chapter

Applications of Nuclear Physics

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In this chapter, we study two means for deriving energy from nuclear reactions:

fission, in which a large nucleus splits into two smaller nuclei, and fusion, in which two small nuclei fuse to form a larger one. In both cases, the released energy can be used either constructively (as in electric power plants) or destructively (as in nuclear weapons). We also examine the ways in which radiation interacts with matter and look at several devices used to detect radiation. The chapter concludes with a discussion of some industrial and biological applications of radiation.



The Cruas Nuclear Power Station, near the town of Montélimar, France, is one of dozens of nuclear power plants around the world that provide energy from uranium. The energy is released from uranium by a process called *fission*, which we discuss in this chapter. The mural, entitled "Aquarius," that appears on the rightmost cooling tower, was created by Jean-Marie Pierret, painted by nine mountaineers, and completed in 2005. (© *Oso Media/Alamy*)

45.1 Interactions Involving Neutrons

Nuclear fission is the process that occurs in present-day nuclear reactors and ultimately results in energy supplied to a community by electrical transmission. Nuclear fusion is an area of active research, but it has not yet been commercially developed for the supply of energy. We will discuss fission first and then explore fusion in Section 45.4. To understand nuclear fission and the physics of nuclear reactors, we must first understand how neutrons interact with nuclei. Because of their charge neutrality, neutrons are not subject to Coulomb forces and as a result do not interact electrically with electrons or the nucleus. Therefore, neutrons can easily penetrate deep into an atom and collide with the nucleus.

A fast neutron (energy greater than approximately 1 MeV) traveling through matter undergoes many collisions with nuclei, giving up some of its kinetic energy in each collision. For fast neutrons in some materials, elastic collisions dominate. Materials for which that occurs are called **moderators** because they slow down (or moderate) the originally energetic neutrons very effectively. Moderator nuclei should be of low mass so that a large amount of kinetic energy is transferred to them in elastic collisions. For this reason, materials that are abundant in hydrogen, such as paraffin and water, are good moderators for neutrons.

Eventually, most neutrons bombarding a moderator become **thermal neutrons**, which means they have given up so much of their energy that they are in thermal equilibrium with the moderator material. Their average kinetic energy at room temperature is, from Equation 21.4,

$$K_{\text{aver}} = \frac{3}{2} k_{\text{B}} T \approx \frac{3}{2} (1.38 \times 10^{-23} \text{ J/K}) (300 \text{ K}) = 6.21 \times 10^{-21} \text{ J} \approx 0.04 \text{ eV}$$

which corresponds to a neutron root-mean-square speed of approximately 2 800 m/s. Thermal neutrons have a distribution of speeds, just as the molecules in a container of gas do (see Chapter 21). High-energy neutrons, those with energy of several MeV, *thermalize* (that is, their average energy reaches K_{avg}) in less than 1 ms when they are incident on a moderator.

Once the neutrons have thermalized and the energy of a particular neutron is sufficiently low, there is a high probability the neutron will be captured by a nucleus, an event that is accompanied by the emission of a gamma ray. This **neutron capture** reaction can be written

$${}^{1}_{0}\mathbf{n} + {}^{A}_{Z}\mathbf{X} \rightarrow {}^{A+1}_{Z}\mathbf{X}^{*} \rightarrow {}^{A+1}_{Z}\mathbf{X} + \gamma$$

Once the neutron is captured, the nucleus ${}^{A+1}_{Z}X^*$ is in an excited state for a very short time before it undergoes gamma decay. The product nucleus ${}^{A+1}_{Z}X$ is usually radioactive and decays by beta emission.

The neutron-capture rate for neutrons passing through any sample depends on the type of atoms in the sample and on the energy of the incident neutrons. The interaction of neutrons with matter increases with decreasing neutron energy because a slow neutron spends a larger time interval in the vicinity of target nuclei.

45.2 Nuclear Fission

As mentioned in Section 44.2, nuclear **fission** occurs when a heavy nucleus, such as 235 U, splits into two smaller nuclei. Fission is initiated when a heavy nucleus captures a thermal neutron as described by the first step in Equation 45.1. The absorption of the neutron creates a nucleus that is unstable and can change to a lower-energy configuration by splitting into two smaller nuclei. In such a reaction, the combined mass of the daughter nuclei is less than the mass of the parent nucleus, and the difference in mass is called the **mass defect.** Multiplying the mass defect by c^2 gives the numerical value of the released energy. This energy is in the form of kinetic energy associated with the motion of the neutrons and the daughter nuclei after the fission event. Energy is released because the binding energy per nucleon of the daughter nuclei is approximately 1 MeV greater than that of the parent nucleus (see Fig. 44.5).

Nuclear fission was first observed in 1938 by Otto Hahn (1879–1968) and Fritz Strassmann (1902–1980) following some basic studies by Fermi. After bombarding uranium with neutrons, Hahn and Strassmann discovered among the reaction products two medium-mass elements, barium and lanthanum. Shortly thereafter, Lise Meitner (1878–1968) and her nephew Otto Frisch (1904–1979) explained what

Neutron capture reaction

(45.1)

Pitfall Prevention 45.1

Binding Energy Reminder

Remember from Chapter 44 that binding energy is the absolute value of the system energy and is related to the system mass. Therefore, when considering Figure 44.5, imagine flipping it upside down for a graph representing system mass. In a fission reaction, the system mass decreases. This decrease in mass appears in the system as kinetic energy of the fission products.



Figure 45.1 A nuclear fission event.



Figure 45.2 Distribution of fission products versus mass number for the fission of ²³⁵U bombarded with thermal neutrons. Notice that the vertical axis is logarithmic.



had happened. After absorbing a neutron, the uranium nucleus had split into two nearly equal fragments plus several neutrons. Such an occurrence was of considerable interest to physicists attempting to understand the nucleus, but it was to have even more far-reaching consequences. Measurements showed that approximately 200 MeV of energy was released in each fission event, and this fact was to affect the course of history.

The fission of ²³⁵U by thermal neutrons can be represented by the reaction

$${}^{1}_{0}n + {}^{235}_{92}U \rightarrow {}^{236}_{92}U^* \rightarrow X + Y + neutrons$$
 (45.2)

where 236 U* is an intermediate excited state that lasts for approximately 10^{-12} s before splitting into medium-mass nuclei X and Y, which are called **fission frag-ments.** In any fission reaction, there are many combinations of X and Y that satisfy the requirements of conservation of energy and charge. In the case of uranium, for example, approximately 90 daughter nuclei can be formed.

Fission also results in the production of several neutrons, typically two or three. On average, approximately 2.5 neutrons are released per event. A typical fission reaction for uranium is

$${}^{1}_{0}n + {}^{235}_{92}U \rightarrow {}^{141}_{56}Ba + {}^{92}_{36}Kr + 3({}^{1}_{0}n)$$
 (45.3)

Figure 45.1 shows a pictorial representation of the fission event in Equation 45.3.

