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Price Discrimination and Inventory Allocation in Bertrand Competition

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Abstract. ***Problem definition*:** It is common practice for firms to deploy strategies based on customer segmentation (by clustering customers into different segments) and price discrimination (by offering different prices to different customer segments). Price discrimination, although seemingly beneficial, can hurt firms in competitive environments. ***Academic/practical relevance*:** It is thus critical for firms to understand when to engage in price discrimination and how to support discriminatory pricing practices with appropriate inventory management strategies. This paper tackles this overarching question through operational lenses by studying the joint impact of price discrimination and the allocation of limited inventory across customer segments. ***Methodology*:** We develop a Bertrand competition game featuring capacity restrictions, quality differentiation, and customer heterogeneity. ***Results*:** We characterize (pure- or mixed-strategy) Nash equilibria for a single-stage game reflecting uniform pricing and for a two-stage inventory-price game reflecting discriminatory pricing along with endogenous inventory allocation. ***Managerial implications*:** We identify three sources of market friction in price competition enabling firms to earn higher profits: capacity limitations, quality differentiation, and customer heterogeneity. Price discrimination eliminates the market frictions from customer heterogeneity, but strategic inventory allocation restores (or strengthens) the market frictions from capacity limitations. As such, price discrimination is only beneficial when combined with optimal inventory allocation across segments. We discuss relevant real-world examples featuring regional price discrimination along with strategic inventory allocation, including fast fashion and vaccines. Otherwise, uniform pricing may outperform discriminatory pricing. Our results thus underscore the critical role of inventory allocation in the design of competitive pricing strategies.

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1. Introduction

Pricing is one of the main levers to boost profits in competitive environments. If firms set high prices, they bear the risk of losing customers to their competitors. Vice versa, if firms set low prices, they may not fully exploit the market potential. To avoid leaving money on the table, firms need to strike a balance between these two strategies. In today's data-rich environment, two common practices to achieve this objective are customer segmentation and price discrimination.

Customer segmentation is enabled by the ubiquity of vast volumes of data on customers' past transactions, loyalty status, demographics, etc. These data offer opportunities to define relevant clusters of customers and refine pricing strategies accordingly. A common segmentation is based on how customers trade off quality and price; price-sensitive customers (PSCs) favor cheaper products, whereas quality-sensitive customers (QSCs) are willing to pay a premium for higher-quality products.

In addition, many firms have the ability to offer different prices to different customer segments through price discrimination. This practice has become common both in e-commerce and brick-and-mortar retail. Examples include regional pricing (i.e., different prices based on geographical location), time-based pricing, and channel-based pricing. A natural question is thus how to set different prices to different segments in the presence of competition. More fundamentally, is it always beneficial to opt for a discriminatory pricing strategy (i.e., offering different prices to different segments) as opposed to a uniform pricing strategy (i.e., charging the same price to all customers)?

Research in economics and marketing has shown that price discrimination is beneficial in monopolistic settings but not necessarily in oligopolistic settings. In other words, price discrimination induces an extra degree of freedom, which although seemingly beneficial, may intensify competition and decrease firms' profits (see, e.g., Thisse and Vives 1988, Corts 1998, Fudenberg and Tirole 2000, Choudhary et al. 2005, Choe et al. 2018). In this paper, we revisit this question through operational lenses by considering capacity restrictions and inventory allocation decisions. Namely, can price discrimination benefit the firms, and if so, under which conditions? How should the firms allocate their inventory across customer segments to design effective price discrimination strategies? Additionally, to what extent do the benefits of price discrimination depend on inventory allocation decisions?

We study these questions by proposing a game-theoretic model of Bertrand competition—one of the most fundamental price competition models—that features capacity restrictions, product differentiation, and customer heterogeneity. Specifically, two firms offer products with different quality levels in a market comprising price-sensitive customers and quality-sensitive customers. Uniform pricing is modeled as a one-stage game, where both firms charge the same price to all customers. Price discrimination is modeled as a two-stage game, where firms first allocate their inventory across the customer segments and then compete on prices in each segment in the second stage. In the first stage, we study a baseline where firms allocate their inventory proportionally to the size of the customer segments (referred to as pro rata allocation) and also, examine the case where firms optimize their inventory on each segment (referred to as endogenous allocation).

This paper builds upon the vast literature on Bertrand competition. In its simplest form, the Bertrand game leads to a "price war," where both firms charge their marginal cost and earn zero profit. The firms, however, can escape from this price war through capacity restrictions and quality differentiation—two sources of *market friction* (Levitan and Shubik 1972, Tirole 1988, Ghose and Gu 2008). This paper extends this literature by uncovering a third source of market friction—customer heterogeneity—and studying the impact of price discrimination combined with inventory allocation in Bertrand competition. Specifically, this paper makes the following contributions.

• Extending Bertrand competition with customer heterogeneity and discrimination. Most studies on price competition between differentiated firms have relied on the Hotelling framework by considering a continuum of customers with heterogenous quality preferences (typically, using a uniform distribution). In this paper, we propose a Bertrand competition model where customers are divided into two discrete segments: price-sensitive customers and quality-sensitive customers. When compared with the continuous price-demand relationships from the Hotelling framework, our model gives rise to discontinuous "winner takes all" dynamics (in which one firm can attract the entire market up to capacity).

• Characterizing Nash equilibria. In our model, the existence of pure-strategy equilibria is not always guaranteed, which contrasts with various price competition games based on continuous demand models, such as the Hotelling framework. Our main technical result establishes the existence and uniqueness of a mixed-strategy Nash equilibrium for the Bertrand competition game featuring customer heterogeneity and quality differentiation. In this setting, both firms need to account for each customer segment, leading to a mixed-strategy equilibrium with overlapping, yet distinct, supports. Furthermore, we report a complete characterization of the Nash equilibria for Bertrand competition games under homogenous (price-sensitive or quality-sensitive) customers, differentiated qualities, and differentiated capacities.

• Examining the impact of customer heterogeneity on firms' performance. We find that under uniform pricing, the firm with the higher-quality product benefits from a more quality-sensitive customer pool, whereas the firm with the lower-quality product benefits from a more heterogenous customer pool (as opposed to a more price-sensitive pool). Thus, discrete customer heterogeneity introduces an additional source of market friction that allows both firms to earn positive profits even under uncapacitated competition. We also find that the expected prices are not monotonic with the proportion of quality-sensitive customers.

• Analyzing the joint impact of price discrimination and inventory allocation. We characterize the subgame-perfect equilibria of the two-stage inventory-price competition game. Our results underscore the critical role of inventory allocation; price discrimination with pro rata inventory allocation hurts both firms relative to uniform pricing, but combining price discrimination with strategic inventory allocation can benefit both firms. Under low demand, uniform pricing dominates price discrimination with pro rata allocation, whereas price discrimination combined with endogenous inventory allocation dominates uniform pricing. Under high demand, uniform pricing dominates price discrimination with pro rata allocation and can even dominate price discrimination with endogenous inventory allocation (because the market friction from capacity restrictions is now ubiquitous). These insights extend to the intermediate moderate-demand regime and are robust to various modeling assumptions.

Our results suggest that, under competition, discriminatory pricing is not necessarily beneficial by itself and must be coupled with strategic capacity allocation across customer segments. Indeed, price discrimination eliminates the market friction from customer heterogeneity, whereas inventory allocation introduces an extra friction by strengthening capacity restrictions—and the latter effect can dominate the former. Practically, these findings suggest that firms need to commit to a capacity allocation strategy prior to engaging in price discrimination, whenever possible. Examples of (regional) price discrimination with such capacity commitment include the following.

- Fast fashion (e.g., Zara versus H&M). Consider two brick-and-mortar clothing retailers that offer rapidly changing collections, with one brand of higher quality than the other.[1] The two retailers operate in several countries, with each country forming a customer segment. Both firms can offer different prices in different locations.[2] Given the short product shelf life and geographical distance between countries, firms must commit inventory levels for each segment separately.
- Vaccines (e.g., Pfizer versus Moderna). The prices of coronavirus disease 2019 vaccines can vary widely across countries, from \$2 to over \$40 per dose.[3] Similar variations occur for other vaccines. Vaccines from different manufacturers naturally have different quality levels (e.g., efficacy and side effects). Again, short shelf times and geographical distance prevent inventory rebalancing.

Vice versa, when firms cannot commit to a prespecified inventory allocation (e.g., retailers using one warehouse to cater to quality-seeking and discount-seeking customers), discriminatory pricing does not necessarily outperform uniform pricing. In other words, competing firms should not blindly adopt price discrimination but only do so if market frictions from inventory restrictions are strong enough to offset the loss in market frictions from customer heterogeneity.

2. Literature Review

This paper relates to the price discrimination literature. Starting from Robinson (1934), many studies have focused on how price discrimination affects social welfare (see, e.g., Schmalensee 1981; Varian 1985; Armstrong and Vickers 1993, 2001; Cowan 2016, Bergemann et al. 2022) and fairness (Cohen et al. 2022). Although discriminatory pricing boosts profits in a monopolistic setting, price discrimination intensifies competition in oligopolies and can hurt firms' profits. This result has been shown in the context of personalized pricing (Thisse and Vives 1988, Chen and Iyer 2002, Choudhary et al. 2005, Choe et al. 2018) and third-degree price discrimination (Shaffer and Zhang 1995, Corts 1998, Fudenberg and Tirole 2000, Esteves 2010). In a setting related to ours, Corts (1998) found that price discrimination leads to lower prices and profits in both segments, and in response, proposed strategic nondiscriminatory commitments.

Research in marketing has identified business environments in which price discrimination can be more profitable than uniform pricing, even under competition. Chen et al. (2001) showed that when targeting customers is hard, price discrimination coupled with improved targeting abilities can lead to higher profits. Empirically, Besanko et al. (2003) and Esteves and Resende (2016) found that discriminatory pricing in the form of customized coupons and targeted advertising can boost profits. Belleflamme et al. (2020) found that personalized pricing can be beneficial when firms have different abilities to identify customer valuations. Our paper complements the literature by adding a new—operational—layer through which firms can benefit from price discrimination under competition: by strategically allocating inventory across segments to create capacity shortages.

We leverage a model of Bertrand competition, one of the most common models, to study price competition. In its original setting, firms sell homogenous products, and customers make purchasing decisions based exclusively on prices (Bertrand 1883). This game admits a unique Nash equilibrium, in which the firms set prices at marginal cost and earn zero profits—an outcome referred to as the price race to the bottom. This undesired outcome vanishes when the firms face capacity constraints or sell differentiated products. In such cases, the firms can attract customers without charging the lowest possible prices, thus earning positive profits. For that reason, capacity constraints and product differentiation are often referred to as *market frictions* (Ghose and Gu 2008).

In a capacitated Bertrand competition, Edgeworth (1925) showed that marginal pricing is no longer an equilibrium. Levitan and Shubik (1972) characterized the equilibrium under the Bertrand–Edgeworth competition. Kreps and Scheinkman (1983) showed that capacity commitments relax price competition and induce higher prices. Brock and Scheinkman (1985) examined the role of capacity restrictions for repeated Bertrand games. Osborne and Pitchik (1986) considered an instance with differentiated but exogenous capacities and found that equilibrium prices decrease with demand. Our results with homogenous price-sensitive customers mirror these findings.

Turning to product differentiation, Hotelling (1929) introduced a seminal model, in which two firms first select their locations in a "linear city" and then compete on prices, with customers trading off prices and distances—a setting often referred to as horizontal differentiation. The game admits a pure-strategy equilibrium in which both firms earn positive profits, highlighting that horizontal differentiation can alleviate the Bertrand price race to the bottom.

Using the Hotelling framework, Gabszewicz and Thisse (1979, 1980) incorporated product differentiation into price competition—referred to as vertical differentiation. Their results identify regimes where only the firm with the higher-quality product earns a positive profit. Shaked and Sutton (1982, 1983) endogenized quality differentiation, showing that firms will offer differentiated products to alleviate price competition. Tirole (1988) presented a price competition model under a continuum of customers with different quality preferences. This model admits a pure-strategy Nash equilibrium, in which even the firm with the lower-quality product can earn a positive profit if customers are sufficiently differentiated. In our paper, we incorporate customer heterogeneity into the traditional Bertrand competition model by considering two customer segments: price-sensitive customers (who make decisions based only on prices) and quality-sensitive customers (who trade off price and quality). The discontinuity of the demand structure can lead to mixed-strategy equilibria. Furthermore, we investigate how the market structure affects the equilibrium and the profits.

