Quiz AS101
Aircraft Propulsion

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Problems

Problem 1

The ratio of flight speed to exhaust velocity for maximum propulsion efficiency is:

A) 0

B) 0.5

C) 1.0

D) 2.0

Problem 2

A turbojet engine is powering a fighter airplane. Its cruise altitude and Mach number are 10 km and 0.86, respectively. The ambient pressure and temperature are 223 K and 0.265 bar, respectively. The exhaust gases leave the nozzle at a speed of 580 m/s and a pressure of 0.75 bar. The outlet area of the exhaust nozzle is A_e = 0.25 m^2. The air mass flow rate is 46 kg/s and the fuel-to-air ratio is 0.023. Determine the specific thrust and the propulsive efficiency. Define the propulsive efficiency as the ratio of thrust power to power imparted to engine airflow.

A) F_{sp} = 300 N·s/kg and η_p = 74.3%

B) F_{sp} = 300 N·s/kg and η_p = 86.9%

C) F_{sp} = 600 N·s/kg and η_p = 74.3%

D) F_{sp} = 600 N·s/kg and η_p = 86.9%

Problem 3

A helicopter is powered by a Rolls-Royce turboshaft engine with a takeoff shaft power of 700 hp. The engine has the following data:

→ Rotor efficiency: 0.74
→ Gearbox efficiency: 0.98
→ Fuel heating value: 45,000 kJ/kg
→ Fuel mass flow rate: 100 kg/h

Determine the thermal efficiency of the helicopter engine.

A) η_p = 20.7%

B) η_p = 30.3%

C) η_p = 40.2%

D) η_p = 50.5%

Problem 4 (Yahya, 1982)

An aircraft flies at 900 km/h. One of its turbojet engines takes in 45 kg/s of air and expands the gases to the ambient pressure. The air-fuel ratio is 38 and the lower calorific value of the fuel is 41 MJ/kg. True or false?

1.( ) The jet velocity for maximum thrust power is greater than 400 m/s.

2.( ) The specific thrust (based on air intake) is greater than 250 m/s.

3.( ) The propulsive efficiency, again assuming maximum conditions, is greater than 75 percent.

4.( ) The thermal efficiency is greater than 10 percent.

5.( ) The TSFC is greater than 0.45 kg/h·N.

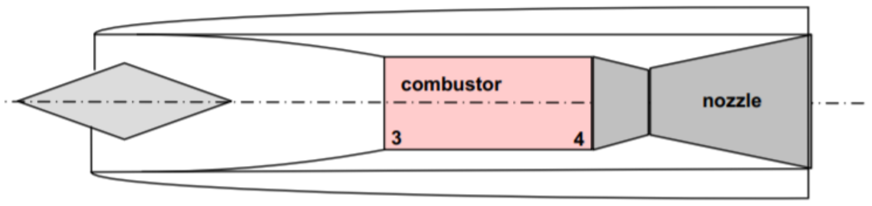
Problem 5

In an ideal gas turbine cycle, the expansion in a turbine is represented by:

A) an isentropic process.
B) an isenthalpic process.
C) an isobaric process.
D) an isochoric process.

Problem 6

Consider a ramjet-powered missile of the general type illustrated below. Fuel is added at a mass flow rate $\dot{m}_f$ while air enters the combustion chamber at a rate $\dot{m}_3$ with a total temperature T_{t3} = 512 K and a Mach number of M_3 = 0.36. The combustor efficiency is 0.90. Assuming that the combustion chamber is of constant area throughout and the heating value of the fuel is 42,500 kJ/kg, find the maximum fuel-to-air ratio that can be sustained by the jet engine. Use γ = 1.33 and $\bar{c_p}$ = 1.15 kJ/kg.

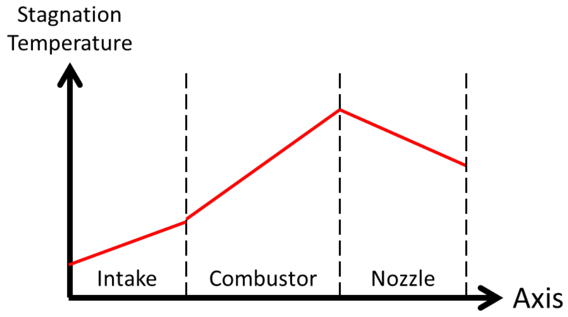


A) $\dot{m}_f/\dot{m}_3 =$ 0.0125
B) $\dot{m}_f/\dot{m}_3 =$ 0.0189
C) $\dot{m}_f/\dot{m}_3 =$ 0.0253
D) $\dot{m}_f/\dot{m}_3 =$ 0.0317

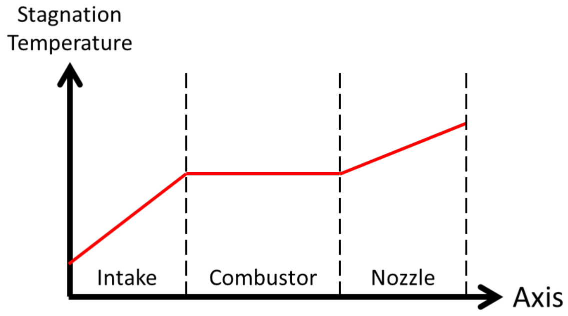
Problem 7

Which of the following graphs shows the correct variation of stagnation temperature along the axis of an ideal ram jet engine?

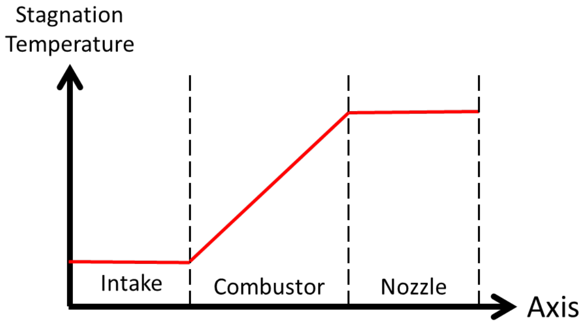
A)



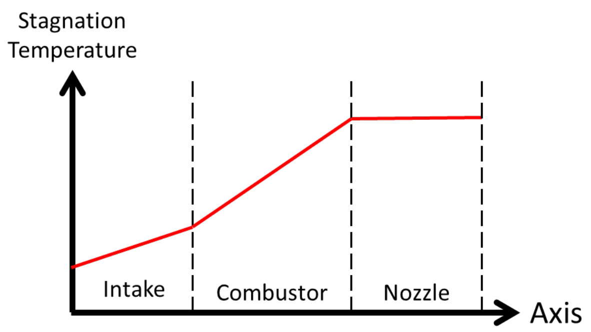
B)



C)



D)



Problem 8 (Modified from Flack, 2005, w/ permission)

A ramjet is traveling at Mach 2.5 at an altitude of 4000 m. Air flows through the engine at 40 kg/s and the fuel that impels the aircraft has a heating value of 42,800 kJ/kg. The burner exit temperature is 1880 K. Use γ = 1.4 and c_p = 1000 J/kg·K. True or false?

1.( ) The total temperature at the diffuser exit is greater than 650 K.
2.( ) The total pressure at the nozzle exit is greater than 1000 kPa.
3.( ) The temperature at the nozzle exit is greater than 820 K.
4.( ) The thrust developed by the engine is greater than 20 kN.
5.( ) The TSFC of the engine is greater than 0.2 kg/h·N.

Problem 9A (Modified from Flack, 2005, w/ permission)

An ideal turbojet flies at sea level at a Mach number of 0.80. It ingests 75 kg/s of air, and the compressor operates with a total pressure ratio of 15. The fuel has a heating value of 41,400 kJ/kg, and the burner exit total temperature is 1430 K. Use γ = 1.4 and c_p = 1000 J/kgK. True or false?

1.( ) The total temperature at the diffuser exit is greater than 310 K.
2.( ) Assuming the gas velocity to be the same as the jet velocity, the diffuser inlet diameter is greater than 0.7 m.
3.( ) The total temperature at the exit of the compressor is greater than 650 K.
4.( ) The total pressure at the exit of the burner is greater than 2500 kPa.
5.( ) The total pressure at the turbine exit is greater than 900 kPa.
6.( ) The nozzle exit diameter is greater than 0.35 m.
7.( ) The thrust produced by the engine is greater than 50 kN.
8.( ) The TSFC of the engine is greater than 0.12 kg/h·N.

Problem 9B

The ideal turbojet, as examined in the previous problem, now has an afterburner with an exit total temperature of 1900 K. True or false?

1.( ) The updated total fuel mass flow is greater than 3 kg/s.
2.( ) The updated nozzle exit diameter is greater than 0.55 m.
3.( ) The updated thrust is more than 60 percent greater than the thrust produced by the engine without an afterburner.
4.( ) The updated TSFC is more than 20 percent greater than the TSFC for an engine without an afterburner.

Problem 9C

Reconsider the turbojet introduced in Problem 9A. The compressor operates at a pressure ratio of 15 and an efficiency of 85 percent. The burner has an efficiency of 88 percent and a total pressure ratio of 0.93, whereas the turbine has an efficiency of 80 percent. A converging nozzle is used, and the nozzle efficiency is 95 percent. The total pressure recovery for the diffuser is 0.93, and the shaft efficiency is 99 percent. As before, assume γ = 1.4 and c_p to be constant at 1000 J/kgK. True or false?

1.( ) The diffuser exit total pressure is decreased by more than 5 percent relatively to the ideal engine.
2.( ) The compressor exit total temperature is increased by more than 15 percent relatively to the ideal engine.
3.( ) The turbine exit total pressure is decreased by more than 50 percent relatively to the ideal engine.

4.( ) The nozzle exit diameter is increased by more than 25 percent relatively to the ideal engine.

5.( ) The thrust is decreased by more than 60 percent relatively to the ideal engine.

6.( ) The TSFC is increased by more than 50 percent relatively to the ideal engine.

Problem 10A (Modified from Flack, 2005, w/ permission)

An ideal turbofan with an exhausted fan flies at sea level at a Mach number of 0.6. The primary mass flow is 61 kg/s and the bypass ratio is 1.25. The compressor pressure ratio is 15, while that of the fan is 3. The fuel used has a heating value of 42,000 kJ/kg, and the burner exit total temperature is 1300 K. Use γ = 1.4 and c_p = 1000 J/kgK. True or false?

1.( ) The diffuser inlet has a diameter greater than 0.75 m.

2.( ) The fan exit total temperature is greater than 405 K.

3.( ) The flow velocity at the exit of the fan nozzle is greater than 550 m/s.

4.( ) The turbine exit total temperature is greater than 850 K.

5.( ) The turbine exit total pressure is greater than 360 kPa.

6.( ) The gas velocity at the exit of the primary nozzle is greater than 650 m/s.

7.( ) The thrust produced by the engine is greater than 60 kN.

8.( ) The TSFC of the engine is greater than 0.05 kg/h·N.

Problem 10B

Reconsider the turbofan introduced in the previous problem. The turbofan now has a mixed fan and flies at sea level at a Mach number of 0.6. The primary mass flow is 61 kg/s and the bypass ratio is 1.25. The compressor pressure ratio is 15. The fuel used has a heating value of 42,000 kJ/kg, and the burner exit total temperature is 1300 K. The Mach numbers and turbine and duct exits are the same. Repeat the engine calculations. Use γ = 1.4 and c_p = 1000 J/kgK.

Problem 11 (Modified from Farokhi, 2014)

A propeller of diameter 2.75 m is in forward flight with speed of 100 m/s in an altitude where the ambient pressure p_a = 40 kPa and density ρ_a = 0.61 kg/m^3. The shaft power delivered to the propeller is 1.1 MW and the shaft angular speed is 940 rpm. Use momentum theory to estimate the performance of the engine. True or false?

1.( ) The air speed at the propeller is greater than 120 m/s.

2.( ) The propeller thrust is greater than 5.5 kN.

3.( ) The propeller efficiency is greater than 60 percent.

4.( ) The propeller torque is greater than 10 kN·m.

Problem 12 (Modified from Flack, 2005, w/ permission)

An ideal turboprop powers an aircraft at sea level at a Mach number of 0.5. The compressor has a pressure ratio of 5.8 and an airflow of 12 kg/s. The burner exit total temperature is 1260 K and the nozzle exit Mach number is 0.9. The heating value of the fuel is 42,500 kJ/kg. Use γ = 1.4 and c_p = 1000 J/kgK. True or false?

1.( ) The work coefficient is greater than 1.20.

2.( ) The thrust power is greater than 3 MW.

3.( ) The SFC is greater than 220 kg/MW·h.

Problem 13 (Modified from Mattingly, 1996, w/ permission)

Consider an axial compressor with the following characteristics.

→ Total temperature at station 1, T_{t1} = 288.2 K
→ Total pressure at station 1, p_{t1} = 101.3 kPa
→ Rotation speed, ω = 1000 rad/s
→ Radius, r = 0.30 m
→ Rotor flow angles, $\alpha_1 = \alpha_3 = 40^{O}$
→ Solidity, σ = 1.0
→ Mass flow rate, $\dot{m}$ = 20 kg/s
→ Flow Mach number, $M_1 = M_3$ = 0.6
→ Velocity ratio, u_2/u_1 = 1.2
→ Total temperature variation, ΔT = 22 K
→ Rotor loss coefficient, ϕ_{cr} = 0.08
→ Stator loss coefficient, ϕ_{cs} = 0.03

Use γ = 1.4 and c_p = 1.0 kJ/kgK. True or false?

