Cobwebbing

Cobwebbing is a graphical technique used to determine the *behaviour* of solutions to a DTDS without calculations.

This technique allows us to sketch the graph of the solution (a set of discrete points) directly from the graph of the updating function.

Cobwebbing

Algorithm:

1. Graph the updating function and the diagonal.

2. Plot the initial value m_0 on the horizontal axis. From this point, move vertically to the updating function to obtain the next value of the measurement. The coordinates of this point are (m_0,m_1).

3. Move horizontally to the point (m_1,m_1) on the diagonal. Plot the value m_1 on the horizontal axis. This is the next value of the solution.

4. From the point (m_1,m_1) on the diagonal, move vertically to the updating function to obtain the point (m_1,m_2) and then horizontally to the point (m_2,m_2) on the diagonal. Plot the point m_2 on the horizontal axis.

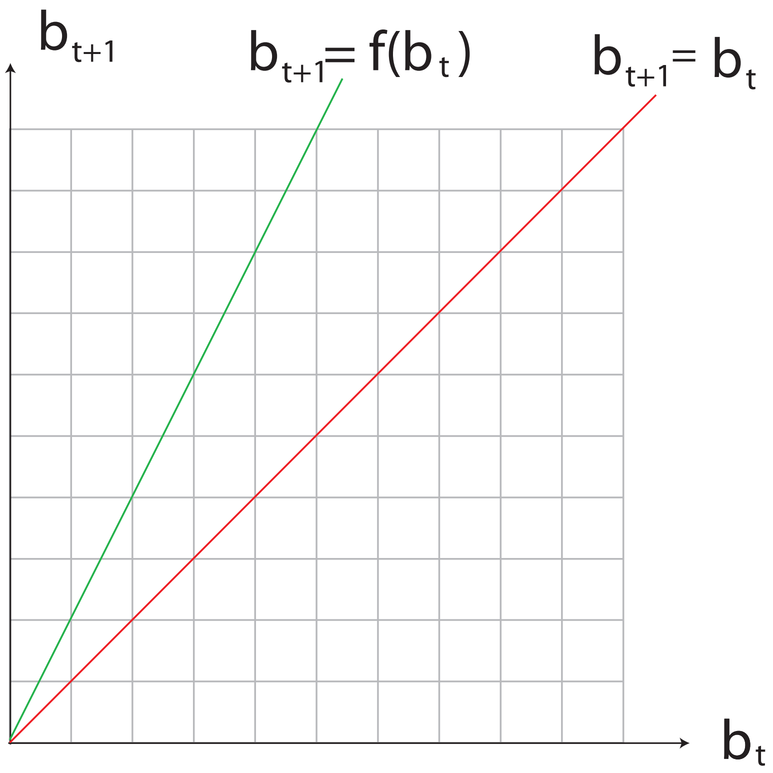
5. Continue alternating (or "cobwebbing") between the updating function and the diagonal to obtain a set of solution points plotted along the horizontal axis.

Cobwebbing

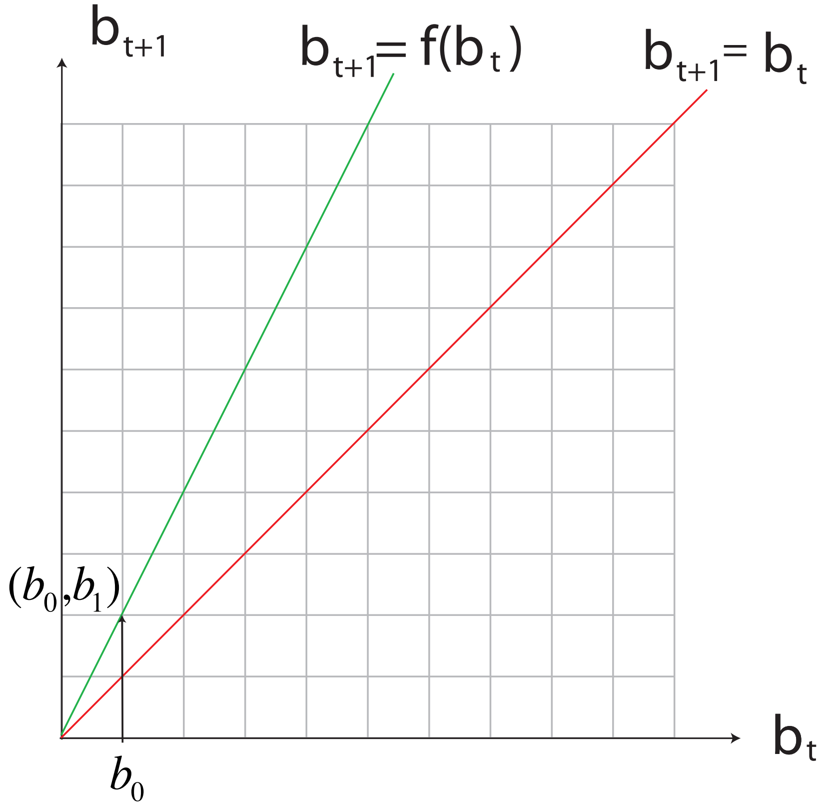
Example:

Starting with the initial condition $b_0 = 1$, sketch the graph of the solution to the system $b_{t+1} = 2b_t$ by cobwebbing 3 steps.

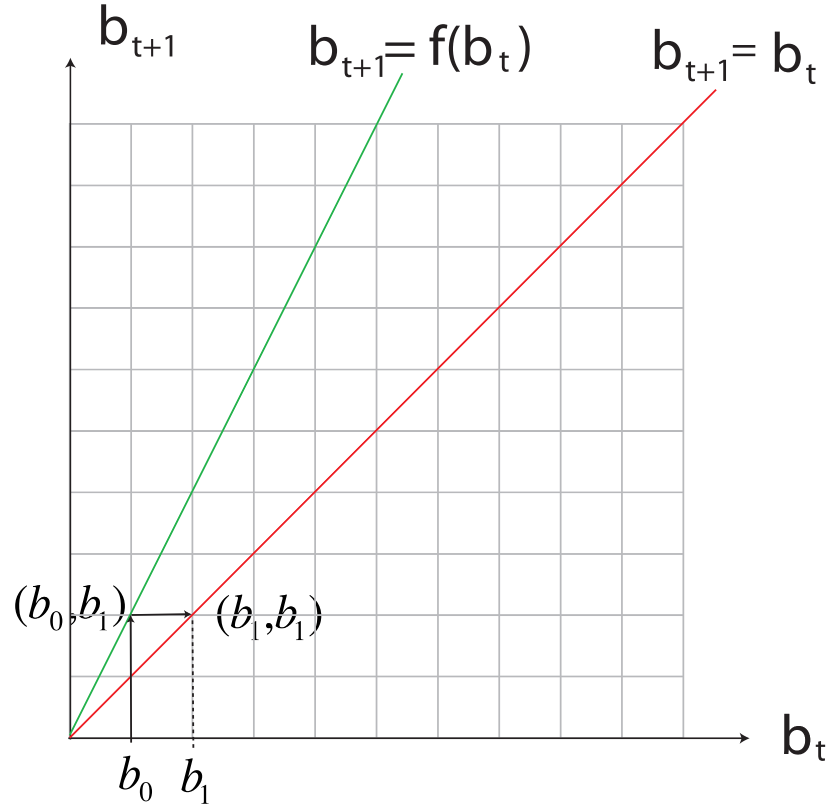
Cobwebbing



Cobwebbing



Cobwebbing



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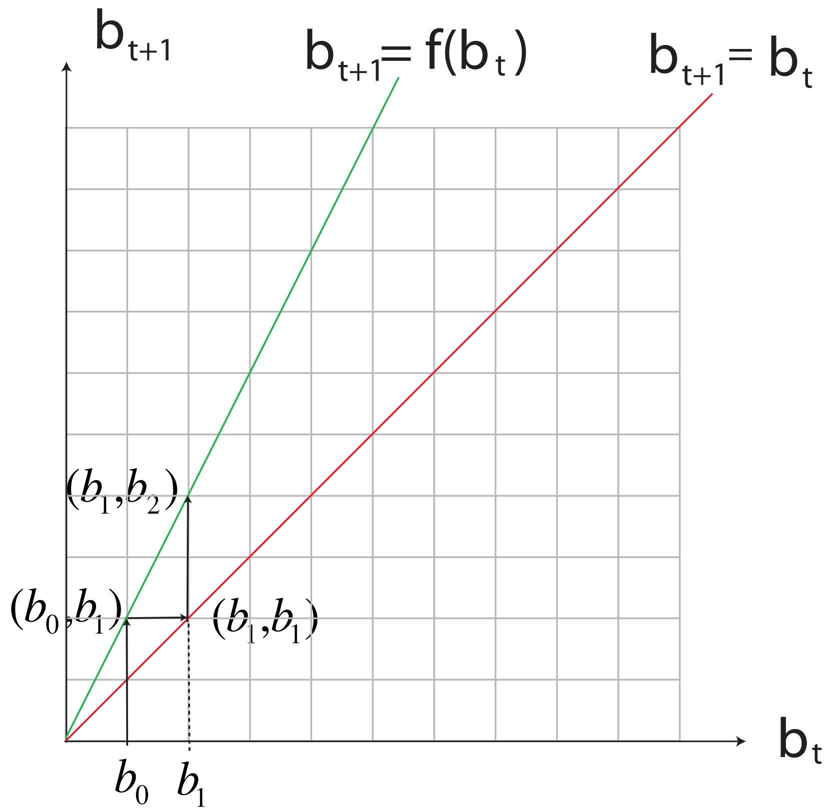
Cobwebbing



