Mode-locking of a CW laser

1 Introduction

Short pulses are useful in a number of applications in optics, e.g., time resolved spectroscopy, sampling of fast electronics, and communications. This has led to a number of mode-locking techniques to obtain such pulses, for example acousto-optic, injection, synchronous-pump, colliding-pulse mode-locking, and Kerr-lens mode-locking. Using the latter two techniques, pulses of less than 100 fs duration can be readily obtained. With particular attention to the design of each intracavity element, pulses as short as 5 fs (which corresponds only to two optical cycles) have been obtained [1].

In this experiment you will mode-lock a continuous wave laser to generate a high repetition rate train of short pulses. The technique used will be a variation of injection mode-locking. A fraction of the laser's output beam will be shifted in frequency by one longitudinal mode and re-injected into the laser, causing locking of the adjacent longitudinal mode, the next mode, and so forth. This yields an equally spaced phase-locked comb of longitudinal modes across the gain curve (in the frequency domain), the Fourier transform of which is a train of short pulses (ideally of length approximately equal to the inverse of the gain bandwidth and at a repetition rate equal to the inverse of the longitudinal mode spacing.)

The main goals of the experiment will be as follows:

- To become familiar with acousto-optic modulators.
- To set up the optical and electronic systems required to mode-lock the laser.
- To obtain stable trains and measure the corresponding laser spectra.
- To demonstrate optically (using a scanning Fabry-Perot interferometer) that competition between the longitudinal modes is eliminated under mode-locked operation, again contrary to the CW case.

2 Equipment Required, Operation Notes

- Spectra-Physics Model 125 He-Ne laser. Cavity length $\approx$ 177.5 cm, maximum CW power $\approx$ 100mW when new. In its present status 30 mW maximum. You must exercise caution.
- Intra-Action AOM-40 acoustooptic Modulator.
- Variable Frequency Synthesizer (40-45 MHz). Provides power output of 1 W rms into 50 Ω.
- Electronic Spectrum Analyzer — a key instrument in tuning the frequency to the right resonance, and to measure the mode bandwidth. To accurately measure the driving frequency.
- Spectra-Physics Model 450 Scanning Fabry-Perot Interferometer. Read Siegman [2], pp. 438-439, 763-765 on scanning interferometry, and appendix B prior to use.
- Fast photodetector (bandwidth $\geq$ 1 GHz).

- Tektronix digital oscilloscope. To be used to record spectra and pulse trains that can be downloaded to a computer.

- Power meter. To compare CW vs. mode locked average power.

3 Theory

3.1 Mode-locking

The theory of mode-locking is covered in a number of readily available general references, thus the following explanation is primarily heuristic, concentrating on qualitative results. Please consult any laser textbook for a more comprehensive discussion.

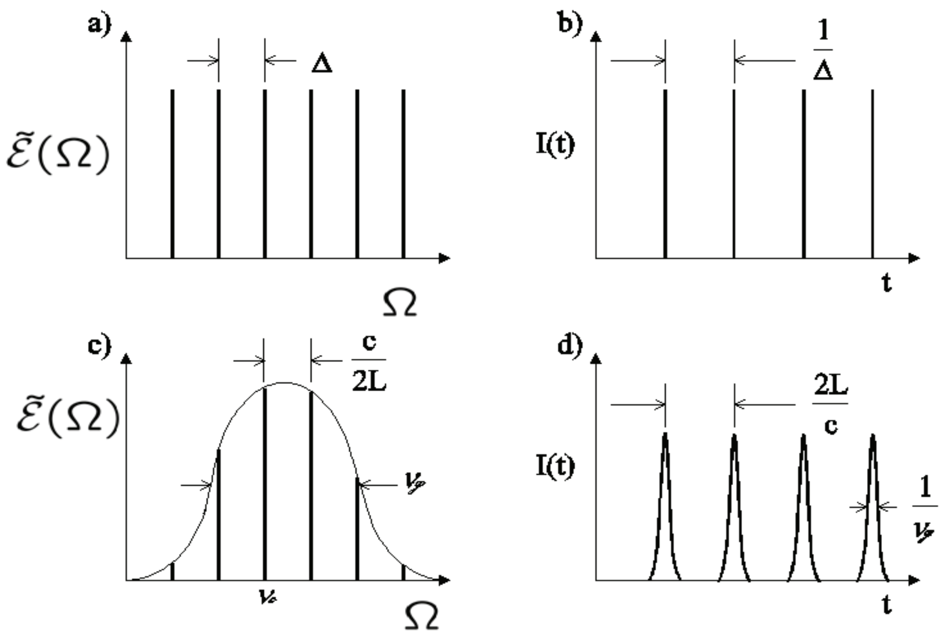

Figure 1: a) Ideal frequency spectrum. b) Time domain corresponding to a). c) Typical mode-locked laser frequency spectrum. d) Time domain corresponding to c).