Only a handful of studies have combined capacity restrictions, quality differentiation, and customer heterogeneity into Bertrand competition. Banker et al. (1988) and Chambers et al. (2006) proposed a two-stage model where firms compete on quality and then, on prices. Acemoglu et al. (2009) developed a two-stage game where firms compete on both capacities and prices. Boccard and Wauthy (2010) characterized the pure-strategy pricing equilibrium in a three-stage game in quality, capacity, and price. In a Stackelberg game in capacity and price between two quality-differentiated firms, Porteus et al. (2010) found that the leader can price relatively low to leave the leftover demand to the follower, hence avoiding direct competition. Our paper also uncovers the strategic use of capacity allocation but from a different perspective—to create inventory shortages as an additional market friction in Bertrand competition in order to increase equilibrium prices.

This paper is also related to the revenue management literature, which dynamically optimizes prices and inventory levels to match supply with demand (see, e.g., Bitran and Caldentey 2003, Talluri and Van Ryzin 2006, Özer and Phillips 2012). Netessine and Shumsky (2005) conveyed that competition leads to higher prices because firms allocate more capacity to high-valuation customers. Levin et al. (2009) presented a stochastic pricing game, where differentiated goods are sold to strategic customers who can time their purchases. Adida and Perakis (2010) considered a continuous-time differential game of pricing and inventory, where differentiated products share production capacity. Martínez-de Albéniz and Talluri (2011) studied a duopoly competition game with homogeneous products under demand uncertainty, and Gallego and Hu (2014) added product differentiation to this problem. Empirically, Cohen et al. (2020) showed that in the airline industry, markups in the fare ladder can increase revenue in the presence of quality differentiation.

In summary, our paper contributes to the literature on Bertrand competition (i) by studying the joint effect of price discrimination and inventory allocation and (ii) by incorporating quality differentiation, capacity differentiation, and customer heterogeneity. From a technical standpoint, this paper characterizes the unique mixed-strategy Nash equilibrium in Bertrand competition with quality differentiation and heterogenous customers and all Nash equilibria in Bertrand competition with homogenous customers, quality differentiation, and capacity differentiation.

3. Preliminaries

3.1. Setting and Assumptions

We consider a noncooperative single-period Bertrand–Edgeworth game between two firms, indexed by $i = 1, 2$.[4] Both firms have access to a fixed inventory I, consistent with the theoretical literature (Levitan and Shubik 1972, Dasgupta and Maskin 1986, Osborne and Pitchik 1986) and the applied literature (Netessine and Shumsky 2005, Behzad et al. 2015, Behzad and Jacobson 2016).

The market is characterized by a deterministic inelastic demand $D > 0$.[5] Firms compete on prices, and Firm i sets a price $p_i \in [0, p_{\max}]$. The upper bound $p_{\max}$ can be interpreted as a proxy for an outside option (e.g., if prices are too high, customers will not purchase from the market) or as a maximal price that firms are allowed to charge (e.g., because of price regulations). From a technical perspective, this assumption simplifies the derivation of our results without affecting our qualitative insights. We assume that both firms face identical marginal costs, normalized to zero without loss of generality.[6] Finally, we assume that $D \le 2I$: that is, the market demand falls below the total market capacity (if $D > 2I$, then $p_1 = p_2 = p_{\max}$ is a strictly dominant strategy).

We denote by $D_i(p_i, p_j)$ the demand served by Firm i when it sets p_i and Firm j sets p_j. We denote by $\tilde{\pi}_i(p_i, p_j)$ the profit function, given by $\tilde{\pi}_i(p_i, p_j) = p_i D_i(p_i, p_j)$. Let F_i be the cumulative distribution function of Firm i's mixed strategy (when Firm i plays a pure strategy, F_i is a step function). We also denote by $Q_{F_i}(p_i)$ the atom probability at p_i associated with strategy F_i, namely

$$\begin{cases} Q_{F_i}(p_i) = F_i(p_i) - \lim\limits_{p \to p_i^-} F_i(p), & \forall p_i \in (0, p_{\max}], \\ Q_{F_i}(0) = F_i(0). \end{cases}$$

We denote by $\pi_i(p_i, F_j)$ Firm i's profit when it sets p_i and Firm j plays according to the strategy F_j. Finally, we denote Firm i's expected profit by $\overline{\Pi}_i$. Mathematically, we have

$$\pi_i(p_i, F_j) = \int_0^{p_{\max}} \tilde{\pi}_i(p_i, p_j)\, \mathrm{d}F_j(p_j), \quad \forall p_i \in [0, p_{\max}],$$
$$\overline{\Pi}_i = \int_0^{p_{\max}} \pi_i(p_i, F_j)\, \mathrm{d}F_i(p_i).$$

We consider a market with two customer segments, a mass D_p of PSCs and a mass D_u of QSCs, so that $D = D_u + D_p$. Price-sensitive customers purchase Firm i's product if $p_i < p_j$ as long as Firm i has sufficient inventory (customers are indifferent between both firms if $p_i = p_j$). Quality-sensitive customers make purchasing decisions by minimizing $(p_i - \alpha\mu_i)$, where μ_i denotes the quality of Firm i's product and $\alpha > 0$ represents the quality sensitivity.[7] Without loss of generality, we assume that Firm 1's product is of lower quality than Firm 2's product (i.e., $\mu_1 < \mu_2$). We define the *quality differential* $\Delta = \alpha(\mu_2 - \mu_1) > 0$. We assume throughout the paper that $\Delta \le 2p_{\max}/5$: that is, the quality differential is not too high (otherwise, Firms 1 and 2 effectively serve different markets). When customers are indifferent between the two firms (i.e., when $p_i = p_j$ for PSC and when $p_1 + \Delta = p_2$ for QSC), each firm will serve half the market. This rationing rule follows the original Bertrand–Edgeworth setting and has been commonly adopted in the literature (e.g., Levitan and Shubik 1972, Kreps and Scheinkman 1983) (Table 1).

3.2. Analytical Setting: Homogenous and Heterogenous Markets

Let us start by characterizing the equilibrium in homogenous markets.

3.2.1. Homogenous Market with Price-Sensitive Customers ($D_p = D$).

We have

$$D_i(p_i, p_j) = \begin{cases} \min(D, I) & \text{if } p_i < p_j, \\ \dfrac{D}{2} & \text{if } p_i = p_j, \\ \max(D - I, 0) & \text{if } p_i > p_j. \end{cases}$$

This setting is consistent with Edgeworth (1925), Levitan and Shubik (1972), and Kreps and Scheinkman (1983), albeit simpler because of our inelastic demand assumption. Proposition 1 reports the Nash equilibrium—a special case of the more-complex models studied in this paper.

Table 1. Table of Notation

Symbol	Description
Parameters	
D	Total market demand
D_p	Demand from PSCs
D_u	Demand from QSCs
I	Total inventory from each firm
$p_{\max}$	Maximum price each firm can charge
Δ	Quality differential of Firm 2, relative to Firm 1
Pricing strategy	
N	Uniform (i.e., nondiscriminatory) pricing
R	Discriminatory pricing with pro rata capacity allocation
E	Discriminatory pricing with endogenous capacity allocation
Decision	
p_i	Price played by Firm i
$F_i(p)$	Firm i's mixed strategy at price p
$Q_{F_i}(p)$	Atom of Firm i's mixed strategy F_i at price p
I_i^p	Firm i's inventory allocated to PSC
I_i^u	Firm i's inventory allocated to QSC
Profit	
$\tilde{\pi}_i(p_i, p_j)$	Firm i's profit given Firm i plays p_i and Firm j plays p_j
$\pi_i(p_i, F_i)$	Firm i's profit given Firm i plays p_i and Firm j plays F_j
$\overline{\Pi}_i$	Firm i's expected profit at equilibrium
$\overline{\Pi}_i^p$	Firm i's expected profit at equilibrium from PSC
$\overline{\Pi}_i^u$	Firm i's expected profit at equilibrium from QSC

Proposition 1. *With homogeneous PSC, the game admits a unique Nash equilibrium.*

- *If $D \le I$, we have $p_1 = p_2 = 0$ and $\overline{\Pi}_1 = \overline{\Pi}_2 = 0$.*
- *If $I < D \le 2I$, the game admits a mixed-strategy Nash equilibrium, with $\overline{\Pi}_1 = \overline{\Pi}_2 = (D - I)p_{\max}$.*

When demand is below each firm's capacity, both firms set $p_i = 0$ and make zero profit—the well-known price race to the bottom. As firms become capacitated (i.e., $D > I$), expected prices increase, allowing both firms to earn positive expected profits. Specifically, each firm's expected profit is $(D - I)p_{\max}$, which is the profit that each firm would secure by playing $p_{\max}$ (we refer to this quantity as the *secured profit from leftovers*). This yields the classical result that capacity constraints introduce market frictions, thus alleviating price competition.

3.2.2. Homogenous Market with Quality-Sensitive Customers ($D_u = D$).

We have

$$D_1(p_1, p_2) = \begin{cases} \min(D, I) & \text{if } p_1 + \Delta < p_2, \\ \dfrac{D}{2} & \text{if } p_1 + \Delta = p_2, \\ \max(D - I, 0) & \text{if } p_1 + \Delta > p_2, \end{cases}$$
$$D_2(p_1, p_2) = D - D_1(p_1, p_2).$$

The Nash equilibrium characterization is reported in Proposition 2.

Proposition 2. *With homogeneous QSC, the following holds.*

- *If $D \le I$, Firm 2 sets $p_2 = \Delta$ (Firm 1 can play various mixed strategies): $\overline{\Pi}_1 = 0$ and $\overline{\Pi}_2 = \Delta D$.*
- *If $I < D < (2 - (\Delta/p_{\max}))I$, the game admits a unique mixed-strategy Nash equilibrium. The expected profits satisfy $\overline{\Pi}_1 = (D - I)p_{\max}$ and $\overline{\Pi}_2 = (D - I)p_{\max} + I\Delta$.*
- *If $D \ge (2 - (\Delta/p_{\max}))I$, the game admits a unique pure-strategy equilibrium with $p_1 = p_2 = p_{\max}$. The expected profits satisfy $\overline{\Pi}_1 = (D - I)p_{\max}$ and $\overline{\Pi}_2 = Ip_{\max}$.*

The specifics of the mixed strategies are reported in EC.1. When the firms are uncapacitated (i.e., $D \le I$), Firm 2 sets its price at Δ to capture the entire market. Thus, Firm 2 can earn a positive profit because of its quality advantage, leaving zero profit to Firm 1. Firm 1 plays a mixed strategy over a range of low prices, acting as a threat to prevent Firm 2 from increasing its price beyond Δ. Once again, this equilibrium reflects the price race to the bottom because both firms end up undercutting each other. Yet, because of the quality differential, Firm 1 can no longer undercut Firm 2 when $p_2 = \Delta$, so Firm 2 ends up capturing the entire market.

Again, as firms become capacitated (i.e., $D > I$), capacity restrictions bring market frictions and raise the equilibrium prices. The setting with QSC, however, brings two main novelties. First, Firm 1 keeps earning $(D-I)p_{\max}$ (i.e., the secured profit from leftovers), but Firm 2 earns an extra profit of $I\Delta$ induced by its quality advantage. Second, when demand exceeds $(2-(\Delta/p_{\max}))I$, both firms play $p_{\max}$, even though the market is not saturated. To see this, note that $D \geq (2-(\Delta/p_{\max}))I$ is equivalent to $(D-I)p_{\max} \geq I(p_{\max}-\Delta)$, meaning that for Firm 1, it becomes more profitable to settle for leftovers at $p_{\max}$ than compete with Firm 2 and serve I customers at $p_{\max}-\Delta$.

3.2.3. Heterogenous Customers. As compared with the baseline setting with homogenous (price-sensitive or quality-sensitive) customers, this paper introduces customer heterogeneity—that is, by considering the case where $D_p < D$ and $D_u < D$. To retain analytical tractability, we focus in the main body of the paper on two regimes identified in Proposition 2: $D \leq I$ (low demand; in Section 4) and $D \geq (2-(\Delta/p_{\max}))I$ (high demand; in Section 5). In Online Appendix D, we establish the robustness of our findings in the moderate-demand regime, namely when $D \in (I, (2-(\Delta/p_{\max}))I)$.

3.3. Paper Road Map

The main technical results of the paper fall into four categories.