Figure 45.2 is a graph of the distribution of fission products versus mass number *A*. The most probable products have mass numbers $A \approx 95$ and $A \approx 140$. Suppose these products are $\frac{95}{39}$ Y (with 56 neutrons) and $\frac{140}{53}$ I (with 87 neutrons). If these nuclei are located on the graph of Figure 44.4, it is seen that both are well above the line of stability. Because these fragments are very unstable owing to their unusually high number of neutrons, they almost instantaneously release two or three neutrons.

Let's estimate the disintegration energy Q released in a typical fission process. From Figure 44.5, we see that the binding energy per nucleon is approximately 7.2 MeV for heavy nuclei ($A \approx 240$) and approximately 8.2 MeV for nuclei of intermediate mass. The amount of energy released is 8.2 MeV – 7.2 MeV = 1 MeV per nucleon. Because there are a total of 235 nucleons in $\frac{235}{92}$ U, the energy released per fission event is approximately 235 MeV, a large amount of energy released in the combustion of one molecule of octane used in gasoline engines is about onemillionth of the energy released in a single fission event!

Quick Quiz 45.1 When a nucleus undergoes fission, the two daughter nuclei are generally radioactive. By which process are they most likely to decay?
(a) alpha decay
(b) beta decay (e⁻)
(c) beta decay (e⁺)

Quick Quiz 45.2 Which of the following are possible fission reactions?

The Energy Released in the Fission of ²³⁵U

Calculate the energy released when 1.00 kg of 235 U fissions, taking the disintegration energy per event to be Q = 208 MeV.

SOLUTION

Conceptualize Imagine a nucleus of ²³⁵U absorbing a neutron and then splitting into two smaller nuclei and several neutrons as in Figure 45.1.

45.1 cont.

Categorize The problem statement tells us to categorize this example as one involving an energy analysis of nuclear fission.

Analyze Because A = 235 for uranium, one mole of this isotope has a mass of m = 235 g.

Find the number of nuclei in our sample in terms of the number of moles n and Avogadro's number, and then in terms of the sample mass m and the molar mass M of ²³⁵U:

$$N = nN_{\rm A} = \frac{m}{M}N_{\rm A}$$

Find the total energy released when all nuclei undergo fission:

$$E = NQ = \frac{m}{M} N_{\rm A}Q = \frac{1.00 \times 10^3 \,\mathrm{g}}{235 \,\mathrm{g/mol}} (6.02 \times 10^{23} \,\mathrm{mol^{-1}})(208 \,\mathrm{MeV})$$
$$= 5.33 \times 10^{26} \,\mathrm{MeV}$$

Finalize Convert this energy to kWh:

$$E = (5.32 \times 10^{26} \,\text{MeV}) \left(\frac{1.60 \times 10^{-13} \,\text{J}}{1 \,\,\text{MeV}}\right) \left(\frac{1 \,\,\text{kWh}}{3.60 \times 10^{6} \,\text{J}}\right) = 2.37 \times 10^{7} \,\text{kWh}$$

which, if released slowly, is enough energy to keep a 100-W lightbulb operating for 30 000 years! If the available fission energy in 1 kg of ²³⁵U were suddenly released, it would be equivalent to detonating about 20 000 tons of TNT.

45.3 Nuclear Reactors

In Section 45.2, we learned that when ²³⁵U fissions, one incoming neutron results in an average of 2.5 neutrons emitted per event. These neutrons can trigger other nuclei to fission. Because more neutrons are produced by the event than are absorbed, there is the possibility of an ever-building chain reaction (Active Fig. 45.3). Calculations show that if the chain reaction is not controlled (that is, if it does not proceed slowly), it can result in a violent explosion, with the sudden release



Figure 45.4 Artist's rendition of the world's first nuclear reactor. Because of wartime secrecy, there are few photographs of the completed reactor, which was composed of layers of moderating graphite interspersed with uranium. A self-sustained chain reaction was first achieved on December 2, 1942. Word of the success was telephoned immediately to Washington, D.C., with this message: "The Italian navigator has landed in the New World and found the natives very friendly." The historic event took place in an improvised laboratory in the racquet court under the stands of the University of Chicago's Stagg Field, and the Italian navigator was Enrico Fermi.



Enrico Fermi Italian Physicist (1901–1954)

Fermi was awarded the Nobel Prize in Physics in 1938 for producing transuranic elements by neutron irradiation and for his discovery of nuclear reactions brought about by thermal neutrons. He made many other outstanding contributions to physics, including his theory of beta decay, the free-electron theory of metals, and the development of the world's first fission reactor in 1942. Fermi was truly a gifted theoretical and experimental physicist. He was also well known for his ability to present physics in a clear and exciting manner.



of an enormous amount of energy. When the reaction is controlled, however, the energy released can be put to constructive use. In the United States, for example, nearly 20% of the electricity generated each year comes from nuclear power plants, and nuclear power is used extensively in many other countries, including France, Japan, and Germany.

A nuclear reactor is a system designed to maintain what is called a self-sustained chain reaction. This important process was first achieved in 1942 by Enrico Fermi and his team at the University of Chicago, using naturally occurring uranium as the fuel.¹ In the first nuclear reactor (Fig. 45.4), Fermi placed bricks of graphite (carbon) between the fuel elements. Carbon nuclei are about 12 times more massive than neutrons, but after several collisions with carbon nuclei, a neutron is slowed sufficiently to increase its likelihood of fission with ²³⁵U. In this design, carbon is the moderator; most modern reactors use water as the moderator.

Most reactors in operation today also use uranium as fuel. Naturally occurring uranium contains only 0.7% of the ²³⁵U isotope, however, with the remaining 99.3% being ²³⁸U. This fact is important to the operation of a reactor because ²³⁸U almost never fissions. Instead, it tends to absorb neutrons without a subsequent fission event, producing neptunium and plutonium. For this reason, reactor fuels must be artificially enriched to contain at least a few percent ²³⁵U.

To achieve a self-sustained chain reaction, an average of one neutron emitted in each ²³⁵U fission must be captured by another ²³⁵U nucleus and cause that nucleus to undergo fission. A useful parameter for describing the level of reactor operation is the **reproduction constant** K, defined as **the average number of neutrons from** each fission event that cause another fission event. As we have seen, K has an average value of 2.5 in the uncontrolled fission of uranium.

A self-sustained and controlled chain reaction is achieved when K = 1. When in this condition, the reactor is said to be **critical.** When K < 1, the reactor is subcritical and the reaction dies out. When K > 1, the reactor is supercritical and a runaway reaction occurs. In a nuclear reactor used to furnish power to a utility company, it is necessary to maintain a value of K close to 1. If K rises above this value, the internal energy produced in the reaction could melt the reactor.