1.( ) The area of the flow annulus at station 1 is greater than 0.18 m^2.
2.( ) The relative total temperature at station 1 is greater than 300 K.
3.( ) The relative total pressure at station 2 is greater than 110 kPa
4.( ) The cascade flow angle β_2 is greater than 25^{o}.
5.( ) The relative velocity at station 2 is greater than 220 m/s.
6.( ) The pressure at station 2 is greater than 75 kPa.
7.( ) The area of the flow annulus at station 2 is greater than 0.08 m^2.
8.( ) The area of the flow annulus at station 3 is greater than 0.16 m^2.
9.( ) The degree of reaction is greater than 0.18.
10.( ) The diffusion factors for the rotor and stator are both <u>less</u> than 0.6.
11.( ) The stage efficiency is greater than 0.92.
12.( ) The polytropic efficiency is greater than 0.85.
13.( ) The stage loading coefficient is greater than 0.32.
14.( ) The flow coefficient is greater than 0.6.

Problem 14 (Modified from Mattingly, 1996, w/ permission)

Consider a centrifugal compressor with the following characteristics.

→ Mass flow rate, $\dot{m}$ = 9 kg/s
→ p_{t1} = 101.3 kPa
→ T_{t1} = 288.2 K
→ Pressure ratio, π_c = 4.0
→ Polytropic efficiency, e_c = 0.86
→ Inlet root diameter, d_{1h} = 20 cm
→ Inlet tip diameter, d_{1t} = 35 cm
→ Outlet diameter of the impeller, d_2 = 50 cm
→ Slip factor, ε = 0.9
→ V_3 = 90 m/s
→ $w_2 = u_1$

Use γ = 1.4 and c_p = 1000 J/kgK. True or false?

1.( ) The rotational speed of the rotor is greater than 15,000 rpm.
2.( ) The relative flow angle β_2 at the hub is greater than 60 degrees.
3.( ) The adiabatic efficiency is greater than 0.8.
4.( ) The Mach number at station 2 is greater than 1.2.
5.( ) The total pressure at station 2 is greater than 480 kPa.
6.( ) The depth of the rotor exit is greater than 6.5 mm.
7.( ) Product $A_3 \times \cos\alpha_3$ is greater than 0.05 m^2.

Additional Information

Table 1 US Standard Atmosphere, 1976 – SI Units

Altitude (m)	Temperature (K)	Pressure (kPa)
0	288.2	101.3
500	284.9	95.47
1000	281.7	89.90
2000	275.2	79.53
3000	268.7	70.16
4000	262.2	61.66
5000	255.7	54.09
6000	249.2	47.25
7000	242.7	41.13
8000	236.3	35.68
9000	229.8	30.82
10,000	223.3	26.51

Figure 1 Ramjet component numbering (Problem 8)

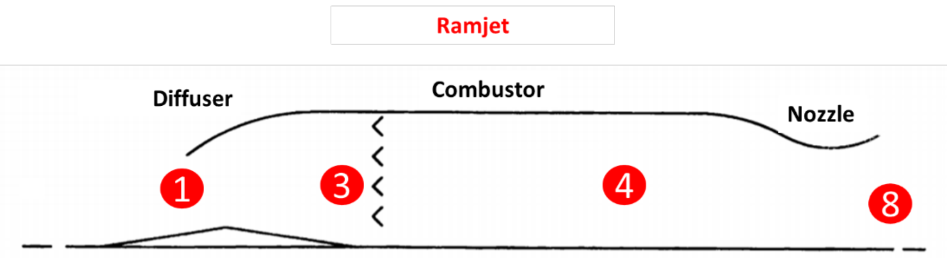


Figure 2 Nonafterburning turbojet numbering (Problem 9A)

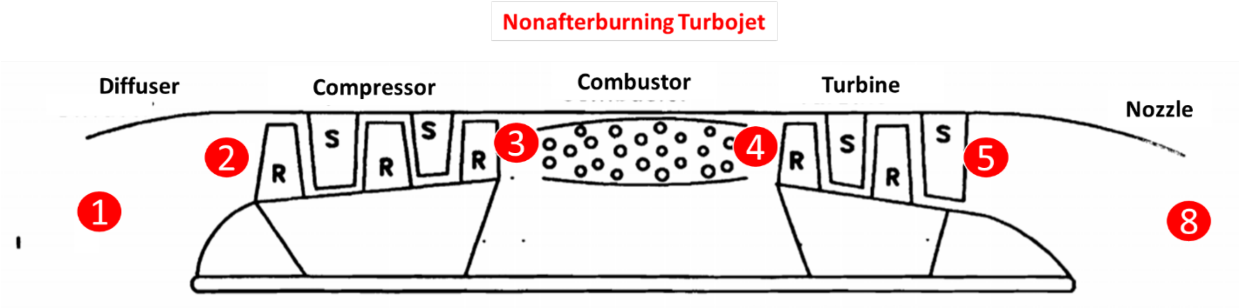


Figure 3 Turbojet with afterburner component numbering (Problem 9B)

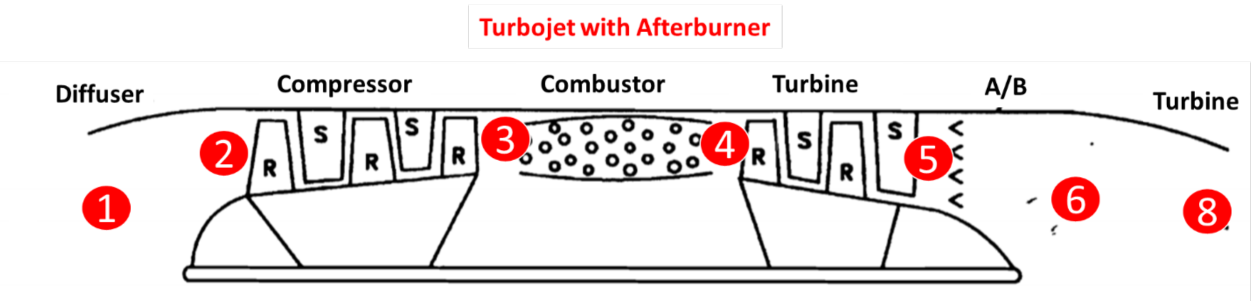


Figure 4 Nonafterburning turbofan with exhausted turbofan component numbering (Problem 10A)

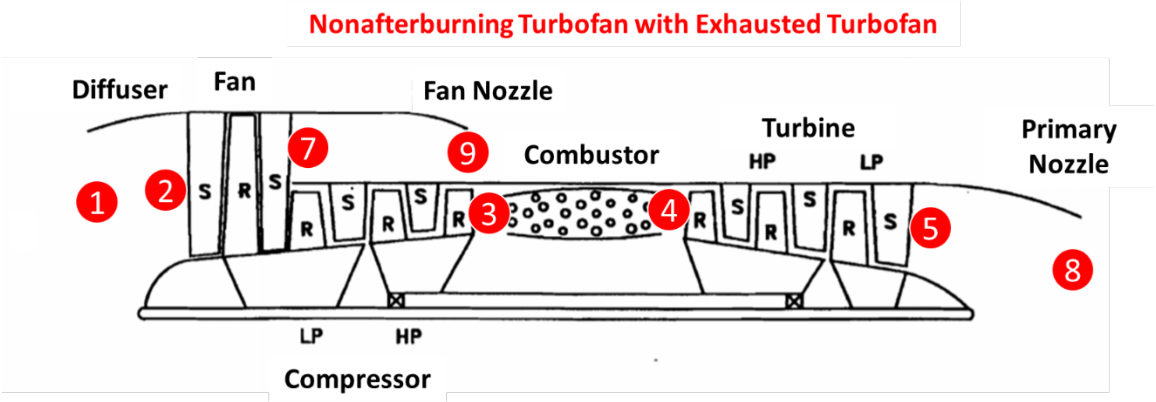


Figure 5 Nonafterburning mixed turbofan component numbering (Problem 10B)

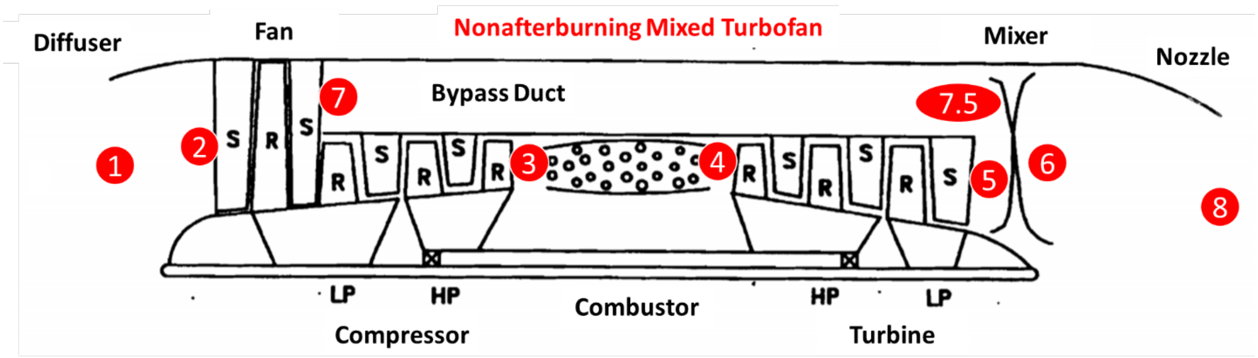


Figure 6 General turboprop component numbering (Problem 12)

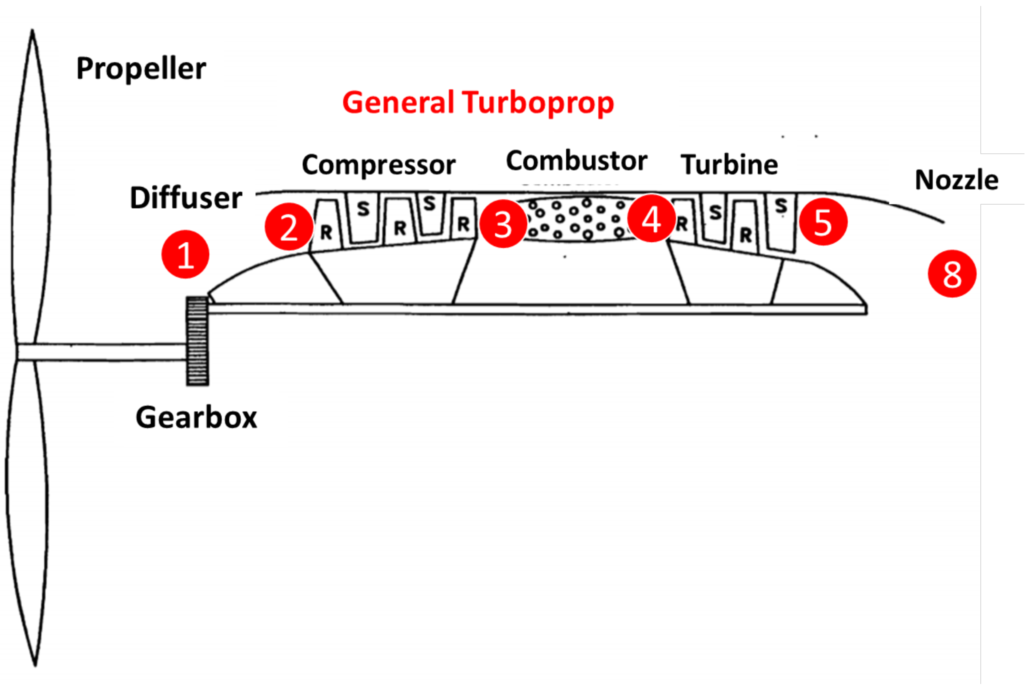
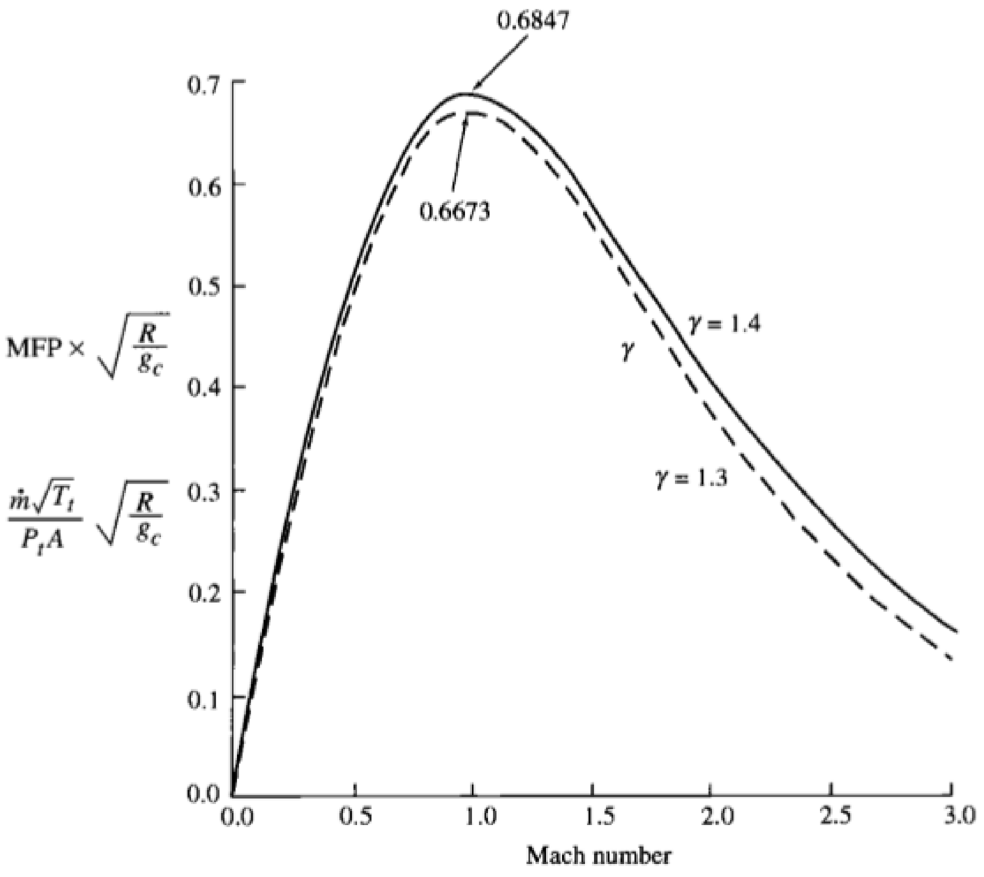