- We characterize the Nash equilibrium (or equilibria) for the following cases: (i) uniform pricing with pooled capacity across both segments (Propositions 3 and 10), (ii) discriminatory pricing with pro rata capacity allocation across segments (Propositions 6 and 12), and (iii) discriminatory pricing with endogenous capacity allocation (Propositions 7 and 13). We thus identify the impact of discriminatory pricing (relative to uniform pricing) and endogenous capacity allocation (relative to pooled capacity and pro rata inventory allocation).[8]
- We uncover that under uniform pricing, the firm with the lower-quality product benefits from customer heterogeneity, whereas the firm with the higher-quality product benefits from more quality-sensitive customers (Propositions 5 and 11). This result identifies customer heterogeneity as an extra source of market friction besides capacity restrictions and quality differentiation, enabling higher prices with a more heterogeneous customer pool (Proposition 4).
- We find that uniform pricing outperforms discriminatory pricing with pro rata capacity allocation but can be dominated by discriminatory pricing with strategic inventory allocation (Propositions 8, 14, and 15). The driver of this result is that discriminatory pricing eliminates the friction from customer heterogeneity but can restore—or even strengthen—the friction from capacity restrictions, enabling both firms to charge higher prices (Proposition 9).
- We highlight the disparate effect of quality differentiation under uniform versus discriminatory pricing (Online Appendix E). Under uniform pricing, a stronger quality differential increases the profit of both firms; the firm with the higher-quality product benefits from a higher quality sensitivity, whereas the firm with the lower-quality product benefits from a higher customer heterogeneity. In contrast, under discriminatory pricing, a stronger quality differential benefits the firm with the higher-quality product but hurts the firm with the lower-quality product.

4. Low-Demand Regime

In this section, we assume that $D \leq I$: that is, each firm would have sufficient capacity to accommodate the entire market. The proofs of our statements are relegated to Online Appendix B and EC.3.

4.1. Uniform Pricing

Because $D \leq I$, the demand structure is given by

$$D_1(p_1,p_2) = \begin{cases} D & \text{if } p_1+\Delta < p_2, \\ \dfrac{D_u}{2} + D_p & \text{if } p_1+\Delta = p_2, \\ D_p & \text{if } p_1 < p_2 < p_1+\Delta, \\ \dfrac{D_p}{2} & \text{if } p_1 = p_2, \\ 0 & \text{if } p_1 > p_2. \end{cases}$$

$$D_2(p_2,p_1) = D - D_1(p_1,p_2).$$

4.1.1. Nash Equilibrium. When customers are heterogeneous, each firm's expected profit includes the profits generated by PSC and QSC. As a result, Firm 1's pricing decision does not solely rely on Firm 2's decision to play p or $p+\Delta$ but on both. Similarly, Firm 2's pricing decision depends on Firm 1's strategies at p and at $p-\Delta$. This complex game structure makes the analysis more intricate; yet, we can still characterize the Nash equilibrium and establish its uniqueness.

Proposition 3. *Under heterogenous customers and low demand (i.e., $D \leq I$), there exists a unique mixed-strategy Nash equilibrium (F_1, F_2) given by*

$$\begin{aligned}
F_1(p_1) &= 1 - \frac{\tilde{p}+\Delta}{p_1+\Delta}, && \forall p_1 \in [\tilde{p}, \hat{p}], \\
F_1(p_1) &= \frac{D - \frac{D_u(\tilde{p}+\Delta)}{p_1}}{D_p}, && \forall p_1 \in [\hat{p}, \tilde{p}+\Delta], \\
F_2(p_2) &= 1 - \frac{\hat{p}}{p_2}, && \forall p_2 \in [\hat{p}, \tilde{p}+\Delta], \\
F_2(p_2) &= \frac{D - \frac{D_p\hat{p}}{p_2-\Delta}}{D_u}, && \forall p_2 \in [\tilde{p}+\Delta, \hat{p}+\Delta],
\end{aligned}$$

where $\tilde{p}$ and $\hat{p}$ are the unique positive (real) solutions of the following system of equations:

$$\frac{\tilde{p}+\Delta}{\hat{p}} - 1 = \frac{\tilde{p}+\Delta}{\hat{p}+\Delta}\frac{D_p}{D_u}, \tag{1}$$

$$\frac{\hat{p}}{\tilde{p}} - 1 = \frac{\hat{p}}{\tilde{p}+\Delta}\frac{D_u}{D_p}. \tag{2}$$

In particular, $\tilde{p}$ and $\hat{p}$ are such that $0 < \tilde{p} < \hat{p} < \tilde{p}+\Delta < \hat{p}+\Delta$.

The expected profits satisfy $\overline{\Pi}_1 = \hat{p}D_p$ and $\overline{\Pi}_2 = (\tilde{p}+\Delta)D_u$.

The equilibrium structure is more sophisticated than before because uniform pricing strategies cater to both customer segments. The proof first shows that Equations (1) and (2) admit a unique solution and then, verifies that the solution from Proposition 3 is a Nash equilibrium. Establishing uniqueness is more challenging. We derive necessary conditions, showing that the supports of both mixed strategies have length Δ. We then retrieve equations akin to Equations (1) and (2) and conclude by noting that the functional form outlined in Proposition 3 is necessary. The assumption that Δ is small relative to $p_{\max}$ is used to ensure that $\hat{p} + \Delta \le p_{\max}$.

Figure 1 depicts both firms' mixed strategies. Recall that with homogeneous PSC, both firms play a pure strategy at $p = 0$, whereas with homogeneous QSC, Firm 2 plays a pure strategy at $p = \Delta$ (Section 3.2). In contrast, when customers are heterogeneous, both firms play mixed strategies with distinct—albeit overlapping—supports of length Δ. Firm 1 plays a continuous strategy over $[\tilde{p}, \tilde{p}+\Delta]$, whereas Firm 2 plays a continuous strategy over $[\hat{p}, \hat{p}+\Delta]$.

The limiting cases converge to the equilibria with homogeneous customers. When $D_u \to 0$ (i.e., the market is dominated by PSC), we have $\tilde{p} \to 0$ and $\hat{p} \to 0$. Consequently, the Nash equilibrium converges to the pure strategy $p_1 = p_2 = 0$, and both firms earn zero profit. When $D_u \to D$ (i.e., the market is dominated by QSC), we have $\tilde{p} \to 0$ and $\hat{p} \to \Delta$, so that Firm 2's strategy converges to the pure strategy $p_2 = \Delta$ (with an expected profit of Δ) and Firm 1's strategy converges to a mixed-strategy identified in the proof of Proposition 2 (with an expected profit of zero).

4.1.2. Pricing Strategies.

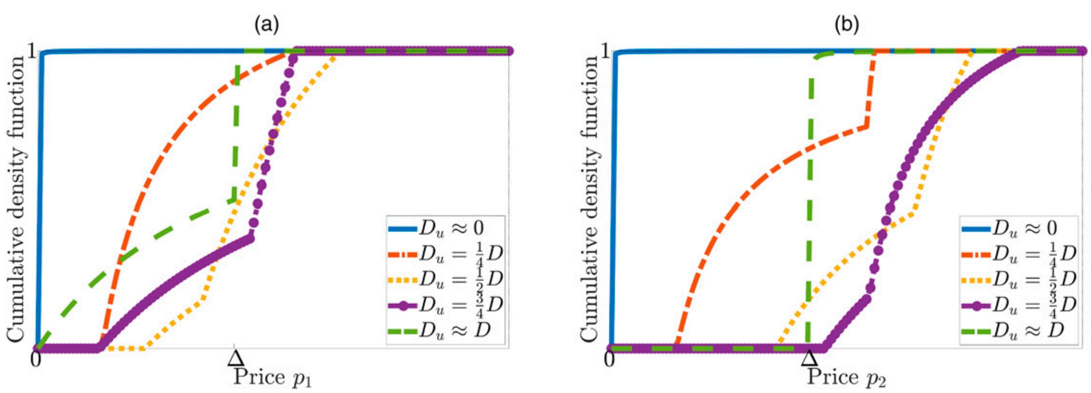
We first note that the expected prices are not monotonic with D_u or D_p (holding D constant). Indeed, both $\tilde{p}$ and $\hat{p}$ increase with D_u over an interval of the form $[0, d]$ and then, decrease with D_u over $[d, D]$ (for a parameter $0 < d < D$). Accordingly, as D_u increases, the cumulative distribution functions of both firms' mixed strategies move first rightward and then leftward (see Figure 1). Proposition 4 establishes this result formally, namely (i) when $d_1 < d_2$ are smaller than 0.5, each firm's mixed strategy when $D_u = d_1$ is stochastically dominated by its mixed strategy when $D_u = d_2$ and (ii) when $d_1 < d_2$ are larger than 0.75, each firm's mixed strategy when $D_u = d_2$ is stochastically dominated by its mixed strategy when $D_u = d_1$.

Proposition 4. *Fix D, D_u, and D_p such that $D_u + D_p = D$. Let $F_{1,d}(\cdot)$ and $F_{2,d}(\cdot)$ be the cumulative distribution functions when $D_u = d$ and $D_p = D - d$. We then have the following.*

- *For any $d_1, d_2 \in [0, 0.5062D]$ such that $d_1 < d_2$,* $\begin{cases} F_{1,d_1}(p) \ge F_{1,d_2}(p), \ \forall p \in [0, p_{\max}], \\ F_{2,d_1}(p) \ge F_{2,d_2}(p), \ \forall p \in [0, p_{\max}]. \end{cases}$
- *For any $d_1, d_2 \in [0.7566D, D]$ such that $d_1 < d_2$,* $\begin{cases} F_{1,d_1}(p) \le F_{1,d_2}(p), \ \forall p \in [0, p_{\max}], \\ F_{2,d_1}(p) \le F_{2,d_2}(p), \ \forall p \in [0, p_{\max}]. \end{cases}$

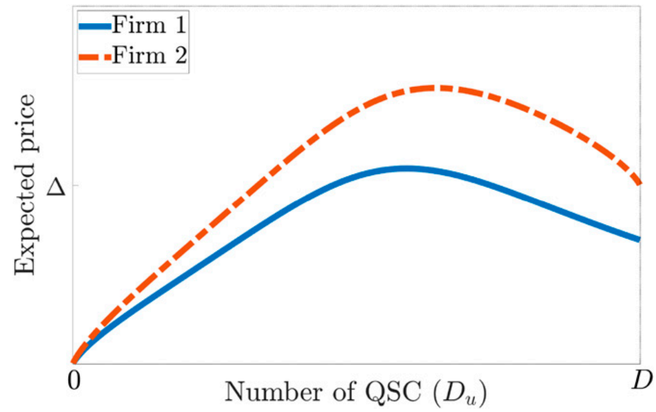
Figure 2 confirms that the expected prices are unimodal with respect to D_u, holding D constant. When the market is mainly composed of PSCs (i.e., D_u is low), both firms play low prices, and all QSCs will choose Firm 2's product. As the market becomes more populated by QSCs, Firm 2 will increase its price. This strategy obviously reduces the expected number of PSCs served by Firm 2, but this loss is offset by a profit gain from QSCs. Stated differently, increasing D_u will lead Firm 2 to be more risk taking. In turn, Firm 1 follows the price increase; because Firm 2 plays higher prices, Firm 1 is likely to attract most PSCs and can thus afford to

Figure 1. (Color online) Mixed-Strategy Nash Equilibrium with Heterogeneous Customers ($D = 20$, $p_{\max} = 20$, $\Delta = 5$)



Notes. (a) Firm 1. (b) Firm 2.

Figure 2. (Color online) Expected Prices as a Function of D_u ($D = 20$, $p_{\max} = 20$, $\Delta = 5$)



charge higher prices. These dynamics explain the increasing part of both curves in Figure 2.

Once D_u reaches a certain threshold, however, a marginal increase in D_u induces a price reduction by Firm 1, which is then followed by Firm 2 (after a second slightly higher threshold value). When the market becomes dominated by QSCs, it then becomes more attractive for Firm 1 to lower its price to avoid the risk of losing the PSC (and also, to become more attractive to the QSC). This strategy obviously reduces per-customer profits and can be viewed as a shift from an optimistic reaction to Firm 2's price increase to a more conservative price-cut strategy. For Firm 2, this strategy poses a threat, so Firm 2 reacts by also lowering its price. These dynamics result in fiercer price competition, reinstalling the price war dynamics of Bertrand competition under homogeneous customers—although Firm 2's price converges to Δ and not to zero, as discussed in Section 3.2.

These findings reveal that prices increase with customer heterogeneity—and not with the proportion of quality-sensitive customers. This result stems from the fact that customer heterogeneity makes it harder for both firms to tailor their pricing strategies to both customer segments, thus creating a new source of market friction to escape from the Bertrand price race to the bottom.

