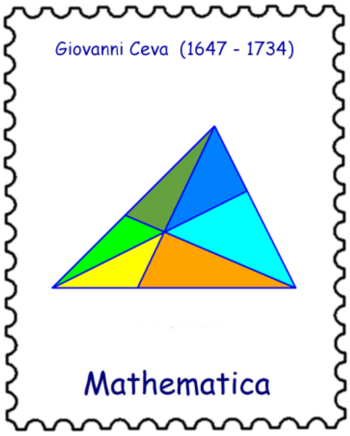
GIOVANNI BENEDETTO CEVA (September 1, 1647 – May 13, 1734)

by HEINZ KLAUS STRICK, Germany



The exact dates of GIOVANNI BENEDETTO CEVA's life were unknown for a long time and it was not until the beginning of this century that the above dates were discovered in old church records. To this day, no portrait of the Italian mathematician is known either.

GIOVANNI was born as the fifth of eight children (three girls, five boys) in the marriage of CARLO FRANCESCO CEVA and PAOLA COLUMBO. Five of his siblings later made a "career" in church service, four entered the Jesuit order.

GIOVANNI's father acquired a fortune through the purchase and sale of land and houses and in addition, there was regular income from his work as a tax collector in the service of the DUKE OF MILAN. After attending the Jesuit College in Milan, where GIOVANNI already showed a particular interest in mathematics and natural sciences, he first worked for his father and took on various administrative tasks in Milan and in Genoa. Finally he entered the service of FERDINANDO CARLO GONZAGA, Duke of Mantua and Montferrat.

His continuing scientific interest led GIOVANNI CEVA to enrol at the University of Pisa in 1670. His professors were also members of the *Mathematical-Physical Academy* in Rome, so CEVA had the opportunity to keep abreast of current developments in mathematics and physics.

First, CEVA set himself the goal of solving the problem of squaring the circle with the help of CAVALIERIan indivisibles. However, after he discovered that all his approaches were useless, he gave up working on this topic.

Nevertheless, he continued to delve into questions of classical Euclidean geometry until he was able to publish his first (and best-known) work *De lineis rectis se invicem secantibus statica constructio* in 1678. He dedicated the book to his employer, who in the meantime had appointed him as the main person responsible for the economy and finances of the duchy.

The book appeared in only one edition and was not widely distributed. As a result, the theorems he discovered were developed again by mathematicians who lived later. It was not until the research of the 19th century on the history of mathematics that the importance of CEVA's contributions were recognised.

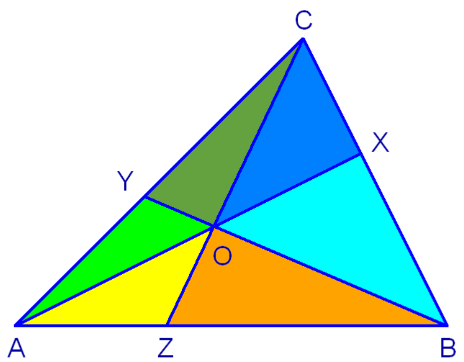
The theorem now named after GIOVANNI CEVA dealt with transversals in triangles.

However, as J B HOGENDIJK discovered in 1995, the subject had already been described in the 11th century by YUSUF AL-MUTAMAN, Emir of Zaragossa.

CEVA's theorem

If you connect the vertices *A, B, C* of a triangle with points *X, Y, Z* which lie on the opposite sides *a, b, c* respectively, then these transversals intersect at a common inner point *O* of the triangle exactly when the following relationship for the aspect ratios is satisfied:

$$\frac{|AZ|}{|ZB|}\cdot\frac{|BX|}{|XC|}\cdot\frac{|CY|}{|YA|}=1.$$



A simple proof is possible with the help of the following theorem (compare EUCLID's *Elements*, I,37 following):

Two triangles with the same base side have the same area if they have the same height.

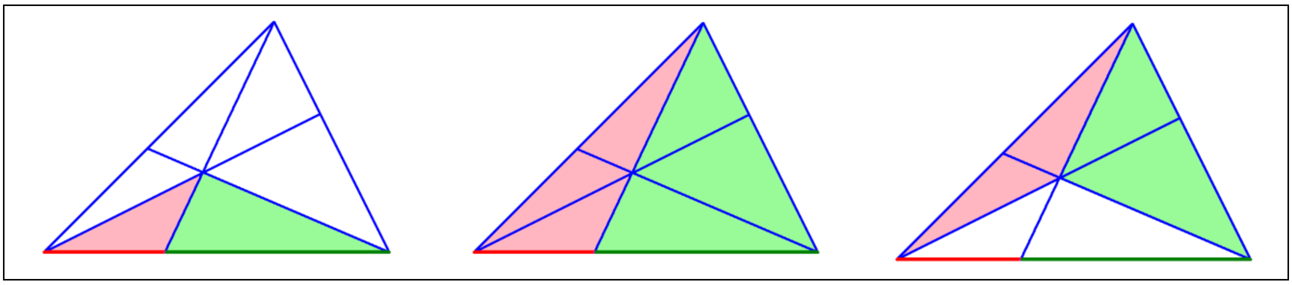
From this it follows that the area of two triangles with the same height are in the same ratio as their base sides. This means here:

$$\frac{|\Delta AZO|}{|\Delta ZBO|}=\frac{|AZ|}{|ZB|}=\frac{|\Delta AZC|}{|\Delta ZBC|},\ \frac{|\Delta BXO|}{|\Delta XCO|}=\frac{|BX|}{|XC|}=\frac{|\Delta BXA|}{|\Delta XCA|}\ \text{and}\ \frac{|\Delta CYO|}{|\Delta YAO|}=\frac{|CY|}{|YA|}=\frac{|\Delta CYB|}{|\Delta YAB|}.$$

