MusMat

Brazilian Journal of Music and Mathematics

Vol. II - No. 1 - May. 2018

MusMat

Brazilian Journal of Music and Mathematics

Vol. II - No. 1

MusMat Research Group (Ed.)

Graduate Program in Music
Universidade Federal do Rio de Janeiro (UFRJ)
Rio de Janeiro - May 2018

MusMat • Brazilian Journal of Music and Mathematics

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MusMat - Brazilian Journal of Music and Mathematics. MusMat Research Group (Ed.). Rio de Janeiro: Universidade Federal do Rio de Janeiro (UFRJ), School of Music, Graduate Program in Music, 2018. Vol.II, No. 1.

ISSN: 2526-3757

Foreword

We are glad to present the first number of 2018's volume of *MusMat – Brazilian Journal of Music and Mathematics*. The seven articles of this issue address different aspects of the multitude of possible intersections between mathematical and musical theories and practice, forming a rich and varied pannel of the current research in these fields. **Guilherme Bertissolo** examines applications of some instances of pitch-class cycles in musical composition. **Charles de Paiva** proposes a model for computational assisted-analysis using deterministic algorithms, which is applied in an analysis of Steve Reich's *Clapping Music*. **Luigi Irlandini** correlates the notion of number/proportions and mythic cosmologies as a key for understanding his own compositional processes. **Luka Marohnić** presents an interesting approach using *Hans van der Laan's plastic number* for the study of Hepokosky and Darcy's *sonata type 3* in Mozart's movements. **Pauxy Gentil-Nunes** exposes the concept of *Partitional Complexes*, as an expansion of his theory, *Partitional Analysis*, with some proposals for hierarchical analysis of musical texture. **Dmitri Tymoczko** introduces the *interable voice-leading schemas*, proposing a new systematical, mathematical approach for analytical exame of voice-leading configurations. **Didier Guigue** presents the elements of an original methodology based on the idea of *sounding partitioning*, which is applied in the analysis of the orchestration of Webern's *Variationen op. 30*.

Carlos Almada
May 2018

Contents

Cycles in Music: Spaces, Experience and Applications in Music Theory and Composition

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Abstract: *This paper focuses the idea of cycle and its approaches in music theory, in order to propose further application to music composition. The impulse for reseaching cycle was made possible through a previous research, in which I proposed a conceptual framework for the relationship between music and movement in Capoeira. Here, I first discuss the idea of cycle, then its theoretical approaches, and finally, some compositional processes based on cycles.*

Keywords: *cycle, composition, creative process.*

Introduction

This paper focuses the idea of cycle and its approaches in music theory, in order to propose further application to music composition. The impulse for researching cycle was made possible through a previous research, in which I proposed a conceptual framework for the relationship between music and movement in Capoeira[1].

In the first section, I discuss the idea of cycle, proposing a definition for the term, in relation to cognition and our bodily experience. In the second section, I propose a short discussion about theoretical approaches to cycle, where music and mathematics are co-implicated. Finally, I discuss some compositional processes based on cycles, in order to exemplify the previous discussion.

I. The notion of cycle: a conceptual territory

Cyclicity is understood here as a property which is somewhat characterized through a cycle[2]. The term cycle holds many meanings. We may consider several approaches to define a cycle.

[1]In my doctoral research – PhD in Music Composition at Universidade Federal da Bahia (Federal University of Bahia – UFBA) – I focused the complex interaction between music and movement, and specially its uses and functions in the creation of compositional processes, taking as reference a context in which it is not possible to establish a clear distinction between them: the Brazilian Capoeira [1]. Capoeira is a combination of music, dance and martial arts that has been developed in Brazil by West African slaves and its descendants and has been exported all over the world. The combination of field work and a critique of existing literature on the interaction between music and movement led to the proposition of a conceptual framework, four concepts which are not mutually exclusive: Cyclicity, Sharpness, Circularity and Surpriseness.

[2]It is important to mention that this approach to cycle in music is based on a previous – and preliminary – effort in a presentation entitled as "A noção de Ciclo em música: concepções e aplicações composicionais" [2], realized at the ANPPOM's Congress, João Pessoa.

From a cognitive point of view, the cycle is an image schema based on our bodily experience[3, p. 362], therefore, it is a basic and easily recognizable concept, since we experience different cycles everyday.

Brower defines cycle in relation to the container image schema, emphasizing its time dimension:

> The CYCLE schema serves to organize our experience of time and the changes by which we measure time [...] The CYCLE schema combines easily with the CONTAINER schema, much of its structure. The circle, being closed, can be conceptualized as a container for motion around its perimeter. [...] [T]emporal containers may be of fixed duration, as in the case of conventional cycles such as minutes, hours, and weeks, or they may be subject to expansion and contraction, as in the case of most bodily cycles. Cycles may also be nested, with larger cycles subsuming smaller ones, producing a temporal hierarchy [...]. We observe [...] opposition in many bodily cycles – the alternation of left and right in walking, in and out in breathing, back and forth in swinging. The alternation may be balanced, so that the halves of the cycle mirror one another exactly. Or the alternation may be asymmetrical, as in the case of most bodily processes of tension and relaxation, with the climax sometimes coming well after the midpoint of the cycle. A wave can assume an infinite variety of forms, and in fact it is this kind of flexibility that allows the schema to play such a pervasive part in our understanding of temporal experience [3, p. 329-30]

Notwithstanding, we propose a conceptual approximation, in order to create a territory for cycle based in three notions (Figure **??**).

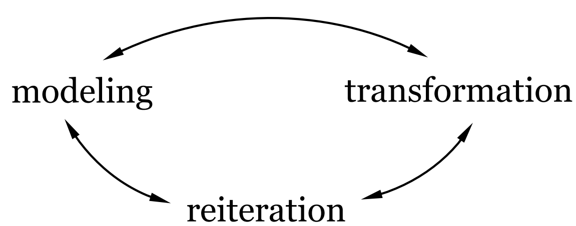


Figure 1: *Notion of cycle based on three concepts*

Modeling is conceived here as a virtual connection between two events which allows us to identify resemblances and correlations between them. In this sense, it is an element capable of promoting the identification of traces, even changing ones, common features of two subsequent events spread in time. In other words, it is a sort of design, schema or contour. This modeling is then continually changing and transforming through time.

Therefore, the definition I would like to propose is: in a cycle, a certain modeling reiterates itself by transformation (a cycle not necessarily implicates a repetition, even tough the repetition is the most literal way of reiteration). As a matter of fact, The idea of cycle is one of the most important schemas for time. Cycles organize our sense of time. In music, cycle plays an important role: cycles of songs, cycles of fifths, interval cycles, and so on, are obvious examples. Nevertheless, the notions of cycle should neither be considered as rigid models nor as chains of literal repetition. Even when they are not assumed, there is no doubt they are sorts of cycles and even the most literal way of representing a cycle.

It is important to mention that there is no linearity or cause/effect relation between the three notions, they permeate each other. In other words, most of the time it is not possible to precisely the beginning and/or the ending, once we are always in the midst of the cycle. Thus, one can only delimit the scope of a cycle through the formalizing interference of an observer. The notions of transformation and reiteration imply temporality, as mentioned by Candace Brower [3]. The cycle is one of the image schemas based on our body experience (p. 328).

Some questions still remain: are there any cycles which don't reiterate? Are there any cycles which don't transform themselves? Are there any cycles with no modelling (more or less literal)? I propose here that the approach of cycle as modelling, transformation and reiteration allows us to understand such nebulous concept.

Concerning the previous research [1], the rhythm, the melodic profiles, the movements and the game itself are cyclic characteristics of Capoeira Regional. For example, Ginga is a basic Capoeira movement which plays a role as a basis for all movements[3].

II. Notions of cycle in some approaches in music theory

Laske's Epistemology of Composition [7] takes the creative process in music as a "Compositional Life Cycle". In this sense, composition is a cycle *per se*. For him, the cycle has four interdependent levels: ideas, materials, implementation, and work. The author discusses the complex network of the creative process, taking both model-based and rule-based composition as complementary approaches.

The notion of cycle plays an important hole in contemporary music theory, especially in *neo-riemannian theory*. Cohn [4] presents a historical overview of this field of study, taking as reference the first edition of Lewin's seminal book [8]. Cohn focuses on Lewin's essay, written in 1982 five years earlier than the book), in order to propose a series of perspectives for music theory.

As a starting point, the *neo-riemannian theory* took as reference models of voice-leading applied to triads, a process also known as triadic post-tonality [4, 11]. Afterward, this model was also applied to other pitch class sets and collections.

It is important to mention the important role of smooth voice-leading and its geometric visualization. Voice-leading is related to pitch-spaces, and in some ways it offers us forms to understand the relationships between aggregates. Morris, for example, asserts [9, p. 95]: "in recent music theory, graphs—nodes connected by lines or arrows – have become important tools for modeling music, musical structures, and compositional systems".

Siciliano [10, p. 222] proposes the maximally smooth cycle of triads as a *toggling*, as expressed in Figure 2.

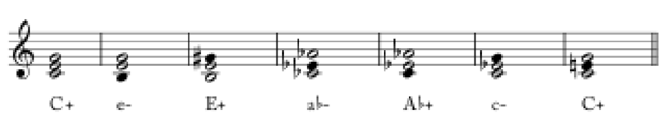


Figure 2: *toggling: maximally smooth voice leading [10, p. 222]*

Notice that the modeling is the half-step voice leading between two subsequent notes. The transformation always results in new perfect triadic forms. Figure 3 shows the toogling in a geometric approach.

Figure 4 shows a cycle of chords which took the pitch class set 026 as a starting point. Note the smoothness of the voice leading, as the modeling consists of whole steps which transform the chords gradually until it gets back to the original chord.

[3]For more examples of cycle in capoeira, its significance, and how the cycle in the context suggested me this approach, see the PhD dissertation [1].

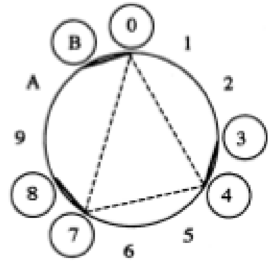
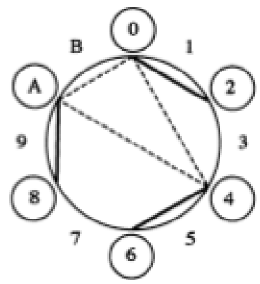


Figure 3: *Toggling in a geometric form: maximally smooth voice leading [10, p. 225]*



(a) Toggling cycle by ic 2.

(b) Voice leading among 3–8[026] trichords in an ic 2 toggling cycle.

Figure 4: *Toggling 026 with a whole step voice leading [10, p. 226]*

The *tonnetz* is a network of triads in a pitch space [4, p. 175], where each triad is related to its homonym and relative forms (Figure 5). This cycle is modeled by four cycles of half-step (marked by circles) in C, C sharp, D and E, presenting its homonym and relative forms in anticlockwise. The central square indicates the *hyper-hexatonic system*.

Straus [11] proposes a series of concepts through which we may discuss atonal voice-leading. His approach establishes tools for connecting different pitch class sets. We point out important notions for cycle, such as Pitch-class voice, Transformational voice leading, Uniformity, Balance, Offset, Consistency, Span, Fuzzy transpositions and Voice-leading smoothness.

In the third edition of his seminal book "Introduction to Post Tonal Theory", Straus also approaches interval cycles[12, p. 154], with examples by Bartók, Ives and Varèse. Unfortunately, he only mentioned simple cycles, with a unique interval. Tymoczko [13, p. 107] also proposes cycles of intervals and its modes of visualization in a geometry of music. In spite of the simple cycles he proposes, Tymoczko's work presents interesting insights for understanding music through geometry[4].

[4]Due to scope issues, this paper focuses on cycle instead of geometry, Tymoczko's approach shall not be discussed here

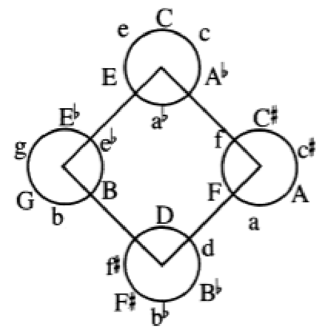


Figure 5: *Tonnetz: hyper-hexatonic system [4, p. 175]*

On the other hand, Gollin [6, p. 143] proposes "compound interval cycles" projected in a pitch space with two or more intervals. Interesting examples are Bartók's *Study Op. 18, 1* (Figure 6) and *Scherzo* from *Suite Op. 14* (Figure 7), in which the modeling is composed respectively by the alternation of major/minor thirds and major thirds/minor seconds. Notice in Figure 8 how cycle 4-5 engenders the octatonic collection (Bartok also extensively uses the 4-5 cycle).

Figure 6: *Cycle of major and minor thirds in Bartók's Study Op. 18, 1 [6, p. 144]*

As I shall further discuss, the idea of projection of multi-aggregate cycles in pitch spaces is a powerful tool for generating material, which allows us, on the one hand, to implode the sonority of a chord by the projection of its constitunt intervals, and, on the other hand, to transform the chord in a pitch space.

In his turn, Morris [9] proposes musical applications of *minimal graph cycles*:

> Graphs may be constructed from other graphs called input relations. Strictly speaking, a relation is a graph of two nodes connected by one or two arrows. However, we will allow input relations to be more complex, assuming they satisfy some context-

in a more detailed way.

Figure 7: *Cycle of major thirds and minor seconds in Scherzo from Suite Op. 14 [6, p. 145]*

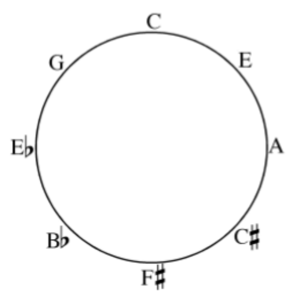


Figure 8: *Cycle 4-5 and the octatonic collection [6, p. 147]*

> sensitive definition of simplicity and/or basic importance. So, input relations can be the graphic representation of ordered or unordered sets, partially ordered sets, cycles, and so forth. Both the input relations and the graphs they construct may or may not be partitioned into disconnected subgraphs [9, p. 100]

Figure 9, Morris shows the graph cycle through which he composed the flute excerpt presented in 10. Notice how the cycle starts in 0 (C), and moves step by step, forming a square in the upper left. Then, he starts the square again and departs to the lower part of the cycle, followed by the right part of the Figure. All the paths which make the composition of the excerpt possible are easily recognizable through the graph.

Finally, it is important to mention the work by Pedro Augusto Dias [5], a PhD Dissertation on combined concentric cycles in structuration of pitch in Thomas Adès. Dias analyzes several modelings of cycles and proposes important insights though visualization tools made possible by the geometry of music. Even the work is focused on Adès' processes, Dias presents and discusses important issues on cycles in music, with several examples both in analysis and composition.

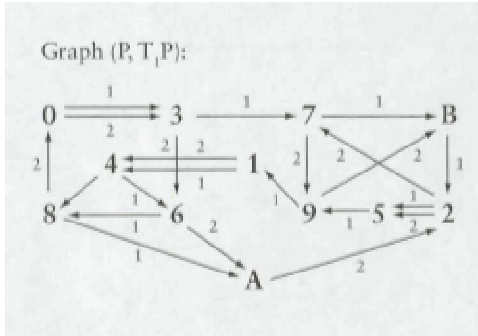


Figure 9: *Morris graph cycle [9, p. 104]*

Figure 10: *Morris' music based on the graph cycle [9, p. 104]*

III. Some applications of cycles in composition

In this section I briefly discuss some applications of cycles in composition[5]. *Fumebianas* is a series of works composed in previous research on the relationship between music and movement in Capoeira [1]. The notion of cycle plays an important role in the process of generation of harmonic material in the series. Interval cycles made possible the creation of harmonic spaces, by the projection of a sonority extracted from the context (pentatonic collection) in relation to the pitch class set 5-16 (especially in its form 03467). Figure 11 shows the smoothness between the two aggregates.

$$
\begin{array}{lcl}
0 & \xrightarrow{0} & 0 \\
2 & \xrightarrow{+1} & 3 \\
4 & \xrightarrow{0} & 4 \\
7 & \overset{0}{\searrow} & 6 \\
9 & \overset{-3}{\nearrow} & 7
\end{array}
$$

Figure 11: *Smoothness between pentatonic collection and the pitch class set 5-16*

The dialog between the two sonorities allowed me to propose cycles of transpositions of 5-16 through interpolations of two subsequent pentatonic intervals. Figure 12 shows these interpolations and how they were constructed, and Figure 13 shows the application of the cycle in

[5]The intent of this section is to provide examples of applications of cycle in composition. I hope just to show a few examples, in order to illustrates the issues previously discussed in the paper. For more detailed analysis, the scores and recordings are available in `https://guilhermebertissolo.wordpress.com/`.

Fumebianas Nº 5.

Figure 12: *Cycles of 5-16 transformed by two subsequent pentatonic intervals*

Figure 13: *Cycles of 5-16 transformed by two subsequent pentatonic intervals - Application in Fumebianas Nº 5*

In Figure 14 I show a sequence of chords based on the projection of the sonorities in a pitch space. It is important to notice the three reiterations of the cycle, each generating different trichords. Figure 15 shows the application of the material in *Fumebianas Nº 5*.

It is possible to generate, gradually transforming, multiple cycles of chords in this space, while maintaining resemblance to the sonority of Capoeira.

In *Fumebianas Nº 4*, I proposed two different interval projections, from 5-16 and pentatonic collection, in order to generate pitch spaces. Thus, I constructed pitch spaces based on cyclic projections of the interval of each set, as shown in Figure 16.

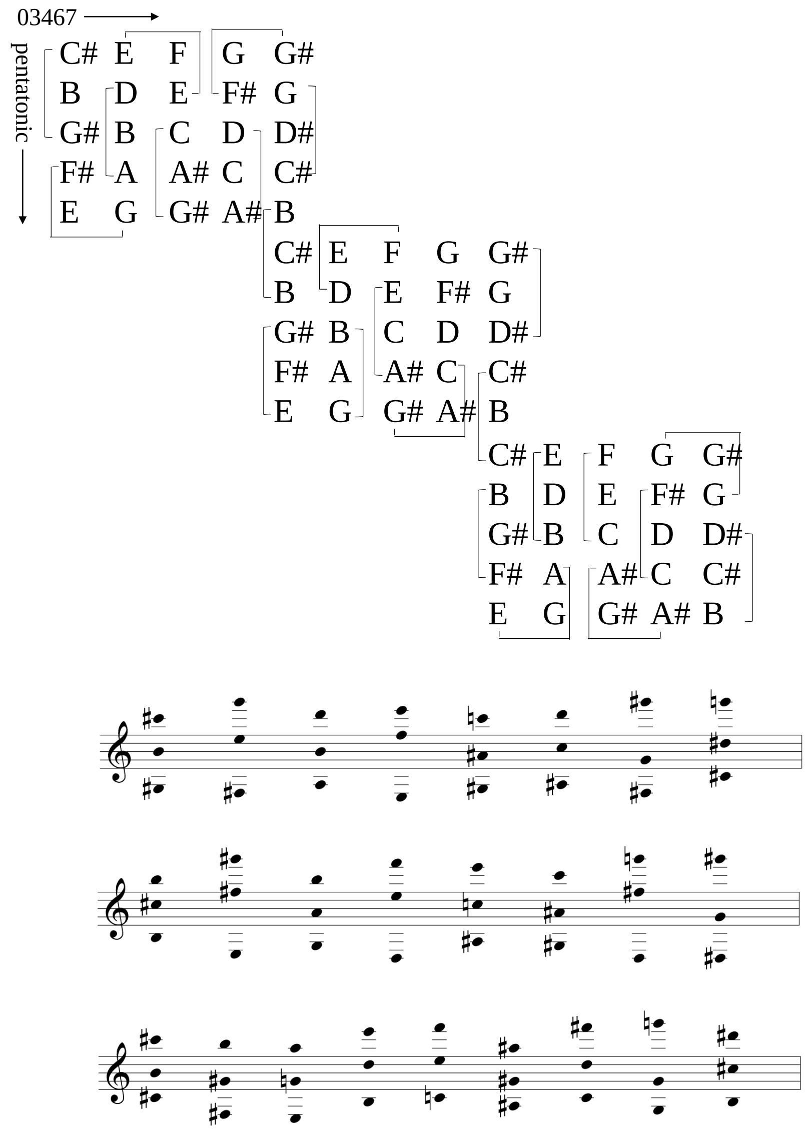


Figure 14: *Creating pitch spaces*

These pitch spaces were the basic map through which the piece moved. For example, I took the modeling 3, 2, 2, 5, 5, 5, 6, 4, 4, 5, 5, 4 and 5 and applied to both cycles, generating different materials from the same path in two different spaces (Figure 17). This type of strategy was applied throughout the series.

Finally, in *Fumebianas Nº 5*, I generated material by an algorithmic process made possible through cycles of superposed patterns of eighth notes. Figure 18 shows the modeling, where different patterns of eighth notes are separated by pauses. These cycles started with five eighth pauses, and then four, and so on, until we get just one eighth pause.

Figures 19 and 20 show the application of the first and last cycle. It is important to notice the gradual intensification process which takes place through the first part of *Fumebianas Nº 5*. As the

Figure 15: *Creating pitch spaces: application in Fumebianas Nº 5*

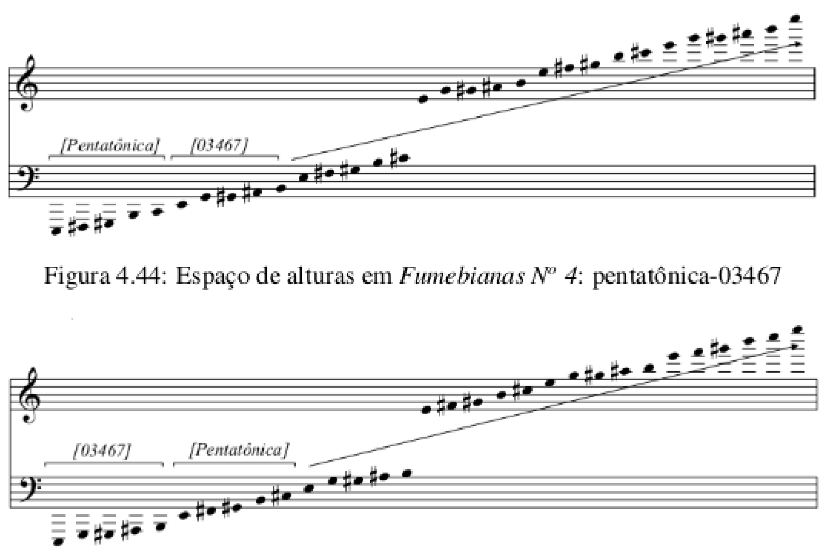


Figure 16: *Two cycles of intervals*

texture gets more and more dense with every new cycle.

IV. Final considerations

Cycles plays an important hole both in composition and music theory. The numerous approaches to cycle allow us to better understand music and its creative process. Cycles are important ways of visualization, once they allow us to think in terms of espacialization.

As I argue above, I believe cycles connect our bodily experience to the "allegedly abstract" pitch spaces. Once we think in terms of metaphorical projections, bodily spaces help us to understand

Figure 17: *Same path in two different pitch spaces*

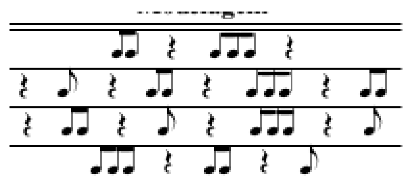


Figure 18: *Cycles of patterns*

Figure 19: *Cycles of patterns: five eighth pauses*

pitch spaces and even create them. I propose that our experience of cycle is a key to understanding music as a complex of experience, cognition, space, geometry and culture.

Figure 20: *Cycles of patterns: one eighth pause*

The concepts of cycle I propose here were inspired by Capoeira's movements. Therefore, cycle has a meaningful cultural significance in this context, and my point of view was strongly impregnated by this significance. The cognitive meaning of cycle as an image schema is fundamental, as expressed by Brower [3, p. 325], since "image schemas that lend coherence to our bodily experience are metaphorically reflected in conventional patterns of melody, harmony, phrase structure, and form".

Finally, I argue that cycles and their graphs in pitch spaces are so capable of making sense to us, and the geometry of music is so significant, due to the power of metaphorical mapping through which the relationship between notes, chords and harmonies take on meaning through our embodied cognition. Ultimately, the space in music is eminently cultural. In other words, we experience music spaces as we perceive bodily spaces, with both cognitive and cultural meaning. Thus, it is fundamental to develop tools for understanding music spaces through visualization, in order to better analyze and create music.

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Rudiments of Simulation-Based Computer-Assisted Analysis Including a Demonstration With Steve Reich's *Clapping Music*

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Abstract: *Based on previous experiments the article presents the most basic principles of a computational approach for musical analysis that through deterministic algorithms aims to reconstruct and then simulate neighboring variants (called instances) of existing musical scores. For that, adequate numerical representations are required, and their use in Computer-Aided Composition (CAC) systems are presented. Numerical sequences mapping to musical elements such as pitches, durations, and articulations may be computed or hardcoded for subsequent transformation, concatenation, and superimposition. They allow the reconstruction of the segments of a given musical score. The rhythmic pattern of Clapping Music can be modeled as a group of beats being progressively deprived of one beat, each group being separated by a rest, and the sequence concatenated with its retrograde. The sequence is subsequently transformed by the successive application of "phase shifts". A graphical interpretation of the piece is introduced using barcodes. Variations are envisaged by manipulating parameter values, each different value corresponding to a specific instance. Usually, parameters reflect compositional choices, but completely arbitrary models are possible. Such is the case of an alternative model of Clapping Music where a number is converted to binary representation and then mapped to rests and beats of eighth-notes. The manipulation of strong parameters modifies structural features of the musical score while weak parameters may only change the way the score is notated. The set of possible simulations gives rise to a space of instances. It can be analyzed through diachronic analysis, where a small group of variations is compared to the original piece, or achronic analysis, where variations are seen as single points in the space.*

Keywords*: Algorithmic composition, Computer-assisted musical analysis, Computational Musicology, Steve Reich .*

I. Introduction

In the course of the past years, I was involved in the algorithmic modeling of a couple of pieces. They included:

- *The Spectral Canon for Colon Nancarrow* by James Tenney [21, 22]
- *Spiegel im Spiegel* by Arvo Pärt [26]
- *Désordre* for piano by György Ligeti [23].

Although very different, they share a strong underlying algorithmic thinking. For this very reason, I chose those pieces as case studies of a kind of computer-assisted analysis that focus on the reconstruction and generation of neighboring variants.

In that I was inspired by the work of French researchers André Riotte and Marcel Mesnage [18, 19]. They are known to have developed a musical analysis based on formal models of compositions from the repertoire [1]. Before them, the usual was to simulate a given style, not a single piece, through the determination of probabilities and the conceptual help of Shannon's *Information Theory* [25]. Following the path opened by Fred Brooks [5], Lejaren Hiller [12] and their compatriot Pierre Barbaud [3], they used computer simulations to validate deterministic models instead. However, the possibility of using the same models to simulate variations was only suggested by Riotte and Mesnage. Figure 1 shows a comparison between the approaches.

In this article, I intend to expose some of the principles that guided me in the elaboration of such computer models. Naturally, the following ideas are still in development, and I expect them to be further expanded in future works. For such a short text, I selected what is most relevant and imperative to understand the general approach.

In the course of the following explanations I will refer to other published experiments but will include the modeling of Steve Reich's *Clapping Music* that was conceived for a pedagogical purpose [2].

Nico Schüler [24] rightly says that musical analysis, and especially when computer-assisted, is often taught and practiced with few or any references to the used methods. This article is also my first answer to that.

II. Theoretical Premise

It was demonstrated that existing musical scores could be rebuilt or generated using computer models and it was also suggested that neighboring variants could also be produced by such procedure:

> [...] for us, to model a musical score is to model the composition process by an algorithm able to reproduce, either the score or the neighboring variants obtained by a different set of parameters.[3] [20].

Expanding on the above definition, we can postulate that *a musical score is one single occurrence of a system's particular configuration.* Neighboring variants are then envisaged by modeling the behavior of such systems and manipulating its parameters values.

The variants are called the *instances* of the piece. Being similar or unlike, the instances are ontologically related. Their study should result in further knowledge about the musical work's inherent attributes and open for new analytic and creative possibilities.

The model is understood *as a computer program that allows the reconstruction of the musical score or some of its specific aspects.* The model inputs are parameter values, and its output is symbolic musical data. Subroutines implement music composition techniques and related musical tasks. Aspects of a musical score that reflect compositional "choices" are implemented as parameters. The model conception is based on the preliminary musical analysis of the chosen piece.

[1] Those included the *Variations for piano, op. 27* by Anton Webern, The *Two-Part Invention No. 1* by J.-S. Bach, and the *Troisième Regard sur L'Enfant Jésus* by Olivier Messiaen.

[2] This text is however intentionally devoid of programming examples. For a complete (Common LISP) implementation of Clapping Music, allowing for the reconstruction of the original piece and generation of variants refer to `https://github.com/charlesdepaiva/Clapping-Music`

[3] [...] modéliser la partition, pour nous, c'est modéliser le processus de composition par un algorithme capable de reproduire, soit la partition, soit des variantes voisines obtenues par un nouveau jeu de paramètres.

The modeling is oriented mostly towards the immanent properties of the musical score (its *neutral level*), rather than how the composer made the score (*poietic level*) or how the piece is perceived by the listener (*esthesic level*) [17, 22].

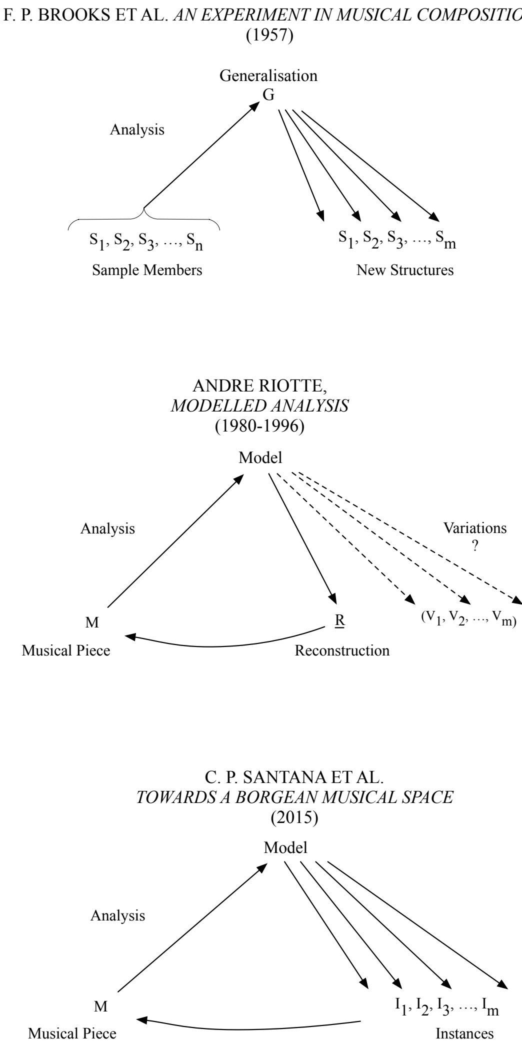


Figure 1: *Diagrammatic representation of three different approaches for modeling and simulation of musical scores. Based on Fred Brooks* An experiment in musical composition *[5]*

To implement the model and run simulations the use of a Computer-Assisted Composition environment (CAC) is of great help. Those include *OpenMusic* [4], *PWGL* [15], and *Common Music / Grace* [28, 29, 30] environments. Those offer a large set of ready-to-use *functions* performing these categories of tasks and where new routines can be constructed from them. The output of the model can be heard as an audio or MIDI file, seen as a musical score and fine-tune edited (see Figure 2).

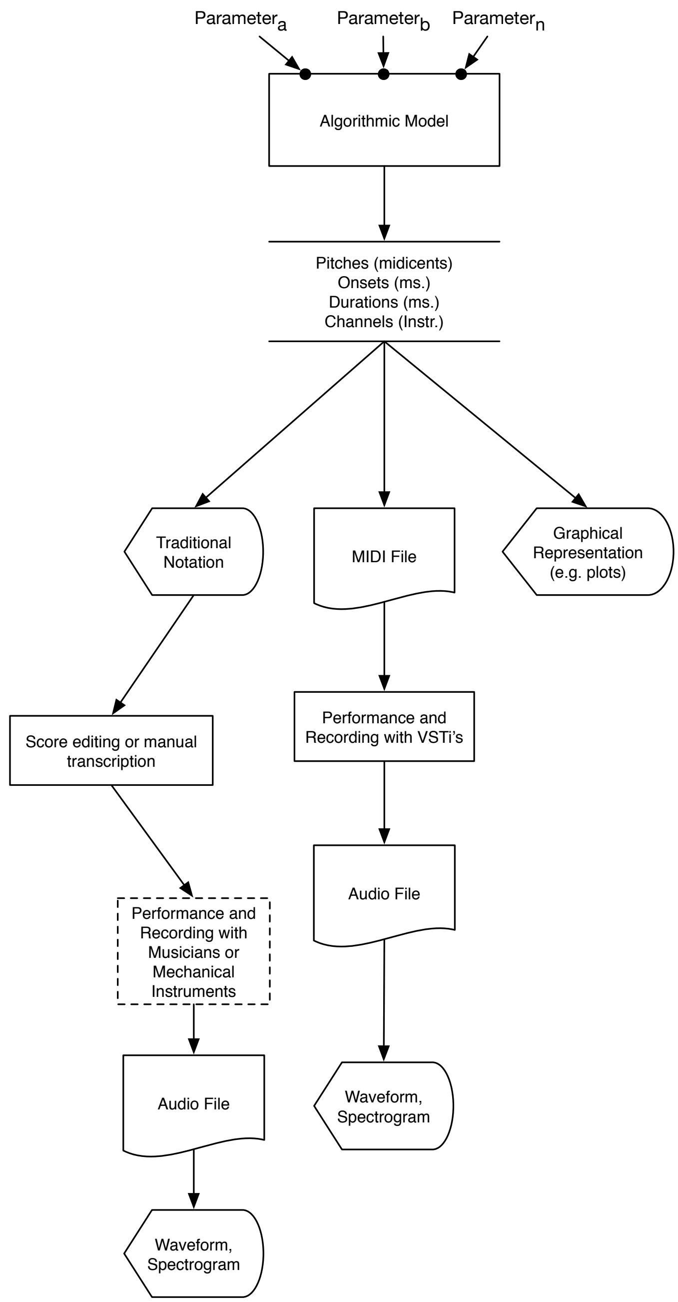


Figure 2: *Flowchart illustrating the symbolic data output by the algorithmic model being explored through different digital formats for representing music.*

III. Numerical Representations

The fundamental step of any computer-assisted analysis is the specification of adequate numerical representation for the musical structures [8]. The musical score is a representation of sound phenomena. The score is a space that suggests and privileges certain operations as a result of its bidimensional aspect. Space and time operations, as transpositions, inversions and retrogradations result from the possibilities offered by the representation itself. In this sense, the numerical representation of (notated) musical structures is a representation of a representation, and the computer model of a score is a model of a model. The numerical representation is the mediation between musical score and computer.

The numerical representation of musical structures refers us back to French composer Olivier Messiaen (1912–1992). With the piece *Modes de valeurs et d'intensités* (1949), he not only laid the foundations of *total serialism*, but also introduced a detailed mapping of different musical dimensions to integer numbers (see Figure 3). This representation allowed Messiaen to operate space and time transformations, used in polyphonic writing (and by the twelve-tone technique), not only upon pitches and durations but also intensities and articulations.

In fact, pitch and durations, because of cultural and historical developments, will be more suitable to this kind of representation and, consequently, to calculations upon them. As we know, most of the Western musical tradition, in which fall our case studies, privileged those two dimensions. On the other hand, intensities, articulations and especially timbre, can be seen as more challenging as they were not explored as much by traditional western music theory. That probably comes from the fact that, from the perspective of performance and perception, those dimensions pose some challenges, although the issue may be better handled in the electroacoustic domain.

Be as it may, one could argue that simulation-based analysis is more suitable to compositional practices that focus chiefly on the pitch and durational dimensions. Additionally, compositional practices where an underlying algorithmic thought already exists, as in some serial pieces and most of the works written by Olivier Messiaen, James Tenney, Arvo Pärt and Steve Reich, among others, may constitute a *corpus* more fitted to be studied by the modeling approach presented here.

Pitch can be represented in hertz, savarts, MIDI number and so on. The MIDI standard assigns an integer number to each key of a standard keyboard (C4=60, C#=61, D=62 . . .). As the MIDI system was not conceived for microtonal music, we can use instead the *midicent* standard, which is the MIDI number multiplied by one hundred (C4 = 6000). In the *midicent* system, one semitone is equal to 100 and one octave is equal to 1200 midicents (see Figure 4). It is used in CAC environments such as OpenMusic [4] and PGWL [15]. In some cases, as in the model of James Tenney's piece [21] and most of *spectral music*, the representation in hertz is needed for some calculations.

One way to represent durations and rhythms is to map the traditional rhythmic figures to a fractional representation. Such representation is already used to formulate measure signatures. For example, 1/8 (or just 8) refers to the eighth-note, while 1/4 refers to the quarter-note and so forth. To represent processes not based on rhythm figures (as in proportional notation) the concept of onset is used. It represents the moment in time (here expressed in milliseconds) where a note is attacked or an event is started. The onset information is complemented by the determination of the same event's duration (see Figure 5).

For dynamics, we use what is already specified by the MIDI standard, a range from 0 (very soft or silence) to 127 (very loud). It can be mapped to scales of different steps to represent music score marks, such as *piano*, *forte*, and so on. For articulations such as *legato*, *non legato* and *staccato*, which have a durational nature, we may think of a scale that goes from 0, a very short interpretation of

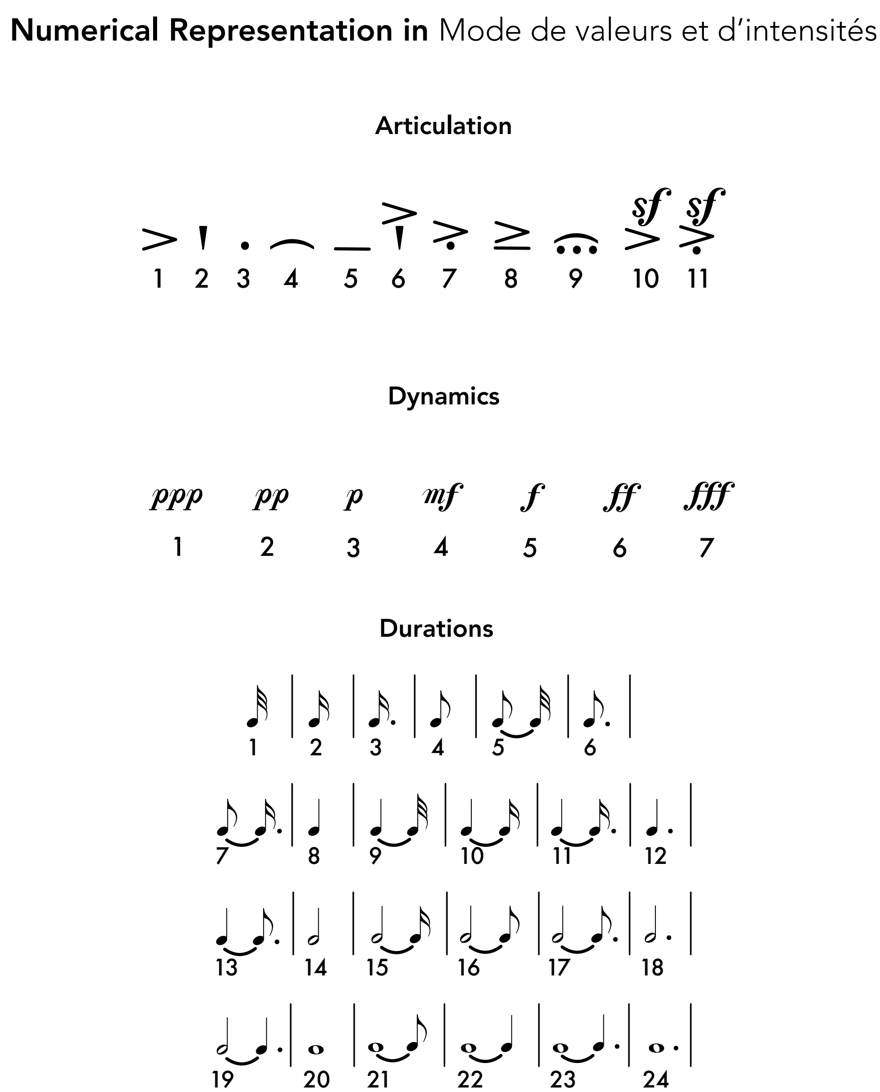


Figure 3: *Numeric representations in* Modes de valuers et d'intensités *(1949) by Olivier Messiaen.*

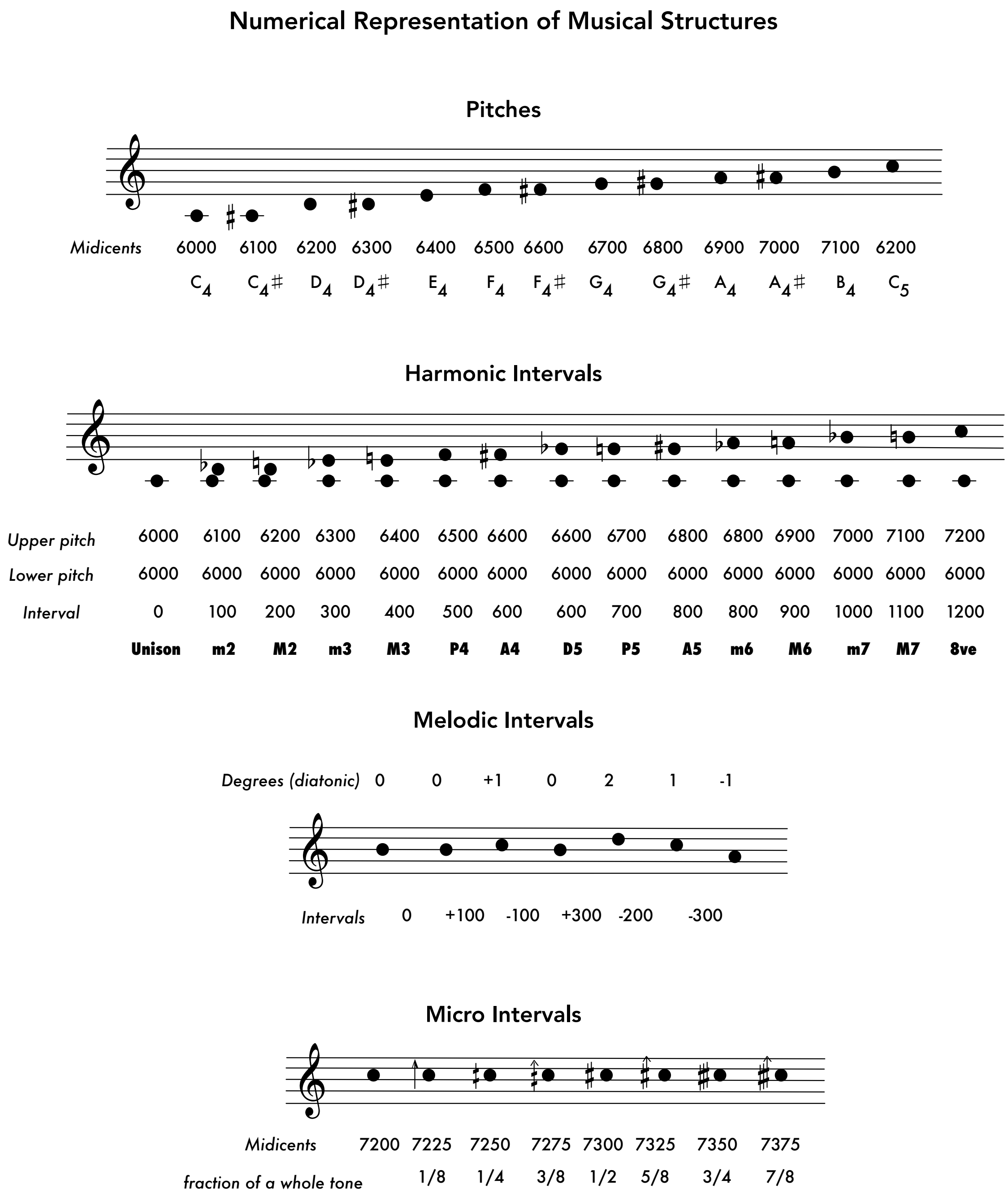


Figure 4: *Numerical representation of pitches as they are used in CAC systems such as* OpenMusic *and* PWGL*. The* midicent *number is the MIDI number multiplied by 100, allowing the use of steps smaller than the semitone.* Common Music / Grace *however uses a decimal MIDI number representation for micro intervals.*

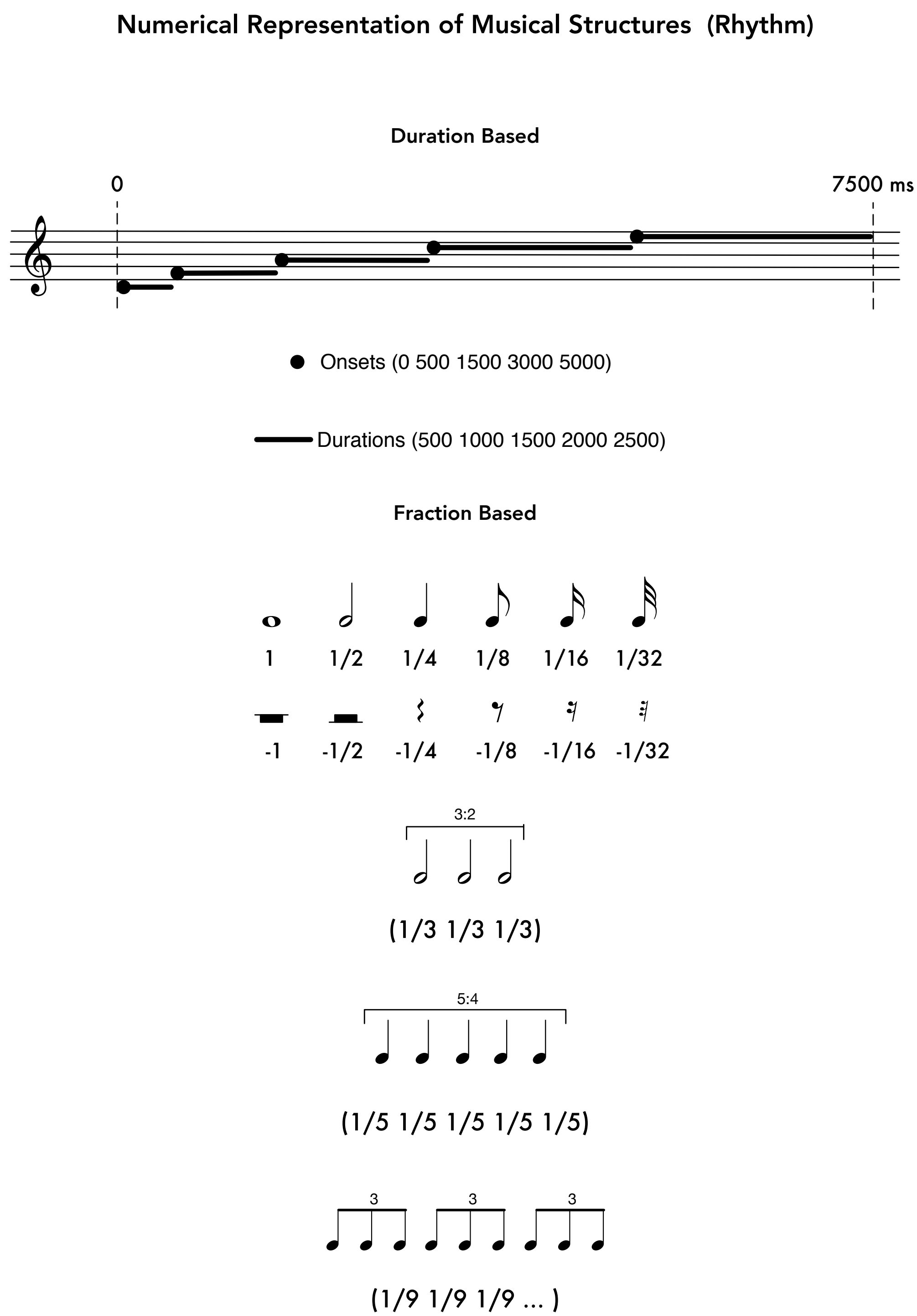


Figure 5: *Numeric Representation of rhythm and durations. Onsets and durations are represented in milliseconds.*

the ’written’ duration as in *staccatissimo*, to 100, as in *legato*, where the duration, if needed, may be prolonged to connect the notes one after another (see Figure 6).