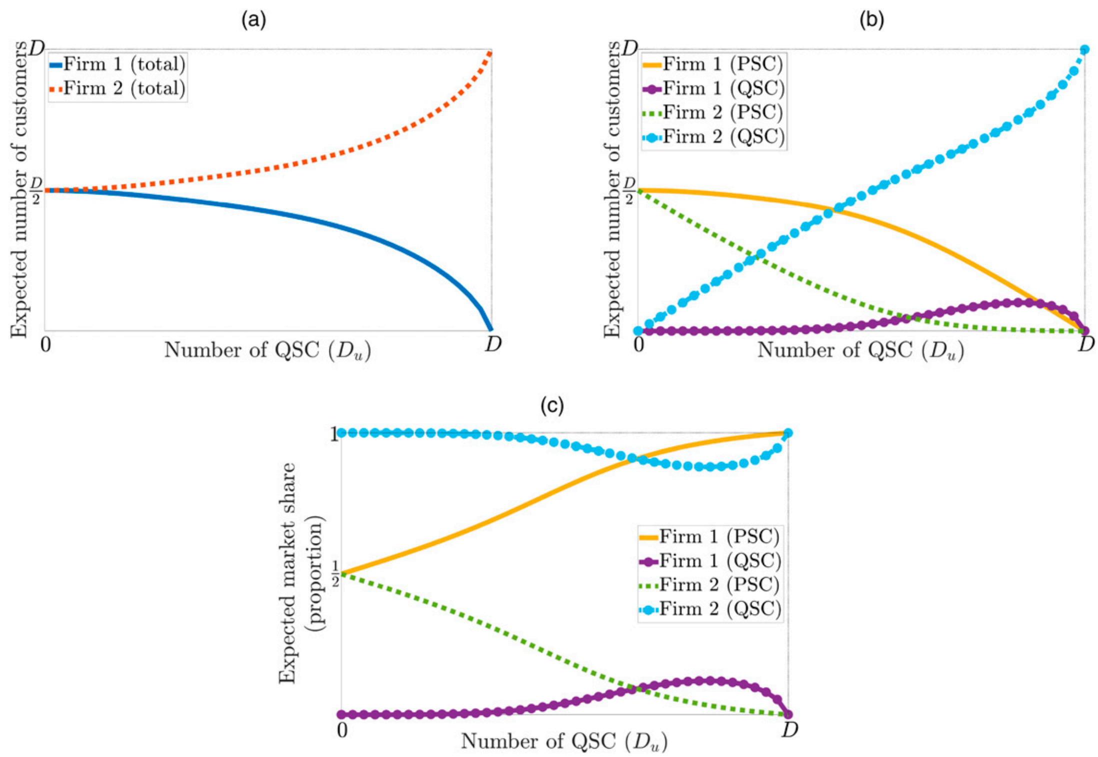
4.1.3. Market Shares. Figure 3(a) plots the expected number of customers served by each firm as a function of D_u. Figure 3(b) breaks it down into PSC and QSC. Figure 3(c) shows each firm's market share (i.e., proportion of customers served) in each segment, also as a function of D_u. In all plots, D is held constant, so that any increase in D_u is compensated by a corresponding decrease in D_p.

As D_u increases, Firm 2's market share increases, whereas Firm 1's market share decreases. This is intuitive; as the overall market becomes more quality sensitive, it provides a more favorable environment for the firm with the higher-quality product. Moreover, as D_u increases, Firm 1's market share among PSC increases. This suggests that increasing D_u induces Firm 2 to design a pricing strategy more tailored to QSC at the expense of PSC; although its market share of QSC is not monotonic, Firm 2 consistently captures close to 90% of QSCs on average regardless of the value of D_u. We next discuss Figure 3 in more detail by considering three regimes.

- When D_u is low, the market is mainly composed of PSC. Starting from $D_u = 0$, both firms play $p = 0$ and split the demand. As D_u increases, Firm 2 sets higher prices than Firm 1; yet, the difference between both firms' prices is small enough so that all QSCs opt for Firm 2. As a result, Firm 2 serves, on average, fewer PSCs but more QSCs, leading to higher prices.
- When D_u is intermediate, customer heterogeneity is strongest. As D_u increases, Firm 1 starts cutting its prices, increasing differentiation between both firms' prices. Thus, Firm 1 may capture some QSCs—although the expected total number of QSCs served by Firm 2 increases.
- When D_u is high, the market is mainly composed of QSC. Firm 1 faces an unfavorable environment and thus, lowers its price to attract QSC and minimize the risk of losing PSC. For high values of D_u, Firm 2 reacts by lowering its price, ultimately serving all QSCs as $D_u \to D$.

4.1.4. Expected Profits. Figure 4 plots expected profits as a function of D_u. Firm 2's expected profit increases with D_u; because it has a higher-quality product, it naturally benefits from a more quality-sensitive customer pool. One may expect, by invoking a mirror argument, that Firm 1's profit would increase as the market becomes more price sensitive (i.e., as D_u decreases). Interestingly, this is not the case; $\overline{\Pi}_1$ is unimodal with D_u. In Proposition 5, we show that Firm 1's expected profit is maximized at $D_u = 0.521D$—that is, when the market comprises a mix of PSC and QSC. Note that the

Figure 3. (Color online) Number of Customers and Market Shares as Functions of D_u ($D = 20$, $p_{\max} = 20$, $\Delta = 5$)



Notes. (a) Total number of customers served by each firm. (b) Number of PSC/QSC served by each firm. (c) Market share of each firm.

profit-maximizing PSC-QSC mix does not depend on the quality differential Δ.

Proposition 5. *$\overline{\Pi}_2$ is increasing with D_u, and $\overline{\Pi}_1$ is maximized when $D_u = 0.521D$.*

This result sheds light on our main finding from this section; the firm with the higher-quality product benefits from a more quality-sensitive customer pool, whereas the firm with the lower-quality product is better off in a more heterogeneous (as opposed to a more price-sensitive) environment. In particular, Firm 1 earns a positive profit with heterogeneous customers—that is, for any $0 < D_u < D$. This is in stark contrast with our earlier results under homogenous customers (either PSC or QSC). Indeed, when $D_u = 0$ or $D_u = D$, Firm 1 earns zero profit in the low-demand regime because of the price war induced by Bertrand competition when $D_u = 0$ and because of Firm 2's ability to capture the entire demand when $D_u = D$. In contrast, when the market comprises a mix of QSC and PSC, Firm 2 can no longer tailor its price to the entire market. Instead, it attempts to balance the conflicting objectives of charging high prices to extract a high profit from QSCs versus offering low prices to attract more PSCs. The resulting mixed strategy played by Firm 2 creates an opportunity for Firm 1, which can charge nonzero prices and successfully attract some customers. As Proposition 5 shows, Firm 1's expected profit is the highest when customer heterogeneity is strongest ($D_u \simeq 0.5$). Stated differently, this result highlights customer heterogeneity as another source of market friction—besides capacity restrictions and quality differentiation.

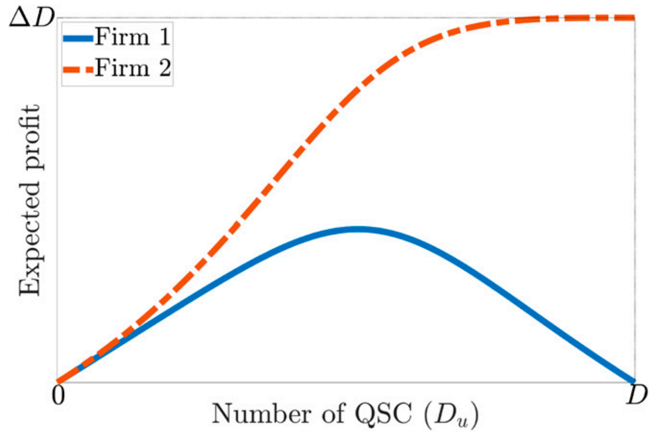
4.2. Discriminatory Pricing

We now assume that firms implement a discriminatory pricing strategy by offering a different price to each customer segment. We assume that firms need to commit to an inventory allocation across the two segments prior to engaging in price competition. As discussed, this setting is motivated by real-world examples, such as fast fashion and vaccines. We study the joint impact of price discrimination and inventory allocation by formalizing a two-stage game, in which inventory management constitutes the first-stage decision and pricing is the second-stage decision.

We denote Firm i's inventory allocated to QSC (PSC) by I_i^u (I_i^p), so that $I_i^u + I_i^p = I$ for $i = 1, 2$. In particular, we consider the following two inventory allocation rules.

- Pro rata capacity allocation. It is the simplest capacity allocation rule, where each firm assigns inventory proportionally to customer mass: that is, $I_i^u = (D_u{}^*I/D)$ and $I_i^p = (D_p{}^*I/D)$ for $i = 1, 2$.
- Endogenous inventory allocation. In this case, each firm allocates inventory strategically by anticipating the subsequent pricing dynamics. The problem is formalized

Figure 4. (Color online) Expected Profits as a Function of D_u ($D = 20, p_{\max} = 20, \Delta = 5$)



as a two-stage game in capacity and prices. We compute the subgame-perfect equilibrium.

This section departs from Section 4.1 in two aspects: price discrimination (as opposed to uniform pricing) and inventory allocation (as opposed to pooled inventory). Under low demand, the two-stage game is identical under pro rata capacity allocation and under pooled capacity (see Online Appendix C), so collectively, our results identify the impact of price discrimination (by comparing uniform pricing with pooled capacity versus discriminatory pricing with pro rata capacity allocation), the impact of endogenous capacity allocation (by comparing discriminatory pricing with pro rata capacity allocation versus endogenous capacity allocation), and their joint impact.

The second stage of the game with endogenous inventory allocation reduces to Bertrand competition under both capacity and quality differentiation with homogeneous customers in each segment. We derive a full equilibrium characterization for this class of games in Online Appendix A, thus augmenting the analysis from Section 3.2 by incorporating asymmetric capacities.

We use the superscripts N, R, and E to represent quantities under uniform (nondiscriminatory) pricing, price discrimination with pro rata capacity allocation, and price discrimination with endogenous inventory allocation, respectively. We use the superscripts u and p to refer to QSC and PSC. Next, we examine the game equilibrium as well as each firm's expected price and profit under price discrimination first with pro rata and endogenous inventory allocation.

4.2.1. Pro Rata Capacity Allocation. Recall that under pro rata capacity allocation, Firm i assigns $I_i^u = (D_u {}^*I/D) \geq D_u$ and $I_i^p = (D_p {}^*I/D) \geq D_p$. Consequently, the Bertrand competition game remains uncapacitated in each segment. We derive the equilibrium by leveraging Propositions 1 and 2.

Proposition 6. *Under pro rata capacity allocation and low demand (i.e., $D \leq I$), Firm 1 earns $\overline{\Pi}_1^{R,u} = 0$, $\overline{\Pi}_1^{R,p} = 0$, and $\overline{\Pi}_1^{R} = 0$, and Firm 2 earns $\overline{\Pi}_2^{R,u} = 0$, $\overline{\Pi}_2^{R,p} = D_u\Delta$, and $\overline{\Pi}_2^{R} = D_u\Delta$.*

Proposition 6 shows that price discrimination is detrimental under pro rata inventory allocation. Indeed, pro rata allocation does not create any market friction; each segment remains uncapacitated, and price competition leads to a price race to the bottom. In the PSC segment, both firms price at marginal cost and thus, earn zero profit. In the QSC segment, Firm 2 captures all customers by charging a price of Δ. Thus, each firm earns the lowest possible profit. This is in sharp contrast with uniform pricing, where customer heterogeneity introduces market frictions even in the absence of capacity restrictions; recall from Section 4.1 that under uniform pricing, Firm 2 cannot effectively cater to both customer segments, hence creating an opportunity for Firm 1 to earn a positive profit. Under price discrimination, however, each firm can tailor its pricing strategy to each customer segment, resulting in a Bertrand price war on each segment. This result is consistent with Corts (1998), who shows that in an uncapacitated environment, price discrimination makes competition fiercer (under homogenous customers).

These results underscore the crucial role of inventory allocation; because price discrimination eliminates the market frictions induced by customer heterogeneity, the firms need to strategically create inventory shortages in order to restore frictions. We study this question next.

4.2.2. Endogenous Inventory Allocation. This discussion provides some intuition on the interactions between inventory allocation and price competition, suggesting that the firms should create a capacity restriction in either segment (i.e., $D_p \leq \min\{I_1^p, I_2^p\}$ and

$D_u \le \min\{I_1^u, I_2^u\}$). Given that the overall demand exceeds the total capacity in the low-demand regime, this implies that the firms should allocate their inventory levels asymmetrically. This is shown formally in Proposition 7.