The basic idea behind pulse generation is illustrated in Fig. 1. Suppose we are given an infinite number of single frequency lasers, emitting at equally spaced frequencies such as to provide the spectrum of Fig. (1a). That is,

$$\tilde{\mathcal{E}}(\Omega) = \sum_{n=-\infty}^{\infty} k\delta\left(\Omega - n\Delta\right) \tag{1}$$

a so-called Dirac comb in the frequency domain, with lines separated by Δ. By taking the Inverse Fourier transform and doing a bit of algebra, we obtain the time domain behavior

$$E(t) = \sum_{n=-\infty}^{\infty} k'\delta\left(t - \frac{n}{\Delta}\right) \tag{2}$$

which is a train of infinitely narrow pulses with repetition rate $1/\Delta$, as illustrated in Fig. (1b).

The closest version of this ideal laser consists in a comb of frequencies provided by a laser and its harmonics [3] of a laser and its Raman generated sidebands [4, 5]. There is more in Eq. (1) than a mere combination of pulses. By writing the field as a some of real delta functions of real amplitude, there is an implicit assumption that all the sources that are combined are *in phase*. This means that all the "cosine" waves of different frequency represented by Eq. (1) have at some time the same value 1. It is thus not sufficient to have created a multitude of cw sources equally spaced in frequency, it is also necessary to adjust their phase, i.e. delay them respectively to each other by appropriate quantities of the order of a fraction of light period. Another, more common approximation of the series of Eq. (1), is provided by the modes of a continuous laser. In both cases, the spectrum is finite, resembling that of Fig. (1c): A finite, Gaussian modulated comb of frequencies given by

$$\tilde{\mathcal{E}}(\Omega) = \sum_{n=-N}^{N} \delta\left(\Omega - \omega_o - n\frac{c}{2L}\right) e^{-2(\Omega-\omega_o)^2/\Omega_g^2} \tag{3}$$

where ω_o is the center frequency of the group of delta-functions, Ω_g its bandwidth. For the case when the delta-functions are represented by modes of a laser cavity, and the longitudinal mode separation $\frac{c}{2L}$ has been substituted for Δ. The amplitude distribution of the modes determine the shape of the pulse in the time domain. If the amplitude distribution is Gaussian, by taking the Inverse Fourier transform it can be shown that the time behavior is

$$\tilde{\mathcal{E}}(t) = k' \sum_{n=-\infty}^{\infty} \exp\left[-2\left(t - n\frac{2L}{c}\right)^2 \Omega_g^2\right] e^{i\omega_0 t}. \tag{4}$$

That is, we obtain a train of Gaussian pulses of width $1/\Omega_g$ at a repetition rate of c/2L as shown in Fig. (1d). Note that the round trip travel time is $\frac{2L}{c}$: in effect, there is one pulse oscillating back and forth in the laser cavity that is output each time it comes to the output coupler.

It was stated above that the pulse width is proportional to the inverse of the gain bandwidth. In fact, the transform limit on the time-bandwidth product is $\Delta\tau\Delta\nu_g = .44$ for the case of Gaussian pulses/Gaussian gain profile.[1] The quantities $\Delta\tau$ and $\Delta\nu_g$ are measured as the full width at half maximum. (See, for example [6].) This particular Gaussian limit applies if the modes are exactly in phase, i.e. the difference in phase between successive modes is zero or a number proportional to 2π. If the difference in phase between successive modes is different from $2N\pi$ (N being an integer 0, 1, 2 ...), then the pulses are broader, and "chirped"; that is their frequency is swept along the pulse.

Question Which property of the Fourier transform explain the statement above?