Several types of reactor systems allow the kinetic energy of fission fragments to be transformed to other types of energy and eventually transferred out of the reactor plant by electrical transmission. The most common reactor in use in the

¹Although Fermi's reactor was the first manufactured nuclear reactor, there is evidence that a natural fission reaction may have sustained itself for perhaps hundreds of thousands of years in a deposit of uranium in Gabon, West Africa. See G. Cowan, "A Natural Fission Reactor," Scientific American 235(5): 36, 1976.



Figure 45.5 Main components of a pressurized-water nuclear reactor.

United States is the pressurized-water reactor (Fig. 45.5). We shall examine this type because its main parts are common to all reactor designs. Fission events in the uranium **fuel elements** in the reactor core raise the temperature of the water contained in the primary loop, which is maintained at high pressure to keep the water from boiling. (This water also serves as the moderator to slow down the neutrons released in the fission events with energy of approximately 2 MeV.) The hot water is pumped through a heat exchanger, where the internal energy of the water is transferred by conduction to the water contained in the secondary loop. The hot water in the secondary loop is converted to steam, which does work to drive a turbine–generator system to create electric power. The water in the secondary loop is isolated from the water in the primary loop to avoid contamination of the secondary water and the steam by radioactive nuclei from the reactor core.

In any reactor, a fraction of the neutrons produced in fission leak out of the uranium fuel elements before inducing other fission events. If the fraction leaking out is too large, the reactor will not operate. The percentage lost is large if the fuel elements are very small because leakage is a function of the ratio of surface area to volume. Therefore, a critical feature of the reactor design is an optimal surface area-to-volume ratio of the fuel elements.

Control of Power Level

Safety is of critical importance in the operation of a nuclear reactor. The reproduction constant *K* must not be allowed to rise above 1, lest a runaway reaction occur. Consequently, reactor design must include a means of controlling the value of *K*.

The basic design of a nuclear reactor core is shown in Figure 45.6. The fuel elements consist of uranium that has been enriched in the 235 U isotope. To control the power level, **control rods** are inserted into the reactor core. These rods are made of materials such as cadmium that are very efficient in absorbing neutrons. By adjusting the number and position of the control rods in the reactor core, the *K* value



Figure 45.6 Cross section of a reactor core showing the control rods, fuel elements containing enriched fuel, and moderating material, all surrounded by a radiation shield.

can be varied and any power level within the design range of the reactor can be achieved.

Quick Quiz 45.3 To reduce the value of the reproduction constant *K*, do you(a) push the control rods deeper into the core or (b) pull the control rods farther out of the core?

Safety and Waste Disposal

The 1979 near-disaster at a nuclear power plant at Three Mile Island in Pennsylvania and the 1986 accident at the Chernobyl reactor in Ukraine rightfully focused attention on reactor safety. The Three Mile Island accident was the result of inadequate control-room instrumentation and poor emergency-response training. There were no injuries or detectable health effects from the event, even though more than one-third of the fuel melted.

Unfortunately, at Chernobyl the activity of the materials released immediately after the accident totaled approximately 1.2×10^{19} Bq and resulted in the evacuation of 135 000 people. Thirty individuals died during the accident or shortly thereafter, and data from the Ukraine Radiological Institute suggest that more than 2 500 deaths could be attributed to the Chernobyl accident. In the period 1986–1997, there was a tenfold increase in the number of children contracting thyroid cancer from the ingestion of radioactive iodine in milk from cows that ate contaminated grass. One conclusion of an international conference studying the Ukraine accident was that the main causes of the Chernobyl accident were the coincidence of severe deficiencies in the reactor physical design and a violation of safety procedures. Most of these deficiencies have since been addressed at plants of similar design in Russia and neighboring countries of the former Soviet Union.

Commercial reactors achieve safety through careful design and rigid operating protocol, and only when these variables are compromised do reactors pose a danger. Radiation exposure and the potential health risks associated with such exposure are controlled by three layers of containment. The fuel and radioactive fission products are contained inside the reactor vessel. Should this vessel rupture, the reactor building acts as a second containment structure to prevent radioactive material from contaminating the environment. Finally, the reactor facilities must be in a remote location to protect the general public from exposure should radiation escape the reactor building.

A continuing concern about nuclear fission reactors is the safe disposal of radioactive material when the reactor core is replaced. This waste material contains long-lived, highly radioactive isotopes and must be stored over long time intervals in such a way that there is no chance of environmental contamination. At present, sealing radioactive wastes in waterproof containers and burying them in deep geologic repositories seems to be the most promising solution.

Transport of reactor fuel and reactor wastes poses additional safety risks. Accidents during transport of nuclear fuel could expose the public to harmful levels of radiation. The U.S. Department of Energy requires stringent crash tests of all containers used to transport nuclear materials. Container manufacturers must demonstrate that their containers will not rupture even in high-speed collisions.

Despite these risks, there are advantages to the use of nuclear power to be weighed against the risks. For example, nuclear power plants do not produce air pollution and greenhouse gases as do fossil fuel plants, and the supply of uranium on the Earth is predicted to last longer than the supply of fossil fuels. For each source of energy—whether nuclear, hydroelectric, fossil fuel, wind, solar, or other—the risks must be weighed against the benefits and the availability of the energy source.

45.4 Nuclear Fusion

In Chapter 44, we found that the binding energy for light nuclei (A < 20) is much smaller than the binding energy for heavier nuclei, which suggests a process that is the reverse of fission. As mentioned in Section 39.9, when two light nuclei combine to form a heavier nucleus, the process is called nuclear **fusion**. Because the mass of the final nucleus is less than the combined masses of the original nuclei, there is a loss of mass accompanied by a release of energy.

Two examples of such energy-liberating fusion reactions are as follows:

$${}^{1}_{1}H + {}^{1}_{1}H \rightarrow {}^{2}_{1}H + e^{+} + \nu$$
$${}^{1}_{1}H + {}^{2}_{1}H \rightarrow {}^{3}_{2}He + \gamma$$

These reactions occur in the core of a star and are responsible for the outpouring of energy from the star. The second reaction is followed by either hydrogen-helium fusion or helium-helium fusion:

$${}^{1}_{1}H + {}^{3}_{2}He \rightarrow {}^{4}_{2}He + e^{+} + \nu$$
$${}^{3}_{3}He + {}^{3}_{3}He \rightarrow {}^{4}_{3}He + {}^{1}_{1}H + {}^{1}_{1}H$$

These fusion reactions are the basic reactions in the **proton-proton cycle**, believed to be one of the basic cycles by which energy is generated in the Sun and other stars that contain an abundance of hydrogen. Most of the energy production takes place in the Sun's interior, where the temperature is approximately 1.5×10^7 K. Because such high temperatures are required to drive these reactions, they are called **thermonuclear fusion reactions.** All the reactions in the proton-proton cycle are exothermic. An overview of the cycle is that four protons combine to generate an alpha particle, positrons, gamma rays, and neutrinos.