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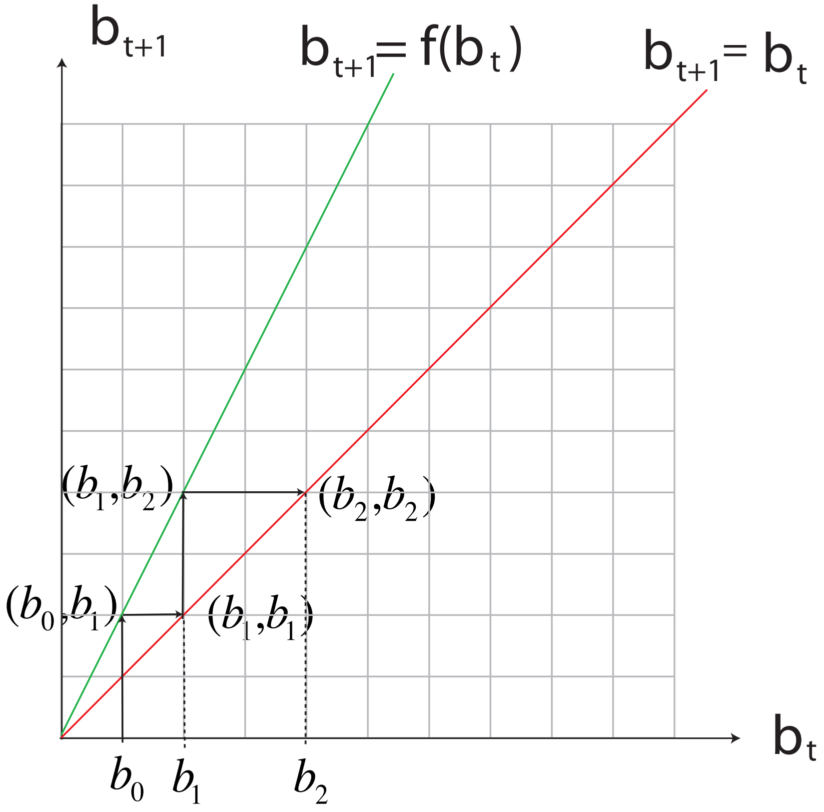

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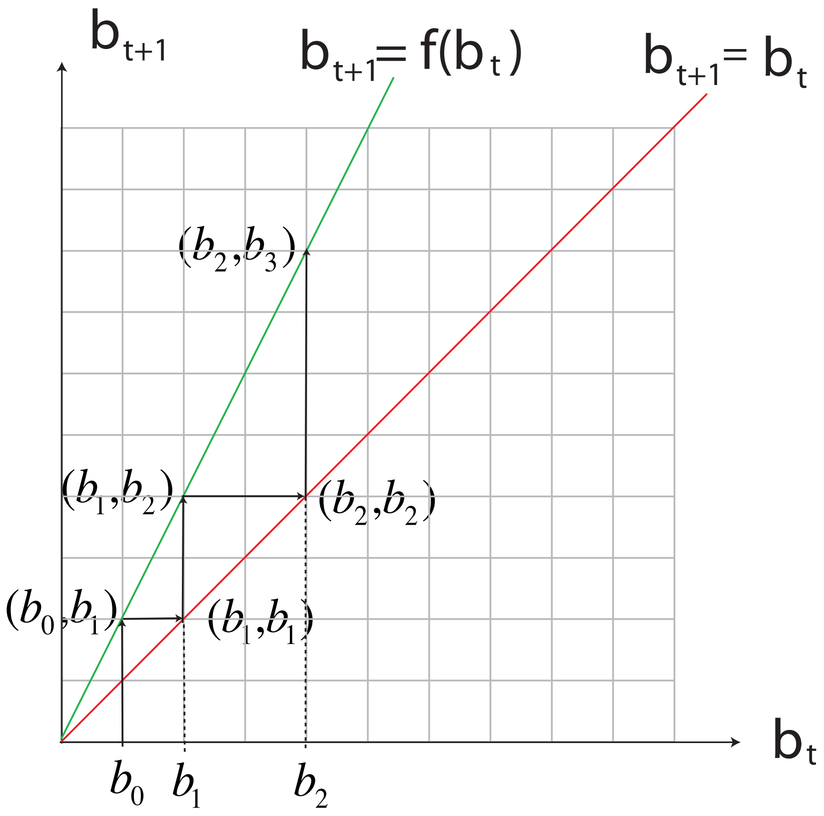

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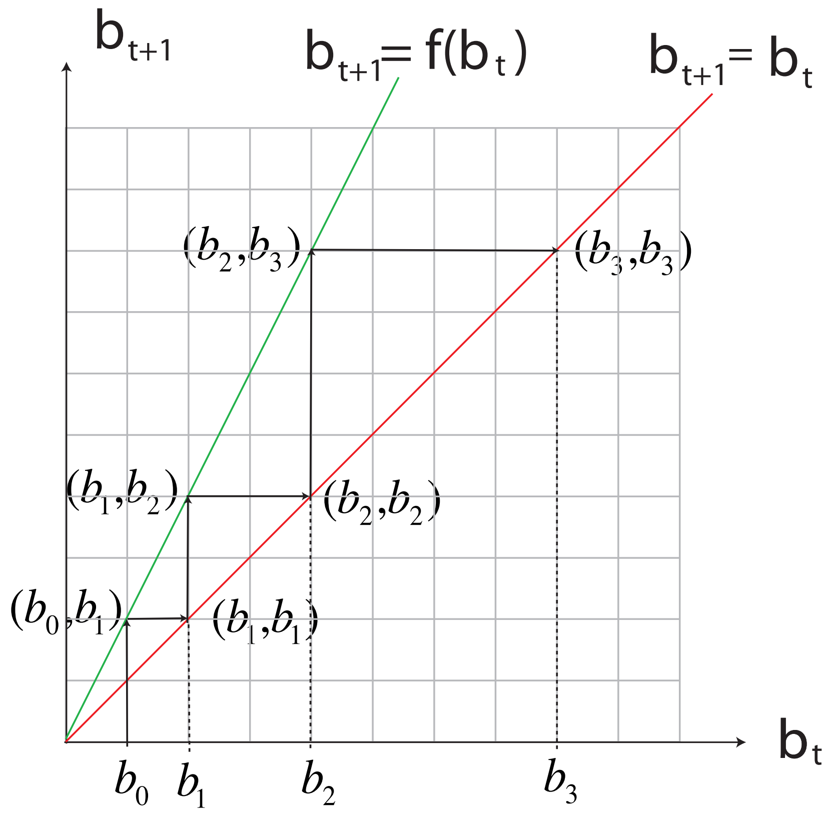
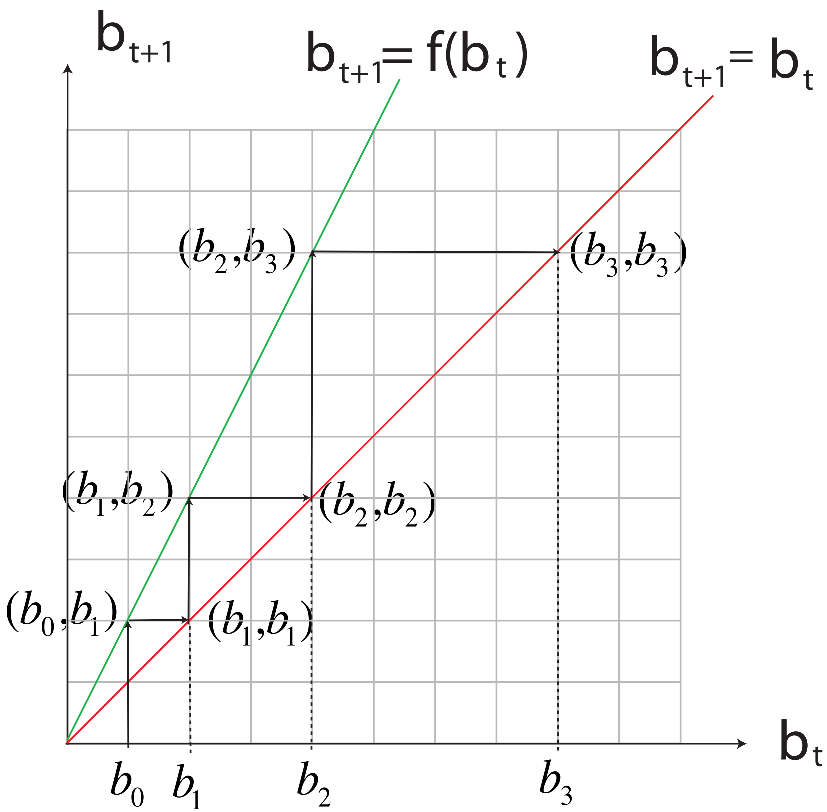
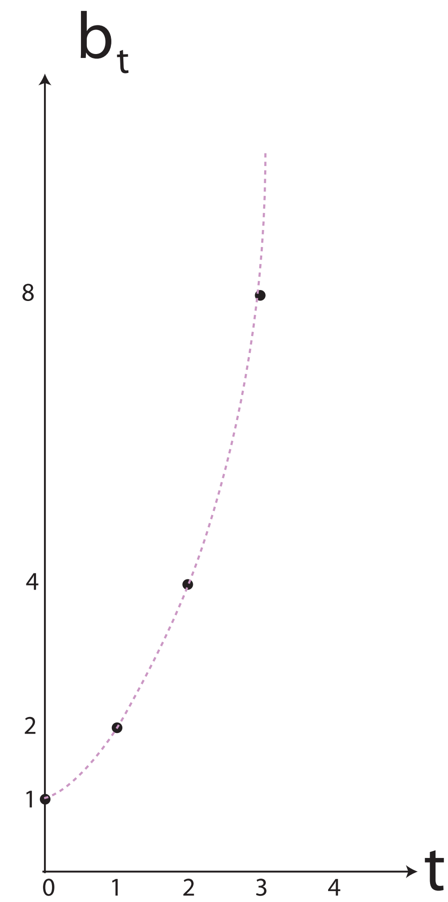


A Solution From Cobwebbing





Cobwebbing

Example:

Consider the DTDS for the methadone concentration in a patient's blood:

$$M_{t+1} = \frac{1}{2} M_t + 1$$

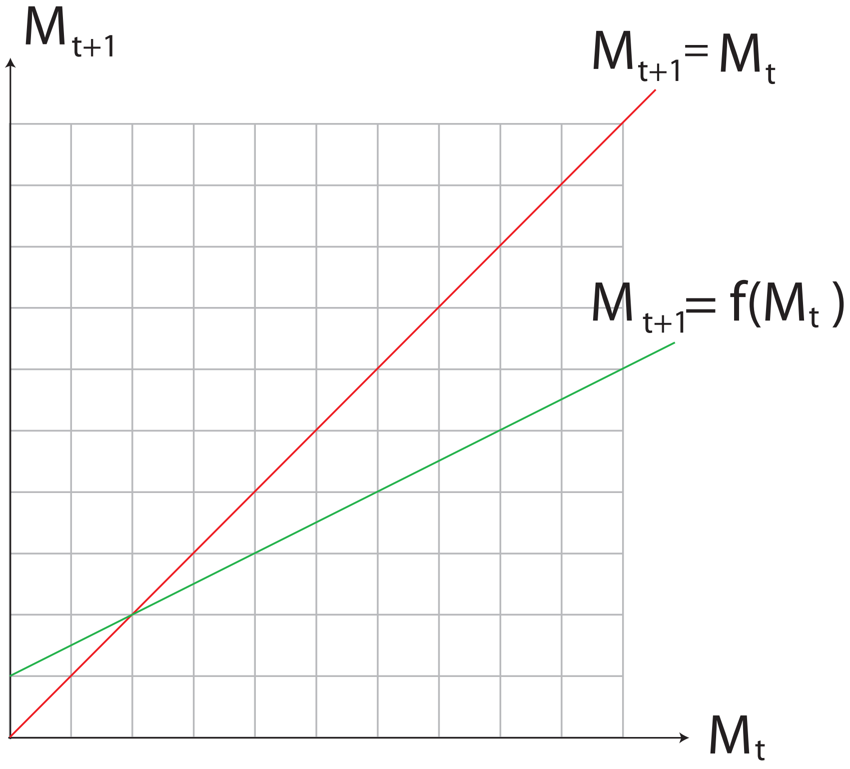
Cobweb for 3 steps starting from

(i) $M_0 = 1$

(ii) $M_0 = 5$

(iii) $M_0 = 2$

Cobwebbing



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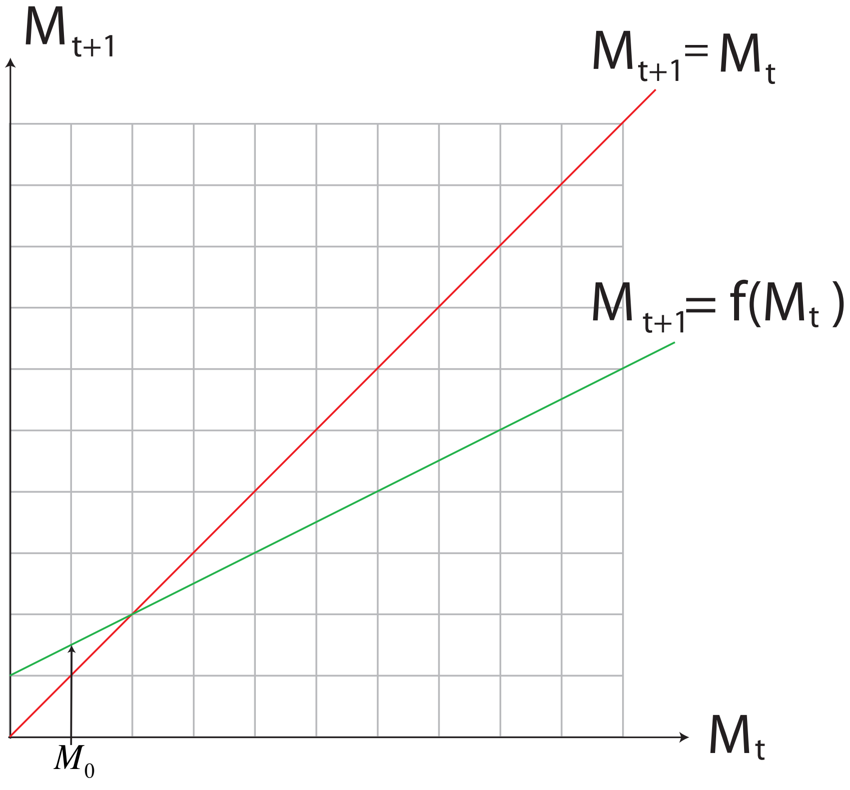

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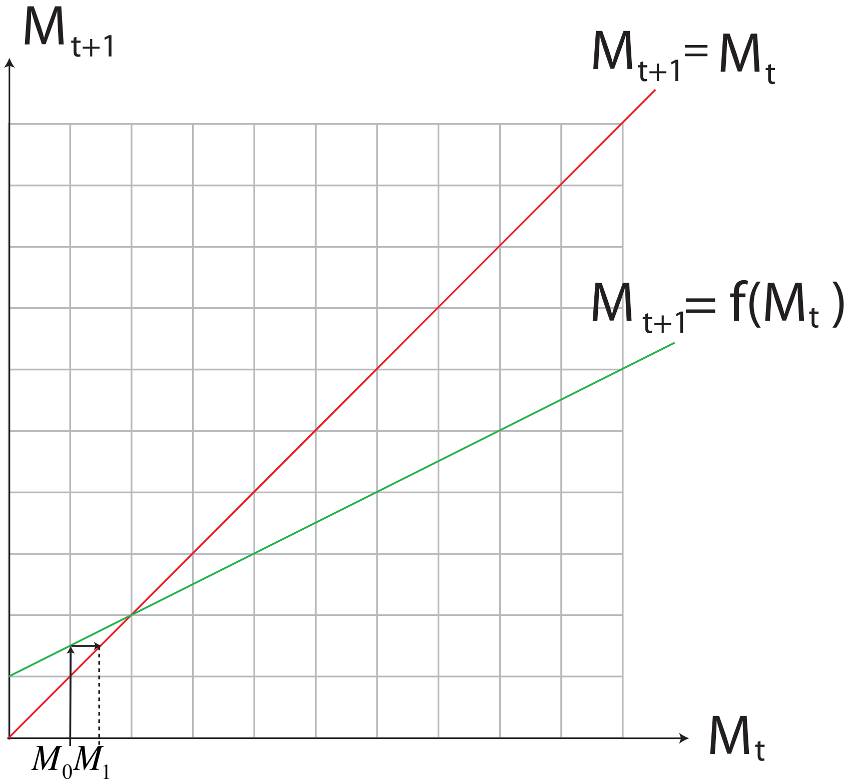

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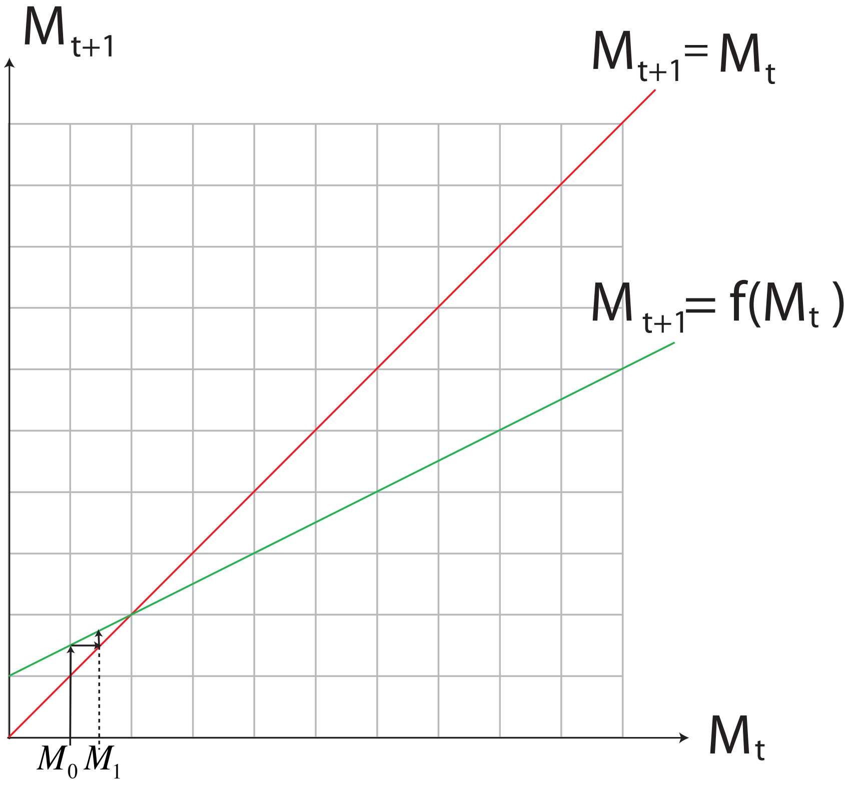

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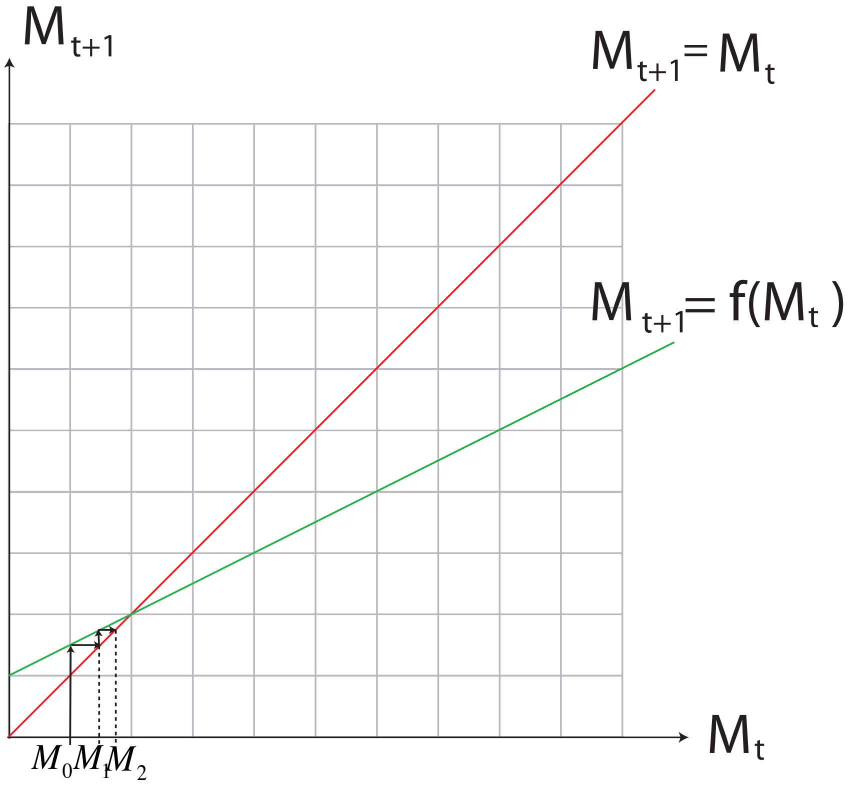