Figure 7 Mass flow parameter versus Mach number (γ = 1.3 and γ = 1.4)



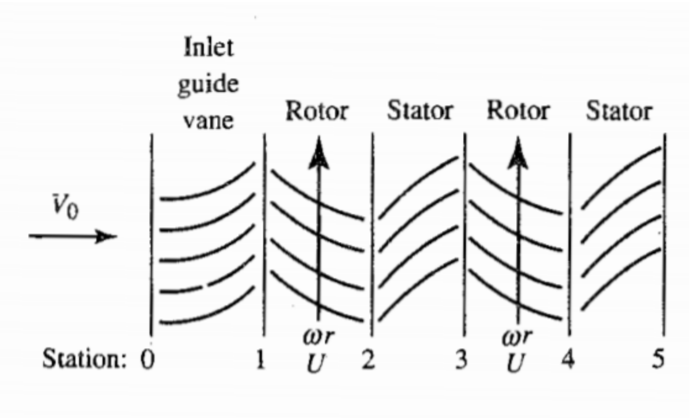
Appendix

Analysis of Axial Flow Compressors (from Mattingly, *Elements of Gas Turbine Propulsion*, 1996, McGraw-Hill)

In the analysis adopted in Problem 13, two different coordinate systems are used: one fixed to the compressor housing (absolute) and the other fixed to the rotating blades (relative). The static (thermodynamic) properties do not depend on the reference frame. However, the total properties do depend on the reference frame. The velocity of a fluid in one reference frame is easily converted to the other frame by the equation

$$V = V_R + \omega r$$

where V is the velocity in a stationary coordinate system, V_R is the velocity in a moving coordinate system, and ωr is the velocity of the moving coordinate system. Consider the compressor stage made up of a rotor followed by a stator as shown in the figure below.



The flow enters the rotor with velocity V_1 (relative velocity V_{1R}) and leaves with velocity V_2 (relative velocity V_{2R}). The rotor is moving upward at velocity ωr. The flow enters the stator with velocity V_2 and leaves with velocity V_3. Rather than keep the axial velocity constant, as is done in many textbooks, this approach permits variation in axial velocity from station to station. The tangential velocity can thus be decomposed as $v_1 = \omega r - u_1 \tan\beta_1 = u_1 \tan\alpha_1$ and $v_2 = \omega r - u_2 \tan\beta_2 = u_2 \tan\alpha_2$, giving

$$c_p\left(T_{t2} - T_{t1}\right) = \frac{(\omega r)^2 u_1}{g_c(\omega r)}\left(\tan\beta_1 - \frac{u_2}{u_1}\tan\beta_2\right)$$

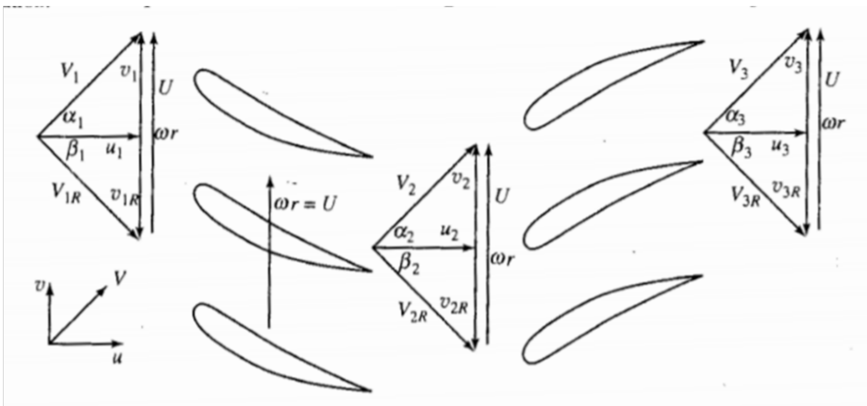
or, equivalently,

$$c_p\left(T_{t2} - T_{t1}\right) = \frac{(\omega r)^2 u_1}{g_c(\omega r)}\left(\frac{u_2}{u_1}\tan\alpha_2 - \tan\alpha_1\right)$$

Hence, the work done per unit mass flow can be determined from the rotor speed (ωr), the velocity ratios (u_1/U and u_2/u_1), and either the rotor cascade flow angles (β_1 and β_2) or the absolute rotor flow angles (α_1 and α_2). The two foregoing equations are useful forms of the Euler equation for compressor stage design and show the dependence of stage work on the rotor speed squared, $(\omega r)^2$.

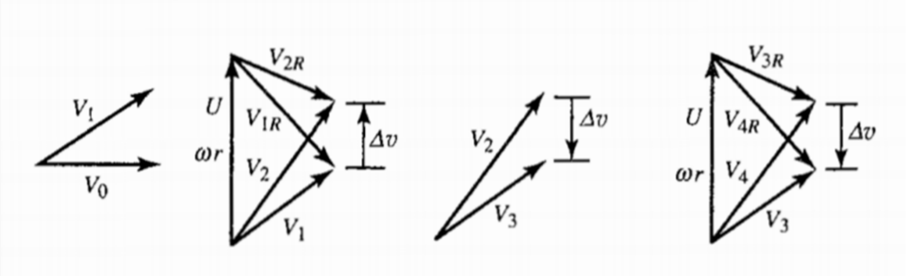
An axial flow compressor stage consists of a rotor followed by a stator, as shown in Figure (a) below. Two compressor stages (which are identical in geometry) are shown in figure (b) below preceded by inlet guide vanes. The velocity diagrams depicted in figure (b) show the absolute velocities entering and leaving the guide vanes, rotor, and stator. In addition, for the rotors, the entering and leaving relative velocities and the rotor tangential velocity are shown. We have assumed, in the diagram, that the axial velocity component is constant.

(a)



	Rotor	Stator
Velocities	$\mathbf{V}_1 = \mathbf{V}_{1R} + \mathbf{U}$	$\mathbf{V}_2 = \mathbf{V}_{2R} + \mathbf{U}$
	$u_1 = V_1 \cos \alpha_1$	$u_2 = V_2 \cos \alpha_2$
	$v_1 = V_1 \sin \alpha_1$	$v_2 = V_2 \sin \alpha_2$
	$u_1 = V_{1R} \cos \beta_1$	$u_2 = V_{2R} \cos \beta_2$
	$v_{1R} = V_{1R} \sin \beta_1 = u_1 \tan \beta_1$	$v_{2R} = V_{2R} \sin \beta_2 = u_2 \tan \beta_2$
	$v_1 + v_{1R} = \omega r = U$	$v_2 + v_{2R} = \omega r = U$

(b)



Referring to Figure (b), we see that the guide vanes act as nozzles through which the static pressure decreases as the air velocity increases, and the fluid is given a tangential (swirl) component in the direction of the rotor velocity. The air leaves the guide vanes with velocity V_1.

The absolute velocity entering the rotor at station 1 is V_1. Subtracting the rotor speed ωr vectorially, we obtain the relative velocity V_{1R} entering the rotor. In the rotor blade row, the blade passages act as diffusers, reducing the relative velocity from V_{1R} to V_{2R} as the static pressure is increased from p_1 to p_2. Combining V_{2R} vectorially with ωr, we get their sum V_2 − the absolute velocity leaving the rotor.

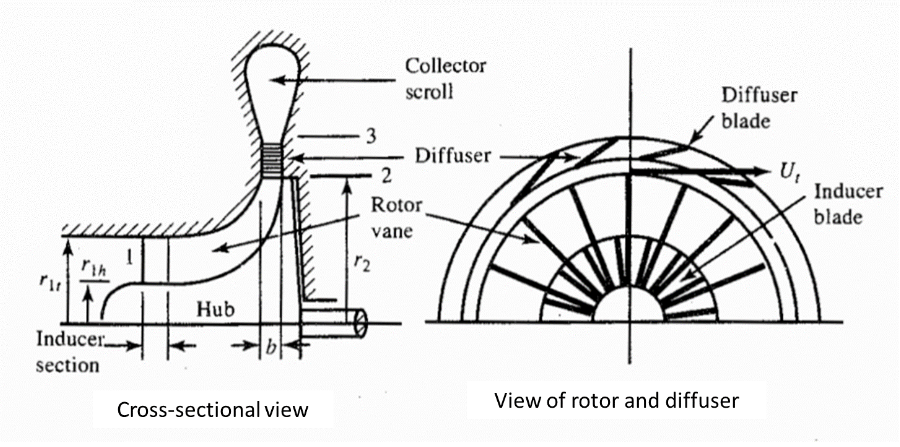
The velocity of the air leaving the rotor and entering the stator at station 2 is V_2. The stator diffuses the velocity to V_3 as the static pressure rises from p_2 to p_3. Since the velocity V_3 entering the rotor at station 3 is identical with V_1 entering the first-stage rotor, we find that the velocity triangle for the second-stage rotor is a repeat of the triangle for the first stage. The effects occurring in each compressor component are summarized in the following table, where +, 0, and − mean increased, unchanged, and decreased, respectively. The table entries assume isentropic flow. In making entries in the table, it is important to distinguish between absolute and relative values. Since total pressure and total temperature depend upon the speed of the gas, they have different values "traveling with the rotor" than for an observer not riding on the rotor. In particular, an observer on the rotor sees a force F (rotor on gas), but it is stationary; hence, in the rider's reference system, the force does no work on the gas. Consequently, the total temperature and total pressure do not change relative to an observer on the rotor as the gas passes through the rotor. An observer not on the rotor sees the force F (rotor on gas) moving at the rate ωr. Hence, to the stationary observer, work is done on the gas passing through the rotor, and the total temperature and total pressure increase.

Property	Inlet guide vanes	Rotor	Stator
Absolute velocity	+	+	−
Relative velocity	n/a	−	n/a
Static pressure	−	+	+
Absolute total pressure	0	+	0
Relative total pressure	n/a	0	n/a
Static temperature	−	+	+
Absolute total temperature	0	+	0
Relative total temperature	n/a	0	n/a

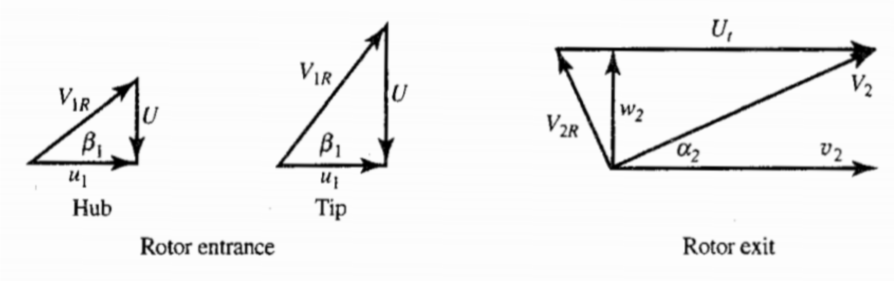
+ = increase − = decrease
0 = unchanged n/a not applicable

Analysis of Centrifugal Flow Compressors

The following figure shows a sketch of a centrifugal-flow compressor with radial rotor (or impeller) vanes. Flow passes through the annulus between r_{1h} and r_{1t} at station 1 and enters the inducer section of the rotor (also called rotating guide vanes). Flow leaves the rotor at station 2 through the cylindrical area of radius r_2 and width b. The flow then passes through the diffuser, where it is slowed and then enters the collector scroll at station 3.



The velocity diagrams at the entrance and exit of the rotor (impeller) are shown below. The inlet flow is assumed to be axial of uniform velocity u_1. The relative flow angle of the flow entering the rotor increases from hub to tip and thus the twist of the inlet to the inducer section of the rotor. The flow leaves the rotor with a radial component of velocity w_2 that is approximately equal to the inlet axial velocity u_1 and a swirl (tangential) component of velocity v_2 that is about 90 percent of the rotor velocity U_t. The diffuser (which may be vaneless) slows the velocity of the flow V_3 to about 90 m/s.



Solutions

P.1 ■ Solution

The propulsive efficiency is maximum when the flight speed is equal to the jet velocity; this condition corresponds to zero thrust, and hence is only a hypothetical case. However, an important conclusion can be drawn from this hypothesis: high propulsive efficiency can be attained by employing jet speeds close to the flight speed, whereas high thrust can be obtained by increasing the flow rate of air or gas through the propulsive device.

★ The correct answer is **C**.