Quick Quiz 45.4 In the core of a star, hydrogen nuclei combine in fusion reactions. Once the hydrogen has been exhausted, fusion of helium nuclei can occur. If the star is sufficiently massive, fusion of heavier and heavier nuclei can occur once the helium is used up. Consider a fusion reaction involving two nuclei with the same value of A. For this reaction to be exothermic, which of the following values of A are impossible? (a) 12 (b) 20 (c) 28 (d) 64

Example 45.2 Energy Released in Fusion

Find the total energy released in the fusion reactions in the proton-proton cycle.

SOLUTION

Conceptualize The net nuclear result of the proton–proton cycle is to fuse four protons to form an alpha particle. Study the reactions above for the proton–proton cycle to be sure you understand how four protons become an alpha particle.

Categorize We use concepts discussed in this section, so we categorize this example as a substitution problem.

Find the initial mass of the system using the atomic mass of hydrogen from Table 44.2:	$4(1.007\ 825\ u) = 4.031\ 300\ u$
Find the change in mass of the system as this value minus the mass of a 4 He atom:	$4.031\ 300\ u\ -\ 4.002\ 603\ u\ =\ 0.028\ 697\ u$
Convert this mass change into energy units:	$E = 0.028 \ 697 \ u \times 931.494 \ MeV/u = 26.7 \ MeV$

This energy is shared among the alpha particle and other particles such as positrons, gamma rays, and neutrinos.

Pitfall Prevention 45.2

Fision and Fusion

The words *fission* and *fusion* sound similar, but they correspond to different processes. Consider the bindingenergy graph in Figure 44.5. There are two directions from which you can approach the peak of the graph so that energy is released: combining two light nuclei, or fusion, and separating a heavy nucleus into two lighter nuclei, or fission.

Terrestrial Fusion Reactions

The enormous amount of energy released in fusion reactions suggests the possibility of harnessing this energy for useful purposes. A great deal of effort is currently under way to develop a sustained and controllable thermonuclear reactor, a fusion power reactor. Controlled fusion is often called the ultimate energy source because of the availability of its fuel source: water. For example, if deuterium were used as the fuel, 0.12 g of it could be extracted from 1 gal of water at a cost of about four cents. This amount of deuterium would release approximately 10¹⁰ J if all nuclei underwent fusion. By comparison, 1 gal of gasoline releases approximately 10⁸ J upon burning and costs far more than four cents.

An additional advantage of fusion reactors is that comparatively few radioactive by-products are formed. For the proton–proton cycle, for instance, the end product is safe, nonradioactive helium. Unfortunately, a thermonuclear reactor that can deliver a net power output spread over a reasonable time interval is not yet a reality, and many difficulties must be resolved before a successful device is constructed.

The Sun's energy is based in part on a set of reactions in which hydrogen is converted to helium. The proton-proton interaction is not suitable for use in a fusion reactor, however, because the event requires very high temperatures and densities. The process works in the Sun only because of the extremely high density of protons in the Sun's interior.

The reactions that appear most promising for a fusion power reactor involve deuterium $\binom{2}{1}H$ and tritium $\binom{3}{1}H$:

${}_{1}^{9}H + {}_{1}^{9}H$	\rightarrow	${}_{2}^{3}\text{He} + {}_{0}^{1}\text{n}$	Q = 3.27 MeV	
${}^{2}_{1}H + {}^{2}_{1}H$	\rightarrow	${}^{3}_{1}H + {}^{1}_{1}H$	Q = 4.03 MeV	(45.4)
${}^{2}_{1}H + {}^{3}_{1}H$	\rightarrow	${}^{4}_{9}\text{He} + {}^{1}_{0}\text{n}$	Q = 17.59 MeV	

As noted earlier, deuterium is available in almost unlimited quantities from our lakes and oceans and is very inexpensive to extract. Tritium, however, is radioactive $(T_{1/2} = 12.3 \text{ yr})$ and undergoes beta decay to ³He. For this reason, tritium does not occur naturally to any great extent and must be artificially produced.

One major problem in obtaining energy from nuclear fusion is that the Coulomb repulsive force between two nuclei, which carry positive charges, must be overcome before they can fuse. Figure 45.7 is a graph of potential energy as a function of the separation distance between two deuterons (deuterium nuclei, each having charge +e). The potential energy is positive in the region r > R, where the Coulomb repulsive force dominates ($R \approx 1$ fm), and negative in the region r < R, where the nuclear force dominates. The fundamental problem then is to give the two nuclei enough kinetic energy to overcome this repulsive force. This requirement can be accomplished by raising the fuel to extremely high temperatures (to approximately 10^8 K, far greater than the interior temperature of the Sun). At these high temperatures, the atoms are ionized and the system consists of a collection of electrons and nuclei, commonly referred to as a *plasma*.

Example 45.3 The Fusion of Two Deuterons

For the nuclear force to overcome the repulsive Coulomb force, the separation distance between two deuterons must be approximately 1.0×10^{-14} m.

(A) Calculate the height of the potential barrier due to the repulsive force.



The Coulomb repulsive force is

The attractive nuclear force is dominant when the deuterons are close together.

Figure 45.7 Potential energy as a function of separation distance between two deuterons. *R* is on the order of 1 fm. If we neglect tunneling, the two deuterons require an energy *E* greater than the height of the barrier to undergo fusion.

SOLUTION

45.3 cont.

Conceptualize Imagine moving two deuterons toward each other. As they move closer together, the Coulomb repulsion force becomes stronger. Work must be done on the system to push against this force, and this work appears in the system of two deuterons as electric potential energy.

Categorize We categorize this problem as one involving the electric potential energy of a system of two charged particles.

Analyze Evaluate the potential energy associated with two charges separated by a distance r (Eq. 25.13) for two deuterons:

(B) Estimate the temperature required for a deuteron to overcome the potential barrier, assuming an energy of $\frac{3}{2}k_{\rm B}T$ per deuteron (where $k_{\rm B}$ is Boltzmann's constant).

 $= 2.3 \times 10^{-14}$ J = 0.14 MeV

 $\frac{3}{2}k_{\rm B}T = 1.1 \times 10^{-14}$ J

SOLUTION

Because the total Coulomb energy of the pair is 0.14 MeV, the Coulomb energy per deuteron is equal to 0.07 MeV = 1.1×10^{-14} J.

