PART ONE
OWN SHIP AT CENTER

## EXAMPLE 1

## CLOSEST POINT OF APPROACH

## Situation:

Other ship $M$ is observed as follows:

| Time | Bearing | Range (yards) | Rel. position |
| :---: | :---: | :---: | :---: |
| 0908. | $275^{\circ}$ | 12,000 | $M_{1}$ |
| 0913. | $270^{\circ}$ | 10,700 | $M_{2}$ |
| 0916. | $266^{\circ} .5$ | 10,000 | $M_{3}$ |
| 0920. | $260^{\circ}$ | 9,000 | $M_{4}$ |

## Required:

(1) Direction of Relative Movement (DRM).
(2) Speed of Relative Movement (SRM).
(3) Bearing and range at Closest Point of Approach (CPA).
(4) Estimated time of Arrival at CPA.

## Solution:

(1) Plot and label the relative positions $M_{1}, M_{2}$, etc. The direction of the line $M_{1} M_{4}$ through them is the direction of relative movement (DRM): $130^{\circ}$
(2) Measure the relative distance (MRM) between any two points on $M_{1} M_{4}$. $M_{1}$ to $M_{4}=4,035$ yards. Using the corresponding time interval (0920-0908 = $12^{\mathrm{m}}$ ), obtain the speed of relative movement (SRM) from the Time, Distance, and Speed (TDS) scales: 10 knots.
(3) Extend $M_{l} M_{4}$. Provided neither ship alters course or speed, the successive positions of $M$ will plot along the relative movement line. Drop a perpendicular from $R$ to the relative movement line at $M_{5}$. This is the CPA: $220^{\circ}, 6,900$ yards
(4) Measure $M_{1} M_{5}: 9,800$ yards. With this MRM and SRM obtain time interval to CPA from TDS scale: 29 minutes. ETA at CPA $=0908+29=0937$.

## Answer:

(1) DRM $130^{\circ}$.
(2) SRM 10 knots.
(3) CPA $220^{\circ}, 6,900$ yards.
(4) ETA at CPA 0937.


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EXAMPLE 1
Scale: Distance 2:1 yd.

## EXAMPLE 2

## COURSE AND SPEED OF OTHER SHIP

## Situation:

Own ship $R$ is on course $150^{\circ}$, speed 18 knots. Ship $M$ is observed as follows:

| Time | Bearing | Range (yards) | Rel. position |
| :---: | :---: | :---: | :---: |
| 1100. | $255^{\circ}$ | 20,000 | $M_{1}$ |
| 1107..................... | $260^{\circ}$ | 15,700 | $M_{2}$ |
| 1114. | $270^{\circ}$ | 11,200 | $M_{3}$ |

## Required:

(1) Course and speed of $M$.

## Solution:

(1) Plot $M_{1}, M_{2}, M_{3}$, and $R$. Draw the direction of relative movement line (RML) from $M_{1}$ through $M_{3}$. With the distance $M_{1} M_{3}$ and the interval of time between $M_{1}$ and $M_{3}$, find the relative speed (SRM) by using the TDS scale: 21 knots. Draw the reference ship vector er corresponding to the course and speed of $R$. Through $r$ draw vector $r m$ parallel to and in the direction of $M_{1} M_{3}$ with a length equivalent to the SRM of 21 knots. The third side of the triangle, em, is the velocity vector of the ship $M: 099^{\circ}, 27$ knots.

## Answer:

(1) Course $099^{\circ}$, speed 27 knots.


## EXAMPLE 2

Scale: Speed 3:1, Distance 2:1 yd.

## EXAMPLE 3

## COURSE AND SPEED OF OTHER SHIP USING RELATIVE PLOT AS RELATIVE VECTOR

## Situation:

Own ship $R$ is on course $340^{\circ}$, speed 15 knots. The radar is set on the $12-$ mile range scale. Ship $M$ is observed as follows:

| Time | Bearing | Range (mi.) | Rel. position |
| :---: | :---: | :---: | :---: |
| 1000... | $030^{\circ}$ | 9.0 | $M_{1}$ |
| 1006... | $025^{\circ}$ | 6.3 | $M_{2}$ |

## Required:

(1) Course and speed of $M$.

## Solution:

(1) Plot $M$ and $M_{2}$. Draw the relative movement line (RML) from $M_{l}$ through $M_{2}$.
(2) For the interval of time between $M_{1}$ and $M_{2}$, find the distance own ship $R$ travels through the water. Since the time interval is 6 minutes, the distance in nautical miles is one-tenth of the speed of $R$ in knots, or 1.5 nautical miles.
(3) Using $M_{1} M_{2}$ directly as the relative vector $r m$, construct the reference ship true vector er to the same scale as $r m\left(M_{1}-M_{2}\right)$, or 1.5 nautical miles in length.
(4) Complete the vector diagram (speed triangle) to obtain the true vector em of ship $M$. The length of em represents the distance ( 2.5 nautical miles) traveled by ship $M$ in 6 minutes, indicating a true speed of 25 knots.

## Note:

In some cases it may be necessary to construct own ship's true vector originating at the end of the segment of the relative plot used directly as the relative vector. The same results are obtained, but the advantages of the conventional vector notation are lost.

## Answer:

(1) Course $252^{\circ}$, speed 25 knots.

## Note:

Although at least three relative positions are needed to determine whether the relative plot forms a straight line, for solution and graphical clarity only two relative positions are given in examples 3,6 , and 7 .


OWN SHIP AT CENTER

EXAMPLE 3
Scale: 12-mile range setting

## EXAMPLE 4

## CHANGING STATION WITH TIME, COURSE, OR SPEED SPECIFIED

## Situation:

Formation course is $010^{\circ}$, speed 18 knots. At 0946 when orders are received to change station, the guide $M$ bears $140^{\circ}$, range 7,000 yards. When on new station, the guide will bear $240^{\circ}$, range 6,000 yards.

## Required:

(1) Course and speed to arrive on station at 1000 .
(2) Speed and time to station on course $045^{\circ}$. Upon arrival on station orders are received to close to 3,700 yards.
(3) Course and minimum speed to new station.
(4) Time to station at minimum speed.

## Solution:

(1) Plot $M_{I} 140^{\circ}, 7,000$ yards and $M_{2} 240^{\circ}, 6,000$ yards from $R$. Draw em corresponding to course $010^{\circ}$ and speed 18 knots. The distance of 5.0 miles from $M_{1}$ to $M_{2}$ must be covered in 14 minutes. The SRM is therefore 21.4 knots. Draw $r_{1} m$ parallel to $M_{1} M_{2}$ and 21.4 knots in length. The vector $e r_{1}$ denotes the required course and speed: $062^{\circ}, 27$ knots.
(2) Draw $e r_{2}$, course $045^{\circ}$, intersecting $r_{l} m$ the relative speed vector at the 21knot circle. By inspection $r_{2} m$ is 12.1 knots. Thus the distance $M_{1} M_{2}$ of 5.0 miles will be covered in 24.6 minutes.
(3) To $m$ draw a line parallel to and in the direction of $M_{2} M_{3}$. Drop a perpendicular from $e$ to this line at $r_{3}$. Vector $e r_{3}$ is the course and minimum speed required to complete the final change of station: $330^{\circ}, 13.8$ knots.
(4) By measurement, the length of $r_{3} m$ is an SRM of 11.5 knots and the MRM from $M_{2}$ to $M_{3}$ is 2,300 yards. The required maneuver time MRM $/ r_{3} m=6 \mathrm{~min}-$ utes.

## Answer:

(1) Course $062^{\circ}$, speed 27 knots.
(2) Speed 21 knots, time 25 minutes.
(3) Course $330^{\circ}$, speed 13.8 knots.
(4) Time 6 minutes.

## Explanation:

In solution step (1) the magnitude (SRM) of the required relative speed vector $\left(r_{l} m\right)$ is established by the relative distance $\left(M_{l} M_{2}\right)$ and the time specified to complete the maneuver ( $14^{\mathrm{m}}$ ). In solution step (2), however, the magnitude ( 12.1 knots) of the resulting relative speed vector $\left(r_{2} m\right.$ ) is determined by the distance from the head of vector em along the reciprocal of the DRM to the point where the required course $\left(045^{\circ}\right)$ is intersected. Such intersection also establishes the magnitude ( 21 knots) of vector $e r_{2}$. The time ( $25^{\mathrm{m}}$ ) to complete the maneuver is established by the SRM (12.1 knots) and the relative distance (5 miles).
In solution step (3) the course, and minimum speed to make the guide plot along $M_{2} M_{3}$ are established by the shortest true vector for own ship's motion that can be constructed to complete the speed triangle. This vector is perpendicular to the relative vector $\left(r_{3} m\right)$.

In solution step (4) the time to complete the maneuver is established by the relative distance ( 2,300 yards) and the relative speed ( 11.5 knots).


## EXAMPLE 5

## THREE-SHIP MANEUVERS

## Situation

Own ship $R$ is in formation proceeding on course $000^{\circ}$, speed 20 knots. The guide $M$ bears $090^{\circ}$, distance 4,000 yards. Ship $N$ is 4,000 yards ahead of the guide.

## Required:

$R$ and $N$ are to take new stations starting at the same time. $N$ is to take station 4,000 yards on the guide's starboard beam, using formation speed. $R$ is to take $N$ 's old station and elects to use 30 knots.
(1) $N$ ' $s$ course and time to station
(2) $R$ 's course and time to station.
(3) CPA of $N$ and $R$ to guide.
(4) CPA of $R$ to $N$.
(5) Maximum range of $R$ from $N$.

## Solution:

(1) Plot $R, M_{l}, M_{2}$, and $N_{l}$. Draw em. From $M_{l}$ plot $N$ 's new station $N M$, bearing $090^{\circ}$, distance 4,000 yards. From $M_{2}$ plot $N_{3}$ bearing $090^{\circ}$, distance 4,000 yards ( $N$ 's final range and bearing from $M$ ). Draw $N_{l} N M$, the DRM of $N$ relative to $M$. From $m$, draw $m n$ parallel to and in the direction of $N_{l} N M$ intersecting the 20 -knot speed circle at $n . N$ 's course to station is vector en: $090^{\circ}$. Time to station $N_{l} N M / m n$ is 6 minutes.
(2) To $m$, draw a line parallel to and in the direction of $M_{1} M_{2}$ intersecting the 30 -knot speed circle at $r$. $R$ 's course to station is vector er: $017^{\circ}$. Time to station $M_{1} M_{2} / r m$ is 14 minutes.
(3) From $M_{l}$ drop a perpendicular to $N_{l} N M$. At CPA, $N$ bears $045^{\circ}, 2,850$ yards from $M$. From $R$ drop a perpendicular to $M_{1} M_{2}$. At CPA, $R$ bears $315^{\circ}$, 2,850 yards from $M$.
(4) From $r$ draw $r n$. This vector is the direction and speed of $N$ relative to $R$. From $N_{l}$ draw a DRM line of indefinite length parallel to and in the direction of $r n$. From $R$ drop a perpendicular to this line. At CPA, $N$ bears $069^{\circ}, 5,200$ yards from $R$.
(5) The intersection of the DRM line from $N_{I}$ and the line $N M N_{3}$ is $N_{2}$, the point at which $N$ resumes formation course and speed. Maximum range of $N$ from $R$ is the distance $R N_{2}, 6,500$ yards.

## Answer:

(1) $N$ 's course $090^{\circ}$, time 6 minutes
(2) $R$ 's course $017^{\circ}$, time 14 minutes.
(3) CPA of $N$ to $M 2,850$ yards at $045^{\circ}$. $R$ to $M 2,850$ yards at $315^{\circ}$.
(4) CPA of $N$ to $R 5,200$ yards at $069^{\circ}$.
(5) Range 6,500 yards.

## Solution Key:

(1) Solutions for changing station by own ship $R$ and ship $N$ are effected separately in accordance with the situation and requirements. The CPAs of $N$ and $R$ to guide are then obtained.
(2) Two solutions for the motion of ship $N$ relative to own ship $R$ are then obtained: relative motion while $N$ is proceeding to new station and relative motion after $N$ has taken new station and resumed base course and speed.

## Explanation:

In solution step (4) the movement of $N$ in relation to $R$ is parallel to the direction of vector $r n$ and from $N_{I}$ until such time that $N$ returns to base course and speed. Afterwards, the movement of $N$ in relation to $R$ is parallel to vector $r m$ and from $N_{2}$ toward that point, $N_{3}$, that $N$ will occupy relative to $R$ when the maneuver is completed.


## EXAMPLE 5

Scale: Speed 3:1; Distance 1:1 yd.

## EXAMPLE 6

## COURSE AND SPEED TO PASS ANOTHER SHIP AT A SPECIFIED DISTANCE

## Situation 1:

Own ship $R$ is on course $190^{\circ}$, speed 12 knots. Other ship $M$ is observed as follows:

| Time | Bearing | Range (yards) | Rel. position |
| :---: | :---: | :---: | :---: |
| 1730.................................. | $153^{\circ}$ | $153^{\circ}$ | 20,000 |
| $1736 . . . . . . . . .$. | 16,700 | $M_{I}$ |  |
|  |  |  | $M_{2}$ |

## Required:

(1) CPA.
(2) Course and speed of $M$.

## Situation 2:

It is desired to pass ahead of $M$ with a CPA of 3,000 yards.

## Required:

(3) Course of $R$ at 12 knots if course is changed when range is 13,000 yards.
(4) Bearing and time of CPA.

## Solution:

(1) Plot $M_{1}$ and $M_{2}$ at $153^{\circ}, 20,000$ yards and $153^{\circ}, 16,700$ yards, respectively, from $R$. Draw the relative movement line, $M_{1} M_{2}$, extended. Since the bearing is steady and the line passes through $R$, the two ships are on collision courses.
(2) Draw own ship's velocity vector $e r_{1} 190^{\circ}, 12$ knots. Measure $M_{I} M_{2}$, the relative distance traveled by $M$ from 1730 to 1736: 3,300 yards. From the TDS scale determine the relative speed, SRM, using 6 minutes and 3,300 yards: 16.5
knots. Draw the relative speed vector $r_{l} m$ parallel to $M_{1} M_{2}$ and 16.5 knots in length. The velocity vector of $M$ is $\mathrm{em}: 287^{\circ}, 10$ knots.
(3) Plot $M_{3}$ bearing $153^{\circ}, 13,000$ yards from $R$. With $R$ as the center describe a circle of 3,000 yards radius, the desired distance at CPA. From $M_{3}$ draw a line tangent to the circle at $M_{4}$. This places the relative movement line of $M\left(M_{3} M_{4}\right)$ the required minimum distance of 3,000 yards from $R$. Through $m$, draw $r_{2} m$ parallel to and in the direction of $M_{3} M_{4}$ intersecting the 12-knot circle (speed of $R$ ) at $r_{2}$. Own ship velocity vector is $e r_{2}$ : course $212^{\circ}$, speed 12 knots.
(4) Measure the relative distance (MRM), $M_{2} M_{3}: 3,700$ yards. From the TDS scale determine the time interval between 1736 and the time to change to new course using $M_{2} M_{3}, 3,700$ yards, and an SRM of 16.5 knots: 6.7 minutes. Measure the relative distance $M_{3} M_{4}: 12,600$ yards. Measure the relative speed vector $r_{2} m$ : 13.4 knots. Using this MRM and SRM, the elapsed time to CPA after changing course is obtained from the TDS scale: 28 minutes. The time of CPA is $1736+6.7+28=1811$.

Note:
If $M$ 's speed was greater than $R$ 's, two courses would be available at 12 knots to produce the desired distance.

## Answer:

(1) $M$ and $R$ are on collision courses and speeds.
(2) Course $287^{\circ}$, speed 10 knots.
(3) Course $212^{\circ}$.
(4) Bearing $076^{\circ}$, time of CPA 1811.


## EXAMPLE 6

Scale: Speed 2:1; Distance 2:1 yd.

## EXAMPLE 7

## COURSE AND SPEED TO PASS ANOTHER SHIP AT A SPECIFIED <br> DISTANCE USING RELATIVE PLOT AS RELATIVE VECTOR

## Situation 1:

Own ship $R$ is on course $190^{\circ}$, speed 12 knots. Other ship $M$ is observed as follows:

| Time | Bearing | Range (mi.) | Rel. position |
| :---: | :---: | :---: | :---: |
| 1730............... | $153^{\circ}$ | 10.0 | $M_{1}$ |
| $1736 . . . . . . . . . . . . . . . . . . . ~$ | $153^{\circ}$ | 8.3 | $M_{2}$ |

## Required

(1) CPA.
(2) Course and speed of $M$.

## Situation 2:

It is desired to pass ahead of $M$ with a CPA of 1.5 nautical miles

## Required:

(3) Course of $R$ at 12 knots if course is changed when range is 6.5 nautical miles.
(4) Bearing and time of CPA

## Solution:

(1) Plot $M_{I}$ and $M_{2}$ at $153^{\circ}, 10.0$ nautical miles and $153^{\circ}, 8.3$ nautical miles, respectively from $R$. Draw the relative movement line, $M_{1} M_{2}$, extended. Since the bearing is steady and the line passes through $R$, the two ships are on collision courses.
(2) For the interval of time between $M_{1}$ and $M_{2}$, find the distance own ship $R$ travels through the water. Since the time interval is 6 minutes, the distance in nautical miles is one-tenth of the speed of $R$ in knots, or 1.2 nautical miles.
(3) Using $M_{l} M_{2}$ directly as the relative vector $r_{l} m$, construct the reference ship true vector $e r_{1}$ to the same scale as $r_{1} m\left(M_{1} M_{2}\right)$, or 1.2 nautical miles in length.
(4) Complete the vector diagram (speed triangle) to obtain the true vector em of ship $M$. The length of em represents the distance ( 1.0 nautical miles) traveled by ship $M$ in 6 minutes, indicating a true speed of 10 knots.
(5) Plot $M_{3}$ bearing $153^{\circ}, 6.5$ nautical miles from $R$. With $R$ as the center describe a circle of 1.5 nautical miles radius, the desired distance at CPA. From $M_{3}$ draw a line tangent to the circle at $M_{4}$. This places the relative movement line of $M\left(M_{3} M_{4}\right)$ the required minimum distance of 1.5 nautical miles from $R$.
(6) Construct the true vector of ship $M$ as vector $e^{\prime} m^{\prime}$, terminating at $M_{3}$. From $e^{\prime}$ describe a circle of 1.2 miles radius corresponding to the speed of $R$ of 12 knots intersecting the new relative movement line $\left(M_{3} M_{4}\right)$ extended at point $r_{2}$. Own ship $R$ true vector required to pass ship $M$ at the specified distance is vector $e^{\prime} r_{2}$ : course $212^{\circ}$, speed 12 knots.
(7) For practical solutions, the time at CPA may be determined by inspection or through stepping off the relative vectors by dividers or spacing dividers. Thus the time of CPA is $1736+6.5+28=1811$.