P.2 ■ Solution

Observe that the ambient pressure is less than the exit pressure. Accordingly, the nozzle is choked and the pressure thrust is not zero. The flight speed is

$$U = M\sqrt{\gamma R T_a} = 0.86 \times \sqrt{1.4 \times 287 \times 223} = 257 \text{ m/s}$$

The specific thrust is determined next,

$$F_{\text{sp}} = \frac{F}{\dot{m}_a} = \left[(1+f)U_e - U\right] + (p_e - p_a)\frac{A_e}{\dot{m}_a}$$

$$\therefore F_{\text{sp}} = \left[(1+0.023)\times 580 - 257\right] + \left[(0.75-0.265)\times 10^5\right] \times \frac{0.25}{46} = \boxed{600 \text{ N}\cdot\text{s/kg}}$$

One expression to use for propulsive efficiency is

$$\eta_p = \frac{2(F/\dot{m}_a)U}{2(F/\dot{m}_a)U + (1+f)(U_e - U)^2}$$

$$\therefore \eta_p = \frac{2\times 600\times 257}{2\times 600\times 257 + (1+0.023)\times(580-257)^2} = \boxed{74.3\%}$$

As mentioned in the foregoing, here the propulsive efficiency is defined as the ratio of thrust power to power imparted to engine airflow. Another way to define propulsive efficiency is to express it as the ratio of thrust power to the rate of kinetic energy added to engine airflow, which brings to

$$\eta_p = \frac{2(F/\dot{m}_a)U}{(1+f)U_e^2 - U^2} = \frac{2\times 600\times 257}{(1+0.023)\times 580^2 - 257^2} = 111\%$$

This bewildering result is inherent to this latter formula, which sometimes yields efficiencies greater than unity.

★ The correct answer is **C**.

P.3 ■ Solution

The thermal efficiency is given by

$$\eta_T = \frac{P_{\text{Total}}}{\dot{m}_f Q_R}$$

where P_{Total} is the total power produced by the engine, $\dot{m}_f$ = 100/3600 = 0.0278 kg/s is the fuel mass flow rate, and Q_R = 45,000 kJ/kg is the heating value of the fuel. Given the rotor efficiency η_R = 0.75, the gearbox efficiency η_G = 0.98, and the shaft power SP = 700 hp, the total power is estimated as

$$P_{\text{Total}} = \eta_R \eta_G SP = 0.75\times 0.98\times 700 = 508 \text{ hp} = 379 \text{ kW}$$

Backsubstituting in the expression for η_T gives

$$\eta_T = \frac{379}{0.0278\times 45{,}000} = \boxed{30.3\%}$$

★ The correct answer is **B**.

P.4 ■ Solution

1. True. The flight velocity is U = 900/3.6 = 250 m/s. For maximum thrust power, the ratio of flight velocity to jet velocity should equal 0.5, which corresponds to a jet velocity such that

$$\frac{U}{c_j} = 0.5 \rightarrow c_j = \frac{250}{0.5} = \boxed{500 \text{ m/s}}$$

2. True. The total mass flow of gas, $\dot{m}$, is given by

$$\dot{m} = \dot{m}_a + \dot{m}_f = \dot{m}_a\left(1 + \frac{\dot{m}_f}{\dot{m}_a}\right)$$

$$\therefore \dot{m} = 45\times\left(1+\frac{1}{38}\right) = 46.2 \text{ kg/s}$$

where we have used the air-fuel ratio = 38. The thrust is determined next,

$$F = \dot{m}c_j - \dot{m}_a U = 46.2\times 500 - 45\times 250 = 11.9 \text{ kN}$$

The specific thrust based on air intake follows as

$$F_{\text{sp}} = \frac{F}{\dot{m}_a} = \frac{11{,}900}{45} = \boxed{264 \text{ m/s}}$$

3. False. The thrust power is the product of thrust and flight velocity. Recall, as before, that c_j/U = 0.5 for maximum thrust power. Accordingly,

$$\eta_P = \frac{1}{1+c_j/U} = \frac{1}{1+1/0.5} = \boxed{66.7\%}$$

4. False. The thermal efficiency is calculated with the relation

$$\eta_T = \frac{\frac{1}{2}\dot{m}\left(c_j^2 - U^2\right)}{\dot{m}_f Q_f}$$

where the fuel mass flow is such that

$$\frac{\dot{m}_a}{\dot{m}_f} = 38 \rightarrow \dot{m}_f = \frac{\dot{m}_a}{38}$$

$$\therefore \dot{m}_f = \frac{45}{38} = 1.18 \text{ kg/s}$$

Backsubstituting in the equation for η_T gives

$$\eta_T = \frac{0.5 \times 46.2 \times \left(500^2 - 250^2\right)}{1.18 \times \left(41 \times 10^6\right)} = \boxed{8.95\%}$$

5. False. The TSFC is given by

$$TSFC = \frac{\dot{m}_f \times 3600}{T} = \frac{1.18 \times 3600}{11{,}900} = \boxed{0.357 \text{ kg/h} \cdot \text{N}}$$

P.5 ■ Solution

The gas expansion in an ideal gas turbine cycle is an isentropic process.

★ The correct answer is **A**.

P.6 ■ Solution

The stagnation temperature ratio across a constant area combustor is given by

$$\frac{T_{t4}}{T_{t3}} = \frac{\gamma_4 M_4^2 W_4}{\gamma_3 M_3^2 W_3}\left(\frac{1+\gamma_3 M_3^2}{1+\gamma_4 M_4^2}\right)^2 \frac{\left(1+\frac{\gamma_4 - 1}{2}M_4^2\right)}{\left(1+\frac{\gamma_3 - 1}{2}M_3^2\right)}$$

which, observing that $\gamma_3 = \gamma_4 = \gamma = 1.33$, becomes

$$\frac{T_{t4} - T_{t3}}{T_{t3}} = \frac{M_4^2}{M_3^2}\left(\frac{3+4M_3^2}{3+4M_4^2}\right)^2 \left(\frac{6+M_4^2}{6+M_3^2}\right) - 1$$

The maximum heat addition occurs when the combustor exit is choked, that is, if $M_4 = 1.0$. Setting $M_4 = 1.0$ and substituting $M_3 = 0.36$ gives

$$\frac{T_{t4\max} - T_{t3}}{T_{t3}} = \frac{\Delta T_{t\max}}{T_{t3}} = \frac{1.0^2}{0.36^2}\left(\frac{3+4\times 0.36^2}{3+4\times 1.0^2}\right)^2 \left(\frac{6+1.0^2}{6+0.36^2}\right) - 1$$

$$\therefore \Delta T_{t\max} = 1.23 T_{t3} = 1.23 \times 512 = 630 \text{ K}$$

Accordingly,

$$\dot{m}_3 \bar{c}_p \left(T_{t4\max} - T_{t3}\right) = \dot{m}_f \eta_b \Delta H \rightarrow \frac{\dot{m}_f}{\dot{m}_3} = \frac{\bar{c}_p \left(T_{t4\max} - T_{t3}\right)}{\eta_b \Delta H}$$

$$\therefore \frac{\dot{m}_f}{\dot{m}_3} = \frac{1.15 \times 630}{0.99 \times 42{,}500} = \boxed{0.0189}$$

★ The correct answer is **B**.

P.7 ■ Solution

The stagnation temperature in a ramjet increases in the combustor and remains unchanged in the inlet and nozzle processes.

★ The correct answer is **C**.

P.8 ■ Solution

1. False. From Table 1, the ambient temperature and pressure for an altitude of 4000 m are T_a = 262.2 K and p_a = 61.66 kPa, respectively. The inlet speed of sound is determined first,

$$a_a = \sqrt{\gamma R T_a} = \sqrt{1.4 \times 287 \times 262.2} = 325 \text{ m/s}$$

The ramjet velocity is then

$$U_a = M_a a_a = 2.5 \times 325 = 813 \text{ m/s}$$

The ambient total temperature is

$$\frac{T_{ta}}{T_a} = \left(1 + \frac{\gamma - 1}{2} M_8^2\right) \rightarrow T_{ta} = T_a \left(1 + \frac{\gamma - 1}{2} M_8^2\right)$$

$$\therefore T_{ta} = 262.2 \times \left(1 + \frac{1.4 - 1}{2} \times 2.5^2\right) = 590 \text{ K}$$

Because the diffuser is adiabatic, the total temperature at the diffuser exit, station 3, is equal to the total ambient temperature,

$$T_{t3} = T_{ta} = \boxed{590 \text{ K}}$$

2. True. The ambient total pressure is determined next,

$$\frac{p_{ta}}{p_a} = \left(1 + \frac{\gamma - 1}{2} M_a^2\right)^{\gamma/(\gamma - 1)} \rightarrow p_{ta} = p_a \left(1 + \frac{\gamma - 1}{2} M_a^2\right)^{\gamma/(\gamma - 1)}$$

$$\therefore p_{ta} = 61.66 \times \left(1 + \frac{1.4 - 1}{2} \times 2.5^2\right)^{1.4/(1.4 - 1)} = 1050 \text{ kPa}$$

From the ideal assumption that the exit pressure matches ambient conditions, we have $p_8 = p_a$. Processes a to 1 (external flow), 1 to 3 (diffuser), and 4 to 8 (nozzle) are all isentropic, while process 3 to 4 (combustor) is isotobaric (constant total pressure). Thus, the total pressure is constant throughout an ideal ramjet and we can write

$$p_{ta} = p_{t1} = p_{t4} = p_{t8}$$

Accordingly, the total pressure at the nozzle exit is $p_{t8} = p_{ta}$ = 1050 kPa.

3. True. For an ideal analysis, the total temperature is constant from the burner, station 4, through the nozzle, station 8 (adiabatic), i.e., $T_{t8} = T_{t4}$ = 1880 K. Furthermore, the Mach numbers at the freestream and exhaust are the same, i.e., $M_a = M_8$ = 2.5. The nozzle exit temperature is then

$$T_8 = \frac{T_{t8}}{\left(1 + \frac{\gamma - 1}{2} M_8^2\right)} = \frac{1880}{1 + \frac{1.4 - 1}{2} \times 2.5^2} = \boxed{836 \text{ K}}$$

The nozzle exit gas velocity is

$$U_8 = M_8 a_8 = M_8 \sqrt{\gamma R T_8}$$

$$\therefore U_8 = 2.5 \times \sqrt{1.4 \times 287 \times 836} = 1450 \text{ m/s}$$

4. True. Since the thrust has no pressure thrust component, the equation to use is simply

$$F = \dot{m}\left(U_e - U_a\right) = 40 \times \left(1450 - 813\right) = \boxed{25.5 \text{ kN}}$$

5. False. Before computing the TSFC, we require the fuel mass flow, $\dot{m}_f$. Given the burner total temperature T_{t4} = 1880 K and noting that, because the diffuser is adiabatic, $T_{t3} = T_{ta}$ = 590 K, it follows that

$$\dot{m}_f = \frac{\dot{m}c_p\left(T_{t4}-T_{t3}\right)}{\Delta H} = \frac{40\times 1.0\times\left(1880-590\right)}{42{,}800} = 1.21\ \text{kg/s}$$

Accordingly, the TSFC is

$$TSFC = \frac{\dot{m}_f}{F} = \frac{1.21}{25{,}500}\times 3600 = \boxed{0.171\ \text{kg/h}\cdot\text{N}}$$

P.9 ■ Solution

→ Part A

1. True. Referring to Table 1, we have p_a = 101.3 kPa and T_a = 288.2 K. To begin, we establish the inlet speed of sound,

$$a_a = \sqrt{\gamma R T_a} = \sqrt{1.4\times 287\times 288.2} = 340\ \text{m/s}$$

which corresponds to a jet velocity of U_a = 0.8 × 340 = 272 m/s. The inlet total temperature is calculated as

$$\frac{T_{ta}}{T_a} = 1+\frac{\gamma-1}{2}M_a^2 \rightarrow T_{ta} = T_a\left(1+\frac{\gamma-1}{2}M_a^2\right)$$

$$\therefore T_{ta} = 288.2\times\left(1+\frac{1.4-1}{2}\times 0.8^2\right) = 325\ \text{K}$$

For an ideal analysis that is adiabatic, the total temperature at the diffuser exit is the same as the inlet total temperature, hence $T_{t2} = T_{ta}$ = 325 K.

2. False. The inlet total pressure is

$$\frac{p_{ta}}{p_a} = \left(1+\frac{\gamma-1}{2}M_a^2\right)^{\gamma/(\gamma-1)} \rightarrow p_{ta} = p_a\left(1+\frac{\gamma-1}{2}M_a^2\right)^{\gamma/(\gamma-1)}$$

$$\therefore p_{ta} = 101.3\times\left(1+\frac{1.4-1}{2}\times 0.8^2\right)^{1.4/(1.4-1)} = 154\ \text{kPa}$$

For an ideal analysis that is isentropic, the inlet pressure at the diffuser is the same as the inlet total pressure, hence $p_{t2} = p_{ta}$ = 154 kPa. For an ideal gas, the density of air can be estimated as

$$\rho_a = \frac{p_a}{RT_a} = \frac{101{,}300}{287\times 288.2} = 1.22\ \text{kg/m}^3$$

Assuming the gas velocity at the diffuser inlet is the same as the jet velocity, the diffuser inlet area is found as

$$A_{\text{in}} = \frac{\dot{m}}{\rho_a U_a} = \frac{75}{1.22\times 272} = 0.226\ \text{m}^2$$

which corresponds to an inlet diameter of 0.536 m.

3. True. Since the compressor is isentropic for an ideal case, the temperature ratio can be calculated as

$$\tau_c = \pi_c^{(\gamma-1)/\gamma} = 15^{(1.4-1)/1.4} = 2.17$$

Accordingly, the total temperature at the exit of the compressor is

$$\tau_c = \frac{T_{t3}}{T_{t2}} \rightarrow T_{t3} = \tau_c T_{t2}$$

$$\therefore T_{t3} = 2.17\times 325 = \boxed{705\ \text{K}}$$

4. False. Because the burner is ideal, the total pressure is constant across the burner, i.e., $p_{t4} = p_{t3}$. Given the pressure ratio π_c = 15, the compressor exit total pressure is found as

$$\pi_c = \frac{p_{t3}}{p_{t2}} \rightarrow p_{t3} = \pi_c p_{t2}$$

$$\therefore p_{t3} = 15 \times 154 = 2310 \text{ kPa}$$

Accordingly, the total pressure at the exit of the burner is p_{t4} = 2310 kPa.