Proposition 7. *Consider the case with endogenous inventory allocation under low demand (i.e., $D \le I$). We define two allocation profiles: $E_A = (I_1^{E_A,u}, I_2^{E_A,u}) = (I - (D_p/2), (1 + (\Delta/p_{\max})/2)D_u)$ and $E_B = (I_1^{E_B,u}, I_2^{E_B,u}) = (((1-\Delta p_{\max})/2)D_u, I - (D_p/2))$. If $D_u \le (D/1 + 2(1-(\Delta/p_{\max}))(\Delta/p_{\max}))$, the game admits two equilibria: E_A and E_B. Otherwise, the game has the unique equilibrium E_A. The expected profits are given by*

- *Equilibrium A*:

$$\overline{\Pi}_1^{E_A} = \underbrace{\frac{I_1^{E_A,p}}{D_p}(D_p - I_1^{E_A,p})p_{\max}}_{\overline{\Pi}_1^{E_A,p}} + \underbrace{(D_u - I_2^{E_A,u})p_{\max}}_{\overline{\Pi}_1^{E_A,u}}$$

$$= \underbrace{\frac{D_p}{4}p_{\max}}_{\overline{\Pi}_1^{E_A,p}} + \underbrace{\frac{1 - \dfrac{\Delta}{p_{\max}}}{2} D_u p_{\max}}_{\overline{\Pi}_1^{E_A,u}},$$

$$\overline{\Pi}_2^{E_A} = \underbrace{(D_p - I_1^{E_A,p})p_{\max}}_{\overline{\Pi}_2^{E_A,p}} + \underbrace{I_2^u\left(\frac{D_u - I_2^{E_A,u}}{D_u}p_{\max} + \Delta\right)}_{\overline{\Pi}_2^{E_A,u}}$$

$$= \underbrace{\frac{D_p}{2}p_{\max}}_{\overline{\Pi}_2^{E_A,p}} + \underbrace{\left(\frac{1 + \frac{\Delta}{p_{\max}}}{2}\right)^2 D_u p_{\max}}_{\overline{\Pi}_2^{E_A,u}}.$$

- *Equilibrium B*:

$$\overline{\Pi}_1^{E_B} = \underbrace{(D_p - I_2^{E_B,p})p_{\max}}_{\overline{\Pi}_1^{E_B,p}} + \underbrace{I_1^u\left(\frac{D_u - I_1^{E_B,u}}{D_u}p_{\max} - \Delta\right)}_{\overline{\Pi}_1^{E_B,u}}$$

$$= \underbrace{\frac{D_p}{2}p_{\max}}_{\overline{\Pi}_1^{E_B,p}} + \underbrace{\left(\frac{1 - \frac{\Delta}{p_{\max}}}{2}\right)^2 D_u p_{\max}}_{\overline{\Pi}_1^{E_B,u}},$$

$$\overline{\Pi}_2^{E_B} = \underbrace{\frac{I_2^{E_B,p}}{D_p}(D_p - I_2^{E_B,p})p_{\max}}_{\overline{\Pi}_2^{E_B,p}} + \underbrace{(D_u - I_1^{E_B,u})p_{\max}}_{\overline{\Pi}_2^{E_B,u}}$$

$$= \underbrace{\frac{D_p}{4}p_{\max}}_{\overline{\Pi}_2^{E_B,p}} + \underbrace{\frac{1 + \frac{\Delta}{p_{\max}}}{2} D_u p_{\max}}_{\overline{\Pi}_2^{E_B,u}}.$$

Proposition 7 suggests that the game admits one or two equilibria. When two equilibria exist, we have $\overline{\Pi}_1^{E_A} + \overline{\Pi}_2^{E_A} = \overline{\Pi}_1^{E_B} + \overline{\Pi}_2^{E_B}$. This implies that there is no Pareto dominance between the two equilibria. Moreover, each equilibrium results in a "divide and conquer" strategy; one firm earns a higher profit than its competitor in one segment and a lower profit in the other segment.

Moreover, one can easily verify that in Equilibrium E_A, $I_1^{E_A,p} < D_p < I_2^{E_A,p}$ and $I_2^{E_A,u} < D_u < I_1^{E_A,u}$ and that in Equilibrium E_B, $I_2^{E_B,p} < D_p < I_1^{E_B,p}$ and $I_1^{E_B,u} < D_u < I_2^{E_B,u}$. As a result, (asymmetric) inventory allocation creates capacity shortages on each segment.

In Equilibrium E_A, Firm 2 allocates most of its inventory to PSC, whereas Firm 1 allocates most of its inventory to QSC. In other words, the firm with the higher-quality product allocates most of its capacity to price-sensitive customers in order to create a capacity shortage for the quality-sensitive segment. Although this may appear counterintuitive at first, the rationale is consistent with our insights from Section 3.2; even though creating a shortage limits the total number of QSC that Firm 2 can serve, it also boosts the equilibrium price by introducing market frictions. In turn, the expected profit $\overline{\Pi}_2^{E_A,u}$ is higher than ΔD_u, resulting in a profit gain for Firm 2 from QSC.

Equilibrium E_B has the opposite structure; Firm 2 allocates most of its inventory to QSC, whereas Firm 1 allocates most of its inventory to PSC. Note that Equilibrium E_B holds for most values of D_u but disappears when D_u is large. This can be explained as follows. Because at Equilibrium E_B, Firm 1 creates a shortage for QSC, Firm 2 can always lower its inventory on the QSC segment to intensify the capacity constraint and raise the QSC price to $p_{\max}$. However, doing so would remove the capacity constraints on the PSC segment, leading to zero profits from PSC. Under this deviation, Firm 2 thus focuses exclusively on QSC. When the PSC segment is relatively small (i.e., when D_u is large), this deviation is profitable because the added profit from a higher price charged to QSC offsets the profit loss because of the uncapacitated price competition on the PSC segment.

4.2.3. Profits Comparison.

Under either equilibrium, both firms earn positive profits from both customer segments. In each segment, one of the firms earns exactly the secured profit from leftovers. At E_A, Firm 1 earns $(D_u - I_2^{E_A,u})p_{\max}$ from QSC, and Firm 2 earns $(D_p - I_1^{E_A,p})p_{\max}$ from PSC; at E_B, Firm 1 earns $(D_p - I_2^{E_B,p})p_{\max}$ from PSC, and Firm 2 earns $(D_u - I_1^{E_B,u})p_{\max}$ from QSC. This is consistent with Section 3, given that each segment comprises homogeneous customers, hence inducing the firms to undercut each other as much as possible.

We now turn to the main insight from this section; under low demand, price discrimination coupled with strategic inventory allocation yields the highest profit for both firms, followed by the uniform pricing strategy. Price discrimination under pro rata capacity allocation yields the lowest profits for both firms. This is illustrated in Figure 5 and proved formally in Proposition 8.

Proposition 8. *Under low demand (i.e., $D \le I$), we have $\overline{\Pi}_i^R \le \overline{\Pi}_i^N \le \overline{\Pi}_i^E$ for $i = 1, 2$ for either equilibrium E_A or E_B.*

Proposition 8 suggests that price discrimination, by itself, does not necessarily benefit firms under competition. Instead, firms only benefit from discrimination if they allocate capacities strategically.

To interpret this result, let us underscore the three sources of market frictions identified in this paper: capacity shortages, quality differentiation, and customer heterogeneity. Uniform pricing involves quality differentiation and customer heterogeneity but not capacity shortages (in the low-demand regime). In contrast, price discrimination eliminates the friction from customer heterogeneity. As a result, under price discrimination with pro rata capacity allocation, both firms perform worst—even worse than under uniform pricing. In contrast, price discrimination with endogenous capacity allocation introduces a market friction from capacity shortages because of asymmetric inventory allocation, therefore improving the profits of both firms.

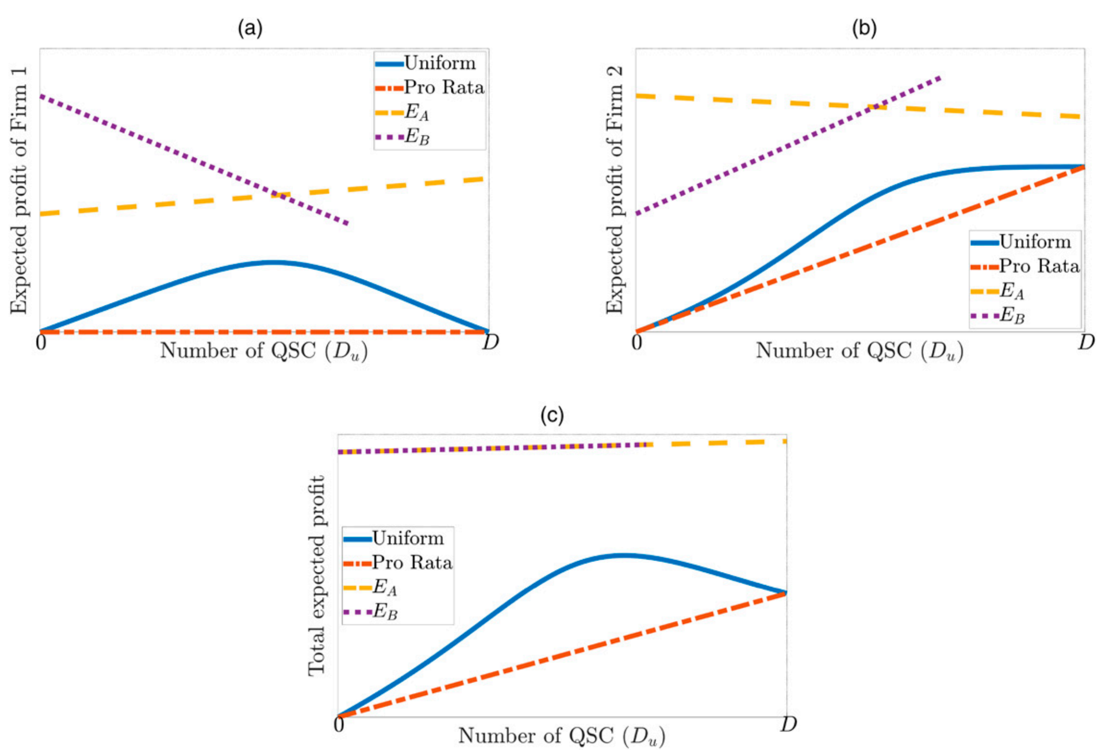
An interesting aspect of these results is that *both* firms are better off under price discrimination with endogenous inventory allocation relative to uniform pricing: that is, $\overline{\Pi}_i^N \le \overline{\Pi}_i^{E_A}$ and $\overline{\Pi}_i^N \le \overline{\Pi}_i^{E_B}$ for $i = 1, 2$. To explain this result, Figure 6 plots the expected prices as a function of D_u. Intuitively, one could think that under discriminatory pricing, firms would reduce the price charged to PSC and increase the price charged to QSC. However, Figure 6 shows that it is not the case. Instead, the expected prices charged to both segments are higher under price discrimination (with endogenous capacity allocation) relative to uniform pricing. This is proved formally in Proposition 9.

Proposition 9. *For $i = 1, 2$ and $E \in \{E_A, E_B\}$, $\mathbb{E}[p_i^{E,u}]$ and $\mathbb{E}[p_i^{E,p}]$ are independent of D_u. Moreover, the following inequalities hold.*

- $\mathbb{E}[p_1^{E_A,u}] > \mathbb{E}[p_1^N]$, $\mathbb{E}[p_1^{E_A,p}] > \mathbb{E}[p_1^N]$, *and* $\mathbb{E}[p_1^{E_B,p}] > \mathbb{E}[p_1^N]$. *If* $\frac{\Delta}{p_{\max}} < 0.3968$, $\mathbb{E}[p_1^{E_B,u}] > \mathbb{E}[p_1^N]$.
- $\mathbb{E}[p_2^{E_A,u}] > \mathbb{E}[p_2^N]$, $\mathbb{E}[p_2^{E_A,p}] > \mathbb{E}[p_2^N]$, $\mathbb{E}[p_2^{E_B,u}] > \mathbb{E}[p_2^N]$, *and* $\mathbb{E}[p_2^{E_B,p}] > \mathbb{E}[p_2^N]$.