## Note:

If the speed of $\operatorname{ship} M$ is greater than own ship $R$, there are two courses available at 12 knots to produce the desired distance.

## Answer:

(1) $M$ and $R$ are on collision courses and speeds.
(2) Course $287^{\circ}$, speed 10 knots.
(3) Course $212^{\circ}$
(4) Bearing $076^{\circ}$, time of CPA 1811.


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EXAMPLE 7
Scale: 12-mile range setting

## EXAMPLE 8

## COURSE AT SPECIFIED SPEED TO PASS ANOTHER SHIP AT MAXIMUM

## AND MINIMUM DISTANCES

## Situation:

Ship $M$ on course $300^{\circ}$, speed 30 knots, bears $155^{\circ}$, range 16 miles from own
hip $R$ whose maximum speed is 15 knots. ship $R$ whose maximum speed is 15 knots.

## Required:

(1) $R$ 's course at 15 knots to pass $M$ at (a) maximum distance (b) minimum distance.
(2) CPA for each course found in (1).
(3) Time interval to each CPA.
(4) Relative bearing of $M$ from $R$ when at CPA on each course.

## Solution:

(1) Plot $M_{l} 155^{\circ}, 16$ miles from $R$. Draw the vector em $300^{\circ}, 30$ knots. With $e$ as the center, describe a circle with radius of 15 knots, the speed of $R$. From $m$ draw the tangents $r_{1} m$ and $r_{2} m$ which produce the two limiting courses for $R$. Parallel to the tangents plot the relative movement lines through $M_{I}$. Course of own ship to pass at maximum distance is $e r_{1}: 000^{\circ}$. Course to pass at minimum distance is $e r_{2}: 240^{\circ}$.
(2) Through $R$ draw $R M_{2}$ and $R M^{\prime}{ }_{2}$ perpendicular to the two possible relative movement lines. Point $M_{2}$ bearing $180^{\circ}, 14.5$ miles is the CPA for course of $000^{\circ}$. Point $M_{2}^{\prime}$ bearing $240^{\circ}, 1.4$ miles is the CPA for course $240^{\circ}$.
(3) Measure $M_{I} M_{2}: 6.8$ miles, and $M_{I} M_{2}^{\prime}: 15.9$ miles. $M$ must travel these relative distances before reaching the CPA on each limiting course. The relative
speed of $M$ is indicated by the length of the vectors $r_{1} m$ and $r_{2} m: 26$ knots. From the TDS scale the times required to reach $M_{2}$ and $M^{\prime}{ }_{2}$ are found: 15.6 minutes and 36.6 minutes, respectively.
(4) Bearings are determined by inspection. $M_{2}$ bears $180^{\circ}$ relative because own ship's course is along vector $e r_{1}$ for maximum CPA. $M_{2}^{\prime}$ bears $000^{\circ}$ relative when own ship's course is $e r_{2}$ for minimum passing distance.

## Note:

This situation occurs only when own ship $R$ is (1) ahead of the other ship and (2) has a maximum speed less than the speed of the other ship. Under these conditions, own ship can intercept (collision course) only if $R$ lies between the slopes of $M_{l} M_{2}$ and $M_{l} M^{\prime}$. Note that for limiting courses, and only for these, CPA occurs when other ship is dead ahead or dead astern. The solution to this problem is applicable to avoiding a tropical storm by taking that course which results in maximum passing distance.

## Answer:

(1) Course (a) $000^{\circ}$; (b) $240^{\circ}$.
(2) CPA (a) $180^{\circ}, 14.5$ miles; (b) $240^{\circ}, 1.4$ miles.
(3) Time (a) 16 minutes; (b) 37 minutes.
(4) Relative bearing (a) $180^{\circ}$; (b) $000^{\circ}$.


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EXAMPLE 8
Scale: Speed 3:1,
Distance 2:1 mi.

## EXAMPLE 9

## COURSE CHANGE IN COLUMN FORMATION ASSURING LAST SHIP IN COLUMN CLEARS

## Situation:

Own ship $D 1$ is the guide in the van of a destroyer unit consisting of four destroyers ( $D 1, D 2, D 3$, and $D 4$ ) in column astern, distance 1,000 yards. $D 1$ is on station bearing $090^{\circ}, 8$ miles from the formation guide $M$. Formation course is $135^{\circ}$, speed 15 knots. The formation guide is at the center of a concentric circular ASW screen stationed on the 4-mile circle.
The destroyer unit is ordered to take new station bearing $235^{\circ}, 8$ miles from the formation guide. The unit commander in $D 1$ decides to use a wheeling maneuver at 27 knots, passing ahead of the screen using two course changes so that the CPA of his unit on each leg is 1,000 yards from the screen.

## Required:

(1) New course to clear screen commencing at 1000
(2) Second course to station.
(3) Bearing and range of $M$ from $D 1$ at time of coming to second course.
(4) Time of turn to second course.
(5) Time $D 1$ will reach new station.

## Solution:

(1) Plot own ship $D 1$ at the center on course $135^{\circ}$ with the remaining three destroyers in column as $D 2, D 3, D 4$. ( $D 2$ and $D 3$ not shown for graphical clarity.) Distance between ships 1,000 yards. Plot the formation guide $M$ at $M_{l}$ bearing $270^{\circ}, 8$ miles from $D 1$. Draw em, the speed vector of $M$. It is required that the last ship in column, $D 4$, clear $M$ by 9,000 yards (screen radius of 4 miles plus 1,000 yards). At the instant the signal is executed to change station, only $D 1$ changes both course and speed. The other destroyers increase speed to 27 knots but remain on formation course of $135^{\circ}$ until each reaches the turning point.

D4's movement of 3,000 yards at 27 knots to the turning point requires 3 minutes, 20 seconds. During this interval there is a 12 knot true speed differential between $D 4$ and the formation guide $M$. Thus to establish the relative position of D4 to M at the instant D4 turns, advance D4 to D4' $\left(3^{\mathrm{m}} 20^{\mathrm{s}} \times 12\right.$ knots $=1,350$ yards). With $D 4^{\prime}$ as a center, describe a CPA circle of radius 9,000 yards. Draw a line from $M_{I}$ tangent to this circle. This is the relative movement line required for $D 4$ to clear the screen by 1,000 yards. Draw a line to $m$ parallel to $M_{1} M_{2}$ intersecting the 27-knot circle at $r_{1}$. This point determines the initial course, $e r_{1}$ : $194^{\circ} .2$.
(2) Plot the final relative position of $M$ at $M_{3}$ bearing $055^{\circ}, 8$ miles from $D 1$. Draw a line from $M_{3}$ tangent to the CPA circle and intersecting the first relative movement line at $M_{2}$. Draw a line to $m$ parallel to and in the direction of $M_{2} M_{3}$. The intersection of this line and the 27-knot circle at $r_{2}$ is the second course required, $e r_{2}: 252^{\circ} .8$.
(3) Bearing and range of $M_{2}$ from $D 1$ is obtained by inspection: $337^{\circ}$ at 11,250 yards.
(4) Time interval for $M$ to travel to $M_{2}$ is $M_{1} M_{2} / r_{1} m=7.8$ miles $/ 23.2$ knots $=$ 20.2 minutes. Time of turn $1000+20=1020$.
(5) Time interval for the second leg is $M_{2} M_{3} / r_{2} m=8.8$ miles $/ 36.5$ knots $=14.2$ minutes. $D 1$ will arrive at new station at 1034 .

## Answer:

(1) Course $194^{\circ}$.
(2) Course $253^{\circ}$.
(3) Bearing $337^{\circ}$, range 11,250 yards.
(4) Time 1020.
(5) Time 1034.


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## EXAMPLE 9

Scale: Speed 3:1, Distance 1:1 mi.

## EXAMPLE 10

## DETERMINATION OF TRUE WIND

## Situation:

A ship is on course $240^{\circ}$, speed 18 knots. The relative wind across the deck is
30 knots from $040^{\circ}$ relative.

## Required:

Direction and speed of true wind.

## Solution:

Plot $e r$, the ship's vector of $240^{\circ}, 18$ knots. Convert the relative wind to apparent wind by plotting $r w 040^{\circ}$ relative to ship's head which results in a true direction of $280^{\circ} \mathrm{T}$. Plot the apparent wind vector (reciprocal of $280^{\circ} \mathrm{T}, 30$ knots) from the end of the vector er. Label the end of the vector $w$. The resultant vector
$e w$ is the true wind vector of $135^{\circ}, 20$ knots (wind's course and speed). The true wind, therefore, is from $315^{\circ}$.

## Answer:

True wind from $315^{\circ}$, speed 20 knots.

## Note:

As experienced on a moving ship, the direction of true wind is always on the same side and aft of the direction of the apparent wind. The difference in directions increases as ship's speed increases. That is, the faster a ship moves, the more the apparent wind draws ahead of true wind.


## EXAMPLE 10

Scale: Speed 3:1

## EXAMPLE 11a

## DESIRED RELATIVE WIND

## (First Method)

## Situation:

An aircraft carrier is proceeding on course $240^{\circ}$, speed 18 knots. True wind has been determined to be from $315^{\circ}$, speed 10 knots.

## Required:

Determine a launch course and speed that will produce a relative wind across the flight deck of 30 knots from $350^{\circ}$ relative ( $10^{\circ}$ port).

## Solution:

Set a pair of dividers for 30 knots using any convenient scale. Place one end of the dividers at the origin $e$ of the maneuvering board and the other on the $350^{\circ}$ line, marking this point $a$. Set the dividers for the true wind speed of 10 knots and place one end on point $a$, the other on the $000^{\circ}$ line (centerline of the ship) Mark this point on the centerline $b$. Draw a dashed line from origin $e$ parallel to
$a b$. This produces the angular relationship between the direction from which the true wind is blowing and the launch course. In this problem the true wind should be from $32^{\circ}$ off the port bow ( $328^{\circ}$ relative) when the ship is on launch course and speed. The required course and speed is thus $315^{\circ}+32^{\circ}=347^{\circ}, 21$ knots.

## Answer:

Course $347^{\circ}$, speed 21 knots.

## Note:

As experienced on a moving ship, the direction of true wind is always on the same side and aft of the direction of the apparent wind. The difference in directions increases as ship's speed increases. That is, the faster a ship moves, the more the apparent wind draws ahead of true wind.


## EXAMPLE 11b

## DESIRED RELATIVE WIND

## (Second Method)

## Situation:

A ship is on course $240^{\circ}$, speed 18 knots. True wind has been determined to be from $315^{\circ}$, speed 10 knots.

## Required:

Determine a course and speed that will produce a wind across the deck of 30 knots from $350^{\circ}$ relative ( $10^{\circ}$ port).

## Solution:

(1) A preliminary step in the desired relative wind solution is to indicate on the polar plotting sheet the direction from which the true wind is blowing. The direction of the true wind is along the radial from $315^{\circ}$.
(2) The solution is to be effected by first finding the magnitude of the required ship's true (course-speed) vector; knowing the true wind (direction-speed) vector and the magnitude ( 30 knots) of the relative wind vector, and that the ship's course should be to the right of the direction from which the true wind is blowing, the vector triangle can then be constructed.
(3) Construct the true wind vector ew.
(4) With a pencil compass adjusted to the true wind ( 10 knots), set the point of the compass on the 30 -knot circle at a point $10^{\circ}$ clockwise from the intersection of the 30 -knot circle with the radial extending in the direction from which the wind is blowing. Strike an arc intersecting this radial. That part of the radial from the center of the plotting sheet to the intersection represents the magnitude of the required ship's true vector ( 21 knots). The direction of a line extend-
ing from this intersection to the center of the arc is the direction of the ship's true vector.
(5) From $e$ at the center of the plotting sheet, strike an arc of radius equal to 21 knots. From $w$ at the head of the true wind vector, strike an arc of radius equal to 30 knots. Label intersection $r$. This intersection is to the right of the direction from which the true wind is blowing.
(6) Alternatively, the ship's true (course-speed) vector can be constructed by drawing vector er parallel to the direction established in (4) and to the magnitude also established in (4). On completing the vector triangle, the direction of the relative wind is $10^{\circ}$ off the port bow.

## Answer:

Course $346^{\circ}$, speed 21 knots.

## Note:

If the point of the compass had been set at a point on the 30 -knot circle $10^{\circ}$ counterclockwise from the radial extending in the direction from which the true wind is blowing in (4), the same magnitude of the ship's true vector would have been obtained. However, the direction established for this vector would have been for a 30 -knot wind across the deck from $10^{\circ}$ off the starboard bow.

* Use that intersection closest to the center of the polar diagram.



## EXAMPLE 11c

## DESIRED RELATIVE WIND

## (Third Method)

## Situation:

A ship is on course $240^{\circ}$ speed 18 knots. True wind has been determined to be from $315^{\circ}$ speed 10 knots.

## Required:

Determine a course and speed that will produce a wind across the deck of 30 knots from $350^{\circ}$ relative ( $10^{\circ}$ port).

## Solution:

(1) A preliminary step in the desired relative wind solution is to indicate on the polar plotting sheet the direction toward which the true wind is blowing. The direction of the true wind is along the radial from $315^{\circ}$.
(2) The solution is to be effected by first finding the magnitude of the required ship's true (course-speed) vector; knowing the true wind (direction-speed) vector and the magnitude ( 30 knots) of the relative wind vector, and that the ship's course should be to the right of the direction from which the true wind is blowing, the vector triangle can then be constructed.
(3) Construct the true wind vector ew.
(4) With a pencil compass adjusted to the true wind ( 10 knots), set the point of the compass on the 30 -knot circle at a point $10^{\circ}$ clockwise from the intersection of the 30 -knot circle with the radial extending in the direction toward which the wind is blowing. Strike an arc intersecting this radial. That part of the radial from the center of the plotting sheet to the intersection represents the magnitude of the required ship's true vector ( 21 knots). The direction of a line extend-
ing from the center of the arc to the intersection with the radial is the direction of the ship's true vector.
(5) From $e$ at the center of the plotting sheet, strike an arc of radius equal to 21 knots. From $w$ at the head of the true wind vector, strike an arc of radius equal to 30 knots. Label intersection $r$. This intersection is to the right of the direction from which the true wind is blowing.
(6) Alternatively, the ship's true (course-speed) vector can be constructed by drawing vector er parallel to the direction established in (4) and to the magnitude also established in (4). On completing the vector triangle, the direction of the relative wind is $10^{\circ}$ off the port bow.

## Answer:

Course $346^{\circ}$, speed 21 knots.

## Note:

If the point of the compass had been set at a point on the 30 -knot circle $10^{\circ}$ counterclockwise from the radial extending in the direction from which the true wind is blowing in (4), the same magnitude of the ship's true vector would have been obtained. However, the direction established for this vector would have been for a 30 -knot wind across the deck from $10^{\circ}$ off the starboard bow.

* Use that intersection closest to the center of the polar diagram.



## PRACTICAL ASPECTS OF MANEUVERING BOARD SOLUTIONS

The foregoing examples and their accompanying illustrations are based upon the premise that ships are capable of instantaneous changes of course and speed. It is also assumed that an unlimited amount of time is available for determining the solutions.
In actual practice, the interval between the signal for a maneuver and its execution frequently allows insufficient time to reach a complete graphical solution. Nevertheless, under many circumstances, safety and smart seamanship both require prompt and decisive action, even though this action is determined from a quick, mental estimate. The estimate must be based upon the principles of relative motion and therefore should be nearly correct. Course and speed can be modified enroute to new station when a more accurate solution has been obtained from a maneuvering board.

Allowance must be made for those tactical characteristics which vary widely between types of ships and also under varying conditions of sea and loading. Experience has shown that it is impractical to solve for the relative motion that occurs during a turn and that acceptable solutions can be found by eye and mental estimate
By careful appraisal of the PPI and maneuvering board, the relative movement of own ship and the guide during a turn can be approximated and an estimate made of the relative position upon completion of a turn. Ship's characteristic curves and a few simple thumb rules applicable to own ship type serve as a basis for these estimates. During the final turn the ship can be brought onto station with small compensatory adjustments in engine revolutions and/or course.

## EXAMPLE 12

## ADVANCE, TRANSFER, ACCELERATION, AND DECELERATION

## Situation:

Own ship $R$ is a destroyer on station bearing $020^{\circ}, 8,000$ yards from the guide $M$. Formation course is $000^{\circ}$, speed 15 knots. $R$ is ordered to take station bearing $120^{\circ}, 8,000$ yards from guide, using 25 knots.

## Required:

(1) Course to new station
(2) Bearing of $M$ when order is given to resume formation course and speed
(3) Time to complete the maneuver.

Solution:
(1) Plot $R$ at the center with $M_{l}$ bearing $200^{\circ}, 8,000$ yards and $M_{2}$ bearing $300^{\circ}, 8,000$ yards. Draw the guide's speed vector em $000^{\circ}, 15$ knots.
By eye, it appears $R$ will have to make a turn to the right of about $150^{\circ}$, accelerating from 15 to 25 knots during the turn. Prior to reaching the new station a reverse turn of about the same amount and deceleration to 15 knots will be required. Assume that $R$ averages 20 knots during each turn.
Using $30^{\circ}$ rudder at 20 knots, a DD calibration curve indicates approximately $2^{\circ}$ turn per second and a 600 yard tactical diameter. Thus, a $150^{\circ}$ turn will re-
quire about 75 seconds and will produce an off-set of about 600 yards. During the turn, $M$ will advance 625 yards ( $1 \frac{1}{4}$ minutes at 15 knots). Plotting this approximate off-set distance on the maneuvering board gives a new relative position of $M_{3}$ at the time the initial turn is completed. Similarly, a new off-set position at $M_{4}$ is determined where $R$ should order a left turn to formation course and reduction of speed to 15 knots.

