Econ674

Economics of Natural Resources and the Environment

Session 9

Renewable Resource

Dynamic Optimization

The Nature of Renewable Natural Resources

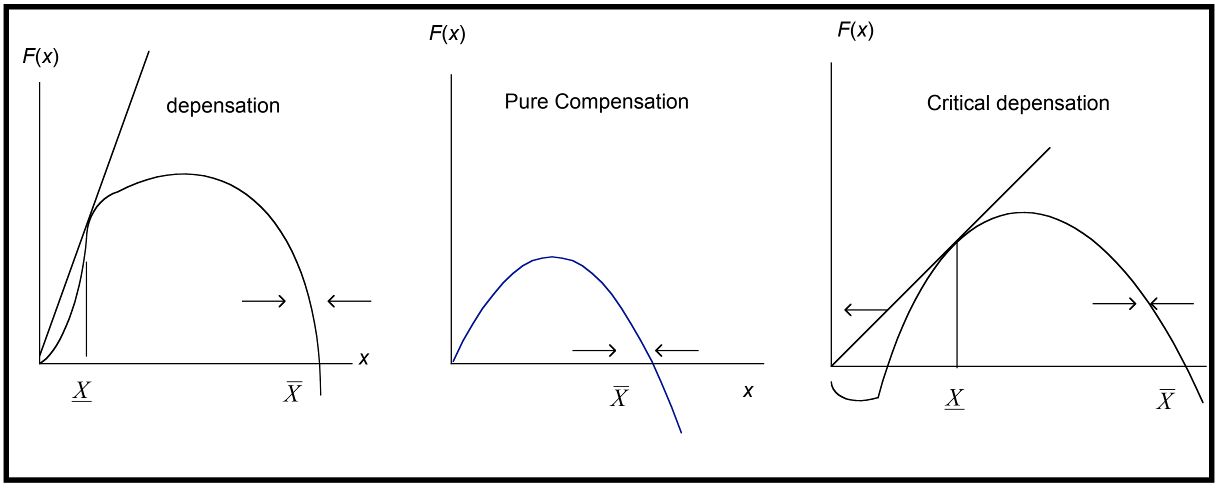
1. Renewable resources: biological entities with the capacity for reproduction and growth or inanimate mass/energy source subject to constant and/or continuous change
2. As noted, modeling dynamics of resources requires defining the state variables i.e., those that adequately describe the state of the resource at a point in time.
3. Let X_t (discrete), $X(t)$ (continuous) denote the size the stock at time t.
4. The natural resource dynamics/without harvesting/ is described by the difference equation (discrete case)

$$X_{t+1} - X_t = F(X_t) \qquad \mathbf{EQ(1)}$$

Assumption: growth is *density dependent*; i.e., changes in the resource depend on current stock size (X_t or $X(t)$).
F() = the growth function, and is defined over the interval X≥0 with two values for which $\underline{X} < \overline{X}$

$$
\begin{aligned}
&F(X) \le 0 && \text{if } 0 \le X \le \underline{X} \\
&F(X) > 0 && \text{if } \underline{X} \le X \le \overline{X} \qquad \mathbf{EQ(3)} \\
&F(X) < 0 && \text{if } \overline{X} \le X
\end{aligned}
$$

Functions that exhibit these properties are shown below:



The characteristics of these functions are:

1. *A purely compensatory growth function*: if $\underline{X} = 0$ And F() is strictly concave=> the relative growth rate r(X) = F(X)/X is decreasing in x.
2. *A depensatory growth function*: if $\underline{X} = 0$ and F() is initially convex then concave (thus it has an inflection point.
3. *Growth functions with critical depensation*: if $\underline{X} > 0$ and F() is initially convex then concave, then $\underline{X}$ is the minimum viable population.

The distinction between compensatory and depensatory growth functions is important in understanding the relationship between yield and effort.