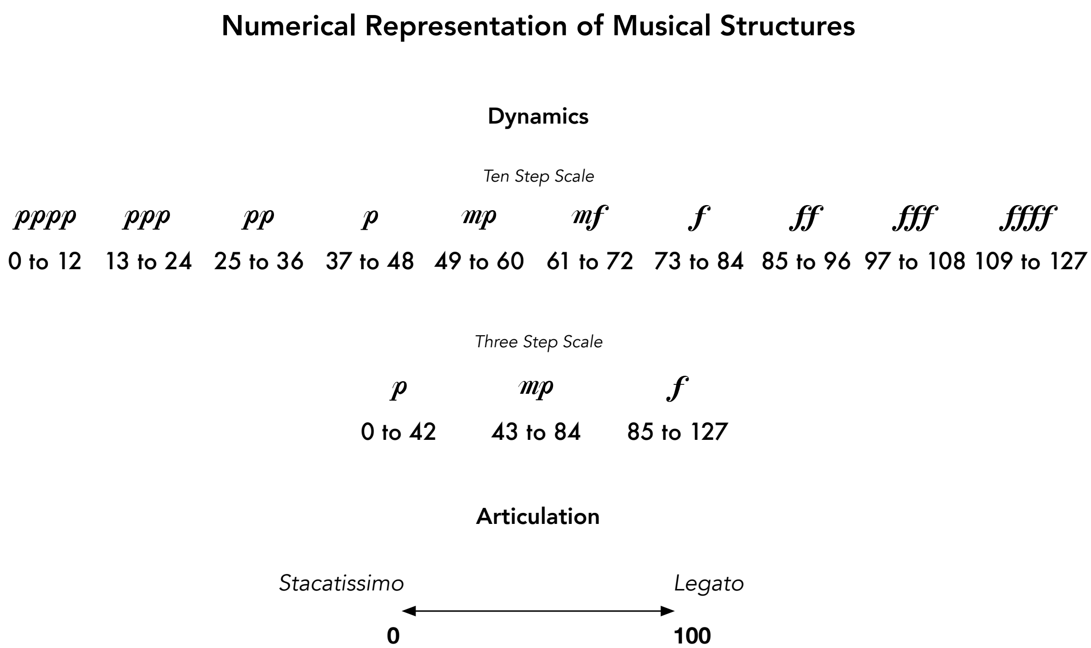


Figure 6: *Numeric Representation: dynamics (from MIDI standard) and articulation (as used in* OpenMusic*).*

IV. Generation of Numerical Sequences

Very often, modeling scores and compositional processes involve the segmentation of sequences of symbolic data, like pitches, durations, etc. More precisely, it is interested in the formalisms that correlate those segments and generate them. For instance, a model can describe the rules from which a sequence, or pattern, is *transformed*, *concatenated* and *superimposed* to reconstruct a given musical score or excerpt. Such is the case when a melody is consistently repeated (concatenation) and transposed (transformation), as in Ligeti’s *Désordre* [30, 14]. That is also the case when a rhythmic pattern is repeated, rotated (transformation) and played in a texture of two voices (superimposed), as it happens in *Clapping Music*.

The model can treat a sequence, or segment of, monolithically, that is to say, consider it as a “given series” and “hardcode” it on the implementation. Or the model can describe how its most basic patterns can be computed and generated. In both cases, such endeavor often overlaps with some domains in discrete mathematics and computer science, especially those of *Formal languages* and *Automata theory* [19, 6]. Those fields are interested in the study of numerical sequences and the different, often abstract, machines that can compute them [31, 1, 9, 16].

In the modeling of James Tenney’s *Spectral Canon for Conlon Nancarrow* there is almost any transformations, and the entire sequence of durations is modeled by a single equation (the sequence is then concatenated with its reverse and superimposed) [21]. On the other hand, the modeling of a twelve-tone piece may be interested only in the row’s transformations and not in how the twelve-tone series itself can be computed.

In short, the modeling process means finding algorithms that can reconstruct and transform numerical sequences which, in turn, are mapped to different musical elements as pitches, durations, dynamics and so on.

Another aspect that can be part of the modeling process is the *generation of music materials*. It is considered as musical material the set of chords, scales, articulations and so on. Broadly speaking it involves the study of "sets", where the sequential or temporal aspects are not considered yet. For instance, it concerns the generation of pitches from the harmonic series for the *Spectral Canon for Colon Nancarrow* [21], or the generation of pitches from the concept of "combinatoric tonality" in György Ligeti's *Désordre* [27, p. 8]. According to the categorization presented by Chemilier [7], after Xenakis, the generation of music materials is part, essentially, of what can be called the *outside time domain*.

Now, to demonstrate the modeling of a rhythmic sequence I will refer to the pattern of *Clapping Music* (also used in *Music for Pieces of Wood*). The pattern can be interpreted as a borrowing from Subsaharan African music or an exercise drum pattern. In any case, we could see it as a result of an algorithmic process where (1) a group of eighth notes is progressively deprived of one beat, (2) each group is separated by a rest, (3) and the current sequence is followed by its retrograde. Figure 7, shows its analysis. This sequence is repeated, transformed, and arranged in a particular form. Figure 8 shows a graphical interpretation of the piece.

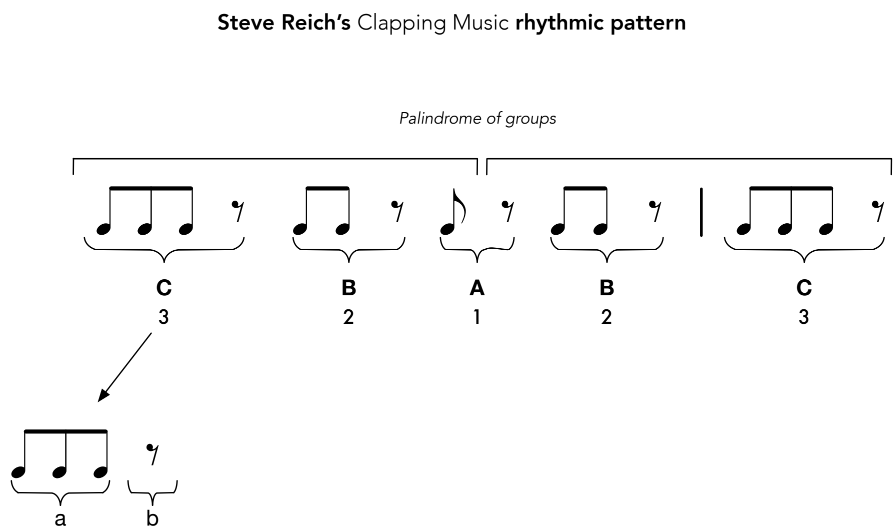


Figure 7: *Analytic decomposition of Steve Reich's* Clapping Music *rhythmic pattern.*

V. Transformations and Operations

Once established the representations, materials, and segmentation, the next step is the determination of the transformations that operate on those numerical structures. Some of the most recognizable transformations of Western music literature can be implemented by simple arithmetic, for example, transpositions can be made by adding to a sequence of numbers (representing a melody or chord) a particular interval. The intervals between pitches can be implemented as the

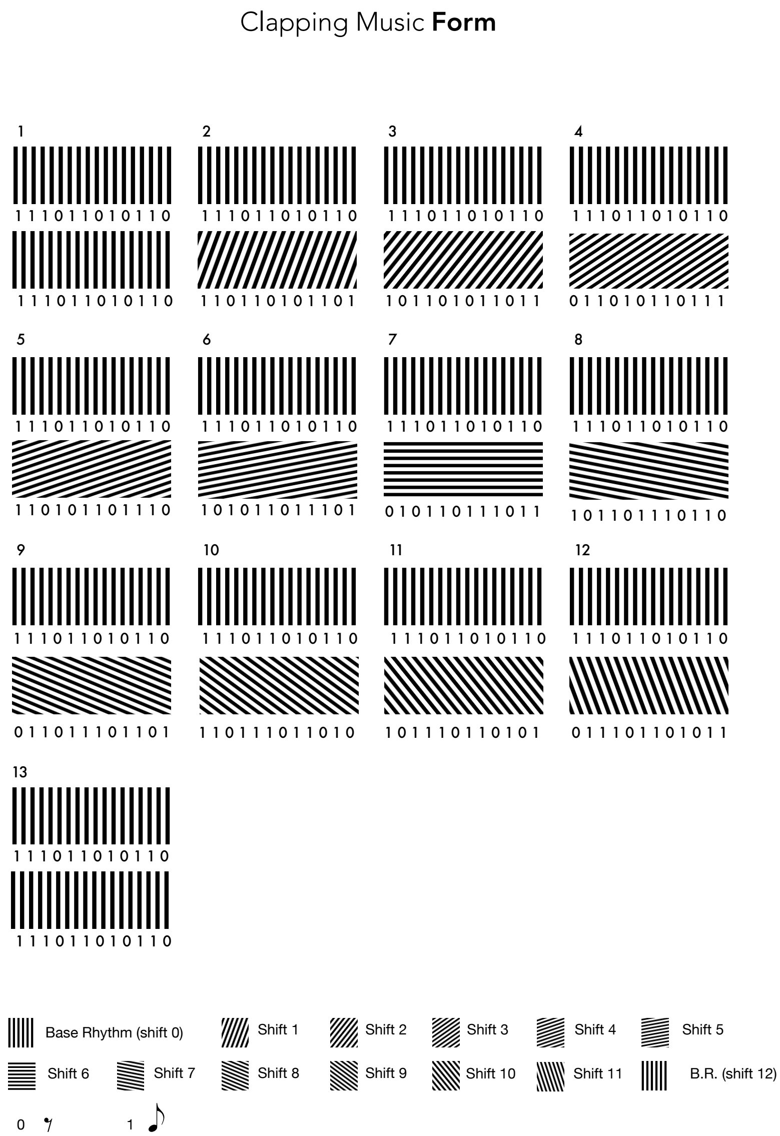


Figure 8: *Musical form of Steve Reich's* Clapping Music*. The rhythmic pattern is numerically represented by zeros (rests) and ones (eighth-notes). While the first voice only repeats the pattern, the second voice successively* transforms *it through the application of* phase shifts *(cf. Figure 10). The repetitions and shifts are also represented by the different 'barcodes'.*

subtraction of highest and lowest notes. Processes as rhythmic augmentation and diminution can be calculated as multiplications of the pattern.

Very often *special transformations* or *operations*, that is, compositional processes particular to a given composer or practice, need to be implemented. An example of special transformation is the *tintinnabulation* used by Arvo Pärt, where a melodic sequence is consistently mapped to the tonic's triad [13]. Another example of special transformation is the technique of Phase Shifting used by Steve Reich. In the specific case of *Clapping Music*, it means the consistent rotation of its rhythmic pattern (Figure 10).

The implementation of special transformations may involve the preliminary study of available literature, as the composer's texts and sketches. Nevertheless, transformations and even complete models could be made from independent, arbitrary generative processes. To demonstrate the modeling by an arbitrary process, in the Figure 9 we see *Clapping Music*'s rhythmic pattern represented through the mapping of rests and eighth-notes to zeros and ones. Then successive *phase shifts* are applied to the that binary sequence (Figure 13). We consider this process as arbitrary because it displays a weak musical thought; the manipulation of its only parameter, a decimal number that is then converted to a binary string, gives very little control of the musical output.

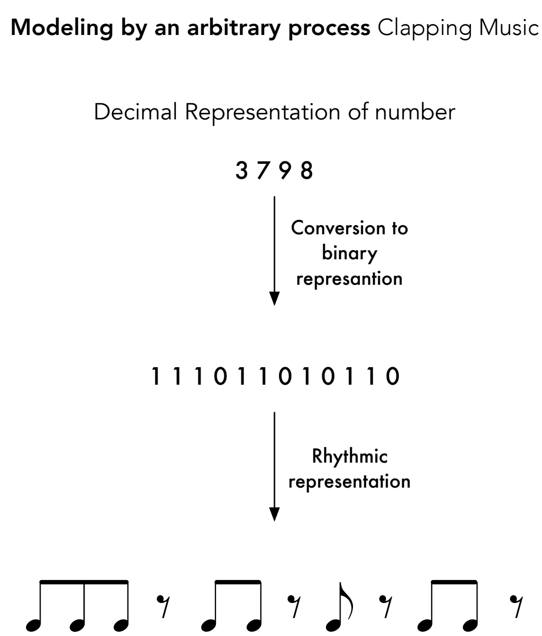
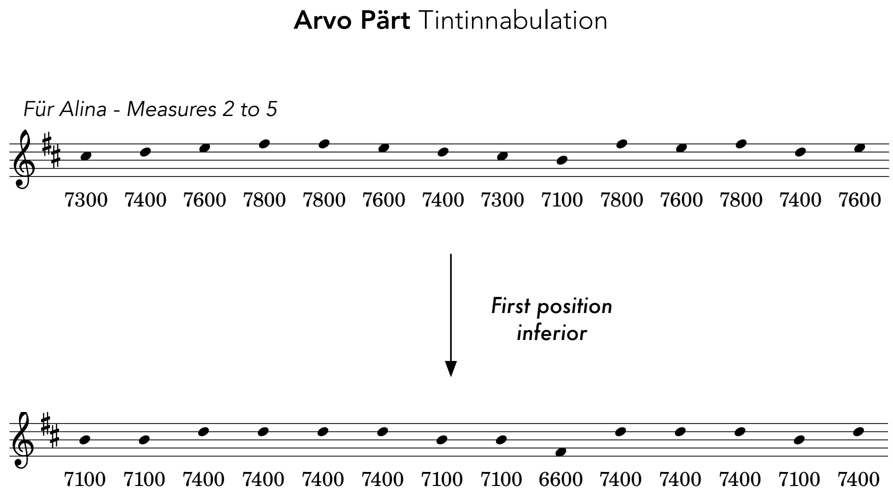


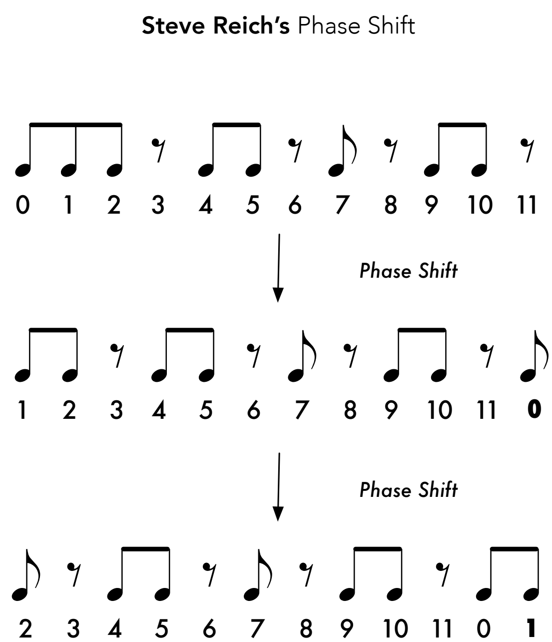
Figure 9: Clapping Music*'s rhythmic pattern. By changing the representation of a decimal number, 3798, to binary and mapping its ones and zeros to beat and rests the pattern can be reconstructed. Applying this same procedure to different decimal numbers can generate new patterns, but there is so little control of the musical output that this procedure is not so different from a random process. Compare with Figure 11.*

VI. Scope of the Model and Its Parameters

In most cases, a model will reproduce a partial section of a piece or one of its specific dimensions. For instance, Rokita [20] modeled only the rhythmic aspects of the first of the *Three Pieces for Clarinet Solo* by Igor Stravinsky. Chemillier [7] modeled only a few measures of Ligeti's *Melodien*.



(a)



(b)

Figure 10: *Examples of special* transformations *or* operations*. In (a) each pitch of a melodic sequence is mapped to the closest one of the tonic's triad. This technique is used in pieces such as* Für Alina *and* Spiegel im Spiegel*. In (b) a rhythmic pattern (from* Clapping Music*) is consistently rotated. When those rotations are superimposed with the original underlying "phase patterns" emerge.*

Occasionally one can conceive an exhaustive model for a piece in all its extension and most of its dimensions (pitch, rhythm, dynamics, etc.), like the model for James Tenney's *Spectral Canon* [21] or most of Riotte and Mesnage examples [19]. Alternatively, the modeling can be concerned uniquely with a particular technique or set of them.

The conception of parameters is an essential step; they control the model's behavior and are responsible for generating neighboring variants.

Usually, the parameters reflect a compositional choice. It can concern aspects like instrumentation, tonality, mode, tunning, and so forth. For example, in a canonic piece, one of the parameters could be the starting point for each voice or how many canonic voices should be written (see [21]). The creation of parameters for a model may depend on the creativity and purposes of the researcher. The determination of the parameters will significantly influence the implementation process and the model's capacity for generating more or less neighboring variants. To exemplify the elaboration of parameters, Figure 11, shows the modeling of *Clapping Music* rhythmic pattern this time using an essentially more musical procedure. Aspects as the number of beats inside rhythmic groups, the inclusion of gaps between groups and the notational figures (in this case the beat-unit) can be considered as parameters (compare with Figure 9).

Parameters may have a strong or weak effect on the simulations of the model. *Strong parameters* change structural features of the modeled piece while *weak parameters* may only transpose the whole structure maintaining the same structural relations between its elements or only change the way the score is notated. In the illustration showed in Figure 11 parameters `length` and `step` are strong parameters that can the structure of the rhythmic pattern. On the other hand, parameter `figure` only changes the speed of the pattern or the way it is notated. In the model of the *Spectral Canon* [21]), the parameter `fundamental` is weak because its effect is only a transposition of the whole pitch structure; its internal relationships are not modified when its value changes.

In the case of a deterministic model, every single parameter value will correspond to a particular output, to a specific *instance* (cf. Figure 12). When we plug the same values to the parameters in successive simulations, we can be sure that the instances produced will also be the same. By using deterministic algorithms we can adjust the model according to the simulation's results, i.e., we can modify the model to shape a determined space of instances.

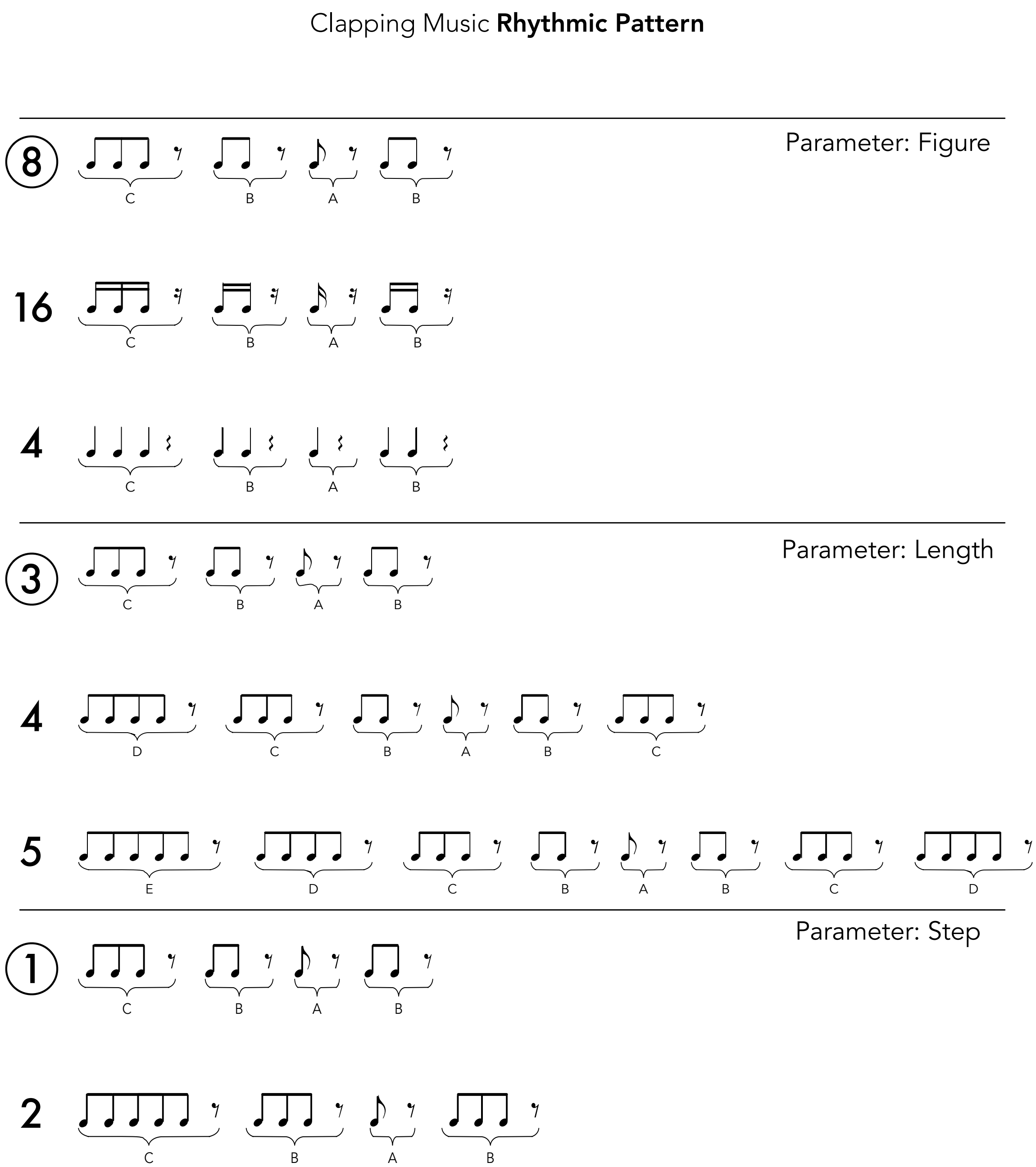


Figure 11: *Conceptualization of parameters for the rhythmic pattern used in* Clapping Music*. The parameter* `figure` *changes the notation and speed of the pattern, 8 meaning an eighth-note. Parameter* `length` *controls the length and the number of beats in a rhythmic group while the parameter* `step` *can insert gaps between the rhythmic groups of the pattern.*

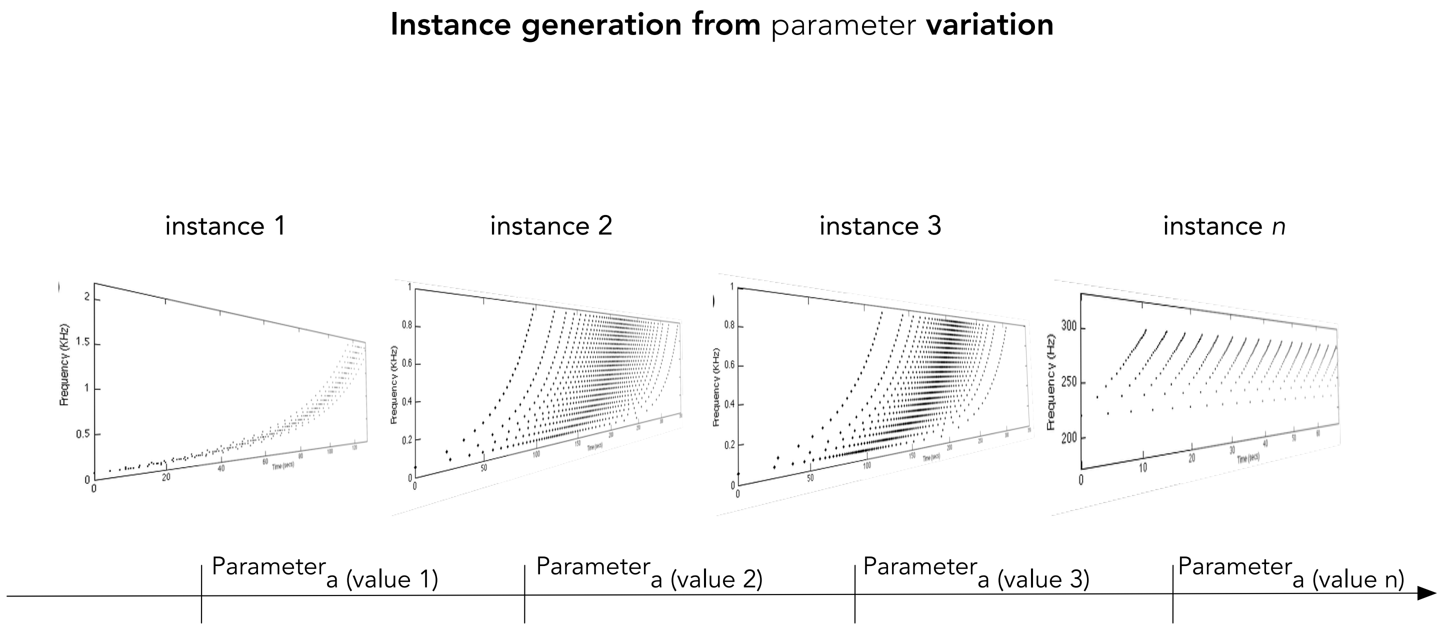


Figure 12: *Conceptualization of neighboring variants generation* (instances) *by plugging different values to a parameter. Each value corresponds to a specific* instance. *The rectangular boxes representing the instances are piano rolls (time x frequency).*

Figure 13: *Different variations obtained from simulating* Clapping Music *from the arbitrary process shown in Figure 9: Each given decimal number is converted to binary and then mapped to beats and rests. The resulting patterns are phase shifted and repeated accordingly to build the original piece and the variations. White boxes represents rests and black boxes eighth-notes. The* ritornellos *are not included in neither the reconstruction or variations.*

The fewer parameters a model has, the higher is its explanatory potential, as a more comprehensive systematization will be required to connect all the generative process with fewer variables. On the other hand, the more parameters a model has, the greater is its potential to generate different instances, thus serving to more creative (compositional) or speculative purposes [2] (Figure 14).

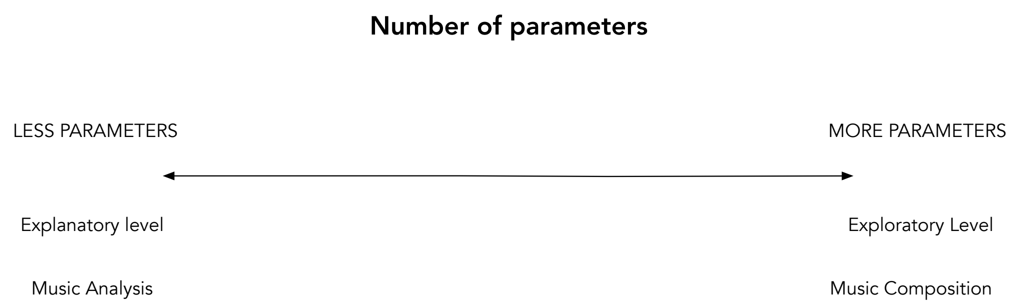


Figure 14: *The less parameters the model has, higher is its explanatory potential. The more parameters a model has, higher is its potential to generate different instances.*

Different models can be conceived for one single piece. The conception of a model depends on the hypothesis and purposes of the researcher. In the same way, different implementations are possible for one single model. Also, as with any computer program, an implementation can have several versions and be developed in several forms (see Fig 15). During the modeling and implementation process, adaptations can continuously be made to better adjust the model's output to the numerical representation of piece being modeled.

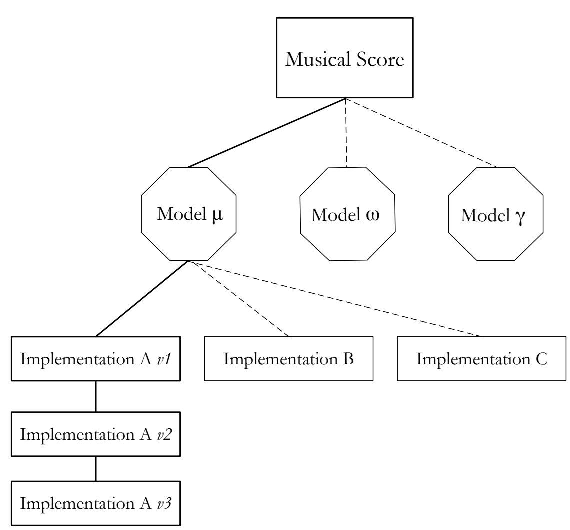


Figure 15: *Modeling and implementation process. Different models can be conceived for one single musical score. A single model, in turn, can be implemented by different implementations.*

In the case of the model presented in [21], we first developed a model with only a few parameters, to test compositional decisions, to understand the underlying principles of the composition, and then we added more parameters as a way to explore the potential of the model itself and not only the features of the original composition. Those type of additional parameter were introduced as *Extended Parameters* Another way to see this process is to think as a composer who first analyses a piece to learn about the aspects he is interested in but wants, in a later moment, to emulate them in a new composition, adding to the model his particular procedures.

VII. The Space of Instances

The set of possible variations for each parameter gives origin to a *space of instances*. One way to analyze this space is to simulate the effect of a specific parameter on the musical features of a sample of different instances. This analysis can be done in two ways, the *diachronic* and *achronic* analysis (see Figure 16), concepts borrowed from the *Sonic Object Analysis Library* (SOAL) written for *OpenMusic* and conceived in a different context [11].

In the *diachronic analysis*, a small sample of different instances is selected, and some features of the musical structures are analyzed. Those features are then compared to the original piece, always considering their evolution through time, throughout its extension. One single instance can be represented as a series of points in a specific time span, as is the case of a *piano roll*. It can be seen as a way to test hypothesis on a limited number of specific instances, where some of its precise details will be taken into account. Diachronic analysis is seen in [22, p. 75] (cf. Figures 5–8).

On the other hand, this method may not be the most appropriate to evaluate the dynamics of a larger sample of the space of instances. The *achronic analysis* supplies this need (see Figure 17). In this analysis, each instance is reduced to one measurement (or descriptor) such as total duration, shortest duration, *ambitus*, mean pitch, and so forth. Individual instances are seen as a single point in a determined space, where the instances' internal time is considered only implicitly. Diachronic analysis is seen in [22, pp. 77–78] (cf. Figures 10–16).

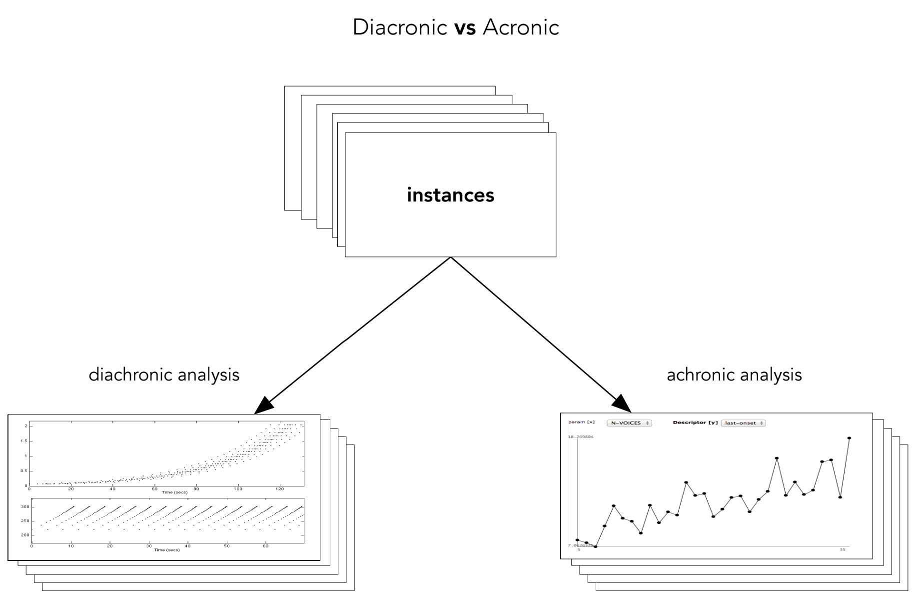


Figure 16: *In Diacronic analysis, one single instance can be represented as a series of points in a specific time span, as in the case of a piano roll. In Acronic analysis, one single instance is seen as a single point in a determined space, where the instances' internal time is considered only implicitly.*

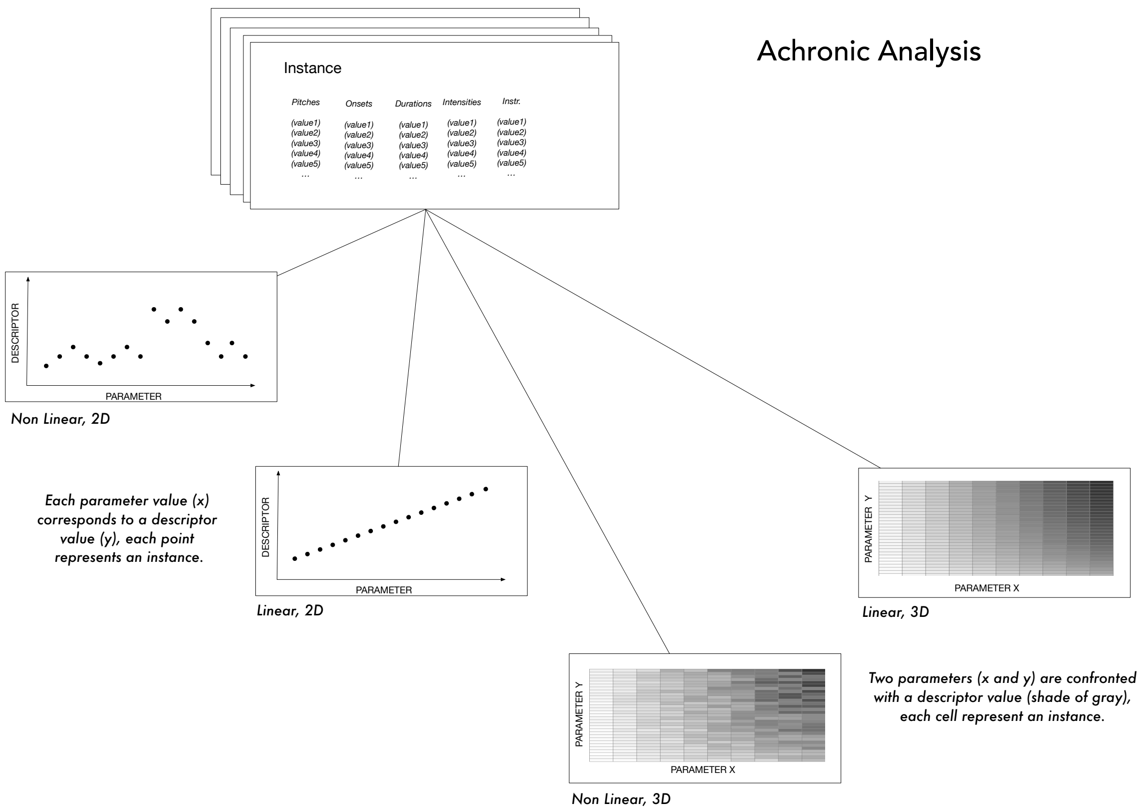


Figure 17: *Some strategies to visualize the space of instances with achronic analysis. The 2D visualization is a Cartesian plane where a subset of values of a given parameter is plotted against a measurement from the resulting instances (such as total duration, or the mean pitch, or the smallest durations, etc). In the 3D visualization subsets of values from two distinct parameters are plotted in a heat-map where a shade of gray represents a measurement from the resulting instances.*

Through *achronic* and *diachronic* analysis, we observed that, as a result of the utilization of deterministic algorithms, there might be a consistent, very linear behavior in such spaces of *instances*. That is to say, changes in the value of parameters lead to proportional responses of the simulation results. For example, if one of the parameters is the first duration, increasing it will make the total duration of the resulting *instance* proportionally bigger. If the parameter is the fundamental frequency (as in the case of a *spectral composition*), we may also expect that increasing it will also increase every frequency (pitch) of the *instances* produced by the simulation.

On the one hand, the linearity observed in the space of instances can be interpreted as a sort of validation, an element of coherence, of the model's behavior and therefore would be a desirable feature from a more musicological and pedagogical perspective. It can be used as a way to further understand specific compositional processes and explore the consequences of particular decisions. On the other hand, from a more creative, speculative attitude such a space of instances could appear as too homogeneous and predictable for some purposes.

We tried to conceive some strategies to break this linearity and introduce more heterogeneity on the space of instances. One method was to generate *perturbations* in a structural element of the model, namely a variable, by multiplying it by a pseudo-random, controlled, number. In this way, the higher the perturbation, the higher is the probability of unexpected simulation results [22, pp. 77–78].

VIII. Final Considerations

The basics of a simulation-based, computer-assisted approach for musical analysis were introduced in this paper. Some of those principles are common to other methodologies, and some are original. The case studies are not too many and come from a somewhat restricted compositional context. Nevertheless, I hope that some of the original concepts here presented will form the basis to bolder and more encompassing experiments.

The modeling of musical scores, with the explicit goal of generating neighboring variants, perhaps is one step further into the direction of a more *creative analysis*, like the one forecast and advocated decades ago by both Luciano Berio and Pierre Boulez. On the other hand, there are indeed still too much to be explored and to be learned from it. The field is open, and there is an infinity of solutions that could be tried to examine the potential for creating new musical forms from a given model.

Unsurprisingly, presenting such a method also poses some challenges. One of them is how to describe the algorithms or model implementations comprehensively without turning it a too tedious, tiresome endeavor. Additionally, the description and evaluation of several *instances* can be too burdensome, as the discussion around just one single neighboring variant can be not easily exhaustible. The automatic estimation of symbolical and psychoacoustic measurements may help in this exploration. Also, much more is needed to support better the visualization of the *space of instances* that a model can produce.

To paraphrase pioneer Stanley Gill [10], the musical results produced by the different models we worked so far "touched" me quite enough. I believe that the favorable results come from having implemented *expressive* parameters that give greater control of the musical result. For instance, some variations of Tenney's *Spectral Canon* or Steve Reich's *Clapping Music* seems to be authentic enough to be conceivably appreciated on its own. For that, the neighboring variants need only to be coherent, to retain a sense of form and achievement.

Finally, I feel that to use the computer to automate the process of discovering and reconstructing patterns, in the context of computational musicology, is not what is more important. It is more desirable to reveal new meanings to already known musical phenomena, to favor *polysemy*, and avoid the trap of reducing musical expressiveness to a fixed, inflexible algorithm.

IX. Acknowledgments

This paper put together research developed at the *Interdisciplinary Nucleus for Sound Studies* (NICS) and the *Institut de Recherche et Coordination Acoustique/Musique* (IRCAM) as part of a doctoral thesis. It was initially funded by the *Institut de Recherche pour le Development* (IRD) and later by the Sao Paulo Research Foundation (FAPESP), grant 2014/08525–8.

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Cosmicizing Sound
music – cosmos – number

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Abstract: *The compositional practice to be described in this article includes number and proportion as decisive factors for the problem of musical form. The role of number and proportion in my music is entirely dependent on the idea of music composition as cosmology, which has two aspects: the "scientific", in which musical form is determined by formative principles or techniques, and the "contemplative", which confers to the music an iconic relationship with mystical or mythic cosmologies such as those of the Pythagoreans and of ancient India Vedas and Vedanta. To better understand the context in which number and proportion relate to musical form in some of my works, I will first develop the ideas of composition as cosmology, composition as cosmogony, and spiral time. After this, I will briefly study the role of proportion in the temporal organization of a few key works, and conclude with some other general considerations concerning number and proportion in contemporary music. The idea of "cosmicizing sound" will surface naturally from the considerations about the poetics and aesthetics of my music.*

Keywords: *Compositional Processes. Formative Principles. Cosmology. Number and Music. Cosmology and Music.*

"(...)music is natural law as related to the sense of hearing."

"(...)and finally we have the impression
of being faced by a work not of man but of Nature."

Anton Webern

Number [1] and proportion take an important role in the compositional processes of my music, not as a simple and arbitrary "application", but as an integral part of its poetics [2], in the sense that several aspects of the composition are shaped and structured by numbers, though *not only* by numbers, from the beginning of the creative process. A poetics, as described by philosopher Luigi Pareyson (1918-1991) in his *I Problemi dell'Estetica* (1966), "is a certain taste converted into a program of art, in which taste is understood as the whole spirituality of an epoch or of a person turned into expectation of art" ([37, p. 26]) Poetics encompasses everything that

[1]This article presents the academic findings of a research stage concluded within the Research Project entitled "Ancient and Non-European Contents in 20th- and 21st Century Music Composition", currently coordinated by myself within the Research Group "Processos Músico-Instrumentais", in the Research Line "Processos Criativos em Interpretação e Composição Musical" of the graduate studies program PPGMUS, at the Universidade do Estado de Santa Catarina (UDESC).

[2]From the Greek *poiesis* – $\pi o \iota \eta \sigma \iota \zeta$, meaning the activity that brings forth something that did not exist before.

determines and defines the creative praxis of a composer, one part of which, in my case, is number and proportion.

Together with number and proportion comes also cosmology. In fact, the relationship between cosmology, number and music is a key feature in a music theory tradition that starts at the very beginnings of Western philosophical thinking, with Pythagorean philosophers, who conceived the universe as being organized by musical ratios, an idea that was called *musica mundis*, music of the spheres, one of the most important cosmological conceptions of Antiquity. The study of the relationship between musical ratios and the universe has persisted, in a way or another, even all the way into the 21st century, when, for example, physicists have associated cosmic microwave background to harmonious sound waves ([21, p. 44]).

This theoretical tradition linking music, number and cosmology reaches a moment of maturity in the 8th- and 9th- centuries Quadrivium, which developed through the educational ideas about music set up by Plato in chapter VII of his *The Republic*, Aristotle (384-322 B.C.E.), St. Augustine of Hippo (354 – 430) and Boethius (ca. 480 – 524). Arithmetic, Geometry, Music and Cosmology (or Astronomy) are the four mathematical disciplines of the Middle Ages that, together with the disciplines of the Trivium (Grammar, Logic and Rhetoric), formed the seven Liberal Arts required for the study of Theology and Philosophy. In this context, music is not discussed as a performance or compositional practice, but is seen as a contemplative science; music, as one of the mathematical disciplines of the Quadrivium is Number in time, while Arithmetic is Number in itself, Geometry is Number in space, and Astronomy is Number in space and time ([17, p. 34]:34).

This essay is an opportunity to focus on the role of number and proportion in the construction of the "musical building" (the whole form or macroform) of a few of my works. Probably, it is about musical architecture, but also describes the role of number and proportion in the production of a type of musical becoming that I call "spiral" because it consists of musical contents that evolve expanding or contracting, like a spiral. Construction and architecture are, in any case, only two aspects, among others, in the creative processes that generate music, more important in certain composers than in others [3] : definitely very important in Karlheinz Stockhausen (1928-2007), constructivism, and Iannis Xenakis (1922-2001), architecture, for example. While music cannot be reduced to constructivism and architecture, they sometimes require the fully focused attention from the composer, the performer, the listener, the musicologist, the theorist, or the student. Therefore, if other aspects—other than number, proportion, cosmology, architecture, form—are not mentioned here, it is because of the chosen focus, and not for lack of recognizing that the activity of composing music involves, at the same time, several kinds of creative processes that may be identified as intuitive, intellectual, emotional, planned, pre-determined, written (as derived from *ècriture*), extemporaneous, improvisatory, experimental, systemic, symbolic, and so on.

The compositional practice to be described here does include number, proportion and a relationship with cosmology that takes spatial and temporal conceptions into the realm of music. To better understand the context in which number and proportion relate to musical form in some of my works, I will first develop the ideas of composition as cosmology and spiral time. After this, I will briefly study the role of proportion in the temporal organization of a few key works, and conclude with some other general considerations concerning number and proportion in contemporary music. The idea of "cosmicizing sound" [4] will surface naturally from the following considerations.

[3] Or, at times more important than at other times, for the same composer.

[4] I decided to re-use the title, *Cosmicizing Sound*, which was first given to a Lecture I presented at the Institute of World Culture in Santa Barbara, California, on 21 August 2004. The text of that Lecture remained unpublished and, although some of its topics are discussed here, only a few fragments have been included, but modified, in the present, much larger article.

From the aesthetic point of view, the sense of beauty (or of aesthetic satisfaction) brought to music by proportion [5] consists in the existence of a single organizing and ordering principle, and of the resulting structural relationships in action within this (micro) cosmos. Important for both poetics and aesthetics is to explore the interdependence between proportion and cosmology and the capacity both (proportion and cosmology) may have to provide coherence to music, a coherence that sometimes comes from outside the limits of music, bringing to the latter a specular or symbolic quality, in the sense that this music reflects a reality exterior to itself. This quality will be studied on the section about contemplative cosmology.

I. Composition as Cosmology

Having briefly considered this historical and theoretical background, and turning now to musical composition as a creative practice, it is natural to say that a composition, "piece" or musical work, seen as an object resulting from a creative process, is comparable to a microcosm, in fact, a musical *kosmos*, in all its integrated space-time constitution. The Greek term *kosmos*($\kappa o \sigma \mu o \zeta$) means *order* and implies holistic ideas of totality and unity. It was applied to the world or universe for the first time by Pythagorean philosophers.

> "*kosmos* does not signify primarily universe. Calling the universe by this name denotes already a particular kind of cosmological interpretation. The original meaning of *κόσπος* is all kind of order whatsoever. The mere fact that Neo-Platonism speaks of *κόσπς νοητός* (intelligible world), or that eventually the state in which reign justice and order— *εύνομία καί δίκη* — is said to be a *κόσπος*, should prevent from interpreting the name, in its ancient and medieval use, according to the limited signification it has been given in modern times, especially under the influence of science." ([1, p. 321], footnote 5).

Applied to music in the context of my composition, the term "cosmos" emphasizes that music is not just the sounds but also the order of sounds, the music's own formative and ordering principles, and implies a more contemplative conception of the cosmos, according to "its ancient and medieval use". For now, however, it is in the sense of cosmos as "all kind of order whatsoever" that I will focus my analogy between music composition and cosmology.