Draw a line to $m$ parallel to and in the direction of $M_{3} M_{4}$ and intersecting the 25 -knot speed circle at $r$. Vector er is the required course of $158^{\circ}$.
(2) When $M$ reaches point $M_{4}$ bearing $299^{\circ}$, turn left to formation course using $30^{\circ}$ rudder and slow to 15 knots.
(3) Time to complete the maneuver is $M_{3} M_{4} / \mathrm{SRM}+2.5$ minutes $=11,050$ yards/39.8 knots +2.5 minutes $=11$ minutes.

## Answer:

(1) Course $158^{\circ}$.
(2) Bearing $299^{\circ}$.
(3) Time 11 minutes.


OWN SHIP AT CENTER

EXAMPLE 12
Scale: Speed 3:1; Distance 1:1 yd.

## COLLISION AVOIDANCE

Numerous studies and the inventive genius of man have provided the mariner with adequate means for virtually eliminating collisions at sea. One of the most significant of these is radar. However, radar is merely an aid, and is no substitute for good judgment coupled with good seamanship. Its use grants no special license in applying the Rules of the Road in a given situation. Properly interpret-
ed, however, the information it does provide the mariner can be of inestimable value in forewarning him of possible danger.

The following example is a practical problem encountered in the approaches to many of the world's busy ports.

## EXAMPLE 13

## AVOIDANCE OF MULTIPLE CONTACTS

## Situation:

Own ship is proceeding toward a harbor entrance about 30 miles to the southeast. Own ship's course $145^{\circ}$, speed 15 knots. Visibility is estimated to be 2 miles. Numerous radar contacts are being made. At the present time, 2235, six pips are being plotted on the PPI scope.

## Problems:

(1) By visual inspection of the PPI (Fig. 1), which of the contacts appear dangerous and require plotting on a maneuvering board? (Radar is set on 20-mile range scale.)
(2) After plotting the contacts selected in (1), what are their CPA's, true courses and speeds? (Fig. 2 is an example.)
(3) Assume the PPI plots indicate all contacts have maintained a steady course and speed during your solution in (2). What maneuvering action, if any, do you recommend? (Fig. 2 shows one possibility.)
(4) Assume that you maneuver at 2238 and all other ships maintain their courses and speeds. What are the new CPA's of the dangerous contacts in (2) above? (Fig. 2 shows a possible solution.)
(5) Assume that all ships maintain course and speed from 2238 until 2300. What will be the PPI presentation at 2300 ? (Fig. 3 is an example.)
(6) At what time would you return to original course and speed or make other changes?

## Solutions:

(1) Ships $E$ and $F$ look dangerous. Their bearings are almost steady and range is decreasing rapidly. $F$ will reach the center in about one half hour. All other
contacts appear safe enough to merely track on the scope. $A$ is closing, but too slowly to be of concern for several hours. $B$ is overtaking at a very slow rate. $C$ should cross well clear astern in about an hour. $D$ is harmless and needs only cursory checks.

| (2) | CPA | Time | Course | Speed |
| :---: | :---: | :---: | :---: | :---: |
|  | Ship F ... 1,700 yds. | 2258 | $069{ }^{\circ}$ | 7.5 knots |
|  | Ship E ... 1,900 yds. | 2338 | $182^{\circ}$ | 14.0 knots |
| (3) Change course to $180^{\circ}$, maintain 15 knots. |  |  |  |  |
| (4) | CPA |  | Time |  |
|  | Ship F ... 6,300 yds. |  | 2250 |  |
|  | Ship E ... 17,700 yds. |  | (Both own ship and $E$ are now on about the same course with |  |
|  |  |  | $E$ drawing very slowly astern. |  |
|  |  |  | CPA thus has little meaning.) |  |

(5) See Fig. 3. $D$ has faded from the scope.
(6) With $F$ well clear at 2300 , a return to original course appears desirable. Apparently $A, B$, and $C$ also are making the same approach and should cause no trouble. The intentions of $E$ are unknown but you have about an hour's time before convergence.


OWN SHIP AT CENTER

EXAMPLE 13 Figure 1
PPI SCOPE (20-mile scale)

OWN SHIP AT CENTER

EXAMPLE 13 Figure 2
Scale: Speed 2:1; Distance 3:1 yd.



OWN SHIP AT CENTER

EXAMPLE 13 Figure 3
PPI SCOPE (20-mile scale)

## EXAMPLE 14

## AVOIDANCE OF MULTIPLE CONTACTS WITHOUT FIRST DETERMINING

THE TRUE COURSES AND SPEEDS OF THE CONTACTS

## Situation:

Own ship $R$ is on course $000^{\circ}$, speed 20 knots. With the relative motion presentation radar set at the $12-$ mile range setting, radar contacts are observed as follows:

|  | Time 1000 <br> Range (mi.) |  |  |
| :--- | :---: | :---: | :---: |
| Bearing | Rel. position |  |  |
| Contact $A$ | $050^{\circ}$ | 9.0 | $A_{I}$ |
| Contact $B$ | $320^{\circ}$ | 8.0 | $B_{I}$ |
| Contact $C$ | $235^{\circ}$ | 8.0 | $C_{I}$ |
|  |  |  |  |
|  |  | Time 1006 |  |
| Contact $A$ | Bearing | Range (mi.) | Rel. position |
| Contact $B$ | $050^{\circ}$ | 7.5 | $A_{2}$ |
| Contact $C$ | $333^{\circ}$ | 6.0 | $B_{2}$ |
|  | $225^{\circ}$ | 6.0 | $C_{2}$ |

## Required:

(1) Determine the new relative movement lines for contacts $A, B$, and $C$ which would result from own ship changing course to $065^{\circ}$ and speed to 15 knots at time 1006.
(2) Determine whether such course and speed change will result in desirable or acceptable CPA's for all contacts.

## Solution:

(1) With the center of the radarscope as their origin, draw own ship's true vectors $e r$ and $e r^{\prime}$ for the speed in effect or to be put in effect at times 1000 and 1006, respectively. Using the distance scale of the radar presentation, draw each vector of length equal to the distance own ship $R$ will travel through the water
during the time interval of the relative plot (relative vector), 6 minutes. Vector $e r$, having a speed of 20 knots, is drawn 2.0 miles in length in true direction $000^{\circ}$; vector $e r^{\prime}$, having a speed of 15 knots, is drawn 1.5 miles in length in true direction $065^{\circ}$.
(2) Draw a dashed line between $r$ and $r^{\prime}$.
(3) For Contacts $A, B$, and $C$, offset the initial plots $\left(A_{l}, B_{l}\right.$, and $\left.C_{l}\right)$ in the same direction and distance as the dashed line $r-r^{\prime} ;$ label each such offset plot $r^{\prime}$.
(4) In each relative plot, draw a straight line from the offset initial plot, $r^{\prime}$, through the final plot ( $A_{2}$ or $B_{2}$ or $C_{2}$ ). The lines $r^{\prime} A_{2}, r^{\prime} B_{2}$, and $r^{\prime} C_{2}$ represent the new RML's which would result from a course change to $065^{\circ}$ and speed change to 15 knots at time 1006.

## Answer:

(1) New DRM of Contact $A 280^{\circ}$

New DRM of Contact $B 051^{\circ}$
New DRM of Contact $C 028^{\circ}$.
(2) Inspection of the new relative movement lines for all contacts indicates that if all contacts maintain course and speed, all contacts will plot along their respective relative movement lines at safe distances from own ship $R$ on course $065^{\circ}$, speed 15 knots.

## Explanation:

The solution method is based upon the use of the relative plot as the relative vector as illustrated in Example 4. With each contact maintaining true course and speed, the $e m$ vector for each contact remains static while own ship's vector is rotated about $e$ to the new course and changed in magnitude corresponding to the new speed.


Scale: 12-mile range setting

## EXAMPLE 15

## DETERMINING THE CLOSEST POINT OF APPROACH FROM THE GEOGRAPHICAL PLOT

## Situation:

Own ship is on course $000^{\circ}$, speed 10 knots. The true bearings and ranges of another ship are plotted from own ship's successive positions to form a geographical (navigational) plot:

| Time | Bearing | Range (mi.) | True position |
| :---: | :---: | :---: | :---: |
| 0200 | $074^{\circ}$ | 7.3 | $T_{l}$ |
| 0206 | $071^{\circ}$ | 6.3 | $T_{2}$ |
| 0212 | $067^{\circ}$ | 5.3 | $T_{3}$ |

## Required:

(1) Determine the Closest Point of Approach.

## Solution:

(1) Since the successive timed positions of each ship of the geographical plot indicate rate of movement and true direction of travel for each ship, each line segment between successive plots represents a true velocity vector. Equal spacing of the plots timed at regular intervals and the successive plotting of the true positions in a straight line indicate that the other ship is maintaining constant course and speed.
(2) The solution is essentially a reversal of the procedure in relative motion solutions in which, from the relative plot and own ship's true vector, the true vector of the other ship is determined. Accordingly, the true vectors from the two true plots for the same time interval, 0206-0212 for example, are subtracted to obtain the relative vector $(\overrightarrow{\mathrm{mm}}=\overrightarrow{\mathrm{em}}-\overrightarrow{\mathrm{er}})$.
(3) The relative (DRM-SRM) vector $r m$ is extended beyond own ship's 0212 position to form the relative movement line (RML).
(4) The closest point of approach (CPA) is found by drawing a line from own ship's 0212 plot perpendicular to the relative movement line.

## Answer:

(1) CPA $001^{\circ}, 2.2$ miles.

## Explanation:

This solution is essentially a reversal of the procedure in relative motion solutions in which, from the relative plot and own ship's true vector, the true vector of the other ship is determined. See Example 3.

## Notes:

(1) Either the time 0200,0206 , or 0212 plots of the other ship can be used as the origin of the true vectors of the vector diagram. Using the time 0200 plot as the origin and a time interval of 6 minutes for vector magnitude, the line perpendicular to the extended relative movement line would be drawn from the time 0206 plot of own ship.
(2) A practical solution for CPA in the true motion mode of operation of a radar is based on the fact that the end of the Interscan (electronic bearing cursor) moves from the point, at which initially set, in the direction of own ship's course at a rate equivalent to own ship's speed. With the contact at this point, initially, the contact moves away from the point in the direction of its true course at a rate equivalent to its speed. Thus, as time passes, a vector triangle is being continuously generated. At any instant, the vertices are the initial point, the position of the contact, and the end of the Interscan. The side of the triangle between the end of the Interscan and the contact is the rm vector, the origin of which is at the end of the Interscan.
The CPA is found by setting the end of the Interscan at the contact, and, after the vector triangle has been generated, extending the $r m$ vector beyond own ship's position of the PPI.


## EXAMPLE 16

## COURSE AND SPEED BETWEEN TWO STATIONS, REMAINING WITHIN A

## Situation:

Own ship $R$ is on station bearing $280^{\circ}, 5$ miles from the guide $M$ which is on course $190^{\circ}$, speed 20 knots.

## Required:

At 1500 proceed to new station bearing $055^{\circ}, 20$ miles, arriving at 1630 . Remain within a 10 -mile range for 1 hour. The commanding officer elects to proceed directly to new station adjusting course and speed to comply.
(1) Course and speed to remain within 10 miles for 1 hour.
(2) Course and speed required at 1600 .
(3) Bearing of $M$ at 1600 .

## Solution:

(1) Plot the 1500 and 1630 positions of $M$ at $M_{I}$ and $M_{3}$, respectively. Draw the relative motion line, $M_{l} M_{3}$, intersecting the 10 -mile circle at $M_{2}$. Draw em Measure $M_{1} M_{2}: 13.6$ miles. The time required to transit this distance is 1 hour
at an SRM of 13.6 knots. Through $m$ draw $r_{1} m 13.6$ knots in length, parallel to and in the direction $M_{1} M_{3}$. Vector $e r_{1}$ is $147^{\circ} .5,16.2$ knots.
(2) Measure $M_{2} M_{3}, 10.3$ miles, which requires an SRM of 20.6 knots for one half hour. Through $m$ draw $r_{2} m$. Vector $e r_{2}$ is $125^{\circ} .5,18.2$ knots.
(3) By inspection, $M_{2}$ bears $226^{\circ}$ from $R$ at 1600.

## Answer:

(1) Course $148^{\circ}$, speed 16.2 knots.
(2) Course $126^{\circ}$, speed 18.2 knots.
(3) Bearing $226^{\circ}$.

## Explanation:

Since own ship $R$ must remain within 10 miles of the guide for 1 hour, $M$ must not plot along $M_{1} M_{2}$ farther than $M_{2}$ prior to 1600 . The required magnitudes of the relative speed vectors for time intervals 1500 to 1600 and 1600 to 1630 together with their common direction are combined with the true vector of the guide to obtain the two true course vectors for own ship.


OWN SHIP AT CENTER

## EXAMPLE 16

Scale: Speed 3:1, Distance 2:1 mi.

## EXAMPLE 17

## COURSE AT MAXIMUM SPEED TO OPEN RANGE TO A SPECIFIED DISTANCE

## IN MINIMUM TIME

## Situation:

Own ship $R$ has guide $M$ bearing $240^{\circ}$, range 12 miles. The guide is on course $120^{\circ}$, speed 15 knots. Own ship's maximum speed is 30 knots.

## Required:

Open range to 18 miles as quickly as possible.
(1) Course at 30 knots.
(2) Time to complete the maneuver.
(3) Bearing of guide upon arrival at specified range.

## Solution:

The key to this solution is to find that relative position $\left(M^{\prime}\right)$ of the guide that could exist before the problems starts in order to be able to draw the RML through the given relative position $\left(M_{I}\right)$ and $M^{\prime}$ to intersect the specified range circle.
(1) Plot $R$ and $M_{l}$. About $R$ describe a circle of radius 18 miles. Draw em. On the reciprocal of $M^{\prime}$ 's course plot $M^{\prime} 9$ miles from $R$.

$$
\frac{\text { Speed of } M}{\text { Speed of } R} \times 18 \text { miles }=9 \text { miles }
$$

Draw a line through $M^{\prime}$ and $M_{l}$ and extend it to intersect the 18-mile range circle at $M_{2}$.
Through $m$ draw $r m$ parallel to and in the direction $M_{1} M_{2}$. The intersection of $r m$ and the 30 -knot speed circle is the course required to complete the maneuver in minimum time. Vector er is $042^{\circ} .6,30$ knots.
(2) SRM is 30.5 knots. MRM is 7.5 miles. Time to complete the maneuver: 14.8 minutes.
(3) Upon reaching the 18 -mile range circle, $M$ is dead astern of $R$ bearing $222^{\circ}$. 6 .

## Answer:

(1) Course $043^{\circ}$.
(2) Time 15 minutes.
(3) Bearing $223^{\circ}$.

## Explanation:

For $R$ to open or close to a specified range in minimum time, $R$ must travel the shortest geographical distance at maximum speed. The shortest distance is along the radius of a circle centered at the position occupied by $M$ at the instant $R$ reaches the specified range circle.

In the "opening range" problem, determine hypothetical relative positions of $M$ and $R$ that could exist before the problem starts. Referring to the geographical plot, assume $R$ starts from position $R^{\prime}$ and proceeds outward along some radius 18 miles in length on an unknown course at 30 knots. If $M$ moves toward its final position at $M_{2}$ along the given course of $120^{\circ}$, speed 15 knots, it should arrive at $M_{2}$ the instant $R$ reaches the 18 -mile circle. At this instant, the problem conditions are satisfied by $R$ being 18 miles distant from $M$. However, own ship's course required to reach this position is not yet known. During the time interval $R$ opens 18 miles at 30 knots, $M$ moves 9 miles at 15 knots from $M^{\prime}$ on $M$ 's track. This provides the needed second relative position of $M^{\prime}$ from $R^{\prime}, 9$ miles bearing $300^{\circ}$. This position is then transferred to the relative plot.


OWN SHIP AT CENTER

EXAMPLE 17
Scale: Speed 3:1; Distance 2:1 mi.

## EXAMPLE 18

## COURSE AT MAXIMUM SPEED TO CLOSE RANGE TO A SPECIFIED DISTANCE

## IN MINIMUM TIME

## Situation:

Own ship $R$ has the guide $M$ bearing $280^{\circ}$, range 10 miles. The guide is on course $020^{\circ}$, speed 15 knots. Own ship's maximum speed is 24 knots.

## Required:

Close range to 2 miles as quickly as possible.
(1) Course at 24 knots.
(2) Time to complete the maneuver.
(3) Bearing of guide upon arrival at the specified range.

## Solution:

The key to this solution is to find that relative position $\left(M^{\prime}\right)$ of the guide that could exist after the problem starts in order to be able to draw the RML through the given relative position $\left(M_{I}\right)$ and $M^{\prime}$ to intersect the specified range circle.
(1) Plot $R$ and $M_{l}$. About $R$ describe a circle of radius 2 miles. Draw em. On $M$ 's course plot $M^{\prime} 1.25$ miles from $R$.

$$
\frac{\text { Speed of } M}{\text { Speed of } R} \times 2 \text { miles }=1.25 \text { miles }
$$

Draw a line through $M^{\prime}$ and $M_{l}$. The intersection of this line and the 2-mile range circle is $M_{2}$.
To $m$ draw a line parallel to and in the direction $M_{l} M_{2}$. The intersection of this line and the 24 -knot speed circle is the course required to complete the maneuver in minimum time. Vector er is $309^{\circ} .8,24$ knots.
(2) SRM is 23.6 knots. MRM is 8.3 miles. Time to complete the maneuver: 21.1 minutes.
(3) Upon reaching the 2-mile range circle, $M$ is dead ahead of $R$ on a bearing $309^{\circ} .8$.

## Answer:

(1) Course $310^{\circ}$.
(2) Time 21 minutes.
(3) Bearings $310^{\circ}$.

## Explanation:

For $R$ to open or close to a specified range in minimum time, $R$ must travel the shortest geographical distance at maximum speed. The shortest distance is along the radius of a circle centered at the position occupied by $M$ at the instant $R$ reaches the specified range circle.
In the "closing range" problem, determine hypothetical relative positions of $M$ and $R$ that could exist after the problem ends. Referring to the geographical plot, assume $R$ starts from position $R_{l}$ and proceeds inward along some radius on an unknown course at 24 knots. If $M$ moves toward its final position at $M_{2}$ along the given course $020^{\circ}$, speed 15 knots, it should arrive at $M_{2}$ the instant $R$ reaches the 2-mile circle. At this instant the problem conditions are satisfied although the solution for own ship's course is not yet known. Assume that $R$ continues on the same course and speed through the 2 miles to the center of the circle while $M$ moves away from the center on course $020^{\circ}$, speed 15 knots. During the time interval $R$ moves these 2 miles at 24 knots, $M$ opens 1.25 miles. This provides the needed second relative position of $M^{\prime}$ from $R^{\prime}: 1.25$ miles, bearing $020^{\circ}$. This position is then transferred to the relative plot.