A simple and best known functional specification for F() is the logistic model. When written as a differential equation, it takes the form:

$$\dot{X} = F(X(t)) = rX(t)(1 - X(t)/K) \qquad \mathbf{EQ(4)}$$

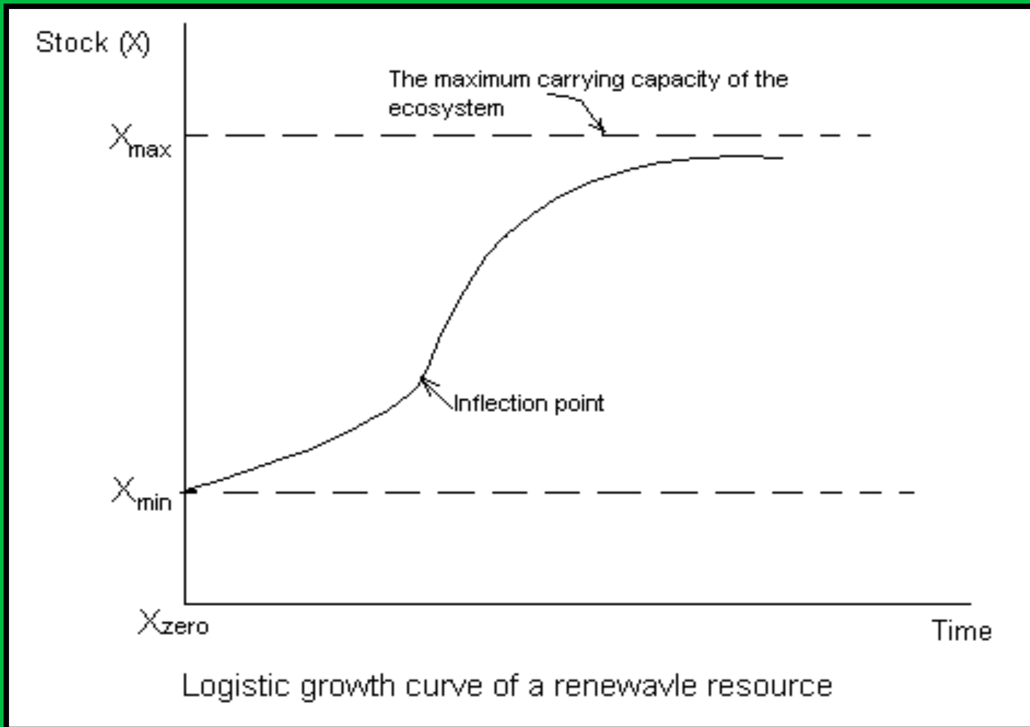
where r = the intrinsic growth rate and K = the environmental carrying capacity.

The explicit solution for X(t) is:

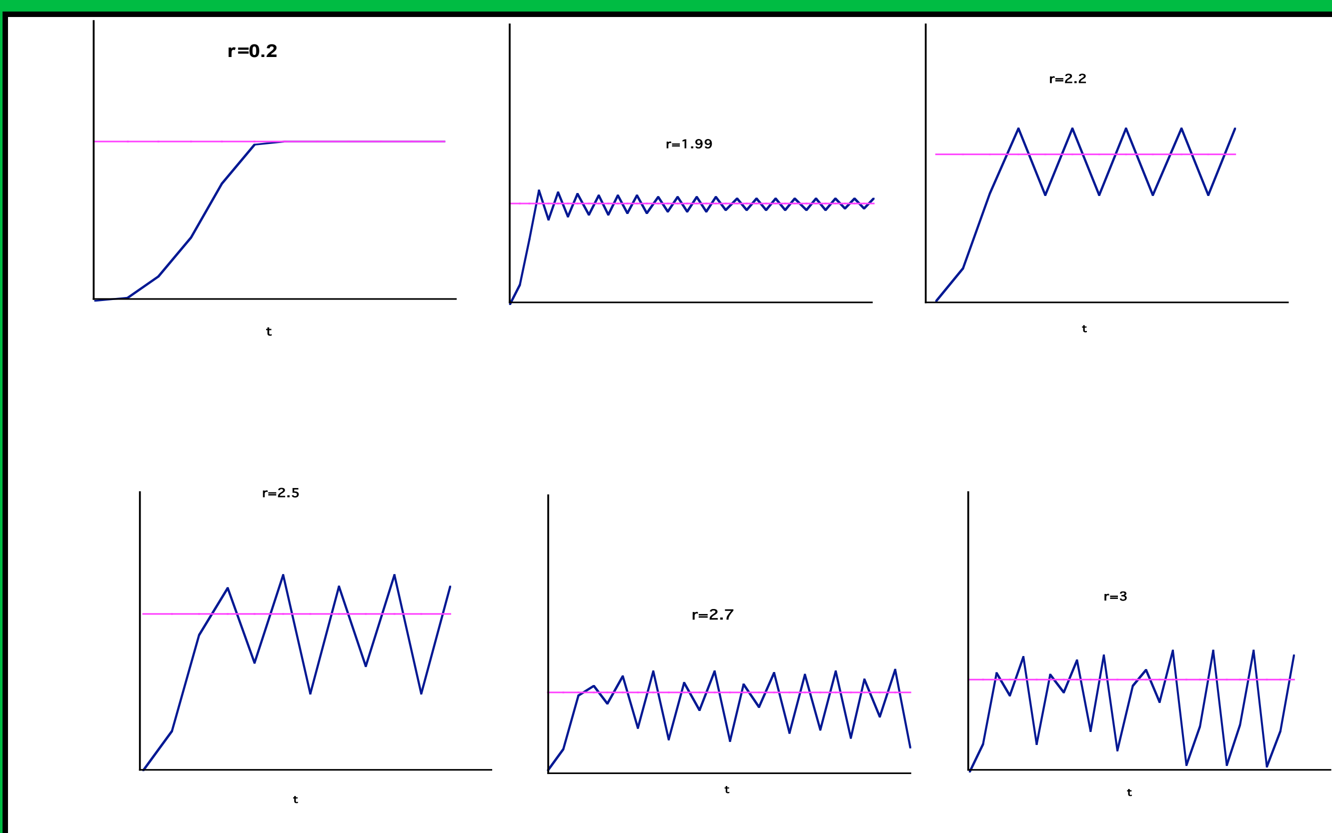
$$X(t) = \frac{K}{(1 - ce^{-rt})}$$

where: $c = (K - x(0))/X(0)$

This can be shown graphically as follows:



For the discrete case, more complex results can arise, depending on the value of r



Production/Yield Functions

The harvest rate per unit of time for a renewable resources is a function of inputs and the stock at time t.
Let E(t) be a measure of inputs referred to as "effort".

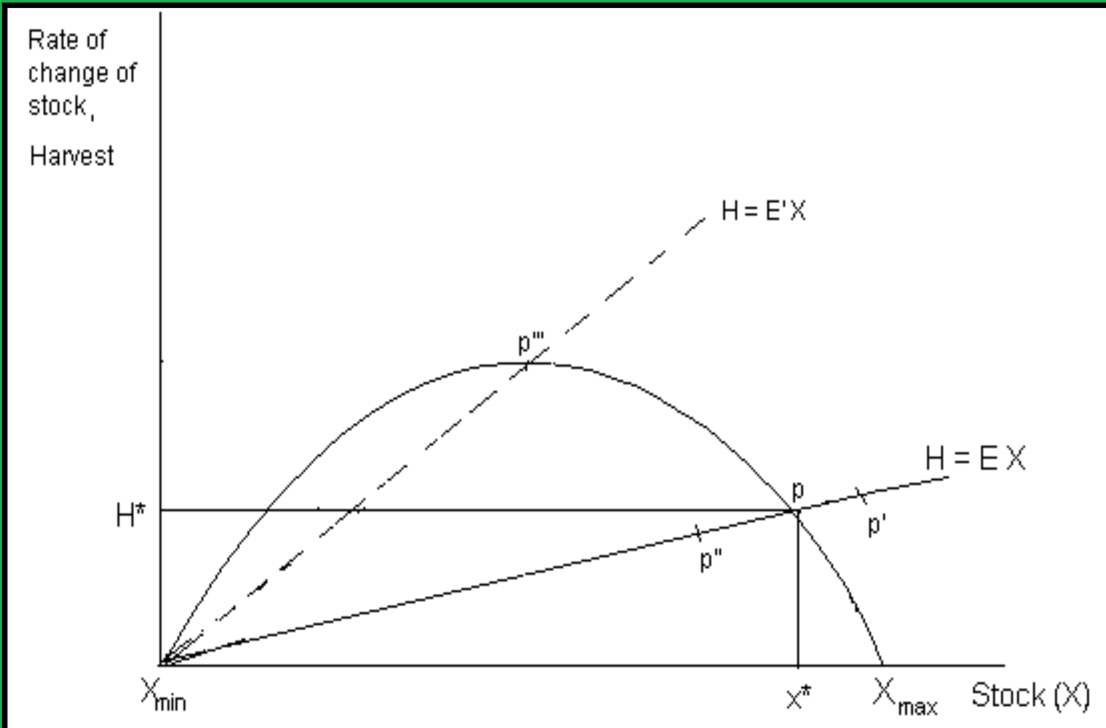
The production function may be written as:
Y(t) = H(E(t), X(t)), where Y(t) is the harvest rate measured in the same units as X(t).