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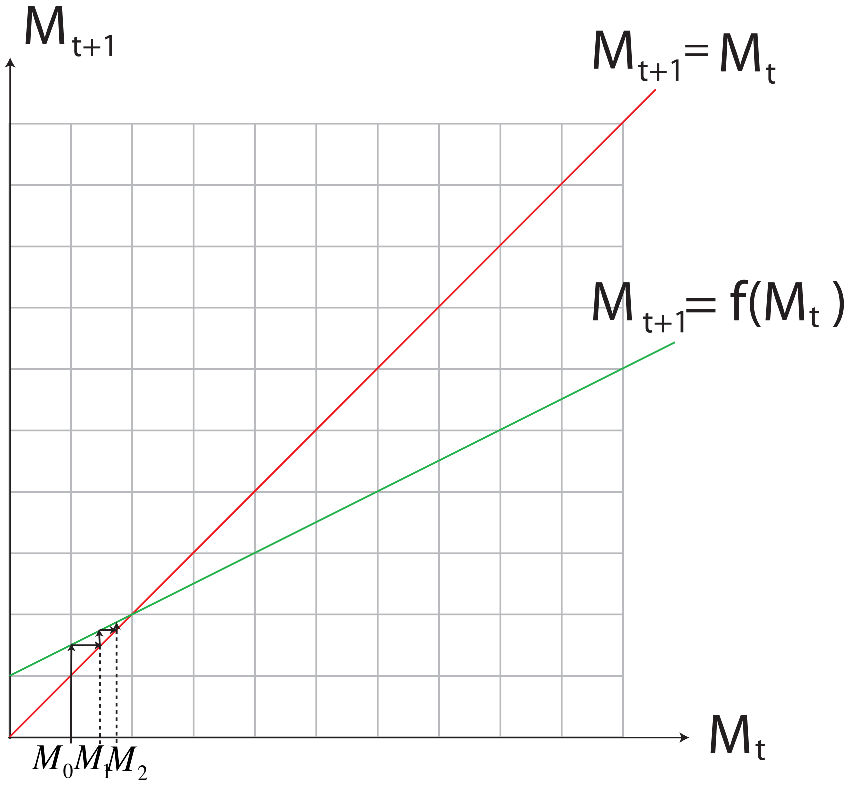

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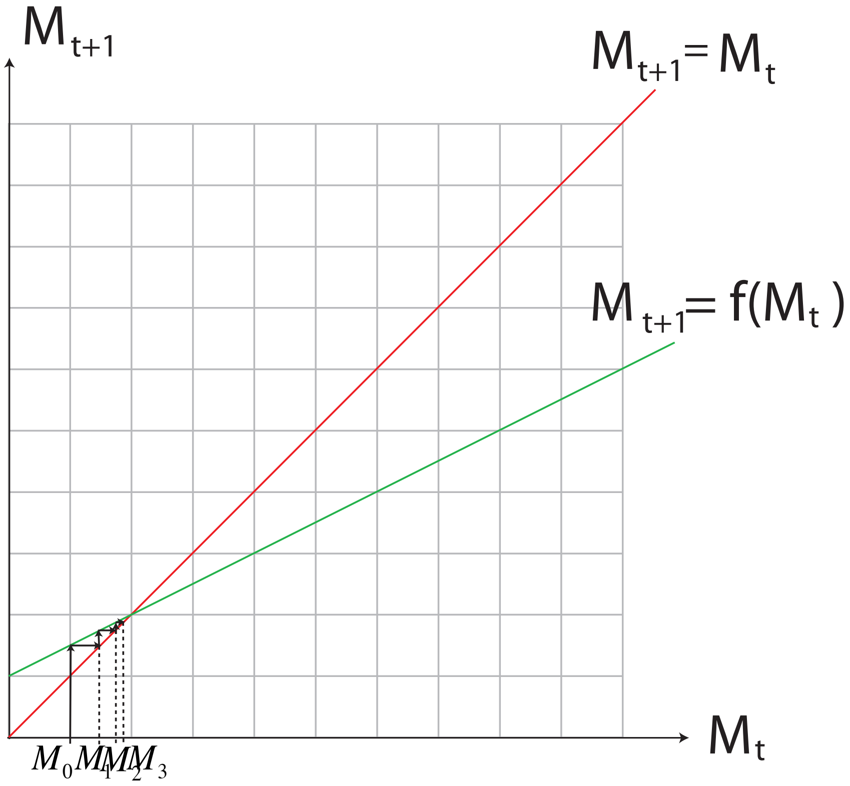

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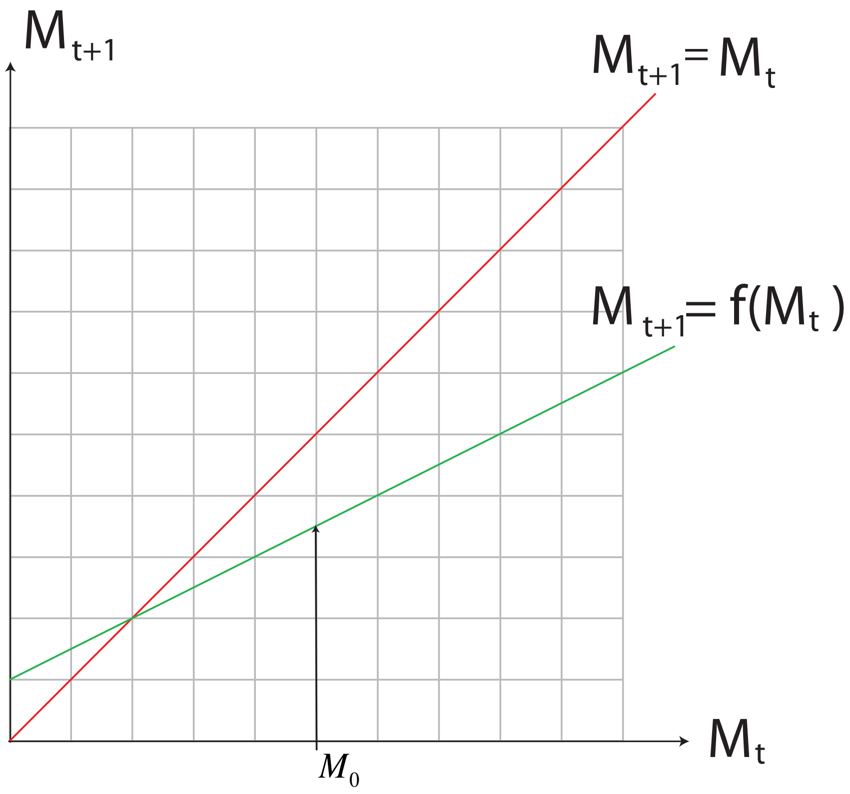

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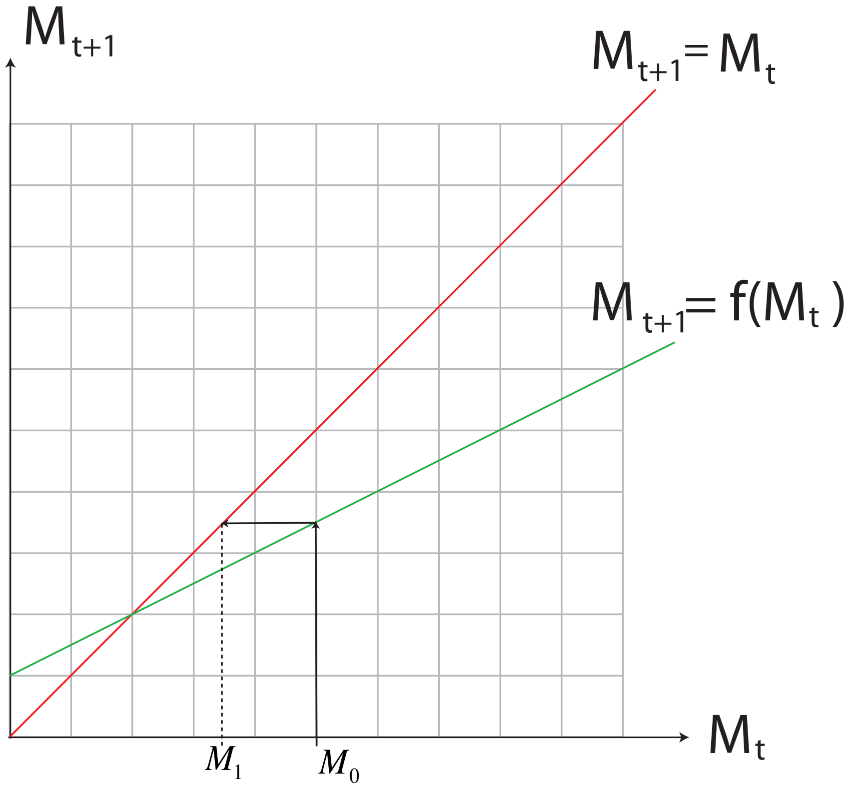

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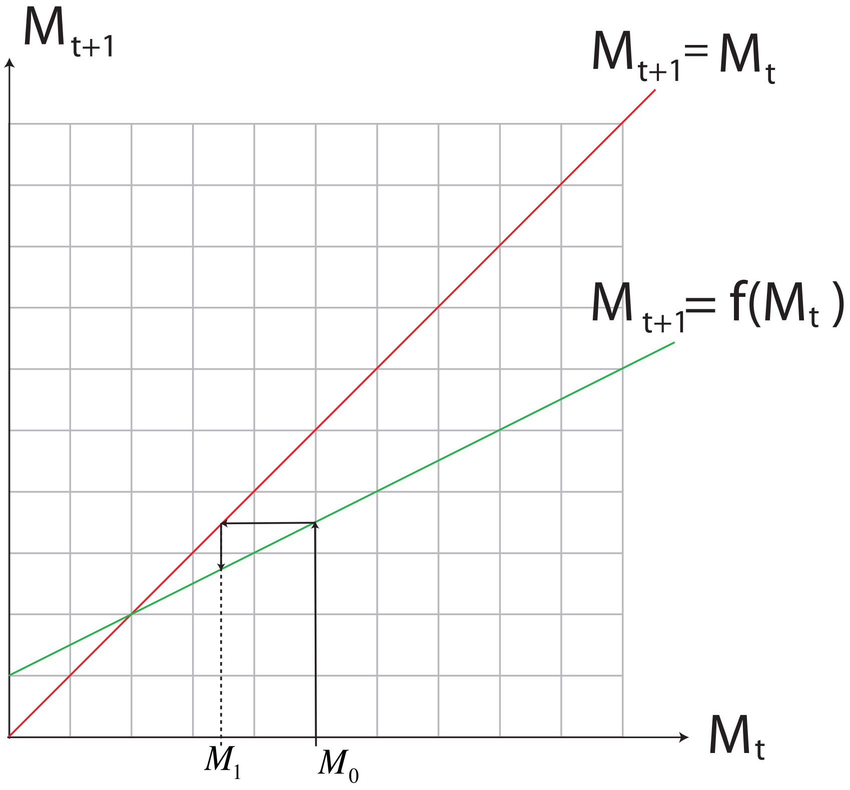