5. False. Performing a shaft energy balance for the ideal case brings to

$$\dot{m}c_p\left(T_{t4} - T_{t5}\right) = \dot{m}c_p\left(T_{t3} - T_{t2}\right) \rightarrow T_{t5} = T_{t4} - \left(T_{t3} - T_{t2}\right)$$

Accordingly, the total temperature at the turbine exit is calculated as

$$T_{t5} = T_{t4} - \left(T_{t3} - T_{t2}\right) = 1430 - \left(705 - 325\right) = 1050 \text{ K}$$

Note that $T_{t5} = T_{t8}$ because the nozzle is adiabatic. For an ideal (isentropic) turbine, the exit pressure follows as

$$\pi_t = \frac{p_{t5}}{p_{t4}} \rightarrow p_{t5} = \pi_t p_{t4} = \tau_t^{\gamma/(\gamma-1)} p_{t4}$$

$$\therefore p_{t5} = 2310 \times \left(\frac{1050}{1430}\right)^{1.4/(1.4-1)} = \boxed{784 \text{ kPa}}$$

6. True. For an ideal (isentropic) nozzle, the total pressure is constant, i.e., $p_{t8} = p_{t5}$ = 784 kPa. At the exit, because for the ideal case the exit pressure matches the ambient pressure, we can write $p_8 = p_a$ = 101.3 kPa. The nozzle exit Mach number is determined next,

$$p_{t8} = p_8\left(1 + \frac{\gamma - 1}{2}M_8^2\right)^{\gamma/(\gamma-1)} \rightarrow M_8 = \sqrt{\left(\frac{2}{\gamma - 1}\right)\left[\left(\frac{p_{t8}}{p_8}\right)^{(\gamma-1)/\gamma} - 1\right]}$$

$$\therefore M_8 = \sqrt{\left(\frac{2}{1.4 - 1}\right)\left[\left(\frac{784}{101.3}\right)^{(1.4-1)/1.4} - 1\right]} = 2.0$$

The exit nozzle temperature is found as

$$T_8 = \frac{T_{t8}}{1 + \frac{\gamma - 1}{2}M_8^2} = \frac{1050}{1 + \frac{1.4 - 1}{2} \times 2.0^2} = 583 \text{ K}$$

We are then in position to compute the nozzle exit velocity,

$$U_8 = M_8\sqrt{\gamma R T_8} = 2.0 \times \sqrt{1.4 \times 287 \times 583} = 968 \text{ m/s}$$

The density of air at the nozzle exit is

$$\rho_8 = \frac{p_8}{RT_8} = \frac{101{,}300}{287 \times 583} = 0.605 \text{ kg/m}^3$$

The nozzle exit area is then

$$A_8 = \frac{\dot{m}}{\rho_8 U_8} = \frac{75}{0.605 \times 968} = 0.128 \text{ m}^2$$

which corresponds to a nozzle exit diameter of 0.404 m.

7. True. Because the nozzle exit pressure is the same as the ambient pressure, the ideal thrust is simply

$$F = \dot{m}\left(U_e - U_a\right) = 75 \times \left(968 - 272\right) = \boxed{52.2 \text{ kN}}$$

8. False. Computing the TSFC requires the fuel mass flow, $\dot{m}_f$, which is such that

$$\dot{m}_f = \frac{\dot{m}c_p\left(T_{t4} - T_{t3}\right)}{\Delta H} = \frac{75 \times 1.0 \times \left(1430 - 705\right)}{41{,}400} = 1.31 \text{ kg/s}$$

Accordingly, the TSFC is

$$TSFC = \frac{\dot{m}_f}{F} = \frac{1.31}{52,200} \times 3600 = \boxed{0.0903\ \text{kg/h} \cdot \text{N}}$$

→ Part B

1. False. All calculations up through the turbine exit are still valid. The afterburner is station 6. The additional fuel flow for the afterburner is

$$\dot{m}_{f,\text{ab}} = \frac{\dot{m}c_p\left(T_{t6} - T_{t5}\right)}{\Delta H} = \frac{75 \times 1.0 \times \left(1900 - 1050\right)}{41,400} = 1.54\ \text{kg/s}$$

The total fuel mass flow is then

$$\dot{m}_{f,\text{t}} = \dot{m}_f + \dot{m}_{f,\text{ab}} = 1.31 + 1.54 = \boxed{2.85\ \text{kg/s}}$$

2. False. Because the inlet total pressure to the nozzle is the same as before, the nozzle exit Mach number continues to be 2.0. However, since the nozzle inlet total temperature is much higher, the exit temperature is correspondingly higher. That is, since the nozzle is adiabatic, $T_{t8} = T_{t6}$ = 1900 K, it follows that

$$T_8 = \frac{T_{t8}}{1 + \frac{\gamma - 1}{2}M_8^2} = \frac{1900}{1 + \frac{1.4 - 1}{2} \times 2.0} = 1360\ \text{K}$$

As a result, the exit speed of sound and exit velocity are also higher, and hence

$$U_8 = M_8 a_8 = M_8\sqrt{\gamma R T_8}$$

$$\therefore U_8 = 2.0 \times \sqrt{1.4 \times 287 \times 1360} = 1480\ \text{m/s}$$

The air density is

$$\rho_8 = \frac{p_8}{RT_8} = \frac{101,300}{287 \times 1360} = 0.260\ \text{kg/m}^3$$

The required nozzle exit area follows as

$$A_8 = \frac{\dot{m}}{\rho_8 U_8} = \frac{75}{0.26 \times 1480} = 0.195\ \text{m}^2$$

which implies a diameter of 0.498 m^2. The presence of the afterburner calls for a larger nozzle. A fixed geometry cannot accommodate the flows with a higher exit temperature. Consequently, a variable geometry becomes necessary.

3. True. The updated thrust is

$$F' = \dot{m}\left(U_e - U_a\right) = 75 \times \left(1480 - 272\right) = \boxed{90.6\ \text{kN}}$$

which corresponds to an increase of 73.6% relatively to the engine without an afterburner.

4. True. The updated TSFC is

$$TSFC' = \frac{\dot{m}_{f,\text{ab}}}{F'} = \frac{2.85}{90,600} \times 3600 = \boxed{0.113\ \text{kg/h} \cdot \text{N}}$$

which corresponds to an increase of 25.1% relatively to the TSFC without an afterburner.

→ Part C

1. True. Since the diffuser is adiabatic, the total temperature at the diffuser exit is such that $T_{t2} = T_{ta}$, with

$$\frac{T_{ta}}{T_a} = 1 + \frac{\gamma - 1}{2}M_a^2 = 1 + \frac{1.4 - 1}{2} \times 0.8^2 = 1.13$$

so that

$$T_{t2} = 1.13 \times 288.2 = 326\ \text{K}$$

In a similar manner, we write the ratio of pressures

$$\frac{p_{ta}}{p_a} = \left(\frac{T_{ta}}{T_a}\right)^{\frac{\gamma}{\gamma-1}} = 1.13^{1.4/(1.4-1)} = 1.53$$

The pressure recovery factor for the diffuser is 0.93. Accordingly,

$$\frac{p_{t2}}{p_a} = \frac{p_{t2}}{p_{ta}}\frac{p_{ta}}{p_a} = \pi_d \frac{p_{ta}}{p_a} = 0.93 \times 1.53 = 1.42$$

Therefore, the diffuser exit total pressure is

$$p_{t2} = 1.42 \times 101.3 = \boxed{144 \text{ kPa}}$$

This is about 6.5 percent less than the result obtained for the ideal engine, p_{t2} = 154 kPa.

2. False. The total pressure exiting the compressor is found as

$$p_{t3} = \pi_c p_{t2} = 15 \times 144 = 2160 \text{ kPa}$$

Ideally, the temperature ratio would be

$$\tau_c' = \pi_c^{(\gamma-1)/\gamma} = 15^{(1.4-1)/1.4} = 2.17$$

and the total temperature at the exit of the compressor would be

$$T_{t3}' = \tau_c' T_{t2} = 2.17 \times 326 = 707 \text{ K}$$

but the compressor efficiency is η_c = 0.85 and hence the compressor total exit temperature becomes

$$\eta_c = \frac{T_{t3}' - T_{t2}}{T_{t3} - T_{t2}} = \frac{707 - 326}{T_{t3} - 326} = 0.85$$

$$\therefore \boxed{T_{t3} = 774 \text{ K}}$$

This corresponds to an increase of 9.8 percent relatively to the result for the ideal engine, T_{t3} = 705 K.

3. True. Writing an energy balance for the shaft, we can determine the turbine exit total temperature,

$$\dot{m}c_p\left(T_{t3} - T_{t2}\right) = \eta_m \dot{m}c_p\left(T_{t4} - T_{t5}\right)$$

$$\therefore \left(T_{t3} - T_{t2}\right) = \eta_m\left(T_{t4} - T_{t5}\right)$$

$$\therefore \left(774 - 326\right) = 0.99 \times \left(1430 - T_{t5}\right)$$

$$\therefore T_{t5} = 977 \text{ K}$$

However, the turbine efficiency is defined as

$$\eta_t = \frac{T_{t4} - T_{t5}}{T_{t4} - T_{t5}'}$$

Given η_t = 0.80, we can determine the ideal total temperature for the turbine exit,

$$0.80 = \frac{1430 - 977}{1430 - T_{t5}'} \rightarrow T_{t5}' = 864 \text{ K}$$

Ideally,

$$\tau_t' = \frac{T_{t5}'}{T_{t4}} = \frac{864}{1430} = 0.604$$

From the definition of turbine efficiency and the total pressure ratio, we find

$$\tau_t' = \pi_t^{(\gamma-1)/\gamma} \rightarrow \pi_t = \tau_f^{\gamma/(\gamma-1)}$$

$$\therefore \pi_t = 0.604^{1.4/(1.4-1)} = 0.171$$

The turbine exit total pressure is then

$$p_{t5} = \pi_t p_{t4}$$

where the burner exit total pressure is

$$p_{t4} = \pi_b p_{t3} = 0.88 \times 2160 = 1900 \text{ kPa}$$

so that

$$p_{t5} = 0.171 \times 1900 = \boxed{325 \text{ kPa}}$$

This corresponds to a decrease of 58.5 percent relatively to the result for the ideal engine, p_{t5} = 784 kPa.

4. False. Since no mixer is present, we have $p_{t5.5} = p_5$ = 325 kPa and $T_{5.5} = T_5$ = 977 K. Further, because no afterburner is present, the inlet total pressure for the primary nozzle is $p_{t6} = p_{t5.5}$ = 325 kPa and the total temperature is $T_{t6} = T_{t5.5}$ = 977 K. Since the engine has a fixed converging nozzle, we must first check if the nozzle is choked. For a choked nozzle, the nozzle exit pressure is

$$p_8^* = p_{t6}\left[1+\frac{1-\gamma}{\eta_n(1+\gamma)}\right]^{\frac{\gamma}{\gamma-1}} = 325 \times \left[1+\frac{1-\gamma}{\eta_n(1+\gamma)}\right]^{\frac{\gamma}{\gamma-1}} = 165 \text{ kPa}$$

Since $p_8^* > p_a$ = 101.3 kPa, the nozzle is choked and the exit Mach number is identically unit. Accordingly, the exit pressure is $p_8 = p_8^*$ = 165 kPa. In view of the result M_8 = 1, the exit total temperature is found as

$$T_8 = \frac{2T_{t6}}{1+\gamma} = \frac{2 \times 977}{1+1.4} = 814 \text{ K}$$

The nozzle exit velocity is then

$$U_8 = \sqrt{2c_p\left(T_{t6}-T_8\right)} = \sqrt{2 \times 1000 \times (977-814)} = 571 \text{ m/s}$$

As a side note, we could compute the speed of sound at the exit,

$$a_8 = \sqrt{\gamma R T_8} = \sqrt{1.4 \times 287 \times 814} = 571 \text{ m/s}$$

with the result that $M_8 = U_8/a_8$ = 1.0, as expected. Since T_8 and p_8 are both known, the air density at the exit can be determined with the ideal gas law,

$$\rho_8 = \frac{p_8}{RT_8} = \frac{165{,}000}{287 \times 814} = 0.706 \text{ kg/m}^3$$

The nozzle exit area is determined next,

$$A_8 = \frac{\dot{m}_8}{\rho_8 U_8} = \frac{75}{0.706 \times 571} = 0.186 \text{ m}^2$$

which corresponds to a nozzle exit diameter of 0.487 m. This is 20.5 percent greater than the nozzle exit diameter for the ideal engine, 0.404 m.

5. False. The total thrust is determined next,

$$F = \dot{m}_8\left(U_8 - U_a\right) + A_8\left(p_8 - p_a\right)$$

$$\therefore F = 75 \times (571-272) + 0.186 \times (165{,}000 - 101{,}300) = \boxed{34.3 \text{ kN}}$$

which corresponds to a decrease of about 34.3 percent relatively to the thrust of the ideal engine, F = 52.2 kN.

6. True. Computing the TSFC requires the fuel mass flow, $\dot{m}_f$, which can be obtained by applying an energy balance at the burner,

$$\Delta H \eta_b \dot{m}_f = \dot{m} c_p\left(T_{t4} - T_{t3}\right) + \dot{m}_f c_p T_{t4}$$

$$\therefore 41{,}400 \times 0.88 \times \dot{m}_f = 75 \times 1.0 \times (1430 - 774) + \dot{m}_f \times 1.0 \times 1430$$

$$\therefore \dot{m}_f = 1.41 \text{ kg/s}$$

so that

$$TSFC = \frac{\dot{m}_f}{F} = \frac{1.41}{34{,}300} \times 3600 = \boxed{0.148 \text{ kg/h} \cdot \text{N}}$$

This corresponds to an increase of 63.9 percent relatively to the TSFC obtained for the ideal engine, $TSFC$ = 0.0903 kg/h·N.