## EXAMPLE 19

## COURSE AT MAXIMUM SPEED TO REMAIN WITHIN A SPECIFIED RANGE

 FOR MAXIMUM TIME
## Situation:

Ship $M$ bears $110^{\circ}, 4$ miles from $R . M$ is on course $230^{\circ}, 18$ knots. Maximum speed of $R$ is 9 knots.

## Required:

Remain within a $10-$ mile range of $M$ for as long as possible.
(1) Course at maximum speed.
(2) Bearing of $M$ upon arrival at specified range.
(3) Length of time within specified range.
(4) CPA.

## Solution:

(1) Plot $R$ and $M$. About $R$ describe circles of radius 9 knots and range 10 miles. Draw em. On $M$ 's course, plot $M^{\prime} 20$ miles from $R$.

$$
\frac{\text { Speed of } M}{\text { Speed of } R} \times 10 \text { miles }=20 \text { miles }
$$

Draw a line through $M^{\prime}$ and $M_{1}$. The intersection of the 10-mile range circle and $M^{\prime} M_{1}$ is $M_{2}$, the point beyond which the specified or limiting range is exceeded. Through $m$ draw $r m$ parallel to and in the direction $M_{1} M_{2}$. The intersection of $r m$ and the 9 -knot speed circle is the course required for $R$, at 9 knots, to remain within 10 miles of $M$. Vector er is $220^{\circ} .8,9$ knots.
(2) Upon arrival at limiting range at $M_{2}, M$ is dead ahead of $R$ bearing $220^{\circ} .8$.
(3) The time interval within specified range is:

$$
\frac{\mathrm{M}_{1} \mathrm{M}_{2}}{\mathrm{rm}}=\frac{12 \text { miles }}{9.1 \text { knots }}=78.8 \text { minutes }
$$

(4) Drop a perpendicular from $R$ to $M_{1} M_{2}$. CPA is $148^{\circ} .9,3.1$ miles.

## Note:

When $R$ 's speed is equal to or greater than that of $M$, a special case exists in which there is no problem insofar as remaining within a specified range.

## Answer:

(1) Course $221^{\circ}$.
(2) Bearing $221^{\circ}$.
(3) Time 79 minutes.
(4) CPA $149^{\circ}, 3.1$ miles.

## Explanation:

As in the "closing range" problem, example 18, determine hypothetical relative positions of $M$ and $R$ that could exist after the problem ends. Referring to the geographical plot, assume $R$ starts from position $R_{l}$ and proceeds inward along some radius on an unknown course at 9 knots. $M$ is on course $230^{\circ}$ at 18 knots. At the instant $M$ passes through $M_{2}, R$ reaches the 10 -mile limiting range at $R_{2}$. At this instant the problem conditions are satisfied although the solution is not yet known. Assume that $R$ continues on the same course and speed the 10 miles to the center of the circle while $M$ moves away from the center on course $230^{\circ}$, speed 18 knots. During the time interval $R$ closes 10 miles at 9 knots, $M$ opens 20 miles at 18 knots. This provides the needed second relative position of $M^{\prime}$ from $R^{\prime}, 20$ miles bearing $230^{\circ}$. This position is then transferred to the relative plot.


## EXAMPLE 19

Scale: Speed 2:1; Distance 2:1 mi.

## EXAMPLE 20

## COURSE AT MAXIMUM SPEED TO REMAIN OUTSIDE OF A SPECIFIED RANGE FOR MAXIMUM TIME

## Situation:

Ship $M$ bears $020^{\circ}, 14$ miles from own ship $R . M$ is on course $210^{\circ}$, speed 18 knots. Maximum speed of $R$ is 10 knots.

## Required:

Remain outside a 10 -mile range from $M$ for as long as possible.
(1) Course at maximum speed.
(2) Bearing of $M$ upon arrival at specified range.
(3) Time interval before reaching specified range.

## Solution:

(1) Plot $R$ and $M_{l}$. About $e$ and $R$, describe circles of radius 10 knots and 10 miles. Draw em. On the reciprocal of $M^{\prime}$ s course, plot $M^{\prime} 18$ miles from $R$.

$$
\frac{\text { Speed of } M}{\text { Speed of } R} \times 10 \text { miles }=18 \text { miles }
$$

Draw a line through $M^{\prime}$ and $M_{1}$ intersecting the 10-mile range circle at $M_{2}$ and $M_{3}$.

To $m$ draw a line parallel to and in the direction of $M_{1} M_{2}$ intersecting the 10knot speed circle at $r_{1}$ and $r_{2} . M_{2}$ and $e r_{1}$ are selected for use in completing the solution. $M_{2}$ is the first point at which limiting range is reached and $r_{l} m$ is the minimum relative speed vector which gives the maximum time. Vector $e r_{1}$ is $175^{\circ} .9,10$ knots.
(2) Upon arrival at limiting range at point $M_{2}, M$ is dead astern of $R$ bearing $355^{\circ} .9$.
(3) The time interval outside of specified range is:

$$
\frac{M_{1} M_{2}}{r_{1} m}=\frac{6.3 \text { miles }}{11.1 \mathrm{knots}}=34.2 \text { minutes }
$$

## Note:

Own ship can remain outside the limiting range indefinitely if $M_{l}$ falls outside the area between two tangents drawn to the limiting range circle from $M^{\prime}$.

## Answer:

(1) Course $176^{\circ}$.
(2) Bearing $356^{\circ}$.
(3) Time 34 minutes.

## Explanation:

To determine a course to remain outside of a given range for maximum time, determine hypothetical relative positions of $M$ and $R$ that could exist before the problem starts. Referring to the geographical plot, assume $R$ starts from position $R^{\prime}$ and proceeds outward along some radius on an unknown course at 10 knots. If $M$ moves toward its final position at $M_{2}$ along the given course $210^{\circ}$, speed 18 knots, it should arrive at $M_{2}$ the instant $R$ reaches the 10 mile circle at $R_{2}$. At this instant the problem conditions are satisfied although the solution for own ship's course is not yet known. During the time interval required for $R$ to move from $R^{\prime}$ to $R_{2}, 10$ miles at 10 knots, $M$ moves from $M^{\prime}$ to $M_{2}, 18$ miles at 18 knots along the given course $210^{\circ}$. This provides the needed second relative positions. $M^{\prime}$ bears $030^{\circ}, 18$ miles from $R^{\prime}$. This position is then transferred to the relative plot.


Scale: Speed 2:1; Distance 2:1 mi.

## USE OF A FICTITIOUS SHIP

The examples given thus far have been confined to ships that have either maintained constant courses and speeds during a maneuver or else have engaged in a succession of such maneuvers requiring only repeated application of the same principles. When one of the ships alters course and/or speed during a maneuver a preliminary adjustment is necessary before these principles can be applied.
This adjustment consists, in effect, of substituting a fictitious ship for the ship making the alteration. This fictitious ship is presumed to:
(1) maintain a constant course and speed throughout the problem (this is the final course and speed of the actual ship).
(2) start and finish its run at times and positions determined by the conditions established in the problem.
For example, the course and speed of advance of a ship zig-zagging are considered to be the constant course and speed of a fictitious ship which departs from a given position at a given time simultaneously with the actual ship, and arrives simultaneously with the actual ship at the same final position. The principles discussed in previous examples are just as valid for a fictitious ship as for an actual ship, both in the relative plot and speed triangle. A geographical plot facilitates the solution of problems of this type.

## EXAMPLE 21

## ONE SHIP ALTERS COURSE AND/OR SPEED DURING MANEUVER

## Situation:

At 0630 ship $M$ bears $250^{\circ}$, range 32 miles. $M$ is on course $345^{\circ}$, speed 15 knots but at 0730 will change course to $020^{\circ}$ and speed to 10 knots.

## Required:

Own ship $R$ takes station 4 miles on the starboard beam of $M$ using 12 knots speed.

1) Course to comply.
(2) Time to complete maneuver

## Solution:

The key to this solution is to determine the 0630 position of a fictitious ship that by steering course $020^{\circ}$, speed 10 knots, will pass through the actual ship's 0730 position. In this way the fictitious ship travels on a steady course of $020^{\circ}$ and speed 10 knots throughout the problem.
(1) Plot $R, M_{1}$, and $M_{3}$. Draw $e m_{l}$ and $e m_{2} / e m f$

Construct a geographical plot with initial position $M_{1}$. Plot $M_{1}$ and $M_{2}, M$ 's 0630-0730 travel along course $345^{\circ}$, distance 15 miles. Plot $M F_{1}$, the fictitious ship's initial position, on bearing $200^{\circ}, 10$ miles from $M_{2} . M F_{1}$ to $M F_{2}$ is the fictitious ship's 0630-0730 travel.

Transfer the relative positions of $M_{l}$ and $M F_{l}$ to the relative plot. $M F_{l} M F_{3}$ is the required DRM and MRM for problem solution. Draw $r m_{2}$ parallel to and in the direction of $M F_{1} M F_{3}$. The intersection of $r m_{2}$ and the 12-knot speed circle is the course, er: $303^{\circ}$, required by $R$ in changing stations while $M$ maneuvers.
(2) The time to complete the maneuver is obtained from the TDS scale using fictitious ship's MRM from $M F_{1}$ to $M F_{3}$ and the SRM of $r m f$.

## Answer:

(1) Course $303^{\circ}$.
(2) Time 2 hours 29 minutes.


Scale: Speed 2:1; Distance 4:1 mi.

## EXAMPLE 22

## BOTH SHIPS ALTER COURSE AND/OR SPEED DURING MANEUVER

## Situation:

At $0800 M$ is on course $105^{\circ}$, speed 15 knots and will change course to $350^{\circ}$, speed 18 knots at 0930 . Own ship $R$ is maintaining station bearing $330^{\circ}, 4$ miles from $M . R$ is ordered to take station bearing $100^{\circ}, 12$ miles from $M$, arriving at 1200.

## Required:

(1) Course and speed for $R$ to comply if maneuver is begun at 0800 .
(2) Course for $R$ to comply if $R$ delays the course change as long as possible and remains at 15 knots speed throughout the maneuver.
(3) Time to turn to course determined in (2).

## Solution:

Since the relative positions of $R$ and $M$ at the beginning and end of the maneuver and the time interval for the maneuver are given, the solution for (1) can be obtained directly from a geographical plot. Solve the remainder of the problem using a relative plot.
(1) Using a geographical plot, lay out $M^{\prime}$ 's 0800-1200 track through points $M_{I}$, $M_{2}$, and $M_{3}$. Plot $R_{1}$ and $R_{3}$ relative to $M_{I}$ and $M_{3}$, respectively. The course of $040^{\circ}$ from $R_{l}$ to $R_{3}$ can be measured directly from the plot. $R$ will require a speed of 10.8 knots to move 43.4 miles in 4 hours.
(This solution can be verified on the relative plot. First, using a geographical plot, determine the 0800 position of a fictitious ship, $M F_{1}$, such that by departing this point at 0800 on course $350^{\circ}, 18$ knots it will arrive at point $M F_{2}$ simultaneously with the maneuvering ship $M . M F_{l}$ bears $141^{\circ}, 41.7$ miles from $M_{l}$. Transfer the positions of $M_{l}$ and $M F_{1}$ to the relative plot. Plot $R$ and $M_{2}$. Draw
the fictitious ship's vector, $e m f_{l}$. To $m f_{l}$ construct the SRM vector parallel to $M F_{1} M F_{2}$ and 13.8 knots in length. Vector $e r_{1}$ is the required course of $040^{\circ}$.)
(2) To find the two legs of $R^{\prime}$ s 0800-1200 track, use a relative plot. Draw $e r_{2}$, own ship's speed vector which is given as $105^{\circ}, 15$ knots. At this stage of the solution, disregard $M$ and consider own ship $R$ to maneuver relative to a new fictitious ship. Own ship on course $040^{\circ}, 10.8$ knots from part (1) is the fictitious ship used. Label vector $e r_{1}$ as $e m f_{2}$, the fictitious ship's vector. From point $r_{2}$ draw a line through $m f_{2}$ extended to intersect the 15 -knot speed circle at $r_{3}$. Draw $e r_{3}$, the second course of $012^{\circ}$ required by $R$ in changing station.
(3) To find the time on each leg draw a time line from $r_{2}$ using any convenient scale. Through $r_{3}$ draw $r_{3} \mathrm{X}$. Through $r_{1}$ draw $r_{1} \mathrm{Y}$ parallel to $r_{3} \mathrm{X}$. Similar triangles exist; thus, the time line is divided into proportional time intervals for the two legs: XY is the time on the first leg: 1 hour 22 minutes. The remainder of the 4 hours is spent on the second leg.

## Answer:

(1) Course $040^{\circ}, 10.8$ knots.
(2) Course $012^{\circ}$.
(3) Time 0922.

## Note:

In the above example, an alternative construction of the time line as defined in the glossary is used so that the line can be drawn to a convenient scale. The proportionality is maintained by constructing similar triangles. See Note with example 24


OWN SHIP AT CENTER

## EXAMPLE 22

Scale: Speed 2:1; Distance 4:1 mi.

## EXAMPLE 23

## COURSES AT A SPECIFIED SPEED TO SCOUT OUTWARD ON PRESENT

## BEARING AND RETURN AT A SPECIFIED TIME

## Situation:

Own ship $R$ is maintaining station on $M$ which bears $110^{\circ}$, range 5 miles. Formation course is $055^{\circ}$, speed 15 knots.

## Required:

Commencing at 1730, scout outward on present bearing and return to present station at 2030. Use 20 knots speed.
(1) Course for first leg.
(2) Course for second leg.
(3) Time to turn.
(4) Maximum distance from the guide.

## Solution:

(1) Plot $R$ and $M_{I}$. Draw em. The DRM "out" is along the bearing of $M$ from $R$. The DRM "in" is along the bearing of $R$ from $M$. Through $m$ draw a line parallel to the DRM's intersecting the 20-knot circle at $r_{1}$ and $r_{2}$. Vector $r_{1} m$ is the DRM "out". Vector $e r_{l}$ is $327^{\circ} .8$, the course "out".
(2) Vector $r_{2} m$ is the DRM "in". Vector $e r_{2}$ is $072^{\circ}$, the course "in".
(3) To find the time on each leg, draw a time line from $r_{l}$ using any convenient scale. Through $r_{2}$ draw $r_{2} \mathrm{X}$. Through $m$ draw $m \mathrm{Y}$ parallel to $r_{2} \mathrm{X}$. Similar triangles exist; thus, the time line is divided into proportional time intervals for the two legs. XY is the time on the first leg, 41 minutes. The remainder of the time is spent on the second leg returning to station.
(4) Range of $M$ when course is changed to "in" leg is 21.7 miles. Initial range $+\left(r_{l} m \times\right.$ time on "out" leg).

## Answer:

(1) Course $328^{\circ}$.
(2) Course $072^{\circ}$.
(3) Time 1811
(4) Distance 21.7 miles.

## Explanation:

Since own ship $R$ returns to present station, relative distances out and in are equal. In going equal distances, time varies inversely as speed:

$$
\frac{\text { time (out) }}{\text { time (in) }}=\frac{\text { relative speed (in) }}{\text { relative speed (out) }}=\frac{r_{1} m(\mathrm{in})}{r_{2} m(\mathrm{out})}
$$

Therefore, the time out part of the specified time $\left(3^{h}\right)$ is obtained by simple proportion or graphically.

As defined in the glossary, the time line is the line joining the heads of vectors $e r_{1}$ and $e r_{2}$. This line is divided by the head of vector $e m$ into segments inversely proportional to the times spent by own ship $R$ on the first (out) and second (in) legs. In the above example an alternative construction is used so that the line can be drawn to a convenient scale. The proportionality is maintained by constructing similar triangles.


Scale: Speed 2:1; Distance 2:1 mi.

## EXAMPLE 24

## COURSES AND MINIMUM SPEED TO CHANGE STATIONS WITHIN

## A SPECIFIED TIME, WHILE SCOUTING ENROUTE

## Situation:

Own ship $R$ bears $130^{\circ}$, 8 miles from the guide $M$ which is on course $040^{\circ}$ speed 12 knots.

## Required:

Proceed to new station bearing $060^{\circ}, 10$ miles from the guide, passing through a point bearing $085^{\circ}, 25$ miles from the guide. Complete the maneuver in 4.5 hours using minimum speed.
(1) First and second courses for $R$
(2) Minimum speed.
(3) Time to turn to second course.

## Solution:

(1) Plot $M_{1}, M_{2}$ and $M_{3}$. Draw em . From $m$ draw lines of indefinite length parallel to and in the direction of $M_{1} M_{2}$ and $M_{2} M_{3}$. Assume that a fictitious ship $M F$, departs $M_{I}$ simultaneously with $M$ and proceeds directly to $M_{3}$ arriving at the same time as $M$ which traveled through $M_{2}$ enroute. The fictitious ship covers a relative distance of 10.5 miles in 4.5 hours. SRM of the fictitious ship is 2.3 knots. To $m$ draw $m f m 2.3$ knots in length parallel to and in the direction of $M_{1} M_{3}$. Vector emf is the true course and speed vector of the fictitious ship. With $m f$ as a pivot, rotate a straight line so that it intersects the two previously drawn lines on the same speed circle. The points of intersection are $r_{1}$ and $r_{2}$. Vector $e r_{l}$ is the course out: $049^{\circ}$. Vector $e r_{2}$ is the course in: $316^{\circ} .9$.
(2) Vectors $r_{1}$ and $r_{2}$ lie on the 17.2 knot circle which is the minimum speed to complete the maneuver.
(3) From $r_{2}$ lay off a 4.5 hour time line using any convenient scale. Draw $r_{1} \mathrm{X}$. Draw $m f$ Y parallel to $r_{l} \mathrm{X}$. The point Y divides the time line into parts that are inversely proportional to the relative speeds $r_{2} m f$ and $r_{1} m f$. XY the time "in" is 51 minutes. $\mathrm{Y} r_{2}$ the time "out" is 3 hours 39 minutes. Time on each leg may also be determined mathematically by the formula MRM/SRM=time.

## Answer:

(1) First course $049^{\circ}$, second course $317^{\circ}$
(2) Speed 17.2 knots.
(3) Time 3 hours and 39 minutes.