A production function used in fishery management can be defined as:
Y(t) = qE(t)X(t), where q>0 (a constant), and is sometimes called a "catchability coefficient." This function is derived on the assumptions that:

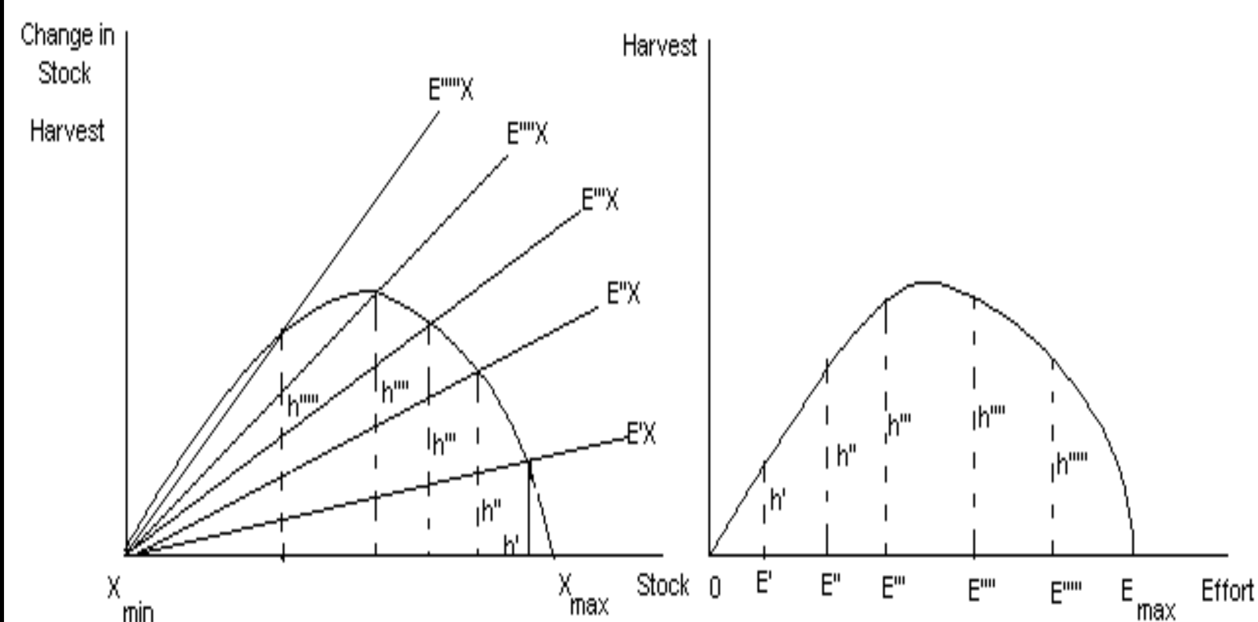
- the catch per effort (Y/E) is proportional to the density of fish in a given water body (the function is referred to as catch-per-unit-effort (CPUE) production function)
- the density of fish is proportion to the abundance X(t).