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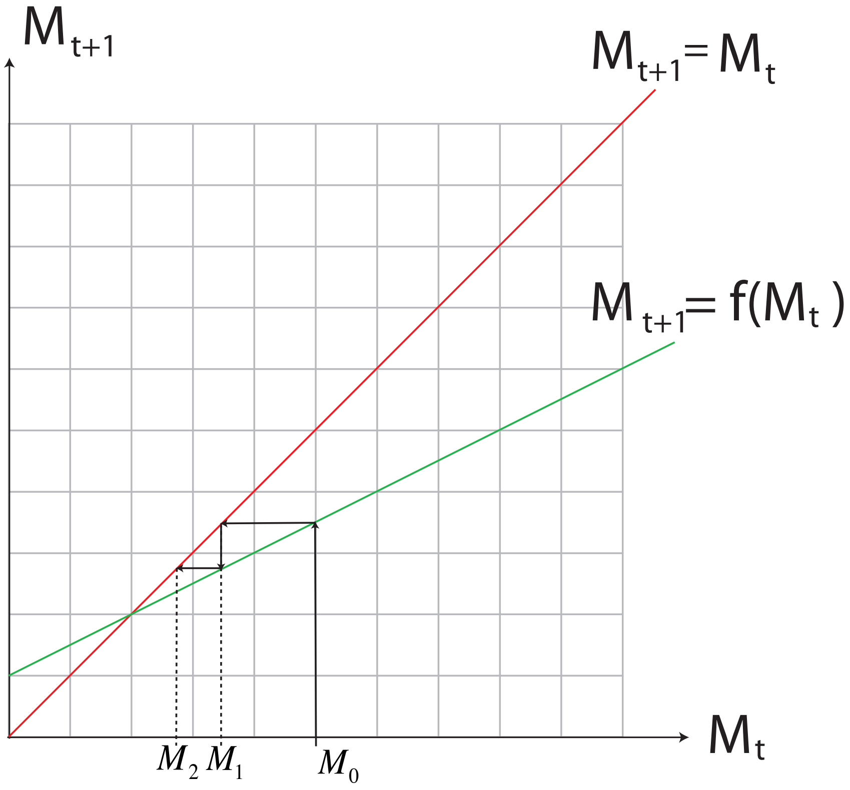

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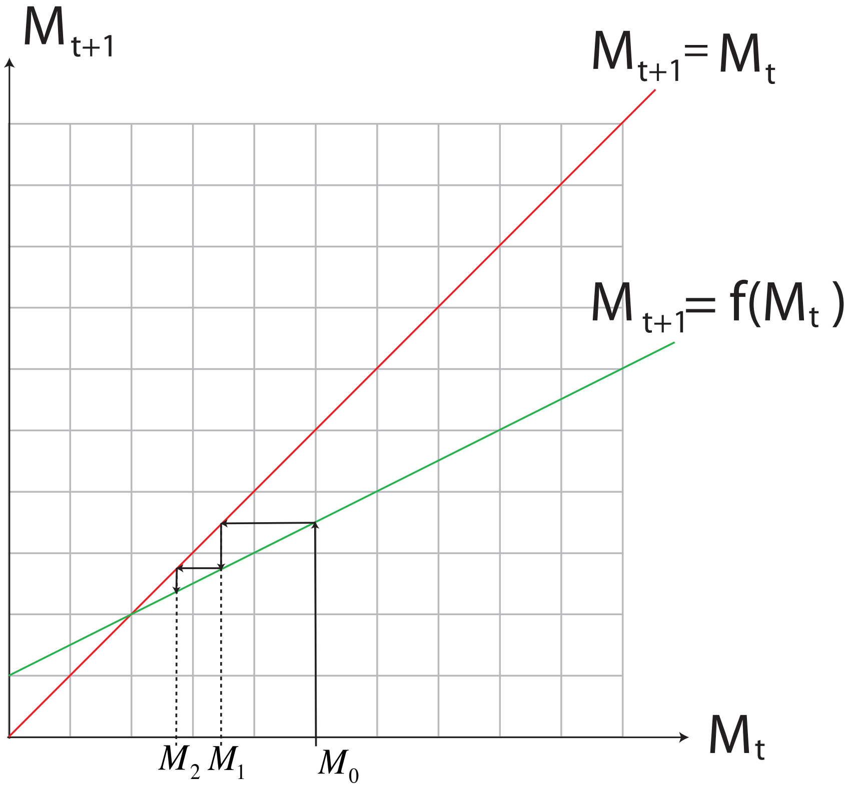

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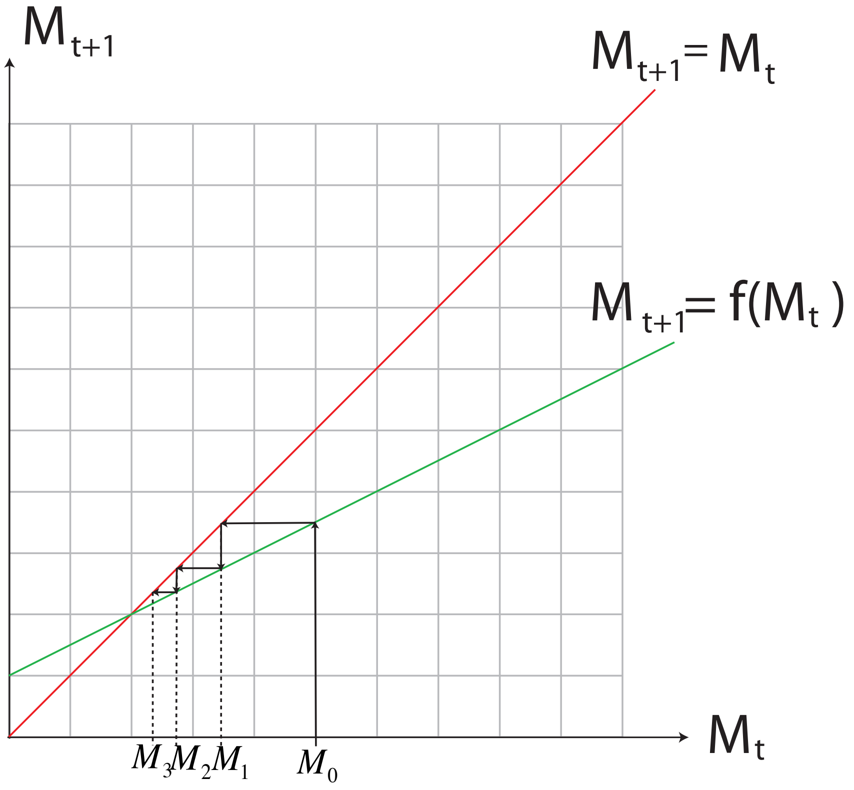

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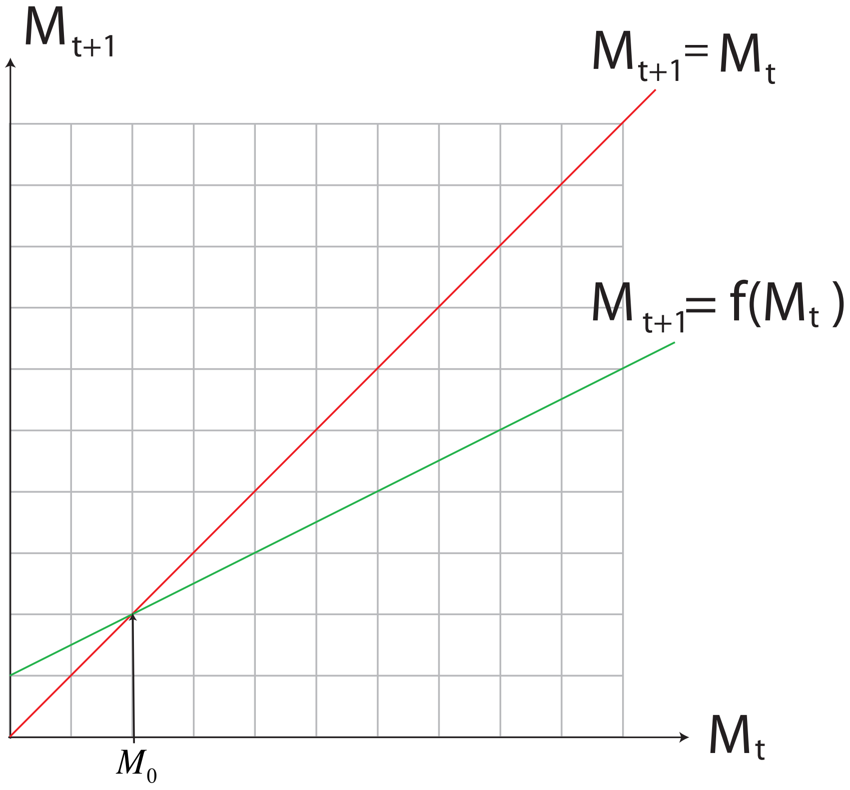


section 3.2

Equilibria

Definition:

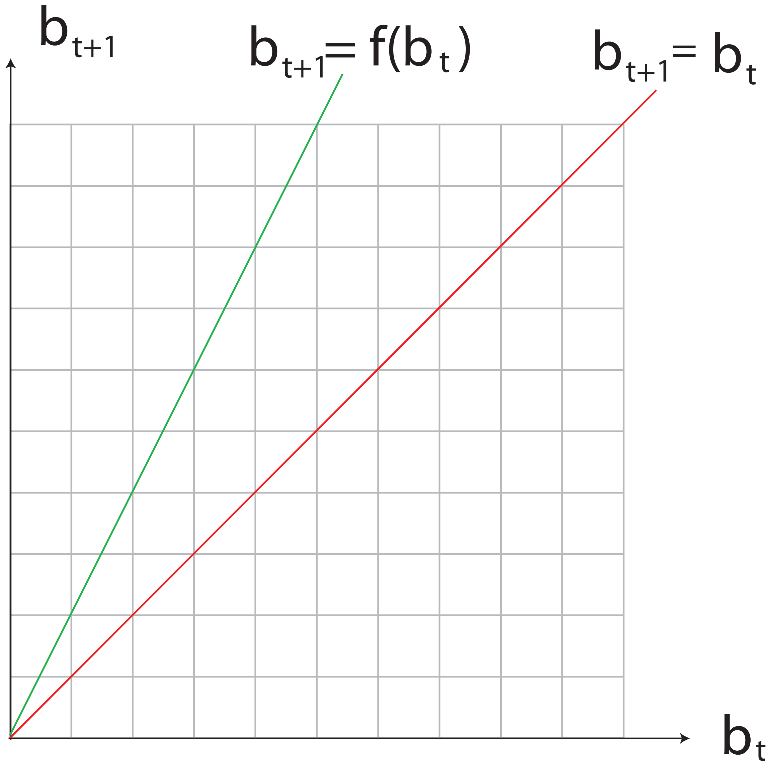
A point m^* is called an equilibrium of the DTDS

$$m_{t+1} = f(m_t)$$

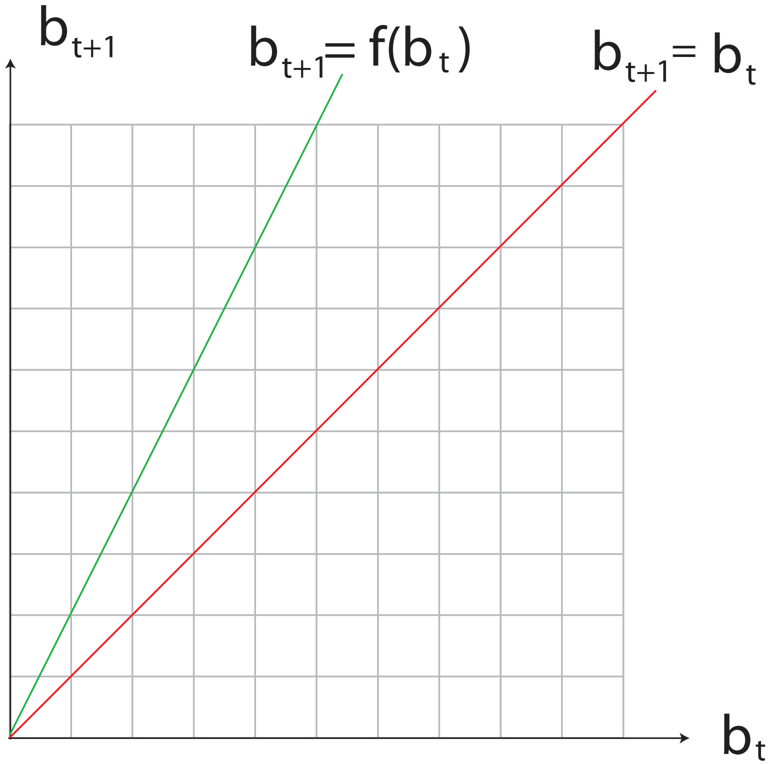
if $f(m^*) = m^*$.

Geometrically, the equilibria correspond to points where the updating function intersects the diagonal.

Equilibria

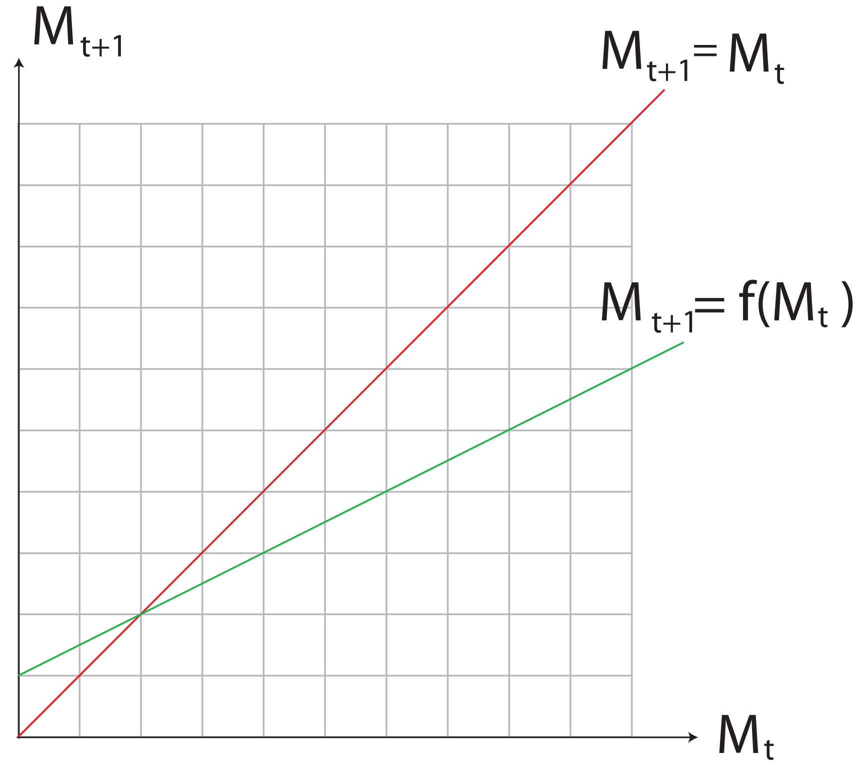


Equilibria



$$b^* = 0$$

Equilibria



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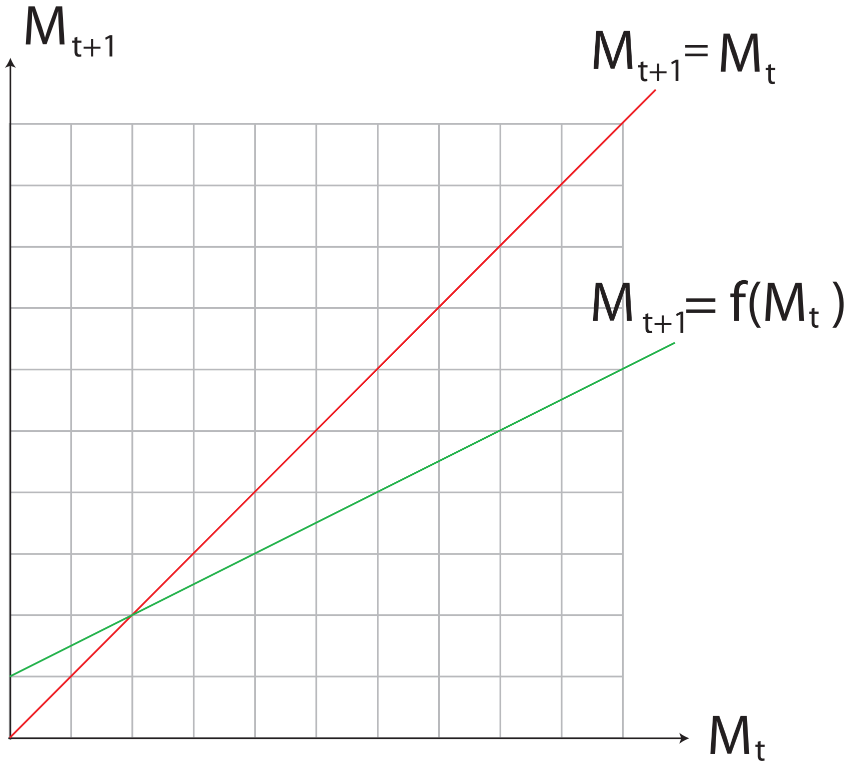
Equilibria



$$M^{*} = 2$$

Solving for Equilibria

Algorithm:

1. Write the equation for the equilibrium.
2. Solve for m^*.
3. Think about the results.

Solving for Equilibria

Examples:

Find the equilibria, if they exist, for each of the following systems.

(a) $M_{t+1} = \frac{1}{2} M_t + 1$

(b) $x_{t+1} = \frac{a x_t}{1 + x_t}$

Cobwebbing

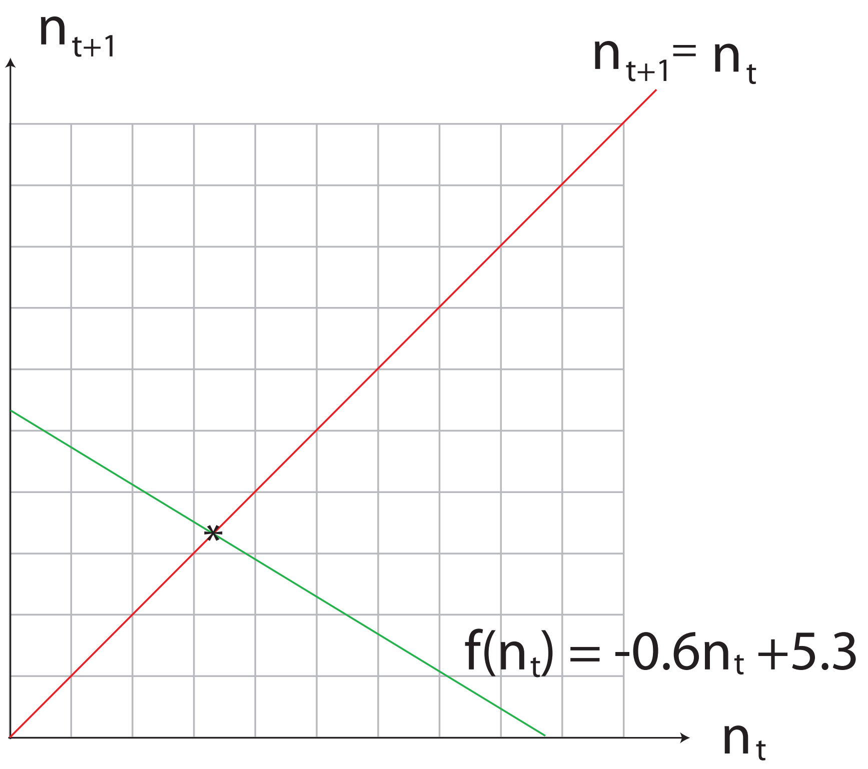
Example:
Consider the DTDS for a population of codfish

$$n_{t+1} = -0.6n_t + 5.3$$

where n_t is the number of codfish in millions and t is time.

Suppose that initially there are 1 million codfish. Determine the equilibria and the behaviour of the population over time by cobwebbing.