## Note:

The time line, as defined in the glossary, is the line joining the heads of vectors $e r_{l}$ and $e r_{2}$ and touching the head of the fictitious ship vector emf. This time line is divided by the head of the fictitious ship vector into segments inversely proportional to the times spent by the unit on the first and second legs.
In the above example, an alternative construction of the time line is used so that the line can be drawn to a convenient scale. The proportionality is maintained by constructing similar triangles.


Scale: Speed 2:1; Distance 3:1 mi.

## EXAMPLE 25

## COURSE, SPEED, AND POSITION DERIVED FROM BEARINGS ONLY

## Situation:

Own ship is on course $090^{\circ}$, speed 15 knots. The true bearings of another ship are observed as follows:

| Time | Bearing |
| :--- | :--- |
| 1300 | $010^{\circ}$ |
| 1430 | $358^{\circ}$ |
| 1600 | $341^{\circ}$ |

At 1600 own ship changes course to $050^{\circ}$ and increases speed to 22 knots. The following bearings of ship $M$ are then observed:

| Time | Bearing |
| :--- | :--- |
| 1630 | $330^{\circ}$ |
| 1730 | $302^{\circ}$ |
| 1830 | $274^{\circ} .5$ |

## Required:

(1) Course and speed of ship $M$.
(2) Distance of $M$ at time of last bearing.

## Solution:

(1) Draw own ship's vector $e r_{1}$.
(2) Plot first three bearings and label in order observed, $B_{1}, B_{2}$, and $B_{3}$.
(3) At any point on $B_{1}$, construct perpendicular which intersects $B_{2}$ and $B_{3}$. Label these points $P_{1}, P_{2}$, and $P_{3}$.
(4) Measure the distance $P_{1}$ to $P_{2}$ and plot point $X$ at the same distance from $P_{2}$ towards $P_{3}$.
(5) From $X$ draw a line parallel to $B_{I}$ until it intersects $B_{3}$. Label this intersection $Y$.
(6) From $Y$ draw a line through $P_{2}$ until it intersects $B_{l}$ at $Z$.
(7) From head of own ship's vector $e r_{l}$, draw a line parallel to $Y Z$. This establishes the DRM on the original course and speed. The head of the em vector of ship $M$ lies on the line drawn parallel to $Y Z$. It is now necessary to find the DRM following a course and/or speed change by own ship. The intersection of the two lines drawn in the direction of relative movement from the heads of own ship's vector establishes the head of vector em .
(8) Following course and speed change made to produce a good bearing drift, three more bearings are plotted; the new direction of relative movement is obtained following the procedure given in steps (3) through (7). The lines drawn in the directions of relative movement from the heads of vector $e r_{1}$ and $e r_{2}$ intersect at the head of the vector em . Ship $M$ is on course $170^{\circ}$ at 10 knots.
(9) From relative vector $r_{2} m$, the SRM is found as 28.4 knots during the second set of observations.
(10) Compute the relative distance traveled during the second set of observations (MRM 56.8 mi.$)$.
(11) On the line $Z Y$ for the second set of observations, lay off the relative distance $Z A$. From $A$ draw a line parallel to $B_{4}$ until it intersects $B_{6}$. Label this point $B$. This is the position of $M$ at the time of the last bearing.

## Answer:

(1) Course $170^{\circ}$, speed 10 knots.
(2) Position of $M$ at $1830: 274^{\circ} .5$ at 61 miles.

## Note:

These procedures are based on bearings observed at equal intervals. For unequal intervals, use the following proportion:
$\frac{\text { Time difference between } B_{1} \text { and } B_{2}}{\text { Distance from } P_{1} \text { to } P_{2}}=\frac{\text { Time difference between } B_{2} \text { and } B_{3}}{\text { Distance from } P_{2} \text { to } X}$.


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EXAMPLE 25
Scale: Speed 3:1; Distance 10:1 mi.

## EXAMPLE 26

## LIMITING LINES OF APPROACH

## (single ship)

## Situation:

Own ship $R$ 's course and speed is $000^{\circ}, 20$ knots. At 0930, both sonar and radar report a contact bearing $085^{\circ}$, distance 22,500. At 0931, radar loses contact and at 0932 sonar loses contact. Last known position was $085^{\circ}$, distance 20,000 Datum error is 1,000 yards.

## Required:

(1) Advanced position.
(2) Limiting lines of approach for submarine with maximum quiet speed of 15 knots.

## Solution:

(1) Plot $R$ at center of maneuvering board and draw the vector "er" $000^{\circ}, 20$ knots.
(2) Plot datum position from own ship $\left(085^{\circ}, 20,000\right.$ yards).
(3) Plot datum error (circle of radius 1,000 yards) around datum.
(4) Compute own ship's advanced position using the formula:
$\frac{\text { Torpedo Firing Range }}{\text { Torpedo Speed }} \times$ Vessel Speed $=\frac{10,000 \mathrm{yds}}{45 \mathrm{kts}} \times 20 \mathrm{kts}=4,444 \mathrm{yds}$
(5) Plot advanced position along own ship's course and speed vector.
(6) Plot Torpedo Danger Zone (10,000 yard circle) around advanced position.
(7) From " $r$ ", describe an arc with a radius of 15 nautical miles (the assumed quiet speed of the submarine).
(8) Draw the tangent vector " $e M q$ " until it intersects the edge of the maneuvering board plotting circle. Do this on both sides of the ship's head. The true bearing of the tangent lines are the limiting lines of approach.
(9) Parallel the tangent vectors " $e M q$ " until they are tangent to the Torpedo Danger Zone to complete the plotting picture.

## Answer:

(1) Advanced position $=4,444$ yards.
(2) Left side limiting line $=310^{\circ}$.

Right side limiting line $=050^{\circ}$.
Limiting lines of approach $=310^{\circ}-050^{\circ}$.

Notes:
(1) Limiting lines of approach are read clockwise.
(2) This example assumes the submarine maintains a constant speed throughout the approach.
(3) The submarine and torpedo data were chosen for example purposes and should NOT be used as real estimates. Consult appropriate intelligence publications for correct data.


# EXAMPLE 26 

Scale: Speed 1:1;
Distance 1:1 mi.

## EXAMPLE 27a

## CONES OF COURSES

Solution: 1

## Situation:

Own ship $R$ is on course $000^{\circ}, 15$ knots. At 1600 , submarine $M$ is reported bearing $325^{\circ}, 40$ miles from $R$. Maximum assumed speed for $M$ is 10 knots.

## Required:

(1) Courses at 10 knots the submarine $M$ will steer to intercept $R$
(2) Time of the first and last intercept opportunities for submarine $M$ against $R$ at the assumed speed of 10 knots.

## Solution:

(1) Plot the 1600 position of the submarine $M 325^{\circ}, 40$ miles from $R$. Draw the vector "er" $000^{\circ}, 15$ knots. From $M$, draw a DRM line to $R$ and from " $r$ " draw the vector " $r m$ " parallel and in the same direction as the DRM. With " $e$ " as the center, describe an arc with radius of 10 knots, the assumed speed of $M$. The points $e m_{l}$ and $e m_{2}$ where the arc intersects the "rm" vector, define the courses at 10 knots that the submarine will steer to intercept $R$. Courses between "em" and "em" are lower assumed speed intercepts and "em $m_{L}$ ", the perpendicular line from $R$ to " $r m$ ", is the course for the lowest possible assumed speed at which the submarine can move and still intercept $R$.
(2) Parallel the "em" and "em" lines as vectors to the 1600 position at $M$ and extend "er" until it crosses these vectors; the area enclosed by these 3 vectors represents the true geographic area through which the submarine will move at or below 10 knots to intercept $R$. The elapsed times to the first (" $t_{1}$ ") and the last (" $\mathrm{t}_{2}$ ") intercept opportunities is obtained by dividing the relative distance at $1600(R M)$ by the respective relative speed (" $r m_{l}$ " and " $r m_{2}$ ").

## Answer:

(1) Courses $024^{\circ}$ to $086^{\circ}$.

$$
\text { (2) }{ }^{\prime} \mathrm{t}_{1} "=\frac{R M}{{ }^{\prime \prime} \mathrm{rm}_{1}{ }^{\prime \prime}}=\frac{40 \mathrm{miles}}{17.5 \mathrm{knots}}=2 \mathrm{hrs} 17 \mathrm{mins}
$$

$$
\begin{gathered}
\mathrm{T}_{1}=1600+{ }^{\prime t} \mathrm{t}_{1} "=1817 \\
" \mathrm{t}_{2} "=\frac{R M}{{ }^{\prime r m_{2}}{ }^{\prime \prime}}=\frac{40 \mathrm{miles}}{7 \mathrm{knots}}=5 \mathrm{hrs} 43 \mathrm{mins} \\
\mathrm{~T}_{2}=1600+{ }^{\prime \mathrm{t}} \mathrm{t}_{2} "=2143
\end{gathered}
$$

## Note:

If the submarine's position involves an error (i.e., datum error) and a main body or convoy formation is present (with an associated Torpedo Danger Zone (TDZ) around it) the DRM from $M$ to $R$ becomes tangential lines drawn from " $r$ " with a high speed and low speed leg corresponding to a forward or aft DRM on the formation.


OWN SHIP AT CENTER

## EXAMPLE 27a

Scale: Speed 3:1;
Distance 10:1 mi.

## EXAMPLE 27b

## CONES OF COURSES

Solution: 2

## Situation:

Own ship $R$ is on course $000^{\circ}, 15$ knots. At 1600 , submarine $M$ is reported bearing $325^{\circ}, 40$ miles from $R$. Maximum assumed speed for $M$ is 10 knots.

## Required:

(1) Courses at 10 knots the submarine $M$ will steer to intercept $R$
(2) Time of the first and last intercept opportunities for submarine $M$ against $R$ at the assumed speed of 10 knots.

## Solution:

(1) Plot the 1600 position of the submarine $M 325^{\circ}, 40$ miles from $R$. Draw the vector "er" $000^{\circ}, 15$ knots. From $M$, draw a DRM line to $R$ and from " $r$ " draw the vector " $r m$ " parallel and in the same direction as the DRM. With " $e$ " as the center, describe an arc with radius of 10 knots, the assumed speed of $M$. The points $E M_{1}$ and $E M_{2}$ where the arc intersects the "rm" vector, define the courses at 10 knots that the submarine will steer to intercept $R$. Courses between " $\mathrm{em}{ }_{1}$ " and " $\mathrm{em} m_{2}$ " are lower assumed speed intercepts and "em$m_{2}$ ", the perpendicular line from $R$ to " $r m$ ", is the course for the lowest possible assumed speed at which the submarine can move and still intercept $R$.
(2) Parallel the "em" and "em" lines as vectors to the 1600 position at $M$ and extend "er" until it crosses these vectors; the area enclosed by these 3 vectors represents the true geographic area through which the submarine will move at or below 10 knots to intercept $R$. The elapsed times to the first (" $t_{1}$ ") and the last (" $\mathrm{t}_{2}$ ") intercept opportunities is obtained by dividing the relative distance at $1600(R M)$ by the respective relative speed (" $r m_{l}$ " and " $r m_{2}$ ").

## Answer:

(1) Courses $024^{\circ}$ to $086^{\circ}$.

$$
\begin{equation*}
\text { " } \mathrm{t}_{1} \text { " }=\frac{R M}{{ }^{\prime r} m_{l}{ }^{\prime \prime}}=\frac{40 \mathrm{miles}}{17.5 \mathrm{knots}}=2 \mathrm{hrs} 17 \mathrm{mins} \tag{2}
\end{equation*}
$$

$$
\mathrm{T}_{1}=1600+{ }^{\prime \prime} \mathrm{t}_{1} "=1817
$$

$$
" \mathrm{t}_{2} "=\frac{R M}{\prime \prime r m_{2}{ }^{\prime \prime}}=\frac{40 \mathrm{miles}}{7 \mathrm{knots}}=5 \mathrm{hrs} 43 \mathrm{mins}
$$

$$
\mathrm{T}_{2}=1600+" \mathrm{t}_{2} "=2143
$$

## Note:

If the submarine's position involves an error (i.e., datum error) and a main body or convoy formation is present (with an associated Torpedo Danger Zone (TDZ) around it) the DRM from $M$ to $R$ becomes tangential lines drawn from " $r$ " with a high speed and low speed leg corresponding to a forward or aft DRM on the formation.


OWN SHIP AT CENTER

EXAMPLE 27b
Scale: Speed 3:1;
Distance 10:1 mi.

## EXAMPLE 28

## EVASIVE ACTION AGAINST A TARGET MOVING AT SLOW SPEED

## Situation:

A vessel possessing a speed advantage is always capable of taking evasive action against a slow-moving enemy. It may be necessary to take evasive action against a slow-moving enemy. For example, when a surface vessel is attempting to evade attack by a submarine.

## Required:

The essence of the problem is to find the course for the maneuvering ship at which no matter how the enemy maneuvers he will not be able to come any closer than distance D (Torpedo/Missile Danger Zone) to the maneuvering ship. In order to accomplish this, the maneuvering ship should press the slow-moving enemy at a relative bearing greater than critical.

## Solution:

Evasive action is graphically calculated in the following manner. The position of the slow-moving enemy vessel $K_{0}$ is plotted on a maneuvering board and the distance it travels from the moment of detection to the beginning of evasive action is calculated:

$$
\mathrm{S}=\mathrm{V}_{\mathrm{k}}\left(\mathrm{~T}_{1}-\mathrm{T}_{0}\right)
$$

where $T_{l}=$ time at which evasive action begins;

$$
T_{0}=\text { time of detection of the enemy. }
$$

The accuracy of determination of the position of the enemy, assumed to be within the datum error zone, (r) is also verified. Then the minimum divergence from the enemy (d) is determined (e.g., 2-3 times the range of fire of torpedoes or 1.5 to 2 times the sonar detection range). Adding up the selected values, with a radius of:

$$
\mathrm{D}_{1}=\mathrm{r}+\mathrm{S}+\mathrm{d},
$$

we have a circle about the initial position of the enemy $K_{0}$.
Constructing a tangent to this circle from the position of the maneuvering ship (point $M_{0}$ ) and, constructing a speed triangle at the point of tangency, we obtain the course of the maneuvering vessel $K m_{1}$ or $K m_{2}$ which the latter must steer in order to avoid meeting the enemy.

## Note:

As a rule, the point of turn to the previous course after taking evasive action is not calculated and the turn is usually executed after the bearing on the point of detection of the slow-moving enemy vessel changes more than $90^{\circ}$.


OWN SHIP AT CENTER

EXAMPLE 28
Scale: Speed 1:1, Distance 1:1 mi.

PART TWO
GUIDE AT CENTER

## EXAMPLE 29

## CHANGING STATION WITH TIME, COURSE, OR SPEED SPECIFIED

## Situation:

Formation course is $010^{\circ}$, speed 18 knots. At 0946 when orders are received to change station, the guide $R$ bears $140^{\circ}$, range 7,000 yards. When on new station, the guide will bear $240^{\circ}$, range 6,000 yards.

## Required:

(1) Course and speed to arrive on station at 1000.
(2) Speed and time to station on course $045^{\circ}$. Upon arrival on station orders are received to close to 3,700 yards.
(3) Course and minimum speed to new station
(4) Time to station at minimum speed.

## Solution:

(1) Plot $M_{1} 320^{\circ}, 7,000$ yards and $M_{2} 060^{\circ}, 6,000$ yards from $R$. Draw er corresponding to course $010^{\circ}$ and speed 18 knots. The relative distance of 10,000 yards from $M_{1}$ to $M_{2}$ must be covered in 14 minutes. SRM is therefore 21.4 knots. Draw $r m_{l}$ parallel to $M_{l} M_{2}$, and 21.4 knots in length. On completing the
vector diagram, the vector $e m_{I}$ denotes the required course and speed: $062^{\circ}, 27$ knots.
(2) Draw $e m_{2}$, course $045^{\circ}$, intersecting the relative speed vector $r m_{l}$ at the 21-knot circle. The length $r m_{2}$ is 12.1 knots. Thus the relative distance $M_{1} M_{2}$ of 10,000 yards will be covered in 24.6 minutes.
(3) Plot $M_{3} 060^{\circ}, 3,700$ yards from $R$ after closing. Through $r$ draw a line parallel to and in the direction of $M_{2} M_{3}$. Drop a perpendicular from $e$ to this line at $m_{3}$. Vector $\mathrm{em}_{3}$ is the course and minimum speed required to complete the final change of station: $330^{\circ}$, 13.8 knots.
(4) By measurement, the length of $r m_{3}$ is an SRM of 11.5 knots; the distance from $M_{2}$ to $M_{3}$ is 2,300 yards. $M_{2} M_{3} / r m_{3}$ is the required maneuver time: 6 minutes.

## Answer:

(1) Course $062^{\circ}$, speed 27 knots.
(2) Speed 21 knots, time 25 minutes.
(3) Course $330^{\circ}$, speed 13.8 knots.
(4) Time 6 minutes.


Scale: Speed 3:1, Distance 1:1 yd.

## EXAMPLE 30

## THREE-SHIP MANEUVERS

## Situation:

Own ship $M$ is in formation proceeding on course $000^{\circ}$, speed 20 knots. The guide $R$ bears $090^{\circ}$, distance 4,000 yards. Ship $N$ is 4,000 yards ahead of the guide.

## Required:

$M$ and $N$ are to take new stations starting at the same time. $N$ is to take station 4,000 yards on the guide's starboard beam using formation speed. $M$ is to take $N$ 's old station and elects to use 30 knots.
(1) $N$ 's course and time to station
(2) $M$ 's course and time to station.
(3) CPA of $M$ and $N$ to guide.
(4) CPA of $M$ to $N$.
(5) Maximum range of $M$ from $N$.