Already in the 1920s, Edgard Varèse (1883-1965) called his music "organized sound". This statement clearly defines the difference between music and sound to be a question of order: he argues that "to stubbornly conditioned ears, anything new in music has always been called noise. But after all what is music but organized noises? And a composer, like all artists, is an organizer of disparate elements. Subjectively, noise is any sound one doesn't like"([43, p. 18]) [6] . Luciano Berio (1925-2003), on the other hand, defines music with an apparently more open statement: "music is that which is heard with the intention of hearing music" ([5, p. 7]) (my translation). In reality, this statement is in no contradiction with Varèse's "music as organized sound", as the intention to hear music is, in itself, a form of sound organization, the act of making sense of a becoming of sounds. Even John Cage's (1912-1992) attitude towards music composition denotes an extreme concern with order, with his indeterminate music always searching for chance procedures to determine ordering principles.

My compositions are microcosms that imitate the macrocosm (the universe), either by simply following the idea of composition as a cosmos and thereby creating their own "kinds of musical

[5] "Proportion" will be used in the place of "number and proportion", since proportion implies number, as it puts numbers in relationship.

[6] This quotation is from a lecture given by Varèse at Yale University in 1962.

order whatsoever", or by imitation of actually existent *conceptions* or images of the macrocosm. In both cases, these conceptions may present a rich variety of roles assumed by number as one of the factors that generate musical phenomena. To explain all this is the goal of this article, but first, it is necessary to establish the meaning of a few important terms.

A cosmology, simply defined, is an image or conception of the cosmos; or, more exactly, "cosmology is the term for the study of cosmic views in general and also for the specific view or collection of images concerning the universe held in a religion or cultural tradition. (...) it relates also to inquiries in the natural sciences" ([6, p. 100]).

The creation process of a composition is, unfortunately, also called "composition" and, to avoid confusion, I will only refer to it as "the compositional process", and reserve the word "composition" to the final microcosm resulting from the compositional process. In the same way, the word "cosmology" could refer to both the universe and the creation process generating this universe, but I will only use the word "cosmogony" to refer to the creation process, and reserve the word "cosmology" to talk about the structures and "laws of nature" that sustain the universe.

It would be fair enough to equate the idea of formative/organizing principles and codes with that of a compositional "system" or compositional technique or method. However, while referring to the composition in the neutral level [7] , it is preferable to speak about principles of immanent order, since this emphasizes the composition as cosmology. "System" is avoided here in favor of the notion of cosmology, which includes contemplative aspects—to be seen further ahead—that the idea of a system tends to exclude, due to its avoidance of anything exterior to itself.

As for "compositional technique" or "method", due to the emphasis of "technique" on the action of a composer, these terms should be reserved for the compositional process, the musical cosmogony, that is, when speaking about the poietic level. Meanwhile, "formative principle", although in Pareyson's sense applies to the act of forming or shaping, should be reserved for the composition in the neutral level, because "principle" is not a composer's action, but the cause of a phenomenon.

The analogy between composition and cosmology first came to me from the awareness that each musical style—and musical culture —has its own set of rules for creating music. These rules (principles of order) result in the way the music sounds, and reflect the way music is conceived and enjoyed. The obvious difference between how a classical Hindustani *rāga* and a Mozart symphony sound denotes two different modes of creation and poetics, of perception and aesthetics, of spirituality and corporeity [8], of mind set and world view. I will use the term "cosmovision" (*Weltanschauung*) to join all these aspects together (mind/spirit/body set, world view, aesthetics and poetics) in a general large idea expressing that which conditions the way human beings act, think, behave, and exist in the world. Music poetics (musical making) belongs to the realm of human action. Cosmovision, in relationship to music, and applied to a whole culture or an individual composer, is

> a great artistic imaginary which covers the composer's (or culture's) entire poetic, aesthetic and technical thought, as well as psychic life and capacity to produce meaning, including his image, perception, cognition and interpretation of the world and his mode of interaction with it. Cosmovision determines the contextuality [9] that is observed in

[7] I am using Jean-Jacques Nattiez's semiological tripartite approach to the discourse about music, by which the work (composition) can be studied from the point of view of the poietic level (the composer's activity), the esthesic level (the listener's activity), and the neutral level (the work itself). The latter "describes the immanent organization of the object, and is said to be neutral because not necessarily pertinent poietically or esthesically" ([35, p. 4]) (my translation).

[8] I mean spirituality very much in the sense given by Pareyson in his definition of poetics, given above, but not only: the soul (psyche) and spirit as integral parts of the human being, should not be forgotten in a discourse about music or its structure. Corporeity should include all physical and body aspects of making music, as well as one's own body perception.

[9] For a discussion of Milton Babbitt's notion of contextuality, see more further ahead.

> 20th- and 21st- centuries musical compositions. ((...)) Cosmovision is the background which allows the formation of a cognitive perspective that generates contents, meanings, and worldly practices" ([27, p. 241]) (my translation).

Cosmovision is reflected in music because it determines the way music is practiced and conceived by a subject (an individual, a community). "Musical systems or languages are always more than organized sounds, vocabularies, and syntaxes. They are instances of the way a specific people understand and relate to the phenomenal world" ([4, p. 215]). It is in the sense of music as a reflection of a conception of the phenomenal world that musical systems or languages can be thought of as musical cosmologies, because a cosmology is just that: a conception of the phenomenal world. I will return to this point further ahead.

Because they contrast greatly with one another, collective cosmovisions that are farther distanced or isolated in space or time from one another (such as serial music and medieval plainsong, jazz and *maculelê*, Javanese gamelan and Romantic sonatas for piano, *candomblé* music and Lakota singing—the examples could go on forever...) are easier to be recognized as such than those that are closer to each other, such as Javanese gamelan and Balinese gamelan, Roman plainchant and Ambrosian chant, Romantic sonatas and Classical sonatas, serial music and free atonal music, maculelê and *maracatu*, American jazz and ECM jazz, candomblé music and Ewe drumming, Lakota singing and Inuit singing, etc. The latter examples are closely related but do not completely share the larger cosmovision they belong to, because they are less general. These musics, due to some particularities, became more distinct within the more general cosmovision and, therefore, they start to be considered as subcategories of the larger one. Otherwise, it would not make sense to distinguish, say, between the contemplative and slow Javanese gamelan from its sanguine and loud sibling, Balinese gamelan, for example. Their musical communities are smaller, i.e., their cosmovisions are no longer shared completely and collectively by so many people in a wider context, because there exist notable differences preventing them to be placed in the same large category, therefore the need for subcategories. This individualization process seems to be an effect of local ecology, kinship, derivation, or cross-fertilization that makes the matter all the more complex, enhancing the particular and individual, rather than the universal and collective.

In the Western world [10], since the 20th century and continuing today, cosmovision tends to become such an increasingly individualized phenomenon. Although composers do share common musical practices, have studied according to mostly the same musical methods, know and enjoy more or less the same repertoire, they also have their own blend of musical vocabularies, tastes, interests, points of reference and of preference, and may act from disparate points of departure, so much that they will tend to conceive music quite differently from each other, and produce highly individualized styles of music. Individualization makes the cultural context more complex and fragmented, and, in an attempt to make sense of this, or out of a "necessity" to label them in some expedite mode of promotion or diffusion, there is a tendency to group composers together in a certain "ism" or genre. Most often than not, the result is a grouping that does not do justice to what composers are actually proposing. As an example of this is the term "contemporary music": one needs to be more explicit about what this term entitles rather than simply using it indiscriminately, so that there will be no misunderstanding about what it stands for.

[10] It seems safe to say that the universe of "20th-century and contemporary composition" is still a "Western world", a "Western music", notwithstanding the international and global scenario in which composers from the entire world (Brazil, Bolivia, Mexico, South Africa, Australia, New Zealand, China, Japan, Korea, to name but a few) join the practice of musical composition, a practice that cannot ignore the European/North American past and present tradition. Western music became something like Tibetan Buddhism: it has spread worldwide, and can no longer be identified with its geographic place of origin.

The task seems to be to embrace individualization as part of our "learning to live together with chaos" [11] . Since there is no longer a necessarily common canon to be followed, composers have found themselves seized by the opportunity to create their own individual canon according to what they *see fit* (world vision). This should not be understood as indulgence and caprice; it has to do with one's own artistic inclinations, instinct, intuition, and consciousness that identify what one wants to become. From this desire to become something comes a consequent choice of education: while education is received in childhood not by one's own choice but from someone else's choice, in a later or more mature age, a person may choose consciously their own continued education and acquire the skills and knowledge that the first education failed to provide. It is like recognizing one's own destiny and place in the world, like the favorable *habitat* not only for surviving but for thriving. I have called this situation of the 20th- and 21st- centuries composers a "cosmological state of things" ([27, p. 243]), a situation that favors composers to come up with individualized ways of organizing sound and music for their creative practices. "Individualized" does not mean always unique and certainly never completely independent from other practices, since *creatio ex nihilo* does not apply to human practices. It means the freedom to choose one's artistic and theoretical references and to dialogue with that chosen segment of history.

The repertory of New Music created since 1945 is a large reservoir of examples of the "cosmological state of things" in which their creators are living. The work of Karlheinz Stockhausen is probably the most consistent example of a poetics in which there is an entirely new creative endeavor for each composition: each piece is born from a specific new way of structuring and organizing sounds. Milton Babbitt (1926-2002), speaking about what he calls "for now, serious music", introduces the term "contextuality", a helpful notion in the recognition of the cosmological *status* of New Music, since it refers to the new and autonomous context created at each new work of a composer. It starts with serial music, where a tone row, being created for each composition, defines the context of each piece always in a different way, turning the piece into a self-referential, autonomous entity.

> "musical compositions of the kind under discussion possess a high degree of contextuality and autonomy. That is, the structural characteristics of a given work are less representative of a general class of characteristics than they are unique to the individual work itself. Particularly, principles of relatedness, upon which depends immediate coherence of continuity, are more likely to evolve in the course of the work than to be derived from generalized assumptions. Here again greater and new demands are made upon the perceptual and conceptual abilities of the listener." ([3, p. 2])

Serial contextuality also means freedom; this was Anton Webern's (1883-1945) feeling about composing: "but now I can invent more freely; everything has a deeper unity. Only now is it possible to compose in free fantasy, adhering to nothing except the row" ([44, p. 59]). Contextuality becomes acute in indeterminate and aleatory music, where each composition is created by a completely new set of rules the result of which is always somewhat different. Somewhat or entirely, as in the case of Earle Brown (1926-2002) and his composition *December 1952*, which is a one-page graphic score with lines and dots to be freely realized by an indeterminate number of musicians, and read in any position (upside or upside down, vertically or horizontally) with no established starting or ending point. As an extreme case of indeterminacy and contextuality, *December 1952* seems to have lost its own contextuality, or this contextuality becomes entirely dependent on the performer's interpretation of the score.

[11] The phrase is one of the many aphorisms by my composition teacher in Brazil, German composer Hans-Joachim Koellreutter (1915-2005): "we must learn to live together with chaos" ("precisamos aprender a conviver com o caos").

Luigi Pareyson's theory of formativity is also helpful in the recognition of the cosmological *status* of the poietic activity of an artist. Applying to music what he says about art, composing music is an act of forming or shaping a form, a doing that, while doing, invents how to do. The creative process is the process of realization of the whole or idea (*forma formante*), which provides the direction of the creative activity since its beginning, guiding the artist "like an omen of the work it desires to actualize" ([38, p. 75])(my translation). It is clear that, in this conception, the *forma formante* not only creates contextuality at each work, but also is what creates the final form (*forma formata*).

The analogy between composition and cosmology may be applied to a musical style as well, or a music period in history, but, because of the individualization process, I mean it mainly in regards to a single composer's production, and more exactly to my own, since it would not be correct to generalize and say that all (or the majority of) composers agree that composition is a cosmology. In fact, I have not found other composers that conceive music as cosmology. Karlheinz Stockhausen comes closest in conception, as exemplified by several statements such as the following: "*Mantra*, as it stands, is a miniature of the way a galaxy is composed (...) As it was being constructed through me, I somehow felt that it must be a very true picture of the way the cosmos is constructed" ([13, p. 242]). Japanese composer Jōji Yuasa (b. 1929), mentions that a "composer's music reflects his individual cosmology, and that this cosmology encompasses both his cultural identity and the collective consciousness of the society which shares his language (...) music is a metaphor or metonymy of a composer's cosmology" ([46, p. 197]). However, his article does not go any further on this idea, except for attributing an important role to language in shaping cultural identity, and stating that a person's sensibility, way of thinking and perceptions are ruled by language. Yuasa's idea corresponds to what I have been saying about cosmovision; in my words, Yuasa's statement is that music reflects a composer's cosmovision. His statement does not proceed to conceiving compositional formative and organizational principles as a musical cosmology. Hans-Joachim Koellreutter speaks of the different aesthetic phases in the history of western music as determined by different levels of human consciousness. This idea certainly has to do with cosmovision but, again, not with musical cosmology as I am proposing here.

As mentioned above, a composition is a microcosm, a universe (*kosmos*) of sounds. But in addition to the sounds themselves, music is also the order of sounds, the whole of formative and ordering principles which put sounds together, creating the mutual relationships and ways of interaction of sounds, in the same way as, in the Greek concept of *kosmos*, natural phenomena are in relationship and interaction creating a tight unit. Sounds must be organized to acquire the *status* of music. Sounds, textures, chords, melodies, sound masses, rhythms, gestures, motifs, phrases, whatever morphological units in a given composition are like natural phenomena in the acoustic universe called "composition". The relationship between them is determined by formative principles or codes. These musical principles and relationships give shape to a universe of sounds; they *are* the laws of nature of this sound world. A principle, in the sense of classical Greek and Latin philosophy, "is a fundamental rule or cause (principle, *arché* - *ἀρχή* – the beginning, which rules and governs everything)" ([39, p. 28]). A formative principle gives shape and form to music. Therefore, there are principles (or causes) that give form/existence to the consequent phenomena observed (or heard) in music, integrating them in a unified whole. Music composition is a cosmology, seen as the *rationale* of its creation.

The analogy between compositional formative principles and laws of nature must be investigated a little closer, since there is a fundamental difference between the two. The term "laws of nature" is generally used in reference to moral, politic, and legal doctrines, but the meaning that relates to music composition, viewed as a *physis* (nature) [12] , a microcosm, comes from its use in the natural sciences, as the attempt to describe, understand and even predict natural phenomena based on the observation of empirical evidence. Newton's Law of Gravity, or Darwin's Law of Natural Selection are good examples of laws of nature in the natural sciences. In this analogy between musical cosmology and the natural sciences, where the composition is identified with an acoustic cosmos, it is possible to speak of a *scientific cosmology of composition* to identify the principles that shape this musical natural world. In this case, the term "laws of nature" are better understood as "physical or scientific law". However, while "physical law" is a scientific generalization based upon repeated experimental observations that describe aspects of the universe, the "laws of nature" of a composition are the principles governing the *generation* of musical forms; they are not obtained or inferred from observation and empirical evidence; laws resulting from these activities would be the findings of musical analysis, and not of musical creation. For this is what musical analysis does when it analyses a composition: it observes the *forma formata* with the goal of describing aspects of that universe of sounds by general ideas inferred by means of that empirical observation. Compositional formative principles are not descriptive, but rather prescriptive; they are invented by the composer, who acts according to them, even when breaking them[13], in order to generate the desired musical result. In this sense, there are moments in the compositional process when techniques, methods, rules and formative principles are tested and evaluated as achieving or not the desired results by means of observation and analysis of the "empirical evidence", i.e., the results obtained by them; some of them might not be what had been "requested and ordered by the whole as *forma formante* along the forming process" ([38, p. 101]) (my translation), in which case they are discarded by the composer, until the right ones are found. The moment of adopting the formative principle that "works", the right technique, is immediately recognized by the composer because it transforms the *forma formante* in the right *forma formata*. In any case, and to conclude, formative principles are not like scientific laws because the latter are descriptive and the former are prescriptive; the term "laws of nature" may only be used with this distinction in mind and, preferably, referring to the results of a musical analysis.

At this point, it would be important to provide concrete examples of formative principles, "laws of nature" at work in a composition. One simple example is the principle of imitation, by which a melody written on a part is written on another part starting after one or two beats from the beginning of the first part, and transposed according to a given interval (for example, an octave above or a fifth below). This technical procedure is, in the poietic level, a group of actions: repeating a melody in another part, transposing, making adjustments, etc, while, from the point of view of the neutral level, that is, the composition itself, it is an immanent principle of organization.

With the purpose of providing more examples of formative principles in my own music, I should refer the reader to the article "Expanded Modal Rhythm" which describes in detail and with examples, part of the temporal/rhythmic organization principles I have been using since 1989.

The above considerations establish composition as cosmology based on two causes: 1) the high contextuality and autonomy of composers' works since 1945, and 2) the simple analogy between music as organized sound and the world as organized matter (a cosmos). In the first cause, as already argued, the composer, in a "cosmological state of things", may create music according to a very particular prospective (cosmovision, world view) and produce a music that is consequently,

[12] There is no word that translates all the meanings of the Greek word physis (*φύσιζ*), but "nature" is a generally accepted one.

[13] My composition teacher in Italy, Franco Donatoni (1927-2000) used to say that, to avoid becoming a bureaucrat, the composer created the codes (*"i codici"*) or laws *in order to break them*. Furthermore, and however, I believe that formative principles may be general enough as to provide a great flexibility in their application, without the composer needing to go against them.

highly individualized and, nonetheless, authentic, valid and contemporary, in the sense that it could not have been produced in another time. It is the research for and creation of one's own techniques and formative principles, and not the simple adoption of the already existing ones, that allows the composer to create a new musical thought, a new musical cosmology. As for the second cause, the analogy musical order/world order consists in the recognition that, because the musical composition is conceived according to the ancient Greek concept of *kosmos*, it reflects the Greek conception that the universe is an ordered, organized whole of phenomena. Therefore, it is possible to affirm that (my) compositions are microcosms that *imitate* nature, the macrocosm (the universe). However, this is not all that is meant by the analogy: there is also the musical imitation of actually existent *conceptions* of the macrocosm that interest me the most.

Before considering this, it is important to understand correctly the idea of art as an "imitation of nature". Even today, in this respect, Aristotle's concept of poetic mimesis (*μίμησιζ*) as "a kind of recreation of reality according to the laws of possibility and likelihood" ([39, p. 168, v. IX]) seems to be the appropriate theoretical reference. It is not the case that the artist copies nature: the painter is not imitating nature for drawing a tree, for example; what is "copied", better say, what is imitated, is the *activity* of nature, in the sense that the artist "imitates nature in the process of creating a world or a whole" ([42]). Or, quoting Anton Webern, who quotes Goethe, "art is a product of nature in general, in the particular form of human nature" ([44, p. 19]), or still, "everything must be just as in Nature, since here (in art), too, Nature expresses herself in the particular form 'man'. That's what Goethe says." ([44, p. 44]).

The whole created by the artist is a microcosm (*μικρόζ κόσμοζ*), a "miniature thing" that encloses the characteristics of something much larger. Theories of the microcosm/macrocosm appear first in history comparing the human being (*minor mundus*) with the universe (*maior mundus*), as they share, in these theories, the same aspects or elements in structure or nature (and vice-versa) ([1, p. 321]). The idea developed since ancient Greece through the Renaissance, and lost momentum due to the advent of modern science [14] , in the philosophy and sciences of the 18th century. It is not an exclusivity of the western world either, as it appears, for example, in Indian religions such as Buddhism and in the Vedic idea of Puruṣa, Cosmic Person (Ṛg Veda X, 90 hymn) [15]. In the realm of the arts, the idea turns the art work into a microcosm.

> "The idea of the microcosm, the notion that the structure of the universe can be reflected on a smaller scale in some particular phenomenon, has always been a favorite in the history of aesthetics. ((...)) Even the famous doctrine of art as the imitation of nature lies in germ in these early cosmologies, if we take imitation in its liberal and true meaning, not as the duplication of isolated things, but as the active attempt to participate in a superior perfection". ([18, p. 6])

Therefore, a musical composition is a microcosm when it reflects the structure of the universe on its own small scale.

Some ethnomusicological studies corroborate this statement, as they identify a relationship between the way music is structured in a music culture and the cosmological conception of that culture, that is, "a way a specific people understand and relate to the phenomenal world": musical form as cosmology. Although ethnomusicologists do not speak of microcosm, it is, in fact, the case of a macrocosm/microcosm equivalence. For example, in their studies of Javanese gamelan music, ethnomusicologists Judith and Alton Becker use the term "iconic" to refer to this relationship. I will adopt this meaning of the word "iconic" hereinafter in relationship to my own music. The

[14] Throughout this article, the expressions "modern science", "modern world", and "Modernity" refer to the historical Modern Era, which starts roughly around the year 1500 and arguably finishes (?) in 1989.

[15] For the relationship of this idea with composition, see the article "Música e Sacrifício",[24].

idea is that a music's power (truth or beauty) is associated with the iconicity (or "naturalness", non-arbitrariness) of the coherence system behind that music. "Iconicity can be defined using Burke's categories [16] as finding the image of something in another realm" ([4, p. 205]). Therefore, finding the image of the cosmos (cosmology) in music makes music iconic with cosmology. This is exactly what Becker describes Javanese gamelan's music structure to be: it is iconic with the Javanese system of calendrical cycles. "In Java, a day is reckoned by describing its position within a number of simultaneous cyclic systems all moving at a different rate or all of different lengths." ([4, p. 209]). In a similar way, gamelan music is ruled by multiple cycles with pitches "coinciding at predictable points in the music system" ([4, p. 208]). The coherence system in case here is that of *coincidence*, for, "as pitches coincide at important structural points in gamelan music, so certain days coincide to mark important moments in one's personal life. One might say that gamelan music is an idea made audible(...)" ([4, p. 210]).

Another example is given by ethnomusicologist Paul Humphreys in his study of ceremonial songs of the Pueblo Indians. In the coherence system that informs the music of these ceremonial songs, the Tewa cosmos is ordered in six levels of spiritual and corporeal existence, with both spatial extremes (the center and the periphery) considered as sacred. *Katcina* dance songs'

> "musical organization has iconic significance in relation to cosmological orientation. ((...)) 'centripetal organization' and the activity of composing songs stand in iconic relationship as well. Song makers must work 'inwards' from large-scale formal requirements through a somewhat more flexible web of prescribed vocable formulas to the melodic content of initial and final sections, only last composing the portion of the song identified as 'song's middle' and which song makers from a number of pueblos acknowledge as 'the hardest part' to compose" ([22, p. 74]).

The ethnomusicological examples above are sufficient to make the point that the traditional music of a given culture is a microcosm; it expresses its own underlying cosmology; it imitates, in its structure and formative principles, the image of the cosmos prevailing in that culture. It does not seem to matter whether or not the iconicity described is a conscious element of its culture's people, their musicians and listeners, as the cosmological foundation of human behaviors is often buried in the unconscious. What matters is that the cosmology does find its reflection in musical form: *form as cosmology*. This "symbolic reflection of cultural meaning", to use the words of ethnomusicologist Alan P. Merriam, shows one of the ways music is symbolic, and helps "to understand music not simply as a constellation of sounds, but rather as human behavior" ([34, p. 258]). Yet, at the same time I bring this citation to provide the academic context (the study of music as culture) in which ethnomusicologists see the relationship between music and cosmology, I feel the "but rather" part of Merriam's statement needs a "correction". Since "but rather" implies "instead" and indicates that the understanding of music as human behavior is a *better* understanding of music than the structural understanding, I prefer to simply state that it is important to understand music in both ways at the same time, as a constellation of sounds *and* as human behavior. Musical structure (music as a "constellation of sounds"), as proven by Becker or Humphreys, has much to say about creative processes and human actions, and to abandon or diminish its study is to waste a good opportunity for advancing the study of music as culture. Therefore, in order to avoid defending the point that cosmovision is defined by society or culture, it is important to stress that, on the contrary, it is culture, as a complex of learned human behavior patterns that *express*es its cosmovision and cosmology. It is not possible to dwell on this subject any longer because the goal of this article is another. However, as in all situations in which there

[16] Kenneth Burke's *The Rhetoric of Religion: Studies in Logology* (1961) describes four realms of referentiality, i.e., realms which words can refer to: the natural, the socio-political, logology (the realm of words about words), and the supernatural.

are two aspects in a possible relationship of cause and effect, it is more likely that the two elements in question (in this case, society/culture and cosmovision) influence each other in a feedback relationship. While agreeing to say that a cosmology is a cultural construct, I am only recognizing that cosmology is *partially* a cultural construct; culture is the realm of manifestation, the temporal and spatial limiting factor or context in which human beings bring to manifestation ideas and objects of art (among other things). Therefore, ideas and objects of art will always be formed in relationship to whatever is already manifest in the culture, but in the creative process—which includes both the formation of (even cosmological) ideas and or artistic objects—it is not the *forma formata* (the finished cultural idea or object of art) that determines what is being created next, but rather, the *forma formante*, the archetype. If it were not so, cultures would be static and unchanging, ideas would never change, and their music would be stuck with the repetition of a limited repertory of pieces.

In addition, following modern science's line of questioning, which limits truth or knowledge exclusively to that which can be proved empirically, one might argue that, for example, the Tewa cosmos of the Pueblo Indians mentioned above in Humphreys' study is a mistaken idea of what the universe "is in reality". For this has been the main concern of modern science, to explain the universe objectively and quantitatively, once and for all. This line of questioning seems out of place.

Firstly, for the purpose of the iconic relationship between music and cosmology to take place, it does not matter whether the cosmological view is scientifically "true" or not: the lack of scientific truth in a given conception does not invalidate its influence on the music, nor the music itself, with which it is iconic. In this context of iconicity, it is irrelevant whether the earth is flat or not, or that it is at the center of the universe or not. Cosmology is an *image* of the cosmos, a set of ideas. The imitation of nature in art is always the imitation of what human beings think nature is. Cosmology is a conception about the universe and not the universe itself, and this conception is, in fact, the order human beings see in the world; *the conception is the cosmos*. Without a cosmic conception, the world is an unintelligible mystery, just like sound without order is not music.

Secondly, because cosmology, as a science of the world, exists since Antiquity, it should not be limited to the conceptions of modern science. The common use of the term "cosmology" currently in the media, popular knowledge and outside of academic circles of the Humanities (Philosophy, Comparative Religion, Anthropology, Mythology) has reduced its meaning to a purely physical description of the cosmos, leading straight to the notions of "space, the last frontier" and the Big Bang theory. Because of this overwhelming propaganda in mass culture, it is easy for the layperson to forget, or even to ignore, that traditional cosmologies did exist before modern science, were not limited to a simple cosmography and should not be dismissed as nonsense.

Once studying the non-scientific cosmologies of Antiquity, European Middle Ages, and non-western cultures from the Middle to the Far East, one not only discovers the existence of several culturally diverse conceptions, but also, that they frequently express a fundamental unity transcending their differences. It is an awesome phenomenon that disparate cosmologies such as those found in Vedanta, Platonism, Pythagoreanism, Sāṃkhya, Sufism and Buddhism, among others do have important points of connection, similarities and even identities. Philosopher Titus Burckhardt (1908-1984), whose main concern is metaphysics, explains, from that prospective, the reason why there is such a variety of cosmological conceptions:

> "Cosmology is thus the science of the world inasmuch as this reflects its unique cause, Being. This reflection of the uncreated in the created necessarily presents itself under diverse aspects, and even under an indefinite variety of aspects, each of which has about it something whole and total, so that there are a multiplicity of visions of the

> cosmos, all equally possible and legitimate and springing from the same universal and immutable principles." ([8, p. 17]).

Although the main object of a traditional cosmology is the world as it exists, it is also concerned with metaphysics, differently from modern science, which is satisfied with the mechanistic explanation of the world.

> "Cosmology as such does not refer to the Absolute or pure Being, but rather to existence, the totality of created and manifested worlds. However, as the cosmos would not exist without its divine origin, and, from the point of view of its essence, it cannot be but a limited image of the divine, cosmology is interested indirectly in the metaphysical truths, those from which it receives its ultimate certainties" ([7, p. 19]) (my translation).

This statement clearly shows the transcendental background of a traditional cosmology, its connection to religious and theological traditions, and its concern with the relationship between the human being and the world, and between them and a "metaphysical truth" or, in other words, the Sacred. In the majority of the cases (in fact, I cannot think of any exception), traditional cosmologies are in accordance with their related religion. Religion may be defined here as a cultural or a people's ensemble of ideas, beliefs and practices that follow their conception (theology) of the Sacred. By its turn, I understand by *sacred* that which religious scholar Rudolf Otto (1869-1937), by derivation of the Latin term *numen* (divine, or non-personified divine presence), defined as the "numinous" in his 1917 book *Das Heilige*: while "((...)) 'the holy' is a category of interpretation and valuation peculiar to the sphere of religion ((...)) it completely eludes apprehension in terms of concepts ((...)) "([36, p. 5]), the numinous is "((...)) 'the holy' *minus* its moral factor ((...)) minus its 'rational' aspect altogether" ([36, p. 6]).

Therefore, traditional cosmologies confer to music other meanings beyond that which, up to now, I have identified as the "scientific cosmology of music", and which generates a rational knowledge (*episteme* - *έπιστήμη*) about the physical aspect of the musical microcosm: its structure, formative principles and compositional techniques. I mean *episteme* very much in the ancient Greek sense of a rational science. In the analogy with a traditional cosmology, I should speak of a *contemplative cosmology of music*, as this assigns a specular or symbolic quality and scope that generate a gnostic knowledge (*gnosis* - *γνῶσιζ*) about the musical microcosmos, i.e., the physics of the musical microcosm, its structure, formative principles and compositional techniques. I do not mean *gnosis* specifically as in the mystical knowledge of certain late Antiquity religious-philosophical groups, which is the immediate reference of the term, but rather in a wider sense, as an intellectual intuition of the numinous.

In order to describe this relationship of the music with the numinous, as opposed to a relationship with the immediate empirical reality, I prefer the term "contemplative" instead of "traditional" or "cultural" cosmology. Until recently, I have used the term "cultural cosmology of music". However, this turns out to be a reductive approach, as "cultural" emphasizes only the relativity of cultural diversities, it only refers to the fact that traditional cosmologies vary according to culture. Also, "cultural", in terms of knowledge, is not in the same level as "scientific". The term "contemplative", or even "gnostic", is a better complement to "scientific" than "cultural" or "traditional", the latter often being used in opposition to "modern".

Contemplation, from the Latin *contemplatio*, means consideration, observation and study, and "is the prolonged insistence of the gaze or thought over a source of wonder or admiration" ([14, p. 568]) (my translation). The Latin root *templum* (a sacred place) indicates that this insistent gaze aims at a mystical *henosis* (ἕνωσις, oneness or unity) with the numinous. German theologist Friedrich Heiler (1892-1967) writes, in 1933, about the definition of contemplation:

> “The clearest definition of contemplation was perhaps presented by St. Thomas Aquinas (...) ‘contemplation is the simple intuition of divine truth, proceeding from a supernatural principle’. Contemplation is spiritual vision, a total vision, directed toward the divine reality in its transcendental totality, a vision that is not voluntarily induced by human activity but that arises without our volition, that—speaking in religious terms—is inspired, infused, bestowed as a divine charisma” ([20]).

Therefore, a contemplative cosmology reflects the mystical or spiritual vision over which it is founded. The idea of a contemplative cosmology of music returns to contemporary musicians an element of poetics, aesthetics and the philosophy of art that has been present in the art of practically all times and of probably every people, and that had been rejected and forgotten, perhaps, only by Modernity: the sacredness of the creative act and the function of art as a vehicle for spiritual truth. It seems fair to say that in the late modern (post-modern?) world, generally speaking, materialistic and utilitarian values have emptied music (and art in general) from any spiritual meaning and reduced it to the *status* of a disposable item, while, in tribal societies, for example and also generally speaking, the sacred is seen in everything, and consequently, even the most banal utensil is crafted as an art piece, because, in fact, they are all sacred objects.

In the same way, the words of the Parisian architect Jean Mignot (fl. 14^{th} and 15^{th}- centuries) in 1398, at the time of the construction of the Milan cathedral, "*Ars sine scientia nihil*" (art without science is nothing), summarize the nature of sacred art and the close relationship between art and science in the Middle Ages, a conception that was beginning to be challenged already at that time [17]. In medieval understanding, the term *ars* means *techné*, the ability to make things, and *scientia* does not mean science as we understand it, but the reason (*ratio*), the theme, the content, or dominant motif of the work, which is spiritual truth ([10, p. 229]). Therefore, in the Middle Ages, art without spiritual content is nothing. The tendency to separate art from science eventually led to the modern scientific view that traditional cosmology’s attempts to explain natural phenomena are naïve or plainly wrong—who could possibly defend the idea that the earth is flat, after science’s discovery that it is round? Traditional cosmologies, as they are not scientific in the modern sense of the word, express themselves by means of allusions, parables, myths, and symbols in a way that is comparable to art.

It is in the spirit of this engagement with existing traditional cosmologies that the analogy between composition and cosmology is finally complete: by means of the musical imitation of actually existent contemplative or mystical *conceptions* of the macrocosm. While the medieval *musicus* was more a cosmologist than a practical musician, concerned with what Boethius called *musica mundana* and the concept of *harmonia tou kosmos*, harmony of the cosmos or universal harmony, the contemporary composer becomes invested by the cosmologist role, similar to that of the *musicus*, by means of associating musical composition with a contemplative cosmology. This, in its turn, does not have to be limited to Pythagorean philosophy, but may be expanded by contemplative cosmologies of other traditions. The *scientific cosmology of composition* and the *contemplative cosmology of composition* stand to each other not in a relationship of contradiction or opposition but in that of complementarity and interdependence.

I should briefly mention that, although it has been my choice to engage exclusively with contemplative cosmologies, there still is the possibility for composers to work at an iconic relationship of their music with contemporary scientific cosmological thought. A good example of this is found in the late works of Hans-Joachim Koellreutter, those he called “essays” instead of “compositions”. Works like *Dharma* (1990) or *Wu-li* (1992) are iconic with certain principles

[17] Coomaraswamy narrates that Mignot’s phrase was the answer he gave to the nascent opinion that “science is one thing and art another”.

of Quantum Physics, which explain the composer's choice for open form and his "relativistic aesthetic of the imprecise and paradoxal". These two works in particular refer, at least in their titles, to concepts which come, respectively, from Indian and Chinese contemplative cosmologies. Koellreutter's permanence in India and Japan from 1964 to 1974 ([28]) seems to have been a decisive period for the development of this tendency to connecting several works to concepts from Buddhism (*Sunyata*, 1968), monistic Vedanta (*Advaita*, 1968), and Zen Buddhism (*Yūgen*, 1970). In fact, aligned with ideas from Fritjof Capra's book *The Tao of Physics*, Koellreutter also saw a connection between Quantum cosmology and Hindu, Buddhist and Taoist philosophies. Curiously, while none of his works' titles indicate a relationship of the music with Quantum Physics, several indicate Buddhist or Hindu mysticism. Conversely, in his discourse about his musical work, Koellreutter was much more eloquent about establishing a connection with Quantum Physics instead of mysticism, as if Quantum Physics provided a safer rhetoric which corroborated, in a scientific way, that which was already present in these mystical traditions (this is, in fact, one of the points defended in Capra's book). Certainly, these musical compositions of Koellreutter display a strong attraction towards the transcendent, numinous Being, and this fact places them in direct relationship with contemplative cosmologies.

Several of my musical works have included one or as many elements as possible, to convey, musically, the symbolic coherence that constitutes or is found in an existing contemplative cosmology. A particularly expressive work in this regard is my *Sacrifício* (1998), for mixed choir *a cappella*, which is composed according to the Vedic idea of sacrifice (*yajña*) as cosmic law, by means of "sacrificial structures", interdependence, complementarity and inversion as formative principles in the music [18]. The role of number in this composition will be described further ahead.

I have a great interest in the philosophical and cosmological thought of pre-Socratic philosophers, especially Pythagoras of Samos (c.570-c.495) and sixth century Heraclitus of Ephesus, (c.535-c.475 B.C.E.), which has helped me engender a number of musical works. In *Madrigal de Fogo* (1996, for mezzo-soprano, strings—vn., vla., vc., db.—and two percussionists), the voice sings, in Greek, selected cosmological fragments by Heraclitus about fire, cosmic fire, and the elements. This piece is a "madrigal" in the sense that it "imitates" the content of the text. The composition as a microcosm establishes an analogy with Heraclitus' conception of cosmos, by which all phenomena are generated by fire, all are transmutations of the primordial element, fire. In the music's universe of sounds, the element that corresponds to fire is rhythm, understood as becoming (*devenir*), since both fire and time live by self-consumption, in the same way as music. The syllabic and quantitative nature of Greek language allows the rhythm of the text to become rhythm in the other instrumental parts [19]. There is no thematicism, but simply a constant transmutation of the words into instrumental sounds, following the Heraclitean cosmological principle sung in the first fragment of the text: "*panton hen kai ex enos panta*", "from All, one; from One, all".

In another work, "*...a natureza ama esconder-se...*" (1989), for oboe, Bb trumpet and cello, the title itself is a fragment by Heraclitus which states that "nature loves to hide", meaning that the real constitution of things, i.e., the formative principle that shapes them, "loves to hide" from plain view. This idea prompt me to conceive a music which sounds according to a certain organization or order principle which is not easily perceived neither clearly apparent. This notion surely has to do with Franco Donatoni's emphasis on *procedimenti nascosti*, hidden compositional techniques or codes that he used in his own works.

[18] As this article is concerned with the role of number or mathematical models applied to composition, I should refer the reader again to the article "Música e Sacrifício" ([24]), which describes the relationship between music form and the Vedic sacrificial cosmology.

[19] See expanded modal rhythmic principles in [23]

At this point, it is necessary to conclude the description of the poetic context in which number and proportion find an application in my music by briefly approaching the ideas of composition as cosmogony and spiral musical time.

II. Composition as Cosmogony

A cosmology always implies a cosmogony, to the extent that it may be very difficult to separate one from the other, the description of the cosmos (cosmology) from the account of its origin (cosmogony or cosmogenesis). As a discipline in the Humanities, cosmogony "has to do with myths, stories, or theories regarding the birth or creation of the universe as an order or the description of the original order of the universe" ([32, p. 94]). Regarding the musical composition as a microcosmos, this corresponds to looking at the creative, compositional process as analogous to a cosmogony. Each composition has its own compositional process, and a detailed description of it would take into consideration the composer's method (from the Greek *methodos*, the path to reach a goal). This means that we would be back to the issue of the ensemble of techniques which, used in a certain sequence of steps (the path) obtains a certain result, even if this path had not been foreseen by the composer. In fact, it may not be clearly foreseen, but, according to the idea of *forma formante*, which guides the creative process, it might be envisioned intuitively. Therefore, this leads back again to cosmology, the formative principles of the neutral level, seen as techniques, in the light of the artist's poietic level.

Such is the conundrum between cosmology and cosmogony. Therefore, the discussion about compositional process as cosmogony must not be about methodology or technique (which belongs to the scientific cosmology of composition), but about its meaning from a contemplative and artistic point of view, which is how the compositional process and the resulting composition "reflect the uncreated in the created", reflect their "unique cause, Being". One could say that what follows is a metaphysic of musical creation.

In "the active attempt to participate in a superior perfection", the contemplative point of view sees the compositional process as a repetition of the creation of the universe, the sacred act *par excellence*. It is a repetition in the human scale, a re-enactment of an act attributed to (a) god, or to a reality that transcends the human being. This idea comes directly from Romanian philosopher and comparative religion scholar Mircea Eliade's (1907-1989) thesis that the cosmogonical myth is a model for every significant human activity including rituals and artistic creation: "every construction or fabrication has the cosmogony as paradigmatic model. The creation of the world becomes the archetype of every creative human gesture, whatever its plane of reference may be" ([15, p. 45]). When I first read, in 1988, in Eliade's 1949 book *The Myth of Eternal Return*, about the re-enactment of Creation that takes place in the process of a city's foundation, for example, the idea was for me perfectly natural and applicable to the work of creating music, even if Eliade was referring to "archaic societies":

> "If the act of the Creation realizes the passage from the nonmanifest to the manifest, or to speak cosmologically, from chaos to cosmos (...) all this beautifully illuminates for us the symbolism of sacred sites (centers of the world), the geomantic theories that govern the foundation of towns, the conceptions that justify the rites accompanying their building. We studied these construction rites and the theories which they imply (...) every creation repeats the pre-eminent cosmogonic act, the Creation of the world" ([16, p. 18]).

As strange as it seems, the notion that the creation myth of an "archaic" society could be a model for the compositional work of myself, a contemporary Brazilian composer, at the time (1988) living in Italy, sounded to me as unquestionable and true. I still wonder to this day how this can be... A couple of years later, another reading that was for me "as lightning in a calm sky" was the Puruṣa Sukta, Ṛg Veda X, 90 hymn, which narrates a Vedic cosmogonic myth by which the universe is created from the sacrifice and dismemberment of Puruṣa, the Cosmic Person: it was, to me, like a clear description of the process of musical composition. . . , and eventually led to the 1998 choral piece *Sacrifício*.

I am probably not the only composer who could talk about their personal experience of this cosmic creation re-enactment through composing music. But I do it while being aware of the sacredness inherent in the process, or aware that I am attributing to it this sacred quality, which ultimately amounts to the same thing. When I am about to start working on a new piece, I feel it as a latent, unmanifest sonic energy or sound that I can visualize or imagine as a sphere, unless it already comes as a musical idea. Most often, the hearing comes later in the process, but initially, I sense this kind of entity that I am willing to bring to the outside world through the intuition of a title, or of a formative principle and technique, or through a musical structure. There is no formula to approaching this sphere. Most of the times, because of a structural *rationale* that seems to dominate my way of thinking, I start planning the macroform, the macrocosm of the piece (the macrocosm of the microcosm...), its proportions, its larger sections. Then, these ideas become material for the microform, i.e., the local structures that the listener experiences directly when listening to the finished work. This is clearly a desire to establish, in music, traditional cosmologies' principle of identity or similarity between the macrocosm (universe) and the microcosm (human being) within the piece itself, i.e., by creating identity or similarity between musical macroform and microform.

My conscience of what the piece is grows gradually. Along the working process, I feel like the explorer of a *terra incognita* that I am creating at the same time that I am exploring it. I think this applies to every composer whose work is born from research of something new. It is through the work of composing, by trial and error, that I become conscious of the sound universe of the composition, therefore, giving to it a meaning and a final form. Probably, as in Pareyson's theory of formativity, the composition is bound to have that final form from the very beginning, the time in which it was still unmanifest, as *forma formante*, which becomes *forma formata* along the process. The composer should just be able to get in the right syntony with *forma formante*'s vibrations. The piece, before it is composed, lives only in potentiality, outside of existence. Unmanifest, it remains part of the undifferentiated absolute until the composer becomes gradually conscious of it by bringing it to light through the compositional process (cosmogony), which consists in the creation and application of the necessary, appropriate and rightful formative principles and structural relationships (scientific cosmology).

This process of creation I have just narrated above is, to a contemplatively inclined mind, sacred in itself simply because it is a poetic act, i.e., an act that brings forth into nature-existence-something that did not exist before. Therefore, this human creative act of composing music shares the same nature of the act of Creation of the world. It turns undifferentiated sound (chaos) into music (cosmos); the composer witnesses in first hand, experiences directly, and actively helps the conduction of the transformation path travelled by *forma formante* from potential to actual, from archetypal to typal. And this is probably what Karlheinz Stockhausen meant by "as it (the piece) was being constructed through me" in a previously presented quotation.

To compose music with this awareness of artistic creation's sacredness and with the intention of conveying an existing contemplative image of the cosmos (cosmology) at the structural level of the music's organizing and formative principles is to *cosmicize sound*. I have borrowed the idea of "cosmicization" from Mircea Eliade:

> "The cosmicization of unknown territories is always a consecration; to organize a space

> is to repeat the paradigmatic work of the gods. ... Establishment in a particular place, organizing, inhabiting it, are acts that presuppose an existential choice—the choice of the universe that one is prepared to assume by 'creating' it. Now, this universe is always a replica of the paradigmatic universe created and inhabited by the gods, hence it shares in the sanctity of the god's work" ([15, p. 32-34]).

As the composer creates and explores the *terra incognita* of a new musical language through work, research, listening, and, last but not least, inspiration, this "land" takes shape and becomes music, home and style: becomes a cosmos. The awareness of this musical process "from nonmanifest to manifest" as being sacred cosmicizes sound, or, in other words, consecrates it (con-secrate = to associate with the sacred): cosmicization "is always a consecration". This is the first part of cosmicization: the consecration of the musical creation act (or the awareness that it is sacred) because it repeats the Creation of the world by transforming, like world into cosmos, sound into music. But to a non-contemplatively inclined mind, this idea will be questioned in this way: *since concert music is not liturgical, how can it be sacred music; where does this power to consecrate come from? With what authority can a composer declare their secular work to be sacred?*

No one would question that Olivier Messiaen's (1908-1992) *La Nativité du Seigneur* (1935) or *Quatour pour la Fin du Temps* (1941) are pieces of sacred music, since the Catholic themes are evident in their titles, although these pieces are concert, non-liturgical music. However, the same can be said of anyone of Messiaen's bird pieces from his *Catalogue des Oisexaus* (1956-8) i. e., that these are sacred music as well, since birds are sacred for Messiaen. And even *Modes des Valeurs et d'Intensités* (1949), an "abstract" piece and almost a technical experiment, can also be seen as sacred, as, in Messiaen, the same musical cosmology (as a group of techniques) generates all of his pieces, whether or not he associated them with religion. The recognition that Messiaen's music is sacred comes from the authority of his religion's faith, which is Catholicism. Therefore, Messiaen's power to consecrate his music comes from a culturally established world religion. However, when Karleinz Stockhausen attributes sacredness to works such as *Stimmung* (1968) or *Inori* (1974) (*inori* is a Japanese word for prayer or adoration), the recognition of this attribution is not—and has not been, historically—so straightforward. The frequent negative criticism of these works exemplifies the rejection, within the concert music world, of "foreign mysticism" and other notions that express the disagreement with a composer's authority to attribute sacredness to their music when they are not members of their own society's predominant religion or when their music does not reflect it. A non-contemplatively inclined mind—one that does not perceive the sacred—will never find a satisfying rational argument that can support the idea that the individual's awareness of the artistic creative act's sacredness is enough in itself to empower the individual to state that their art is sacred.

What to say about the case in which the composer does not valuate or is unaware of their own creative act as being a sacred act? Can one speak of cosmicization in this case? One uses the adjective "sacred" to emphasize a *perceived* quality ("sacred") of the substantive "act". Is this quality still there, in the substantive, when one is unaware of it or cannot perceive it? I will leave these questions open... However, the individual does not live in isolation from culture and society. Ideas about the sacred will necessarily be confronted with cultural ideas about the sacred, found in world's religions, in other people's experiences of the sacred. The ideas of an individual will never come "out of the blue, and, for this reason, for the composer that considers their creative act as sacred, their authority comes exactly from this perception".