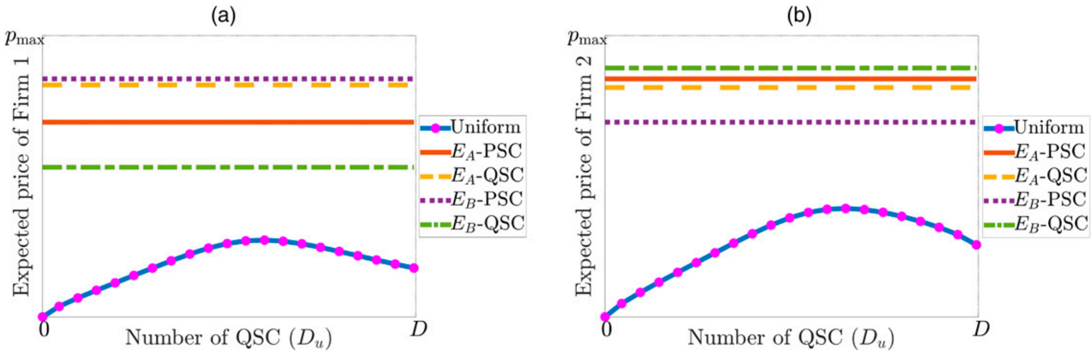
Proposition 9 suggests that market frictions from capacity restrictions have a stronger impact on prices than market frictions from customer heterogeneity. In other words, capacity restrictions induce price surges in Bertrand competition even with homogenous customers, so both firms are guaranteed significant profits from leftover demand. In contrast, under unrestricted

Figure 5. (Color online) Profits Comparisons Under Low Demand ($D = 20$, $p_{\max} = 20$, $\Delta = 7$)



Notes. (a) Firm 1's profits. (b) Firm 2's profits. (c) Total profits (Firm 1 and Firm 2).

Figure 6. (Color online) Expected Prices Comparisons Under Low Demand (Parameters: $D = 20$, $p_{\max} = 20$, $\Delta = 5$)



Notes. (a) Firm 1's expected prices. (b) Firm 2's expected prices.

capacity with heterogenous customers, the firms still face the risk of losing the entire market if their price is too high. Ultimately, price discrimination with endogenous inventory allocation induces equilibrium prices and higher profits relative to uniform pricing (despite creating capacity constraints).

We conclude with some market-level remarks. First, total profits (across Firm 1 and Firm 2) are highest under discriminatory pricing with endogenous capacity allocation; moreover, under uniform pricing, the total profits are maximized with heterogenous customers (Figure 5(c)). This mirrors our firm-level findings, indicating an overall alignment between industry-level and firm-level incentives. At the same time, the settings leading to higher profits (e.g., uniform pricing under customer heterogeneity and discriminatory pricing with endogenous capacity allocation) come at the expense of charging higher prices to customers (Figure 6). Although surplus-maximizing analyses fall outside the scope of this paper (because our framework does not include a customer utility component), our results highlight the impact of market frictions created by quality differentiation, customer heterogeneity, and capacity restrictions on prices and producer surplus.

In summary, price discrimination is not necessarily beneficial under price competition because it eliminates the market frictions from customer heterogeneity. However, firms can increase their profits by adopting a price discrimination strategy combined with strategic inventory allocation across customer segments—by reintroducing capacity constraints for each customer segment.

5. High-Demand Regime

We now assume that $D \geq (2 - (\Delta/p_{\max}))I$: that is, customer demand is close to the overall capacity of both firms. The proofs of our statements are relegated to EC.4. Most of the insights established in the low-demand case continue to hold under high demand. Online Appendix D further establishes the robustness of our findings numerically under moderate demand: that is, $D \in (I, (2 - (\Delta/p_{\max}))I)$.

5.1. Uniform Pricing

The demand structure is given by

$$D_1(p_1, p_2) = \begin{cases} I & \text{if } p_1 + \Delta < p_2, \\ \min\left\{\dfrac{D_u}{2} + D_p, I\right\} & \text{if } p_1 + \Delta = p_2, \\ D_p + \min\{I - D_p, 0\} & \\ + \max\{D_u - I, 0\} & \text{if } p_1 < p_2 < p_1 + \Delta, \\ D - \min\left\{D_u + \dfrac{D_p}{2}, I\right\} & \text{if } p_1 = p_2, \\ D - I & \text{if } p_1 > p_2. \end{cases}$$

$$D_2(p_2, p_1) = D - D_1(p_1, p_2).$$

The firms' profits depend on whether $D_p \geq I$ or $D_p < I$, whether $D_u \geq I$ or $D_u < I$, whether $D_p + (D_u/2) \geq I$ or $D_p + (D_u/2) < I$, and whether $D_u + (D_p/2) \geq I$ or $D_u + (D_p/2) < I$ (because of demand rationing). We say that the market is dominated by PSC (QSC) when $D_p \geq I$ ($D_u \geq I$). In either of these cases, the game is equivalent to the earlier settings with homogenous customers (Section 3.2). Indeed, when $D_p \geq I$, Firm 1's and Firm 2's market shares only depend on whether $p_1 < p_2$, $p_1 = p_2$, or $p_1 > p_2$; similarly, when $D_u \geq I$, Firm 1's and Firm 2's market shares only depend on whether $p_1 + \Delta < p_2$, $p_1 + \Delta = p_2$, or $p_1 + \Delta > p_2$. In between, when $D_u \in (D - I, I)$, we can still characterize the Nash equilibrium. This is reported in Proposition 10.

Proposition 10. *Under heterogenous customers and high demand (i.e., $D \geq (2 - (\Delta/p_{\max}))I$), there exists a unique mixed-strategy Nash equilibrium (F_1, F_2) characterized as follows.*

- *When $D_u \leq D - I$ (or equivalently, $D_p \geq I$), the unique Nash equilibrium is identical to the one in Proposition 1 for $D \geq I$. The expected profits satisfy $\overline{\Pi}_1 = \overline{\Pi}_2 = (D - I)p_{\max}$.*
- *When $D_u \geq I$, the unique Nash equilibrium is $p_1 = p_2 = p_{\max}$. The expected profits satisfy $\overline{\Pi}_1 = (D - I)\, p_{\max}$ and $\overline{\Pi}_2 = Ip_{\max}$.*

- *When* $D_u \in (D-I, D)$, *we define* $\underline{p} = \frac{D_u}{I} p_{\max}$. *The unique Nash equilibrium is given by*

$$F_1(p_1) = \frac{I - \frac{D_u p_{\max}}{p_1}}{I - D_u}, \ \forall p_1 \in [\underline{p}, p_{\max}],$$
$$F_2(p_2) = \frac{D_p - \frac{D_u D_p}{I}\frac{p_{\max}}{p_2}}{I - D_u}, \ \forall p_2 \in [\underline{p}, p_{\max}),$$
$$Q_{F_2}(p_{\max}) = 1 - \frac{D_p}{I}.$$

The expected profits satisfy $\overline{\Pi}_1 = \frac{D_u D_p}{I} p_{\max}$ *and* $\overline{\Pi}_2 = D_u p_{\max}$.

When $D_u \in (D-I, D)$, both firms play mixed strategies in $[\underline{p}, p_{\max}]$, where $\underline{p}$ is defined such that Firm 2 is indifferent between serving I customers at $\underline{p}$ or D_u customers at $p_{\max}$. Figure 7 depicts the cumulative distribution functions of both firms' mixed strategies under different values of D_u. In this example, starting from $D_u = 8$ (i.e., $D_p = I = 12$), we obtain the same equilibrium as in the setting with homogenous PSC; at the other extreme, when $D_u = I = 12$, we obtain the same equilibrium as in the setting with homogenous QSC. In between, each firm's price increases with D_u (in a stochastic dominance sense). Moreover, for any value of D_u, Firm 2 plays higher prices than Firm 1 (also in a stochastic dominance sense). Because Firm 2 is guaranteed to serve QSC, it can charge higher prices than Firm 1; in contrast, Firm 1 needs to play lower prices in order to increase its likelihood to serve PSC. These two results are formally summarized in Corollary 1.

Corollary 1. *For any* $d_1, d_2 \in [0, D]$ *with* $d_1 < d_2$, *each firm's mixed strategy when* $D_u = d_1$ *is stochastically dominated by its mixed strategy when* $D_u = d_2$. *Additionally, for any value of* D_u, *Firm 1's mixed strategy is stochastically dominated by Firm 2's mixed strategy.*

We observe an important difference between the low- and high-demand regimes. In the low-demand regime, each firm's expected price was unimodal with D_u. Specifically, both firms were decreasing their prices as D_u increased when QSCs account for the majority of the market. This switch was motivated by both firms' incentives to compete for QSC and by the concerns of losing all the customers to the competitor. However, in the presence of capacity constraints (i.e., in the high-demand regime), both firms already set high prices in $[p_{\max} - \Delta, p_{\max}]$, so that all QSCs always prefer Firm 2's product. As a result, both firms effectively compete only for PSC. This difference results in monotonic prices played by both firms as a function of D_u in the high-demand regime.

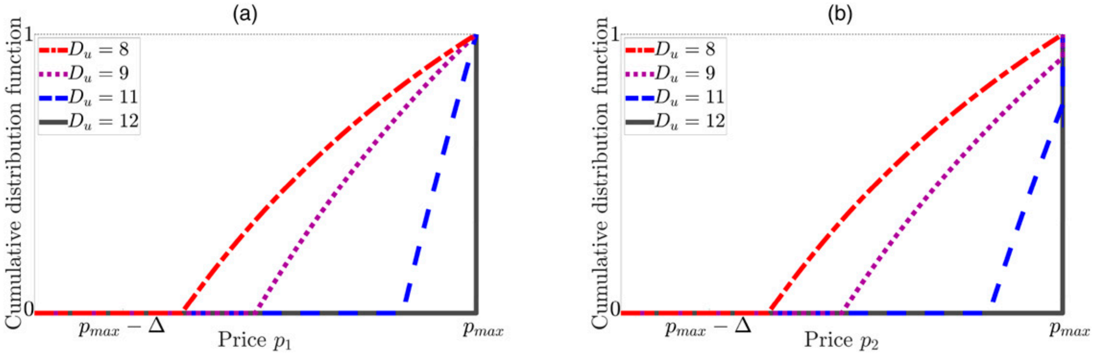
Figure 8 plots the expected prices and profits as a function of D_u. Figure 8(a) illustrates Corollary 1, showing that each firm's expected price increases when D_u increases from $D - I$ to I and that Firm 2 plays a higher expected price than Firm 1. Figure 8(b) confirms our finding from Section 4.1; Firm 2 earns a higher expected profit as the proportion of QSC increases, whereas Firm 1 earns a higher expected profit when the customer pool is more heterogenous—as opposed to more price sensitive. In the high-demand regime, Firm 1's expected profit is the highest when the proportions of PSC and QSC are exactly equal. This result stems from similar reasons as in the low-demand regime. When the market is dominated by PSC, Firm 2 charges lower prices to serve more customers overall, and Firm 1's expected profit is $(D-I)p_{\max}$. When the market is dominated by QSC, Firm 2 charges $p_{\max}$ (because it will always capture I customers), and Firm 1's expected profit remains $(D-I)p_{\max}$. However, in between, Firm 2 cannot effectively cater to both customer segments simultaneously, thus providing an opportunity for Firm 1 to exploit customer heterogeneity and earn a strictly higher profit than $(D-I)p_{\max}$. This result is formalized in Proposition 11.

Proposition 11. $\overline{\Pi}_2$ *is increasing with* D_u, *and* $\overline{\Pi}_1$ *is maximized when* $D_u = 0.5D$.

5.2. Discriminatory Pricing

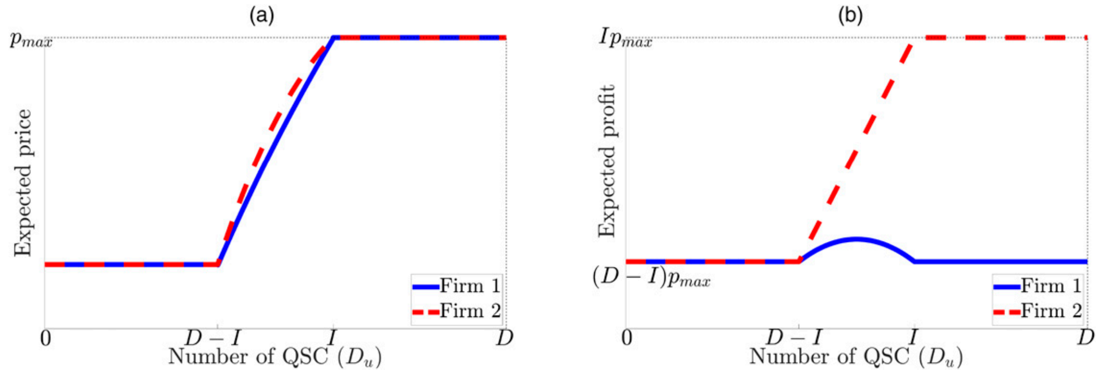
We now investigate the competition dynamics under price discrimination. Note at the outset a major difference with the low-demand regime, in that when $D \geq (2 - (\Delta/p_{\max}))I$, competition remains capacitated regardless of the firms'

Figure 7. (Color online) Mixed-Strategy Nash Equilibrium with Heterogeneous Customers ($D = 20$, $I = 12$, $p_{\max} = 20$, $\Delta = 8$)



Notes. (a) Firm 1. (b) Firm 2.