Mode-locked lasers have a number of modes that can be anything from 5 [7] to a few million [1] in the case of ultrashort pulses. A example of a pulse made of 5 frequencies that add constructively at one point, destructively at other points is shown in Fig. 2. We will call $t = 0$ the instant at which the waves that constitute the pulse add at their crest. The waves of increasing frequency that are added on top of each other have the same amplitude. The sum of these waves is shown with a thick red line. At the common crest ($t = 0$), the sum of the electric field of all these waves adds up to 5 $\times$ the field of a single wave, which implies that the intensity is 25 $\times$ that of a single wave. There is no violation of energy conservation, because after a few optical cycles, the fields add up to nearly zero. Because the amplitude distribution in frequency

[1] $\nu_g = \Omega_g/(2\pi)$.

is square, the envelope of the pulse in time is a sinc. Conversely, square pulse generation has been demonstrated with the same number of modes, but distributed as a sinc [7].

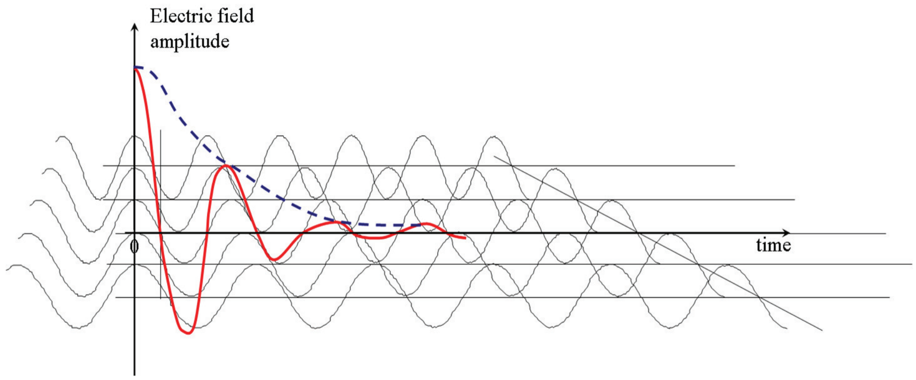

Figure 2: Five waves are added on top of each other, such that at a given time all the crests coincide.

We are generally taught in introductory laser courses that most lasers have spectra reminiscent of Fig. (1c), which leads to the question: "Why are most lasers CW rather than pulsed (mode-locked)?" The main reason is, as pointed out above, that the modes have to be locked in phase, and oscillate with comparable amplitude. In the case of homogeneously broadened gain media, the strongest mode tends to saturate the entire gain line, preventing weaker modes from lasing (note that most laser media are homogeneously broadened.) Another difficulty is that generally the modes are not equally spaced, because of the dispersion associated with the gain line.

Question Justify the above statement that the modes may not be equally spaced.

To achieve mode-locking, it is thus necessary to lock the frequency separation of the longitudinal modes to $c/2L$ and to lock their phases. In this experiment this will be accomplished by picking off a bit of the outcoupled light, shifting it up (or down) in frequency by $c/2L$, then reinjecting this into the laser cavity. The effect will be to lock the frequency and phase of the next higher (lower) mode, which on the next pass will induce locking of the next mode, and so forth. To maintain the mode-locking once pulses have been generated, the delay between the output of a pulse from the laser and the reinjection of the frequency shifted pulse must equal the round trip travel time for light in the cavity. This is the same as saying that the frequency shifted pulse must be reinjected coincident with the intracavity pulse.

3.2 Acoustooptic deflector

The aforementioned frequency shift will be generated by an acousto-optic modulator/deflector. The theory of the A-O deflection of light is covered very well in [8] and [9], and is outlined below.

Suppose we have a sound wave of circular frequency Ω_A traveling through a solid medium for which the speed of sound is v_A. The stress induced in the solid by the acoustic wave will cause a small variation of the refractive index $n(\vec{x})$ in the form of a traveling grating with spacing $\Lambda = 2\pi v_A/\Omega_A$, or wave vector $\vec{K}_A = 2\pi/\Lambda$. Referring to Fig. 3, suppose a light beam is incident on this grating at angle θ_i and diffracted at an angle θ_d. The relationship between λ, Λ, θ_i, θ_d, etc. may be obtained directly by considering a particle picture wherein an incident photon of circular frequency ω_i interacts with a phonon to produce a diffracted photon of frequency

$$\omega_d = \omega_i \pm \Omega_A \tag{5}$$

to satisfy conservation of energy. The $\pm$ corresponds to a Doppler blue (red) shift induced by the traveling acoustic wave. Momentum conservation requires that

$$\vec{k_d} = \vec{k_i} + \vec{K}_A, \tag{6}$$

where $\vec{k_i}$ and $\vec{k_d}$ are the incident and diffracted photon wavevectors. Taking the projection on $\hat{K}$ of the momentum conservation equation yields

$$\frac{omega_d n}{c} \sin\theta_d = -\frac{\omega_i n}{c} \sin\theta_i + \frac{\Omega_A}{v_A}. \tag{7}$$