Changes in the resource stock must reflect growth and harvest: $X_{t+1} - X_t = F\left(X_t\right) - Y_t$

From this we derive a sustained yield function: This refers to an equilibrium level that expresses the sustainable harvest (yield) as a function of effort. The sustained yield means that X, Y, and E remain constant over time:



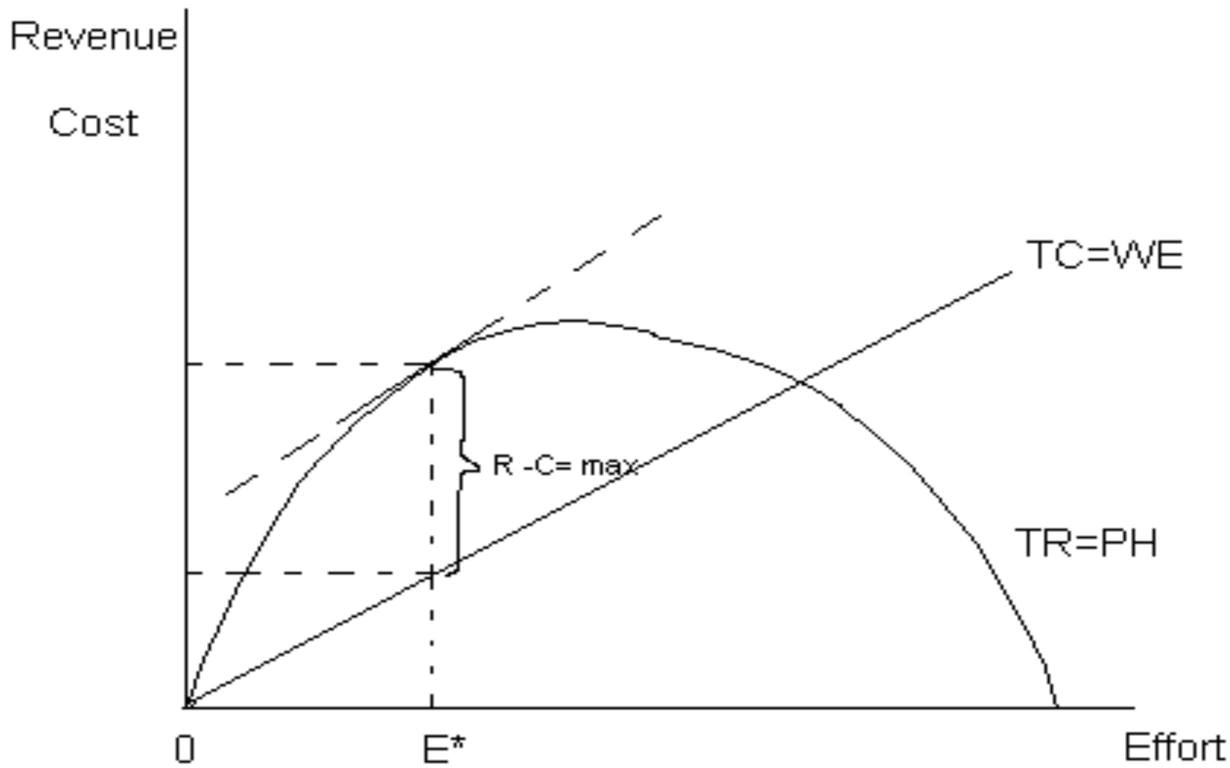
Effort - growth relationship



Transformation from growth - harvest to effort - harvest functions

Managing Renewable Resource Stocks:

The Static Efficient Sustainable Yield by a sole owner



Profit maximization condition

Derivation of the Maximum Sustainable Yield (MSY):

The maximum sustainable yield (MSY): a common objective of renewable resource management, which has been to maintain the standing stock X(t)=X so as to afford the MSY

With continuous time this means maximizing the sustained yield Y=F(X), which requires F'(X)=0.

For the logistic growth function,

$F'(x)=0$ at $X_{MSY}=K/2$ and $Y_{MSY}=rK/4$.

Alternatively, one can maximize the yield function $Y=Y(E)$ with respect to E, but the results are the same.

Economists tend to view renewable resources simply as a type of capital asset that should be managed so as to maximize its value to society. If the objective is to maximize the present value of a renewable resource,the static rent maximization approach is not optimal. Instead, the problem needs to be reformulated as:

Maximize $\Sigma \rho^t U(X_t, Y_t)$,
subject to $X_{t+1} - X_t = F(X_t) - Y_t$ and where X_0 is given.