Music is certainly capable of "sharing the sanctity of the god's work of cosmic creation" with renewed force when its formative principles are intimately connected to an existing, traditional contemplative image of the cosmos. We have seen it in the examples of Pueblo Indians and Javanese gamelan above. This image becomes manifest in music, thus re-enacting cosmic creation by repetition or mimesis of that cultural sacred image of the cosmos. In other words, music is capable of conveying cosmological contents when it is iconic with that cosmology, or when a composer organizes music by formative principles similar, analogous or same as the cosmological formative principles (believed to be) in operation within a given cosmology.

Therefore, it is by conveying a traditional contemplative cosmology that composers are finally or ultimately invested with the authority to consecrate their music as sacred, an authority that, in the view of those who do not perceive the sacredness of artistic creation, was lacking. Thus, composers (and artists in general) may no longer be accused of simply inventing a personal cosmology or a personal symbolism. In fact, in Eliade's archaic societies, the foundation and building of a city always follows the traditional cosmology of the people, thus enabling them to consecrate or cosmicize territories as a "replica of the god's work"; the god's work remains the model and primordial reference.

As shown before, the traditional music of a society tends to be iconic with its cosmology. Iconicity had been defined as "naturalness" and "non-arbitrariness" in Judith Becker's article on Javanese gamelan because "music systems are instances of the way a specific people understand and relate to the phenomenal world" ([4, p. 215]). Again, playing the role of the devil's advocate, a question could be rightfully raised about this relationship between a contemporary composer and a traditional cosmology: *what is natural and non-arbitrary about the music of a contemporary composer who obviously does not live in the traditional cosmology his music is supposed to be iconic with?*

The only answer to this question is that such composer should make all effort to establish that traditional cosmology as their own, to fully engage with it in all possible aspects of life and work, so that it won't be just an intellectual abstraction or an arbitrary choice, but a lived one. In fact, it is a choice because one sees in it truth and sense. It is "the existential choice of the universe that one is prepared to assume by 'creating' it", to repeat Eliade's words on cosmicization. Contemplative cosmologies contrast sharply with the contemporary regular, most commonly stereotyped, urban, modern, consumerist, individualist way of life. To go against that could turn into a daunting or even impossible task: it would involve taking personal choices in a very opposed direction to the "natural" tendencies of contemporary society. However, if one seeks coherence and integration between life, knowledge and art, it is not because of the seeming difficulty of the path that one should refrain from travelling it. Study in depth of the different contemplative traditions is one of the paths to attaining a full understanding and assimilation of their contents; to the willing researcher, these traditional contents are easily accessible nowadays, the idea that they are remote, difficult to find and *passé* being an entirely equivocated prejudice. From study comes the possibility of these contents being applied to one's life, not as a nostalgic return to the past, or a superficial reproduction of behaviors, but rather as part of their assimilation into a new, albeit individual, cosmovision. Evidently, an "individual cosmovision" is not conceivable without it sharing its content with cultural, collective cosmovision(s), for the simple reason that, as mentioned before, the individual does not live in a bubble, isolated from any contact whatsoever with cultural practices. A traditional cosmology can become, today, an integral part of a contemporary cosmovision and, in this sense, the music composed accordingly will be iconic with it.

III. Spiral Musical Time

The spiral is a complex symbol that human beings have drawn or carved for centuries since the Paleolithic; it appears in the Vedic, Yogic, Tantric, Hermetic, Celtic, Christian, Taoist, Maori, and Cabbalistic traditions, to name but a few, each emphasizing perhaps different aspects of the spiral, but always revealing it as an archetypal symbol of the creative processes' great strength and emanation. The spiral is also seen in natural phenomena as growth processes such as waves, hurricanes, shells, animals, plants, the moon phases, the seasons becoming, and the sun's ecliptic path. "Growth" must be understood as intensification in either of the two complementary directions of the spiral—coiling and uncoiling—by which the expanding, outward, movement represents cosmogony, creation and multiplication of phenomena, towards manifestation or birth, while the contracting, inward movement, represents eschatology, destruction, retraction from multiplicity of manifestation, decline or death ([11, p. 156]).

When the musical composition as *forma formata*, sense form, the microcosmos, takes the spiral as a constructive principle of its whole macroform, it coincides with the spiral and leads back to the archetype (*forma formante*, not sense form). The spiral becomes *present*, not only as the formative principle but also as the musical time experienced by the listener, when it establishes, in organized sounds, its cyclic and intensifying trajectory. The music becomes a symbol of the spiral and, consequently, also of the archetype.

Apparently, the spiral seems to be a spatial symbol, as it has been traditionally represented visually in petroglyphs, sculptures, architecture, paintings, drawings, etc, and, therefore, is most often *seen* than heard. However, it is naturally not only spatial, but temporal as well, since its line denotes a progressive process of transformation in time. The spiral is a space-time continuum. Furthermore, depending on its own momentum, this identity macroform/spiral results in a more or less gradual musical becomingness. The speed of the spiral can be as intense as a vertiginous vortex or as gradual and slow as not to be immediately noticeable. In this respect, two ideas are important guidelines for my conception of *spiral form composition or composition in spiral time*: one from American composer Steve Reich (b. 1933) and another from Hungarian composer György Ligeti (1923-2006).

The idea of music as a gradual process originates in Reich's short 1968 essay, *Music as a Gradual Process*, in which he talks about a music that is, literally, the process. For him, this process must be perceivable, this being the reason why Reich's music of that time is repetitive. This idea is also important, in a similar way, in my spiral forms as it is desired and intended that the spiral as a formative process/principle be aurally recognizable and perceivable. However, there is neither minimalism nor repetitive music in my spiral form compositions, since they aim to the accumulative effect of the spiral's *different return*. Reich's idea that "once the process is set up and loaded it runs by itself" ([40, p. 33]) is, to a certain extent, present in these spiral works, but only insofar the general principle needs to be preserved and needs to function as Ligeti's idea of "notional compositional structure". Ligeti suggested that the composition needs to provide the listener with a notion of its structure and of the formative principles through sound, by ear, without the recourse to complex analysis and intellectual effort over the musical score. His exact words are: "in working out a notional compositional structure the decisive factor is the extent to which it can make its effect directly on the sensory level of musical perception" ([30, p. 31]). His point of view is diametrically opposed to Franco Donatoni's poetics of hidden procedures.

Musical time, understood as musical form *as it enfolds in time* —and, therefore, as experienced by the listener—acquires a direction, a teleological sense, when it is organized as a spiral. In this case, musical time is linear and cyclical at the same time. Cyclical time [20], which is a special case of temporal circularity in music, turns over itself and, as it does so, it does not progress any further and remains ultimately static. Spiral time, which is another special case of temporal circularity, has the cyclical auto-referentiality combined with a linear aspect, which consists in the production of difference by the accumulative effect of its outward or inward tendency. The spiral turns around itself but, when it arrives at the "same" point, this is no longer the same point: it

[20] For a reflection on cyclical musical time, see my article "Tempo musical cíclico no *Miserere mei, Deus* de Gregorio Allegri ([25])

is the same in another level of realization. The spiral's *telos* (τέλος , end or purpose) lies in the infinite, either when expanding or contracting, and it consists in its own ordering tendency: *the process is the purpose*; this will become clear with the following musical examples. However, as a composition cannot last forever, the spiral form ends by simple interruption of the process, either because, if expanding, it would not make aesthetic sense for it to continue indefinitely, or because, if contracting, the process had arrived at its center, which is always a point of origin and of no manifestation, which requires the music to stop.

The spiral as a dynamic structure, constructive principle or musical process appeared for the first time in my 1984 violin and piano duo *De Natura* but, at that time, I was not aware of it as a spiral. In fact, it is composed of two alternating or entangled spirals, one contracting, the other expanding, characterized by two kinds of contrasting textures: the contracting spiral is multilayered or polyphonic, and the expanding is monodic, a growing unison in both instruments. Both spirals use Fibonacci numbers to control their expansion or contraction: the contracting spiral starts with a section of 144 eighth notes, then 89, 55, 34, 21, 13, 8, while the expanding spiral growths from 8 eighth notes to 13, 21, ... until 144. Therefore, the macroform is sectioned by the numbers 144 – 8 – 89 – 13 – 55 – 21 etc.

The spiral has been a conscious object of my compositional research since 1988, beginning with the piano solo work *Pralāya.* Since then, several other compositions have created different spiral temporalities: musical time organized by the cyclic, continuous and intensified winding /unwinding transformation process, which is characteristic of the spiral. In the same way that diverse spiral lines may be defined by different geometries and mathematics, innumerable are the ways by which musical time may appear to be expanding or contracting as a spiral.

A characteristic of spiral musical time is that it most often applies to the whole macroform of a work, and only a few works have explored spiral time only in certain parts or sections. Although it is a continuous movement, spiral musical time can be articulated by *phrase cycles*. The expression "phrase cycle" tries to relate musical "phrases" with the context of spiral time, but it should not be understood as "phrases" in western classical/romantic morphology, which belongs to a linear and dialectic time context. The expression is abbreviated to "cycle", and is meant to simply indicate a structure of musical course (or duration) which has, like the "phrase", a complete sense, within the context of spiral temporality. This complete sense, therefore, is reached by the completion of a spiral turn or cycle. After this completion, the spiral starts over, but at another level of manifestation. The moment between completion and re-start may or may not include a break (interruption, *caesura*, *fermata*) in the movement.

Two aspects of spiral time are naturally mathematical: duration and gnomonic growth. Duration can be ruled by pre-established ratios either in a linear time composition or in a spiral time composition. In the latter case, the macroform is the total duration of the spiral, and the sections are the durations of its expanding or contracting cycles. The rate of growth or decrease of the length (duration) of each spiral cycle defines such proportion, as will be seen in the musical examples ahead. Gnomonic growth, on its turn, is more specific of spiral time, as it is a form of "growth by accretion or accumulative increase, in which the old form is contained within the new (...) all figures which grow by gnomonic expansion create intersections upon which spirals can be drawn" ([29, p. 65-66]). Lawlor provides an ancient definition of a gnomon according to mathematician Hero of Alexandria (ca. 10 C.E. – ca. 70 C.E.): "a gnomon is any figure which, when added to an original figure, leaves the resultant figure similar to the original", therefore, just like the modern fractals.

Gnomonic growth appears in the construction of Hindu temples, in the horns of rams and antelopes, mollusk shells, in the Pythagorean *tetraktys* and other triangular numbers, in rectangular numbers, and in several geometrical processes of recursive self-similarity such as the infinite accumulation of alternating pentagrams and pentagons (in figure 1), or the equally infinite process of marking a square inside golden rectangles (figure 2). Gnomonic growth is certainly not the only form of growth or increment that I have applied to my compositions, but it does provide a sense of beauty as in mathematics as in art, especially in relation to the principle of macroform/microform identification. The role of proportion in gnomonic structures is decisive for the rhythmic quality of the perceived process.

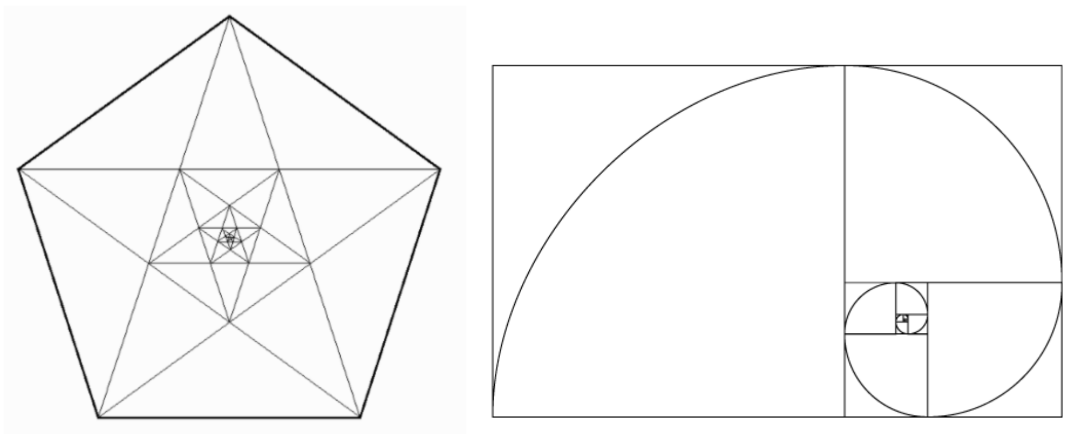

Figure 1: *Left: The diagonals of a pentagon form a pentagram. The intersections of the pentagram form a smaller pentagon. Draw again the diagonals of the smaller pentagon, ad infinitum. Right: Draw a square inside a golden rectangle. Inside the smaller rectangle, draw another square, ad infinitum. A logarithmic spiral is formed by linking the opposed corners of the squares by a continuous curved line).*

By presenting the ideas of spiral time and composition as cosmogony before tackling the subject of number and proportion in composition, I expect to have made easier to discuss the musical examples, because in their majority, these proportions define the durations of spiral cycles and of the whole macroformal spiral. I also thought that presenting first the poetic context in which number and proportion find their musical application in my own music would result in a better understanding of why these proportions are brought into the music.

IV. Number and Music

Number and proportion appear in my music at all levels of temporal organization, from the microform to the macroform. Since 1989, I have made extensive use of a set of formative rhythmic principles designed to generating ametric textures, complex polyrhythm, cross-rhythms and certain specific qualities of rhythmic flow defined by syllabic rhythms ruled by numerical ratios. I call this set of principles *expanded modal rhythm* because they greatly expand the kind of organization found in the rhythmic modes of 12^{th}- century polyphony of the Notre Dame School. A full description of the general principles of expanded modal rhythm is available in a previous article ([**?**]), in which my main concern was to explain what modal rhythm is and what is "expanded" about it. These principles do organize local rhythms (microform) as well as the work as a whole (macroformal design).

Expanded modal rhythm ([24]) is based on the idea that durations are either long (—) or short (∪), and that they can be quantified by a temporarily fixed — : ∪ (long : short) ratio involving any whole number of time units (*chronos protos*). Notre-Dame rhythmic modes not only fit in this description, since they were mostly constructed at the 2:1 ratio, but also indicated that their formative principles could be expanded. The appearance of long and short durations in Notre Dame polyphony was related to Greek feet such as *íambos*, *dactylos* etc. In my music, I work with rhythmic patterns called "syllabic meters", which are combinations (or permutations) of any given number of durations, called syllables. Therefore, the Greek feet are only a small part of this collection of patterns, obtainable from combinatoriality. As an example, the six-syllable meter — ∪ — — ∪ ∪ can be assigned a 2:1 ratio with the quarter note as *chronos protos* resulting in the rhythm of Example 1. The same pattern, with a 4:3 ratio and the eighth note as time unit is shown in Example 2. It is the combination of ratio syllabic structures alternating long and short syllables that forms a rhythmic mode, and a mode should be understood as exactly what it is: a mode, a manner, therefore, a quality, a rhythmic quality, a quality of movement.

a) b)

Figure 2: *a) Syllabic meter — ∪ — — ∪ ∪ (2:1) ♩ ; b) Syllabic meter — ∪ — — ∪ ∪ (4:2) ♩.*

At the same time that the application of numbers to music implies quantification, it also brings quality to music. This quality is the qualitative aspect of the number: two-ness, three-ness, etc. The proportion, as it puts two (or more) quantities in relationship, creates relativity between these quantities to each other. Proportion, "or Reason (ratio) is the comparison, correspondence or relation that exists between a number and another" ([33, p. 6]). As in the Quadrivium, Music is Number in Time. While this quality of rhythmic movement is immediately perceived aurally at the microformal level, as shown in the examples above, it is still perceived in the macroformal level, albeit less directly, or in a different manner, since the length and content of durations is much bigger. I should now concentrate on the role of proportion in the macroform, i.e., compositional design, as much more about the relationship between number and microformal rhythms has already been shown in the article on expanded modal rhythm.

When I start working on a new piece, one of my first compositional actions is to attribute a ratio to the largest (and also the smaller) sections of the work's macroform: to determine (or pre-determine) how many parts will constitute the piece, and what is the balance or proportion between their durations is a decision that pertains to an abstract and numerical level, since the music has not yet been composed, but only its "size" and the size of its parts is being decided. What the macroformal ratio *suggests* in terms of its numerical relationship should be explored by the *content* of these pre-determined durations of sections.

The simplest macroformal ratio is 1:1, and it is found in the piece *Sacrifício*, for choir, which is iconic with the cosmogonic Vedic myth of Puruṣa and the philosophy of the *Brāhmanas* and *Upaniṣads*. The macroform is symmetrical and is governed by a 1:1 ratio, since the first and second parts have exactly the same duration. A dotted quarter note rest on measure 186 at the exact center of the music separates the first half of the piece from the second. This rest is the arrival of the first section's spiral contraction towards the point of non-manifestation, silence, *bindu*. The second part is a spiral expansion towards multiplicity of phenomena. In this way, the macroform shares the same cosmological symbolism of god Shiva's drum, the *damaru*, which has the shape of an hourglass [21]. Because the piece is sung in Sanskrit, the microformal rhythm of vocal parts respects the long and short syllables of the Sanskrit language even when, in the music, syllables assume values different from 2:1, which is Sanskrit's original long-to-short ratio.

[21] The article "Música e Sacrifício" ([24]) describes this subject at length.

There is a static quality about the 1:1 ratio. This is easily perceived in the microformal level, as equal values result rhythmically in a series of regular pulses. Depending on how long this pulsating rhythm goes on, it will, eventually, reveal itself not only as regular but as static, unless it changes. The richer are the combination of different values in a line, the more varied and, therefore, kinetic is the line's *rhythm*. Seen from another angle, the pulsation, in all measured music, is the simplest, most primordial level of rhythmic organization, one that remains "behind" or "underneath" varying rhythms. As pulses are grouped in meters (2-beat, 3-beat, 4-beat meters), a second level of regularity is generated: meter, which is, in itself, a higher level beat. If meter remains unchanged for the duration of the macroform, meter is no longer just regular, but becomes non-moving, a static element. In this case, because it never changes, meter moves from the *en-temps* to the *hor-temps* category [22]. In the other hand, if meter changes frequently, it tends to disappear, because there will be no regularity at its level. This is when longer morphological units will take over the reference role for measuring this higher (metric) level flow or rhythm.

As a ratio governing the macroform of two equal sections, the 1:1 ratio is static for a number of reasons. First, because there are only two occurrences of the "1" duration (as in *Sacrifício*), this results in the simplest case of a binary relationship: that of equality, a sense of perfect balance, like two equal weights on a scale. This, however, is what the ratio means as a numerical relationship; this is what the ratio *suggests* to what will happen in the musical time. As it concerns the macroform, each part is really a section of music, during which *something* happens, this "something" being the *content* of their duration. At least in theory, it is possible to imagine that, even in a macroform divided in a 1:1 ratio, the sections could be out of balance because of the effect produced by the music filling that duration.

In the spiral contraction that makes the first section of *Sacrifício*, the music gradually loses momentum, while the opposite happens with the second section, which gains momentum with its spiral expansion and multiplication of rhythmic and polyphonic activities. Other elements in the pitch organization and morphology concur to create a sense of balance suggested by the 1:1 ratio, even with the opposite momentum qualities of each section. In fact, the whole conception of sacrifice as a creative principle consists in the interdependent or complementary balance of the two aspects of a same coin, just exactly in the same way that breathing alternates inspiration and expiration or objective time alternates days and nights.

A more familiar example of 1:1 macroformal ratio is found in Johan Sebastian Bach's *Goldberg Variations*. The Aria and practically all of the thirty Variations are made of two sections of exactly the same duration, each with sixteen measures [23]. Bach respects the 1:1 ratio and symmetry by creating music in the first part that is balanced by the music of the second part, by several means such as morphological *quadratura*, motivic consistency and tonal route (the route from one tonal region to another).

An elementary aspect of music composition is that whatever the composer intended to be sections of a piece should actually *result* in sections: this is produced by creating *contrast* between the content of the sections, using characteristic materials and processes for each one. A certain level of ambiguity may be desirable and there is no need to be didactic, as the listener is capable of perceiving such contrasts, just as long as the sections are *notional compositional structures*, as mentioned before.

[22] *En-temps* and *hors-temps* are terms created by Iannis Xenakis to differentiate music as a phenomenon in time from music as a set of formative principles ([45, p. 68]).

[23] Variations 3, 9, 21 and 30 have two eight-measure sections, a different 1:1 ratio. Variation 16, an Overture, contrasts with all other Variations with its 2:1 ratio: the first section, in 2/2 has 16 measures, while the second section, in 3/8 has 32 measures. Because the eight note is performed as a triplet and not as a regular eighth note, each 3/8 measure of the second section corresponds to one quarter note in the 2/2 of the first. It follows that four 3/8 measures (one hypermeasure) are equivalent to one 2/2 measure, resulting, therefore, in eight hypermeasures or in a 2:1 macroformal ratio for Variation 16.

The macroform of the first piece composed using expanded modal rhythm, *"... a natureza ama esconder-se..."* (1989), was defined, as I was mainly concerned with Greek feet, as *kretykos* (– ∪ –) at the ratio 2:1. Therefore, the piece has three sections, starting with a large one, followed by a short and ending again with a long. Long sections are twice as long as the short section. The rhythmic lines are constructed using a sequence of Greek feet, which, because they are used reiteratively, is called a *time cycle*. In the second piece with expanded modal rhythm, *Mojave* (1989), for piano and two percussionists, I started to use the time cycle as a time unit for measuring the macroform. Used in this manner, the time cycle was called a "theoretical cycle" because it is not audible as a rhythm in the music, but serves the main purpose of measuring the macroform (it is not the case to dwell on the other purposes at this moment). *Mojave* is clearly articulated in two sections, the first kinetic, the second static, and the "theoretical cycle" corresponds to the duration of 21 eighth notes [24]. The ratio is 33:15, meaning that there are 33 cycles in the first, longer section, and 15 in the second (corresponding to a ratio of 2,2:1). However, because of the tempo change between sections (the first is at quarter note MM = 72 and the second at MM = 54), the actual proportion between sections is 1,65:1, which would correspond to 24,75:15 in terms of "theoretical cycles". One of the basic principles is to only use cycles in their entire length, so this shows that the cycle as a time unit is subordinate to tempo. Later on, I decided to call "theoretical cycles" by the Sanskrit name tāla, which comes from classical Indian music; the Indian *tāla* has similarities with the *tāla* in my music. For the purpose of this article, it is enough to understand the *tāla* as a time unit measuring the macroform and the ratios of its sections.

In the following section of this article I will describe in greater detail the ratios at work in a few key compositions: *Pythagoras* (2001), *Metagon* (2008), *Phoînix* (2010), *Triskelion* (2015), and *Santuário de Baleias (Whale Sanctuary)* (2016).

Pythagoras

Pythagoras, for tenor recorder solo, is iconic with Pythagorean cosmology, for it applies to music composition the original Pythagorean idea of cosmos, in the specific meaning by which "the world is order when it is harmony and number, which keeps the Whole unified and such as it is" ([39, p. 62, v. IX]) (my translation). I have mentioned before that number is such a fundamental element in Pythagorean cosmology. In fact, in Pythagoreanism, number was the principle of all things; "All is number", would have said Pythagoras. Number "is the principle, the source, and the root of all things", writes Theon of Smyrna ([17, p. 21]).

> "... the so-called Pythagoreans, the first to be absorbed in mathematics, not only advanced this particular science, but, having been brought up on it, they believed that its principles are the principles of all things. Now, of these principles, numbers are naturally the first. As a result, they seemed to see in numbers, rather than in fire, earth and water, many similarities to things as they are and as they come to be: for one sort of modification of numbers, so to speak, is justice; another, soul and mind; still another, opportunity; and so forth. Musical modes and relations, too, they saw in terms of numbers. And all other matters appeared to be ultimately of the nature of numbers; and numbers were for them the primary natures. In view of all this, they took the elements of numbers to be the elements of all things, and the whole heaven to be harmony and number. They were adept at finding numbers and harmonies, both in patterns of change and in the structure of parts. And they organized and unified

[24] This is actually one of the possible forms of the rhythmic cycle used for the construction of rhythmic lines, and results from the 4:2 ratio combined with the sixteenth note as *chronos protos*.

> the whole arrangement of the heavens to exhibit its harmony. And if they discovered defects anywhere, they invented the necessary additions in order to make their whole system hang together perfectly." ([2, p. 15, Book A, 5, 985b])

This rather long citation from Aristotle's *Metaphysics* was included here because it is not only a description of Pythagorean ideas about number, but also a criticism of them. Aristotle's rationalism is in great contrast with Pythagorean's thought: it is with Aristotle that number starts acquiring the sense it has for us now, i.e., that of mere quantification, an operation of the mind, an abstraction. However, for the Pythagoreans, number is "a real thing, actually, the most real of all things, which, as such, can be the constitutive principle of all other things" ([39, p. 80, v. 1]) (my translation). It is important to read, instead of Aristotle's, the surviving texts from actual Pythagorean philosophers such as Archytas (first half of 4th century B.C.E.), Iamblichus (c. 250 – c. 325 C.E.), or other anonymous ones preserved in the writings of other authors such as Photius (c. 820-891 C.E.), so that one may grasp the Pythagorean point of view and the right frame of mind to understand them. In fact, Aristotle's way of looking at the "so-called" Pythagoreans leads, in the best cases, to appreciating them as having been "probably the first to recognize the abstract concept that the basic *forces* in the universe may be expressed through the language of mathematics" ([31, p. 31]) and nothing more.

The macroform of *Pythagoras* represents the Pythagorean *tetraktys* (the sequence of the first whole numbers, adding to the perfect number 10, the decad) and is divided in four spiral movements: *Monas*, *Katharsis*, *Theoria* and *Theosis*. The connection established between each movement and the *tetraktys* is that these movements are in the ratio 1:2:3:4. The sum total of these durations is 1 + 2 + 3 +4 = 10, as in the "*tetraktys* of the decad". According to the Pythagorean oath, the *tetraktys* expresses

> "the Pythagorean conception of the process by which the One goes out into the manifold world. The *tetractys* is not only a symbol of static relations linking the various parts of the cosmos: it contains also the cosmogonical movement of life, evolving out of primal unity the harmonized structure of the whole. It is a fountain of ever-flowing life" ([12, p. 207]).

English classical scholar Francis M. Cornford (1874-1943) also explains that "Pythagoras regarded (the *tetraktys*) as 'the nature of number, because all men, whether Hellenes or not, count up to ten, and, when they reach it, revert again to unity'." This reversion symbolizes the life cycle, by which mortals are born, grow, reach maturity, grow old and pass away and are re-born: "Nature causes them to come to their goal in her region of darkness, and then back again out of the darkness they come round in mortal form, by alternation of birth and repayment of death" ([12, p. 208]).

Another important concept is that of the Monad, *monas*, which is both the cosmos as a whole and the principle that generates everything in the cosmos. In fact, the Monad, or Unity, the One, is not a number, but the principle behind Number: "numbers—especially the first ten—may be seen as manifestations of diversity in a unified continuum" ([17, p. 21]). As an example of such continuum, the One as the source of all numbers, Fideler suggests a circle in which to inscribe various polygons, or the vibrating monochord string as unity, which sounds, when plucked, the complete harmonic series.

The Monad is represented by the 10-note melody that starts the first movement (*Monas*) and is called "nucleus": the nucleus is the source of everything else in the piece. The creative power of the number One is reflected in the outward spiral motion of the first movement, which, although subtle, increases melodic manifestation. The other three movements, *Katharsis* (purification), *theoria* (contemplation of the first principles, i.e., numbers), and *theosis* (union with God) relate to the three paths of spiritual development in ancient Pythagoreanism and, since these are the individual's attempts of reconnection with the creative principle, are represented, in the music by spiral structures with inward direction. While *Monas* and *Theosis* use the same melodic nucleus, *Katharsis* and *Theoria* each use "their own" nucleus form, but these are derived forms from the original nucleus in *Monas*, as shown in the Fig. 3.

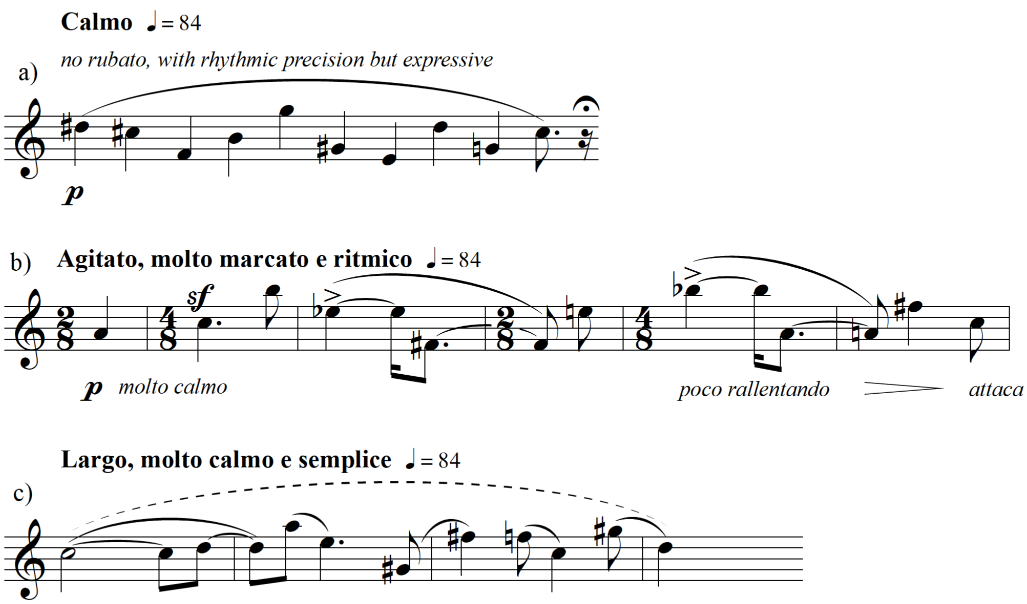

Figure 3: *Melodic nucleus in each movement of Pythagoras (2001): a) Monas; b) Katharsis; c) Theoria.*

Because the first movement is the Monad, it represents the number One. The second movement, *Katharsis*, as the number Two, is twice as long as *Monas*. It is ruled by the dualism of long and short notes, loud and soft intensities, and the sense of struggle or strife. The nucleus melody is placed at the end of each phrase cycle. As the cycles approach the end of the spiral, the nucleus appears gradually less ornamented, until, at the end, it is stated twice, reaching its pure, completely unornamented form at the very last time. The third movement, *Theoria*, is three times as long as *Monas*. As representing the number Three, it overcomes the struggling tone of dualism of the previous movement by means of its tranquil fluidity. Synthesis over long/short rhythmic dualism is achieved by the creation of a new level of temporal organization, namely, the three-beat meter which, in the a-metric context of the piece, is a new feature. A sense of temporality, of past, present and future is established by the melodic nucleus placement in the middle of each phrase cycle (and no longer in the end, as in *Katharsis*). Therefore, each cycle is formed by an antecedent, followed by the nucleus, and by a consequent. Antecedent and consequent are structurally related by complementary formative principles. As their durations shrink, all that remains, at the end of *Theoria*, is the nucleus, stated three times, and reaching its pure form at the very last time. The fourth movement, *Theosis*, is four times as long as *Monas*. The number Four, as well as the concept of *theosis*, represents completion and union with God. For this reason, it is the movement that most resembles *Monas*. Each phrase cycle is a rhythmically augmented and heavily ornamented version of the *Monas* nucleus. As the spiral contracts, each cycle becomes shorter and less ornamented, until at the end, the nucleus is stated four times, reaching, at the very last time, the final identification with the pure, original melodic nucleus, as a drop of water dissolves in the ocean.

In *Pythagoras*, the tāla works basically as a measure unit to control the ratios involved between movements and phrase cycles. One tāla has the length of the nucleus, i.e., ten quarter-notes. The first movement, Monas, is a microcosm of the four-movement macroform, as it has four phrase cycles growing in the same ratio 1:2:3:4 as the macroform's movements 1:2:3:4 ratio, and this is easily seen by the duration of notes in the melody: the nucleus is stated in Monas' phrase cycle A as 10 quarter notes. Cycle B is formed by ten half notes, with a slight ornamentation; cycle C further increases ornamentation and time values become dotted half notes; finally, cycle D, is formed by ten whole notes. In terms of tāla-s [25], this leads to the Table 1:

Table 1: *Duration of phrase cycles in* Monas

Monas	**phase cycle A**	**phase cycle B**	**phase cycle C**	**phase cycle D**
number of tāla-s and durations in ♩	1 = 10♩	2 = 20♩	3 = 30♩	4 = 40♩

Consequently, the entire duration of the *Monas* spiral is equal to 100 quarter notes on the metronome mark of MM = 96. Here, the phrase cycles are not in accordance with Fibonacci numbers, but only with the *tetraktys*.

The length of each phrase cycle in the other movements is ruled by the golden ratio phi, ϕ, as expressed by Fibonacci numbers. The Fibonacci sequence, which is fixed in the numbers 1, 2, 3, 5, 8, 13, 21, 34, 55, etc, and the 1:2:3:4 ratio between movements are not commensurate elements. There is a different solution for each movement to the problem of fitting together the Fibonacci sequence and the 1:2:3:4 ratio between movements.

The duration of the *Katharsis* spiral is equal to 200 quarter notes (twice as long as *Monas*). It is easier to see the relation with Fibonacci numbers by looking at the total as equal to 400 eighth notes. *Katharsis* contracts in five phrase cycles according to the first five Fibonacci numbers in retrograde order, because the cycles are decreasing in length: 8, 5, 3, 2 and 1. Multiplication of these numbers by 20 leads to the number of eighth notes in ϕ ratio: 160, 100, 60, 40 and 20. Table 2 shows the actual durations of the phrase cycles in the Katharsis spiral to be very close to these numbers in ϕ ratio:

What Table 2 does not show is that there are two consecutive statements of the 20 eighth-note-long nucleus, the first as the last 20 eight-notes of phrase cycle D and the second (and final) as phrase cycle E. In *Katharsis*, each phrase cycle is formed by a melodic line followed by the nucleus ending the cycle. The melodic line is progressively shortened, but the nucleus remains intact in length, decreasing only in ornamentation. When cycle E arrives, there is nothing left of the melodic line, and this results in the two consecutive statements of the nucleus. In fact, the sum total of all phrase cycles (A+B+C+D+E, i.e., 170 + 104 + 64 + 42 + 20) is equal to 400 eight notes, as it should be, and the two consecutive statements of the nucleus conclude the movement as reminding the listener that *Katharsis* is about the number Two.

Table 3 shows the actual durations of the phrase cycles in the *Theoria* spiral. Initially, in order to determine the duration of each phrase cycle, the Fibonacci numbers were multiplied by 30

[25] I use the plural form tāla-s by adding a hifen and an "s" because the correct plural of this Sanskrit word is not "tālas".

Table 2: *Duration of phrase cycles in Katharsis*

Katharsis	**phase cycle A**	**phase cycle B**	**phase cycle C**	**phase cycle D**	**phase cycle E**
Actual durations in ♪	170♪	104♪	64♪	42♪	20♪
Fibonacci numbers *x* 20	160	100	60	40	20
Actual number of *tāla-s*	8.5	5.2	3.2	2.1	1
Fibonacci numbers	8	5	3	2	1

(resulting in 8x30=240, 5x30=150, 3x30=90, 2x30=60 and 1x30=30). However, this series of numbers is, again, incommensurable with the prescribed total duration of 300 quarter notes (thrice *Monas*), or 600 eight notes for *Theoria*'s macroform. The actual durations were arrived at so as to result in a sum total of 540 eighth notes, since phrase cycle F would, by necessity, include three consecutive statements of the 20 eighth note-long nucleus, which is equal to 20x3=60 eighth notes. These three consecutive statements reassure to the listener, the number Three. 540 + 60 = 600 eighth notes. Therefore, the value of 226 eighth notes of phrase cycle multiplied by ϕ = 0.618 results in 139.668, which is rounded up to 140, the duration of phrase cycle B. By the same procedure, multiplying 140 by 0.618 result in 86.52 (rounded up to 87 for phrase cycle C), and so on until 33 is obtained for cycle E. Notice that the value of 20 eighth notes of the nucleus could never be reached by this treatment of the Fibonacci numbers.

Table 3: *Duration of phrase cycles in* Theoria

Theoria	**phase cycle A**	**phase cycle B**	**phase cycle C**	**phase cycle D**	**phase cycle E**	**phase cycle F**
Actual durations in ♪	226♪	140♪	87♪	54♪	33♪	20♪
Actual number of *tāla-s*	11.3	7	4.35	2.7	1.1	–

Therefore, the actual durations of *Theoria*'s phrase cycles are not Fibonacci numbers, but they stand in a ratio closer to the ϕ ratio than Fibonacci numbers do. The actual number of *tāla-s* also reflects the ϕ ratio: 11.3 x 0.618 = 6.9834, which rounds up to 7.0, and so on and so forth for the other *tāla* values.

The *Theosis* spiral was composed according to a particular idea: the ten-note *Monas* nucleus is the material for each phrase cycle. Heavily augmented and ornamented at the first cycle, it is gradually shortened until it reaches its original form in the last cycle, when it is, then, stated four times consecutively, to provide the final sense of conclusion of the work. Table 4 shows the resulting durations for each phrase cycle obtained from the augmentation principle by which, in each cycle, each note of the nucleus is augmented from one quarter note to a Fibonacci number, starting with 13 for cycle A, 8 for cycle 8, 5 for C etc.

Table 4: *Duration of phrase cycles in* Theosis

Theosis	**phase cycle A**	**phase cycle B**	**phase cycle C**	**phase cycle D**	**phase cycle E**	**phase cycle F**
Actual durations in ♪	130 ♪	80 ♪	50 ♪	30 ♪	20 ♪	10 ♪
Actual number of *tāla-s*	13	8	5	3	2	1

By this principle, the sum total of phrase cycles A, B, C, D and E is equal to 310 quarter notes. The four consecutive statements of the nucleus in cycle F is equal to 40 quarter notes. The *Theosis* spiral, therefore, has a total of 350 quarter notes, and not 400, as the 1:2:3:4 ratio would require. However, the ratio is still at work by a change in tempo: the first three movements were all at metronome mark MM = 96. The *Theosis* spiral, with its 350 quarter notes at MM = 84, has the same duration as 400 quarter notes in MM = 96, i.e., 250 seconds (4’10”), four times the 62.5 seconds (1’2.5”) of *Monas*.

Metagon

Written for the Zen Buddhist bamboo flute, the *shakuhachi*, *Metagon* (2008) is a solo composition of approximately fifteen minutes of duration [26]. A metagon (Fig. 1) is a spiral line formed by open polygons. It starts with a triangle, which is circumscribed by a square, then a pentagon, an hexagon, and so forth, possibly indefinitely. Each polygon is “open” in the sense that its last side does not close the figure, making it possible for the outer figures to circumscribe the inner ones with no lines touching each other.

Metagon’s macroform determines a single finite segment of a possibly infinite melodic, rhythmic and textural tendency which is the formative principle represented by the metagon, the “metagon principle”, the process by which each polygon has one side added in relationship to the one immediately circumscribed by itself. *Metagon*’s macroform entirely respects this principle as applied to temporal organization: the piece is a monody line segmented in phrase cycles (each is a turn in the spiral line) which gradually expand in duration: they become longer in the same ratio as the principle suggested by the metagon’s figure: initially, a polygon with three sides, then with four, then five, etc. This relationship occurs here with the number of “measures” which, deprived of any metrical meaning, simply serve as a unit for the measurement of the phrase cycles: the first cycle lasts two measures, the second three, then four, five, until the last, with twenty seven. *Metagon*’s macroformal spiral has a total of twenty six phrase cycles.

The spiral tendency suggested by the metagon is also applied to pitch content, i.e., space. The monody starts with a single pitch, and gradually includes the greatest multiplicity of pitches available in the instrument. Tonal centricity (the center is C, third space in treble clef) does not

[26] More about Metagon (including the full score) is found in “Tempo Espiral em Metagon” (IRLANDINI, 2013).

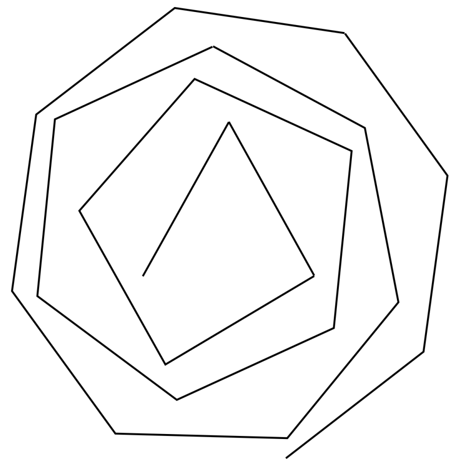

Figure 4: *Metagon.*

imply tonalism, but simply a central point in a gradually expanding textural pitch space. Pitches are included accumulatively one by one in each phrase cycle, by adding a new pitch a half-step above or below the last new note. Explained differently, in relationship to the central C, pitches are added first by the upper semitone (C♯), then the lower semitone (B natural), next by the upper interval 2 (the whole tone, D), then the lower (B♭), next by upper interval 3, and so on and so forth, gradually expanding the melodic *tessitura* and the textural space.

Phoînix

I wrote the three pieces of the series *Bestiarium, v. I* between the years 2008 and 2013. *Phoînix* was the second piece to be completed, although it is number one in the series. Each of the compositions in the *Bestiarium* explores musical analogies with mythological and symbolic aspects of animals. The following discussion will concentrate on the role of the ϕ ratio in the macroformal plan of the composition, which is not in spiral form.

Figure 2 shows that *Phoînix* is divided in three large sections preceded by an eight measure-long Introduction which is not part of the ϕ calculations. Section I, with 183 quarter notes of duration, is in a ϕ ratio with sections II + III, both summing up a duration of 297 quarter notes. 183:297 = 0.61616162, which is very close to ϕ = 0,618. The ratio between sections III and II is 114:183 = 0.62295082, also approximately close ϕ.

Furthermore, the inner subdivisions of sections are also in an approximate ϕ ratio: in section I, (I1 + I2):I3 = 113:70 = 1.61428571, and I1:I2 = 70:43 = 1.62790698. The same occurs with the inner subdivisions of sections II and III.

The values involved here (297, 183, 114 or 113, 70 and 43 or 44) are approximately in the ϕ ratio in the same way as the Fibonacci numbers express the ϕ ratio approximately. In fact, the point of departure was the creation of a number sequence similar to Fibonacci's. The sequence is based in the same principle of adding the two previous numbers to form the next, starting arbitrarily with the values 1 and 3:

$$[1 - 3 - 4 - 7 - 9 - 16 - 27] - \mathbf{43} - \mathbf{70} - \mathbf{113} - \mathbf{183} - \mathbf{296} - \mathbf{479}$$

Only the terms in bold font are used in the composition because those are the ones that express the ϕ ratio more closely. In fact, the ratio between 3 and 4 is 0.75, between 4 and 7 is 0.5714

Table 5: Phoînix *macroform*

Introduction **(34 ♩)**	**I (183 ♩)**			**II (183 ♩)**			**III (13 ♩)**	
	I_1	I_2	I_3	II_1	II_2	II_3	III_1	III_2
	70	43	70	70	43	70	70	43
c. 1-8	c. 9-42	c. 43-62	c. 63-99	c. 100-134	c. 135-154	c. 155-190	c. 191-212	c. 213-227
A	B	D	E	G	I	K	N	P
Pesante	Tranquilo		Tranquilissimo	Energico e preciso			Pesante e ossessivo	
			ϕ_1			ϕ_2		ϕ_3
				ϕ				
	183			**296**				

(...), and between 43 and 27 is 0.62790698. Only from terms 43 and 70 is that the ratio starts to be acceptably approximate to the ϕ ratio (43:70 = 0.61428571), and, from then on, they become progressively closer to ϕ = 0.618. The same occurs with the Fibonacci sequence, where only with terms 8 and 13 does the ratio become very close to ϕ = 0.618 (8:13 = 0.61538462).

Notice that the last measure in the music, a 4/4 measure with two sounds in the percussion (lion's roar followed by the tam-tam), does not count for the purpose of ratios between sections. The two sounds are meant to be as a final period mark. If computed, that measure would add four extra quarter notes to the 43 that make section III_2.

Triskelion

The archetypical sense of wholeness brought about by tri-partition is shown by the fact that so much music in three parts or for three musicians has been composed throughout the centuries. "The Triad is the form of the completion of all things" (Nichomachus of Gerasa, c. 100 a.D.), and this is confirmed by the three-movement sonata form, the Hindu *trimurti*, or the unity of the circle, which depends on its center, radius and circumference. The *triskelion* (τρισκέλιον) is one more tri-unity (trinity), each of its parts being a spiral.

Triskelion, for piano, viola and percussion (bass marimba, large tam-tam and *djembé*) explores relationships between three elements: three movements, three musicians, three simultaneous musical layers, three percussion instruments. More specifically related to the *triskelion* shape is the fact that each movement evolves as a spiral, first expanding from silence, then, reversing direction, contracting into silence. Each spiral/movement has three expanding and three contracting cycles, at the fixed ratio of 1:2:3 for the expansion and 3:2:1 for the contraction, thus preserving the rotational symmetry of the drawn triple spiral line in the formal relationship of the three musical movements. As a consequence, all movements have the same duration, forming the proportion 1:1:1. Furthermore, each instrument predominates in the texture of each movement: the piano in the first, the viola in the second, and the *djembé* in the third.

Figure 5: *Triple spiral on the entrance stone of the Neolithic mound at Newgrange, Ireland (photo: Luigi Irlandini).*

There is no fixed symbolic association of each movement or musician to any one of the three meanings that have been associated to the *triskelion*, or to any specific interpretation of it. I was directly inspired by the pre-Celtic triple spirals carved on the entrance stone and inside the inner chamber of the Neolithic mound at Brú na Bóinne (Newgrange, Ireland) (see the three spirals at the tip of the stone inthe left corner of figure 4, above). The original meaning of these triple spirals is unknown but, to me, from their placement on a tomb monument constructed according to the movements of the sun, it is cosmic and relates to the human being's journey into and out of existence. It is suggestive of life in three stages (youth, maturity and old age), a tripartite cosmology (Earth, Sea and Sky), or even as the tri-unity of Man, Earth and Sky represented by the Shintoist *triskelion*-shaped *mitsudomoe* often painted on *taiko* drum from Japan. Therefore, the 1:1:1 ratio in the music was suggested by the fact that the three spirals in the Newgrange monument are approximately of the same size, and each has three cycles or layers of curves.

Santuário de Baleias (Whale Sanctuary)

Composed for soprano saxophone and strings orchestra during the months of September and October 2016, *Whale Sanctuary (Santuário de Baleias)* originates from imagining and wishing that all oceans would be kept in their pristine condition, free from industrial hunting and waste; an arguably naïve, but legitimate wish. If the oceans were kept in this way, they would become a sanctuary, a protected environment, therefore a sacred space, for the existence of not only whales but the entire maritime fauna and flora... The idea of composing a "sanctuary" implied, again, a visualization of music as space, and a treatment of formal proportions and symmetries such as those found in temple construction and the architecture of sacred spaces.

