 is not a soln b/c plugging it in makes the equ undefined!

Ex: $(3x+7)^2 = (\sqrt{x-2})^2$

Step: Square both sides to kill radical

$$9x^2 + 42x + 49 = x - 2$$

$$9x^2 + 41x + 51 = 0$$

Step: Solve the quad. It will have some soln.

Spse $x = -1$ is soln to the quadratic, then would be soln to the original eqn?

Ex: Absolute value equs.

$|x^2 - 6| = 9$

Step 1: Spse $x^2 - 6$ is positive

$\Rightarrow x^2 - 6 = 9$

$\Rightarrow x^2 = 15$

$\Rightarrow x = \pm\sqrt{15}$

We need to check that $x^2 - 6$ is pos or not for $x = \pm\sqrt{15}$

$(\pm\sqrt{15})^2 - 6 = 15 - 6 > 0$

Step 2: Spse $x^2 - 6$ is neg.

$\Rightarrow -x^2 + 6 = 9$

$\Rightarrow x^2 = -3$

Squares are pos, so no such x satisfies this.

Step 3: We plug in our answers to double ✓ that they work.

Section 1.8: Solving inequalities

Rmk: An ineq. is an eqn but instead of having "=" it involves "≤", "<", "≥", ">"

ie $3x > 1$

ie $3x + 7 \leq 19$

ie $x^{1/4}/(x-1) \leq x^2 - 3x + \pi$

Soln are values of x that make the ineq hold.

ie $3x > 1 \Rightarrow x > \frac{1}{3}$

$\Rightarrow$ soln. $(\frac{1}{3}, +\infty) = \{x \mid x > \frac{1}{3}\}$.

ie $3x + 7 \leq 19 \Rightarrow 3x \leq 12$

$\Rightarrow x \leq 4$,

$\Rightarrow$ soln $(-\infty, 4]$, $\{x \mid x \leq 4\}$.