Question Since $\Omega_A \ll \omega$ and $v_A \ll c$, it is not clear where this relation leads us. Can you figure it out? Find an (approximate) expression relating θ_d and θ_i. It can be argued that the diffraction will be most efficient when all photons in the diffracted beam interfere constructively. From simple geometric considerations, this requires that $\theta_i = \theta_d$ in addition to the equation you just derived. An alternative way to discuss the diffraction is to look at a snapshot of the acoustic wave, see Fig. 3. The amplitude distribution of the acoustic wave gives rise to a modulation of the stress in the material and of the refractive index. The diffraction of the light wave can now be understood as Bragg reflection at a refractive index grating of period Λ.

For the total deflection angle $\Theta = \theta_i + \theta_d = 2\theta_i = 2\theta_d$ derive

$$\Theta = 2\sin^{-1}\left(\frac{\lambda\Omega_A}{4n\pi v_A}\right). \tag{8}$$

For the acoustooptic modulator which will be used in this experiment, $1/nv_A$=.26 $\frac{\text{mrad}}{\mu\text{m MHz}}$ (n and v_A are not given independently for our device.) For $\lambda = 633$ nm and $F = 42$ MHz, $\Theta = 6.9$ mrad $= 23'$.

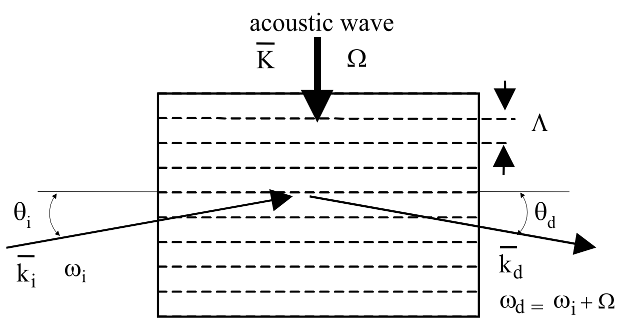

Figure 3: Acoustic-optic deflection of light. The frequency of the diffracted beam is up-shifted in this geometry.

4 Outline of Procedure

The setup for the modelocking experiment is according to Fig. 4. Note that the mirror RRM must be on the frequency-shifted (deflected) beam. Either up- or down-shifted beams may be used. The unshifted beam is easily identified by disconnecting the RF; it will be the only one that remains. A concave mirror of radius R $\approx$ 5-10 m is preferred for mode matching. (The other beam folding mirrors should be flat.) The unshifted beam is diverted to the various diagnostic devices.