P.10 ■ Solution

→ Part A

1. True. For sea level, we have p_a = 101.3 kPa and T_a = 288.2 K. Applying the ideal gas law yields ρ_a = 1.22 kg/m³. The inlet speed of sound follows as

$$a_a = \sqrt{\gamma R T_a} = \sqrt{1.4 \times 287 \times 288.2} = 340 \text{ m/s}$$

which corresponds to a jet velocity of U_a = 0.6 × 340 = 204 m/s. The diffuser inlet area is then

$$A_{\text{in}} = \frac{\dot{m}(1+\alpha)}{\rho_a U_a} = \frac{65 \times (1+1.25)}{1.22 \times 204} = 0.588 \text{ m}^2$$

which implies an inlet diameter of 0.865 m.

2. True. The fan exit is labeled station 7. Because the fan is isentropic, the fan exit total temperature is found as

$$\frac{T_{t7}}{T_{t2}} = \left(\pi_f\right)^{(\gamma-1)/\gamma} \rightarrow T_{t7} = \left(\pi_f\right)^{(\gamma-1)/\gamma} T_{t2}$$

$$\therefore T_{t7} = T_{ta}\left(\pi_f\right)^{(\gamma-1)/\gamma}$$

where we have made the substitution T_{t2} = T_{ta} because the total temperature at the exit of the diffuser equals the total ambient temperature, which in turn is calculated to be

$$T_{ta} = T_a\left(1 + \frac{\gamma-1}{2} M_a^2\right) = 288.2 \times \left(1 + \frac{1.4-1}{2} \times 0.6^2\right) = 309 \text{ K}$$

so that

$$T_{t7} = 309 \times 3^{(1.4-1)/1.4} = \boxed{423 \text{ K}}$$

3. False. The exit of the fan nozzle is labeled station 9. For an ideal fan nozzle, the exit pressure matches the ambient pressure, i.e., p_9 = p_a = 101.3 kPa. Since the nozzle is isentropic, the total pressure remains constant through the nozzle, or p_{t9} = p_{t7}. Pressure p_{t7} is found as

$$\pi_f = \frac{p_{t7}}{p_{t2}} \rightarrow p_{t7} = \pi_f p_{t2}$$

and, since p_{t2} = p_{ta}, we have

$$p_{t2} = p_{ta}$$

$$\therefore p_{ta} = p_a\left(1 + \frac{\gamma-1}{2} M_a^2\right)^{\gamma/(\gamma-1)} = 101.3 \times \left(1 + \frac{1.4-1}{2} \times 0.6^2\right)^{1.4/(1.4-1)} = 129 \text{ kPa}$$

so that

$$p_{t7} = \pi_f p_{t2} = 3.0 \times 129 = 387 \text{ kPa}$$

As stated above, p_{t9} = p_{t7} = 387 kPa. However, we know that

$$p_{t9} = p_9\left(1 + \frac{\gamma-1}{2} M_9^2\right)^{\gamma/(\gamma-1)}$$

Solving for the Mach number and substituting brings to

$$p_{t9} = p_9\left(1+\frac{\gamma-1}{2}M_9^2\right)^{\gamma/(\gamma-1)} \rightarrow M_9 = \sqrt{\left(\frac{2}{\gamma-1}\right)\left[\left(\frac{p_{t9}}{p_9}\right)^{(\gamma-1)/\gamma}-1\right]}$$

$$\therefore M_9 = \sqrt{\left(\frac{2}{1.4-1}\right)\times\left[\left(\frac{387}{101.3}\right)^{(1.4-1)/1.4}-1\right]} = 1.53$$

Observing that $T_{t9} = T_{t7}$ = 423 K, the exit static temperature is determined as

$$T_9 = \frac{T_{t9}}{1+\frac{\gamma-1}{2}M_9^2} = \frac{423}{1+\frac{1.4-1}{2}\times 1.53^2} = 288 \text{ K}$$

The nozzle exit static temperature matches the ambient temperature, as expected. The exit velocity is

$$U_9 = M_9 a_9 = M_9\sqrt{\gamma R T_9} = 1.53\times\sqrt{1.4\times 287\times 288.2} = \boxed{521 \text{ m/s}}$$

4. False. Performing a power balance on the shaft yields

$$\dot{m}c_p\left(T_{t4}-T_{t5}\right) = \dot{m}c_p\left(T_{t3}-T_{t2}\right)+\alpha\dot{m}c_p\left(T_{t7}-T_{t2}\right)$$

$$\therefore\left(T_{t4}-T_{t5}\right) = \left(T_{t3}-T_{t2}\right)+\alpha\left(T_{t7}-T_{t2}\right)$$

Before proceeding, we require the compressor temperature T_{t3}. Since τ_c = $\pi_c^{(\gamma-1)/\gamma}$ = $15^{(1.4-1)/1.4}$ = 2.17, it follows that

$$T_{t3} = T_{t2}\tau_c = 309\times 2.17 = 671 \text{ K}$$

Accordingly,

$$\left(1300-T_{t5}\right) = \left(671-309\right)+1.25\times\left(423-309\right)$$

$$\therefore \boxed{T_{t5} = 796 \text{ K}}$$

5. False. Since the ideal turbine is isentropic, the turbine exit total pressure may be obtained as

$$p_{t5} = p_{t4}\pi_t$$

Observing that $p_{t3} = p_{t4}$, the compressor exit total pressure is found as

$$p_{t3} = \pi_c p_{t2} = 15\times 129 = 1940 \text{ kPa}$$

so that

$$p_{t5} = p_{t4}\pi_t \rightarrow p_{t5} = p_{t4}\tau_t^{\gamma/(\gamma-1)}$$

$$\therefore p_{t5} = 1940\times\left(\frac{796}{1300}\right)^{1.4/(1.4-1)} = \boxed{349 \text{ kPa}}$$

6. True. Since the primary nozzle is isentropic, we can write

$$p_{t8} = p_8\left(1+\frac{\gamma-1}{2}M_8^2\right)^{\gamma/(\gamma-1)}$$

which can be solved for M_8 to yield

$$M_8 = \sqrt{\left(\frac{2}{\gamma-1}\right)\left[\left(\frac{p_{t8}}{p_8}\right)^{(\gamma-1)/\gamma}-1\right]} = \sqrt{\left(\frac{2}{1.4-1}\right)\left[\left(\frac{349}{101.3}\right)^{(1.4-1)/1.4}-1\right]} = 1.46$$

The primary nozzle exit static temperature follows as

$$T_8 = \frac{T_{t8}}{1+\frac{\gamma-1}{2}M_8^2} = \frac{796}{1+\frac{1.4-1}{2}\times 1.46^2} = 558 \text{ K}$$

The exit gas velocity is then

$$U_8 = M_8 a_8 = M_8\sqrt{\gamma R T_8} = 1.46\times\sqrt{1.4\times 287\times 558} = \boxed{691\text{ m/s}}$$

7. False. We are now in position to establish the thrust, which is given by

$$F = \dot{m}\left(U_8 - U_a\right) + \dot{m}_s\left(U_9 - U_a\right) = \dot{m}\left(U_8 - U_a\right) + \alpha\dot{m}\left(U_9 - U_a\right)$$

$$\therefore F = 61\times\left(691-204\right) + 1.25\times 61\times\left(521-204\right) = \boxed{53.9\text{ kN}}$$

8. True. Before computing the TSFC, we require the fuel mass flow, which is calculated as

$$\dot{m}_f = \frac{\dot{m}c_p\left(T_{t4}-T_{t3}\right)}{\Delta H} = \frac{61\times 1.0\times\left(1300-671\right)}{42{,}000} = 0.914\text{ kg/s}$$

so that

$$TSFC = \frac{\dot{m}_f}{F} = \frac{0.914}{53{,}900}\times 3600 = \boxed{0.061\text{ kg/h}\cdot\text{N}}$$

→ Part B

When comparing this problem with the preceding problem, it can be seen that the only difference is the addition of the mixed fan. All of the given conditions are identical. The parameters that remain unchanged are listed below.

p_a = 101.3 kPa	T_a = 288.2 K
U_a = 204 m/s	$T_{ta} = T_{t2}$ = 309 K
a_a = 340 m/s	$p_{t3} = p_{t4}$ = 1940 kPa
$p_{ta} = p_{t2}$ = 129 kPa	τ_c = 2.17
T_{t3} = 671 K	

Before anything else, we must establish the fan pressure ratio π_f, which in turn requires the total temperature ratio τ_f,

$$\tau_f = \frac{\tau_c + \left(\dfrac{T_{ta}}{T_a}\right)\left(\dfrac{T_a}{T_{t4}}\right)\tau_c\left(1+\alpha-\tau_c\right)}{1+\left(\dfrac{T_{ta}}{T_a}\right)\left(\dfrac{T_a}{T_{t4}}\right)\tau_c\alpha}$$

$$\therefore \tau_f = \frac{2.17+\left(\dfrac{309}{288.2}\right)\left(\dfrac{288.2}{1300}\right)\times 2.17\times\left(1+1.25-2.17\right)}{1+\left(\dfrac{309}{288.2}\right)\left(\dfrac{288.2}{1300}\right)\times 2.17\times 1.25} = 1.34$$

so that, for an ideal (isentropic) fan, we can write $\pi_f = 1.34^{1.4/(1.4-1)} = 2.79$, which is quite close to the exhausted fan case. The total temperature exiting the fan is

$$T_{t7} = \tau_f\times T_{t2} = 1.34\times 309 = 414\text{ K} = T_{t7.5}$$

Also, the total pressure exiting the fan is

$$p_{t7} = \pi_f\times p_{t2} = 2.79\times 129 = 360\text{ kPa} = p_{t7.5}$$

Because the duct is ideal (adiabatic), the total temperature is constant, that is, $T_{t7.5} = T_{t7}$ = 414 K. Furthermore, since the duct is ideal (isentropic), the total pressure is also constant, i.e., $p_{t7.5} = p_{t7}$ = 360 kPa. Next, a power balance is applied to the turbine, giving

$$\dot{m}c_p\left(T_{t4}-T_{t5}\right) = \dot{m}c_p\left(T_{t3}-T_{t2}\right) + \alpha\dot{m}c_p\left(T_{t7}-T_{t2}\right)$$

$$\therefore\left(T_{t4}-T_{t5}\right) = \left(T_{t3}-T_{t2}\right) + \alpha\left(T_{t7}-T_{t2}\right)$$

$$\therefore\left(1300-T_{t5}\right) = \left(671-309\right) + 1.25\times\left(414-309\right)$$

$$\therefore T_{t5} = 807\text{ K}$$

For an isentropic turbine, we can write

$$p_{t5} = p_{t4}\left(\tau_t\right)^{\gamma/(\gamma-1)} = p_{t4}\left(\frac{T_{t5}}{T_{t4}}\right)^{\gamma/(\gamma-1)} = 1940\times\left(\frac{807}{1300}\right)^{1.4/(1.4-1)} = 366 \text{ kPa} \approx 360 \text{ kPa}$$

that is, the total pressure at the turbine exit should match the duct exit total pressure, $p_{t7.5}$. Next, let us consider the mixer. For this component, the exit total temperature is found with a balance on the energy equation on the assumption that the exit temperature is uniform. The pertaining equation is

$$T_{t5.5} = \frac{\alpha T_{t7.5} + T_{t5}}{\alpha + 1} = \frac{1.25\times 414 + 807}{1.25+1} = 589 \text{ K}$$

The total pressure remains constant in an ideal mixer, and hence $p_{t5.5} = p_{t5} =$ 360 kPa. Next, consider the nozzle. For an ideal nozzle, the exit total temperature is the same as the inlet value, $T_{t8} = T_{t5.5} = 589$ K. The total pressure can be determined with the relation

$$p_{t8} = p_8\left(1+\frac{\gamma-1}{2}M_8^2\right)^{\gamma/(\gamma-1)}$$

which can be solved for the Mach number to give

$$M_8 = \sqrt{\left(\frac{2}{\gamma-1}\right)\left[\left(\frac{p_{t8}}{p_8}\right)^{(\gamma-1)/\gamma} - 1\right]} = \sqrt{\left(\frac{2}{1.4-1}\right)\left[\left(\frac{360}{101.3}\right)^{(1.4-1)/1.4} - 1\right]} = 1.48$$

Accordingly, the exit temperature is

$$T_8 = \frac{T_{t8}}{1+\frac{\gamma-1}{2}M_8^2} = \frac{589}{1+\frac{1.4-1}{2}\times 1.48^2} = 410 \text{ K}$$

and the exit gas velocity is

$$U_8 = M_8 a_8 = M_8\sqrt{\gamma R T_8} = 1.48\times\sqrt{1.4\times 287\times 410} = 601 \text{ m/s}$$

Finally, since the nozzle exit pressure matches the ambient pressure, the thrust is given by

$$F = (1+\alpha)\dot{m}\left(U_8 - U_a\right) = (1+1.25)\times 61\times(601-204) = \boxed{54.5 \text{ kN}}$$

which corresponds to an increase of one percent relatively to the configuration with no mixed fan. Lastly, we compute the TSFC. The fuel mass flow continues to be $\dot{m} =$ 0.914 kg/s. Accordingly,

$$TSFC = \frac{\dot{m}_f}{F} = \frac{0.914}{54{,}500}\times 3600 = \boxed{0.0604 \text{ kg/h}\cdot\text{N}}$$

which implies a decrease of 0.98 percent relatively to the configuration with no mixed fan.

