Math 160B - Winter 2022 - Class Work - In Lecture, ~~January~~ 18 13 (No upload today.)

Name:

PID:

1. True or false? If true, give an algorithm. If false, what is the difficulty in constructing an algorithm? Let $R, S, R_i, \ldots$ be subsets of Σ^* where $\Sigma^* = \{0,1\}$ (or subsets of $\mathbb{N}$). Let $f : \Sigma^* \to \Sigma^*$ (or $f : \mathbb{N} \to \mathbb{N}$).

(It is OK use either semidecidability or computable enumerability. Typically it easier to use computable enumerability for hypotheses, and to use semidecidability for conclusions.)

(a) If R and S are c.e. (computably enumerable), then $R \cup S$ is c.e. ← Run R and S in parallel.

(b) If R and S are c.e. (computably enumerable), then $R \cap S$ is c.e. ← Same but output only things that both R and S output

(c) If R and S are c.e. (computably enumerable), then $R \backslash S$ is c.e. ← $R \cap \bar{S}$ ("No")

(d) If f is computable and R is decidable, is $S := \{w : R(f(w))\}$ decidable?

(e) If f is computable and R is c.e., is $S := \{w : R(f(w))\}$ c.e?

(f) If each R_i is decidable, then $\bigcup_i \{i\} \times R_i$ is c.e. ($i \in \mathbb{N}$)

If (c) was true.. S is c.e. $\Rightarrow$ $\bar{S}$ is c.e.

Hence S is c.e. $\Rightarrow$ S is decidable.

For (d), (e) $S := \{w : R(f(w))\}$

f is an example of a "many one reduction" from S to R.

(d) - Yes. run algorithm for f, then for R.

(e) Yes, same idea, use the algorithm that semidecide Rs to get an algorithm that semidecides S

(f) "No" $\bigcup_i \{i\} \times R_i = \bigcup_i \{\langle i, w\rangle : w \in R_i\} = \{\langle i, w\rangle : w \in R_i\}$

Take any $X \subseteq \mathbb{N}$. Let $R_i = \emptyset$ if $i \notin X$

$R_i = \Sigma^*$ if $i \in X$.

each R_i is decidable

But $\{\langle i, w\rangle : w \in R_i\} = \{\langle i, w\rangle : i \in X\}$

- There are uncountably many $X \subseteq \mathbb{N}$

- There are countably many c.e. sets.