Math 30200 / CS 38000, Winter 2024: Assignment 1
Due Friday, January 19th

1. Let $A = \{e : \Phi_e(e)\!\downarrow = 0\}$ and $B = \{e : \Phi_e(e)\!\downarrow \neq 0\}$. Note that A and B are disjoint. Show that A and B are *computably inseparable*, i.e., that there is no computable set C such that $A \subseteq C$ and $B \cap C = \emptyset$.

2. Write img f for the image of a function f, i.e., $\{n : \exists k\, f(k) = n\}$. Give an example of a total computable function f such that img f is not computable. Then show that if f and g are computable functions such that img f is the complement of img g, then both img f and img g are computable.

3. Show that $\emptyset' = \{e : \Phi_e(e)\!\downarrow\}$ is not an index set. *[Hint: Define a computable function f such that for all i, we have that $f(i) \neq i$ and $\Phi_{f(i)}$ behaves differently on i than on other indices. Then use the Recursion Theorem.]*

4. Read Section 1.7.2 in Soare. Do Exercises 1.7.8 and 1.7.9 in Soare. [You may assume Exercise 1.7.7.]