In a steady-state, the first-order conditions collapse to three equations in three unknowns, X, Y, and λ. After some manipulation, $\rho\lambda$ can be eliminated, leaving the following two-equation system:

$$F'(X) + (\partial U(.)/\partial X)/\partial U(.)/\partial Y = \delta \qquad \textbf{EQ 5}$$

$$\text{and } Y=F(X) \qquad \textbf{EQ 6}$$

Equation 5 is the fundamental equation for maximizing the present value of a renewable resource: with steady-state levels of X and Y, the resource's own rate of return (LSH) is equated to the discount rate (RHS).

An economic interpretation of equation 5:

- LHS: the first term is the marginal net growth rate while the second is the "marginal stock effort" that measures the marginal value of the stock relative to the marginal value of the harvest.
- The two terms on the LHS sum to the "resource's internal rate of return"

By the implicit function theorem, equation 5 implies a curve Y=h(X) and we can plot this along with Y=F(x) to identify the optimal levels of X* and Y*.

Let us assume the asset behaves according to a logistic growth function, $F(X_t)=rX_t(1=X_t)/K$), then the production function $Y_t=H(X_t, E_t) = qX_tE_t$, and the cost function can be noted as $C_t=cE_t$.

We can solve the production function for $E_t=Y_t/(qX_t)$ and substitute this into the cost equation to obtain the optimal cost function $C_t=cY_t/(qX_t)$.

Assuming a constant price and constant marginal cost, we can define U(X,Y) as profit, which is $pY_t-cY_t/(qX_t)=[p-c/(qX_t)]Y_t$. The derivative of the net growth function is $F'(X_t)=r(1-2X_t/K)$. Using this derivative and evaluating the partials in equation 5 at the steady-state, equation 5 then becomes:

$$r(1-2X/K)+cY[X(pqX-c] = \delta \qquad \textbf{EQ 7}$$

Solving equation 7 for Y we obtain:

$Y = h(X) = (X(pqX-c)[\delta-r(1-2X/K)]/c$.

The value of h(X) depends on the entire set of bio-economic parameters c, δ, K, p, q and r, while the maximum sustainable yield(MSY) and X_{MSY} only depend on r and K.

The intersection of h(X and F(X) could result in the optimal stock lying above or below XMSY=K/2.

The optimal stock will be less than K/2 if the 'marginal stock effect' is less than the discount rate.

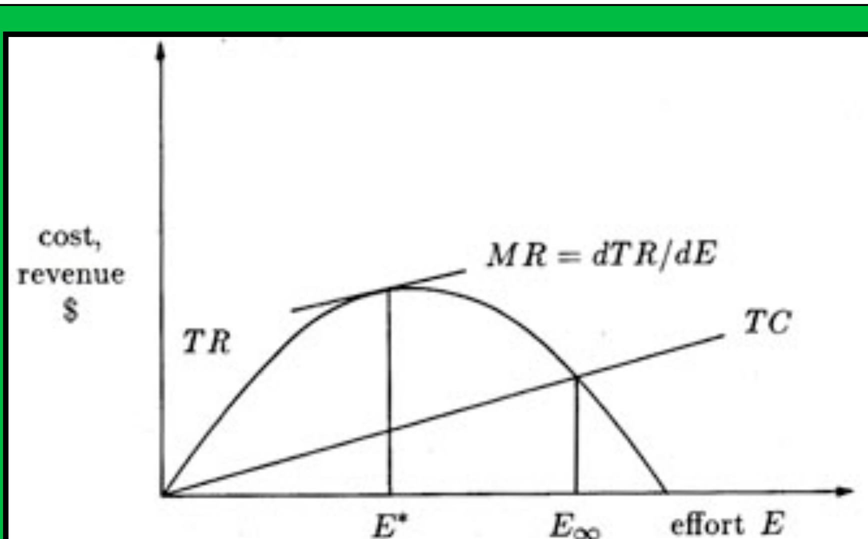
The optimal stock will be greater than K/2 if smaller renewable stocks significantly increase cost. In such a case, it is optimal to maintain a large stock even greater than the MSY stock.

Open Access Common Property Resource (CPR) Choices

Many renewable resources can be classified as "common property", I.e., no individual or firm possesses exclusive rights to exploit them.