In order to establish a concrete analogy between the oceans and the musical space-time continuum, I determined the durations of the five musical sections to be proportional to the real surface areas of the five oceans. After finding such information [27], I attributed the value

[27] I used Google and found the following oceanic surfaces: Pacific: 165,250,000 km^2; Atlantic: 106,400,000 km^2; Indic: 73,560,000 km^2; Antartic: 20,330,000 km^2; Artic: 14,060,000 km^2. These numbers may vary depending on the source. Although I could not find again my initial source, the oceanic surfaces indicated at http://www.whatarethe7continents.com/the-worlds-five-great-oceans/ are very similar to those above: Pacific: 165,200,000 km^2; Atlantic: 106,400,000 km^2; Indic: 73,556,000 km^2; Antartic: 20,327,000 km^2; Artic: 13,986,000 km^2.

1 to the smallest ocean (the Artic) and, after calculating the other values proportionally for the other oceans, arrived at the values 1.0; 1.4; 5.2; 7.6 and 11.7. Out of the need to simplify them, I rounded them up or down to more manageable numbers, first obtaining the values 1.0; 1.5; 5.0; 8.0 and 12.0. The compositional process involved many changes in the use of these values because several aspects of the composition had to be served by these numbers. In the first place, the macroform divided in five sections according to the syllabic meter ∪ – – – ∪ has each syllable representing the oceans in the increasing order of their surfaces: Artic, Indic, Atlantic, Pacific and Antartic, therefore, 1; 5; 8; 12; 1.5. This needed also to be coherent with the macroformal time unit, the *tāla*, for which I finally found the value of 10 quarter-notes. In addition to this, each of the five sections is divided in the same macroformal syllabic meter, which becomes a middle-form structure, i.e., the subdivision of each macroformal syllable. Each middle-form syllable contains a spiral expansion of its musical materials, and the growth rate is the same as that of the oceanic surfaces. At some point, it was also inevitable to notice the similarity of the numbers 1; 1.5; 5; 8; 12 with Fibonacci numbers, 1; 2 (or 3), 5, 8 and 13. For reasons that have to do with satisfying all formal needs into a one coherent system, I rounded the numbers to meet the Fibonacci series, therefore arriving finally to the macroformal durations 1; 5; 8; 13; 3 (not 2, which would have been the closest approximation to 1.5, if this had been the only matter in question). The growth rate of each middle-form syllable along the five sections, expressed in terms of how many quarter-notes they contain is 30, 150, 240, 390 and 90. This, in terms of the number of *tāla-s* that constitute each section is 3, 5, 8, 13 and 3.

V. Conclusion

The importance of number and proportion for music composition in the 21st century is probably a matter of personal choice. Different composers in different periods have not only rejected or embraced the idea (and practice) of allowing number to determine some aspect of their music but also have conceived differently *how* number determines music; and the reasons for this are of a poetic nature.: poetic, because it is directly related to the composer's "program of art". Musical works may be, even if only partially, at specific aspects of its construction, determined by numerical relationships. In fact, hadn't aspects of a work been conceived firstly as number relations, the resulting music would have changed considerably: music would indeed sound differently. While putting "number ahead of sound" may feel to some composers as something unmusical, something alien to the nature of musical creation and inspiration, for others, music and number have always had an intimate relationship which can still be explored and practiced today. In my case, the use of number does not harm my compositional freedom, and there remain plenty of opportunities for spontaneity within the given musical materials.

The presence of number by itself does not guarantee the resulting work to be good music. In fact, depending on how it is used, there is a good chance number can cause a certain stiffness in the resulting music, as well as a rigidity in the compositional process and, consequently, in the compositional mind. Composers who assign a creative role to number in their music are, in the best cases, fully aware of this danger. After all, music is not number; number and music are different things, and a proof of this is that Music in the Quadrivium was not concerned with musical composition. However, were there no good reasons at all for assigning a defining, creative and active role to number in composition, this practice would not have been so present throughout western music's history, not to speak of the role of number in music theory, which has been important since Antiquity. Again, that there may be actually "good" reasons for number to come before music in its process of creation is an appraisal for which there is no consensus; it is a matter of personal choice; it depends on one's poetics, the way one composes, and it depends on the numerical relationships themselves, since not all of them are "musical".

For this reason, I have not brought to this article a historical review of musical works in which number takes a defining role in music. The discussion was mainly concerned with my own poetics, and with the exposition of the particular motivations for such a relationship between music and number.

Concerning the cosmicization of sound, there still remains one observation. Until now I have only spoken about the composer in the process of artistic/musical creation, and it would not be appropriate to exclude the performer, the musician who actually brings the composition to physically vibrate in the air. Performance corresponds to the final stage of cosmic creation re-enactment; it gives music a physical reality. The French word used for the moment in which the musician plays the music composed by someone else, *réalisation*, means production, actualization, and expresses the idea that a composition becomes real only when it is played, performed, i.e., when it is created in a physical, sounding way. The results of those formative principles acquire a brief, however concrete existence; it is only when it is performed that the music becomes an acoustic reality, a concrete universe of sounds. Until this moment, the composition has remained *in potentia*, locked inside the musical score, a text, a musical reality in fossilized form, or an encrypted message a-waiting to be deciphered. This work of deciphering is done by the performer, during the stage of their work called *interpretation*.

According to the traditional conception in western music culture, in this model [28] of musical creation, composer and performer stand in different hierarchical levels within the creation process, the composer being on a higher level than the performer. This seems to be a logical consequence of the actions taken by each artist: the composer writes the text, the performer reads it. It is possible to make an analogy between the roles of composer and performer with the role of divine creators in creation myths from the world. Ethnomusicologist Marius Schneider (1903-1982) made an extensive comparative study of several sonic cosmologies, creation myths according to which the world is created from sound, and their relationship with music, and was able to

> "(...) distinguish an Omnipotent God from another god who is assigned to create the world. The Omnipotent God never gets involved in the action: he only has the idea of creation and limits himself to "utter" with an almost imperceptible voice, the name of a lesser god, whom he assigns the execution of his own idea. (...) in America, the god of thunder, the "big screamer", executes the work of creation commanded by the great Manitu. However, this lesser god, who is more properly the creator, still is too high to occupy himself with the creation of a material world. In order to finish his work he delegates a demiurge (*coyote* or *transformer* according to the English and American ethnologists) and assigns him the partial materialization of the acoustic world. This assistant, who sometimes is some sort of lunatic, not always is a faithful server." ([41, p. 23-24]) (Translated by the author).
>
> In this context, which finds corporeity and materiality as the lowest level of creation, even when manifestation should be seen as the crowning of Creation, the composer would be the lesser god, being gifted with the power of imagination and craftsmanship to create sounding microcosmos (in the plural), and the performer would be the demiurge or *trickster*, who deals directly with matter and creates music by playing an instrument or singing. This analogy does not intend to deify the composer neither the performer, neither makes an apology of their egos. In fact, painters, composers, architects, doctors usually suffer of the God complex without the need of any theoretical support. Neither does this analogy intend to offend the performer by calling them "a

[28] There are other models, such as the composer/performer, or the improviser.

lunatic". I bring this analogy only because it lives in western music culture, in the way artists are frequently compared to gods or geniuses, and because it finds an echo in the divine figures of worldwide creation myths. If the humanly created universe "shares in the sanctity of the god's work", according to Mircea Eliade, every creative act is sacred, divine, and it seems logical to connect the facts in this way: that a composer receives what is uttered by the Omnipotent God, and that the performer brings to realization what the composer was capable to hear from what had been uttered.

From all this, what is really meaningful and important is to recognize that artistic creation happens through the artist, composer or performer, who is a channel capable of shaping that energy that wants to be formed, the *forma formante*, nature. For Jean Cocteau (1889-1963), "a poet is in a way the work-hand whose act engages a self more profound than himself which he doesn't know too well, mysterious forces which inhabit him and which he knows poorly" ([9]). The artist is just a physical agent helping the process to take place, or, to use Anton Webern's words (themselves quoting Goethe) from a 1933 lecture, "man is only the vessel into which is poured what 'nature in general' wants to express" ([44, p. 15]).

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A Study of Variation in Temporal Structure of Sonata Form

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Abstract: *In this paper it is shown that the concept of the Eighteenth-Century Sonata form, under certain conditions, implies exact constraints to its temporal structure, which are essential to keep its inner proportions balanced. The plastic number of Hans van der Laan appears to be closely related to the concordance of lengths of the vital parts of a sonata-form movement of type 3 on Hepokoski and Darcy scale. Furthermore, a probabilistic model of basic variation in the structure of such movement is developed from scratch and empirically justified by analyzing instrumental works of Wolfgang Amadeus Mozart.*

Keywords: *Sonata form. Perception. Plastic number. Morphic number. Lognormal distribution*

I. Introduction

Sonata-form is a simple, yet very potent concept. Besides hundreds of musical pieces written in the last couple of centuries, there also exist dozens of serious books and papers about the subject. We show that it can still be a subject for original research.

In this paper we try to find an exact formulation of certain restrictions[1] inherent to the temporal structure of the Eighteenth-Century Sonata form. More precisely, we ask ourselves what restrictions are necessarily imposed to the ratios between lengths of distinct parts of a sonata-form movement, which we call *inner proportions*. Knowing these restrictions, one would be able to answer questions of type "how long should this part be when compared to the one that follows it?", for example. It would also be possible to explain what exactly means that inner proportions of a sonata-form movement are balanced.

To find the restrictions mentioned above we need to apply a mathematical treatment, presented in Section II, which is similar to the methodology used by Hans van der Laan to develop his unique theory of architectonic space.

Section III is devoted to Sonata form. It begins with a brief introduction to its structure, followed by formulating a set of necessary restrictions applying to it, using the concept of ground ratio formulated in Section II. As we show next, these restrictions imply that the plastic number of Hans van der Laan is tightly related to the temporal organization of a sonata-form movement. These theoretical findings, formulated as a probabilistic model, are justified by performing a simple empirical study based on the set of sonata-form movements from the instrumental opus of W. A. Mozart. Finally, an example of using the plastic number in structural analysis of a sonata-form movement is given.

[1] For an example of such restriction see [11, p. 280].

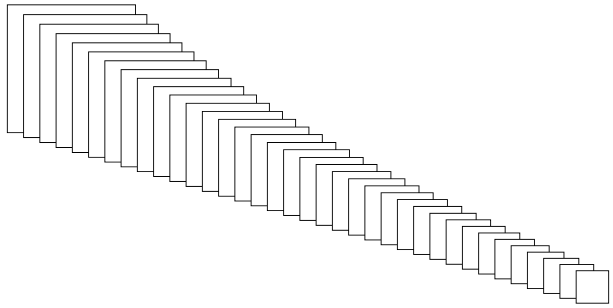

Figure 1: *36 squares forming a geometric sequence*

II. Perception of Spatial Size

Size is one of the fundamental properties of spatial objects. Human sensory system identifies size of an object with one of the available one-dimensional quantities related to that object, namely width, length and breadth. As those are easily compared with one another, the sensory system instinctively assigns the largest of these quantities to represent its size.

Therefore, we may define size of an object to be equal to the length of the longest edge of the smallest cuboid containing it. Size $s > 0$ should be visualized as a thin bar (stick) of length s. Hence we will use the term *length* when referring to size in the rest of this text.

In this section we study how the sensory system interprets relations between different sizes. The goal is to develop a mathematical treatment which is necessary for making assumptions and drawing conclusions later in this text.

i. Automatic Classification of Lengths

When two lengths are exposed simultaneously, the sensory system in our brain automatically attempts to relate them and to generate a valuable information for the conscious sphere of mind. Basically, it decides whether one of the lengths is significantly longer than the other. If that is not the case, they are considered “equal” in a sense of both being (possibly noticeably different) elements of the same class (level) of size. Such a class is called *type of size* [16, p. 55].

For any length ℓ_0 the interval L, consisting of all lengths $\ell \geq \ell_0$ which are "equal" to ℓ_0, represents one type of size. The length $\ell_1 = \sup L$ is the smallest length clearly different than ℓ_0. Dom Hans van der Laan (1904–1991), a Dutch architect, devised a simple and easily reproducible experiment [16, p. 49] to illustrate the concept of type of size. He prepared 36 cardboard squares of different sizes such that the sides of every two consecutive squares differ in length by 4% and thus forming a geometric sequence[2] (see Figure 1). Van der Laan would randomly scatter the squares on the table and ask someone to take out the group of the largest ones. He claimed that it would contain exactly seven squares every time. This action could be repeated until only the smallest square remains, thus dividing the squares into five consecutive groups, each containing seven members and representing one type of size. The smallest square represents an unit; its side is, by design, equal to the difference between sides of the largest members of the first two groups (see Figure 2). The largest members of five groups together with the unit represent six consecutive members of a geometric sequence with quotient $1.04^7 \approx 1.316$.

[2]The relative difference of 4% was obtained as the result of an auxiliary experiment in which a 50 cm long strip of paper needed to be cut in two halves. The experiment was conceived to measure the precision of eye judgement.

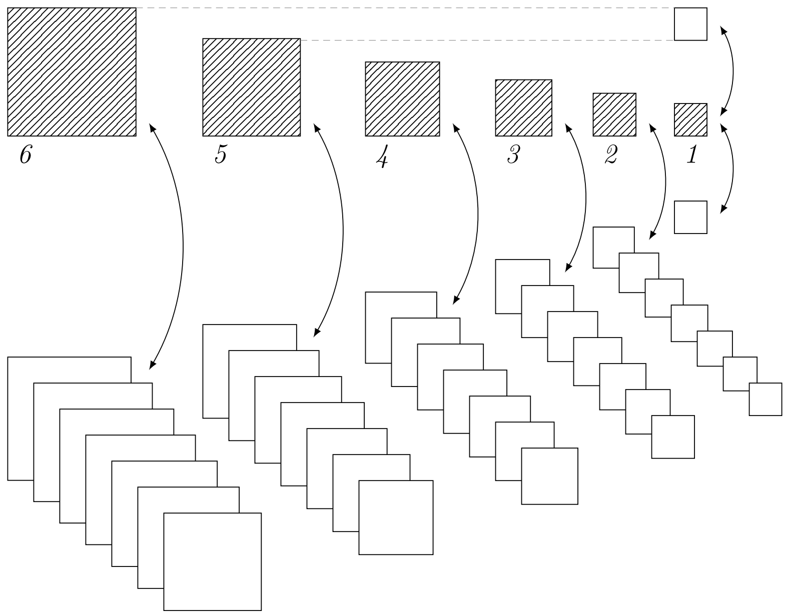

Figure 2: *36 squares sorted by size in six groups*

ii. Ground Ratio

Relating lengths to each other is the only way for the sensory system to "measure" reality. Namely, it does not possess an intrinsic reference value, i.e. an unit, so it cannot do any measuring on its own[3]. However, when two lengths are perceived, the difference between them becomes the source of meaningful information. This is a consequence of Weber–Fechner law [2, p. 83] which states that the sensation s corresponding to a physical stimulus of intensity I is proportional to the logarithm of I [3, p. 90]:

$$s = k \ln I.$$

Here $k > 0$ is a constant, called the Weber fraction [2, p. 83], which is a property of the type of stimulus. Now, the difference d between two sensations s_1 and s_2 of stimuli with intensities I_1 and I_2 is dimensionless, since the unit in the stimulus intensity domain gets cancelled:

$$\delta = |s_2 - s_1| = |k \ln I_2 - k \ln I_1| = k \left| \ln \frac{I_2}{I_1} \right|. \tag{1}$$

In particular, given a positive number r, the relation between any two lengths with ratio equal to r will always appear to be the same since the perceived difference $\delta = k \,|\ln r|$ is the same in each case. Furthermore, there exists a threshold $\delta_0 > 0$ such that two lengths belong to the same type of size if and only if their perceived difference does not exceed δ_0, i.e. the ratio of the longer length to the shorter does not exceed

$$\lambda = \mathrm{e}^{\delta_0 / k},$$

as follows from (1). The constant λ will be called the *ground ratio* in the rest of this text. Clearly, $\lambda > 1$ because δ_0 and k are both positive.

[3]Of course, we are all able to remember some concrete quantities used in everyday life, but resorting to these notions of physical units represents an act of the conscious sphere of mind, the influence of which we tend to ignore in this text.

Definition II.1. Lengths ℓ_1 and ℓ_2 such that $\ell_1 \leq \ell_2$ are *effectively equal* if $\ell_2/\ell_1 \leq \lambda$. When $\ell_2/\ell_1 \geq \lambda$ holds, ℓ_2 is said to be *significantly longer* than ℓ_1.

Remark II.2. In the case when $\ell_2 = \lambda\, \ell_1$, length ℓ_2 is both significantly longer than and effectively equal to ℓ_1 in the sense of Definition II.1. It is the legitimate case characterized by a specific type of balance which emphasizes the clarity of relation between lengths ℓ_1 and ℓ_2.

Given a base length m_0, called the *unit*, the elements of geometric sequence

$$(m_n)_{n\geq 0}, \quad \text{where } m_n = m_0\, \lambda^n \tag{2}$$

are called *measurements*. Every two consecutive measurements delimit a type of size.

iii. Margin

For any fixed length ℓ there is an unique type of size $[\ell_1, \ell_2]$ such that ℓ is perceived as its center. Therefore, ℓ should seem equally distant from ℓ_1 and ℓ_2. The distance between ℓ and ℓ_1 corresponds to the difference $\ell - \ell_1$; but, as the sensory system only interprets the relations between perceived quantities and not the quantities themselves, $\ell - \ell_1$ must be perceived relative to some available reference length. In present case ℓ is the only such length. Analogously, the distance between ℓ and ℓ_2, i.e. the difference $\ell_2 - \ell$, is perceived relative to ℓ, so we have

$$\frac{\ell - \ell_1}{\ell} = \frac{\ell_2 - \ell}{\ell} \implies 1 - \frac{\ell_1}{\ell} = \frac{\ell_2}{\ell} - 1 \implies 2 = \frac{\ell_1 + \ell_2}{\ell} \implies \ell = \frac{\ell_1 + \ell_2}{2}.$$

Hence ℓ is the arithmetic mean [6, p. 4.1] of ℓ_1 and ℓ_2. Now there exists $\Delta\ell > 0$ such that $\ell_1 = \ell - \Delta\ell$ and $\ell_2 = l + \Delta\ell$. Since ℓ_1 and ℓ_2 delimit a type of size, it follows

$$\frac{\ell + \Delta\ell}{\ell - \Delta\ell} = \lambda.$$

Rewriting the above equation, one obtains

$$\frac{\Delta\ell}{\ell} = \frac{\lambda - 1}{\lambda + 1}. \tag{3}$$

Displacement $\Delta\ell$, which obviously depends solely on ℓ, is called the *margin* of ℓ [16, p. 55]. All lengths obtained by changing the given length for values less than or equal to its margin belong to the same type of size and are *practically equal* to ℓ. In other words, the difference between such length and ℓ is *negligible* to ℓ.

The margin of ℓ represents the smallest length which can be related to ℓ. An interval $[\Delta\ell, \ell]$ represents one *order of size* [16, p. 55]. Furthermore, the interval $[m, M]$, where m is equal to the margin of ℓ and ℓ is equal to the margin of M, is called the *scope* of ℓ. It consists of two consecutive orders of size.

The following result characterizes effective equality in terms of negligibility.

Theorem II.3. *Two lengths are effectively equal if and only if their difference is negligible compared to their sum.*

Proof. Let ℓ_1 and ℓ_2 be two arbitrary lengths. As the statement is obviously valid for $\ell_1 = \ell_2$, we can assume that $\ell_2 > \ell_1$ without a loss of generality. First, we assume that $\ell_2 - \ell_1$ is negligible to $\ell_1 + \ell_2$, i.e. that the former does not exceed the margin of the latter:

$$\ell_2 - \ell_1 \leq \Delta(\ell_1 + \ell_2). \tag{4}$$

Dividing the inequality (4) by $\ell_1 + \ell_2$ and using (3), we obtain

$$\frac{\ell_2 - \ell_1}{\ell_1 + \ell_2} \leq \frac{\Delta(\ell_1 + \ell_2)}{\ell_1 + \ell_2} = \frac{\lambda - 1}{\lambda + 1},$$

which readily simplifies down to $\frac{\ell_2}{\ell_1} \leq \lambda$, meaning that ℓ_1 and ℓ_2 are effectively equal.

On the other hand, assuming that (4) is false and reasoning analogously, we conclude that $\frac{\ell_2}{\ell_1} > \lambda$, which means that ℓ_1 and ℓ_2 cannot belong to the same type of size. That completes the proof. □

iv. Derived Measurements

Using (3), for any two consecutive measurements ℓ_1 and $\ell_2 = \lambda\, \ell_1$ we obtain

$$\ell_2 - \Delta\ell_2 = \ell_2 \left(1 - \frac{\Delta\ell_2}{\ell_2}\right) = \lambda\, \ell_1 \left(1 - \frac{\lambda - 1}{\lambda + 1}\right) = \ell_1\, \frac{2\,\lambda}{\lambda + 1} =$$
$$= \ell_1\, \frac{(\lambda + 1) + (\lambda - 1)}{\lambda + 1} = \ell_1 \left(1 + \frac{\lambda - 1}{\lambda + 1}\right) = \ell_1 \left(1 + \frac{\Delta\ell_1}{\ell_1}\right) = \ell_1 + \Delta\ell_1.$$

Hence $\ell_2' = \ell_2 - \Delta\ell_2$ is the only length which is practically equal to both ℓ_1 and ℓ_2, so it may be readily identified with any of the two. It therefore represents the perceived point of balance between the given measurements. Moreover, ℓ_2' coincides with the harmonic mean [6, p. 4.18] of ℓ_1 and ℓ_2 since

$$\ell_2 - \Delta\ell_2 = \ell_2 \left(1 - \frac{\lambda - 1}{\lambda + 1}\right) = \frac{2\,\ell_2}{\lambda + 1} = \frac{2\,\ell_1\,\ell_2}{\ell_1\,(\lambda + 1)} = \frac{2\,\ell_1\,\ell_2}{\ell_1 + \lambda\,\ell_1} = \frac{2\,\ell_1\,\ell_2}{\ell_1 + \ell_2} = \frac{2}{\frac{1}{\ell_1} + \frac{1}{\ell_2}}.$$

Generally, for any two lengths ℓ_1 and ℓ_2 such that ℓ_2 is significantly longer than ℓ_1 there is a unique length ℓ which is perceived as the natural point of balance between ℓ_1 and ℓ_2, i.e. as being equally distant from both lengths. These distances are equal to differences $\ell - \ell_1$ and $\ell_2 - \ell$, which must be taken relative to ℓ_1 and ℓ_2, respectively, as these are the only available reference lengths. Therefore,

$$\frac{\ell - \ell_1}{\ell_1} = \frac{\ell_2 - \ell}{\ell_2} \implies \frac{\ell}{\ell_1} - 1 = 1 - \frac{\ell}{\ell_2} \implies \frac{\ell\,(\ell_1 + \ell_2)}{\ell_1\,\ell_2} = 2 \implies \ell = \frac{2\,\ell_1\,\ell_2}{\ell_1 + \ell_2} = \frac{2}{\frac{1}{\ell_1} + \frac{1}{\ell_2}},$$

i.e. ℓ coincides with the harmonic mean of ℓ_1 and ℓ_2.

The sequence of measurements (2) is naturally interpolated with another geometric sequence with ratio λ, denoted by $(m_n')_{n \geq 1}$, where $m_n' = m_n - \Delta m_n$ is called the *derived measurement* corresponding to m_n. The two sequences, taken together, embody all three Pythagorean means; given $n \geq 1$,

- m_n is the geometric mean [6, p. 4.15] of m_{n-1} and m_{n+1},
- m_{n+1}' is the geometric mean of m_n' and m_{n+2}',
- m_n is the arithmetic mean of m_n' and m_{n+1}',
- m_n' is the harmonic mean of m_{n-1} and m_n.

v. Plastic Number

As the ground ratio is a fixed numeric constant, we ask ourselves what its exact value is. The universality of the concept suggests that its origin is environmental. Indeed, a length can be perceived only in the context of an object whose size it represents; but every perceivable object exists within a certain realm, or environment, which itself has certain properties. While the value of ground ratio may be approximated by conducting the experiment presented in Section i, as van der Laan has demonstrated, its exact value can only be deduced from these properties.

The deductive approach was used by van der Laan in his study of the relations between sizes of physical (spatial) objects [16]. He derived the value of ground ratio by using the fact that the space containing these objects is three-dimensional, showing that λ is equal to the single real root ψ of the trinomial $x^3 - x - 1$. Its exact value is

$$\psi = \frac{\sqrt[3]{108 + 12\sqrt{69}} + \sqrt[3]{108 - 12\sqrt{69}}}{6} \approx 1.324718$$

which is called the *plastic number*. It generates the system of measurements shown in Figure 3, which consists of two consecutive orders of size. The larger order of size contains eight measurements $\mathrm{I}_1, \mathrm{I}_2, \ldots, \mathrm{I}_8$ (blue bars), each one representing a type of size. The smallest measurement I_1, called the *unit*, is equal to the margin of the largest measurement I_8. The other, smaller order of size contains eight measurements $\mathrm{II}_1, \mathrm{II}_2, \ldots, \mathrm{II}_8$ (red bars), where II_k is equal to the margin of I_k: thus $\mathrm{I}_1 = \mathrm{II}_8$. The whole system represents the scope of the unit I_1.

Van der Laan's system of measurements can be used to approximate the Weber fraction corresponding to the visual length stimulus. Since II_1 is the smallest length non-negligible to I_1, which is the margin of I_8, it follows that II_1 approximates the just-noticeable-difference (JND) threshold [2, p. 37–38] associated with I_8. The corresponding Weber fraction is equal to the ratio of II_1 to I_8, i.e. $\psi^{-14} \approx 0.0195$. This is coherent with values, reported by Weber himself, "[...] of about 0.01 or 0.02. The majority of later studies yield similar values [...]" [15, p. 344].

vi. Morphic Numbers

A real number $x > 1$ such that $x - 1 = x^{-m}$ and $x + 1 = x^n$ for some positive integers m and n is called the *morphic number*. A geometric sequence based on morphic number has certain additive properties (their definitions depend on values m and n), i.e. some members of the sequence may be computed by adding/subtracting other members. The above definition may be generally interpreted as follows. Given an unit quantity $x_0 = 1$, assume that x is the smallest quantity significantly greater than x_0. Then x is equal to the corresponding ground ratio. If x is a morphic number, then the difference $x - x_0$ between two quantities, as well as their sum $x + x_0$, belongs to the same system of measurements as x does. This allows the entire system to be reconstructed from any n consecutive measurements using only addition and subtraction.

It can be shown that only two morphic numbers exist [1], namely the plastic number (for $m = 4$ and $n = 3$) and the golden ratio

$$\varphi = \frac{\sqrt{5} - 1}{2} \approx 1.618033$$

(for $m = 1$ and $n = 2$).

Additive properties of the system of measurements based on the plastic number are illustrated in Figure 3. For example, in a sequence of four consecutive measurements the sum of the smallest two is equal to the largest; similarly, in a sequence of six consecutive measurements the difference between the largest two is equal to the smallest. There is also an additive rule which combines authentic and derived measurements: in a sequence of four consecutive measurements, the first one is twice smaller than the derived measurement corresponding to the last.

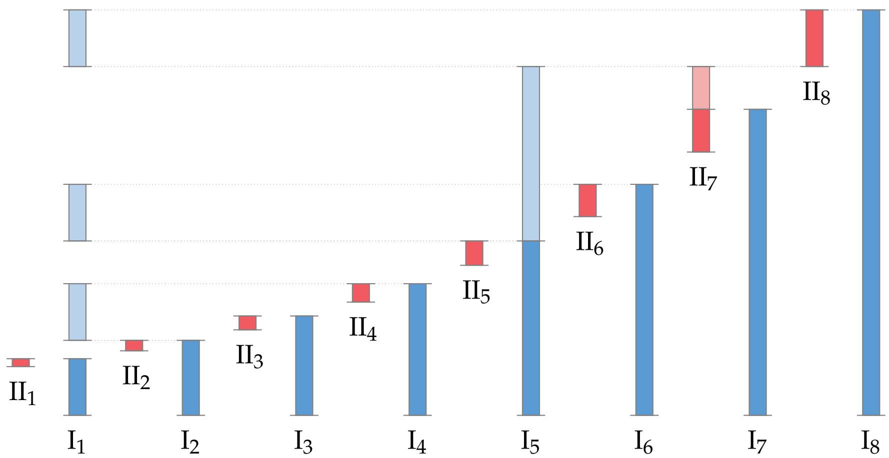

Figure 3: *system of measurements based on the plastic number*

The analogous system based on the golden ratio has similar (albeit not as rich) additive properties. However, the golden ratio is not related to our perception of three-dimensional reality [16, p. 75]. Also, in [18, p. 138] we find:

> For van der Laan, the Golden Section in its application is nothing more than an artificial concept to order matter, as abstract as the discrete quantity of mathematical numbers. Because of its abstract nature, it proves inadequate when brought into relation to concrete and singular reality, since it remains on the level of analysis.

According to van der Laan, the plastic number ratio directly grew from discernment (the human ability to differentiate sizes) and from the necessity of relations [18, p. 138]. As such, it would be an improvement over the golden ratio [19, p. 1].

Nevertheless, many authors still consider φ to be an important proportion in art, architecture and music; see, for example, [11], [12] and [4].

III. Sonata Form

Sonata form is the central musical concept of the Classical period and one of the most important ideas in the history of Western music. During the Classical period it was conceived in two parts: the Exposition (A), in which certain themes are introduced, and the Development and Recapitulation (B) in which the themes are developed and revisited [11], as shown in Figure 4.

Hepokoski and Darcy list five types of sonata-form movements [7]. Type 1 features no Development section and is often used in slow and more peaceful movements. Type 2 is characterized by eliding the end of the Development with the beginning of the Recapitulation, making it difficult to determine a clear bound between them. Types 4 and 5 refer to sonata rondos and concerto sonata movements, respectively. In this paper we focus on type 3, which features full Exposition, Development, and Recapitulation. From now on, the term "sonata-form movement" will refer to that type.

The Exposition is divided in two parts establishing the two different but well-blending tonalities, called the *primary key* and the *secondary key* (the latter usually being either the dominant key for a major primary key or the parallel key for a minor primary key). Often these two tonalities are expressed by mutually contrasting groups of thematic material called the *first subject* and the

Figure 4: *Sonata form*

second subject. We will assume that the two subjects are distinct and disjoint; however, it is not always the case, as they may sometimes blend into each other partially.

The end of the Exposition is marked with a cadence in secondary key, called the *essential expositional closure* (EEC) [7]. Sometimes, however, a short section may follow EEC before the onset of the Development. In this text we consider such a passage to be part of the Exposition.

The end of the Development is marked with a cadence in primary key, called the *essential structural closure* (ESC) [7]. It may be followed by a short passage before the onset of the Recapitulation. We consider such a passage to be part of the Development.

The length of each of three major parts of a sonata-form movement, as well as the length of each subject, is always well-defined with respect to the above conventions. Now we can study how the lengths of different parts are, in general, related to each other. We may use tools developed in the previous section to do so, as the length of each part can be numerically expressed as a number of measures by doing simple counting.

Due to the fact that we are able to memorize, it is not difficult to imagine perceiving a relation between two distinct chunks of music played in succession. However, we may not assume the value of ground ratio *a priori*; we have to obtain it by studying the general properties of Sonata form which organizes the time flow.

From now on, we use the symbols a, b, c, s_1 and s_2 to denote the lengths of parts A and B, the Development and the two subjects, respectively.

i. Inherent Restrictions

The Sonata form itself imposes certain restrictions [11, p. 280]. Hence some of its key aspects may be expressed in form of a set of four structural "rules" (we will call them *propositions*) based on the concept of ground ratio.

The first restriction is related to the shape of Exposition which is determined by the proportion of lengths of two subject groups. The main premise is that the two necessary belong to different keys which are considered equally important in the course of Exposition. Therefore, if the second group was much longer than the first group it would shadow out the importance of the primary key. On the other hand, if it was much smaller, it would be shadowed by the primary key. As the two subject groups have equally demanding tasks of establishing the respective tonalities, effectively equal amounts of time for them to do so should be granted, implying the following statement.

Proposition III.1. *The lengths of two subject groups in the Exposition are effectively equal.*

The Development represents a passage which "renders the established tonal tension[4] more fluid an complex [, . . .] typically [initiating] more active, restless, or frequent tonal shifts—a sense of comparative tonal instability. Here one gets the impression of a series of changing, coloristic

[4]Key displacement at the end of the Exposition results in an unresolved tension.

moods or tonal adventures [...] with shadowed, melancholy, or anxious connotations" [7, ch. 2, p. 7]. As the general characteristic of the Development is its tonal instability, it should not dominate over the parts of well-established tonality. Hence the Development should not be significantly longer than both subjects. It should not be significantly shorter than both of them either, since there would be no time for it to build tension, most often by developing thematic material or modulating through distant keys, and to include a retransition before the Recapitulation; it would seem all too tight and undeveloped to the listener who expects a meaningful contrast to the Exposition. This implies the second important inherent restriction of Sonata form, stated below.

Proposition III.2. *The length of the Development is effectively equal to the length of at least one subject group.*

Discussing Mozart's style in context of Sonata form, D. F. Tovey says that "the return to the tonic [the beginning of Recapitulation] always has the effect of being accurately timed" [17, p. 215]. Onset of the Recapitulation is indeed the crucial moment in the course of a sonata-form movement; it is better to say that it is the moment in which the Development ends, i.e. in which its length becomes definitive. The listener's mind therefore has the data required to guess of how long the entire movement should be, which in turn makes possible to determine whether the Development has the "right" length, i.e. is "accurately timed". Namely, as a reflection of the Exposition is expected to follow, the mind naturally assumes that length of the Recapitulation equals that of the Exposition. Hence $b = a + c$, i.e. $c = b - a$. Since Sonata form has a distinct ternary shape, we assume that c is not negligible to the expected length $a + b$ of the whole movement. Now Theorem II.3 implies that b is (expected to be) significantly longer than a. Therefore, to prevent the Development of being too short, we acknowledge the following restriction.

Proposition III.3. *The Exposition with Development is significantly longer than the former.*

The Development can be realized in a myriad of ways once the subject material is presented in the Exposition. A composer should have the highest possible degree of freedom to express his or her ideas within the central section. In particular, it should be possible for the length of Development to vary considerably when compared to the length of Exposition (enough to discourage an educated listener from trying to guess it). In order to measure the variation, we introduce a property called the *central magnitude*:

$$\mu = \frac{\max c}{\min c},$$

where $\max c$ and $\min c$ are equal to the longest and shortest Development possible (for an arbitrary but fixed a) such that conditions stated in propositions III.2 and III.3 hold. The central magnitude measures the "amount of variation" in length of the Development. Its value is an answer to the question "how many times is the longest possible development section longer than the shortest one?". To ensure the maximal amount of variation, we state the following condition, which is naturally imposed as inherent to Sonata form.

Proposition III.4. *The central magnitude has to be as high as possible.*

ii. Parametrization of Shape

The ratio c/a, i.e. the relative length of Development with respect to the length of Exposition, is crucial for the shape of a sonata-form movement. Therefore it will be called the *shape parameter* in the subsequent text. In this section we use propositions III.1, III.2, III.3 and III.4 to compute the exact value of ground ratio and subsequently determine the range of the shape parameter. Without a loss of generality we may assume that $s_2 \geq s_1$.

Lemma III.5. *For $s_2 \geq s_1$ the following inequalities hold:*

$$\frac{s_2}{s_1} \leq \lambda, \quad \frac{s_1}{c} \leq \lambda \quad \textit{and} \quad \frac{c}{s_2} \leq \lambda. \tag{5}$$

Proof. The first inequality readily follows from Proposition III.1. The statement of Proposition III.2 is equivalent to

$$\frac{s_1}{\lambda} \leq c \leq \lambda\, s_1 \quad \text{or} \quad \frac{s_2}{\lambda} \leq c \leq \lambda\, s_2. \tag{6}$$

But, since $\frac{s_2}{s_1} \leq \lambda$ and $\lambda > 1$, we obtain

$$\frac{\frac{s_2}{\lambda}}{\lambda\, s_1} = \frac{1}{\lambda^2} \cdot \frac{s_2}{s_1} \leq \frac{1}{\lambda^2} \cdot \lambda = \frac{1}{\lambda} < 1,$$

implying $\frac{s_2}{\lambda} < \lambda\, s_1$. Because of that, the statement (6) is equivalent to

$$\frac{s_1}{\lambda} \leq c \leq \lambda\, s_2,$$

implying $\frac{s_1}{c} \leq \lambda$ and $\frac{c}{s_2} \leq \lambda$. That completes the proof. □

Theorem III.6. *The ground ratio for Sonata form is equal to the plastic number ψ.*

Proof. Multiplying the first two inequalities in (5) yields $\frac{s_2}{c} \leq \lambda^2$. Hence from Lemma III.5 follows

$$\frac{a}{c} = \frac{s_1}{c} + \frac{s_2}{c} \leq \lambda + \lambda^2. \tag{7}$$

On the other hand, multiplying the first and the third inequality in (5) yields $\frac{c}{s_1} \leq \lambda^2$. Therefore, Lemma III.5 also implies

$$\frac{a}{c} = \frac{s_1}{c} + \frac{s_2}{c} \geq \frac{1}{\lambda} + \frac{1}{\lambda^2} = \frac{\lambda+1}{\lambda^2}. \tag{8}$$

Inverting the inequalities (7) and (8) yields

$$\frac{1}{\lambda+\lambda^2} \leq \frac{c}{a} \leq \frac{\lambda^2}{\lambda+1}. \tag{9}$$

Furthermore, Proposition III.3 implies

$$\frac{c}{a} = \frac{a+c-a}{a} = \frac{a+c}{a} - 1 \geq \lambda - 1. \tag{10}$$

Let $m_1(\lambda) = \frac{1}{\lambda+\lambda^2}$, $m_2(\lambda) = \lambda - 1$ and $M(\lambda) = \frac{\lambda^2}{\lambda+1}$. It is obvious that, for positive λ, the function m_1 is strictly decreasing and the function m_2 is strictly increasing. As both m_1 and m_2 are continuous, the fact that $m_2(1) = 0 < \frac{1}{2} = m_1(1)$ implies that there exists an unique $\lambda_0 > 1$ such that $m_1(\lambda_0) = m_2(\lambda_0)$. Using the equality $1 + \psi = \psi^3$, which follows from the fact that the plastic number is the root of polynomial $x^3 - x - 1$, we readily check that $\lambda_0 = \psi$. Therefore, from (9) and (10) follows

$$m(\lambda) \leq \frac{c}{a} \leq M(\lambda), \quad \text{where } m(\lambda) = \begin{cases} m_1(\lambda), & 1 < \lambda < \psi, \\ m_2(\lambda), & \lambda \geq \psi. \end{cases} \tag{11}$$

Now we can compute the central magnitude:

$$\mu = \frac{\max c}{\min c} = \frac{\frac{1}{a} \max c}{\frac{1}{a} \min c} = \frac{\max \frac{c}{a}}{\min \frac{c}{a}} = \frac{M(\lambda)}{m(\lambda)}.$$

Using (11), it follows

$$\mu = \begin{cases} \frac{M(\lambda)}{m_1(\lambda)} = \lambda^3, & 1 < \lambda < \psi, \\ \frac{M(\lambda)}{m_2(\lambda)} = \frac{\lambda^2}{\lambda^2-1}, & \lambda \geq \psi. \end{cases} \tag{12}$$

According to Proposition III.4, μ must have the highest possible value. Differentiating μ with respect to λ using (12) yields $\frac{\mathrm{d}\,\mu}{\mathrm{d}\,\lambda} = 3\,\lambda^2 > 0$ for $1 < \lambda < \psi$ and $\frac{\mathrm{d}\,\mu}{\mathrm{d}\,\lambda} = -\frac{2\,\lambda}{(\lambda^2-1)^2} < 0$ for $\lambda > \psi$. Therefore μ is strictly increasing for $1 < \lambda < \psi$ and strictly decreasing for $\lambda > \psi$. It follows that μ, being a continuous function of variable λ on $(1, +\infty)$, attains the maximum value for $\lambda = \psi$. □

We have shown that the essential features of Sonata form imply that the ground ratio must be equal to the plastic number. Therefore, inner proportions of the temporal structure of Sonata form are organized with respect to the same system (imposed by the nature itself) used to organize proportions in a spatial, architectonic structure.

Corollary III.7. *The central magnitude is equal to* ψ^3.

Corollary III.8. *The shape parameter may vary between* ψ^{-4} *and* ψ^{-1}.

Corollary III.9. *The Development is significantly shorter than the Exposition*[5].

The bounds established by Corollary III.8 should not be interpreted as literary as our theoretical deduction suggests. Instead of forcing the shape parameter between some fixed bounds, we should ask ourselves how its value is distributed in probabilistic sense. Hence let us denote $X = {}^{c}/_{a}$, $X > 0$. Now we use the interval $\mathcal{T} = [\psi^{-4}, \psi^{-1}]$ to deduce the probability distribution of the continuous random variable X.

It should be noted that X may attain any positive value in our model. However, $X \in \mathcal{T}$ has to be much more probable than $X \notin \mathcal{T}$. Therefore it is reasonable to assume that the probability distribution function for X is bell-shaped, peaking somewhere near the center of $\mathcal{T}$, i.e. the average of its endpoints $X_l = \psi^{-4}$ and $X_u = \psi^{-1}$. Since X is dimensionless value represented as a ratio, the appropriate averages are geometric and harmonic mean [6, p. 4.17–4.18]. Indeed, the geometric mean $X_g = \sqrt{X_l\,X_u}$ satisfies

$$\frac{X_g}{X_l} = \frac{X_u}{X_g}.$$

Hence the relation between lengths $a\,X_g$ and $a\,X_l$ is the same as the relation between $a\,X_u$ and $a\,X_g$, where $a > 0$ is arbitrary. In other words, ranges $[X_l, X_g]$ and $[X_g, X_u]$ are perceived as being equally wide, so the possibilities $X_l \leq X \leq X_g$ and $X_g \leq X \leq X_u$ should be equally probable. Now, as $\ln X_g$ is the arithmetic mean of $\ln X_l$ and $\ln X_u$, the probability density curve of $\ln X$ appears to be symmetric, so we simply assume that $\ln X$ is normally distributed with mean $\ln X_g$. Therefore the distribution of X is lognormal [5, p. 1–2.]:

$$X \sim \ln \mathcal{N}(\mu, \sigma). \tag{13}$$

Now it follows $X_g = \mathrm{Med}[X]$, which yields [5, p. 9]

$$\mathrm{e}^{\mu} = X_g \implies \mu = \ln X_g. \tag{14}$$

In Section iv we showed that the naturally perceived middle between two fixed lengths coincides with their harmonic mean. In particular, this is valid for (relative) lengths X_l and X_u,

[5] D. F. Tovey also points this out by saying that the Development in a sonata-form movement of Mozart is generally "short" compared to the Exposition [17, p. 215].

implying that their harmonic mean $X_h = \frac{2\,X_l\,X_u}{X_l+X_u}$ is the natural center of the interval $\mathcal{T}$ and a measure of central tendency of X. In statistics, the main measures of central tendency are the mean (expectation), median and mode [6, p. 4.1]; since it was already stated that $\mathrm{Med}[X] = X_g$, it remains to identify X_h with the mode $\mathrm{Mode}[X]$ or the mean $\mathrm{E}[X]$. But $\mathrm{Mode}[X] < \mathrm{Med}[X] < \mathrm{E}[X]$ holds for any lognormal distribution [5, p. 9] and $X_h < X_g$ [6, p. 4.20], hence we assume $X_h = \mathrm{Mode}[X]$. Because $\mathrm{Mode}[X] = \mathrm{e}^{\mu-\sigma^2}$ [5, p. 9], we have

$$X_h = \mathrm{e}^{\mu-\sigma^2} \implies \ln X_h = \mu - \sigma^2.$$

Using (14) it follows

$$\sigma = \sqrt{\ln \frac{X_g}{X_h}}.$$

As $X_g = \psi^{-5/2}$ and $X_h = \frac{2}{\psi+\psi^4}$, we have

$$\mu = \ln X_g = -\frac{5}{2}\ln\psi = -5\ln\sqrt{\psi}$$

and

$$\frac{X_g}{X_h} = \frac{1+\psi^3}{2\,\psi^{3/2}} = \frac{\psi^{-3/2}+\psi^{3/2}}{2} = \frac{1}{2}\left(\mathrm{e}^{\frac{3}{2}\ln\psi} + \mathrm{e}^{-\frac{3}{2}\ln\psi}\right) = \cosh\left(\frac{3}{2}\ln\psi\right) = \cosh(3\ln\sqrt{\psi}).$$

Letting $\omega = \ln\sqrt{\psi}$, we finally obtain

$$\mu = -5\,\omega \approx -0.7029989, \quad \sigma = \sqrt{\ln(\cosh(3\,\omega))} \approx 0.2940039. \tag{15}$$

Parameters in (15) define the probability distribution of X. Now, using the `plnorm` function from computer software R [13], we compute the probability $P(X_l \le X \le X_u) = 0.8486196$, which means that probability for $X \notin \mathcal{T}$ is practically equal to 15%.

Proposition III.10. *Let $\kappa = \frac{1}{2}\sqrt{1+\psi^{-3}}$. Then the expected value and the standard deviation of X are*

$$\mathrm{E}[X] = \frac{\kappa\sqrt{2}}{\psi^{7/4}} \approx 0.5169651 \quad \textit{and} \quad \mathrm{SD}[X] = \frac{\kappa\,(\psi^{3/2}-1)}{\psi^{5/2}} \approx 0.1553341.$$

Proof. The expectation and the variance of X are [5, p. 9]

$$\mathrm{E}[X] = \mathrm{e}^{\mu+\frac{1}{2}\sigma^2} \quad \text{and} \quad \mathrm{Var}[X] = \mathrm{e}^{2\mu+\sigma^2}\left(\mathrm{e}^{\sigma^2}-1\right),$$

hence

$$\mathrm{SD}[X] = \sqrt{\mathrm{Var}[X]} = \mathrm{e}^{\mu+\frac{1}{2}\sigma^2}\sqrt{\mathrm{e}^{\sigma^2}-1}.$$

Using (15), we define

$$F_1 = \mathrm{e}^{\mu} = \psi^{-5/2} \quad \text{and} \quad F_2 = \mathrm{e}^{\sigma^2} = \cosh(3\,\omega) = \frac{1}{2}\left(\psi^{3/2}+\psi^{-3/2}\right).$$

Now we have

$$\mathrm{E}[X] = F_1\sqrt{F_2} \quad \text{and} \quad \mathrm{SD}[X] = F_1\sqrt{F_2^2 - F_2}.$$

The statement follows by rewriting the above equations using the equality $1+\psi = \psi^3$. □

iii. An Empirical Study

The probability distribution (13) applies to sonata-form movements of type 3 in general, defining the probability $P(p \leq X \leq q)$ for arbitrary $0 < p < q$. It would be meaningless trying to apply it to a single sonata-form movement. However, given a large number of such movements written by the same composer, a statistical analysis may be performed in order to compare the empirical distribution of X to the theoretical one.