Set this energy equal to the average energy per deuteron:

 $T = \frac{2(1.1 \times 10^{-14} \text{ J})}{3(1.38 \times 10^{-23} \text{ J/K})} = 5.6 \times 10^8 \text{ K}$

 $U = k_e \frac{q_1 q_2}{r} = k_e \frac{(+e)^2}{r} = (8.99 \times 10^9 \,\mathrm{N \cdot m^2/C^2}) \,\frac{(1.60 \times 10^{-19} \,\mathrm{C})^2}{1.0 \times 10^{-14} \,\mathrm{m}}$

(C) Find the energy released in the deuterium-deuterium reaction

 $^{2}_{1}H + ^{2}_{1}H \rightarrow ^{3}_{1}H + ^{1}_{1}H$

SOLUTION

Solve for *T*:

The mass of a single deuterium atom is equal to 2.014 102 u. Therefore, the total mass of the system before the reaction is 4.028 204 u.

Find the sum of the masses after the reaction:	$3.016\ 049\ u + 1.007\ 825\ u = 4.023\ 874\ u$
Find the change in mass and convert to	$4.028\ 204\ u - 4.023\ 874\ u = 0.004\ 33\ u$
energy units:	$= 0.004 33 \text{ u} \times 931.494 \text{ MeV/u} = 4.03 \text{ MeV}$

Finalize The calculated temperature in part (B) is too high because the particles in the plasma have a Maxwellian speed distribution (Section 21.5) and therefore some fusion reactions are caused by particles in the high-energy tail of this distribution. Furthermore, even those particles that do not have enough energy to overcome the barrier have some probability of tunneling through. When these effects are taken into account, a temperature of "only" 4×10^8 K appears adequate to fuse two deuterons in a plasma. In part (C), notice that the energy value is consistent with that already given in Equation 45.4.

WHAT IF? Suppose the tritium resulting from the reaction in part (C) reacts with another deuterium in the reaction

$$^{2}_{1}H + ^{3}_{1}H \rightarrow ^{4}_{2}He + ^{1}_{0}n$$

How much energy is released in the sequence of two reactions?

continued

45.3 cont.

Answer The overall effect of the sequence of two reactions is that three deuterium nuclei have combined to form a helium nucleus, a hydrogen nucleus, and a neutron. The initial mass is $3(2.014\ 102\ u) = 6.042\ 306\ u$. After the reaction, the sum of the masses is $4.002\ 603\ u + 1.007\ 825\ u + 1.008\ 665 = 6.019\ 093\ u$. The excess mass is equal to $0.023\ 213\ u$, equivalent to an energy of 21.6 MeV. Notice that this value is the sum of the Q values for the second and third reactions in Equation 45.4.



Figure 45.8 Power generated versus temperature for deuteriumdeuterium (D–D) and deuteriumtritium (D–T) fusion. When the generation rate exceeds the loss rate, ignition takes place.



Figure 45.9 The Lawson number $n\tau$ at which net energy output is possible versus temperature for the D–T and D–D fusion reactions.

The temperature at which the power generation rate in any fusion reaction exceeds the loss rate is called the **critical ignition temperature** $T_{\rm ignit}$. This temperature for the deuterium–deuterium (D–D) reaction is 4×10^8 K. From the relationship $E \approx \frac{3}{2}k_{\rm B}T$, the ignition temperature is equivalent to approximately 52 keV. The critical ignition temperature for the deuterium–tritium (D–T) reaction is approximately 4.5×10^7 K, or only 6 keV. A plot of the power $P_{\rm gen}$ generated by fusion versus temperature for the two reactions is shown in Figure 45.8. The straight green line represents the power $P_{\rm lost}$ lost via the radiation mechanism known as bremsstrahlung (Section 42.8). In this principal mechanism of energy loss, radiation (primarily x-rays) is emitted as the result of electron–ion collisions within the plasma. The intersections of the $P_{\rm lost}$ line with the $P_{\rm gen}$ curves give the critical ignition temperatures.

In addition to the high-temperature requirements, two other critical parameters determine whether or not a thermonuclear reactor is successful: the **ion density** *n* and **confinement time** τ , which is the time interval during which energy injected into the plasma remains within the plasma. British physicist J. D. Lawson (1923–2008) showed that both the ion density and confinement time must be large enough to ensure that more fusion energy is released than the amount required to raise the temperature of the plasma. For a given value of *n*, the probability of fusion between two particles increases as τ increases. For a given value of τ , the collision rate between nuclei increases as *n* increases. The product $n\tau$ is referred to as the **Lawson number** of a reaction. A graph of the value of $n\tau$ necessary to achieve a net energy output for the D–T and D–D reactions at different temperatures is shown in Figure 45.9. In particular, **Lawson's criterion** states that a net energy output is possible for values of $n\tau$ that meet the following conditions:

$$n\tau \ge 10^{14} \text{ s/cm}^3$$
 (D–T) (45.5)
 $n\tau \ge 10^{16} \text{ s/cm}^3$ (D–D)

These values represent the minima of the curves in Figure 45.9.

Lawson's criterion was arrived at by comparing the energy required to raise the temperature of a given plasma with the energy generated by the fusion process.² The energy $E_{\rm in}$ required to raise the temperature of the plasma is proportional to the ion density n, which we can express as $E_{\rm in} = C_1 n$, where C_1 is some constant. The energy generated by the fusion process is proportional to $n^2\tau$, or $E_{\rm gen} = C_2 n^2 \tau$. This dependence may be understood by realizing that the fusion energy released is proportional to both the rate at which interacting ions collide ($\propto n^2$) and the confinement time τ . Net energy is produced when $E_{\rm gen} > E_{\rm in}$. When the constants C_1 and C_2 are calculated for different reactions, the condition that $E_{\rm gen} \ge E_{\rm in}$ leads to Lawson's criterion.

²Lawson's criterion neglects the energy needed to set up the strong magnetic field used to confine the hot plasma in a magnetic confinement approach. This energy is expected to be about 20 times greater than the energy required to raise the temperature of the plasma. It is therefore necessary either to have a magnetic energy recovery system or to use superconducting magnets.



Figure 45.10 (a) Diagram of a tokamak used in the magnetic confinement scheme. (b) Interior view of the closed Tokamak Fusion Test Reactor (TFTR) vacuum vessel at the Princeton Plasma Physics Laboratory. (c) The National Spherical Torus Experiment (NSTX) that began operation in March 1999.

Current efforts are aimed at meeting Lawson's criterion at temperatures exceeding T_{ignit} . Although the minimum required plasma densities have been achieved, the problem of confinement time is more difficult. The two basic techniques under investigation for solving this problem are magnetic confinement and inertial confinement.

Magnetic Confinement

Many fusion-related plasma experiments use **magnetic confinement** to contain the plasma. A toroidal device called a **tokamak**, first developed in Russia, is shown in Figure 45.10a. A combination of two magnetic fields is used to confine and stabilize the plasma: (1) a strong toroidal field produced by the current in the toroidal windings surrounding a doughnut-shaped vacuum chamber and (2) a weaker "poloidal" field produced by the toroidal current. In addition to confining the plasma, the toroidal current is used to raise its temperature. The resultant helical magnetic field lines spiral around the plasma and keep it from touching the walls of the vacuum chamber. (If the plasma touches the walls, its temperature is reduced and heavy impurities sputtered from the walls "poison" it, leading to large power losses.)