Figure 8. (Color online) Expected Prices and Profits as Functions of D_u ($D = 20, I = 12.5, p_{\max} = 20, \Delta = 8$)



Notes. (a) Expected prices. (b) Expected profits.

inventory allocation decisions. Therefore, price discrimination weakens but does not eliminate the market frictions from capacity restrictions; vice versa, endogenous inventory allocation can only strengthen as opposed to restore these market frictions.

Unlike in the low-demand regime, pooled capacity and pro rata capacity allocation are no longer strictly equivalent under uniform pricing. For consistency with the previous section, we consider discriminatory pricing with pro rata capacity allocation as a benchmark, which is natural and easy to implement. In Online Appendix C, we consider the additional benchmark of uniform pricing with pro rata capacity allocation. We show that the results under uniform pricing with either pooled capacity (Section 5.1) or pro rata capacity allocation (Online Appendix C) yield similar insights. Thus, this paper identifies the impact of price discrimination (by comparing uniform pricing with pooled capacity and pro rata capacity allocation versus discriminatory pricing with pro rata capacity allocation), the impact of endogenous capacity allocation (by comparing discriminatory pricing with pro rata capacity allocation versus endogenous capacity allocation), and their joint impact.

5.2.1. Pro Rata Capacity Allocation. The inventory levels allocated to the QSC segment satisfy $(D_u - I_2^u)p_{\max} \geq I_1^u(p_{\max} - \Delta)$. Thus, both firms will set their price at $p_{\max}$, so that Firm 1 earns $(D_u - I_2^u)p_{\max}$ and Firm 2 earns $I_2^u p_{\max}$; in the PSC segment, both firms will play a mixed strategy and earn $(D_p - I_1^p)p_{\max}$ (Section 3.2). This is reported in Proposition 12.

Proposition 12. *Under pro rata capacity allocation and high demand (i.e., $D \geq (2 - (\Delta/p_{\max}))I$), Firm 1 earns $\overline{\Pi}_1^{R,u} = (1-(I/D))\, D_u p_{\max}$, $\overline{\Pi}_1^{R,p} = (1-(I/D))D_p p_{\max}$, and $\overline{\Pi}_1^{R} = (D-I)p_{\max}$, and Firm 2 earns $\overline{\Pi}_2^{R,u} = (I/D)D_u p_{\max}$, $\overline{\Pi}_2^{R,p} = (1-(I/D))D_p p_{\max}$, and $\overline{\Pi}_2^{R} = (D-I)p_{\max} + ((2I/D)-1)D_u p_{\max}$.*

As before, Firm 1 earns exactly the secured profit from leftovers, $(D-I)p_{\max}$. In contrast, Firm 2 earns an extra $\left(\frac{2I}{D}-1\right)D_u\Delta$ because of its quality advantage. Moreover, Firm 2's profit increases linearly with D_u, suggesting that more quality-sensitive customers create a more favorable environment for the firm with the higher-quality product (under pro rata capacity allocation).

5.2.2. Endogenous Inventory Allocation. Because of the capacity constraints, the game admits three types of equilibria. Equilibria E_A and E_B are analogous to the two equilibria from the low-demand regime. The third one, Equilibrium E_C, admits a complex representation but yields similar insights as Equilibrium E_A (details are omitted for conciseness).

Proposition 13. *We define $a = (\Delta/p_{\max})$. Under endogenous inventory allocation and high demand (i.e., $D \geq (2-(\Delta/p_{\max}))I$), there exist three types of equilibria.*

- *Equilibrium E_A always exists and is given by*

$$E_A = \left(I_1^{E_A,u}, I_2^{E_A,u}\right) = \begin{cases} \left(I - \dfrac{D_p}{2}, \dfrac{1+\dfrac{\Delta}{p_{\max}}}{2}D_u\right), \\ \qquad \text{if } D_u \leq 2I - D, \\ \left(I - \dfrac{D_p}{2}, \dfrac{D_u + \dfrac{\Delta}{p_{\max}}\left(I - \dfrac{D_p}{2}\right)}{2}\right), \\ \qquad \text{if } 2I - D < D_u < \left(\dfrac{2p_{\max}}{\Delta} - 1\right)(2I - D), \\ \left(I - \dfrac{D_p}{2}, I - \dfrac{D_p}{2}\right), \\ \qquad \text{if } \left(\dfrac{2p_{\max}}{\Delta} - 1\right)(2I - D) \leq D_u \leq D. \end{cases}$$

- *Equilibrium E_B is given by $E_B = \left(I_1^{E_B,u}, I_2^{E_B,u}\right) = \left((1-(\Delta/p_{\max}))D_u/2, I - (D_p/2)\right)$. E_B exists if and only if $D_u < \tilde{D}_u$, where $\tilde{D}_u$ is the unique positive root of the following polynomial*:

$$(a^4 - 3a^3 + 2a^2 + a)D_u^2 + (4aD - 2a^2D - 10aI + 6a^2I)D_u + (D^2 - 4ID + 4I^2) = 0.$$

In particular, a necessary condition for E_B to exist is $D_u < 0.112D$.

- *When $I - \frac{D_p}{2} < (D_u/(a + 2 - 2\sqrt{a}))$, Equilibrium E_C exists, under which both firms charge $p_{\max}$ to QSC.*

As in the low-demand regime, Equilibrium E_A results in more inventory allocated to QSC by Firm 1 and more inventory allocated to PSC by Firm 2. The converse is true for Equilibrium E_B; Firm 1 allocates more inventory to PSC, and Firm 2 allocates more inventory to QSC. As before, Equilibrium E_B exists only for small values of D_u. Indeed, as D_u increases, Firm 2 may find a profitable deviation by lowering its inventory for QSC so that $I_2^u = D_u - (1 - (\Delta/p_{\max}))I_1^u$, bringing more market frictions to the QSC competition and leading both firms to charge $p_{\max}$.

The new equilibrium, E_C, is the outcome of such a deviation.[9] This equilibrium did not exist under low demand because it shifts some capacity from QSC to PSC, and under low demand, PSC competition becomes uncapacitated; in turn, Firm 1 restricts its own capacity allocated to PSC, leading to Equilibrium E_A. Under high demand, however, capacity constraints are such that PSC competition remains capacitated under E_C, and both firms make positive profits from PSC.

From this reasoning, Equilibria E_B and E_C cannot coexist. Indeed, Equilibrium E_C exists only when D_u is higher than a certain threshold. In contrast, Equilibrium E_B exists only for (very) small values of D_u ($D_u < 0.112D$). This is because PSC competition remains capacitated under high demand, so Firm 2 is more likely to deviate from E_B. Note that there exist instances where neither Equilibrium E_B nor E_C exist, in which case the game has E_A as its unique equilibrium.

5.2.3. Profits Comparison. Because E_A always exists, E_B only exists for small values of D_u, and E_C leads to similar insights as E_A, we focus on equilibrium E_A and index it by E.

As in the low-demand regime, one of the two firms will earn exactly the secured profit from leftovers in each segment—a result from competitive pricing. More precisely, we have the following:

$$\overline{\Pi}_1^E = \underbrace{\frac{I_1^p}{I_2^p}(D_p - I_1^p)p_{\max}}_{\overline{\Pi}_1^{E,p}} + \underbrace{(D_u - I_2^u)p_{\max}}_{\overline{\Pi}_1^{E,u}},$$

$$\overline{\Pi}_2^E = \underbrace{(D_p - I_1^p)p_{\max}}_{\overline{\Pi}_2^{E,p}} + \underbrace{\begin{cases} I_2^u\left(\dfrac{D_u - I_2^u}{D_u}p_{\max} + \Delta\right), & \text{if } D_u \le 2I - D, \\ I_2^u\left(\dfrac{D_u - I_2^u}{I_1^u}p_{\max} + \Delta\right), & \text{if } 2I - D < D_u < \left(\dfrac{2p_{\max}}{\Delta} - 1\right)(2I - D), \\ I_2^u p_{\max}, & \text{if } \left(\dfrac{2p_{\max}}{\Delta} - 1\right)(2I - D) \le D_u \le D. \end{cases}}_{\overline{\Pi}_2^{E,u}}$$

Again, pro rata capacity allocation is consistently dominated by endogenous inventory allocation (see Proposition 14). As opposed to low demand, competition remains capacitated. Yet, firms still benefit from endogenous inventory allocation to strengthen capacity shortages in both segments.

Proposition 14. *Under high demand, we have $\overline{\Pi}_i^R \le \overline{\Pi}_i^E$ for $i = 1, 2$.*

We turn to the profit comparison between $\overline{\Pi}^E$ (price discrimination with endogenous inventory allocation) and $\overline{\Pi}^N$ (uniform pricing). Unlike the low-demand regime, there is no strict dominance between these two quantities—the most profitable strategy depends on the value of the demand, specifically whether $D \in [(2-(\Delta/p_{\max}))I, (4p_{\max} - 3\Delta)/(2p_{\max} - \Delta)I]$ or $D \in [(4p_{\max} - 3\Delta)/(2p_{\max} - \Delta)I, 2I]$, and on the balance of PSC and QSC. The formal result is stated in Proposition 15. Figures 9 and 10 depict the firms' profits and expected prices, respectively, when $D \in [(2 - (\Delta/p_{\max}))I, (4p_{\max} - 3\Delta)/(2p_{\max} - \Delta)I]$.

Proposition 15. *When comparing the profits with and without price discrimination, we have the following.*

- *For Firm 1, $\overline{\Pi}_1^E$ is decreasing with D_u, with the minimum value $(D - I)p_{\max}$. There exist two thresholds $\hat{D}_1$ and $\hat{D}_2$ such that $\overline{\Pi}_1^E < \overline{\Pi}_1^N$ if and only if $D_u \in (\hat{D}_1, \hat{D}_2)$.*
- *For Firm 2, $\overline{\Pi}_2^E$ is decreasing with D_u, with the maximum value $Ip_{\max}$. If $(4p_{\max} - 3\Delta)/(2p_{\max} - \Delta)I \le D \le 2I$, then $\overline{\Pi}_2^E \ge \overline{\Pi}_2^N$. If $(2 - (\Delta/p_{\max}))I \le D < (4p_{\max} - 3\Delta)/(2p_{\max} - \Delta)I$, then there exist two thresholds $\tilde{D}_1$ and $\tilde{D}_2$ such that $\overline{\Pi}_2^E < \overline{\Pi}_2^N$ if and only if $D_u \in (\tilde{D}_1, \tilde{D}_2)$.*

Proposition 15 suggests that price discrimination (with endogenous inventory allocation) can still be beneficial. Yet, there exist cases where uniform pricing yields the highest profits for both firms, namely when demand is not too high (relative to capacity) and market heterogeneity is strong (i.e., D_u is around half of the market). This contrasts the low-demand regime, where price discrimination with endogenous inventory allocation consistently dominated uniform pricing.

In Corollary 2, we derive analytical bounds on the profit ratios under the different pricing strategies. This result shows that under high demand, the differences between price discrimination and uniform pricing remain bounded within 6%–27%. This stems from the capacity constraints, which ensure a minimum level of profitability for both firms, regardless of the pricing strategy. Moreover, the impact of the pricing strategy is more substantial for Firm 2 (between −27% and +13%) than for Firm 1 (±6%–7%). This again underscores that the quality differential provides greater opportunities for Firm 2 to leverage each pricing strategy, whereas Firm 1's strategy remains more constrained, and as a result, its profits remain closer to the secured profits from leftovers.

Corollary 2. *Under high demand, we have* $\overline{\Pi}_1^N/\overline{\Pi}_1^E \in [15/16, 16/15)$ *and* $\overline{\Pi}_2^N/\overline{\Pi}_2^E \in [0.731, 1.13)$.