In this section we use a set of sonata-form movements from the instrumental (solo, chamber and orchestral) opus of Wolfgang Amadeus Mozart, which represents a peak of elegance in Western classical music. There are several objective reasons to choose Mozart:

1. a large amount of unified data is at our disposal (see [9]) due to his immense productivity,
2. the quality of his work is uniform; there are no mediocre pieces in his opus,
3. his work is constantly praised for the perfectness of temporal proportions; several distinguished authors are quoted in [11, p. 276]:

> ... the genius of [W. A. Mozart] is manifested in form and balance. His music has been revered, among other things, for its "beautiful and symmetrical proportions" [34, p. 217]. In 1853, Henri Amiel opined that "the balance of the whole is perfect" [I, p. 54]. Hanns Dennerlein described Mozart's music as reflecting the "most exalted proportions," and the composer himself as having "an inborn sense for proportions" [quoted in 7, p. 1], a thought echoed by H. C. Robbins Landon [20, p. 268]. Eric Blom wrote that Mozart had "an infallible taste for saying exactly the right thing at the right time and at the right length" [5, p. 265].

The total of 188 sonata-form movements of type 3 can be extracted from the instrumental part of Mozart's opus as published in [9]; these are listed in Table 1, each of them denoted by the respective Köchel catalog number together with position of the movement in question (expressed in roman numerals) within the corresponding piece.

As noted before, the Exposition and the Development are always well defined in a sense that their lengths can be unambiguously determined. Thus the lengths a (of the Exposition) and c (of the Development) are given for every movement listed in Table 1.

Mathematical expectation of X with respect to the data set $\mathcal{D} = \{c_k/a_k : 1 \leq k \leq 188\}$ with a_k, c_k given in k-th row of Table 1, is

$$\overline{X} = \frac{1}{188} \sum_{k=1}^{188} \frac{c_k}{a_k} \approx 0.5145557.$$

This differs for less than 0.5% from the value $\mathrm{E}[X]$ given in Proposition III.10. In particular, $\mathrm{E}[X]$ represents a statistical measure of Mozart's choice of shape parameter. The important fact is that it was not necessary to include any information from Mozart's music to compute it.

To compare empirical probability distribution of data from $\mathcal{D}$ with the lognormal distribution with parameters given in (15) we apply the Kolmogorov-Smirnov test, available in R as `ks.test` [13]. The test gives p-value of 0.3882 which is significantly greater than the suggested rejection threshold 0.1 (or commonly used 0.05), indicating that there exist strong evidence to support the null hypothesis (that data from $\mathcal{D}$ is log-normally distributed). Next we use the function `fitdistr` available in R [13] to fit a lognormal distribution to data using the method of maximum-likelihood. We obtain the distribution $\ln\mathcal{N}(\mu_f, \sigma_f)$, where

$$\mu_f = -0.71041299 \quad \text{and} \quad \sigma_f = 0.29738620. \tag{16}$$

Table 1: *188 instrumental sonata-form movements composed by W. A. Mozart*

Köchel	*a*	*c*
Piano sonatas		
279 I	38	19
279 II	28	14
279 III	56	30
280 I	56	26
280 II	24	12
280 III	77	30
281 I	40	29
281 II	46	12
282 I	15	6
282 III	39	22
283 I	53	18
283 II	14	9
283 III	102	69
284 I	51	20
309 I	58	35
310 I	49	30
310 II	31	22
311 I	39	39
330 I	58	29
330 III	68	27
332 I	93	39
332 III	90	57
333 I	63	30
333 II	31	19
457 I	74	25
533 I	102	43
533 II	46	26
545 I	28	13
570 I	79	53
576 I	58	40
Piano four-hands		
381 I	30	21
381 III	70	27
358 I	45	11
521 I	84	52
Two pianos		
448 I	80	29
448 II	48	22
Sonatas for organ		
68	26	10
144	27	24
145	37	12
212	30	10
241	35	12
224	45	21
225	50	19
244	46	21
245	35	13
263	35	11
274	32	21
328	41	19
Piano trios		
254 I	81	52
254 II	14	9½
496 I	78	38
502 I	82	35
542 I	101	34
548 I	62	41
548 II	32	23
564 I	41	36
Piano quartets		
478 I	99	41
493 I	95	52
493 II	46	23
Flute quartets		
285 I	65	34
285a I	34	14
285b I	66	44

Köchel	*a*	*c*
370 I	63	34
Violin sonatas		
301 I	84	44
302 I	68	38
304 I	84	28
305 I	73	27
306 I	74	37
306 II	34	23
296 I	68	31
378 I	82	31
376 I	47	26
377 I	51	31
380 I	58	40
454 I	52	24
481 I	92	47
526 I	100	39
526 II	41	15
403 I	24	10
372 I	67	59
547 II	78	37
Bassoon and cello duo		
292 I	46	18
292 II	22	8
Basset horn trios		
439b I/I	51	20
439b III/I	37	20
439b IV/I	28	16
Quintets		
452 I	45	16
452 II	43	30
581 I	79	38
407 I	56	16
407 II	44	25
String Quintets		
174 I	86	34
174 IV	94	68
515 I	151	53
516 I	94	38
593 I	80	43
614 I	86	38
String quartets		
155 I	53	18
156 I	71	38
157 I	52	22
158 I	45	29
159 I	29	15
160 III	54	32
168 I	41	21
169 I	36	36
171 III	10	8
171 IV	71	20
172 I	52	19
172 IV	80	32
173 I	45	19
387 I	56	51
421 I	41	28
458 I	90	47
428 I	68	32
458 I	90	47
458 IV	133	65
464 I	87	74
464 IV	80	64
465 I	84	48
465 IV	136	62
499 I	98	43
575 I	77	39
589 I	71	59
590 I	74	37
590 IV	133	51

Köchel	*a*	*c*
String trio		
563 I	73	37
String duos		
423 I	48	33
424 I	70	43
424 II	80	43
Marches		
189	27	12
237	28	12
Divertimentos		
113 I	27	14
113 IV	50	20
251 I	40	27
213 I	30	10
188 I	14	8
188 II	24	12
240 I	42	22
240 IV	66	24
252 I	18	7
270 I	51	14
136 I	36	28
136 III	58	28
137 II	25	14
137 III	44	20
138 I	35	18
205 I	37	15
334 I	83	42
334 IV	33	16
Serenades		
239 I	25	17
525 I	55	20
388 I	94	35
361 I	76	48
250 I	91	46
203 I	54	35
203 VIII	109	44
185 I	77	22
Symphonies		
48 I	33	26
110 I	69	23
112 I	54	16
112 II	25	12
114 I	59	20
114 IV	73	28
124 I	36	36
128 I	53	31
129 I	46	21
130 I	62	21
199 I	58	28
183 I	82	34
200 I	67	23
201 I	76	30
201 II	38	14
201 IV	61	40
202 I	78	34
202 IV	79	47
204 I	67	21
425 I	103	40
425 II	36	29
425 IV	163	68
504 I	106	65
504 IV	151	64
543 I	117	41
543 IV	104	48
550 I	100	65
550 II	52	21
550 IV	124	82
551 I	120	68
551 IV	157	67

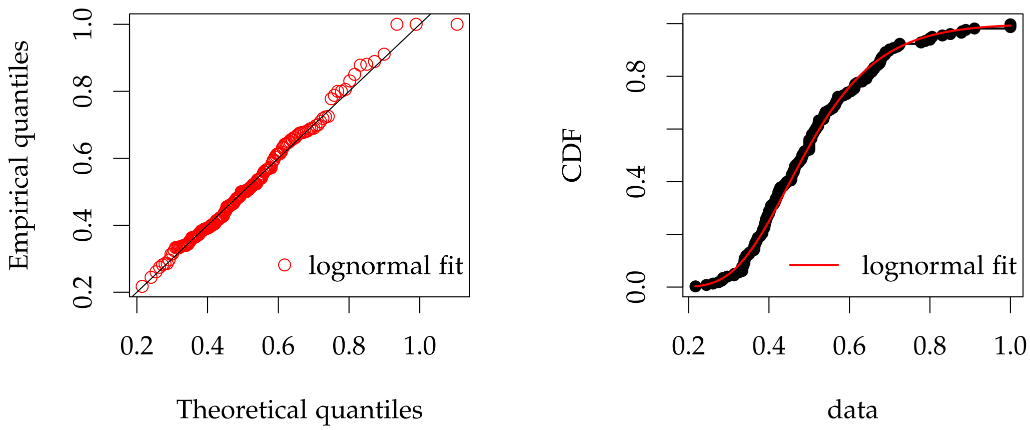

Figure 5: *empirical distribution of X compared to the fitted lognormal: Q–Q plot (left) and CDF plot (right)*

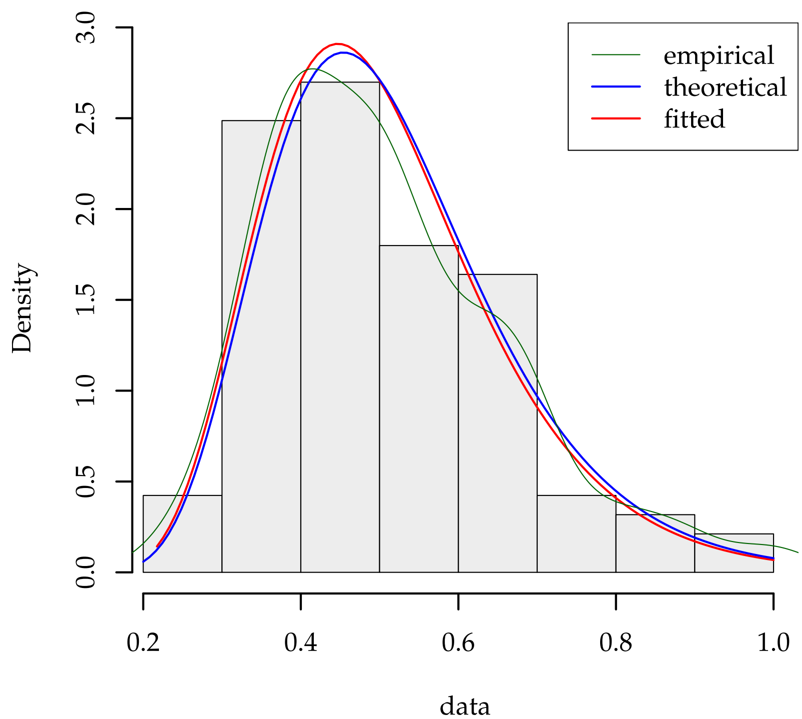

Figure 6: *probability distribution of X*

These values come very close to those given in (15), with relative errors of 1.05% and 1.15%, respectively. The Q–Q and CDF plots shown in Figure 5 present no reason to suspect that data from $\mathcal{D}$ is not log-normally distributed. Figure 6, which includes the histogram and probability density curve corresponding to the empirical data from $\mathcal{D}$, shows that fitted and theoretical distributions of X are practically coincidental.

The results of this simple study suggest that purely theoretical consequences of the inherent restrictions in Sonata form as presented in Section i indeed act as practical necessities, at least in Mozart's work. We can say that this observation justifies our theoretical deductions; since the probability distribution of X depends on two independent parameters, it would be virtually impossible for both values from (15) to coincide with the respective values in (16) so closely simply by chance.

iv. Scope as a Temporal Frame

The notion of scope defined in Section iii may be used to construct a natural time frame for a sonata-form movement. To avoid risking negligibility, the total length of the piece, as well as the length of its opening phrase, should be contained within the scope defined by the length of its first subject, which is the first self-contained part of the movement. In other words, the opening phrase, the first subject and the whole movement should correspond to measurements II_1, I_1 and I_8 from the system shown in Figure 3. These relations are particularly sharp in, for example, first movements of Mozart's piano sonatas K. 279 (C major) and K. 281 (B-flat major). In K. 279 the first subject, ending with half cadence after 15 measures, defines the scope in which the smallest length is 2 measures long and the largest length is 107 measures long. That fits tightly around the span between the length of the opening motive (2 measures) and the total length (100 measures). In K. 281, the first subject is 16 measures long: lengths of the opening motive (2 measures) and the whole movement (109 measures long) again fits within the scope of the first subject, ranging from 2 to 115 measures.

v. An Example of Formal Analysis

The shape of a particular sonata-form movement may be analyzed using the measurements from the system of van der Laan, shown in Figure 3. A remarkable example is the first movement from Mozart's Piano Sonata in B-flat major, KV 333. It is schematically shown in Figure 7 (the black bar).

Let the length of the whole movement be associated with the largest measurement I_8. Then I_1, I_2, I_3 and I_5 approximate lengths of the first subject, the Development, the second subject and the Recapitulation, respectively. The first subject is divided in two parts by the cadence in measure 10; these parts correspond to the margins of I_5 and I_6. The first subject is recapitulated in measures 93–118 and the second subject (including the final closure) in measures 118–165. These two parts correspond to the derived measurements of I_2 and I_4, denoted by Ia_2 and Ia_4, respectively. The measurements are shown as colored bars in Figure 7.

The opening phrase is $3^3/_4$ measures long (including the upbeat). It is only slightly larger than II_1, which is approximately equal to $3^1/_4$ measures. Therefore the temporal frame of this movement can be practically identified with the scope of I_1.

It should be noted that Proposition III.1 does not apply here. It is due to the fact that there are two consecutive closures in F major contained in measures 22–63. However, it does not matter as the transition between two subjects cannot be unambiguously determined. For example, one could claim that the transition happens between measures 22 and 38, in which case the subject lengths would be treated as being practically equal.

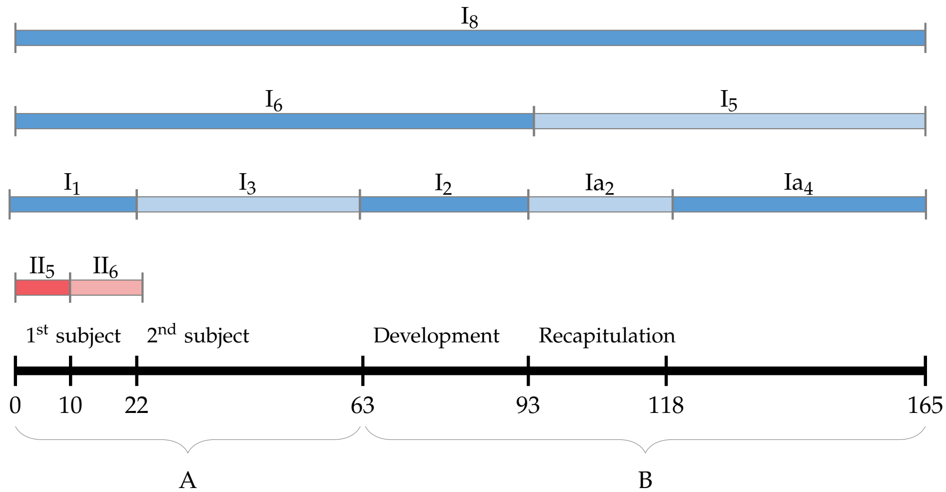

Figure 7: *W. A. Mozart: Piano Sonata in B-flat major KV 333, first movement*

It is obvious that the Recapitulation practically equal to the Exposition, which is coherent to the listener's expectation (forming with the onset of Recapitulation) that the two should be of the same length. It also should be noted that the Recapitulation is prolonged as much as it could be while staying practically equal to the Exposition.

The onset of Recapitulation divides the movement in ratio $93/72 \approx \psi$, while the end of Exposition divides it in ratio $102/63 \approx \varphi$. Hence both morphic numbers are built in the structure, defining the two most prominent moments in the course of movement. Relative errors of the above approximations are equal to 2.50% and 0.06%, respectively.

Although the structure of the movement illustrated in Figure 7 appears as it was consciously designed by combining measurements based on the plastic number (including the derived ones), chances that it actually happened are next to nothing. Namely, Mozart could not know about the plastic number as it was first discovered in 1928 [10] by Hans van der Laan[6]. As this excludes the possibility of a thoughtful mathematical design, it could only happen as a consequence of the plastic number indeed being a natural necessity [18, p. 138]. Many more examples of analyzing piano sonatas of Mozart in a similar way can be given; plastic number tends to appear frequently in his work.

The appearance of the golden ratio is significant as it indicates that measurements from the system of van der Laan can be combined to approximate it:

$$\frac{\mathrm{I}_2 + \mathrm{Ia}_2 + \mathrm{Ia}_4}{\mathrm{I}_1 + \mathrm{I}_3} \approx \varphi.$$

Relative error of this approximation is equal to 0.16%.

IV. Conclusion

We have shown some exact structural aspects of Sonata form can be deduced from its very concept. Length c of the Development can be chosen by simply using a generator of log-normally

[6]According to some sources, the plastic number was actually discovered in 1924 by a French engineer Gérard Cordonnier, who called it the *radiant number* [14, p. 9].

distributed random variable. Since theoretically any value $c > 0$ could be generated, inherent upper and lower bound ψ^{-4} and ψ^{-1} of the shape parameter $X = {}^{c}/_{a}$ also have to be taken into consideration. There need to exist strong musical reasons for X not being between these bounds. According to our probabilistic model it should happen in about 15% of cases. Indeed, we find $X \notin \mathcal{T} = [\psi^{-4}, \psi^{-1}]$ in about 13% of movements from Table 1.

A strong relation between Sonata form and the plastic number is established by our findings. The fact that the plastic number takes the role of ground ratio may characterize Sonata form as a "spatial" construction within the temporal domain. This may be one of the reasons for its popularity. Also, the plastic number acts as a bridge between music and architecture, from which it originated.

Results of the empirical study described in Section iii also give additional credit to Mozart (if that is even possible). Namely, an analysis of his work shows that proportions of the movements from Table 1 are coherent with the abstract concept of Sonata form. In fact, his work may serve as an detailed illustration of the concept, exploring all possible combinations of well-balanced proportions. In particular, values of the shape parameter for movements in Table 1 densely cover the whole theoretical range $\mathcal{T}$. Taking only a set of movements from piano sonatas, we find the shortest development section in the first movement of Piano Sonata in G major, KV 283 ($X \approx 0.34$) and the longest one[7] in the first movement of Piano Sonata in B-flat major, KV 281 ($X = 0.725$). The corresponding values of the shape parameter are almost equal to the extreme values $\psi^{-4} \approx \frac{1}{4}$ and $\psi^{-1} \approx \frac{3}{4}$. Although Mozart's treatment of Sonata form may appear rigid and uniform at first (compared to, for example, Beethoven's), we have shown that his choices of the shape parameter are as diverse as possible.

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Nestings and Intersections between Partitional Complexes

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Abstract: *The formalization of musical texture is the main objective of* Partitional Analysis*. Each integer partition corresponds to a specific textural configuration and is used as a tool to organize and systematize the work with textures through the compositional process.* Partitional complexes*, on the other hand, are sets of partitions, observed in the analysis of musical excerpts, that work in tune to create stable temporal domains where a referential partition projects, extends or presents itself as dominant. The number of partitions and complexes for a certain instrumental, vocal or electronic medium is finite and implies nestings and intersections that can provide important information about textural possibilities available to the composer. In the present work, the relationships established between distinct partitional complexes are discussed, as well as the characterization of an hierarchy related to the number of total choices that each complex offers to the composer.*

Keywords: *Partitional Analysis. Partitional Complexes. Musical Texture. Musical Analysis. Theory of Integer Partitions.*

Musical texture is a diffuse concept and is often interpreted in distinct and even contradictory ways in its various uses ([8]). Two major interpretations have overcome since the term was coined in the early twentieth century, and today the term has acquired many meanings, due to an extreme diversification of sonorous and instrumental resources that occurred from the 1950s to the present day, with the popularization of recording techniques and electronic and digital instruments.

The first interpretation (*texture-plot*), inherited from the traditional classification of texture in generalized categories of *monophony*, *homophony*, *polyphony* and *heterophony*, refers to the combination of vocal or instrumental parts, based mainly on compositional production (*poiesis*) and covering music based on notes.

The second one (*texture-sonority*), developed from the instrumentation and orchestration manuals, is mainly concerned with perception (*esthesis*) and the sound quality of the timbral combinations, as well as sonological researches.

In any case, the texture is still a sub-formalized field at the present and needs further investigation and systematization

One of the pioneering and most influential authors in the work of formalizing the texture is Wallace Berry ([3], pp. 184-199). Berry proposed a coding of the textured configurations taking into account two forces, which he calls *quantitative* and *qualitative* curves. The quantitative curve is determined by the number of sounding components at each time point (called by him as *density-number*), while the qualitative curve is the result of the evaluation of dependence and independence relations between vocal or instrumental parts, determined by the congruence or contrast between them.

The representation of the qualitative curves is given by stacked numbers, corresponding to the various lines and blocks involved in a given point of time (Figure 1). It is curious that, as a pioneer, Berry did not concern with the investigation of the set of these configurations *per se*, independent of the analytical applications that are presented in his book. Berry's representations are, in fact, finite, and can be read through the *Theory of Integer Partitions* simply as *partitions*, that is, representations of integers through sums of integers (called, then, *parts*).

Figure 1: *Textural configurations in Milhaud, first sonnet for choir* A Peine si le coeur, images et figures *([3], p. 187).*

Partitional Analysis (*PA* for short), on the other hand, is an original contribution that can be viewed as a radical expansion of Berry's work, which, in addition to offering the exhaustive taxonomy of the field of textural configurations, as well as its topological and metric mapping, presents formal structures that can be applied to various fields of texture, such as melodic texture, orchestration, form, among many others. In this sense, these formal structures gain as much or even greater importance than the original application as creative tools, because they end up describing a deep level of the organization of musical flows and the possibilities of discourses based on simultaneous, even non-musical, temporal transformations. A much broader field to explore.

In the present work, the textural possibilities available to the composer (as he makes his choices about textural configurations) will be approached. This interaction between the composer's choices and the possibilities that open up at each stage of his actions is referred inside PA as a *compositional game*.

It will be shown later that each textural configuration has a specific number of possible realizations, which can significantly impact compositional thinking. In addition, since partitions always work in sets, also specific, the intersections between different *Partitional Complexes* (that is, sets of partitions that work in tune to make up a global partition) also turn important, by defining how successive partitional domains interact.

Partitional Analysis

The main feature of *Partitional Analysis* ([16], [13]) and its main distinction from Berry's *Textural Analysis* lies in the understanding of how partitions are established, that is, through *binary relations* ([13], p. 33-38).

Every composer of concert music has gone through the experience of searching for a musical interval within a harmonic structure, either in choral exercises, or in the process of analysis of the harmonic content of an instrumental chord, for instance. This exhaustive search can eventually be mandatory. For instance, in a choir, comparing the voices, (e. g., SA, ST, SB, AT, AB, TB) to be assured of all intervals. In this case, the number of assessments is provided by the two-by-two combination of the number of voices, a very well known mathematical function.

In the domain of textural configurations, it is not just the intervals that are evaluated, but the quality of *collaboration* or *counterposition*, which will produce, after all, the individualization of textural elements, like lines and blocks, and, at the same time, the vertical differentiation from one to another element. In the present work, we will consider just the rhythmic congruence or counterposition, determined by the combination of point of attack (or onset) and duration - what is called in PA by *Rhythmic Partitioning* ([13], p. 35 *et seqs*, [7]) .

For each textural configuration, there is a pair of indices corresponding to the total number of found *collaborations* and *counterpositions* between the sounding components (in the case of the string quartet, these components could be each musician, playing in ordinary mode, as an example). This counting reflects how much there is homogeneity and agreement between the sound sources, on one hand; on the other, how much there is diversity, disagreement. These two indices are called in PA by *agglomeration* and *dispersion* indices, respectively, or, for short, *(a, d)*.

Once the pair of indices *(a, d)* is established, two graphical tools are elaborated to visualize the textural progressions in a particular piece or musical work: the *indexogram* and *partitiogram*.

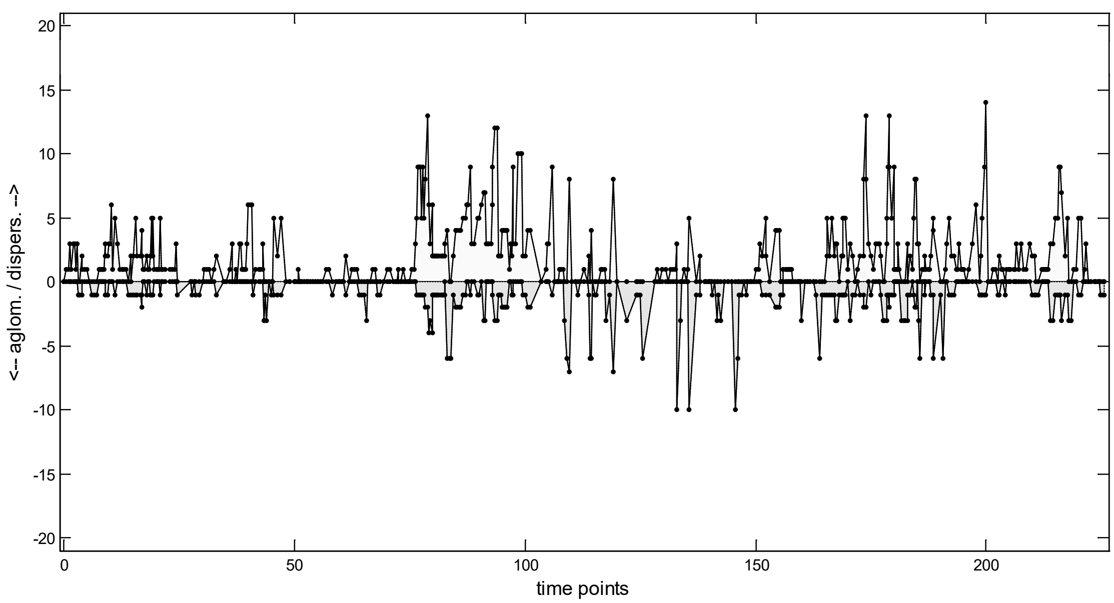

Figure 2: *Indexogram of* Le Marteau sans Maître – I - avant "L'Artisanat furieux" *([4], [12], p. 3). The peaks in dispersion area indicate points of maximum polyphony, while the peaks in agglomeration area mark the more massive blocks. The two indices are partially independent, as variety and mass can assume many combinations. Graph produced by Parsemat® ([15]).*

The Indexogram shows the individual progression of each index over time, updated in each new attack or time point where any event occurs (usually, the onset of a single note or chord). The graph is splitted horizontally in its median portion, so that agglomeration index is presented with negative signal. The intention is to allow the visualization of the interval between indices, in a wave chart-like design.

Partitiogram presents all the partitions referring to a certain density-number, plotted in a plane according to their indices of agglomeration (x axis) and dispersion (y axis). Besides being an exhaustive taxonomy of all available textural configurations (called in PA as *lexset*), it is also a topological representation of its metrics (which reveal adjacencies, proximity, and degrees of similarity or kinship between partitions).

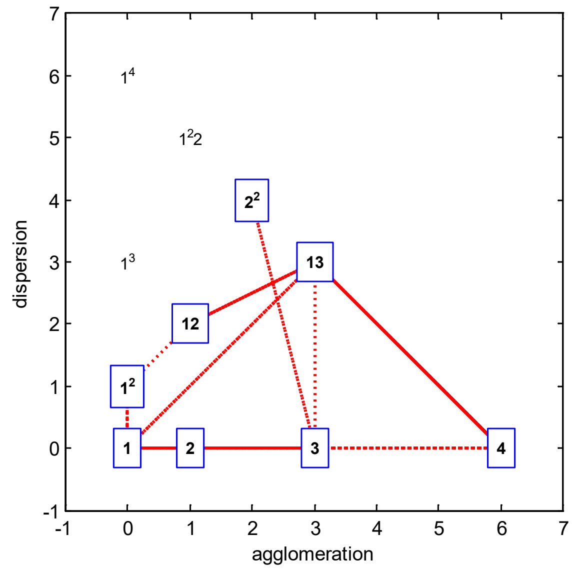

Figure 3: *Partitiogram of Leo Brower's* Estudio 20 *from* Estudios Sencillos *for guitar ([6]). Partitions are mapped by agglomeration and dispersion indices. Used partitions from lexset of integer 4 are marked in blue squares and connections are established between successive configurations presented in the piece. In this case, there are three unused partitions (1^3), ($1^2 2$) and (1^4) – precisely the most polyphonic ones. Graph produced by Parsemat® ([15]).*

Since it represents all the available vocabulary for a particular instrumental, vocal or sound environment, it is possible to read the texture in specific excerpts or works as a sequence of partitions, considering that they are finite and therefore treatable from a compositional point of view.

The adjacency relationships found in the partitiogram can be classified according to their intrinsic qualities, thus forming networks of operators. divided in two general categories: simple and compound.

Simple operators (*resizing* and *revariance*) are the very basis of the construction of texture itself. Compound ones (*transfer* and *concurrence*) are combinations of simple operators and are necessary to explain some adjacencies in PA diagrams. For each one, it is assigned a letter for easier reference.

Resizing (m) a part means to change its thickness. The positive resizing implies the inclusion of more sounding components to a block, making it "fatter"; the negative resizing is, on the contrary, the thickening of a part, subtracting a sounding component from a block.

Revariance (v) is the changing of variety (number of parts) inside a textural configuration.

Positive revariance implies adding an unitary part to the partition and negative revariance means subtracting an unitary part from it.

Transference (t) arises when resizing and revariance are applied together, but with opposite signals (positive resizing with negative revariance, and vice-versa). The consequence is that one sounding component is displaced from a part to another, without affecting the overall density-number. This kind of operation is very common in traditional concert music.

Concurrence (c), on the other side, is the consequence of articulation of resizing and negative revariance, with the same signals (positive resizing and revariance, or the opposite). This causes increment of the distinctions between parts (blocks become more massive and lines are multiplied) and the change in the global density-number. In this way, concurrence is different of the former operators, in the sense that it is not and relation of adjacency. The concept is included here, anyway, because it is relevant to describe some musical situations, where the contrast between successive textural configurations are the rule (for instance, in some *avant-garde* styles).

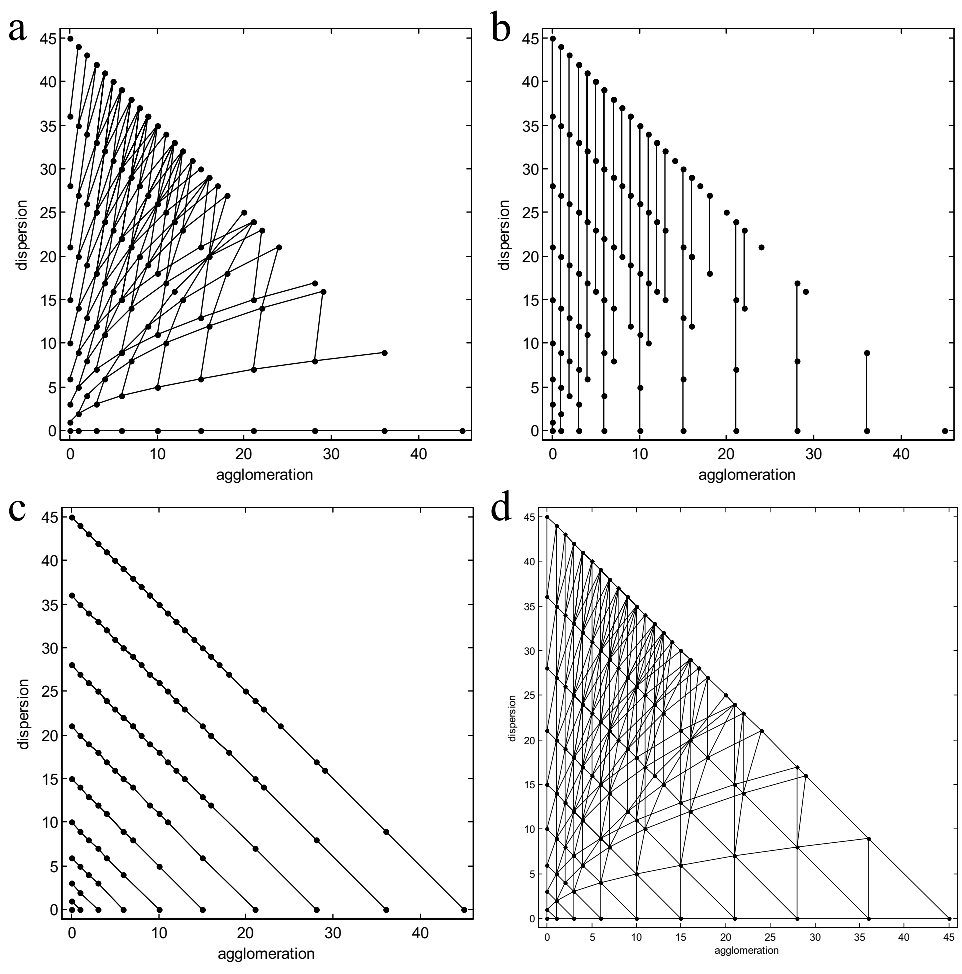

Figure 4: *Networks of operators for* $dn = 10$*:* mnet *(a),* vnet *(b),* tnet *(c), and the overall combination of the three basic nets (d). Each point corresponds to a partition or group of h-related partitions ([19], p. 70). Graph produced by Parsemat® ([15]).*

According to the used operators, some networks are formed, each with a different type of connection, transformation or syntax ([8]).

The three basic networks, constructed from the resizing *(m)*, revariance *(v)* and transfer *(t)* operators respectively, are presented.

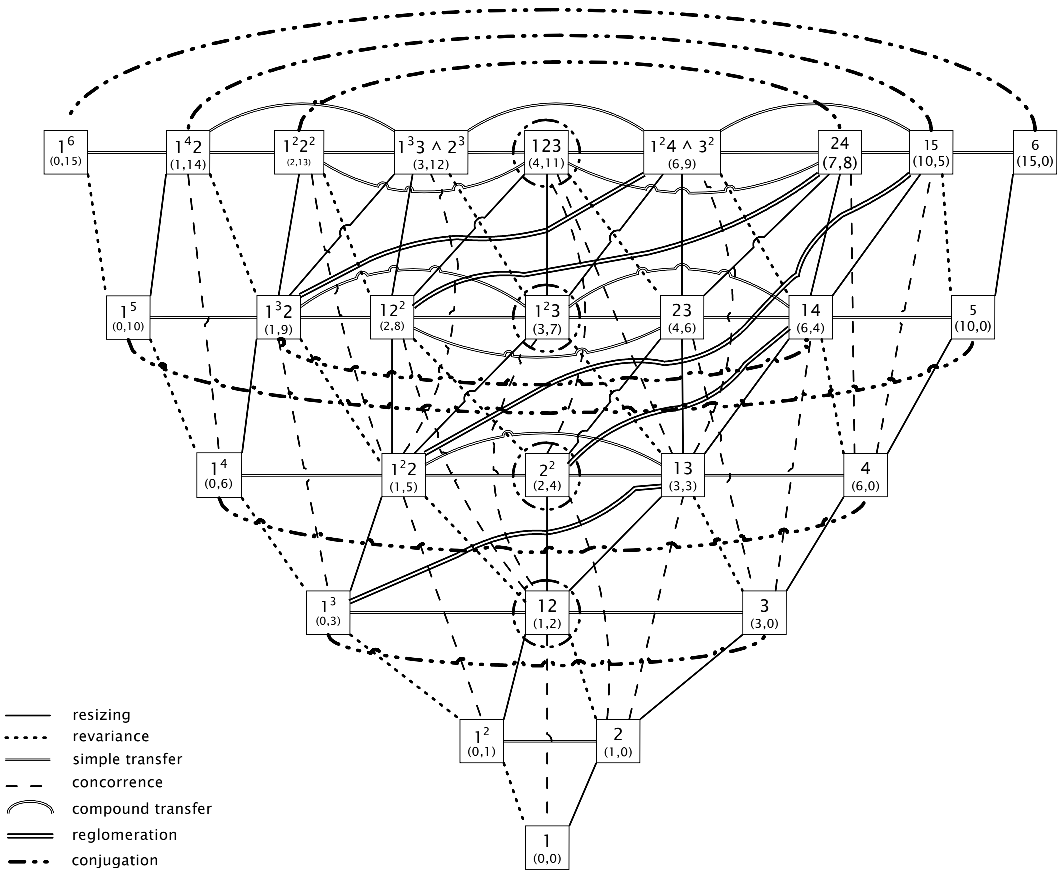

Figure 5: *Partition Young Lattice for $dn = 6$. Basic operators (resizing, revariance, [simple] transfer and concurrence)are presented along with compound and auxiliary ones. The overall resultant net can be used by the composer as a board game, where each type of move means a kind of textural transformation ([19], p. 69)*

Mnet is a fractal structure, whose lines always start with polyphonic configurations, which have their parts gradually resized, and which allows the bifurcation in some points, when there is more than one part available for the resizing. This operator is the one that brings for the partitiogram the greatest irregularity and unpredictability, especially in densities greater than 6.

Vnet is a more predictable network, where each row maintains the fixed agglomeration index, only with more parts being added. For this reason, vertical lines are formed, which start whenever there is a new configuration imbricated by the resizing.

Tnet is a network delimited by density-numbers, that is, all elements components of the same line have the same number of sounding components. Up to density-number 6, tnet presents itself in a linear, simple way. From there, bifurcations make the network increasingly complex, as some partitions start to establish multiple transfers. This unfortunately is not visible in the tnet graph, as the lines are superimposed.

Partitiogram can be read also as a *Hasse Diagram*. This graph is constructed with the basic elements of a list or taxonomy, connected by the relation of inclusion, in a bottom-up arrangement.

Each element can be docked in its superior connected neighbour (and in all the connected superior elements) as a subgroup. This graph can show all the relations of a set with maximum economy of information. The representation of a lexset of a number, for example, the integer 4, in a Hasse diagram, shows 11 elements ([2], p. 108) .

Partitional Young Lattice (*PYL*), on the other side, is an adaptated visualization of the original Hasse Diagram for integer partitions. It includes, beyond the partitions themselves, the subscription of agglomeration and dispersion indices and the qualification of the relations of inclusion as operators ([13], [19]).

In PA, concepts are generally observed as tools within compositional games (even though they may be used in many other ways, for example in musical analysis or hermeneutics). In this sense, both the partitiogram and PYL are seen as a phase space or a board game, respectively, where trajectories are traced, as the composer progresses in his creative work.

There are currently some important theoretical expansions and applications of PA in musical analysis and composition. They just fall outside the scope of this paper.

Partitional Complexes

Partitional Complexes can be defined as a bunch of partitions that cooperate to set a referential partition domain.

The main ideia is that the independence between parts transcends the simple contraposition. In a texturally diverse environment, independence is built when there is complete autonomy between parts. This situation presupposes a certain detachment among parts, which can cause eventual congruences, as a result of fortuitous movements that come to occur without detriment of the global textural conception.

For an organic realization of a given textural configuration, it is necessary to take this dynamics into account, organizing the textural thinking into hierarchical layers.

One of the possibilities of organizing textural configurations comes from the consideration of:

- *Subpartitions*: those textures that are revealed in the partial or incomplete presentations of a referential partition;
- *Subsums*: eventual congruences that their parts offer, constituted by all the sums of the parts of the referential partition.
- *Subsums of subpartitions*: when both processes can be applied concurrently.

As an example, the partition (1^3) or (1.1.1) have, as incomplete presentations of its parts, the subpartitions (1) and (1.1). The subsums are (2) and (3), resulting from the sums of (1.1) and (1.1.1), respectively. There are a subsum of subpartitions, (1.2), where two parts are summed and one preserved. Finally, the partitional complex of (1^3) has six elements: $\{(1^3), (1), (1.1), (2), (3), (1.2)\}$. This is equivalent to say that constructing a three voice polyphony implies in the articulation of some or all the partitions of the complex.

We then have a distinct development of each ingredient for each partition, thus defining a differentiated number of choices for the composer. Massive partitions are the most limited, including only themselves in its partitional complex, while polyphonic ones offer the most numerous alternatives, encompassing all the partitions for the correspondent density-number. Among the massive and polyphonic partitions, there are those partitions that mix blocks and lines together (Table 1).

The information presented in Table 1 can be arranged in a Hasse Diagram through the combination of two relationships: "is subpartition" and "is subsum" (Figure 6). Complexes can be constituted by following a top-down direction from the chosen referential partition through all

Table 1: *Partitional complexes for quartets: referential partitions, with their cardinalities, density-number, subpartitions, subsums, subpartitions of subsums, partitional complex, and the cardinality of the partitional complex [20], p. 123, [9], p. 35-36.*

referential partition	card.	DN	subpartitions (Sp)	subsums (Ss)	subpartitions of subsums	partitional complex	complex card.
(1)	1	1	-	-	-	(1)	1
(2)	1	2	-	-	-	(2)	1
(1^2)	2	2	(1)	(2)	-	(1^2), (1), (2)	3
(3)	1	3	-	-	-	(3)	1
(1.2)	2	3	(1), (2)	(3)	-	(1.2), (1), (2), (3)	4
(1^3)	3	3	(1), (1^2)	(2), (3)	(1.2)	(1^3), (1), (1^2), (2), (3), (4), (1.2)	6
(4)	1	4	-	-	-	(4)	1
(1.3)	2	4	(1), (3)	(4)	-	(1.3), (1), (3), (4)	4
(2^2)	2	4	(2)	(4)	-	(2^2), (2), (4)	3
$(1^2 2)$	3	4	(1), (1^2), (1.2), (2)	(3), (4)	(1.3), (2^2)	$(1^2 2)$, (1), (1^2), (1.2) (2), (3), (4), (2^2), (1.3)	9
(1^4)	4	4	(1), (1^2), (1^3)	(2), (3), (4)	(1.2), (1.3), (2^2), $(1^2 2)$	(the whole lexset)	11

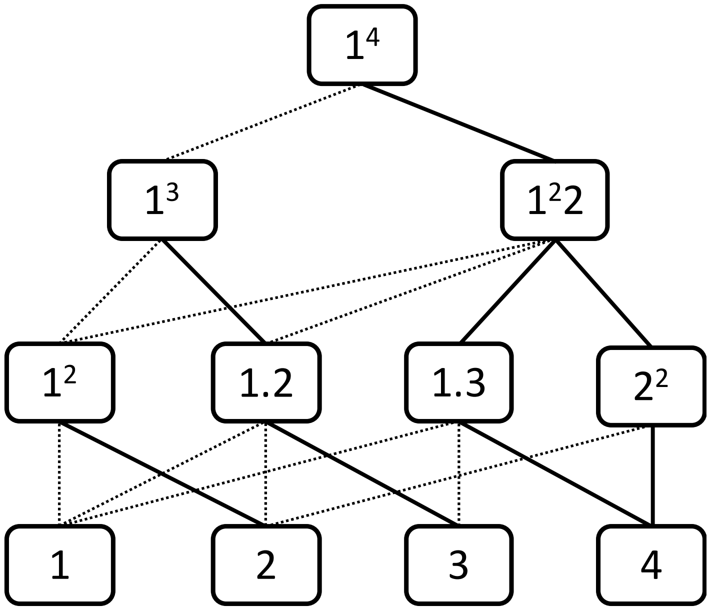

Figure 6: *Partitions for quartet, arranged in partitional complexes. Each partition constitutes its own complex by gathering all top-down connections departing from it. Lines are read bottom-up: dotted lines indicate* subpartitions *and full lines indicate* subsums*. Four levels of crescent textural complexity are constituted.*

connections below. From this point of view, it turns clear that for a quartet (density-number 4) there are four levels of gradual complexity, read bottom-top. Once more, massive partitions – (1), (2), (3) and (4) – stay alone inside its own complexes, as stated before. Complexes of polyphonic partitions – (1^2), (1^3) and (1^4), on the other hand, embrace all partitions from integer 1 to its own density number. The arrangement of Figure 6 shows also some imbalanced distribution, as the partition ($1^2 2$) have four immediate bottom connections (and 13 in total), while (1^3) has only two (7 in total). Each connection corresponds to an available path for more or less (depending on the number of moves in the graph) parsimonious transformation between textural configurations. In this specific case (partitional complexes), the transformations occurs with the ommision or addition of a part (line or block), or the merging of existent parts without any subtraction.

Intersection, nesting and partitional complexes

The introit of the tenth piece from Schoenberg's *Pierrot Lunaire* (*Raub*) is a remarkable example of an organization of textural language through a hierarchical arrangement. There are two distinct regions, the first (mm. 1-2.3), with more rarefaction and discontinuity, with loose notes in staccato; the second (mm.2.2.2-3), more continuous and repetitive, with coordination between the majority of the attacks of all instruments.

Change from one region to the other occurrs smoothly, as the flute begins the second region while the strings are still finishing the first one, drawing a quite diagonal division line between the two domains.

Inside the atomic level of partitions, there are a sudden change of behaviour as well. The sequence of partitions of the first region orbits around a limited set of configurations, all belonging to the partition complex of (1.1.4), yet the referential partition in fact does not appear at all:

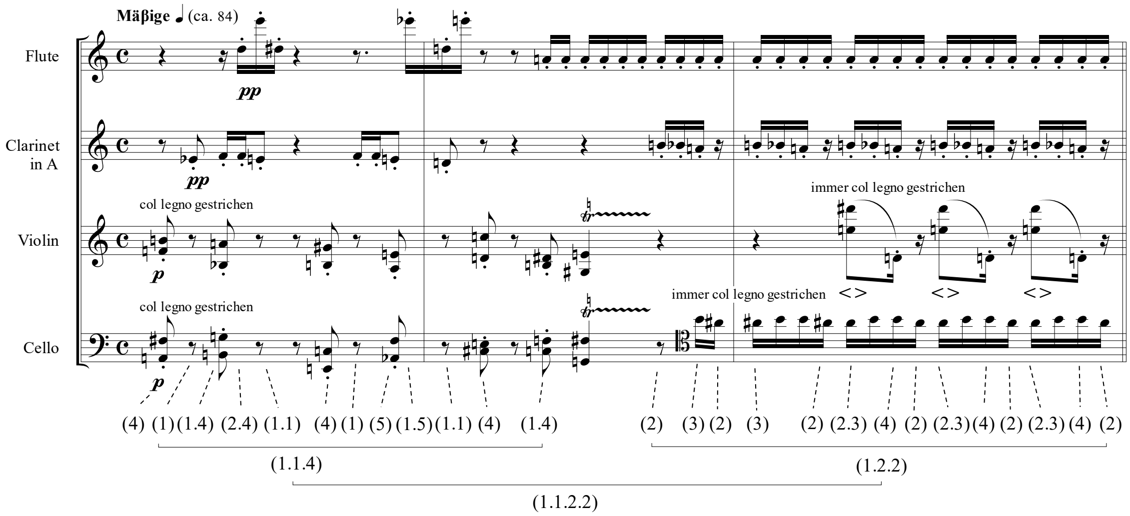

Figure 7: *Partition complexes in Schoenberg's* Raub, *from* Pierrot Lunaire *([21], p. 89, mm. 1-3)*

subpartitions (1), (1.1), (1.4), (4); subsums (1.5), (5) and (2.4).

The emergence of the referential partition (1.1.4) is not surprising at all, considering the motivic role of the wind instruments against the massive blocks of the strings on the initial measures of the piece. What is more striking here is that it was not necessary, at any point of the region, to present this relations literally, through a real partition. That is, an concrete realization of a polyphony with extense lines with attacks and prolongations sustained by blocks configured to working all together. On the contrary, the referential partition is, in this little region, quite virtual.