## Solution:

(1) Plot $R$ at the center with $M_{l}$ at $270^{\circ}, 4,000$ yards; $M_{2}$ and $N_{l}$ at $000^{\circ}, 4,000$ yards. Draw er $000^{\circ}, 20$ knots. From $R$ plot $N$ 's new station $N R$, bearing $090^{\circ}$, distance 4,000 yards. In relation to $R, N$ moves from $N_{l}$ to $N R$. From $r$, draw a line parallel to and in the direction of $N_{l} N R$ and intersecting the 20 -knot speed circle at $n . N$ 's course to station is vector en: $090^{\circ}$. Time to station $N_{l} N R / r n$ is 6 minutes.
(2) In relation to $R, M$ moves from $M_{1}$ to $M_{2}$. From $r$, draw $r m$ parallel to and in the direction of $M_{1} M_{2}$ and intersecting the 30 -knot speed circle at $m$. $M$ 's course to station is vector $\mathrm{em}: 017^{\circ}$. Time to station $M_{1} M_{2} / \mathrm{rm}$ is 14 minutes.
(3) From $R$ drop a perpendicular to $N_{l} N R$. At CPA, $N$ bears $045^{\circ}, 2,850$ yards from $R$. From $R$ drop a perpendicular to $M_{1} M_{2}$. At CPA, $M$ bears $315^{\circ}, 2,850$ yards from $R$.
(4) In relation to $M, N$ travels from $N_{1}$ to $N_{2}$ to $N_{3}$. Plot $N_{3}$ bearing $135^{\circ}, 5,700$ yards from $M_{l}$. From point $m$ draw the relative speed vector $m n$. Draw a relative movement line from $N_{l}$ parallel to and in the same direction as $m n$. When $N$ arrives on new station and returns to base course the relative speed between $M$ and $N$ is the same as $r m$. From $N_{3}$ draw a relative movement line parallel to and in the same direction as $r m$. These lines intersect at $N_{2}$. From $M_{l}$ drop a perpendicular to line $N_{1} N_{2}$. At CPA, $N$ bears $069^{\circ}, 5,200$ yards from $M$.
(5) The point at which $N$ resumes formation course and speed $N_{2}$, is the maximum range of $N$ from $M ; 6,500$ yards.

## Answer:

(1) $N$ 's course $090^{\circ}$, time 6 minutes.
(2) M's course $017^{\circ}$, time 14 minutes
(3) CPA: $N$ to $R 2,850$ yards at $045^{\circ} ; M$ to $R 2,850$ yards at $315^{\circ}$.
(4) CPA of $N$ to $M 5,200$ yards at $069^{\circ}$.
(5) Range 6,500 yards.

## Explanation:

In solution step (4), the movement of $N$ in relation to $M$ is parallel to the direction of vector $m n$ and from $N_{l}$ until such time that $N$ returns to base course and speed. Afterwards, the movement of $N$ in relation to $M$ is parallel to vector $r m$ and from $N_{2}$ toward that point, $N_{3}$, that $N$ will occupy relative to $M$ when the maneuver is completed.


## EXAMPLE 30

Scale: Speed 3:1; Distance 1:1 yd.

## EXAMPLE 31

## COURSE AND SPEED TO PASS ANOTHER SHIP AT A SPECIFIED DISTANCE

## Situation:

At 1743 own ship $M$ is on course $190^{\circ}$, speed 12 knots. Another ship $R$ is observed bearing $153^{\circ}, 13,000$ yards on course $287^{\circ}$, speed 10 knots. It is desired to pass ahead of $R$ with a CPA of 3,000 yards.

## Required:

(1) Course of $M$ at 12 knots.
(2) Bearing of $R$ and time at CPA.

## Solution:

(1) Plot $R$ at the center of $M_{l}$ bearing $333^{\circ}, 13,000$ yards from $R$. Draw the other ship's vector $\operatorname{er} 287^{\circ}, 10$ knots. With $R$ as a center, describe a circle of radius 3,000 yards. From $M_{1}$ draw a line tangent to the circle at $M_{2}$. This satisfies
the requirement of passing with a CPA of 3,000 yards from $R$. From $r$ draw a line parallel to and in the same direction as $M_{I} M_{2}$, intersecting the 12-knot speed circle at $m$. Draw em, own ship's vector $212^{\circ}, 12$ knots.
(2) From $R$ drop a perpendicular to $M_{2}$. When own ship reaches $M_{2}, R$ will bear $076^{\circ}$. Measure the relative distance $M_{1} M_{2}, 12,600$ yards, and the relative speed vector $r m, 13.4$ knots. Using this distance and speed, the elapsed time to CPA is obtained from the TDS scale: 28 minutes. The time at CPA is $1743+28$ $=1811$.

## Answer:

(1) Course $212^{\circ}$
(2) Bearing $076^{\circ}$, time at CPA 1811.


GUIDE AT CENTER

EXAMPLE 31
Scale: Speed 2:1 Distance 2:1 yd.

## EXAMPLE 32

## COURSE AT SPECIFIED SPEED TO PASS ANOTHER SHIP AT MAXIMUM

## AND MINIMUM DISTANCES

## Situation:

Ship $R$ on course $300^{\circ}$, speed 30 knots, bears $155^{\circ}$, range 16 miles from own ship $M$ whose maximum speed is 15 knots.

## Required:

(1) $M$ 's course at 15 knots to pass $R$ at (a) maximum distance, (b) minimum distance.
(2) CPA for each course found in (1).
(3) Time interval to each CPA.
(4) Relative bearing of $R$ from $M$ when at CPA on each course.

## Solution:

(1) Plot $M_{I} 335^{\circ}, 16$ miles from $R$. Draw the vector er $300^{\circ}, 30$ knots. With $e$ as the center, draw a circle with radius of 15 knots, the speed of $M$. From $r$ draw the tangents $r m_{1}$ and $r m_{2}$ which produce the two limiting courses for $M$. Parallel to the tangents plot the relative movement lines from $M_{l}$. Course of own ship to pass at maximum distance is $e m_{l}: 000^{\circ}$. Course to pass at minimum distance is $e m_{2}: 240^{\circ}$.
(2) Through $R$ draw $R M_{2}$ and $R M_{2}$ perpendicular to the two possible relative movement lines. $R$ bearing $180^{\circ}, 14.5$ miles from $M_{2}$ is the CPA for course of $000^{\circ}$. $R$ bearing $240^{\circ}, 1.4$ miles from $M_{2}^{\prime}$ is the CPA for course $240^{\circ}$.
(3) Measure $M_{1} M_{2}: 6.8$ miles, and $M_{1} M_{2}^{\prime}: 15.9$ miles. $M$ must travel these relative distances before reaching the CPA on each limiting course. The relative
speed of $M$ is indicated by the length of the vectors $r m_{l}$ and $r m_{2}: 26$ knots. From the TDS scale the times required to reach $M_{2}$ and $M_{2}^{\prime}$ are found: 15.6 minutes and 36.6 minutes, respectively.
(4) Bearings are determined by inspection. $R$ bears $180^{\circ}$ relative because own ship's course is along vector $e m_{I}$ for maximum CPA. $R$ bears $000^{\circ}$ relative when own ship's course is $e m_{2}$ for minimum passing distance.

## Note:

This situation occurs only when own ship $M$ is (1) ahead of the other ship and (2) has a maximum speed less than the speed of the other ship. Under these conditions, own ship can intercept (collision course) only if $R$ lies between the slopes of $M_{l} M_{2}$ and $M_{l} M^{\prime}$. Note that for limiting courses, and only for these, CPA occurs when other ship is dead ahead or dead astern. The solution to this problem is applicable to avoiding a tropical storm by taking that course which results in maximum passing distance.

## Answer:

(1) Course (a) $000^{\circ}$; (b) $240^{\circ}$.
(2) CPA (a) $180^{\circ}, 14.5$ miles; (b) $240^{\circ}, 1.4$ miles.
(3) Time (a) 16 minutes; (b) 37 minutes.
(4) Relative bearing (a) $180^{\circ}$; (b) $000^{\circ}$.


Scale: Speed 3:1; Distance 2:1 mi.

## EXAMPLE 33

## COURSE CHANGE IN COLUMN FORMATION ASSURING LAST SHIP IN COLUMN CLEARS

## Situation:

Own ship $D 1$ is the guide in the van of a destroyer unit consisting of four destroyers ( $D 1, D 2, D 3$, and $D 4$ ) in column astern, distance 1,000 yards. $D 1$ is on station bearing $090^{\circ}, 8$ miles from the formation guide $R$. Formation course is $135^{\circ}$, speed 15 knots. The formation guide is at the center of a concentric circular ASW screen stationed on the 4-mile circle.
The destroyer unit is ordered to take new station bearing $235^{\circ}, 8$ miles from the formation guide. The unit commander in $D 1$ decides to use a wheeling maneuver at 27 knots, passing ahead of the screen using two course changes so that the CPA of his unit on each leg is 1,000 yards from the screen.

## Required:

(1) New course to clear screen commencing at 1000
(2) Second course to station.
(3) Bearing and range of $R$ and $D 1$ at time of coming to second course.
(4) Time of turn to second course.
(5) Time $D 1$ will reach new station.

## Solution:

(1) Plot the formation guide $R$ at the center. Plot own ship $D 1$ bearing $090^{\circ}, 8$ miles from $R$. Plot the remaining three destroyers in column astern of $D 1$, distance between ships 1,000 yards. Draw er , the speed vector of $R, 135^{\circ}, 15$ knots. It is required that the destroyer column clear $R$ by a minimum of 9,000 yards (screen radius of 4 miles plus 1,000 yards). At the instant the signal is executed, only $D 1$ changes both course and speed. The other destroyers increase speed to 27 knots but remain on formation course of $135^{\circ}$ until each reaches the turning point. Advance $R$ along the formation course the distance $R$ would move at 15
knots while $D 4$ advances to the turning point at 27 knots. The distance is equal to:

$$
\frac{\text { Speed of } R}{\text { Speed of } D 4} \times 3,000 \text { yards }=1,666 \text { yards }
$$

Draw a circle of radius 9,000 yards about the advanced position of the guide $R^{\prime}$. Draw a line from $D 1$ (the turning point) tangent to the circle. This is the relative movement line required for $D 4$ to clear the screen by 1,000 yards on the first leg. Draw a line from $r$ parallel to this line and intersecting the 27 -knot circle at $m_{l}$. This produces $e m_{l}$, the initial course of $194^{\circ} .2$.
(2) Plot the final relative position of $D 1$ at $D 1^{\prime}$ bearing $235^{\circ}, 8$ miles from $R$. Draw a line from $D 1^{\prime}$ tangent to the 9,000 yard circle and intersecting the first relative movement line at $D 1^{\prime \prime}$. Draw a line parallel to and in the direction of $D 1 " D 1$ ' from $r$. The intersection of this line and the 27-knot circle at $m_{2}$ is the second course required, $e m_{2} 252^{\circ} .8$.
(3) Bearing and range of $R$ from $D 1^{\prime \prime}$ is $337^{\circ}$ at 11,250 yards.
(4) Time interval for $D 1$ to travel to $D 1^{\prime \prime}$ is: $D 1 D 1^{\prime \prime} / r m_{l}=7.8$ miles $/ 23.2$ knots
$=20.2$ minutes. Time of turn $1000+20=1020$.
(5) Time interval for the second leg is: $D 1^{\prime \prime} D 1^{\prime} / r m_{2}=8.8$ miles $/ 36.5$ knots $=$ 14.2 minutes. $D 1$ will arrive at new station at 1034 .

## Answer:

(1) Course $194^{\circ}$
(2) Course $253^{\circ}$.
(3) Bearing $337^{\circ}$, range 11,250 yards.
(4) Time 1020.
(5) Time 1034.


GUIDE AT CENTER

EXAMPLE 33
Scale: Speed 3:1,
Distance 1:1 mi.

## PRACTICAL ASPECTS OF MANEUVERING BOARD SOLUTIONS

The foregoing examples and their accompanying illustrations are based upon the premise that ships are capable of instantaneous changes of course and speed. It is also assumed that an unlimited amount of time is available for determining the solutions.
In actual practice, the interval between the signal for a maneuver and its execution frequently allows insufficient time to reach a complete, graphical solution. Nevertheless, under many circumstances, safety and smart seamanship both require prompt and decisive action, even though this action is determined from a quick, mental estimate. The estimate must be based upon the principles of relative motion and therefore should be nearly correct. Course and speed can be modified enroute to new station when a more accurate solution has been obtained from a maneuvering board.

Allowance must be made for those tactical characteristics which vary widely between types of ships and also under varying conditions of sea and loading. Experience has shown that it is impractical to solve for the relative motion that occurs during a turn and that acceptable solutions can be found by eye and mental estimate
By careful appraisal of the PPI and maneuvering board, the relative movement of own ship and the guide during a turn can be approximated and an estimate made of the relative position upon completion of a turn. Ships' characteristic curves and a few simple thumb rules applicable to own ship type serve as a basis for these estimates. During the final turn the ship can be brought onto station with small compensatory adjustments in engine revolutions and/or course.

## EXAMPLE 34

## ADVANCE, TRANSFER, ACCELERATION, AND DECELERATION

## Situation:

Own ship $M$ is a destroyer on station bearing $020^{\circ}, 8,000$ yards from the guide $R$. Formation course is $000^{\circ}$, speed 15 knots. $M$ is ordered to take station bearing $120^{\circ}, 8,000$ yards from guide, using 25 knots.

## Required:

(1) Course to new station.
(2) Bearing of $R$ when order is given to resume formation course and speed.
(3) Time to complete the maneuver.

Solution:
(1) Plot $R$ at the center with $M_{1}$ bearing $020^{\circ}, 8,000$ yards and $M_{2}$ bearing $120^{\circ}, 8,000$ yards. Draw guide's vector, er, $000^{\circ}, 15$ knots.
By eye, it appears $M$ will have to make a turn to the right of about $150^{\circ}$, accelerating from 15 to 25 knots during the turn. Prior to reaching the new station a reverse turn of about the same amount and deceleration to 15 knots will be required. Assume that $M$ averages 20 knots during each turn.
Using $30^{\circ}$ rudder at 20 knots, a DD calibration curve indicates approximately $2^{\circ}$ turn per second and a 600 yard diameter. Thus, a $150^{\circ}$ turn will require about

75 seconds and will produce a transfer of about 600 yards. During the turn, $R$ will advance 625 yards ( $1^{1 / 4}$ minutes at 15 knots). Plotting this approximate offset distance on the maneuvering board gives a new relative position of $M_{3}$ at the time the initial turn is completed. Similarly, a new off-set position at $M_{4}$ is determined where a left turn to formation course and reduction of speed to 15 knots should be ordered.

Draw a line from $r$ parallel to $M_{3} M_{4}$ and intersecting the 25 -knot speed circle at $m$. Vector em is the required course of $158^{\circ}$.
(2) When $M$ reaches point $M_{4}$ with $R$ bearing $299^{\circ}$, turn left to formation course using $30^{\circ}$ rudder and slow to 15 knots.
(3) Time to complete the maneuver is $M_{3} M_{4} / \mathrm{SRM}+2.5$ minutes $=11,050$ yards $/ 39.8$ knots +2.5 minutes $=11$ minutes.

## Answer:

(1) Course $158^{\circ}$.
(2) Bearing $299^{\circ}$.
(3) Time 11 minutes.


## EXAMPLE 34

Scale: Speed 3:1, Distance 1:1 yd.

## MANEUVERING BY SEAMAN'S EYE

In many circumstances it is impossible to use a maneuvering board in the solution of relative movement problems. When the distance between old and new stations is short and well abaft the beam, it may be impractical to attempt to complete the theoretically required turns and travel along an $M_{I} M_{2}$ path. In such cases, a reduction in speed, fishtailing, or various modifications of a fishtail may be required.
In the following example, it is assumed that a destroyer type ship is proceeding at formation speed and using standard rudder which yields a perfect turning circle of 1,000 yards diameter and 3,150 yards circumference. It is also assumed that a $13 \%$ reduction in speed is produced by large turns.
Based upon these assumptions, a ship using a $45^{\circ}$ fishtail either side of formation course will fall behind old station by about 400 yards. By using a $60^{\circ}$
fishtail, it will drop back about 700 yards. Approximate distances for any amount of course change can be computed if desired; however, the above quantities used as thumb rules should be sufficient. Repeated application of either will produce larger "drop backs" and also offer the advantage of not using excessive sea room.

If it is desired to move laterally as well as fall back, a turn of $45^{\circ}$ to one side only and then immediate return to original course will produce a 300 yard transfer and a 200 yard drop back.

If time is not a consideration and the relative movement line is relatively very short, a reduction in speed may prove most desirable.

## EXAMPLE 35

Situation:
Own ship $M$ is on formation course $225^{\circ}$, speed 15 knots, with guide $R$ bearing $000^{\circ}, 3,000$ yards.

## Required:

Take station 2,000 yards broad on the port beam of the guide.

## Solution:

An attempt to solve this problem by normal maneuvering board procedures will prove impractical. $M_{2}$ is directly astern of $M_{1}$ at a distance of 2,150 yards.

Any combination of course changes in an attempt to travel a line from $M_{l}$ to $M_{2}$ will result in own ship falling far astern of the new station. Even a simple $360^{\circ}$ turn will drop own ship back 3,600 yards, almost twice the desired movement.

By fishtailing $60^{\circ}$ to either side using courses of $165^{\circ}$ and $285^{\circ}$ three times per side, own ship will drop straight back approximately 2,000 yards, within 150 yards of station. Final adjustment to station can be effected by normal station keeping maneuvers such as rapidly shifting the rudder between maximum positions or reduction in engine revolutions.


CHANGING STATIONS BY FISHTAIL METHOD

EXAMPLE 35

## EXAMPLE 36

## FORMATION AXIS ROTATION—GUIDE IN CENTER

## Situation:

The formation is on course $240^{\circ}$, speed 15 knots. The formation axis is $130^{\circ}$. The guide is in station Zero and own ship is in station 6330. The OTC rotates the formation axis to $070^{\circ}$. Stationing speed is 20 knots.

## Required:

(1) Course at 20 knots to regain station relative to the new formation axis, $070^{\circ}$.

## Solution:

(1) Mark the initial and new formation axes at $130^{\circ}$ and $070^{\circ}$, respectively Plot the guide's station in the center (station Zero) and label as $R$. Plot own ship's initial position $M_{l}$ on circle 6 in a direction from the formation center
$330^{\circ}$ relative to the initial formation axis. Draw er corresponding to guide's course $240^{\circ}$ and speed 15 knots.
(2) Plot own ship's new position $M_{2}$ oriented to the new axis. The original station assignments are retained, except the stations are now relative to the new axis.
(3) Draw the direction of relative movement line (DRM) from $M_{I}$ through $M_{2}$.
(4) Through $r$ draw a line in the direction of relative movement intersecting the 20-knot circle at $m$.
(5) Own ship's true vector is em: course $293^{\circ}$, speed 20 knots

## Answer:

(1) Course $293^{\circ}$ to regain station relative to the new axis.