One major breakthrough in magnetic confinement in the 1980s was in the area of auxiliary energy input to reach ignition temperatures. Experiments have shown that injecting a beam of energetic neutral particles into the plasma is a very efficient method of raising it to ignition temperatures. Radio-frequency energy input will probably be needed for reactor-size plasmas.

When it was in operation from 1982 to 1997, the Tokamak Fusion Test Reactor (TFTR, Fig. 45.10b) at Princeton University reported central ion temperatures of 510 million degrees Celsius, more than 30 times greater than the temperature at the center of the Sun. The $n\tau$ values in the TFTR for the D–T reaction were well above 10^{13} s/cm³ and close to the value required by Lawson's criterion. In 1991, reaction rates of 6 × 10¹⁷ D–T fusions per second were reached in the Joint European Torus (JET) tokamak at Abington, England.

One of the new generation of fusion experiments is the National Spherical Torus Experiment (NSTX) at the Princeton Plasma Physics Laboratory and shown in Figure 45.10c. This reactor was brought on line in February 1999 and has been running fusion experiments since then. Rather than the doughnut-shaped plasma of a tokamak, the NSTX produces a spherical plasma that has a hole through its center. The major advantage of the spherical configuration is its ability to confine the plasma at a higher pressure in a given magnetic field. This approach could lead to development of smaller, more economical fusion reactors.

An international collaborative effort involving the United States, the European Union, Japan, China, South Korea, India, and Russia is currently under way to build a fusion reactor called ITER. This acronym stands for International Thermonuclear Experimental Reactor, although recently the emphasis has shifted to interpreting "iter" in terms of its Latin meaning, "the way." One reason proposed for this change is to avoid public misunderstanding and negative connotations toward the word *thermonuclear*. This facility will address the remaining technological and scientific issues concerning the feasibility of fusion power. The design is completed, and Cadarache, France, was chosen in June 2005 as the reactor site. Construction will require about 10 years, with fusion operation projected to begin in 2018. If the planned device works as expected, the Lawson number for ITER will be about six times greater than the current record holder, the JT-60U tokamak in Japan. ITER is expected to produce 1.5 GW of power, and the energy content of the alpha particles inside the reactor will be so intense that they will sustain the fusion reaction, allowing the auxiliary energy sources to be turned off once the reaction is initiated.

Example 45.4 Inside a Fusion Reactor

In 1998, the JT-60U tokamak in Japan operated with a D–T plasma density of 4.8×10^{13} cm⁻³ at a temperature (in energy units) of 24.1 keV. It confined this plasma inside a magnetic field for 1.1 s.

(A) Do these data meet Lawson's criterion?

SOLUTION

Conceptualize With the help of the third of Equations 45.4, imagine many such reactions occurring in a plasma of high temperature and high density.

Categorize We use the concept of the Lawson number discussed in this section, so we categorize this example as a substitution problem.

Evaluate the Lawson number for the JT-60U:

 $n\tau = (4.8 \times 10^{13} \text{ cm}^{-3})(1.1 \text{ s}) = 5.3 \times 10^{13} \text{ s/cm}^{3}$

This value is close to meeting Lawson's criterion of 10^{14} s/cm³ for a D–T plasma given in Equation 45.5. In fact, scientists recorded a power gain of 1.25, indicating that the reactor operated slightly past the break-even point and produced more energy than it required to maintain the plasma.

(B) How does the plasma density compare with the density of atoms in an ideal gas when the gas is under standard conditions (T = 0°C and P = 1 atm)?

SOLUTION

Find the density of atoms in a sample of ideal gas by evaluating $N_{\rm A}/V_{\rm mol}$, where $N_{\rm A}$ is Avogadro's number and $V_{\rm mol}$ is the molar volume of an ideal gas under standard conditions, $2.24 \times 10^{-2} \text{ m}^3/\text{mol}$:

 $\frac{N_{\rm A}}{V_{\rm mol}} = \frac{6.02 \times 10^{23} \text{ atoms/mol}}{2.24 \times 10^{-2} \text{ m}^3/\text{mol}} = 2.7 \times 10^{25} \text{ atoms/m}^3$ $= 2.7 \times 10^{19} \text{ atoms/cm}^3$

This value is more than 500 000 times greater than the plasma density in the reactor.

Inertial Confinement

The second technique for confining a plasma, called **inertial confinement**, makes use of a D–T target that has a very high particle density. In this scheme, the confinement time is very short (typically 10^{-11} to 10^{-9} s), and, because of their own inertia, the particles do not have a chance to move appreciably from their initial positions. Therefore, Lawson's criterion can be satisfied by combining a high particle density with a short confinement time.

Laser fusion is the most common form of inertial confinement. A small D–T pellet, approximately 1 mm in diameter, is struck simultaneously by several focused, high-intensity laser beams, resulting in a large pulse of input energy that causes the surface of the fuel pellet to evaporate (Fig. 45.11). The escaping particles exert a third-law reaction force on the core of the pellet, resulting in a strong, inwardly moving compressive shock wave. This shock wave increases the pressure and density of the core and produces a corresponding increase in temperature. When the temperature of the core reaches ignition temperature, fusion reactions occur.

One of the leading laser fusion laboratories in the United States is the Omega facility at the University of Rochester in New York. This facility focuses 24 laser beams on the target. Currently under construction at the Lawrence Livermore National Laboratory in Livermore, California, is the National Ignition Facility. The research apparatus there will include 192 laser beams that can be focused on a deuterium–tritium pellet. Construction was completed in early 2009, and a test firing of the lasers in March 2009 broke the megajoule record for lasers for the first time, delivering 1.1 MJ to a target. Fusion ignition tests are planned for 2010.

Fusion Reactor Design

In the D-T fusion reaction

$${}^{2}_{1}H + {}^{3}_{1}H \rightarrow {}^{4}_{2}He + {}^{1}_{0}n \qquad Q = 17.59 \text{ MeV}$$

the alpha particle carries 20% of the energy and the neutron carries 80%, or approximately 14 MeV. A diagram of the deuterium-tritium fusion reaction is shown in Active Figure 45.12. Because the alpha particles are charged, they are primarily absorbed by the plasma, causing the plasma's temperature to increase. In contrast, the 14-MeV neutrons, being electrically neutral, pass through the plasma and are absorbed by a surrounding blanket material, where their large kinetic energy is extracted and used to generate electric power.

One scheme is to use molten lithium metal as the neutron-absorbing material and to circulate the lithium in a closed heat-exchange loop, thereby producing steam and driving turbines as in a conventional power plant. Figure 45.13 (page 1388) shows a diagram of such a reactor. It is estimated that a blanket of lithium approximately 1 m thick will capture nearly 100% of the neutrons from the fusion of a small D–T pellet.