Cobwebbing



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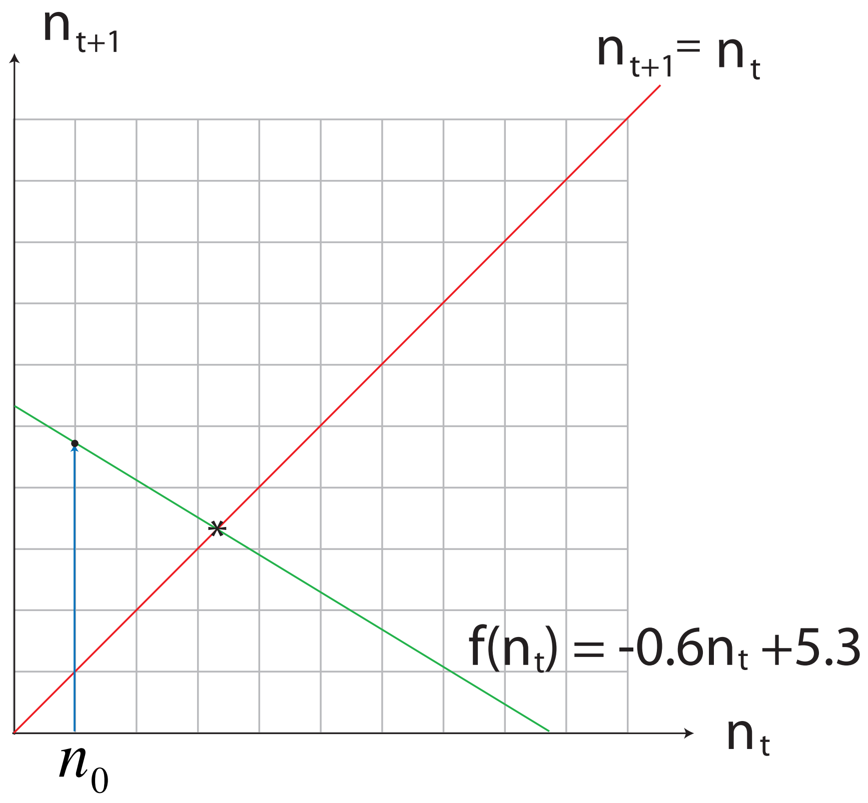

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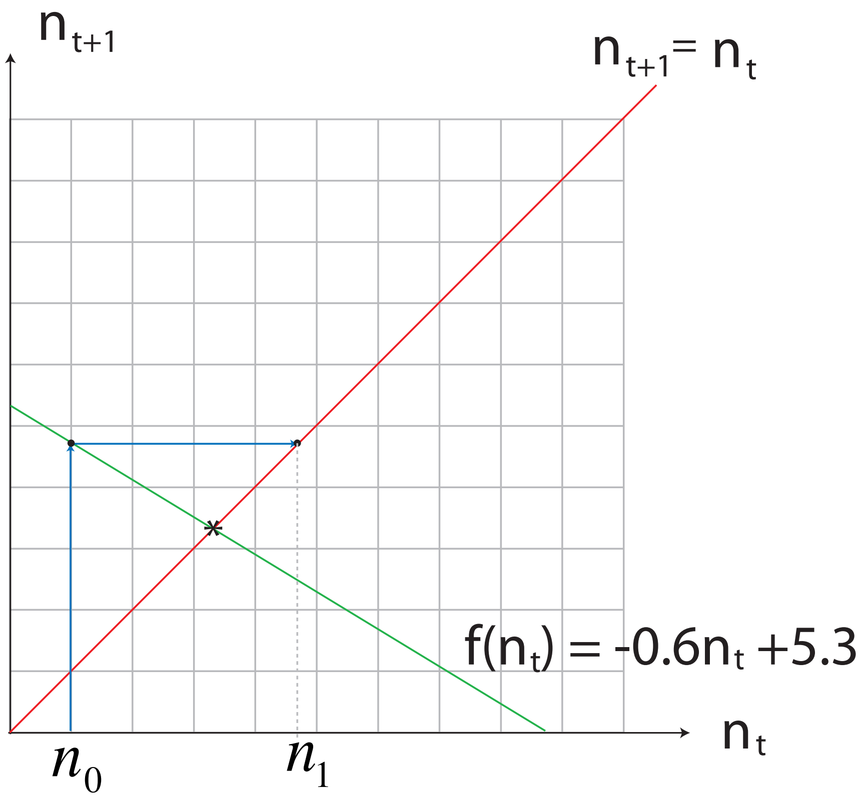

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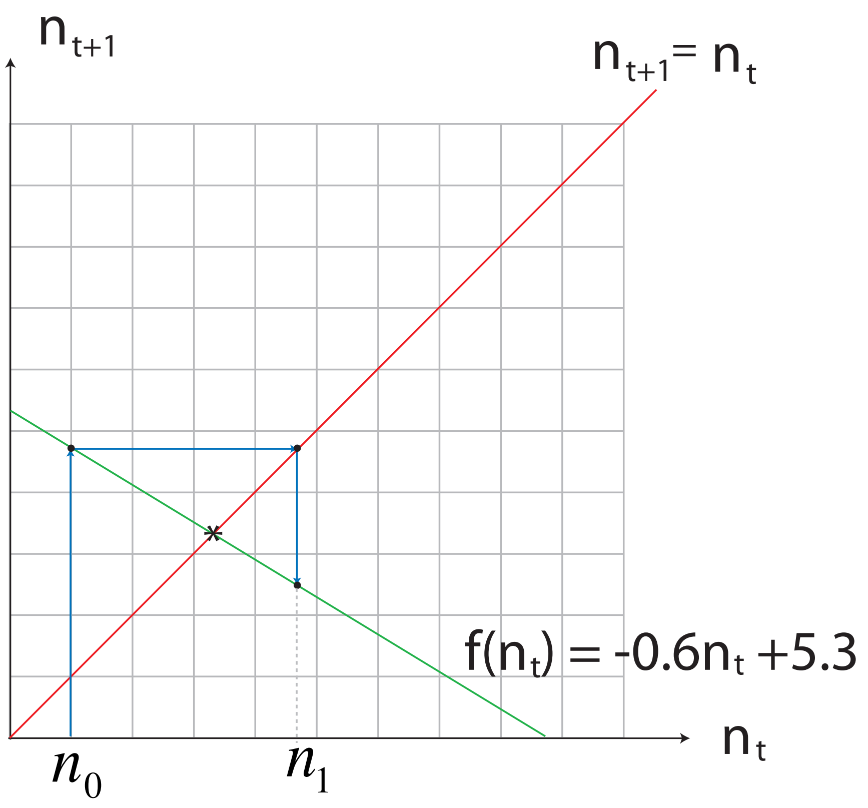

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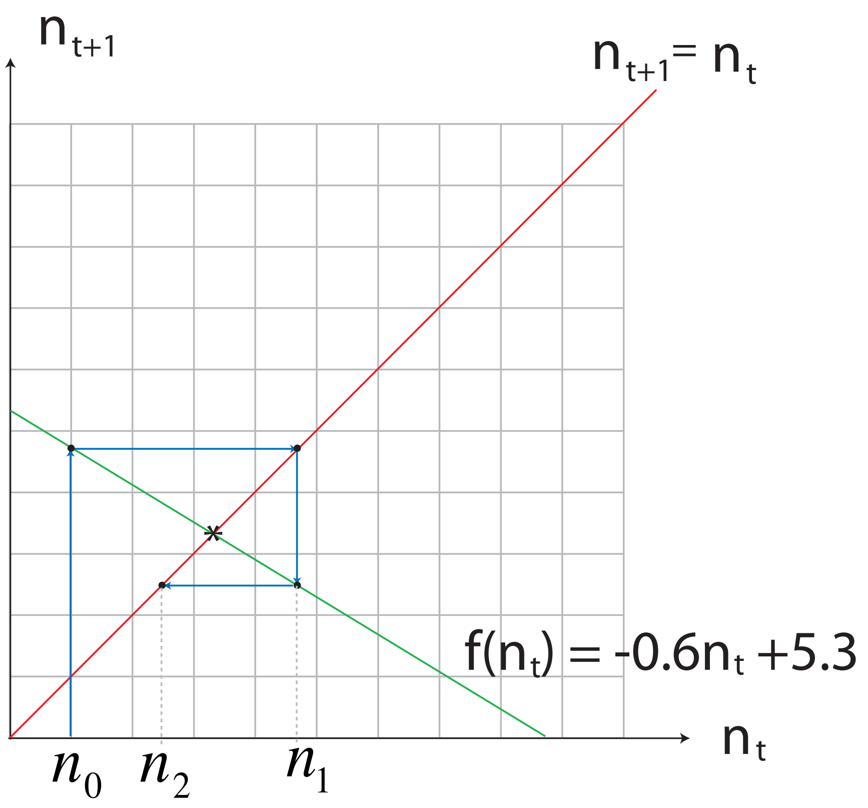

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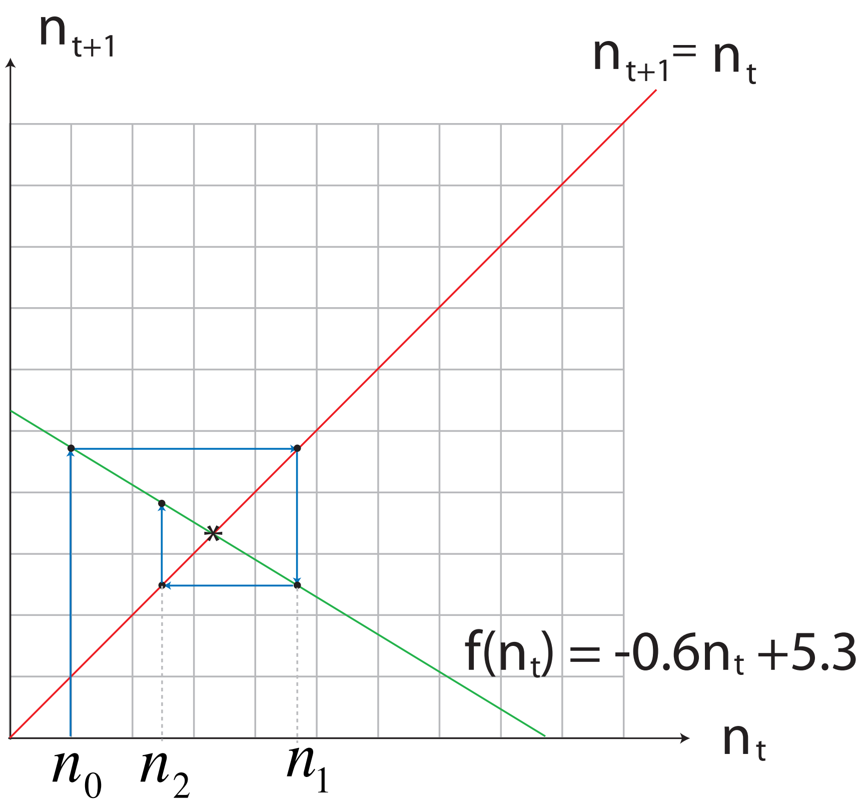
Cobwebbing