GUIDE AT CENTER

## EXAMPLE 36

Scale: Speed 3:1; Distance 1:1 thousands of yds.

## EXAMPLE 37

## FORMATION AXIS ROTATION-GUIDE OUT OF CENTER

## FORMATION CENTER KEPT IN CENTER OF PLOT

## Situation:

The formation is on course $275^{\circ}$, speed 18 knots. The formation axis is $190^{\circ}$. The guide is in station 3030 and own ship is in station 7300. The OTC rotates the formation axis to $140^{\circ}$. Stationing speed is 20 knots.

## Required:

(1) Course at 20 knots to regain station relative to the new formation axis, $140^{\circ}$.

## Solution:

(1) Mark the initial and new formation axes at $190^{\circ}$ and $140^{\circ}$, respectively. Plot the guide's initial station $R_{I}$ on circle 3 in a direction from the formation center $30^{\circ}$ relative to the initial formation axis. Plot own ship's initial station $S_{l}$ on circle 7 in a direction from the formation center $300^{\circ}$ relative to the initial
formation axis. Draw er corresponding to guide's course $275^{\circ}$ and speed 18 knots.
(2) Plot the guide's new station $R_{2}$ oriented to the new formation axis; plot own ship's new station $S_{2}$ oriented to the new formation axis.
(3) Measure the bearings and distances of $S_{l}$ and $S_{2}$ from $R_{l}$ and $R_{2}$, respectively.
(4) From the center, plot the bearing and distance of $S_{I}$ from $R_{l}$ as $M_{I}$ and the bearing and distance of $S_{2}$ from $R_{2}$ as $M_{2}$.
(5) Since the line from $M_{1}$ to $M_{2}$ represents the required DRM for own ship to regain station relative to the new axis, draw a line through $r$ in the direction of relative movement.
(6) Own ship's true vector is em: course $291^{\circ}$, speed 20 knots.

## Answer:

(1) Course $291^{\circ}$ to regain station relative to the new axis.


## GUIDE OUT OF CENTER

## EXAMPLE 37

Scale: Speed 3:1; Distance 1:1 thousands of yds.

## EXAMPLE 38

## FORMATION AXIS ROTATION—GUIDE OUT OF CENTER

## Situation:

The formation is on course $275^{\circ}$, speed 18 knots. The formation axis is $190^{\circ}$ The guide is in station 3030 and own ship is in station 7300. The OTC rotates the formation axis to $140^{\circ}$. Stationing speed is 20 knots.

## Required:

(1) Course at 20 knots to regain station relative to the new formation axis, $140^{\circ}$.

## Solution:

(1) Mark the initial and new formation axes at $190^{\circ}$ and $140^{\circ}$, respectively. Plot the guide's station $R_{I}$ on circle 3 in a direction from the formation center $30^{\circ}$ relative to the initial formation axis. Plot own ship's station $M_{l}$ on circle 7 in a direction from the formation center $300^{\circ}$ relative to the initial formation axis. Draw er corresponding to guide's course $275^{\circ}$ and speed 18 knots.
(2) Plot the guide's station, $R_{2}$, oriented to the new formation axis. Plot own ship's position $M_{3}$ oriented to the new axis. The original station assignments are retained, except the stations are now relative to the new axis.
(3) Shift the initial position of own ship's station at $M_{I}$ in the direction and distance of the fictitious shift of the guide to its position relative to the new axis. Mark the initial position so shifted as $M_{2}$.
(4) Draw the direction of relative movement lines (DRM) from $M_{2}$ through $M_{3}$.
(5) Through $r$ draw a line in the direction of relative movement intersecting the 20 -knot circle at $m$.
(6) Own ship's true vector is em: course $291^{\circ}$, speed 20 knots.

## Answer:

(1) Course $291^{\circ}$ to regain station relative to the new axis.

## Explanation:

Since the guide does not actually move relative to the initial formation center while maintaining course and speed during the formation maneuver, all initial positions of stations in the formation must be moved in the same direction and distance as the fictitious movement of the guide to its new position.


## GUIDE OUT OF CENTER

## EXAMPLE 38

Scale: Speed 3:1; Distance 1:1 thousands of $y d s$.

## EXAMPLE 39

## COURSE AND SPEED BETWEEN TWO STATIONS, REMAINING WITHIN A SPECIFIED RANGE FOR SPECIFIED TIME INTERVAL ENROUTE

## Situation:

Own ship $M$ is on station bearing $280^{\circ}, 5$ miles from the guide $R$ on formation course $190^{\circ}$, speed 20 knots.

## Required:

At 1500 own ship $M$ is ordered to proceed to new station bearing $055^{\circ}, 20$ miles, arriving at 1630 and to remain within a 10-mile range for 1 hour. The commanding officer elects to proceed directly to new station, adjusting course and speed as necessary to comply with the foregoing requirements.
(1) Course and speed to remain within 10 miles for 1 hour.
(2) Course and speed required at 1600 .
(3) Bearing of $R$ at 1600 .

## Solution:

(1) Plot the 1500 and 1630 positions of $M$ at $M_{I}$ and $M_{3}$, respectively. Draw the relative motion line, $M_{1} M_{3}$, intersecting the 10 -mile circle at $M_{2}$. Draw er. Measure $M_{1} M_{2}: 13.6$ miles. The time required to transit this distance is 1 hour
at an SRM of 13.6 knots. Through $r$ draw $r m_{l} 13.6$ knots in length, parallel to and in the direction $M_{1} M_{3}$. Vector $e m_{1}$ is $147^{\circ} .5,16.2$ knots.
(2) Measure $M_{2} M_{3}, 10.3$ miles, which requires an SRM of 20.6 knots for one half hour. Through $r$ draw $r m_{2}$. Vector $\mathrm{em}_{2}$ is $125^{\circ} .5,18.2$ knots.
(3) By inspection, $R$ bears $226^{\circ}$ from $M_{2}$ at 1600 .

## Answer:

(1) Course $148^{\circ}$, speed 16.2 knots.
(2) Course $126^{\circ}$, speed 18.2 knots.
(3) Bearing $226^{\circ}$.

## Explanation:

Since own ship $M$ must remain within 10 miles of the guide for 1 hour, $M$ must not plot along $M_{1} M_{2}$ farther than $M_{2}$ prior to 1600 . The required magnitudes of the relative speed vectors for time intervals 1500 to 1600 and 1600 to 1630 together with their common direction are combined with the true vector of the guide to obtain the two true course vectors for own ship.


GUIDE AT CENTER

EXAMPLE 39
Scale: Speed 3:1,
Distance 2:1 mi.

## EXAMPLE 40

## COURSE AT MAXIMUM SPEED TO OPEN RANGE TO A SPECIFIED

## DISTANCE IN MINIMUM TIME

## Situation:

Own ship $M$ has guide $R$ bearing $240^{\circ}$, range 12 miles. The guide is on course $120^{\circ}$, speed 15 knots. Own ship's maximum speed is 30 knots.

## Required:

Open range to 18 miles as quickly as possible.
(1) Course at 30 knots.
(2) Time to complete the maneuver.
(3) Bearing of guide upon arrival at specified range.

## Solution:

The key to this solution is to find that relative position $\left(M^{\prime}\right)$ of the guide that could exist before the problem starts in order to be able to draw the RML through the given relative position $\left(M_{I}\right)$ and $M^{\prime}$ to intersect the specified range circle.
(1) Plot $R$ and $M_{l}$. About $R$ describe a circle of radius 18 miles. Draw er. Along $R$ 's course plot $M^{\prime} 9$ miles from $R$.

$$
\frac{\text { Speed of } R}{\text { Speed of } M} \times 18 \text { miles }=9 \text { miles }
$$

Draw a line through $M^{\prime}$ and $M_{I}$ and extend it to intersect the 18-mile range circle at $M_{2}$.
From $r$ draw $r m$ parallel to and in the direction $M_{1} M_{2}$. The intersection of $r m$ and the 30 -knot speed circle is the course required to complete the maneuver in minimum time. Vector em is $042^{\circ} .6,30$ knots.
(2) SRM is 30.5 knots. MRM is 7.5 miles. Time to complete the maneuver: 14.8 minutes.
(3) Upon reaching the 18 -mile range circle, $R$ is dead astern of $M$ bearing $222^{\circ} .6$.

## Answer:

(1) Course $043^{\circ}$.
(2) Time 15 minutes.
(3) Bearing $223^{\circ}$.

## Explanation:

For $M$ to open or close to a specified range in minimum time, $M$ must travel the shortest geographical distance at maximum speed. The shortest distance is along the radius of a circle centered at the position occupied by $R$ at the instant $M$ reaches the specified range circle.

In the "opening range" problem, determine hypothetical relative positions of $M$ and $R$ that could exist before the problem starts. Referring to the geographical plot, assume $M$ starts from position $M^{\prime}$ and proceeds outward along some radius 18 miles in length on an unknown course at 30 knots. If $R$ moves toward its final position at $R_{2}$ along the given course of $120^{\circ}$, speed 15 knots, it should arrive at $R_{2}$ the instant $M$ reaches the 18 -mile circle. At this instant, the problem conditions are satisfied by $M$ being 18 miles distant from $R$. However, own ship's course required to reach this position is not yet known. During the time interval $M$ opened 18 miles at 30 knots, $R$ moved 9 miles at 15 knots from $R^{\prime}$ to $R_{2}$.

$$
\frac{\text { Speed of } M}{\text { Speed of } R} \times 18 \text { miles }=9 \text { miles }
$$

This provides the needed second relative position of $M^{\prime}$ from $R^{\prime}, 9$ miles bearing $120^{\circ}$. This position is then transferred to the relative plot.


Scale: Speed 3:1; Distance 2:1 mi.

## EXAMPLE 41

## COURSE AT MAXIMUM SPEED TO CLOSE RANGE TO A SPECIFIED DISTANCE IN MINIMUM TIME

## Situation:

Own ship $M$ has the guide $R$ bearing $280^{\circ}$, range 10 miles. The guide is on course $020^{\circ}$, speed 15 knots. Own ship's maximum speed is 24 knots.

## Required:

Close range to 2 miles as quickly as possible.
(1) Course at 24 knots.
(2) Time to complete the maneuver.
(3) Bearing of guide upon arrival at the specified range.

## Solution:

The key to this solution is to find that relative position $\left(M^{\prime}\right)$ of the guide that could exist after the problem starts in order to be able to draw the RML through the given relative position $\left(M_{I}\right)$ and $M^{\prime}$ to intersect the specified range circle.
(1) Plot $R$ and $M_{1}$. About $R$ describe a circle of radius 2 miles. Draw er, guide's speed vector $020^{\circ}, 15$ knots. On reciprocal of $R$ 's course plot $M^{\prime} 1.25$ miles from $R$.

$$
\frac{\text { Speed of } R}{\text { Speed of } M} \times 2 \text { miles }=1.25 \text { miles }
$$

Draw a line through $M^{\prime}$ and $M_{1}$. The intersection of this line and the 2-mile range circle is $M_{2}$.
From $r$ draw a line parallel to and in the direction $M_{1} M_{2}$. The intersection of this line and the 24-knot speed circle at $m$ is the course required to complete the maneuver in minimum time. Vector em $309^{\circ} .8,24$ knots.
(2) SRM is 23.6 knots. MRM is 8.3 miles. Time to complete the maneuver: 21.1 minutes.
(3) Upon reaching the 2-mile range circle, $R$ is dead ahead of $M$ on a bearing $309^{\circ} .8$.

## Answer:

(1) Course $310^{\circ}$
(2) Time 21 minutes.
(3) Bearing $310^{\circ}$.

## Explanation:

Referring to the geographical plot, assume $M$ starts from position $M_{l}$ and proceeds inward along some radius on an unknown course at 24 knots. If $R$ moves toward its final position at $R_{2}$ along the given course $020^{\circ}$, speed 15 knots, it should arrive at $R_{2}$ the instant $M$ reaches the 2 -mile circle. At this instant the problem conditions are satisfied although the solution for own ship's course is not yet known. Assume that $M$ continues on the same course and speed through the 2 miles to $M^{\prime}$ at the center of the circle while $R$ moves away from the center on course $020^{\circ}$, speed 15 knots. During the time interval that $M$ moves these 2 miles at 24 knots, $R$ opens 1.25 miles.

$$
\frac{\text { Speed of } R}{\text { Speed of } M} \times 2 \text { miles }=1.25 \text { miles }
$$

This provides the needed second relative position of $M^{\prime}$ from $R^{\prime}: 1.25$ miles, bearing $200^{\circ}$. This position is then transferred to the relative plot.


## EXAMPLE 41

Scale: Speed 3:1; Distance 1:1 mi.

## EXAMPLE 42

## COURSE AT MAXIMUM SPEED TO REMAIN WITHIN A SPECIFIED RANGE

 FOR MAXIMUM TIME
## Situation:

Ship $R$ bears $110^{\circ}, 4$ miles from $M . R$ is on course $230^{\circ}, 18$ knots. Maximum speed of $M$ is 9 knots.

## Required:

Remain within a 10 -mile range of $R$ for as long as possible.
(1) Course at maximum speed.
(2) Bearing of $R$ upon arrival at specified range.
(3) Length of time within specified range.
(4) CPA.

## Solution:

(1) Plot $R$ at the center and $M_{I}$ bearing $290^{\circ}, 4$ miles from $R$. About $R$ describe arcs of radius 9 knots and 10 miles. Draw er $230^{\circ}, 18$ knots. Along the reciprocal of $R$ 's course, plot $M^{\prime} 20$ miles from $R$.

$$
\frac{\text { Speed of } R}{\text { Speed of } M} \times 10 \text { miles }=20 \text { miles }
$$

Draw a line through $M^{\prime}$ and $M_{I}$. The intersection of $M^{\prime} M_{I}$ and the 10-mile range circle is $M_{2}$, the point beyond which the specified or limiting range is exceeded. Through $r$ draw a line parallel to and in the direction $M_{1} M_{2}$. The intersection of this line at point $m$ on the 9 -knot speed circle is the required course to remain within 10 miles of $R$. Vector em is $220^{\circ} .8,9$ knots.
(2) Upon arrival at limiting range at $M_{2}, R$ is dead ahead of $M$ bearing $220^{\circ} .8$
(3) The time interval within specified range is:

$$
\frac{M_{1} M_{2}}{r m}=\frac{12 \text { miles }}{9.1 \text { knots }}=78.8 \text { minutes }
$$

$$
\text { (4) Drop a perpendicular from } R \text { to } M_{1} M_{2} \text {. CPA is } 148^{\circ} .9,3.1 \text { miles. }
$$

## Note:

When $M$ 's speed is equal to or greater than that of $R$, a special case exists in which there is no problem insofar as remaining within a specified range.

## Answer:

(1) Course $221^{\circ}$.
(2) Bearing $221^{\circ}$.
(3) Time 79 minutes.
(4) CPA $149^{\circ}, 3.1$ miles.

## Explanation:

As in the "closing range" problem, example 39, determine hypothetical relative positions of $M$ and $R$ that could exist after the problem ends. Referring to the geographical plot, assume $M$ starts from position $M_{I}$ and proceeds inward along some radius on an unknown course at 9 knots. $R$ is on course $230^{\circ}$ at 18 knots. At the instant $R$ passes through $R_{2}, M$ reaches the 10 -mile limiting range at $M_{2}$. At this instant the problem conditions are satisfied although the solution is not yet known. Assume that $M$ continues on the same course and speed for 10 miles to the center of the circle while $R$ moves away from the center on course $230^{\circ}$, speed 18 knots. During the time interval $M$ closes 10 miles toward the center, $R$ opens 20 miles at 18 knots.

$$
\frac{\text { Speed of } R}{\text { Speed of } M} \times 10 \text { miles }=20 \text { miles }
$$

This then gives us the needed second relative position of $R^{\prime}$ from $M^{\prime}, 20$ miles bearing $230^{\circ}$. This position is then transferred to the relative plot.


GUIDE AT CENTER

EXAMPLE 42
Scale: Speed 2:1; Distance 2:1 mi.

## EXAMPLE 43

## COURSE AT MAXIMUM SPEED TO REMAIN OUTSIDE OF A SPECIFIED

 RANGE FOR MAXIMUM TIME
## Situation:

Ship $R$ bears $020^{\circ}, 14$ miles from own ship $M . R$ is on course $210^{\circ}$, speed 18 knots. Maximum speed of $M$ is 9 knots.

## Required:

Remain outside a 10 -mile range from $R$ for as long as possible.
(1) Course at maximum speed.
(2) Bearing of $R$ upon arrival at specified range.
(3) Time interval before reaching specified range.

## Solution:

(1) Plot $R$ at the center and $M_{l}$ bearing $200^{\circ}, 14$ miles from $R$. About $R$, describe circles of radius 9 knots and 10 miles. Draw er $210^{\circ}$, speed 18 knots. Along $R$ 's course, plot $M^{\prime} 20$ miles from $R$.

$$
\frac{\text { Speed of } R}{\text { Speed of } M} \times 10 \text { miles }=20 \text { miles }
$$

Draw a line through $M^{\prime}$ and $M_{1}$ intersecting the 10-mile range circle at $M_{2}$. Through $r$ draw a line parallel to and in the direction of $M_{1} M_{2}$ intersecting the 9 -knot speed circle at $m$. Completion of the speed triangle produces em , the required course of $184^{\circ} .2$ at 9 knots.
(2) Upon arrival at limiting range at point $M_{2}, R$ is dead astern of $M$ bearing $004^{\circ}$.2.
(3) The time interval outside of specified range is:

$$
\frac{M_{1} M_{2}}{r m}=\frac{5.2 \text { miles }}{10.7 \text { knots }}=30 \text { minutes }
$$

## Note:

Own ship can remain outside the limiting range indefinitely if $M_{1}$ falls outside the area between two tangents drawn to the limiting range circle from $M^{\prime}$ and if $R$ remains on the same course and speed.

## Answer:

(1) Course $184^{\circ}$.
(2) Bearing $004^{\circ}$.
(3) Time 30 minutes.