The second region brings partitions which would not be compatible with the previous complex - for instance, (3) and (2.3). But all the presented partitions can be ascribed to another referential partition - in this case, (1.2.2), with subpartition (2) and subsums (3), (2.3) and (4). The elements that induce this result are identifiable as well - the clarinet, assuming a more independent role, due to a periodic interruption, distinguishing itself from the flute and cello, which are at this point filling in a layer of continuous successive attacks. The violin has a more independent role, with the isolated articulation of two notes in a more distant register.

The constitution of the complex can be observed in more detail by the way the partitions are vertically structured. For instance, the alternation between (2) and (3) at the beginning of second region is due to the gesture of clarinet, sometimes participating in the block, sometimes absent. This indicates a textural structure that tends to (1.2), which would be a subpartition of (1.2.2), but in fact is not explicitly stated as such. Similarly, the partition (2.3) is the result of the interval of the violin (2), opposed to the simultaneity formed by the winds and cello (3), in the moments where the clarinet is mixed in the block.

Looking further, the two complexes – (1.1.4) and (1.2.2) – are, in its turn, a subpartition and a subsum, respectively, of (1.1.2.2). This super-complex is also never stated literally, but it can be seen in the overall instrumental partition, in the first region – flute (1), clarinet (1), violin in double stops (2) and cello in double stops (2). In the second region, we have the clarinet motive (1), the insistent notes of flute and cello (2), the double stop of violin in treble register (2) and, finally, the open string of the violin, that can be thought as an independent layer (1), in a lower register.

All these relations are presented then in a partial hierarchical graph of the complex (1.1.2.2), where the two branches of referential partitions (1.1.4) and (1.2.2) generate the remaining ones

(Figure 8).

The branches are also not equally balanced, with the first region, represented by the branch (1.1.4), more populated than the second one, correspondent to partition (1.2.2). This implies that the involved transformation from first to last region is descendent in terms of information – there are more redundancy and confination inside the second branch.

This inference is also supported by formal features. In the first region, blocks articulated by the strings are superimposed and followed by a brief polyphony played by the winds, and that structure is repeated once with little variation in the temporal interval between its elements, at a time distance of nearly five eighths relative to the first presentation.

The second region, in turn, presents more repetitions of shorter modules. The pattern of winds and cello is constructed gradually, lasting three beats and, after presented in its complete form, is stated four times, three of them with a repeated pattern of the violin. This foreground repetition is reflected too in the sequence of partitions, that exhibits for three times the pattern $<(2.3)(4)(2)>$. In the first region, due to the displacement of the elements in the repetition, there are no recognizable patterns in the surface flux of partitions, which grants a bigger amount of information.

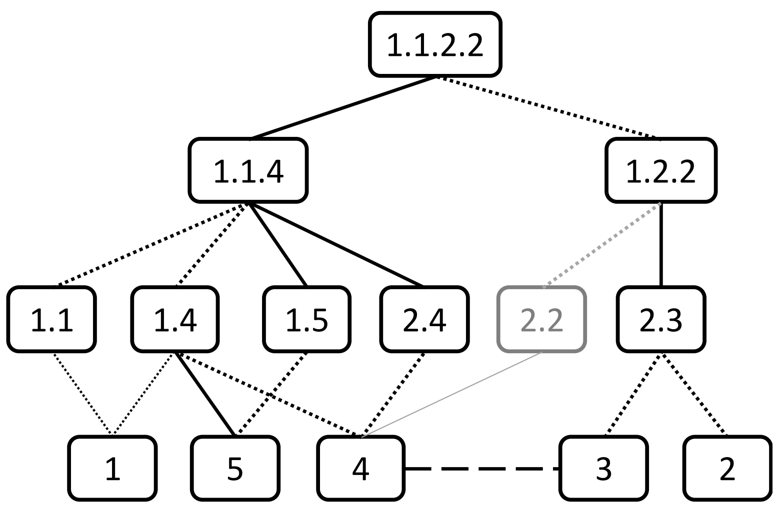

Figure 8: *Partial presentation of partitional complex (1.1.2.2) in Pierrot Lunaire, X - Raub ([21], p. 89).*

The only relation that is not motivated by internal operations of the complex occurs between partitions (3) and (4). Partition (4), specifically, do not fit very well in the frame of second region (1.2.2), because the intermediary partition, (2.2), a subpartition of (1.2.2), is absent, and this gap between (4) and (1.2.2) creates a disconnection between the two configurations. The partition (4) arises when an extraneous pitch is added (just the open string of the violin) to the sounding block (3). This operation is, in fact, not a relation inside a complex, but a real transformation – a simple *resizing* from (3) to (4).

A return to the Berry's example ([3], p. 187) can bring some insights about the deep interaction of partitions and complexes. Berry used an observation window corresponding to the measure, which arises some analytical questions – for example, the binary metric structure of the *fugato*, that leads the author to disregard some combinations that occurs in shorter durations, like, for instance, the convergence between S and A in measure 4, or the beginning of the *tutti* in measure 6, which is registered by Berry only in measure 7 (Figure 1). Some papers was addressed on this subject, trying to explain the cognitive process of defining intuitively what partitions are more important or valid, covering, without expressive results, some hyphotesis like "the most prominent ones" (in terms of peaks of dispersion or agglomeration), or "the more extensive in terms of temporal durations" ([10], [14])

Assessing the configurations with a more refined window (as PA re-evaluate the configuration for each attack, detecting, exhaustively, all used partitions) leads to a more complex and counter-intuitive result. On the other way, it brings some information that is not accessible through an intuitive appreciation.

Applying the concept of complexes can bring some enlightment, even in a very know structure as the *fugato*, where each partial complex gradually blooms from the previous one.

Figure 9: *Partitions and complexes in Milhaud,* A Peine si le coeur, images et figures*. Every complex is encompassed by the coming one, in a nested structure. Referential partition* (1^3*) is not explicitly articulated.*

The interaction between the complexes lead to a embracing structure (1^4), which is compatible with the polyphonic language proposed by the composer. One of the intriguing questions that arouse from the comparation of Berry's analysis and the PA results was the complete absence of the partition (1^3) in the micro-surface of the texture. Even if we consider the traditional gesture of gradual accrual of voices in *fugato* style, a cognitive basis to sustain this so naturally placed conclusion was considered as necessary. The concept of partitional complex can answer this question with ease.

The main point, in this case, is that the construction of partition (1^3) cannot be confined just to the observation of measure 3, where in fact it is not present. It is just the accumulation of all partitions that were articulated until this point that constitutes the complex. The partitions (3) and (1.2) are not sufficient to characterize the complex (1^3). If the piece were summarized by a discourse based on the third measure, certainly it would not be possible to understand the complex as such, but as a complex (1.2), instead.

After the progression reaches its apex, the gradual agglomeration that follows occurs within the complex (1^4), being just an possibility of the writing for voice quartet or choir. In fact, this simplification, as Berry states, is very common in endings and leads to closure.

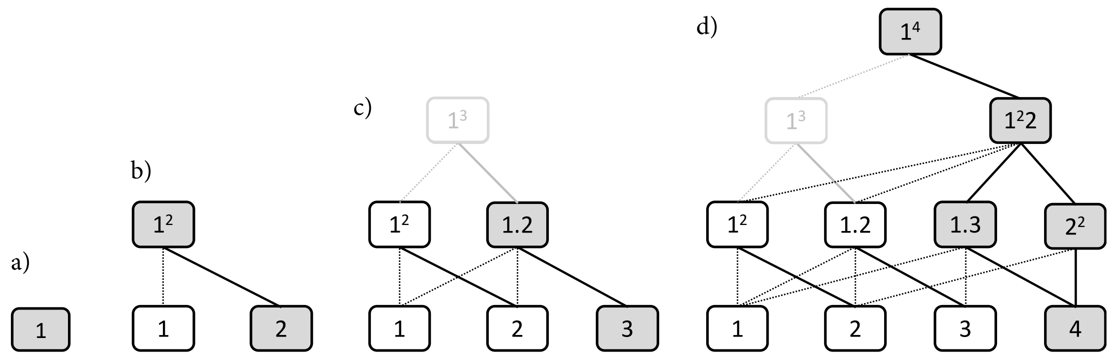

Figure 10: *Sucession of partitional complexes in Milhaud,* A Peine si le coeur, images et figures*: (1), m. 1 (a); (1^2), mm. 1 through 1.3.2 (b); (1^3), mm. 1 through 3.3.1 (c); (1^4), all the excerpt (d). Referential partition (1^3) is never explicitly articulated.*

The middle section of the second movement (mm. 12-28) of Gyorgy Ligeti's *Bagatelles* for woodwind quintet ([17]) is an example of construction of a dramatic curve based mainly in textural development. This time, all regions are very well defined and all referential partitions are stated very clearly (FIgure 11).

There are four domains, referring to partitions (1^2), (1^32), (1^22) and (1^5). They are grouped in two large segments with progressive accrual, each one with two domains.

First segment lasts for 10 measures. It begins with an sparse texture, with only two voices in a calm polyphony, that is suddenly filled with long notes in the fifth to seventh measures, until all the instruments are presented. After that, the initial texture returns, with a profile slightly more prominent.

The second segment lasts for eight measures, where the overall density is noticeably greater than previous one. It begins with a more dramatic tensioning, articulated through the gradual narrowing of the recursive motives, creating superpositions that make the texture also more weighted. The arrival of the massive partition (5) is the apex, from which the following configurations constitute a dissolution.

As the example of Schoenberg, we have also, in the second segment, greater redundancy caused by cyclic return of a partitional segment, in this case the sequence $<(3)(1.2)(1^22)>$. That repetition is clearly used to articulate saturation and to value the arrival of the climax.

Referential partition of first region is (1^2) and, in fact, it stays as the only significant one, as the (1) is only a departure to start the structure. In this sense, the region is very static.

The second region has (1^32), denser partition, found in the local apex. Flute and clarinet are working together, with some sintony with the horn, because of dynamics (whose consideration would cause a trigger of partition (1^23), belonging also to the local complex)

Third region (1^22) seems as a recession compared to the previous complex, but this occurs, as stated before, for balancing further crescendo of the fourth region. A sequence is repeated, just to prepare the arrival of fourth region. In this sense, this complex is a prefix for the last one.

The last region is far more complex, as it articulates the most disperse partition, (1^5), and begins the descent to the more agglomerated one, (5). This contrast is just the main resource to give to this region its dramatic character. The dissolution just uses two partitions kin to (5) in terms of agglomeration. The imbalance of the parts, in this case, have an important role to the rest of the music to come, that will be entirely structured in blocks of (2) and (3), being the (2) constituted by unisons, most of the time.

Figure 11: *Sucession of partitional complexes in Ligeti*, Sechs Bagatellen für Bläserquintett *(II, mm. 12-28)*

Conclusions

Different variety of choices provided by each refential partition can be read as degrees of liberty that a composer have in his creative process. Building a section of a piece with a massive partition – for instance, (4) – implies in a restriction that would be lighter with some other partition, like ($1^2 2$). Obviously, the composer always have the freedom to explore other qualities or possibilities he has at hand - harmonies, timbres, rhythms, among many others; but within the specific field of rhythmic partitioning, the distinction is substantial and certainly has a considerable impact on other aspects of compositional work.

In this sense, each partition offers a different potential to develop parsimounious relations inside its complex.

Relations of the various levels of nesting and intersections create an hierarchy comparable to the Schenkerian concepts of *foreground*, *middleground* and *fundamental structure*. Here is a rich field to be explored further by analysts and composers.

There are at the present moment some research being made inside MusMat Research Group concerning this type of analysis, and considering also other kinds of partitional organization, drawn from observation of textural repertoire.

Applications of the partitional complexes to diverse partitionings, like *melodic partitioning*, *event partitioning*, *spectral partitioning* and others ([13]) are in course. Each partitioning has its own idiosyncrasy and has to be evaluated from scratch in this respect, since each handled material has its own nature.

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Iterable Voice-Leading Schemas

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Abstract: *An iterable voice-leading schema combines a voice leading with a permutation that determines how the voice leading is to be reapplied. These structures model a wide range of repeating musical patterns from the Renaissance to the present day.*
Keywords*: Voice leading. Sequence. Round. Counterpoint. Orbifold. Geometry.*

This paper uses simple mathematics to analyze a not-so-simple collection of musical patterns in which a single voice leading is repeatedly reapplied. The resulting collection encompasses classical sequences as a special case, while also including a wider range of phenomena familiar from other musical styles. Mastering these repeatable patterns is an important component of contrapuntal expertise.

The mathematical background is adapted from previous work ([3],[4],[5],[1]): *pitches* are points in a line $\mathbb{R}$, with integer-valued pitches being *scale tones*; a scale here acts as a both a coordinate system and a metric whose unit is the *scale step*. *Pitch classes* are points in the circle $\mathbb{R}/c$ with c the (integral) size of the octave. A *path in pitch class space* is an ordered pair (p, r) with p a pitch class and r a real number indicating how that pitch moves; this lifts to a directed line-segment $p \rightarrow p{+}r$ in the pitch space $\mathbb{R}$. In this context, paths in pitch-class space can be understood either as points in the circle's tangent space or as homotopy classes of paths in the circle itself. Paths can be related by transposition or inversion, with $\mathbf{T}_x((p,r)) \equiv (\mathbf{T}_x(p), r)$ and $\mathbf{I}_x((p,r)) \equiv (\mathbf{I}_x(p),\text{-}r)$. A *voice leading* is a multiset of paths in pitch-class space, determining how the notes of one chord move to those of another; these are described colloquially by phrases such as "C major moves to F major by keeping the root fixed, moving E up by semitone to F, and G up by two semitones to A." A *transpositional voice leading* is one in which every path has the same real number—moving all its notes in the same direction by the same number of scale steps.

A voice leading $\mathcal{V} = A \rightarrow B$ defines a *transpositional voice-leading schema* $\boldsymbol{\mathcal{V}}$ that can be uniquely applied to any transposition of the initial chord, so long as it contains no pitch-class duplications and is not transpositionally symmetrical: $\boldsymbol{\mathcal{V}}(\mathbf{T}_x(A)) \equiv \mathbf{T}_x(\mathcal{V})$. When B is transpositionally related to A we can therefore reapply the voice-leading schema $\boldsymbol{\mathcal{V}}$ in a chain, generating a repeating musical pattern that sends each note n cycling through chordal elements:

$$n, \mathbf{T}_x(\varphi(n)), \mathbf{T}_{2x}(\varphi^2(n)), \mathbf{T}_{3x}(\varphi^3(n)), ..., \mathbf{T}_{ix}(n)) \tag{1}$$

with i the order of the permutation, so that $\varphi^i(n) = n$. When B is a transposition of A, then the permutation φ is uniquely determined by the voice leading (so long as both chords are suitably nonredundant). In the general case, where A is symmetrical or B is not related to A, we have to supply the permutation φ explicitly. (Geometrically, the permutation φ contains information about the path along which the vector $\mathcal{V}$ is parallel-transported from point A to B.) We therefore define an *iterable voice leading schema* $\boldsymbol{\mathcal{V}}_{\mathbf{I}}$ as a pair $(\mathcal{V},\varphi)$ with $\mathcal{V}$ a voice-leading and φ a permutation acting on the musical voices, allowing us to iterate the schema in analogy to (1).

Figure 1 shows iterated voice-leading schemas spanning more than four centuries. The *period* of a schema is the minimum number p such that the voice leading connecting chord 1 to chord 1 + p moves every voice by the same interval modulo the size of the scale; this is shown by the brackets on Figure 1. The *wraparound voice leading* connects the chord at the start of one period to the chord at the start of the next: a sequence is *transpositional* if this voice leading is transpositional (Figure 1b-d); if not, we have a *contrary-motion sequence* where the relative distance of voices changes by one or more octaves with each period (Figures 1a and 2).

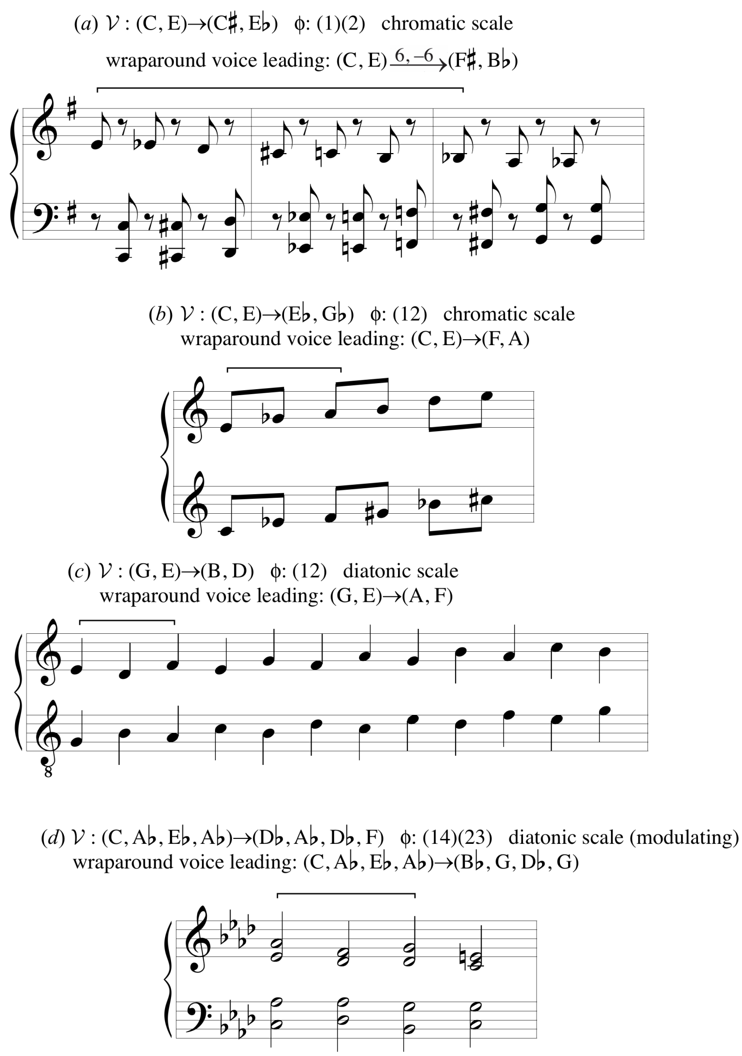

Figure 1: . *Iterated voice-leading schemas. (a) Beethoven Op. 90, I, mm. 105–107; (b) a central intervallic pattern in Stravinsky's* Firebird*; (c) a passage from the Sanctus of Josquin's Mass* L'Ami Baudichon*, mm. 14ff.; and (d) a reduction of the descending-fifth sequence in the development section of the first movement of Beethoven's Op. 2 no. 1. The cyclic notation (12) indicates that the music of voice 1 in the first chord passes to voice 2 in the second (counting from bottom to top), with the music of voice 2 passing to voice 1.*

Such sequences generally produce canons, with the nature of φ determining the structure of the canonic voices. When φ has a single cycle, each voice articulates the same pattern of intervals, forming a single canon as in Figure 1b-d. When φ has two cycles, repeated applications produce a *double canon* with two distinct groups of canonically related voices, as in Figures 1d and 2; more generally an n-cycle permutation produces an n-fold canon. (In the limiting case, where the schema uses n distinct cycles to connect n-voice chords, each voice progresses along its own interval independent of the others.) In Renaissance music, iterated voice-leading schemas tend to link adjacent chords (Figure 3); in classical music, they frequently connect nonadjacent sonorities (Figure 4).

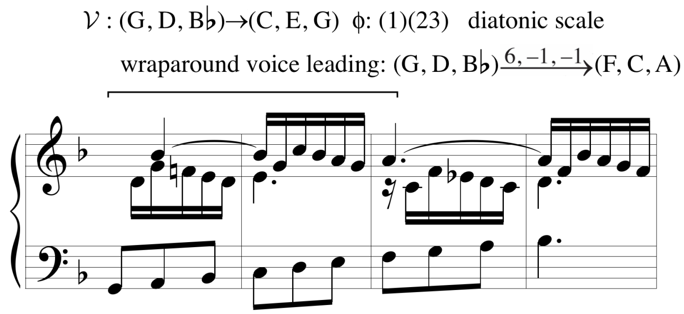

Figure 2: *A contrary-motion sequence in the first F-major fugue from the* Well-Tempered Clavier, *mm. 56ff.*

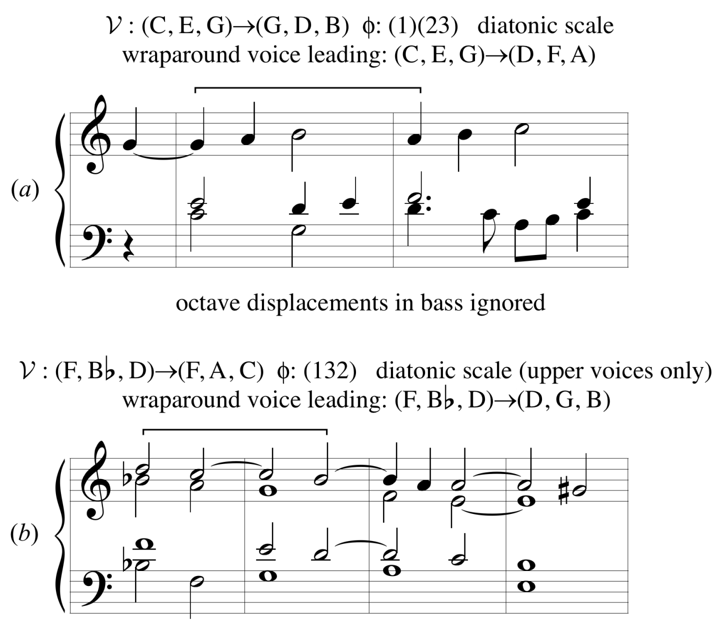

Figure 3: *Iterated voice leadings in (a) the Sanctus of Palestrina's Mass* Ave Regina Coelorum, *m. 19ff. and (b) the Sanctus of Palestrina's Mass* Spem in Alium, *mm. 93ff., presenting six successive ascending fifths in a row.*

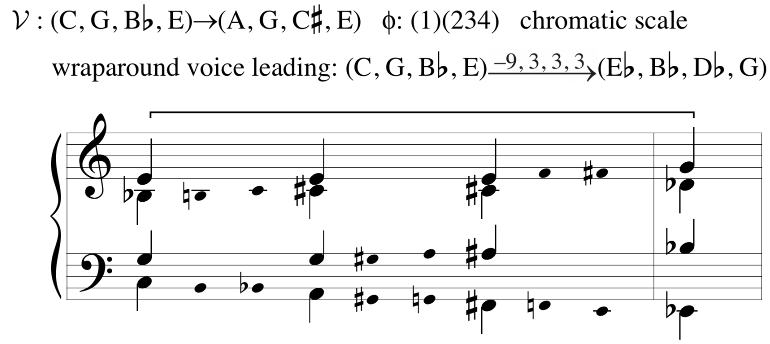

Figure 4: *The "Omnibus sequence," a common Romantic contrary-motion pattern [6].*

Many familiar musical patterns can be analyzed using this framework. In some cases, these structures have traditional music-theoretical names: for instance, a *round* is an iterable voice-leading schema whose generating voice leading is of the form $A \rightarrow A$, connecting a chord to itself (Figure 5). Similarly, previous theorists have explored *wedges* generated by the combination of a nontranspositional voice leading $\mathcal{V}$ with trivial permutation φ, so that all voices move along their own individual paths (e.g. Figure 1a, [2, p. 124ff.]). Sequences are *canonic* when φ is nontrivial (as in all but one of the preceding examples), and *noncanonic* otherwise (Figure 1a, Figure 6 below). A final possibility is a variable sequence in which either the transposition or the permutation changes over the course of the sequence: for instance, in Figure 6 V^6_5–I progressions descend by three thirds and one second, returning to their initial position after four units rather than seven; here the voice leading from C to F is *individually T-related* to the previous voice leadings [4], with the second chord being one step too high. More remarkable is Figure 7, where Bach changes the permutation while preserving the voice leading.

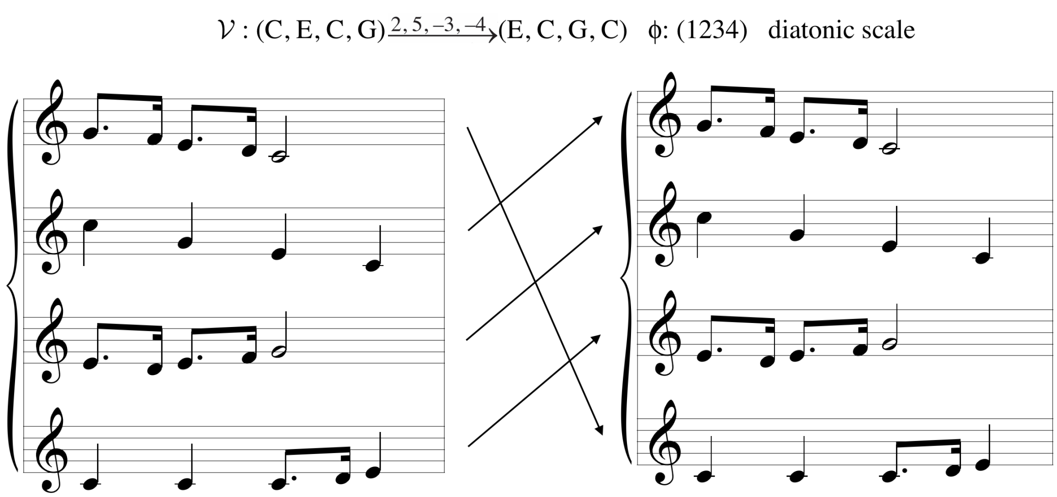

Figure 5: *The round "Row, row, row your boat."*

Figure 6: *A variable sequence in Beethoven's Op. 31 no. 3, I, mm. 68–70.*

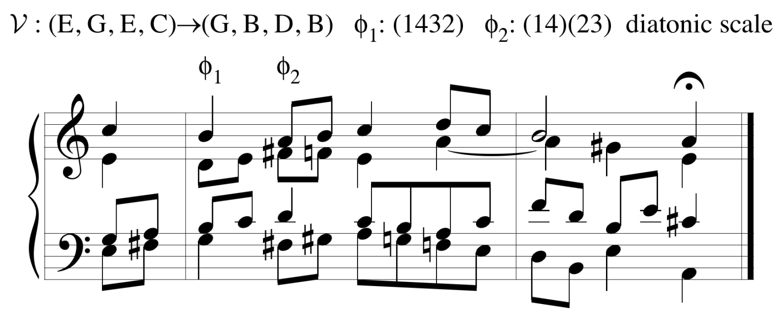

Figure 7: *The final phrase of Bach's chorale "Ach lieben Christen, seid getrost" (BWV 256, Riemenschneider 31).*

All of which is fairly clear when set out in abstract, mathematical form. However, I can testify that even an analytically minded musician can spend a lifetime working with iterated voice-leading patterns without clearly understanding their general structure.

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The Function of Orchestration in Serial Music: The Case of Webern's *Variations* Op. 30 and a Proposal of Theoretical Analysis

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Abstract: *Webern's* Variationen op. 30 *constituted a well-known milestone in the consolidation of serialism as a compositional technique. It has been the target of a large number of investigations focused on the way the composer developed a broader concept. As could it not be otherwise, his orchestral design is also closely tied to his structural concerns. However, it appears to lack a systematization of the composer's orchestration principles. Thus, to propose an analysis of Webern's orchestration, one need to elaborate an* ad hoc *method, virtually starting from scratch. This paper aims to describe the main points of this method in its current experimental stage. At the same time, we point to some conclusions about Webern's orchestration according to his aesthetics.*

Keywords*: 20th Century musicology. Webern. Variations op. 30. Textural analysis. Analysis of orchestration.*

1. The *Variations* op. 30 by Webern

Webern's *Variationen op.30* of 1940 (WEBERN, 1956) constituted a milestone in the consolidation of serialism as a compositional technique. For this reason it has been the target of a large number of investigations, seeking the scrutinization of the peculiar way in which the composer absorbed the technique inherited from Schoenberg to develop a broader concept. This includes a rethinking of elements of the German tradition of composition, especially when trying to achieve a synthesis between the principles, formerly mutually excludent, of permanent variation and cyclic form—in this case, which is named by Webern as the *adagio form*—and the parsing of the total chromatic into smaller, musically significant units. This Webern's last row is precisely known by the complex manner with which it operates multiple symmetries. Generated from a motive of four notes, this row is systematically subdivided into tetrachordal subsets (Figure.1), labeled as O1, O2 and O3. However, Kathryn Bailey specifically demonstrates how the *Gestalt* of Webern has also a rhythmic order ([1], p. 224). Indeed, the serial material is supported

*I thank the MusMat research team who provided the English translation and the LaTeX' typeset of this paper and Charles de Paiva (NICS/Mus[3]) for his contribution to the mathematical model and its implementation in the OpenMusic's SOAL library.

by two initial rhythmic motives enunciated in the first three bars and then declined, in order to permeate the entire work in all of its moments. In other words, with Webern the rhythmic patterns acquire as much functionality as the pitch ones.

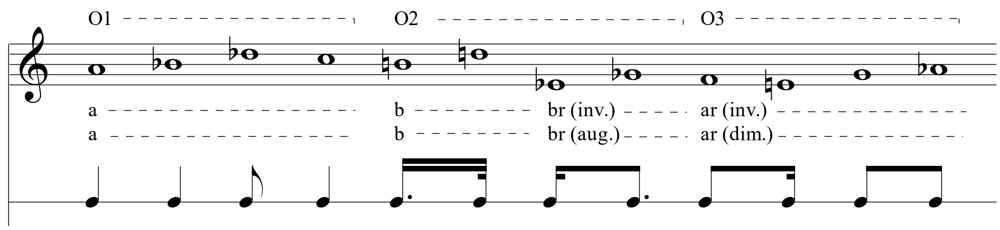

Figure 1: *Original row and basic motives of the* Variations *([1]:224)*

As could it not be otherwise, his orchestral design is also closely tied to his structural concerns, because the orchestral polyphony corresponds "to the extreme point of a motivic-thematic work in which the whole group of voices participates at each moment"([4], p. 31)[1], which necessarily leads to an orchestration with high "timbral chromaticism"([11], p. 426, 434)[2]. However, we conclude that the ways by which the orchestration strategies integrate the process of his compositional project still lack a specific research. Moreover, there is not apparently a systematization of the composer's orchestration principles.[3] There are several reasons for this: first, the composer himself has little reported on this aspect, although, as we can remember, at least considering his orchestration of Bach's *Ricercare*—or, rather, his "analytical instrumentation" ([4])—how this dimension becomes relevant for the consolidation of his serialism. Another reason is the lack of a methodological apparatus that can be compared to the solidity of the tools that a musicologist keeps in hand to turn evident the logical organization of other dimensions, particularly those of pitch and rhythmic patterns, even though Hallis points its relative fragility in the case of Webern.[4] Thus, to propose an analysis of Webern's orchestration is equivalent to elaborate an *ad hoc* method virtually starting from scratch. This paper aims to describe the main points of this method in its current experimental stage.[5] Faced with the magnitude of this task, it is needless to say that we will only come to some insights about the subject. We expect, however, that this first step may encourage the continuation of the project.

[1]Dahlhaus refers to Schoenberg.

[2]This author employs the term "chromaticism" in its etymological meaning, referring thus to the "color" of the sound, that is, to the timbre.

[3]The thesis of Jinho Kim ([11]) is one of the few studies that make some deeper investigation about the question of timbre in the *Variationen*. This author grounds his approach on the question of the setting of relational databases, which is followed by his own interpretation.

[4]"Octatonic collections, the whole-tone scale, symmetry relations, all of them are evident in many of his atonal works [...]. However, his letters, annotations, and conferences do not give support to the idea that he used intentionally them when structured his music " ([9], p. vi).

[5]The publication about the theoretical-analytical model that is presented here is currently being prepared for the Editions IRCAM/Delatour. The procedure described in this article is aided by a computational support developed for this purpose by our research group Mus3: the library SOAL– *SonicObjectAnalysisLibrary*, for the environment *OpenMusic*, proposes a series of functions that incorporate the equations and other calculations explicitly or implicitly described here, and in particular the soal-texture-complexity function that specifically addresses the method of textural analysis (GUIGUE, 2016). Last version at: http://git.nics.unicamp.br/mus3-OM/soal4/tags

2. Seven "Sonic States"

Both the genesis and the Variations' formal framework are well known, including through data informed by the composer himself. In a famous letter addressed to Hildegard Jones, Webern comments: "Here are six notes enunciated in a certain way by succession and rhythm and what follows...it is nothing more than this form, always present !!! [...] All these metamorphoses of the first form give birth to the 'theme'. This new unity, in turn, passes through several metamorphoses, which, merged into a new unity, results in the form of the whole" ([16], preface).[6] Webern organized the overall format of the piece in seven sections (which are intended to meet, as already pointed out, the format of a three-part adagio):

- Introduction (mm. 0-20);
- Theme 1 (mm. 21-55);
- Transition 1 (mm. 56-73);
- Theme 2 (mm. 74-109);
- Recapitulation / "in the manner of a development" (mm. 110-134);
- Transition 2 (mm. 135-145); and
- Coda (mm. 146-180).

However, as properly pointed by Kim, who is based on the same source, "the division of the work into seven sections is inserted not without difficulties or contradictions within the scheme of the form of variation, in the sense that both the coordination of the introduction with the theme, and of the theme with the first variation, are not completely successful" ([11], p. 377). Hence, Kim prefers to support, as Makis Solomos ([13]) had done for opus 21, a structuring based on the "perception of a succession of seven sounding states" corresponding to the seven sections of the macro-form ([11], p. 377).

He describes these states by means of some dimensions that resemble those with which we have approached the notion of sonority as a structuring element of composition ([7]). More specifically, as summarized in Figure 2: the *ambitus*; the relative achronic density (which Kim describes as "the density of notes relative to range"); something that approaches what we define as relative diachronic density (in this case, Kim counts the number of notes per measure[7]). He also evaluates the number of notes per section, the number of different notes per section, and the number of bars. It also addresses the intensity, albeit in a concise way, since it is only limited to the three broad levels ([11], p. 423). In the following chart, we normalize the data provided by Kim to produce a visualization of the configuration of each section, according to some of his criteria. Kim's representation of the evolution of the *ambitus* per section is reproduced in Figure 2, bottom.

These surveys bring statistical data that allow us to observe the low density of sounds per unit of time (*diachronic density*), which would be "compensated" by a much higher *achronic* density, within a generally broad range—where the third section differs from the rest by its somewhat narrower tessiture. But these data are not sufficient for a more effective analysis of the impact of parsing of sonorities on formal configuration. A second step includes an investigation of the orchestration strategies adopted by the composer. Kim analyzes these by means of calculation of *timbral entropy*, assuming that the more the music has timbral information, the more the entropy value rises. This would imply greater activity or complexity, towards disorder. He also collects this information from the mapping of the frequency of appearance of each instrument of the

[6]Declaration that perfectly corroborates Bailey's analytical approach.

[7]See our definition of these two types of density ([7], p. 396,399). See also [8], p. 5, 12), under the name of *spatial-density* and *events-density*, respectively.

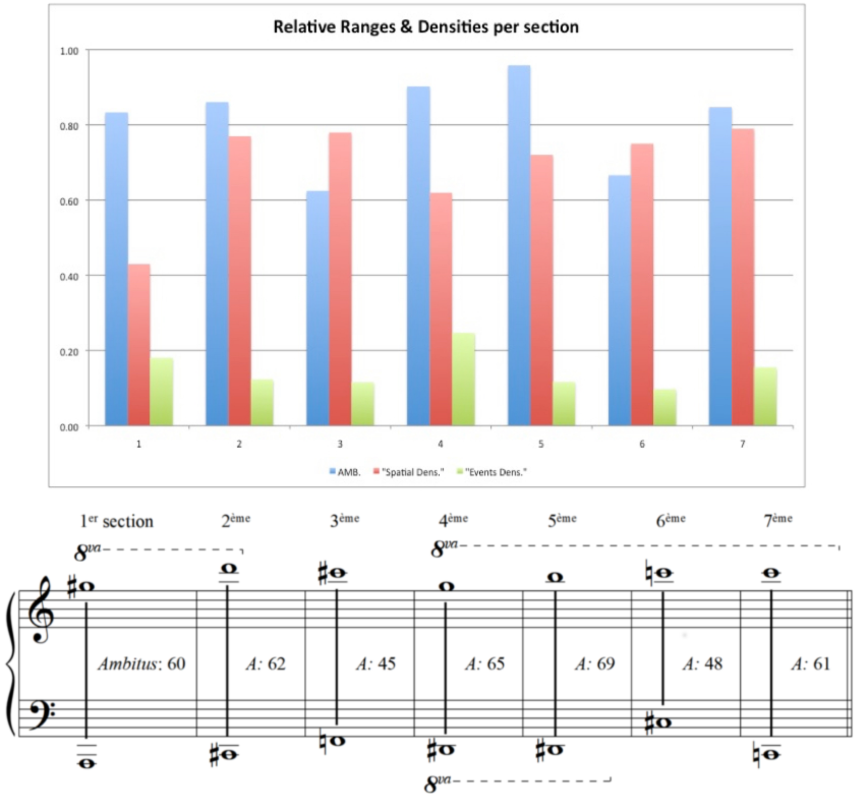

Figure 2: *Some sound characteristics of the* Variations*, section by section. Histograms (top): the* ambitus *(AMB.), blue; the* ***relative achronic density*** *("Spatial Dens."), red; the* ***relative diachronic density*** *("Events Dens."), green; bottom: the ambitus of each main formal section, according to Kim [11], p. 395-396.*

nomenclature (Ibid., p. 310). Kim then applies the entropy-calculation formula (Equation 1) based on Shannon-Weaver model ([12]).[8]

$$P = -(P_1 log_2 P_1 + P_2 log_2 P_2 + P_3 log_2 P_3 + ... + P_n log_2 P_n) = -\Sigma(P_i log_2 P_i) \qquad (1)$$

Albeit revealing some aspects peculiar to Webern's aesthetics, the analysis through entropy only suggests tendencies—to *order* and *predictability*, or to chaos and surprise. Thus, it seems us too generic. Moreover, in order to assemble his database, Kim only takes into account the instruments indicated in the nomenclature, thus neglecting the multiple details of timbral variation prescribed by the composer during the course of the work (*mutes*, *pizzicatos*, etc.). We will not, however, discard the entropy information. It will be applied to our method of gathering information about the sounding resources employed, during the retrieving of the instrumental resources data, as it will be described in the next session of this paper.

[8]We retrieved this formula in order to implement the function *relative entropy* in the SOAL library ([8]:22).

Sonic Res.(SRI): bars>	**56**	**58**	**59**	**60**	**62**	**63**	**65**	**67**	**68**	**69**	**70.3**	**71.2**	**72**	**73**
Fl.	1			1		1	1		1	1	1			
Fl. ***flatterzung***														
Ob.	1			1		1	1		1	1	1			
Cl.[B♭]	1			1		1	1		1	1	1		1	1
Bass Cl.				1		1	1		1	1	1			
Hn.[F]														
Hn.[F] ***sord.***	1	1		1		1	1			1		1		
Tpt.[C]														
Tpt.[C] ***sord.***	1	1		1		1	1			1		1		
Tbn.														
Tbn. ***sord.***	1	1		1		1	1			1		1		
Basstuba														
Basstuba ***sord.***				1		1	1			1		1		
Celesta					4				4					
Hp.		4			4				4	4				
Hp. ***harm. fing..***														
Timp.														
Timp. trill														
Vn. I ***solo arco***		1												1
Vn. I ***solo pizz.***													1	
Vn. I ***solo pizz. sord.***														
Vn. I ***div. pizz.***						1								
Vn. I ***tutti arco***								2		2				
Vn. I ***tutti arco sord.***														
Vn. I ***tutti arco harm.***														
Vn. I ***tutti pizz.***														
Vn. I ***tutti pizz. sord.***														
[etc.]														
D.B. ***pizz.***														
n. Sonic Resources (nSR)	9	5	3	8	2	16	8	4	8	11	8	8	2	2
((partition)(crit.))	((3 3 3) (2))	((3 1 1) (2))	((3) (5))	((4 4) (1))	((1 1) (1))	((4 4 4 4) (1))	((4 4) (1))	((2 2) (1))	((4 2 1 1) (1))	((4 4 2 1) (1))	((4 4) (1))	(4 4) (1))	((1 1) (1))	((1 1) (1))
WNR	0,67	0,49	0,34	0,64	0,21	0,85	0,64	0,43	0,64	0,74	0,64	0,64	0,21	0,21
(-a)	-9	-3	-3	-12	0	-24	-12	-2	-7	-13	-12	-12	0	0
(d)	27	7	0	16	1	96	16	4	21	42	16	16	1	1
(-a + d)	18	4	-3	4	1	72	4	2	14	29	4	4	1	1
RVC	0,17	0,03	-0,02	0,04	0,01	0,72	0,04	0,02	0,14	0,29	0,04	0,04	0,01	0,01
SRC	1	0,39	0,25	1	0,16	1	1	0,34	0,62	0,98	0,47	0,41	0,16	0,16

Table 1: *An excerpt of the mapping of* Variations' *sonic resources from mm. 58 to 73 (top line; bar numbers also label the Local Sonic Setups); Left Column:* ***Sonic Resources Index*** *(the full list contains 78 itens); Other columns: The sonic resources content of each* ***Setup****; integer corresponds to the number of simiultaneous notes the instrument plays; Line 'n.* ***Sonic Resources*** *(****nSR****): number of Sonic Resources per Setup. Other lines antecipate analytic data which will be described later in the paper.*

3. An analysis of the instrumental partitioning

3.1. The sonic resources index

The choice of the instrumental colors determined *a priori* by the composer seems to be a good starting point for the analytical process. The procedure consists, in a first quantitative stage, of identifying in the score the complete range of the instrumental sonorities listed by the composer, establishing a Sonic Resources Index (SRI), which is the set of sonic resources that arise along the work (see Table 1, left column). This index has the format of a list that corresponds to the nomenclature of the instrumental parts indicated by the composer, provided in the caput of an orchestral score (**OP**, **Orchestral Part**s), with the addition of all indications and information, textual or symbolic, that aim at producing a differentiated sonority in the instruments or instrumental groups, which are indicated by the composer, either in the caput or inside the score. These correspond, in particular, to *divisi and soli* indications, and modalities of modifications of the sound by mechanical means (mutes or others) or specific expanded techniques (*flatterzunge*, etc.). The **SRI** index then encompasses the universe of tone colors, or Sonic Resources, from which the composer will extract subsets along the work, called **Local Sonic Setups (LSS)**, or, in short, **Setups**.

3.2. Local Sonic Setup and instrumental distribution

A **Local Sonic Setup** is a particular instrumental configuration at any given moment. Composed of one or more **Sonic Resources**, they form the core of the composer's orchestration strategy. A new **setup** is identified each time the composer changes the instrumental distribution, adds or modifies a timbre. The number of sonic resources identified in the *Variations*, starting from an initial nomenclature of 15 **Orchestral Parts**, increases to 78, due to the diversified use of the sonic modulation resources in the strings (*arco, pizz., sordina, sul ponticello, tremolo, harmonics*, and various modalities of *divisi* and *soli*), and the permanent alternation of brasses between normal and muted sounds. A survey of SRs in the *Variations* is presented in Table 2 (left column).

The complete chart can be used to map the distribution of the instruments, therefore allowing inferences about prevalent sonorities. In this work, the three higher woodwinds stand out due to their recurrence (they very often play together, even in unison, as will be seen), as well as the muted trumpet. In fact, the graph (Figure 3) shows clearly how evident is Webern's preferential use of the mutes in the brasses. In contrast, due to the large amount of sonic transformation techniques employed, each **Setup** involving the string instruments has a low reiteration rate.

An integer, corresponding to the number of notes that an instrument plays simultaneously, has been inserted for each **Sonic Resource** (**SR**) active in each Setup , which in turn are summed to reach a global value (see Table 2, line "n. Sonic Resources"). Each Setup is then classified according to the index Weighted Number of Resources (WNR) (Table 1, line WNR) that corresponds to the ratio between the number of SRs it contains and the total number of SRs that could be used. The procedure employed for reaching this number consists of establishing, on the one hand, the amount of instruments prescribed by the composer, ordered by instrumental sections, and, on the other hand, the total number of sonic effects that each instrument is called to perform during the course of the work. From this double list is extracted the minimum value for each instrumental section. Indeed, of the two things one: either the number of sonic effects is smaller than the number of instruments in the section (in this case, the number of SR corresponds to the number of different effects of timbre) or is the number of instruments that is smaller than the number of requested effects. In the latter case it is this number that imposes its own restriction. Table 1 shows the application of this procedure in the work.

Nomenclature	**#instr.**	**#SR**	**Min.**
Fl	1	2	1
ob	1	1	1
cl	1	1	1
Hn	1	2	1
Tp	1	2	1
Tbn	1	2	1
Tub	1	2	1
Cel	1	1	1
Hp	1	2	1
Timp	1	2	1
vln I solo	1	3	1
Vln I Tutti(div.)	2	5	2
Vln II solo	1	2	1
Vln II Tutti(div.)	3	5	3
Va solo	1	4	1
Va Tutti (div.)	3	5	3
Vc solo	1	3	1
Vc Tutti (div.)	3	5	3
CB	1	3	1
TOTAL	**26**	**52**	**26**

Table 2: *Number of Sonic Resources available at once.*

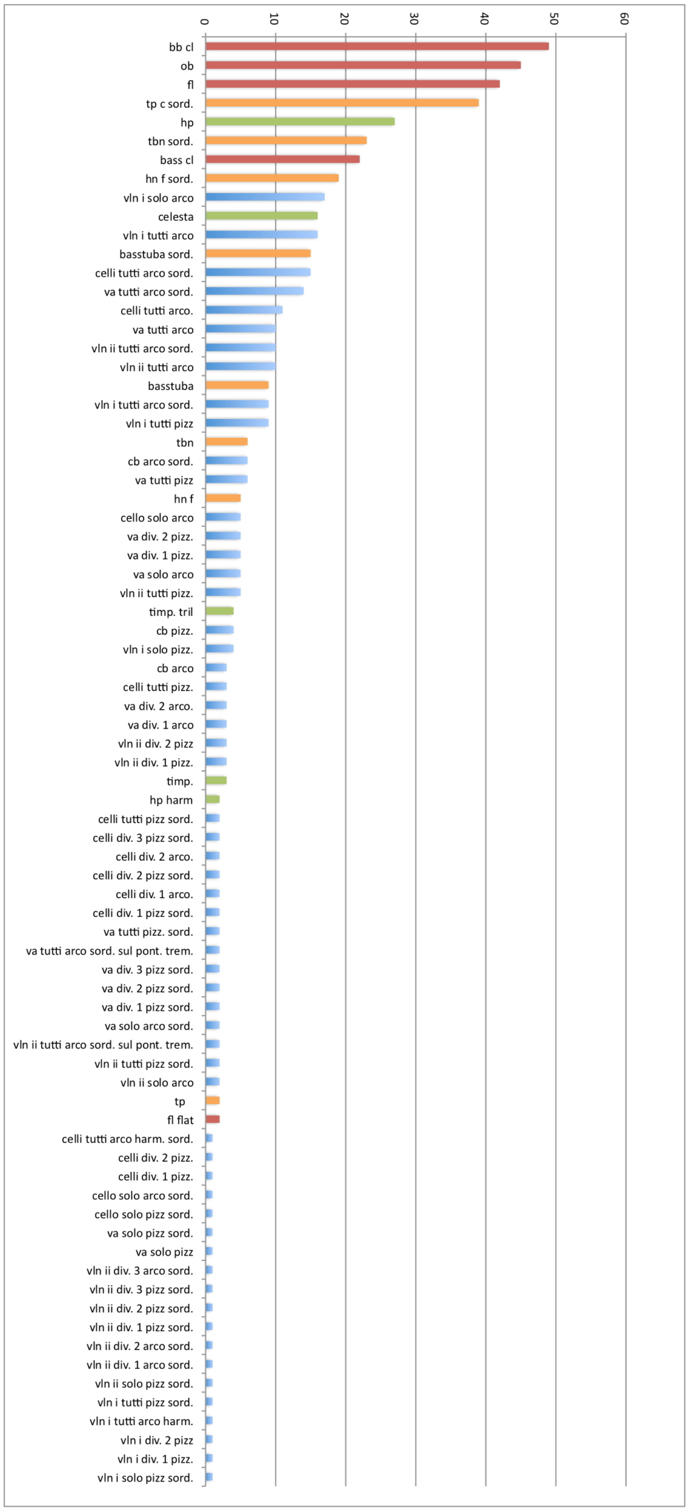

Figure 3: *The* ***Sonic Resources Index*** *of* Variations, *ordered (top-bottom) by recurrence in* ***Setups***

The formula we choose to normalize these two values (the SR number of each LSS and the constant denominator) is logarithmic. The logarithmic curve introduces a compensatory equilibrium, giving more visibility to the setups that use less resources - since we them to be the most usual ones in the work - and to approximate as much as possible those setups that are close to the maximum to the value of 1. Thus, for each LSS, we have one value for WNR (Equation 2).

$$WNR = \frac{ln(nRS)}{ln(nSRI)} \tag{2}$$

The graph (Figure 4) shows the evolution of the WNR in *Variations*, in histograms. We perceive a process of progressive densification when arriving at Transition 1, with one high point in m.63, which accumulates 16 SRs (WNR = 0.85, the largest Setup of the work). This is followed by a sound depression that happens when the composer exposes Theme 2. A second process of sound incrementation takes place from the Reprise, to reach the other climax of the piece, at the end of the Coda, m.168, with its 15 SRs (WNR = 0.83).