By subtracting the numerator and denominator of the terms on the outside of the triangle, we get the following

$$\frac{|AZ|}{|ZB|}=\frac{|\Delta AOC|}{|\Delta BCO|},\ \frac{|BX|}{|XC|}=\frac{|\Delta ABO|}{|\Delta AOC|},\ \frac{|CY|}{|YA|}=\frac{|\Delta BCO|}{|\Delta ABO|}\ \text{and thus finally for the product}$$

$$\frac{|AZ|}{|ZB|}\cdot\frac{|BX|}{|XC|}\cdot\frac{|CY|}{|YA|}=\frac{|\Delta AOC|}{|\Delta BCO|}\cdot\frac{|\Delta ABO|}{|\Delta AOC|}\cdot\frac{|\Delta BCO|}{|\Delta ABO|}=1.$$



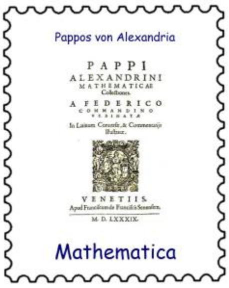
A special case of the theorem is when the points *X, Y, Z* are each in the middle of the sides ("The side bisectors of a triangle intersect at a point").

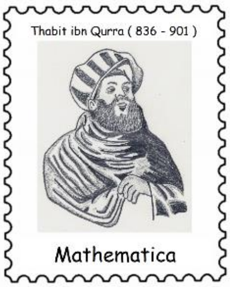
Also, the proofs of the theorems about the common points of the bisectors, the altitudes and the perpendiculars can be traced back to CEVA's theorem.

That this theorem is valid can also be justified – according to CEVA – as follows: If one thinks of different point masses in the three corner points *A, B, C* of the triangle, then the centres of gravity of two neighbouring masses each lie in the points *X, Y, Z*. The connecting lines of these points with the respective opposite corner point are lines of gravity that intersect in one point, the centre of gravity of the system.

In his work, CEVA also deals with a property equivalent to the theorem given above, which he himself had not known until then. Today this is called the *Theorem of MENELAUS*.

The mathematician MENELAUS, who came from Alexandria, lived in Rome around 100 AD. Only one book by him has survived (in Arabic translation). CLAUDIUS PTOLEMY and PAPPUS OF ALEXANDRIA refer to various other works by MENELAUS in their writings, as do numerous mathematicians of the Islamic Middle Ages, including THABIT IBN QURRA, ABU ARRAYHAN AL-BIRUNI and NASIR AL-DIN AL-TUSI.



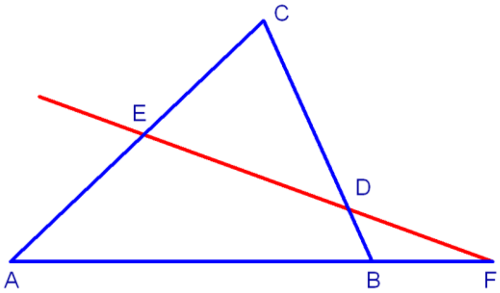


Theorem of Menelaus

If one draws a straight line that intersects the sides *a, b, c* of a triangle *ABC* or their extension(s) at the points *D, E, F*, then we have:

$$\frac{|AF|}{|FB|} \cdot \frac{|BD|}{|DC|} \cdot \frac{|CE|}{|EA|} = -1 .$$

Here we define: $|FB| = -|BF|$ and so on.



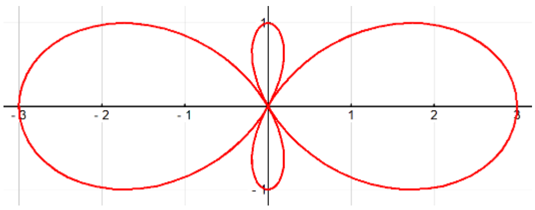
In the course of the following years, GIOVANNI CEVA wrote further works on mathematics and physics:

- *Opuscula mathematica* (1682, classical geometry and hydrodynamics: for example the parallelogram of forces, velocity measurement in/of fluids),
- *Geometria motus* (1692, investigation of curves),
- *Tria problemata geometris proposita* (1710, geometry),
- *De re nummeraria quod fieri potuit, geometrice tractata* (1711, application of mathematics to economic problems, for example real and nominal value of currencies, connection with the quantity of circulating money),
- *Opus hydrostaticum* (1728, summary of his writings on hydrodynamics).

In 1685 GIOVANNI CEVA married. His marriage to CECILIA VECCHI produced seven children. In 1686 the duke appointed him professor of mathematics at the University of Mantua, a post he held until the end of his life.

When the Habsburgs annexed the Duchy of Mantua in 1707 and the Duke was expelled, CEVA also took an oath of allegiance to the new ruler and was commissioned by him to build new fortifications, among other things. In the 1710s, he was able to prevent the city of Bologna from implementing its plans to divert the Reno River, which flows through Bologna, and connect it with the Po River by means of several expert opinions (and public pamphlets).

In 1734, French troops conquered Mantua and an epidemic broke out. Whether these events are causally related to CEVA's death is not known.



Note: The so-called *Cycloid of Ceva* is not named after GIOVANNI CEVA, but after his one year younger brother TOMMASO, who was a professor of mathematics and rhetoric at a Jesuit college near Milan.

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