## Explanation:

To determine a course to remain outside of a given range for maximum time, determine hypothetical relative positions of $M$ and $R$ that could exist before the problem starts. Referring to the geographical plot, assume $M$ starts from position $M^{\prime}$ and proceeds outward along some radius on an unknown course at 9 knots. If $R$ moves toward its final position $R_{2}$ along the given course $210^{\circ}$, speed 18 knots, it should arrive at $R_{2}$ the instant $M$ reaches the 10 -mile circle at $M_{2}$. At this instant the problem conditions are satisfied although the solution for own ship's course is not yet known. During the time interval required for $M$ to move from $M^{\prime}$ to $M_{2}, 10$ miles at 9 knots, $R$ moves from $R^{\prime}$ to $R_{2}, 20$ miles at 18 knots along the given course $210^{\circ}$.

$$
\frac{\text { Speed of } R}{\text { Speed of } M} \times 10 \text { miles }=20 \text { miles }
$$

This provides the needed second relative position, $M^{\prime}$ bearing $210^{\circ}$, 20 miles from $R^{\prime}$. This position is then transferred to the relative plot.


GUIDE AT CENTER

EXAMPLE 43
Scale: Speed 2:1; Distance 2:1 mi.

## USE OF A FICTITIOUS SHIP

The examples given thus far in PART TWO have been confined to ships that have either maintained constant courses and speeds during a maneuver or else have engaged in a succession of such maneuvers requiring only repeated application of the same principles. When one of the ships alters course and/or speed during a maneuver, a preliminary adjustment is necessary before these principles can be applied.
This adjustment consists, in effect, of substituting a fictitious ship for the ship making the alteration. This fictitious ship is presumed to:
(1) maintain a constant course and speed throughout the problem (this is the final course and speed of the actual ship).
(2) start and finish its run at times and positions determined by the conditions established in the problem.

For example, the course and speed of advance of a ship zig-zagging are considered to be the constant course and speed of a fictitious ship which departs from a given position at a given time simultaneously with the actual ship, and arrives simultaneously with the actual ship at the same final position. The principles discussed in previous examples are just as valid for a fictitious ship as for an actual ship, both in the relative plot and speed triangle. A geographical plot facilitates the solution of this type.

## EXAMPLE 44

## ONE SHIP ALTERS COURSE AND/OR SPEED DURING MANEUVER

## Situation:

At 0630 ship $R$ bears $250^{\circ}$, range 32 miles. $R$ is on course $345^{\circ}$, speed 15 knots but at 0730 will change course to $020^{\circ}$ and speed to 10 knots.

## Required:

Own ship $M$ take station 4 miles ahead of $R$ using 12 knots speed.
(1) Course to comply.
(2) Time to complete maneuver.

## Solution:

Determine the 0630 position of a fictitious ship $F$ that, by steering course $020^{\circ}$ at speed 10 knots, will pass through the 0730 position simultaneously with the actual ship. In this way the fictitious ship travels on a steady course of $020^{\circ}$, speed 10 knots throughout the problem.
(1) Construct a geographical plot with $R$ and $R_{l}$ the 0630 and 0730 positions respectively of ship $R$ moving along course $345^{\circ}$ at 15 knots. Plot $F$, the 0630
position of the fictitious ship bearing $200^{\circ}, 10$ miles from $R_{l}$. By measurement, $F$ bears $304^{\circ}, 8.8$ miles from $R$. Transfer this position to a relative plot with $R$ at the center.

Plot own ship at $M_{l}$ bearing $070^{\circ}, 32$ miles from $R$. Draw erf, the fictitious ship's vector, $020^{\circ}, 10$ knots. Lay off own ship's final position, $M_{2}, 4$ miles ahead of $F$ along its final course $020^{\circ}$. Draw the relative movement line $M_{I} M_{2}$ and, parallel to it, construct the relative speed vector from $r f$ to its intersection with the 12 -knot circle at $m$. This produces em the required course of $316^{\circ}$.
(2) The time to complete the maneuver can be obtained from the TDS scale using MRM of 36.4 miles and SRM of 11.8 knots which gives a time of 3.1 hours

## Answer:

(1) Course $316^{\circ}$
(2) Time 3 hours 6 minutes


GUIDE AT CENTER

## EXAMPLE 44

Scale: Speed 2:1; Distance 4:1 mi.

## EXAMPLE 45

## BOTH SHIPS ALTER COURSE AND/OR SPEED DURING MANEUVER

## Situation:

At $0800 R$ is on course $105^{\circ}$, speed 15 knots and will change course to $350^{\circ}$, speed 18 knots at 0930 . Own ship $M$ is maintaining station bearing $330^{\circ}, 4$ miles from $R . M$ is ordered to take station bearing $100^{\circ}, 12$ miles from $R$, arriving at 1200.

## Required:

(1) Course and speed for $M$ to comply if maneuver is begun at 0800 .
(2) Course for $M$ to comply if $M$ delays the course change as long as possible and remains at 15 knots speed throughout the maneuver.
(3) Time to turn to course determined in (2).

## Solution:

Since the relative positions of $R$ and $M$ at the beginning and end of the maneuver and the time interval for the maneuver are given, the solution for (1) can be obtained directly from a geographical plot. Solve the remainder of the problem using a relative plot.
(1) Using a geographical plot, lay out $R$ 's 0800-1200 track through points $R_{I}$, $R_{2}$, and $R_{3}$. Plot $M_{l}$ and $M_{3}$ relative to $R_{l}$ and $R_{3}$, respectively. The course $040^{\circ}$ from $M_{I}$ to $M_{3}$ can be measured directly from the plot. $M$ will require a speed of 10.8 knots to move 43.4 miles in 4 hours.

This solution may be verified on a relative plot by means of a fictitious ship. First, using a geographical plot, determine the 0800 position of a fictitious ship that, by steering $350^{\circ}$, speed 18 knots, will pass through the 0930 position simultaneously with $R$. At 0800 own ship at $M_{l}$ bears $322^{\circ}, 45.7$ miles from the fictitious ship at $F_{1}$. Transfer these positions to a relative plot, placing $F$ at the
center. Plot own ship's 1200 position at $M_{3}$ bearing $100^{\circ}, 12$ miles from $F$. Draw the fictitious ship's vector $\operatorname{erf}_{1} 350^{\circ}, 18$ knots. From $r f_{1}$, construct the relative speed vector parallel to $M_{1} M_{3}$ and 13.8 knots in length. (MRM of 55.2 miles/4 hours $=13.8$ knots.) Draw em $m_{l}$, the required course of $040^{\circ}, 10.8$ knots.
(2) To find the two legs of $M$ 's 0800-1200 track, use a relative plot. Draw em ${ }_{2}$, own ship's vector which is given as $105^{\circ}, 15$ knots. At this stage of the solution, disregard $R$ and consider own ship $M$ to maneuver relative to a new fictitious ship. Own ship on course $040^{\circ}, 10.8$ knots from part (1) is the fictitious ship used. Label vector $e m_{1}$ as $e r f_{2}$, the fictitious ship's vector. From point $m_{2}$ draw a line through $r f_{2}$ extended to intersect the 15 -knot speed circle at $m_{3}$. Draw em ${ }_{3}$, the second course of $012^{\circ}$ required by $M$ in changing station.
(3) To find the time on each leg draw a time line from $m_{2}$ using any convenient scale. Through $m_{3}$ draw $m_{3} X$. Through $m_{l}$ draw $m_{l} \mathrm{Y}$ parallel to $m_{3} X$. Similar triangles exist; thus, the time line is divided into proportional time intervals for two legs. XY is the time on the first leg: 1 hour 22 minutes. The remainder of the 4 hours is spent on the second leg.

## Answer:

(1) Course $040^{\circ}, 10.8$ knots.
(2) Course $012^{\circ}$.
(3) Time 0922.

## Note:

In the above example, an alternative construction of the time line as defined in the glossary is used so that the line can be drawn to a convenient scale. The proportionality is maintained by constructing similar triangles. See Note with example 47.


Scale: Speed 2:1, Distance 4:1 mi.

## EXAMPLE 46

## COURSES AT A SPECIFIED SPEED TO SCOUT OUTWARD ON PRESENT

 BEARING AND RETURN AT A SPECIFIED TIME
## Situation:

Own ship $M$ is maintaining station on the guide $R$ which bears $110^{\circ}$, range 5 miles. Formation course is $055^{\circ}$, speed 15 knots.

## Required:

Commencing at 1730, scout outward on present bearing and return to present station at 2030. Use 20 knots speed.
(1) Course for first leg.
(2) Course for second leg.
(3) Time to turn.
(4) Maximum distance from the guide.

## Solution:

(1) Plot $R$ at the center and $M_{l}$ bearing $290^{\circ}, 5$ miles from $R$. Draw er $055^{\circ}$, 15 knots. The DRM "out" is along the bearing of $M$ from $R$. The DRM "in" is along the bearing of $R$ from $M$. Through $r$ draw a line parallel to the DRM's and intersecting the 20 -knot circle at $m_{l}$ and $m_{2}$. Vector $r m_{l}$ is the DRM "out". Vector $\mathrm{em} m_{l}$ is $327^{\circ} .8$, the course "out".
(2) Vector $r m_{2}$ is the DRM "in". Vector $e m_{2}$ is $072^{\circ}$, the course "in".
(3) To find the time on each leg, draw a time line from $m_{l}$ using any convenient scale. Through $m_{2}$ draw $m_{2} \mathrm{X}$. Through $r$ draw $r \mathrm{Y}$ parallel to $m_{2} \mathrm{X}$. Similar triangles exist; thus, the time line is divided into proportional time intervals for the two legs. XY is the time on the first leg, 41 minutes. The remainder of the time is spent on the second leg returning to station.
(4) Range of $R$ when course is changed to "in" leg is 21.7 miles. Initial range $+\left(r m_{1} \mathbf{x}\right.$ time on "out" leg $)$.

## Answer:

(1) Course $328^{\circ}$.
(2) Course $072^{\circ}$.
(3) Time 1811.
(4) Distance 21.7 miles.

## Explanation:

Since own ship $R$ returns to present station, relative distances out and in are equal. In going equal distances, time varies inversely as speed:

$$
\frac{\text { time }(\text { out })}{\text { time }(\text { in })}=\frac{\text { relative speed (in) }}{\text { relative speed (out) }}=\frac{r m_{1}(\mathrm{in})}{r m_{2}(\text { out })}
$$

Therefore, the time out part of the specified time $\left(3^{h}\right)$ is obtained by simple proportion or graphically.

As defined in the glossary, the time line is the line joining the heads of vectors $e m_{l}$ and $e m_{2}$. This line is divided by the head of vector $e r$ into segments inversely proportional to the times spent by own ship $R$ on the first (out) and second (in) legs. In the above example an alternative construction is used so that the line can be drawn to a convenient scale. The proportionality is maintained by constructing similar triangles.


Scale: Speed 2:1; Distance 2:1 mi.

## EXAMPLE 47

## COURSES AND MINIMUM SPEED TO CHANGE STATIONS WITHIN A

 SPECIFIED TIME, WHILE SCOUTING ENROUTE
## Situation:

Own ship $M$ bears $130^{\circ}$, 8 miles from the guide $R$ which is on course $040^{\circ}$, speed 12 knots.

## Required:

Proceed to new station bearing $060^{\circ}, 10$ miles from the guide, passing through a point bearing $085^{\circ}, 25$ miles from the guide. Complete the maneuver in 4.5 hours using minimum speed.
(1) First and second courses for $M$.
(2) Minimum speed.
(3) Time to turn to second course.

## Solution:

(1) Plot $M_{1}, M_{2}$ and $M_{3}$ at $130^{\circ}, 8$ miles; $085^{\circ}, 25$ miles; and $060^{\circ}, 10$ miles from $R$, respectively. Draw er $040^{\circ}, 12$ knots. From $r$ draw lines of indefinite length parallel to and in the direction of $M_{1} M_{2}$ and $M_{2} M_{3}$. Assume that a fictitious ship, $F$, departs $M_{l}$ simultaneously with $M$ and proceeds directly to $M_{3}$ arriving at the same time as $M$ which traveled through $M_{2}$ enroute. The fictitious ship covers a relative distance of 10.5 miles in 4.5 hours. SRM of the fictitious ship is 2.3 knots. Through $r$ draw $r r f$, the relative speed vector, 2.3 knots parallel to and in the direction of $M_{1} M_{3}$. Vector erf is the true course and speed vector of the fictitious ship. With $r f$ as a pivot, rotate a straight line so that it intersects the two previously drawn lines on the same speed circle. The points of intersec-
tion are $m_{l}$ and $m_{2}$. Vector $\mathrm{em} m_{l}$ is the course out: $049^{\circ}$. Vector $\mathrm{em} m_{2}$ is the course in: $316^{\circ} .9$.
(2) Points $m_{l}$ and $m_{2}$ lie on the 17.2 knot circle which is the minimum speed to complete the maneuver.
(3) From $m_{2}$ lay off a 4.5 hour time line using any convenient scale. Draw $m_{l} \mathrm{X}$. Draw $r f \mathrm{Y}$ parallel to $m_{l} \mathrm{X}$. The point Y divides the time line into parts that are inversely proportional to the relative speeds $r f m_{1}$ and $r f m_{2}$. XY the time "in" is 51 minutes. $\mathrm{Y} m_{2}$ the time "out" is 3 hours 39 minutes. Time on each leg may also be determined mathematically by the formula MRM/SRM = time.

## Answer:

(1) First course $049^{\circ}$, second $317^{\circ}$.
(2) Speed 17.2 knots.
(3) Time 3 hours and 39 minutes.

Note:
The time line, as defined in the glossary, is the line joining the heads of vectors $e m_{l}$ and $e m_{2}$ and touching the head of the fictitious ship vector erf. This time line is divided by the head of the fictitious ship vector into segments inversely proportional to the times spent by the unit on the first and second legs.
In the above example, an alternative construction of the time line is used so that the line can be drawn to a convenient scale. The proportionality is maintained by constructing similar triangles.


# EXAMPLE 47 

Scale: Speed 2:1; Distance 3:1 mi.

## EXAMPLE 48

## LIMITING LINES OF APPROACH

## Situation

A circular formation of ships 4 miles across, with guide $R$ the center is proceeding on course $000^{\circ}, 15$ knots. An enemy torpedo firing submarine is suspected to be in a position some distance ahead of the formation with a maximum speed capability corresponding to modes of operation of:

| Submerged (SU) speed: | 5 knots |
| :--- | ---: |
| Quiet (Q) speed: | 8 knots |
| Snorkel (SN) speed: | 10 knots |
| Surfaced (S) speed: | 12 knots |

## Note:

The maximum speeds above were chosen for example purposes and should NOT be used as real estimates. Consult appropriate intelligence publications on individual submarines for correct speeds

## Required:

(1) Construct Limiting Lines of Submerged Approach (LLSUA).
(2) Construct Limiting Lines of Quiet Approach (LLQA).
(3) Construct Limiting Lines of Snorkel Approach (LLSNA).
(4) Construct Limiting Lines of Surfaced Approach (LLSA).

## Solution:

(1) Plot $R$ at the center of the maneuvering board and draw the vector "er" $000^{\circ}, 15$ knots. Construct the TDZ for the assumed effective torpedo firing range (e.g., 5 miles) and torpedo speed (e.g., 30 knots). From " $r$ " describe an arc (with radius of 5 knots), the assumed submerged speed. Draw the tangent vector "emsu" to the arc and parallel this vector to the TDZ. By extending the parallel vector until it intersects the formation course vector, the other limiting line to the TDZ can be constructed (the area enclosed by the Limiting Lines of Submerged Approach (LLSUA) and the aft perimeter of the TDZ defines the submarine Danger Zone). Solutions (2) through (4) use the similar construction principles as in solution (1) to construct the LLQA, LLSNA and LLSA using their respective assumed speeds

## Note:

This construction assumes the submarine maintains a constant speed throughout the approach.


EXAMPLE 48
Scale: Speed 2:1,
Distance 4:1 mi.

## EXAMPLE 49

## TORPEDO DANGER ZONE (TDZ)

## Situation:

A circular formation of ships 4 miles across, with guide $R$ at the center is proceeding on course $000^{\circ}$, at 15 knots. An enemy torpedo carrying submarine is suspected of being in the area with weapon parameters of:

$$
\begin{array}{ll}
\text { Maximum effective torpedo firing range: } 5 \text { miles } \\
\text { Speed: } & 30 \text { knots }
\end{array}
$$

## Required:

Torpedo Danger Zone (TDZ)

## Solution:

Plot $R$ at the center of the maneuvering board. Calculate the formation's advanced position (i.e., $R$ 's future position along the formation direction of advance if a torpedo is fired when $R$ was located at board center) by:

$$
\text { Advanced Position }=\frac{\begin{array}{c}
\text { Maximum Effective Torpedo } \\
\text { Firing Range } \times \text { Formation Speed }
\end{array}}{\text { Torpedo Speed }}
$$

Label this position AP and plot the formation around AP. Construct the TDZ outer boundary by plotting points at a distance equal to the maximum effective torpedo firing range (e.g., 5 miles) from the perimeter of the formation. The area enclosed is the TDZ relative to the formation in its original position around $R$.

## Note:

The torpedo range and speed were chosen for example purposes only and should not be used as real estimates. Consult appropriate intelligence publications on individual submarine torpedoes for correct ranges and speeds.


GUIDE AT CENTER

EXAMPLE 49
Scale: Speed 3:1,
Distance 2:1 mi.

## EXAMPLE 50

## MISSILE DANGER ZONE (MDZ)

## Situation:

A circular formation of ships 4 miles across with guide $R$ at the center is proceeding on course $000^{\circ}$ at 15 knots. An enemy missile carrying submarine is suspected of being in the area with weapon parameters of:

Maximum effective missile firing range: Speed:

20 miles
600 mph

## Required:

Missile Danger Zone (MDZ)

## Solution:

Plot $R$ at the center of the maneuvering board. Since the enemy's missile travels at 40 times the formation's speed, the formation will not appreciably advance during the missile's time of flight. The missile's maximum effective firing range ( 20 miles) is added to the perimeter of the formation and plotted around the formation. The area enclosed is the MDZ.

## Note:

The missile range and speed were chosen for example purposes only and should not be used as real estimates. Consult appropriate intelligence publications on individual submarine missiles for correct ranges and speeds.


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EXAMPLE 50
Scale: Speed 3:1,
Distance 8:1 mi.

Answers:


Scale: Speed :1;
Distance :1 thousands of yds.

Answers:


Scale: Speed :1;
Distance :1 thousands of yds.

Answers:


Scale: Speed :1;
Distance :1 thousands of yds.

Answers:


Scale: Speed :1;
Distance :1 thousands of yds.

Answers:


Scale: Speed :1;
Distance :1 thousands of yds.

Answers:


Scale: Speed :1;
Distance :1 thousands of yds.