The capture of neutrons by lithium is described by the reaction

$$_{0}^{1}n + _{3}^{6}Li \rightarrow _{1}^{3}H + _{2}^{4}He$$

where the kinetic energies of the charged tritium ${}_{1}^{3}$ H and alpha particle are converted to internal energy in the molten lithium. An extra advantage of using lithium as the energy-transfer medium is that the tritium produced can be separated from the lithium and returned as fuel to the reactor.

Advantages and Problems of Fusion

If fusion power can ever be harnessed, it will offer several advantages over fissiongenerated power: (1) low cost and abundance of fuel (deuterium), (2) impossibility



Figure 45.11 In inertial confinement, a D–T fuel pellet fuses when struck by several high-intensity laser beams simultaneously.



Deuterium–tritium fusion. Eighty percent of the energy released is in the 14-MeV neutron.

Figure 45.13 Diagram of a fusion reactor.



of runaway accidents, and (3) decreased radiation hazard. Some of the anticipated problems and disadvantages include (1) scarcity of lithium, (2) limited supply of helium, which is needed for cooling the superconducting magnets used to produce strong confining fields, and (3) structural damage and induced radioactivity caused by neutron bombardment. If such problems and the engineering design factors can be resolved, nuclear fusion may become a feasible source of energy by the middle of the twenty-first century.

45.5 Radiation Damage

In Chapter 34, we learned that electromagnetic radiation is all around us in the form of radio waves, microwaves, light waves, and so on. In this section, we describe forms of radiation that can cause severe damage as they pass through matter, such as radiation resulting from radioactive processes and radiation in the form of energetic particles such as neutrons and protons.

The degree and type of damage depend on several factors, including the type and energy of the radiation and the properties of the matter. The metals used in nuclear reactor structures can be severely weakened by high fluxes of energetic neutrons because these high fluxes often lead to metal fatigue. The damage in such situations is in the form of atomic displacements, often resulting in major alterations in the properties of the material.

Radiation damage in biological organisms is primarily due to ionization effects in cells. A cell's normal operation may be disrupted when highly reactive ions are formed as the result of ionizing radiation. For example, hydrogen and the hydroxyl radical OH^- produced from water molecules can induce chemical reactions that may break bonds in proteins and other vital molecules. Furthermore, the ionizing radiation may affect vital molecules directly by removing electrons from their structure. Large doses of radiation are especially dangerous because damage to a great number of molecules in a cell may cause the cell to die. Although the death of a single cell is usually not a problem, the death of many cells may result in irreversible damage to the organism. Cells that divide rapidly, such as those of the digestive tract, reproductive organs, and hair follicles, are especially susceptible. In addition, cells that survive the radiation may become defective. These defective cells can produce more defective cells and can lead to cancer.

In biological systems, it is common to separate radiation damage into two categories: somatic damage and genetic damage. *Somatic damage* is that associated with any body cell except the reproductive cells. Somatic damage can lead to cancer or can seriously alter the characteristics of specific organisms. *Genetic damage* affects only reproductive cells. Damage to the genes in reproductive cells can lead to defective offspring. It is important to be aware of the effect of diagnostic treatments, such as x-rays and other forms of radiation exposure, and to balance the significant benefits of treatment with the damaging effects.

Damage caused by radiation also depends on the radiation's penetrating power. Alpha particles cause extensive damage, but penetrate only to a shallow depth in a material due to the strong interaction with other charged particles. Neutrons do not interact via the electric force and hence penetrate deeper, causing significant damage. Gamma rays are high-energy photons that can cause severe damage, but often pass through matter without interaction.

Several units have been used historically to quantify the amount, or dose, of any radiation that interacts with a substance.

The **roentgen** (R) is that amount of ionizing radiation that produces an electric charge of 3.33×10^{-10} C in 1 cm³ of air under standard conditions.

Equivalently, the roentgen is that amount of radiation that increases the energy of 1 kg of air by 8.76×10^{-3} J.

For most applications, the roentgen has been replaced by the rad (an acronym for *radiation absorbed dose*):

One **rad** is that amount of radiation that increases the energy of 1 kg of absorbing material by 1×10^{-2} J.

Although the rad is a perfectly good physical unit, it is not the best unit for measuring the degree of biological damage produced by radiation because damage depends not only on the dose but also on the type of the radiation. For example, a given dose of alpha particles causes about ten times more biological damage than an equal dose of x-rays. The **RBE** (relative biological effectiveness) factor for a given type of radiation is **the number of rads of x-radiation or gamma radiation that produces the same biological damage as 1 rad of the radiation being used.** The RBE factors for different types of radiation are given in Table 45.1. The values are only approximate because they vary with particle energy and with the form of

Radiation	RBE Factor
X-rays and gamma rays	1.0
Beta particles	1.0 - 1.7
Alpha particles	10-20
Thermal neutrons	4-5
Fast neutrons and protons	10
Heavy ions	20

.....

 TABLE 45.2
 Units for Radiation Dosage

Quantity	SI Unit	Symbol	Relations to Other SI Units	Older Unit	Conversion
Absorbed dose	gray	Gy	= 1 J/kg $= 1 J/kg$	rad	1 Gy = 100 rad
Dose equivalent	sievert	Sv		rem	1 Sv = 100 rem

the damage. The RBE factor should be considered only a first-approximation guide to the actual effects of radiation.

Finally, the **rem** (radiation equivalent in man) is the product of the dose in rad and the RBE factor:

Radiation dose in rem 🕨

Dose in rem \equiv dose in rad \times RBE (45.6)

According to this definition, 1 rem of any two types of radiation produces the same amount of biological damage. Table 45.1 shows that a dose of 1 rad of fast neutrons represents an effective dose of 10 rem, but 1 rad of gamma radiation is equivalent to a dose of only 1 rem.

Low-level radiation from natural sources such as cosmic rays and radioactive rocks and soil delivers to each of us a dose of approximately 0.13 rem/yr. This radiation, called *background radiation*, varies with geography, with the main factors being altitude (exposure to cosmic rays) and geology (radon gas released by some rock formations, deposits of naturally radioactive minerals).

The upper limit of radiation dose rate recommended by the U.S. government (apart from background radiation) is approximately 0.5 rem/yr. Many occupations involve much higher radiation exposures, so an upper limit of 5 rem/yr has been set for combined whole-body exposure. Higher upper limits are permissible for certain parts of the body, such as the hands and the forearms. A dose of 400 to 500 rem results in a mortality rate of approximately 50% (which means that half the people exposed to this radiation level die). The most dangerous form of exposure for most people is either ingestion or inhalation of radioactive isotopes, especially isotopes of those elements the body retains and concentrates, such as ⁹⁰Sr.