P.11 ■ Solution

1. False. Recalling that

$$\frac{P_p}{\left(\frac{1}{2}\rho_a V_0^2\right)V_0 A_p} = \frac{1}{2}\left(\frac{V_1}{V_0}\right)^3 + \frac{3}{2}\left(\frac{V_1}{V_0}\right)^2 - \frac{1}{2}\left(\frac{V_1}{V_0}\right) - \frac{3}{2}$$

we have, on the left-hand side,

$$\frac{1{,}200{,}000}{\left(\frac{1}{2}\times 0.61\times 100^2\right)\times 100\times\underbrace{\left(\frac{\pi\times 2.75^2}{4}\right)}_{=5.94\text{ m}^2}} = 0.662$$

which brings to the cubic equation

$$\frac{1}{2}\left(\frac{V_1}{V_0}\right)^3 + \frac{3}{2}\left(\frac{V_1}{V_0}\right)^2 - \frac{1}{2}\left(\frac{V_1}{V_0}\right) - \frac{3}{2} = 0.662 \rightarrow 0.5\left(\frac{V_1}{V_0}\right)^3 + 1.5\left(\frac{V_1}{V_0}\right)^2 - 0.5\left(\frac{V_1}{V_0}\right) - 2.16 = 0$$

The only valid solution to this equation is V_1/V_0 = 1.15, which makes the far downstream speed V_1 = 115 m/s. From momentum theory, the air speed at the propeller is the average of forward speed and the far downstream speed; that is,

$$V_p = \frac{V_0 + V_1}{2} = \frac{100+115}{2} = \boxed{108 \text{ m/s}}$$

2. True. The propeller thrust is calculated as

$$F_p = \rho_0 A_p V_p \left(V_1 - V_0\right) = 0.61 \times 5.94 \times 108 \times \left(115 - 100\right) = \boxed{5.87 \text{ kN}}$$

3. False. The propeller efficiency is given by

$$\eta_p = \frac{F_p V_0}{P_p} = \frac{5870 \times 100}{1.1 \times 10^6} = \boxed{0.534}$$

4. True. The propeller torque can be obtained from the definition of shaft power, namely,

$$P_p = \tau_p \omega \rightarrow \tau_p = \frac{P_p}{\omega}$$

$$\therefore \tau_p = \frac{1.1 \times 10^6}{940 \times \dfrac{2\pi}{60}} = \boxed{11.2 \text{ kN} \cdot \text{m}}$$

P.12 ■ Solution

1. False. At sea level, we have T_a = 288.2 K and p_a = 101.3 kPa. The freestream velocity is

$$U_a = M_a a_a = M_a \sqrt{\gamma R T_a}$$

$$\therefore U_a = 0.5 \times \sqrt{1.4 \times 287 \times 288.2} = 170 \text{ m/s}$$

and the inlet total temperature is

$$T_{ta} = T_a \left(1 + \frac{\gamma - 1}{2} M_a^2\right) = 288.2 \times \left(1 + \frac{1.4 - 1}{2} \times 0.5^2\right) = 303 \text{ K}$$

which is also equal to the compressor inlet total temperature T_{t2} because the process is adiabatic. The total pressure at the inlet is, in turn,

$$p_{ta} = p_a \left(1 + \frac{\gamma - 1}{2} M_a^2\right)^{\gamma/(\gamma-1)} = p_a \left(\frac{T_{ta}}{T_a}\right)^{\gamma/(\gamma-1)}$$

$$\therefore p_{ta} = 101.3 \times \left(\frac{303}{288.2}\right)^{1.4/(1.4-1)} = 121 \text{ kPa}$$

Given the compressor total pressure ratio π_c, the total pressure at the compressor exit is found as

$$p_{t3} = \pi_c p_{t2}$$

However, the process is isentropic for an ideal diffuser, so that $p_{t2} = p_{ta}$ = 121 kPa. Thus,

$$p_{t3} = 5.8 \times 121 = 702 \text{ kPa}$$

For an isentropic compressor, we can write

$$\tau_c = \pi_c^{(\gamma-1)/\gamma} = 5.8^{(1.4-1)/1.4} = 1.65$$

whence we can compute the total temperature at the compressor exit as

$$T_{t3} = \tau_c \times T_{t2} = 1.65 \times 303 = 500 \text{ K}$$

The fuel flow rate is given by

$$\dot{m}_f = \frac{\dot{m} c_p \left(T_{t4} - T_{t3}\right)}{\Delta H} = \frac{12 \times 1.0 \times \left(1260 - 500\right)}{42{,}500} = 0.215 \text{ kg/s}$$

and the exit total pressure is the same as that for the inlet of an ideal burner, that is, $p_{t4} = p_{t3}$ = 702 kPa. Next, observing that p_{t8} = 101.3 kPa and M_8 = 0.9, the nozzle exit total pressure is calculated as

$$p_{t8} = p_8\left(1+\frac{\gamma-1}{2}M_8^2\right)^{\gamma/(\gamma-1)} = 101.3\times\left(1+\frac{1.4-1}{2}\times 0.5^2\right)^{1.4/(1.4-1)} = 120 \text{ kPa}$$

For an ideal nozzle, the total pressure is constant (isentropic flow), and thus $p_{t5} = p_{t8}$ = 120 kPa. Accordingly, the total pressure ratio for the turbine is

$$\pi_t = \frac{p_{t5}}{p_{t4}} = \frac{120}{702} = 0.171$$

and the total temperature ratio follows as

$$\tau_t = \pi_t^{(\gamma-1)/\gamma} = 0.171^{(1.4-1)/1.4} = 0.604$$

so that the turbine total temperature becomes

$$T_{t5} = \tau_t T_{t4} = 0.604\times 1260 = 761 \text{ K}$$

At this point, we are ready to include the propeller in our discussion. The nozzle temperature parameter is

$$\tau_5 = \frac{T_{ta}}{T_a}\tau_c\tau_t = \frac{303}{288.2}\times 1.65\times 0.604 = 1.05$$

The work coefficient is given by

$$C_{W_e} = \frac{T_{t4}}{T_a}\left(1-\frac{\tau_5}{\frac{T_{ta}}{T_a}\tau_c}\right) - \left(\frac{T_{ta}}{T_a}\right)(\tau_c-1) + (\gamma-1)M_a^2\left[\sqrt{\frac{\left(\frac{T_{t4}}{T_a}\right)\left(\frac{T_a}{T_{ta}}\right)\left(\frac{\tau_5-1}{\tau_c}\right)}{\left(\frac{T_{ta}}{T_a}\right)-1}}-1\right]$$

Inserting our data yields

$$C_{W_e} = \frac{1260}{288.2}\left[1-\frac{1.05}{\left(\frac{303}{288.2}\right)\times 1.65}\right]$$

$$-\left(\frac{303}{288.2}\right)(1.65-1) + (1.4-1)\times 0.5^2\left[\sqrt{\frac{\left(\frac{1260}{288.2}\right)\left(\frac{288.2}{303}\right)\left(\frac{1.05-1}{1.65}\right)}{\left(\frac{303}{288.2}\right)-1}}-1\right]$$

$$\therefore C_{W_e} = \underbrace{1.04}_{\text{propeller}} + \underbrace{0.06}_{\text{jet}} = \boxed{1.10}$$

Note that about 95 percent of the thrust work or power comes from the propeller.

2. True. The total thrust power is given by

$$P = C_{W_e}\dot{m}h_a = C_{W_e}\dot{m}c_pT_a = 1.10\times 12\times 1000\times 288.2 = \boxed{3.8 \text{ MW}}$$

3. False. The specific fuel consumption is calculated as

$$SFC = \frac{\dot{m}_f}{P} = \frac{0.215}{3.8}\times 3600 = \boxed{204 \text{ kg/MW}\cdot\text{h}}$$

P.13 ■ Solution

1. False. To begin, we compute properties upstream of the first rotor.

$$T_1 = \frac{T_{t1}}{1+\frac{\gamma-1}{2}M_1^2} = \frac{288.2}{1+\frac{1.4-1}{2}\times 0.6^2} = 269 \text{ K}$$

$$a_1 = \sqrt{\gamma R T_1} = \sqrt{1.4\times 287\times 269} = 329 \text{ m/s}$$

$$V_1 = M_1 a_1 = 0.6 \times 329 = 197 \text{ m/s}$$

$$u_1 = U_1 \cos\alpha_1 = 197 \times \cos 40^\circ = 151 \text{ m/s}$$

$$v_1 = U_1 \sin\alpha_1 = 197 \times \sin 40^\circ = 127 \text{ m/s}$$

$$p_1 = \frac{p_{t1}}{\left(1 + \frac{\gamma - 1}{2} M_1^2\right)^{\gamma/(\gamma-1)}} = \frac{101.3}{\left(1 + \frac{1.4-1}{2} \times 0.6^2\right)^{1.4/(1.4-1)}} = 79.4 \text{ kPa}$$

Referring to Figure 7, the mass flow parameter $MFP(M_1) \times \sqrt{R}$ = 0.58 and, accordingly,

$$MFP(M_1) = \frac{MFP(M_1)\sqrt{R}}{\sqrt{R}} = \frac{0.58}{16.9} = 0.0343$$

The area of the flow annulus follows as

$$A_1 = \frac{\dot{m}\sqrt{T_{t1}}}{p_{t1}\cos\alpha_1 \times MFP(M_1)} = \frac{20 \times \sqrt{288.2}}{101{,}300 \times \cos 40^\circ \times 0.0343} = \boxed{0.128 \text{ m}^2}$$

2. False. Given the rotor velocity ωr = 1000 × 0.30 = 300 m/s, we have

$$v_{1R} = \omega r - v_1 = 300 - 127 = 173 \text{ m/s}$$

from which we find the cascade flow angle

$$\beta_1 = \tan^{-1}\frac{v_{1R}}{u_1} = \tan^{-1}\frac{173}{151} = 1.15 \rightarrow \beta_1 = 49^\circ$$

The relative velocity at station 1 follows as

$$V_{1R} = \sqrt{u_1^2 + v_{1R}^2} = \sqrt{151^2 + 173^2} = 230 \text{ m/s}$$

and the Mach number is

$$M_{1R} = \frac{V_{1R}}{a_1} = \frac{230}{329} = 0.699$$

The relative total temperature is then

$$T_{t1R} = T_1\left(1 + \frac{\gamma-1}{2} M_{1R}^2\right) = 269 \times \left(1 + \frac{1.4-1}{2} \times 0.6^2\right) = \boxed{288 \text{ K}}$$

3. False. The relative total pressure at station 1 is given by

$$p_{t1R} = p_1\left(\frac{T_{t1R}}{T_1}\right)^{\gamma/(\gamma-1)} = 79.4 \times \left(\frac{288}{269}\right)^{1.4/(1.4-1)} = 101 \text{ kPa}$$

In turn, the relative total pressure at station 2 is calculated as

$$p_{t2R} = p_{t1R}\left(\frac{p_{t2R}}{p_{t1R}}\right) = p_{t1R}\left[1 - \phi_{\text{cr}} \frac{\gamma M_{1R}^2/2}{\left(1 + \frac{\gamma-1}{2} M_{1R}^2\right)^{\gamma/(\gamma-1)}}\right]$$

$$\therefore p_{t2R} = 101 \times \left[1 - 0.08 \times \frac{1.4 \times 0.699^2/2}{\left(1 + \frac{1.4-1}{2} \times 0.699^2\right)^{1.4/(1.4-1)}}\right] = \boxed{99.0 \text{ kPa}}$$

4. True. The relative total temperature is the same for stations 1 and 2, that is, $T_{t2R} = T_{t1R}$ = 288 K. The total temperature at the station in question is

$$T_{t2} = T_{t1} + \Delta T = 288.2 + 22 = 310 \text{ K}$$

The cascade flow angle β_2 is determined next,

$$\tan\beta_2 = \frac{u_1}{u_2}\left[\tan\beta_1 - \frac{c_p}{\omega r u_1}\left(T_{t2} - T_{t1}\right)\right]$$

$$\therefore \tan\beta_2 = \frac{1}{1.2}\times\left[\tan 49^\circ - \frac{1000}{300\times 151}\times\left(310 - 288.2\right)\right] = 0.558$$

$$\therefore \boxed{\beta_2 = 29.2^\circ}$$

5. False. Velocity component u_2 is calculated as

$$u_2 = \frac{u_2}{u_1}u_1 = 1.20\times 151 = 181 \text{ m/s}$$

so that

$$v_{2R} = u_2 \tan\beta_2 = 181\times\tan 29.2^\circ = 101 \text{ m/s}$$

and

$$V_{2R} = \sqrt{u_2^2 + v_{2R}^2} = \sqrt{181^2 + 101^2} = \boxed{207 \text{ m/s}}$$

6. True. Velocity component v_2 is computed as

$$v_2 = \omega r - v_{2R} = 300 - 101 = 199 \text{ m/s}$$

and can be used to establish flow angle α_2, that is,

$$\alpha_2 = \tan^{-1}\frac{v_2}{u_2} = \tan^{-1}\frac{199}{181} \rightarrow \alpha_2 = 47.7^\circ$$

The velocity at station 2 is given by

$$V_2 = \sqrt{u_2^2 + v_2^2} = \sqrt{181^2 + 199^2} = 269 \text{ m/s}$$

The temperature at station 2 follows as

$$T_2 = T_{t2} - \frac{V_2^2}{2c_p} = 310 - \frac{269^2}{2\times 1000} = 274 \text{ K}$$

and the pressure therein is, accordingly,

$$p_2 = p_{t2R}\left(\frac{T_2}{T_{t2R}}\right)^{\gamma/(\gamma-1)} = 99.0\times\left(\frac{274}{288}\right)^{1.4/(1.4-1)} = \boxed{83.2 \text{ kPa}}$$