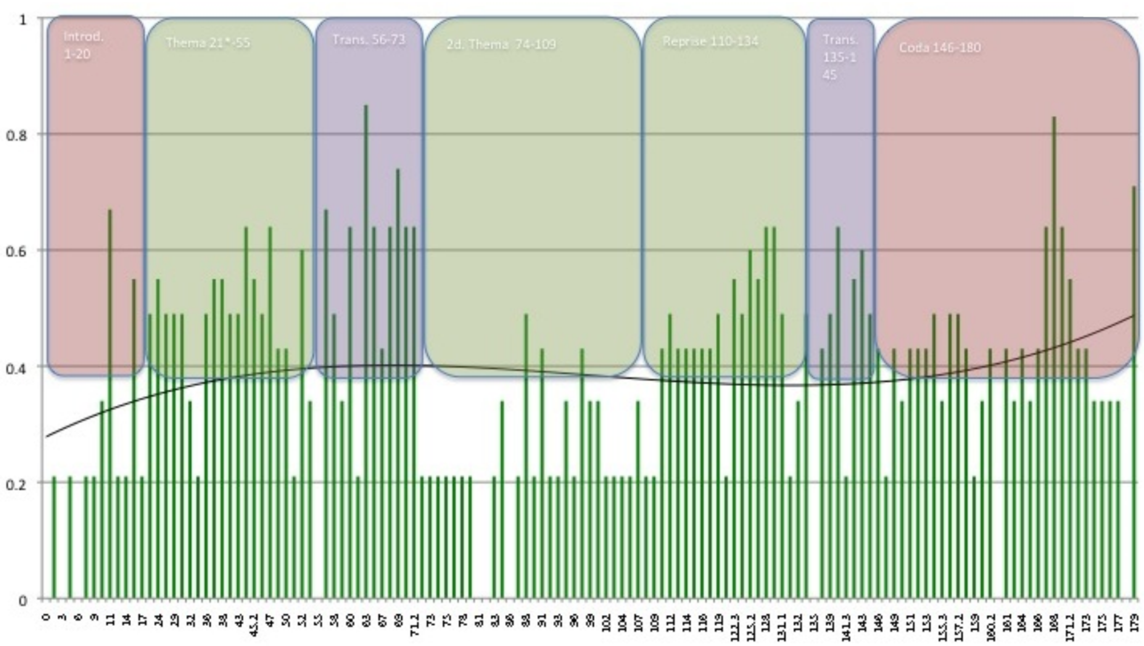

Figure 4: *Evolution of the WNR along the* Variations*,* ***Setup*** *by* ***Setup****. Background colors identify the 7 formal sections.*

However, the numerous histograms of value 0.21 indicate that the most frequent formation is that contains only two instruments. Considering that Webern works with a universe of 78 possible timbres, it is clear that there is an orchestral strategy that considers each sound resource individually, each instrument being able to become an agent that defines the articulation of the work.

Another data, obtained by means of an *ad hoc* pattern-recognition algorithm, showed that 127 of the 137 instrumental combinations are used only once during the course of the work, and the remaining 10 are used twice each. Thus, a secondary application of the principle of non-repetition, which stimulates an investigation into the treatment of sonority from the point of view of Webern's economy, is revealed.

This sound discontinuity would have to be reflected in the calculation of entropy, as this is a way of evaluating the rate of unpredictability of occurrence of data - a more obvious result of the principle of non-repetition. In fact, this calculation reveals an interesting behavior, as seen in the reading of the graph (Figure 5), which summarizes the average number of SRs used per section, in absolute numbers. It confirms the processes we have already discussed regarding the initial accumulation followed by a central depression at the moment of Theme 2 (Figure 5, histogram 4). The calculation of the entropy average, in turn, in the lower graph, brings another image of the orchestral structuring, which can be synthesized by a bipolarization between two states: a relative state of order - Introduction, Theme 2, Transition 2 [9] - destabilized by another, of relative chaos, this reaching its peak at the end of the piece. In fact, in the comparative observation of our survey of SRs used, respectively, in Theme 2 (mm.74-109) and Coda (mm.146-179 – Figure 6. See also Figure 7), one can verify that, in this last section, in spite of accelerated changes of Setups (it reaches three changes in a single measure: 160), the composer does not repeat any configuration. A strategy of *fuite en avant* that asks the listener for a continuous updating of his perception of the work.

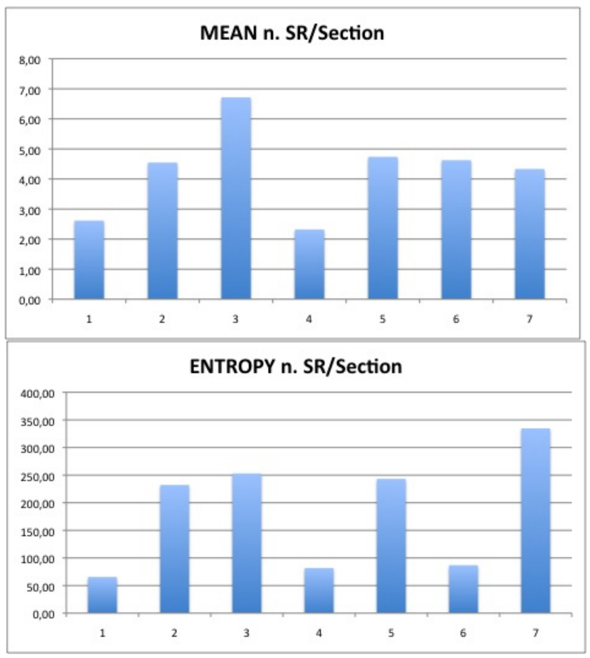

Figure 5: *Top: mean number of* ***Sonic Resources*** *per Section; bottom: entropy value of the number of* ***Sonic Resources*** *per Section*

This effect, however, is counterbalanced in some way at the primary level, where the rhythmic motive values are increased (Figure 7). The rate of motivic changes, therefore, is slower than that of the sound settings. In other words, there is a compensatory gap between the respective dynamics of these two levels of articulation.

Theme 2, on the other hand, maintains similar instrumental formulas (the relative constancy of the flute and clarinets in the first half generates a certain sound stability that highlights motives), less resources involved (less information, therefore, for the listener) and a little slower change rate (some **Setups** last even more than one bar). The entropy rate translates in some way an

[9] The structural weight of this section may have to be relativized because of its brief duration (10 bars only).

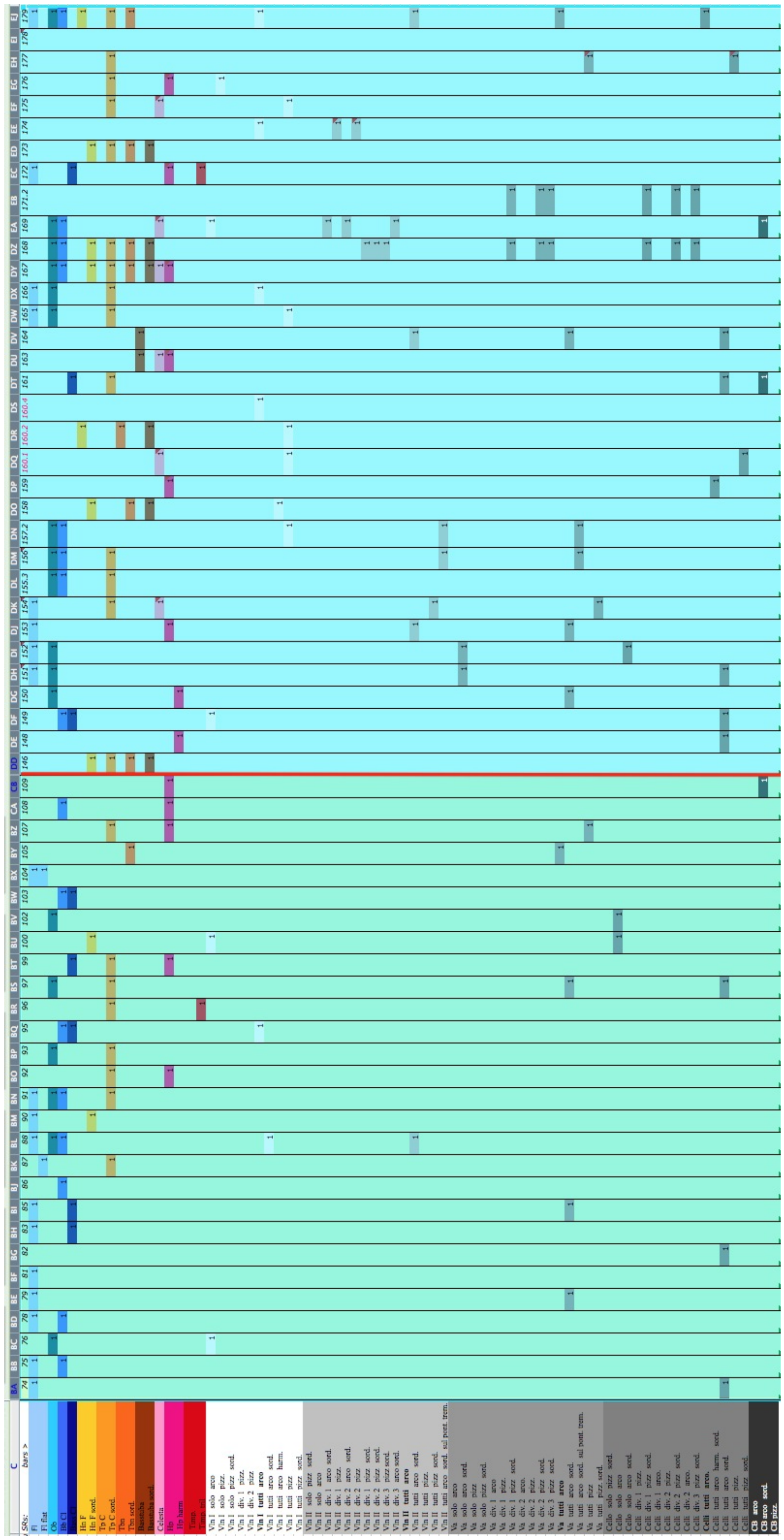

Figure 6: *A comparison of the orchestration of Theme 2 (left side, bars/setups 74-109) and Coda (right side, bars/setups 146-179)*

Figure 7: ***Variations***, *mm. 148-152 (apud Universal Edition, p. 25). Red squares identify the sequence of Local Sonic Setups (circles draw attention to tutti/solo permutations), while blue lines help to follow the subsets (tetrachords) of the serie.*

orchestration strategy directly linked to the expression of different and contrasting instances of the formal structure. But before moving further on these considerations, it is necessary to incorporate a second aspect of orchestration.

3.3. Instrumental Density and Textural Complexity—Relative Voicing Complexity

The impact the manipulation of orchestral resources can have on the formal dynamics in the time axis, which we have just shown, is affected by the way with which the composer organizes them into more or less autonomous streams. This distribution characterizes what is conventionally called texture, a dimension that signals the composer's personal style of orchestration, since it reveals his or her way of negotiating instrumental individualities and more or less stratified sonic masses. In terms of orchestral writing, Webern is known for his economy and pointillism, which our analysis highlighted. The expectation, then, is to be, in most of the time, in front of fine and transparent textures.

To support this hypothesis, we developed an algorithm, *Relative Voicing Complexity* (RVC), based on *Partitional Analysis*, a theory proposed by Pauxy Gentil-Nunes ([5]), which refines the methodological proposal of Wallace Berry ([2], p. 184). The more the instrumental parts are *agglomerated* - that is, the more they form homophony and/or homorhythmy - the more the texture becomes "simpler", and the reverse when dispersed. In the model proposed by Gentil-Nunes, the possible partitions for any integer are arranged in a vector format departing from the agglomeration units. In this way, a set of five instruments, for example, presents a lexical-sum of 18 partitions, which can be represented as follows:

lex(5) = (5), (4.1), (3.2), (3.1.1), (2.2.1), (2.1.1.1), (1.1.1.1.1), (4), (3.1), (2.2), (2.1.1), (1.1.1.1),
(3), (1.1.1), (2), (1.1), (1)

in which the partition $(1.1.1.1.1)$ indicates that all "voices" are independent, as in a polyphony, thus qualifying the texture as "complex", and 5, that all instruments play a chord or are in unison, with a texture resultant described as "simple" ([5], p. 16). Partition (1.4), for example, would typically indicate a soloist accompanied by a four-voice homophonic harmony. Figure 3, line ((Partition) (crit.)), shows the format we use so that the parsing analysis is intelligible for OpenMusic and SOAL, encoded in Common Lisp [10].

The calculation of agglomeration and dispersion indices from the partitions survey and their vectorialization starts from the counting of the total number T of the binary relations between the nSRs (the *n* sound features of a *setup*), e.g., the two-by-two combination of nSR, according to a formula borrowed from Tucker's combinatorial analysis (Equation 3; see [6], p. 2 and [14], p. 181).

$$\begin{aligned} T_2 \colon \mathbb{N}^* &\to \mathbb{N} \\ n &\mapsto \frac{n(n-1)}{2} \end{aligned} \tag{3}$$

This function (Equation 3) allows us to enumerate the agglomeration and dispersion relations of each partitioning. Indeed,

> when Berry attributes these indices to the musical text, he is implicitly dividing the set of total relations (T) into relations of contrast and identity, since the constitution of

[10] Cf. [8] for a more detailed explanation on the format.

> the real components is done in terms of relations of identity and the differentiation is accomplished through relations of contrast. From this observation, we can infer that the sum of the relations of identity and contrast in a given textural configuration will always be equal to T. ([6], p. 3).

The agglomeration index *(a)* corresponds to the sum of all binary combinations of the sound resources of each real component (Equation 4), where r is the number of real components and *r* is the number of sound resources of each real component separately ([6], p. 4 - Equation 4)

$$a\colon \mathbb{N}^r \to \mathbb{N}$$
$$(a_0 \ldots\ a_{r-1}) \mapsto \sum_{i=0}^{r-1} T_2(a_i) \tag{4}$$

In practice, it is enough to apply to each real component the equation T.
The dispersion index (d) is the result of the difference between T and (a) ([6], p. 4).

$$d\colon \mathbb{N}^r \to \mathbb{N}$$
$$(a_0 \ldots\ a_{r-1}) \mapsto T_2(\rho) - a(a_0\ \ldots\ a_{r-1}) \tag{5}$$

A pair of indices *a* and *d* is then obtained. The visual arrangement in the form of an indexogram contributes to the interpretation of the dynamics of the textural configurations in the time axis. The indices a and d are symmetrically displayed around zero, by inverting the sign of a. This inversion has another virtue: when added to d, it forms the sum I, then

$$\mathcal{I}(a_0 \ldots a_{r-1}) = (d-a)(a_0 \ldots a_{r-1}) \tag{6}$$

This produces a synthesis evaluation that shows the tendency of the texture, either towards agglomeration (when the sum is negative) or dispersion (positive sum). The values also provide a dynamic curve of this trend. In other words, if, as we have argued above, it is the dispersion rate that determines the complexity of a texture, the integration of the calculation of its agglomeration rate allows for a finer calibration.

To place I on a single axis of relative complexity ([7], p. 40 *et seq.*), what we call *Relative Voicing Complexity*, we normalize I by dividing it by the T value of the setup that has the highest number of sound resources, that is, by the largest number of binary relations possible in a given set of setups. A configuration whose dispersion index would be equal to this number would, in fact, represent the greatest possible complexity in the context. The RVC index is then obtained (Equation 7)

$$\mathrm{RVC}(a_0 \ldots a_{r-1}) = \frac{\mathcal{I}(a_0 \ldots a_{r-1})}{T_2(\rho_{\max})}. \tag{7}$$

In Table 1, one can observe the results of these equations for the mentioned section of Webern, respectively lines -a, d and d - a, corresponding to I.

Partitional Analysis, however, does not stipulate *a priori* the criteria to determine the lexicon of a given partition. Still based, in part, on Berry ([2], p. 193), we have observed that the dispersion of the voices in the textures of *Variation* are generated by:

1. Heterorhythmy: i. e., divergence or asynchrony between rhythmic structures;

2. Heterodirectionality: voices progress into different directions.

We believe that the first agent has more impact on the perception of a sound fission than the second. The second is chosen when the first is idle or when it appears as predominant in the given context. Thus, the instrumental **Setups** are weighted not only by the degree of agglomeration/dispersion of their component parts, but also depending on the method by which this is brought about. In Webern, serial assumption turns heterorhythmy into a systemic feature - due, particularly, to techniques derived from the counterpoint and canon - which, through analysis, proves to be a permanently active agent of dispersion [11] . It follows from these same assumption that the probability of occurrence of massive agglomerations is quite low, which in fact is the case, as shown in the indexogram of Figure 8.

In fact, statistically, Webern privileges fine textures or even "null" ones: from 137 **Setups** 42 contain only one or two SRs. In this sense, the second Theme, unlike the first (to which we will return), stands out since it is made up almost exclusively of Setups of one or two instruments, never agglomerated (see Figure 9, left). However, the dispersion index does not decrease in proportion to the increase in sound resources, as often happens in the orchestral writing of the nineteenth century: on the contrary, it remains almost always high, so that even when he uses many instruments, the composer still avoids to agglomerate them. The most remarkable examples of this procedure are presented in mm. 140-141 and 179. If the Setup of m. 179 still agglutinates three woodwinds forming the partition (3.1.1.1.1.1.1.1.1), the Setup of mm. 140-141 does disperse the eight voices. (Figure 9, right). Because of this configuration, these two Setups are qualified with the highest rate of relative complexity of the work.

The agglomeration of the three high woodwinds, by the way, is a very common solution of orchestration that Webern uses to highlight some motive. For instance, it marks, just from the start, the most salient point of the Introduction (mm. 11-12, see Figure 14 at the end of this paper). In this moment, the Setup is partitioned into two instrumental subsets, in which six string voices constitute a chord that punctuates the unison formed by the three woodwinds. The climax of the Reprise, located between mm. 125 and 131, also brings the main motive to the same trio, in similar partitions and high complexity. And it is still a unison of this trio that will give an end to the work, at m. 179 (Figure 9).

In a brief instant, Webern joins the three main instrumental groups in a homophonic texture and calls them to a dialogue: in this way, he provides a specific sonority to the first Transition (mm. 56-63). This sonority is caused by the most unusual (and exceptional) partitions up to this point, like (3.3.3) (m. 56) or (4.4.4.4) (m. 63) (Figure 10).

Another extremely important aspect of textural organization of this piece derives from the systematic outline of the series in tetrachords: for all chordal agglomerations of the work are grouped into four instrumental parts. Mm. 60-63, we just comment, are a good example of how Webern put against each other the homophonic tetrachords, subsets of versions of the series. But even more remarkable in this view is the First Theme, only section of the piece that draws an accompanied soloist type partitions (Figure 11) [12] . This 'classicism' of the thematic presentation - in a direction which clearly refers to a Schoenbergian practice [13] , and, beyond this, of course, the common practice of music from previous periods - is unique to this theme, showing how Webern valued the traditional conventions in moments where he thought they constituted the best

[11] Heterodirectionality was only taken into consideration in m. 9, and especially in mm. 56-58, in which it acts in an ostensive fashion (see Figure 10).

[12] We adopt the convention O, I, R, RI for naming the conventional transformations of the row. If transposed, a letter "T" will be added with a subscript integer corresponding to the transposition applied (in number of semitones). Lowercases (a , b, ...) refer eventually to rhythmic motives according to the nomenclature by Balley ([1]), with the letter r being reserved to label retrogradation. Information about augmentation and diminution were omitted for clarity. Because the characteristic Werbenian circular construction it is evident that there are alternative possible nomenclature for a given pitch group.

[13] Cf. his *Variations* op. 31.

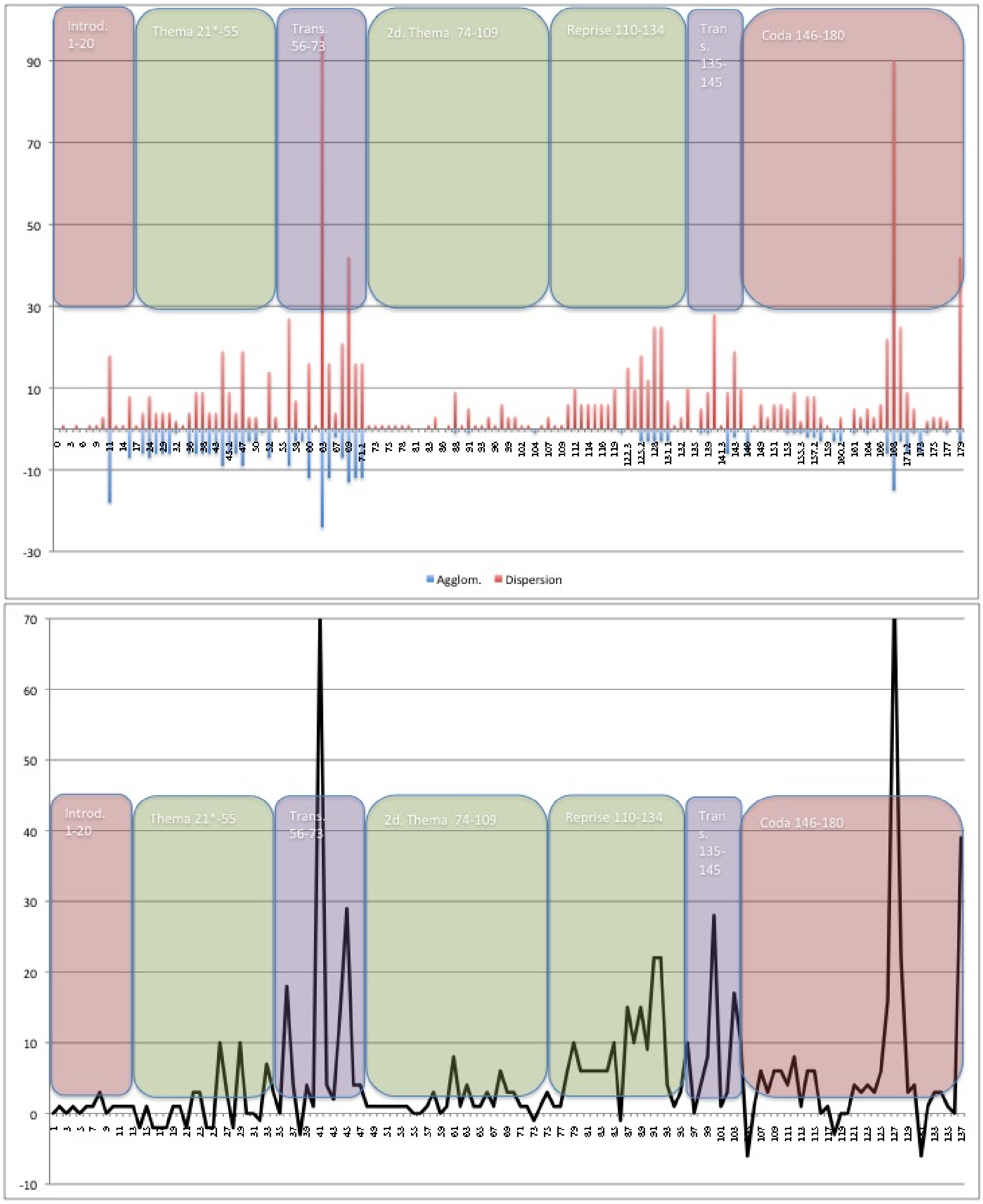

Figure 8: *Top:* indexograms, *with* agglomeration *(blue) and* dispersion *(red) indices, section per section; bottom: Sum* I.

Figure 9: *Comparing textures (mm. 78-81, 140-141, 179-180 (*apud *Universal Edition). The **Setups Complexity** values will be explained in the last section of the paper. "Part." shows the* partitional analysis.

Figure 10: *An excerpt (mm. 56-63) of the Transition 1, where textures present agglomerated subsets (*apud *Universal Edition, p. 10-11).*

solution for the realization of musical ideas. In this sense, he did take care of tuning, as before, in the climax of the Introduction (that we will cover ahead), the two levels: when the goal is to emphasize the thematic focus, the orchestration can return to its historical function.

Figure 11: *Beginning of Theme 1, mm. 9-17 (apud Universal Edition, p. 5).*

The stability of this textural organization is unique: in the beginning of this theme occurs the most extended Setup of the piece, with five-bar long (mm.19-24). This sounding staticity (formed by a muted brass quartet below the solo violin) provokes a sudden break on the constantly changing sonorities that Webern tried to accustom the listener since the beginning of the piece. Undoubtedly, this is a strategy for calling the attention for the *incipit* of the solo violin's theme. It is still maintained in the following three bar-long Setup (in which the brasses are substituted by the woodwind quartet, with the violin being reinforced by the second violins group), and in a lesser extent in some other moments in the same section. The structure of this "harmonic accompaniment" deserves to be examined, because of its exclusivity. It is formed by a sequence of twelve tetrachords, shown in reduction in Figure 12, which distributes the chords along the horizontal plan, proportionally according to their approximate durations.[14]

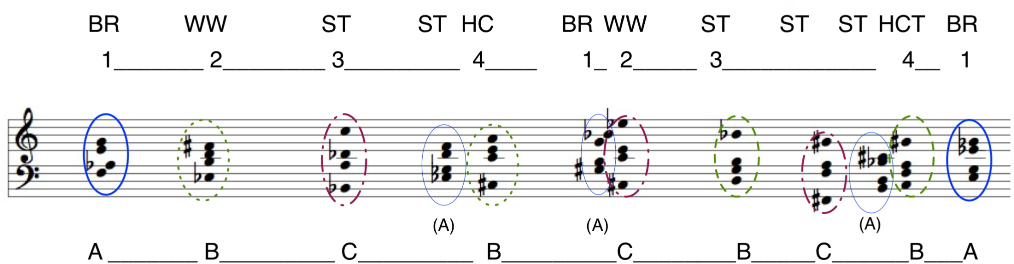

Figure 12: *Harmonic sequence of Theme 1 (mm. 21-55).*

The tetrachords are reduced to three types of intervallic combination. Type A is formed through superimposition of intervals [3 6 3].[15] Type B includes a perfect triad [16] , and type C forms the most dissonant superimposition [11 4 11]. As one can observe in Figure 12, Webern adopts a regular chord sequence ABCBCBCBA.[17] The last A chord is identical to the first one, transposed a semitone lower.

This material is reused in the remaining passages in which chords are present. The Transition initiates just after the Theme, with a restatement of chord A in its original version (m.56, strings - Figure 10). The same chord is also present in mm.69 and 168, as it will be later detailed (Figure 15).

Back to the theme, now considering it under the perspective of orchestral distribution, we obtain another sequence, which is expressed in Figure 12 through numbers: "1" represents the brasses (BR), "2" the woodwinds (WW), "3" the strings (ST), and "4" the combination harp + celesta (HC) + timpani (HCT). This time the sequence is still more linear: < 1 2 3 4 1 2 3 4 1 >. We observe that the return of the A chord at the end corresponds to the returning of the same sound Setup (brasses). Such stability is undoubtedly the most notable aspect of this theme structure. Moreover, Webern's music genuinely unfolds through striking adjacent contrasts of instrumentation, as it will be seen below.

[14] Durations are roughly calculated in number of eigth-notes, from one chord to the following.
[15] That is, superimposition of two minor thirds at the distance of tritone.
[16] The triad is successively presented in root position, first, and second inversion.
[17] Due to their very short duration, the "passing" type-A chords of mm.30-1 (strings), 36 (brasses), and 46-7 (strings) are not considered.

3.4. SRC—Local Sonic Setups Relative Complexity

Needless to say, these two sets of data, which we have just explored in terms of their structuring function – the number of sound resources used (WNR) and the way in which they interact (RVC) – are absolutely interdependent. Indeed, the number of voices in which a texture can stratify depends, of course, on the number of instruments involved. At the same time, it is more likely that a small number of instruments will generate more polyphony than the mass of a large orchestra. Therefore, the complexity of voicing (RVC) makes perfect sense as a qualitative modulator of the quantities of resources identified in each Setup.

In the practice of the experimental method we apply, RVC weights multiply those of WNR, in a kind of metaphor of the frequency modulation process, or, describing more specifically: the result of this multiplication is added to the value of WNR. The weight of the modulator may eventually be adjusted, up or down. For this analysis, we leave this weight neutral (= 1). The result sets what we will call *Local Sonic Setup Relative Complexity*, simplified acronym SRC. From which,

$$SRC = WNR.RVC_p \tag{8}$$

In which p is the weight (in %) of the RVC modulator.

The reader will notice that the procedure described does not take into account the duration of the setups. In effect, these are segmented on the basis of their sound configuration. Therefore, the setups go on as long as the sound configuration does not change. This procedure can generate segments of very dissimilar durations. Furthermore, there is no doubt that the time factor can be decisive in the appreciation, or sensation, of the relative impact of a structural unit on the whole. However, we have decided not to take this parameter into account in this essay, although we plan in the future to incorporate it into our model. For, since the Webernian agogic is extremely dynamic and flexible in its prescriptions, it makes it innocuous or inaccurate to deduce realistic temporal proportions from the score. It is therefore necessary, in order to incorporate this dimension, to enter into the study of the recorded performances of the work, from which the interpretative solutions adopted for the management of time can be extracted, a task that would exceed the scope of this article.

3.5. Orchestration as structuring agent

We will close this essay with more global analytical considerations, woven from the analysis of the relative complexity of setups. The SRC is represented in Figure 13 in the form of a sequence of histograms, section by section. Contains trend lines (3rd or 6th order polynomial functions).

We have already identified, based on our previous observations, an orchestral script elaborated in order to characterize each section by some kind of sonority, through different instrumental distributions. In Figure 13, the green histograms table (MEAN SRC / section), averaging the relative complexities of **Setups** by section, shows a general tendency of economy of resources - the highest average does not reach 50%—with the "chamber-like" treatment (even soloistic) that we have already discussed, in the Introduction (Figure 5, histograms 1) and in the second Theme (Figure 5, histograms 4). The latter forms a sound depression between the two adjacent sections, which correspond to the denser orchestration moments. In this sense, this section constitutes an axis after which the sections follow in inverse order of complexity of the sections that precede it. At this level of abstraction, however, the sensitive impact of a cycle like this remains extraordinarily diluted. There are other impacts on the surface that most efficiently capture the attention of the listener.

It is interesting to observe that the Introduction and the transitional sections are developed through great contrasts of sound complexity: the musical time is very bumpy, the sound renovation

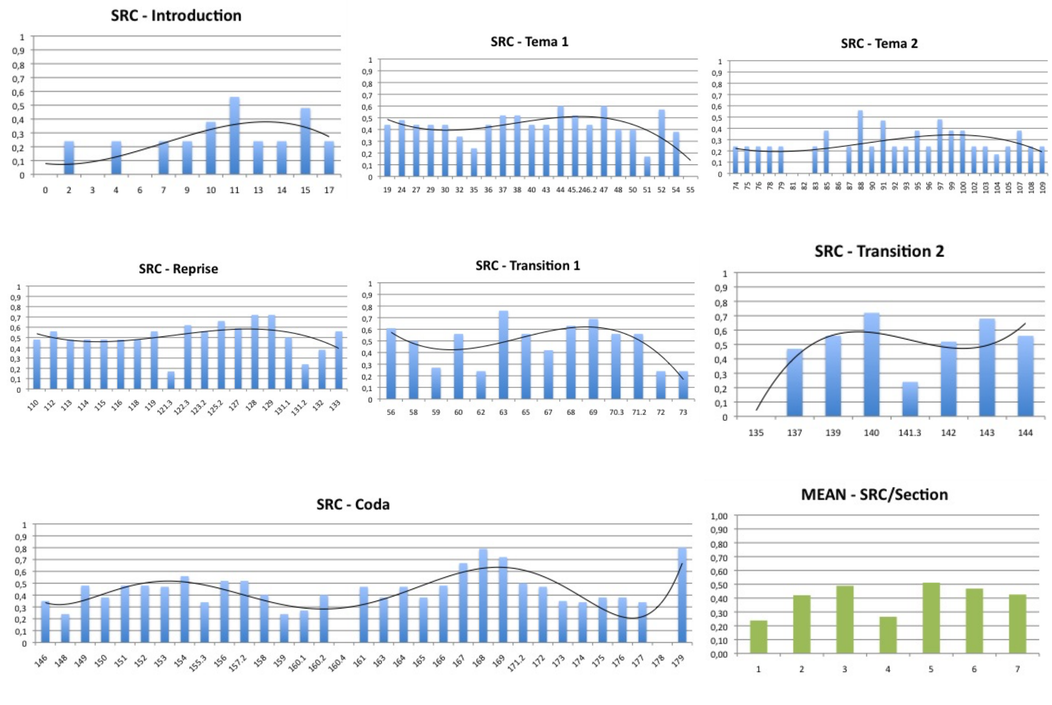

Figure 13: *Histograms of the sequence of **SRC** (**Local Sonic Setups Relative Complexity**) for the whole work, section per section. Green histograms: The mean of **SRC** values per section.*

is permanent, the sonority of the work remains in constant instability, moving abruptly from solos to duos and suddenly to almost *tutti*, and so on. However, at the same time, it is revealed, through the trend lines, a certain directionality in the succession of sonorities, markedly in the Introduction, which obeys a classic format of growth of complexity followed by resimplification, by the structural parallelism between Theme 1 and the Transition that follows it (we have seen that they also share similar chords and textures), in Transition 2 with its central sound reduction, and by the double focus of growth in the Coda, terminated by abrupt contrast from the silence.

As a more detailed example, Figure 14 shows the heart of the Introduction (mm. 6-17). The excerpt is constructed from the cross-overlapping of tetrachords of the four forms of the series supported by elaborations of the two rhythmic motifs a and b (as indicated by colored circles, symbols, and arrows). Although sound transformations are concomitant with the rapidity of tetrachordal changes, a complex dialectic is established between the primary level of organization of these tetrachords and the articulation of orchestral timbres. We note that, in mm. 7-8, *I3ar* and *RI3ar* are divided into 2 successive Setups, which have in common the sonority of the 1st violins and as complementar sonority the Harp, for the first, and the double basses, for the second, in a kind of sound symmetry. We note that it is the same tetrachord (the third) and that I and R have the same rhythmic motion (ar) in phase shift. The following 3 Setups (mm.10, 11, and 13) support two complete forms of the series, Inverted and Retrograde, in alternating distribution between strings, brasses and woodwinds. Different from what usually happens in the rest of the work, in this section the two planes are in synchrony. In other words, the sonorities are exchanged at the same time as the tetrachords, and the complexity curve of the Setups accompanies and sustains the formal logic of tension, climax and rest. In effect, the section culminates with the Setup of

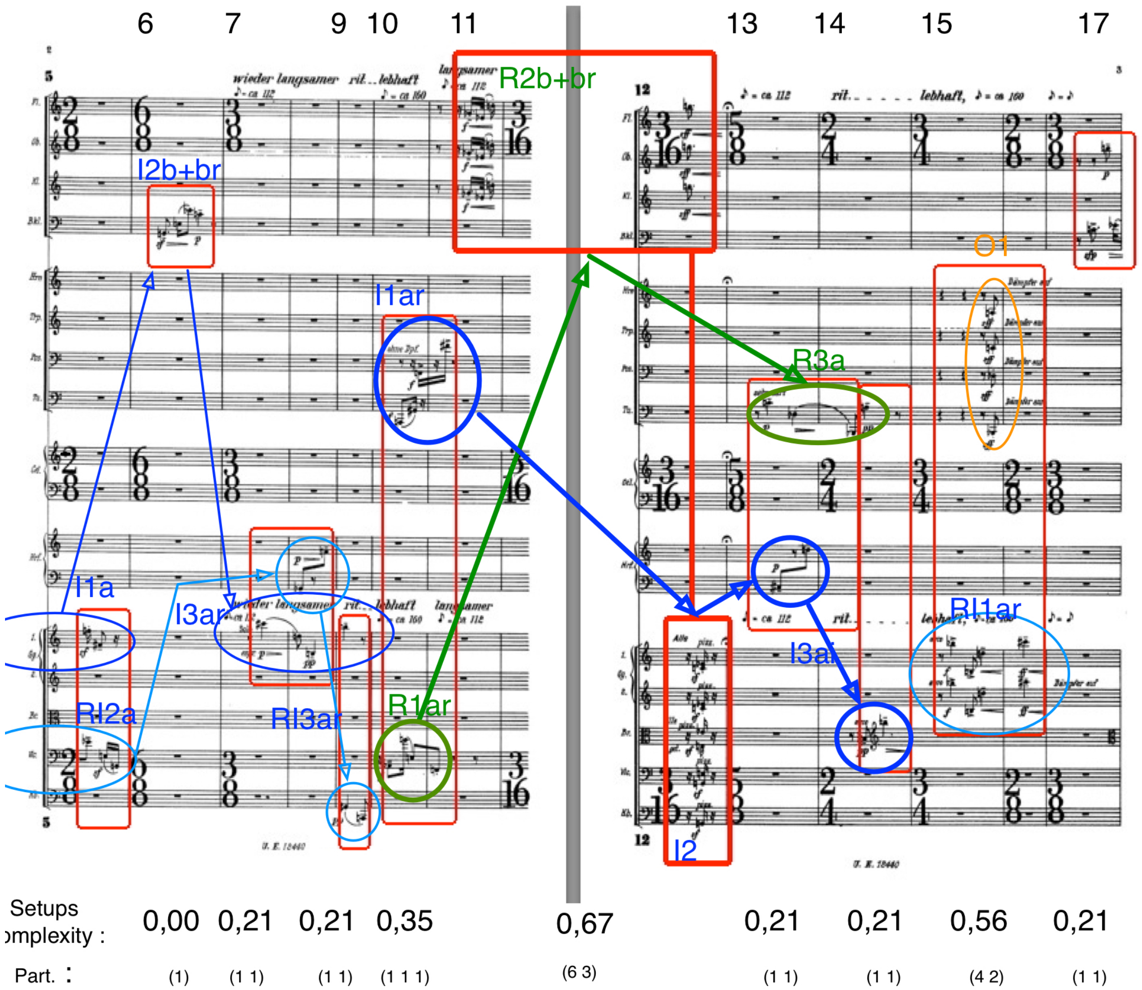

Figure 14: *Examples of the dialectic between serial organization and orchestration, mm. 5-17 (apud Universal Edition, p. 2-3).*

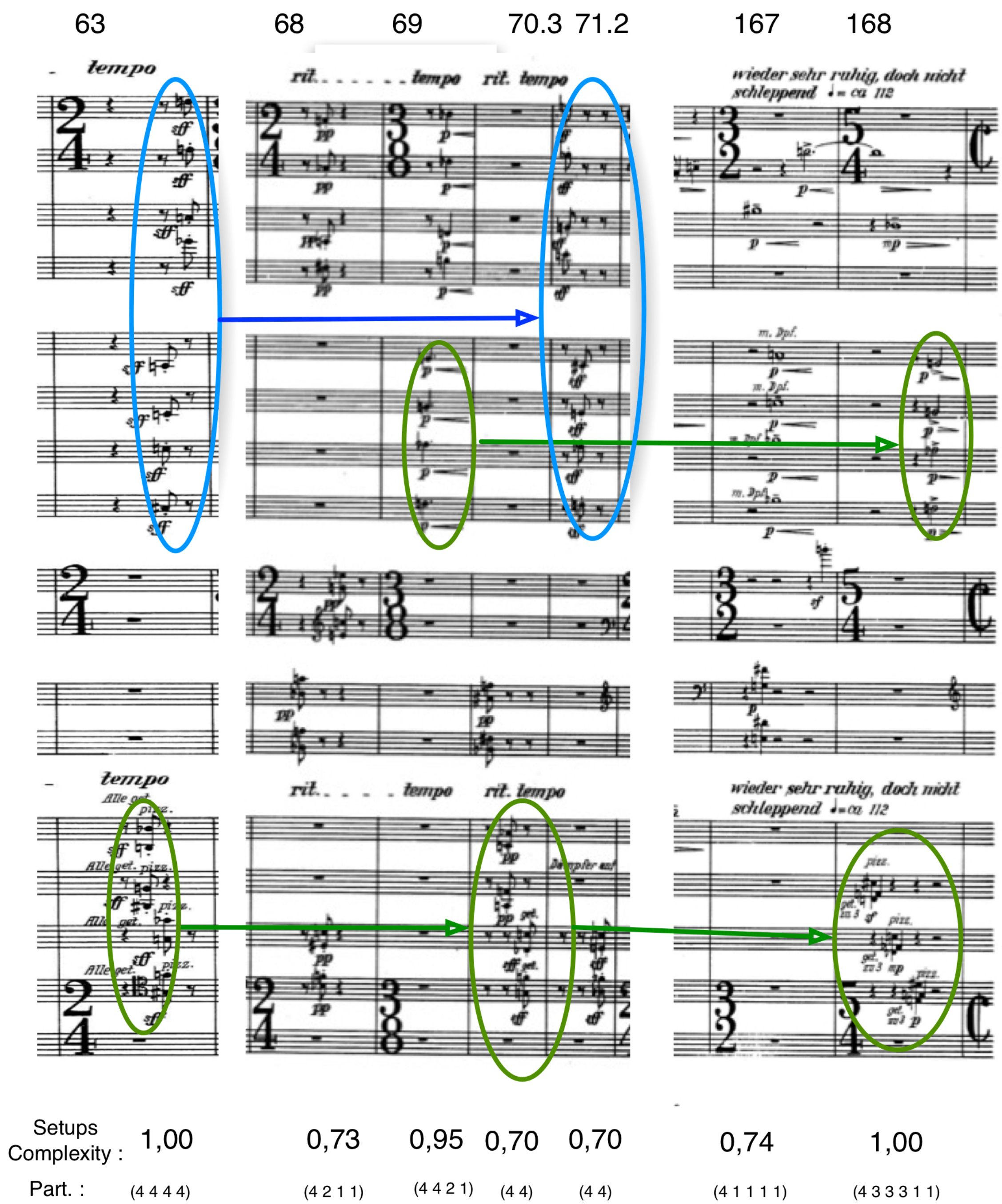

Figure 15: *Most dense* ***Setups*** *of the work. Circles and arrows emphasize similarities of material: mm. 63, 68-71, 167-168 (apud Universal Edition).*

mm. 11-12, which for the first time requests 11 SRs in strong *tutti*, opposing a striking unison in the woodwinds - an orchestral solution structuring in this piece, as we have already mentioned - to a *pizzicato* in the strings. These convergences are precisely reserved for moments in which it is necessary to perceive some structural framework: we have already observed, that they focus, essentially, on the exposition of Theme 1.

Transition 1, on the other hand, concentrates two exceptionally dense **Setups**, since they involve 16 and 11 sound resources, respectively (mm. 63 and 69, Figure 15). They constitute moments of greater sonic impact. This impact will be far fetched at the end of the Coda, in a point also culminating, as if it were a reminiscence: it is m. 168, requesting 15 SRs. In fact, there are many points in common between the three most salient sound objects of the work. We have already pointed out the most apparent: the repetition of chord A in mm. 69 and 168 - which consolidates its structural function. In the case, it is played in the two occasions by the brasses, but the second time, the dynamics is reversed (from p growing to p decreasing): a subtle mirroring, indeed. In addition, the strings always work as a sound punctuation, by means of *pizzicato* or short chords. Woodwind quartet and brass quartet answer each other (in inverted order in mm. 70.3-71), with brief chords of which the *secondo* one forms a species of echo to the first one. The harp always intervenes with tetrachords, with emphasis on that intervallic structure of type C, the most dissonant (in mm.68 and 167).

4. Conclusion

In short, we have tried, through these analytical tools still in the stage of experimentation, to show how Webern's serial economy interacts in the plane of the composition of the orchestral sonorities. We have only approached here the orchestration in its symbolic stage: what the composer lets us know, by means of the score, of his intentions at this level. A second essential step would be to evaluate the results produced by analyzing the sound footprints of the work. Only then could we validate, in the perception level, the sound rhetoric predicted by the composer. In our discussion about Debussy, we had shown that raw materials, motifs, or cells tended to "manifest as amorphous elements, the scourge of the sound atmosphere", diluting "within the systems of articulation of sonorities" ([7], p. 96). We would be inclined to say that Webern radicalizes this logic by freezing the integrality of his primary material in a single intervallic organization (the series) and in two rhythmic patterns. It is not difficult to reach the conclusion that the extraordinary dynamics of this work lies in the ways in which the composer uses this static material to configure sound units that are renewed every moment: we have shown here many examples of the procedures adopted. If Webern's music resists to abstraction by maintaining an organic bond with nature, inherited from romanticism, as argued by Julian Johnson ([10], p. 212 *et seq.*), the art of organizing sounds through the manipulation of instrumental resources, then becomes the locus of the Webernian aesthetics.

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