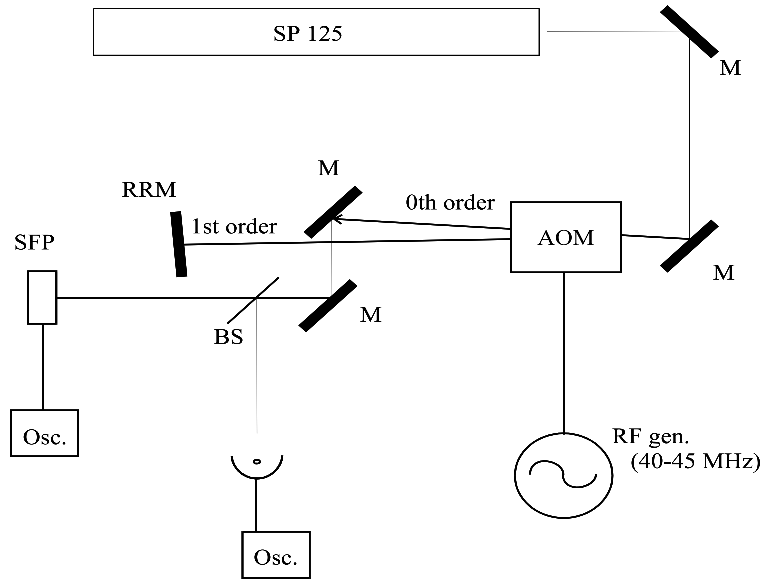

Figure 4: Experimental layout. (SP 125= Spectra Physics 125 He-Ne Gas Laser, AOM=Action AOM acousticoptic modulator, RF gen.=RF frequency synthesizer, RRM=retroreflecting mirror, SFP= scanning Fabry-Perot interferometer, Osc.=oscilloscope, SA=spectrum analyzer).

For initial experimentation, the extra-cavity optical path length should be nearly equal ($\pm$2cm) to the optical cavity length L so that the reflected, frequency-shifted pulses will be coincident with the intracavity traveling pulse. Similarly, extracavity optical path lengths of $L/2, 3L/2$,..., will yield mode locking with two pulses in the cavity; $L/3, 2L3, 4L/3$,...will give three intracavity pulses, etc. This will be demonstrated later in the experiment. Measure the length of the cavity, and set RRM at the corresponding position.

From the laser cavity length, calculate the frequency required for mode locking (remember the double pass geometry.) Set the frequency synthesizer to this value, and connect to the A-O modulator.

Locate the deflected beam and rotate the modulator to maximize its intensity. Position the curved mirror in its path. Careful alignment of this mirror is required to retro-reflect the beam directly into the laser. Note that it is the diffracted part of the retroreflected beam that is to be injected into the laser. Observe the signal with the spectrum analyzer. You should identify the mode beating frequency, the frequency of the synthesizer, and possibly a plasma oscillation frequency modulating the laser beam amplitude. Note the frequency and bandwidth of the mode-beating. Tune the frequency of the synthesizer to coincide with the mode-beating frequency. You should see the mode-beating line narrow by several orders of magnitude, indicating mode-locking. The mode-locking will be confirmed by observing a stable pattern of modes with the scanning Fabry-Perot.

Once the laser is mode-locked use the Fabry-Perot to compare the longitudinal mode spectrum under mode locked operation with that under CW operation. Measure the ratio of the beat note bandwidth in mode-locked and cw operation. Use the digital oscilloscope and the PC to download data files showing the pulse train and the corresponding spectrum. Measure the gain-bandwidth from the mode locked plots. Using the longitudinal mode spacing of the laser for calibration, measure the free spectral range of the scanning Fabry-Perot.

Compare the average power of the laser under mode-locked and CW operation. Is there any difference? From this and previous data, calculate the pulse energy and peak power. Do think about the manner in which you can suppress mode-locking without affecting otherwise the laser configuration.

Determine the mode-locking bandwidth by reducing the driving frequency to the point where the laser cannot be forced to mode-lock, and then increasing the frequency to find the high frequency mode-locking limit. Measure these limits with a frequency counter. How does this bandwidth compare with the longitudinal mode spacing?

After completing the above procedures, move RRM to any fractional-distance position (e.g., $L/2, 2L/3, 4L/3$ etc.) of your choosing. Obtain mode locked operation, and, time permitting, repeat the above procedures (at least qualitatively.) Confirm that the number of intracavity pulses is as would be expected, and obtain a plot of the time domain behavior. Compare any data with that obtained previously.

5 Summary

Upon completing this experiment, you should have (circumstances permitting) most of the following data in hand:

- Plots of the scanning Fabry-Perot spectra for the CW and mode- locked cases.
- A plot of the time domain behavior under mode-locking for the case of one pulse in the cavity, and also for multiple pulses in the cavity if possible. Discuss the temporal resolution of the detection system (detector and oscilloscope).
- Measurements of the CW and ML laser power.
- Measurements of the locking bandwidth l
- Measurement of the longitudinal mode spacing (i.e., of the driving frequency for mode-locking.)

From these, you can now determine the following:

- The gain bandwidth of a HeNe laser.
- Qualitative behavior of the longitudinal mode spectrum under CW and ML operation (discuss the implications thereof.)
- Estimate the pulse length.
- The energy per pulse and the peak optical power.
- Knowing the mode spacing of the laser, measure the free spectral range of the scanning Fabry-Perot.
- The optical path length of the laser cavity.

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