7. True. The speed of sound at station 2 is

$$a_2 = \sqrt{\gamma R T_2} = \sqrt{1.4\times 287\times 274} = 332 \text{ m/s}$$

and the Mach number is

$$M_2 = \frac{V_2}{a_2} = \frac{269}{332} = 0.810$$

while the relative Mach number is

$$M_{2R} = \frac{V_{2R}}{a_2} = \frac{207}{332} = 0.623$$

The total pressure at station 2 is given by

$$p_{t2} = p_2\left(\frac{T_{t2}}{T_2}\right)^{\gamma/(\gamma-1)} = 83.2\times\left(\frac{310}{274}\right)^{1.4/(1.4-1)} = 128 \text{ kPa}$$

Referring to Figure 7, the value of $MFP\times\sqrt{R}$ that corresponds to Mach number M_2 is approximately 0.65. Accordingly,

$$MFP\left(M_2\right) = \frac{MFP\left(M_2\right)\sqrt{R}}{\sqrt{R}} = \frac{0.65}{\sqrt{287}} = 0.0384$$

The area of the flow annulus follows as

$$A_2 = \frac{\dot{m}\sqrt{T_{t2}}}{p_{t2}\cos\alpha_2 \times MFP(M_2)} = \frac{20\times\sqrt{310}}{128{,}000\times\cos 47.7^{\circ}\times 0.0384} = \boxed{0.106\ \text{m}^2}$$

8. False. The total temperature at station 3 is the same as that at station 2, and equals

$$T_{t3} = T_{t2} = T_{t1} + \Delta T_t = 288.2 + 22 = 310\ \text{K}$$

The temperature at station 3 is

$$T_3 = \frac{T_{t3}}{1+\dfrac{\gamma-1}{2}M_3^2} = \frac{310}{1+\dfrac{1.4-1}{2}\times 0.6^2} = 289\ \text{K}$$

The total pressure at station 3 is

$$p_{t3} = p_{t2}\left(\frac{p_{t3}}{p_{t2}}\right) = p_{t2}\left[1-\phi_{\text{cs}}\frac{\gamma M_2^2/2}{\left(1+\dfrac{\gamma-1}{2}M_2^2\right)^{\gamma/(\gamma-1)}}\right]$$

$$\therefore\ p_{t3} = 128\times\left[1-\frac{0.03\times 1.4\times 0.810^2/2}{\left(1+\dfrac{1.4-1}{2}\times 0.810^2\right)^{1.4/(1.4-1)}}\right] = 127\ \text{kPa}$$

The pressure at the station in question, in turn, is

$$p_3 = p_{t3}\left(\frac{T_3}{T_{t3}}\right)^{\gamma/(\gamma-1)} = 127\times\left(\frac{289}{310}\right)^{1.4/(1.4-1)} = 99.4\ \text{kPa}$$

Next, we compute some flow properties for station 3,

$$a_3 = \sqrt{\gamma R T_3} = \sqrt{1.4\times 287\times 289} = 341\ \text{m/s}$$

$$V_3 = M_3 a_3 = 0.6\times 341 = 205\ \text{m/s}$$

$$u_3 = V_3\cos\alpha_3 = 205\times\cos 40^{\circ} = 157\ \text{m/s}$$

$$v_3 = V_3\sin\alpha_3 = 205\times\sin 40^{\circ} = 132\ \text{m/s}$$

The mass flow parameter $MFP(M_3) = MFP(M_1) = 0.0343$, so that

$$A_3 = \frac{\dot{m}\sqrt{T_{t3}}}{p_{t3}\cos\alpha_3 \times MFP(M_3)} = \frac{20\times\sqrt{310}}{127{,}000\times\cos 40^{\circ}\times 0.0343} = \boxed{0.106\ \text{m}^2}$$

9. True. The degree of reaction for a calorically perfect gas is given by

$${}^{o}R_c = \frac{T_2-T_1}{T_3-T_1} = \frac{274-269}{289-269} = \boxed{0.25}$$

10. True. The diffusion factor for the rotor is calculated as

$$D_r = 1-\frac{V_{2R}}{V_{1R}} + \frac{|v_{1R}-v_{2R}|}{2\sigma V_{1R}} = 1-\frac{207}{230}+\frac{|173-101|}{2\times 1.0\times 230} = \boxed{0.257}$$

In sequence, the diffusion factor for the stator is computed as

$$D_s = 1-\frac{V_3}{V_2} + \frac{|v_2-v_3|}{2\sigma V_2} = 1-\frac{205}{269}+\frac{|199-132|}{2\times 1.0\times 269} = \boxed{0.362}$$

11. False. The stage efficiency is obtained with the relation

$$\eta_s = \frac{(p_{t3}/p_{t1})^{(\gamma-1)/\gamma}-1}{(T_{t3}/T_{t1})-1} = \frac{(127/101.3)^{(1.4-1)/1.4}-1}{(310/288.2)-1} = \boxed{0.882}$$

12. True. The polytropic efficiency is given by

$$e_c = \frac{(\gamma-1)}{\gamma}\frac{\ln(p_{t3}/p_{t1})}{\ln(T_{t3}/T_{t1})} = \frac{1.4-1}{1.4}\times\frac{\ln(127/101.3)}{\ln(310/288.2)} = \boxed{0.886}$$

13. False. The stage loading coefficient is given by

$$\psi = \frac{c_p \Delta T_t}{(\omega r)^2} = \frac{1000\times 22}{300^2} = \boxed{0.244}$$

14. False. The flow coefficient is given by

$$\Phi = \frac{u_1}{\omega r} = \frac{151}{300} = \boxed{0.503}$$

The results are summarized in the following table. The given data are highlighted in orange.

	1	1R	2R	2	3
Total Temperature (K)	288.2	288	288	310	310
Temperature (K)	269	269	274	274	289
Total Pressure (kPa)	101.3	101	99	128	127
Pressure (kPa)	79.4	79.4	83.2	83.2	99.4
Mach Number	0.6	0.699	0.623	0.81	0.6
Flow Velocity (m/s)	197	230	207	269	205
Velocity Component *u* (m/s)	151	151	181	181	157
Velocity Component *v* (m/s)	127	173	101	199	132
Flow Angle α (deg)	40	-	-	47.7	40

P.14 ■ Solution

1. True. The rotor velocity is given by

$$U_t^2 = \frac{c_p T_{t1}}{\varepsilon}\left(\pi_c^{(\gamma-1)/\gamma e_c} - 1\right) = \frac{1000\times 288.2}{0.9}\times\left(4.0^{(1.4-1)/1.4\times 0.86} - 1\right) = 187{,}000$$

$$\therefore U_t = 432 \text{ m/s}$$

The rotational frequency of the rotor follows as

$$N = \frac{60U_t}{\pi d_2} = \frac{60\times 432}{\pi\times 0.5} = \boxed{16{,}500 \text{ rpm}}$$

2. False. The mass flow parameter for station 1 is

$$MFP(M_1) = \frac{\dot{m}\sqrt{T_{t1}}}{p_{t1}A_1} = \frac{9\times\sqrt{288.2}}{101{,}300\times\left[\pi\times\left(0.175^2 - 0.10^2\right)\right]} = 0.0233$$

so that the product $MFP(M_1)\times\sqrt{R} = 0.0233\times\sqrt{287} = 0.395$. Mapping this quantity onto Figure 7, we read a Mach number $M_1 = 0.40$. Accordingly, the rotor inlet velocity is calculated as

$$u_1 = V_1 = \sqrt{2c_p T_{t1}\left(1 - \frac{1}{1+\frac{\gamma-1}{2}M_1^2}\right)} = \sqrt{2\times 1000\times 288.2\times\left(1 - \frac{1}{1+\frac{1.4-1}{2}\times 0.4^2}\right)} = 134 \text{ m/s}$$

Velocity components v_{1Rh} (hub) and v_{1Rt} (tip) are determined as

$$v_{1Rh} = \frac{d_{1h}}{d_2}U_t = \frac{20}{50}\times 432 = 173 \text{ m/s}$$

$$v_{1Rt} = \frac{d_{1t}}{d_2}U_t = \frac{35}{50}\times 432 = 302 \text{ m/s}$$

The relative flow angle at the hub is then

$$\beta_{1h} = \tan^{-1}\frac{v_{1Rh}}{u_1} = \tan^{-1}\frac{173}{134} \rightarrow \boxed{\beta_{1h} = 52.2^\circ}$$

In a similar manner, we establish the relative flow angle at the tip as

$$\beta_{1t} = \tan^{-1}\frac{v_{1Rt}}{u_1} = \tan^{-1}\frac{302}{134} \rightarrow \beta_{1t} = 66.1^{\circ}$$

3. True. The total temperature at station 3 is given by

$$T_{t3} = T_{t2} = T_{t1} + \frac{\varepsilon U_t^2}{c_p} = 288.2 + \frac{0.9 \times 432^2}{1000} = 456\ \text{K}$$

The adiabatic efficiency is then

$$\eta_c = \frac{\left(p_{t3}/p_{t1}\right)^{(\gamma-1)/\gamma} - 1}{\left(T_{t3}/T_{t1}\right) - 1} = \frac{4^{(1.4-1)/1.4} - 1}{\left(456/288.2\right) - 1} = \boxed{0.835}$$

4. False. Velocity component v_2 is computed as

$$v_2 = \varepsilon U_t = 0.9 \times 432 = 389\ \text{m/s}$$

Further, $w_2 = u_1 = 134$ m/s. Accordingly,

$$V_2 = \sqrt{w_2^2 + v_2^2} = \sqrt{134^2 + 389^2} = 411\ \text{m/s}$$

The Mach number at station 2 is then

$$M_2 = \sqrt{\left(\frac{2}{\gamma - 1}\right)\left(\frac{T_{t2}}{T_{t2} - V_2^2/2c_p} - 1\right)} = \sqrt{\left(\frac{2}{1.4 - 1}\right)\left[\frac{456}{456 - 411^2/\left(2 \times 1000\right)} - 1\right]} = \boxed{1.07}$$

We may also determine flow angle α_2,

$$\alpha_2 = \tan^{-1}\frac{w_2}{v_2} = \tan^{-1}\frac{134}{389} \rightarrow \alpha_2 = 19.0^{\circ}$$

5. False. Next,

$$\frac{p_{t3s}}{p_{t1}} = \left(\frac{T_{t3}}{T_{t1}}\right)^{\gamma/(\gamma-1)} = \left(\frac{456}{288.2}\right)^{1.4/(1.4-1)} = 4.98$$

The total pressure p_{t3} is

$$\frac{p_{t3}}{p_{t1}} = \pi_c \rightarrow p_{t3} = 4.0 \times 101.3 = 405\ \text{kPa}$$

whence we can write the ratio

$$\frac{p_{t2}}{p_{t3s}} = \frac{p_{t3}}{p_{t2}} = \sqrt{\frac{p_{t3}/p_{t1}}{p_{t3s}/p_{t1}}} = \sqrt{\frac{4.0}{4.98}} = 0.896$$

Since $p_{t3s} = 4.98 \times p_{t1}$ total pressure at station 2 follows as

$$\frac{p_{t2}}{p_{t3s}} = 0.896 \rightarrow p_{t2} = 0.896 \times p_{t3s}$$

$$\therefore\ p_{t2} = 0.896 \times 4.98 \times 101.3 = \boxed{452\ \text{kPa}}$$

6. True. Referring to Figure 7, we see that $MFP(M_2) \times \sqrt{R}$ is about 0.68, and hence

$$MFP\left(M_2\right) = \frac{MFP\left(M_2\right)\sqrt{R}}{\sqrt{R}} = \frac{0.68}{\sqrt{287}} = 0.0401$$

so that

$$A_2 = \frac{\dot{m}\sqrt{T_{t2}}}{p_{t2} \times MFP\left(M_2\right) \times \cos\alpha_2} = \frac{9 \times \sqrt{456}}{452{,}000 \times 0.0401 \times \cos 19.0^{\circ}} = 0.0112\ \text{m}^2$$

and the width b is, accordingly,

$$b = \frac{A_2}{\pi d_2} = \frac{0.0112}{\pi \times 0.5} = 0.00713\ \text{m} = \boxed{7.13\ \text{mm}}$$

7. False The Mach number at station 3 is calculated as

$$M_3 = \sqrt{\left(\frac{2}{\gamma - 1}\right)\left(\frac{T_{t3}}{T_{t3} - V_3^2/2c_p} - 1\right)} = \sqrt{\left(\frac{2}{1.4-1}\right)\left[\frac{456}{456 - 90^2/(2\times 1000)} - 1\right]} = 0.212$$

which, with reference to Figure 7, corresponds to a product $MFP(M_3) \times \sqrt{R}$ of 0.24. Thus,

$$MFP(M_3) = \frac{MFP(M_3)\sqrt{R}}{\sqrt{R}} = \frac{0.24}{\sqrt{287}} = 0.0142$$

and, lastly,

$$A_3 \cos\alpha_3 = \frac{\dot{m}\sqrt{T_{t3}}}{p_{t3} \times MFP(M_3)} = \frac{9 \times \sqrt{456}}{405{,}000 \times 0.0142} = \boxed{0.0334\ \text{m}^2}$$

Some of the results obtained are summarized in the table below. The given data are highlighted in orange.

	1	**2**	**3**
Total Temperature (K)	288.2	456	456
Total Pressure (kPa)	101.3	452	405
Mach Number	0.4	1.07	0.212
Flow Velocity (m/s)	134	411	90
u/w	134	134	-
Velocity Component v (m/s)	0	389	-
Flow Angle α (deg)	0	19	-

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