The general conclusion is that common property resources tend to be over-exploited. Yet this rule is often misunderstood. One result is that policymakers often have failed to achieve desired or stated objectives.

An early model of a common property resource is due to Gordon (1954), with the property that is follows the yield-effort curve of the Schaefer model and assumes a constant fish price p and effort cost c. Under these assumptions, TR = pY, and TC = cE, as functions of effort E.



Gordon argues that:

- the optimum point is at E = E*, where MR = MC,
- the CPR will reach an equilibrium at E = E_∞, where TR = TC (or, AR = AC).
- Rent, TR - TC, (which is maximized at E*) is totally "dissipated" at E = E_∞ and
- the resource stock will be seriously depleted, since $E^*>E_{MSY}$.

Though he argues much about the fact that $E^*<E_{MSY}$, dynamic analysis shows that this conclusion depends on the assumption of zero discounting, which Gordon makes tacitly rather than explicitly in his analysis.
It is always true, even for $\delta>0$, that the optimal level is less than E_∞ ,which in fact corresponds to $\delta \to \infty$
The argument behind this prediction is simple: sustained effort level will not occur at:

- E > E_∞ as producers will lose money and give up production, or
- E < E_∞ as producers make more than their opportunity costs, cE, attracting more producers (or existing producers to expand their effort).

Equilibrium can be established only if E = E_∞
Gordon calls this bionomic equilibrium of the common property fishery.
To express this in terms of theory, we have:

$X = F(X) - Y$ **EQ 8.**
$Y = xEX$ **EQ 9.**
$U(X,E) = pY - cE = (pqX - c)E$ **EQ 10.**

Bionomic equilibrium occurs when net returns U(X,E) are zero.

Using equation 7, we have X = X_∞ = c/pq **EQ 11.**

Thus X_∞ is the stock level at which net returns from fishing become zero.

The corresponding bionomic yield and effort levels are obtained from equations 8 and 9, $Y_\infty = F(X_\infty)$ and $E_\infty = Y_\infty / qX_\infty = F(X_\infty)/qX_\infty$

Extinction of Species

Could common-property exploitation drive a renewable resource stock to extinction?

According to equation11, we have $X_\infty > 0$; if also $F(X_\infty) > 0$ then bionomic equilibrium also is a biological equilibrium at $X = X_\infty$

But if $F(X_\infty) < 0$ (i.e., $X_\infty < \underline{X}$ = the minimum viable population), then extinction will result, although exploitation will cease when X(t) falls below .

The fact that X_∞ is positive is a result of the form of the production function used Y = qEX.

This implies that the catch per unit effort Y/E = >0 as X =>0. Since price p is also assumed constant, it follows that revenues will fall below effort costs at low but positive X levels. In fact, this model assumes that the cost of driving the resource to extinction is infinite a rather strong assumption. But even where the marginal cost is finite, policy instruments can be used to forestall extinction.

Optimal Forestry Rotation

Let $V(t)$ denote the "stumpage" value of a given strand of trees at age t years.
Typically we have:

$V(t) = 0$ for $O<=t<=t_1$ and
$V(t)>0$ for $t>t_1$.

If the forest is clear-cut at age t, net revenue will be
$V(t) - c$,
where c is the total cost of clear-cutting and replanting.
After clear-cutting a new rotation is begun and we wish to determine the optimal rotation period $t = T$.

We assume a discount rate δ, and suppose that all parameters remain constant over time (e.g., soil productivity, price, discount rate, and replanting cost). Given these assumptions, from the principle of optimality it follows that all rotation periods will have the same length T.

The total net present value (TNPV) of all future forest cuttings can be defined as:

$$J=(V(T)-c)\ (e^{-\delta T}+e^{-2\delta T}+\ldots)=(V(T)-c)/(e^{\delta T}-1)$$

The optimal rotation period T is then obtained by setting $\delta J/\delta T = 0$. The resulting equation is:

$$V'(T)/(V(T)-c) =\delta/(1-e^{-\delta T})$$

This is called the Faustmann (1949) equation. Since the growth rate of forests usually is quite low, the rotation period is highly sensitive to the discount rate. The average yield also is sensitive to the discount rate.