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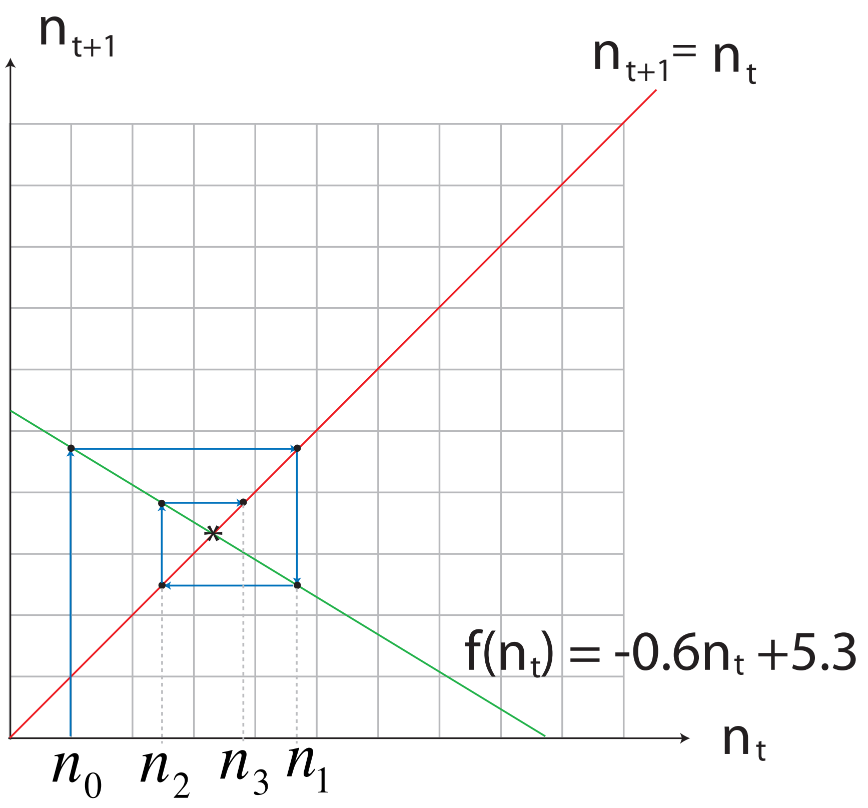

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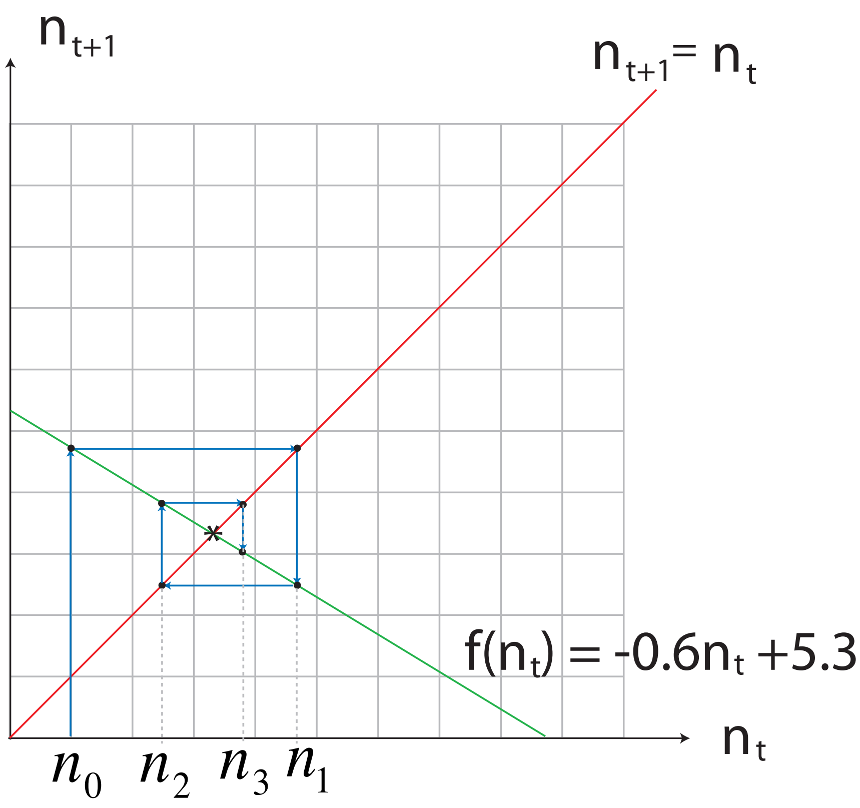

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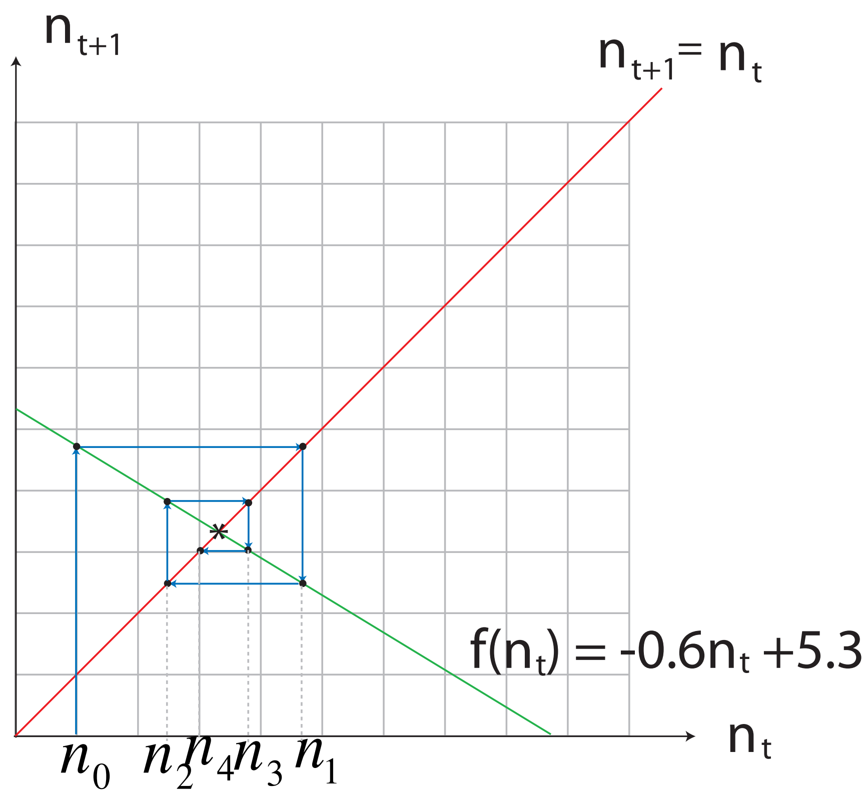
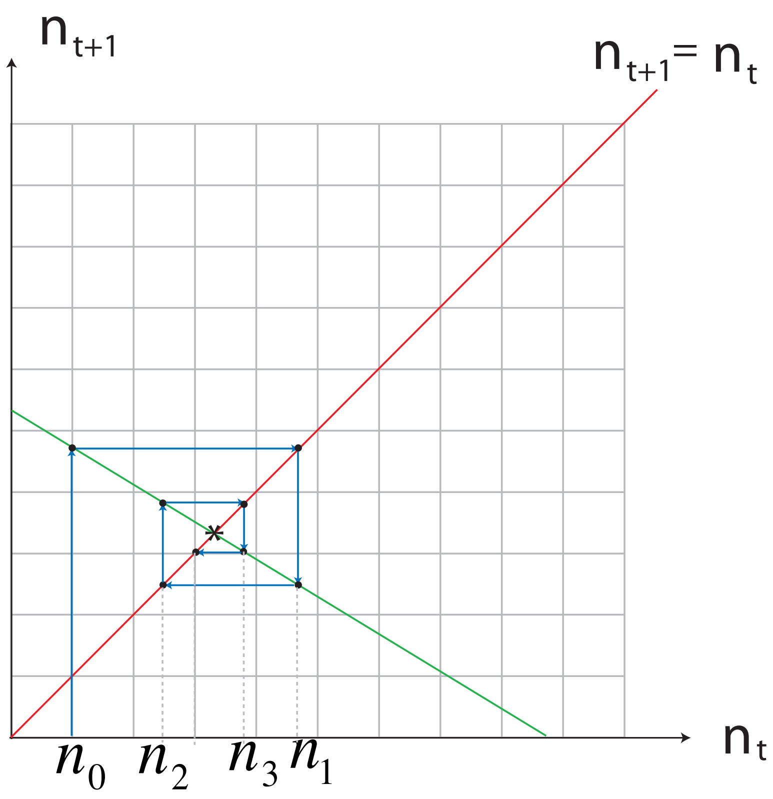
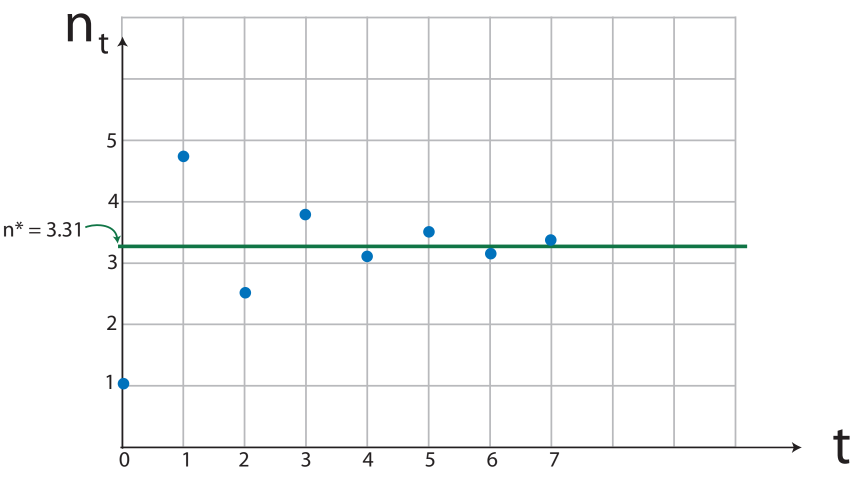


A Solution From Cobwebbing



Solution:



Stability of Equilibria

An equilibrium is *stable* if solutions that start near the equilibrium move closer to the equilibrium.

An equilibrium is *unstable* if solutions that start near the equilibrium move away from the equilibrium.