We next investigate the impact of price discrimination (with endogenous capacity allocation) versus uniform pricing as a function of D_u (based on Figure 9). The observations are threefold.

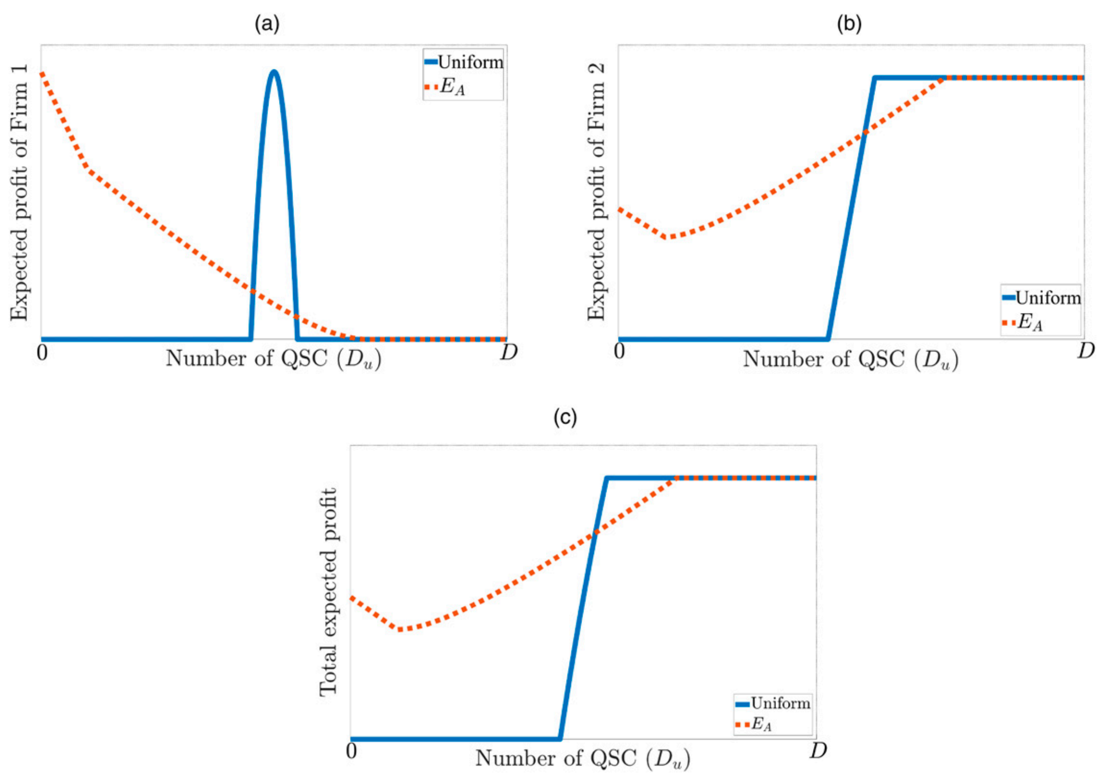
• When D_u is small, both firms earn higher profits under price discrimination. Because firms cannot reduce their inventory in both segments, they cannot charge high prices in both segments simultaneously. Most customers being price sensitive, both firms focus on PSC and end up charging higher prices to PSC than to QSC—a surprising outcome, given that QSCs place a premium on quality. In other words, firms use inventory allocation as a strategic lever to create capacity shortages (hence, more market frictions) in the most prevalent segment—here, the PSC segment. In turn, both firms earn higher profits than uniform pricing.

• When D_u is large, under either pricing strategy, both firms play $p_{\max}$, Firm 1 earns $(D-I)p_{\max}$, and Firm 2 earns $Ip_{\max}$. As D_u increases, both firms increase their prices, regardless of the pricing strategy—discriminatory pricing or uniform pricing. Recall that both firms play $p_{\max}$ under the uniform pricing strategy. Under price discrimination, both firms focus on QSC and hence, allocate no spare capacity to PSC. As a result, both firms can charge $p_{\max}$ to PSC (because of capacity restrictions) and can also charge $p_{\max}$ to QSC (because of quality sensitivity).

• When D_u is intermediate, there exists instances where Firm 1 earns a higher profit under uniform pricing; for Firm 2, there exist values of D such that uniform pricing yields a higher profit than price discrimination. Recall that for Firm 1, $\overline{\Pi}_1^N$ is the highest under strong customer heterogeneity: that is, for intermediate values of D_u (Section 5.1). At the same time, $\overline{\Pi}_1^E$ decreases as D_u increases, hence guaranteeing an interval in which $\overline{\Pi}_1^E < \overline{\Pi}_1^N$.

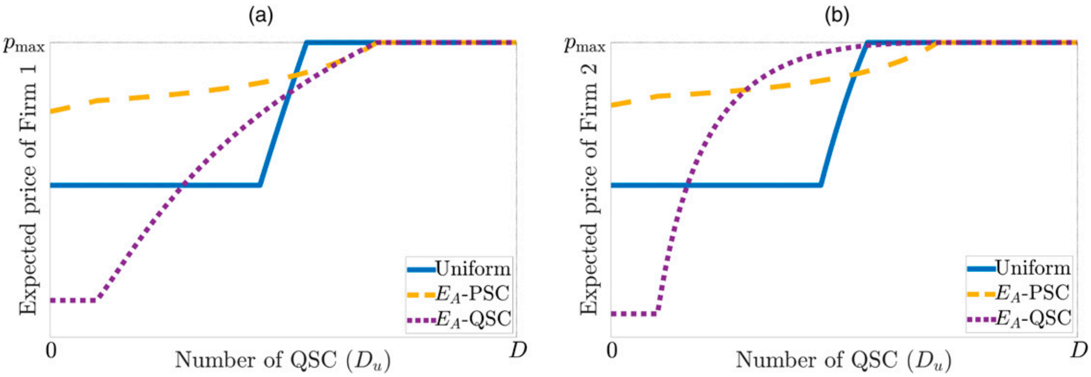
Next, Firm 2's profit increases with D_u under either strategy, but the profit sources are different. Under uniform pricing, Firm 2 focuses on QSC and charges $p_{\max}$ when $D_u \geq I$. Under price discrimination, Firm 2's profit comes from both segments. When $D \in \left[(4p_{\max}-3\Delta)/(2p_{\max}-\Delta)I, 2I\right]$, the capacity constraints are so strong that competition is more likely to lead to a price of $p_{\max}$, so that $\overline{\Pi}_2^E \geq \overline{\Pi}_2^N$. In contrast, when $D \in [(2-(\Delta/p_{\max}))I,\ (4p_{\max}-3\Delta)/(2p_{\max}-\Delta)I]$, Firm 2 can only charge $p_{\max}$ for the highest values of D_u, but there exist intermediate values of D_u where Firm 2 can charge $p_{\max}$ under uniform pricing but not under price discrimination; so, ultimately, uniform pricing

Figure 9. (Color online) Expected Profits When $\left(2-\frac{\Delta}{p_{\max}}\right)I \leq D < \frac{4p_{\max}-3\Delta}{2p_{\max}-\Delta}I$ ($D = 20$, $p_{\max} = 20$, $I = 11$, $\Delta = 5$)



Notes. (a) Firm 1. (b) Firm 2. (c) Firm 1 + Firm 2.

Figure 10. (Color online) Expected Prices When $\left(2-\frac{\Delta}{p_{\max}}\right)I \le D < \frac{4p_{\max}-3\Delta}{2p_{\max}-\Delta}I$ ($D=20$, $p_{\max}=20$, $I=11$, $\Delta=5$)



Notes. (a) Firm 1. (b) Firm 2.

can outperform price discrimination (as shown in Figure 9).

At the industry level, Figure 9 shows that total profits exhibit a similar pattern as Firm 2's profits (because Firm 1's profit variations are comparatively small). Price discrimination can, therefore, still be beneficial at the industry level, but there exist cases where uniform pricing is more profitable. As in the low-demand regime, price discrimination can have adverse effects on customers, who will pay higher prices at the equilibrium. Under high demand, we also observe a mismatch between discriminatory pricing outcomes and customer preferences. More precisely, price-sensitive customers may pay a higher price than under uniform pricing (Figure 10), and at the same time, quality-sensitive customers may face capacity restrictions from the firm with the high-quality product.

In summary, under high demand, there is no strict dominance between uniform and discriminatory pricing. Because of capacity constraints, uniform pricing secures a high level of baseline profits for both firms. Price discrimination (when combined with endogenous inventory allocation) is still generally beneficial; yet, firms may be better off under uniform pricing when the market demand is not too high and under strong customer heterogeneity. Regardless, price discrimination with exogenous pro rata capacity allocation performs consistently worst, as in the low-demand regime.

6. Conclusion

This paper studies Bertrand competition in the presence of product differentiation, customer heterogeneity, and price discrimination. The technical results of this paper fall into four categories. First, we established the existence and uniqueness of the Nash equilibrium of the price competition game featuring customer heterogeneity and quality differentiation. Second, we considered price competition games featuring homogenous (price-sensitive or quality-sensitive) customers, quality differentiation, and capacity differentiation. Third, we characterized the subgame-perfect equilibria of the two-stage inventory-price competition game under price discrimination. Fourth, we conducted extensive comparisons between expected prices and expected profits under uniform and discriminatory pricing (with either pro rata or endogenous inventory allocation).

Our results shed light on the impact of customer heterogeneity and price discrimination on firms' pricing strategies and resulting profits. First, we conveyed that the firm with the higher-quality product benefits from a more quality-sensitive market, whereas the firm with the lower-quality product benefits from a more heterogeneous customer pool (as opposed to a more price-sensitive pool). This result underscores that customer heterogeneity introduces a market friction that enables firms to escape from the price race to the bottom. Altogether, three sources of market frictions are present in our Bertrand competition game: capacity restrictions, quality differentiation, and customer heterogeneity. Second, price discrimination can only be beneficial under competition if coupled with strategic inventory allocation. Namely, price discrimination eliminates the market frictions induced by customer heterogeneity and can thus be detrimental under competition. As such, strategic inventory allocation plays a critical role in price discrimination by restoring (or strengthening) market frictions from capacity restrictions on each customer segment.

In summary, recent advances in data collection, customer segmentation practices, and price discrimination capabilities create opportunities for firms to design tailored pricing strategies and increase their profits. At the same time, price discrimination can end up being detrimental under competition. We show that when competing firms can commit to a capacity allocation strategy prior to engaging in price discrimination (a setting motivated by real-world examples featuring regional pricing and local inventories, such as fast fashion and vaccines), firms should comprehensively revisit their inventory management strategies concomitantly with designing their pricing strategies. Otherwise, uniform pricing may be preferable—that is, price discrimination, although seemingly beneficial, should not be blindly adopted. Ultimately, we hope that these findings will open new research avenues to further understand the trade-offs between discriminatory and uniform pricing under competition and the critical role of capacity management in this context.

Acknowledgments

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Endnotes

[1] See https://www.businessinsider.com/hm-zara-compared-photos-details-2018-5.

[2] See, for example, https://us.fashionnetwork.com/news/zara-worldwide-pricing-strategy-revealed-by-study,544318.html.

[3] See https://www.beckershospitalreview.com/pharmacy/covid-19-vaccine-prices-vary-greatly-among-countries.html and https://www.pharmaceutical-technology.com/features/covid-19-vaccine-pricing-varies-country-company/.

[4] Our focus on a noncooperative setting is consistent with the literature on Bertrand competition. From a practical standpoint, this is motivated by antitrust laws requiring that companies establish prices without colluding with competitors; in the United States, for example, the Sherman Anti-Trust Act (enacted in 1890) prohibits any agreement among competitors to fix prices, rig bids, or engage in any other form of anticompetitive activities.

[5] We show in EC.6 that demand stochasticity does not alter the structure of the game.

[6] In EC.5, we extend our model to combine the results under customer heterogeneity from this paper with the results under cost asymmetry from the literature (Blume 2003, Kartik 2011, Demuynck et al. 2019).

[7] In EC.7, we extend our model to a more general setting by defining two segments with different levels of quality sensitivity. The resulting game reduces to our homogenous setting in Section 3.2 (when α_A and α_B are close) or to our heterogenous setting in Section 4.1 (when α_A and α_B are farther apart).

[8] As established in Online Appendix C, pooled capacity and pro rata inventory allocation either are identical (under low demand) or generate similar insights (under high demand) when pricing is uniform.

[9] In fact, there exist infinitely many equilibria that share the same features as E_C, with both firms charging $p_{\max}$ to QSC and Firm 1 earning $(D-I)p_{\max}$. With some abuse of notation, we denote this set of equilibria as E_C.

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