This discussion has focused on measurements of radiation dosage in units such as rads and rems because these units are still widely used. They have, however, been formally replaced with new SI units. The rad has been replaced with the *gray* (Gy), equal to 100 rad, and the rem has been replaced with the *sievert* (Sv), equal to 100 rem. Table 45.2 summarizes the older and the current SI units of radiation dosage.

45.6 Radiation Detectors

Particles passing through matter interact with the matter in several ways. The particle can, for example, ionize atoms, scatter from atoms, or be absorbed by atoms. Radiation detectors exploit these interactions to allow a measurement of the particle's energy, momentum, or charge and sometimes the very existence of the particle if it is otherwise difficult to detect. Various devices have been developed for detecting radiation. These devices are used for a variety of purposes, including medical diagnoses, radioactive dating measurements, measuring background radiation, and measuring the mass, energy, and momentum of particles created in high-energy nuclear reactions.

In the early part of the 20th century, detectors were much simpler than those used today. We discuss three of these early detectors first. A **photographic emulsion** is the simplest example of a detector. A charged particle ionizes the atoms in an emulsion layer. The particle's path corresponds to a family of points at which



Figure 45.14 (a) Artificially colored bubble-chamber photograph showing tracks of particles that have passed through the chamber. (b) This research scientist is studying a photograph of particle tracks made in a bubble chamber at Fermilab. The curved tracks are produced by charged particles moving through the chamber in the presence of an applied magnetic field. Negatively charged particles deflect in one direction, and positively charged particles deflect on direction.

chemical changes have occurred in the emulsion. When the emulsion is developed, the particle's track becomes visible. A **cloud chamber** contains a gas that has been supercooled to slightly below its usual condensation point. An energetic particle passing through ionizes the gas along the particle's path. The ions serve as centers for condensation of the supercooled gas. The particle's track can be seen with the naked eye and can be photographed. A magnetic field can be applied to determine the charges of the particles as well as their momentum and energy. A device called a **bubble chamber** uses a liquid (usually liquid hydrogen) maintained near its boiling point. Ions produced by incoming charged particles leave bubble tracks, which can be photographed (Fig. 45.14). Because the density of the detecting medium in a bubble chamber is much higher than the density of the gas in a cloud chamber, the bubble chamber has a much higher sensitivity.

More contemporary detectors involve more sophisticated processes. In an **ion chamber** (Fig. 45.15), electron-ion pairs are generated as radiation passes through a gas and produces an electrical signal. Two plates in the chamber are connected to a voltage supply and thereby maintained at different electric potentials. The positive plate attracts the electrons, and the negative plate attracts positive ions, causing a current pulse that is proportional to the number of electron-ion pairs produced when a particle passes through the chamber. When an ion chamber is used both to detect the presence of a particle and to measure its energy, it is called a **proportional counter**.

The **Geiger counter** (Fig. 45.16 on page 1392) is perhaps the most common form of ion chamber used to detect radioactivity. It can be considered the prototype of all counters that use the ionization of a medium as the basic detection process. A Geiger counter consists of a thin wire aligned along the central axis of a cylindrical metallic tube filled with a gas at low pressure. The central wire is maintained at



Figure 45.15 Simplified diagram of an ion chamber.



Figure 45.16 (a) Diagram of a Geiger counter. (b) A scientist uses a Geiger counter to make a measurement.



Figure 45.17 The STAR detector at the Relativistic Heavy Ion Collider at Brookhaven National Laboratory.

a high positive electric potential (approximately 10^3 V) relative to the tube. When a high-energy particle resulting, for example, from a radioactive decay enters the tube through a thin window at one end, some of the gas atoms are ionized. The electrons removed from these atoms are attracted toward the central wire, and, in the process, they ionize other atoms in their path. This sequential ionization results in an *avalanche* of electrons that produces a current pulse. After the pulse has been amplified, it can either be used to trigger an electronic counter or be delivered to a loudspeaker that clicks each time a particle is detected. Although a Geiger counter easily detects the presence of a particle, the energy lost by the particle in the counter is *not* proportional to the current pulse produced. Therefore, a Geiger counter cannot be used to measure the energy of a particle.

A semiconductor-diode detector is essentially a reverse-bias p-n junction. Recall from Section 43.7 that a p-n junction passes current readily when forwardbiased and prohibits a current when reverse-biased. As an energetic particle passes through the junction, electrons are excited into the conduction band and holes are formed in the valence band. The internal electric field sweeps the electrons toward the positive (*n*) side of the junction and the holes toward the negative (*p*) side. This movement of electrons and holes creates a pulse of current that is measured with an electronic counter. In a typical device, the duration of the pulse is 10^{-8} s.

A scintillation counter usually uses a solid or liquid material whose atoms are easily excited by radiation. The excited atoms then emit photons when they return to their ground state. Common materials used as scintillators are transparent crystals of sodium iodide and certain plastics. If the scintillator material is attached to a photomultiplier tube (Section 40.2), the photons emitted by the scintillator can be detected and an electrical signal produced.

Both the scintillator and the semiconductor-diode detector are much more sensitive than a Geiger counter mainly because of the higher density of the detecting medium. Both measure the total energy deposited in the detector, which can be very useful in particle identification. In addition, if the particle stops in the detector, both instruments can be used to measure the total particle energy.

Track detectors are devices used to view the tracks of charged particles directly. High-energy particles produced in particle accelerators may have energies ranging from 10^9 to 10^{12} eV. Therefore, they often cannot be stopped and cannot have their energy measured with the detectors already mentioned. Instead, the energy and momentum of these energetic particles are found from the curvature of their path in a magnetic field of known magnitude and direction.

A **spark chamber** is a counting device that consists of an array of conducting parallel plates and is capable of recording a three-dimensional track record. Evennumbered plates are grounded, and odd-numbered plates are maintained at a high electric potential (approximately 10 kV). The spaces between the plates contain an inert gas at atmospheric pressure. When a charged particle passes through the chamber, gas atoms are ionized, resulting in a current surge and visible sparks along the particle path. These sparks may be photographed or electronically detected and the data sent to a computer for path reconstruction and determination of particle mass, momentum, and energy.

Newer versions of the spark chamber have been developed. A **drift chamber** has thousands of high-voltage wires arrayed through the space of the detector, which is filled with gas. The result is an array of thousands of proportional counters. When a charged particle passes through the detector, it ionizes gas molecules and the ejected electrons drift toward the high-voltage wires, creating an electrical signal upon arrival. A computer detects the signals and reconstructs the path through the detector. A large-volume, sophisticated drift chamber that has provided significant results in studying particles formed in collisions of atoms is the Solenoidal Tracker at RHIC (STAR). (The acronym RHIC stands for Relativistic Heavy Ion Collider, a facility at Brookhaven National Laboratory that began operation in 2000.) This type of drift chamber is called a **time projection chamber**. A photograph of the STAR detector is shown in Figure 45.17.

45.7 Uses of Radiation

Nuclear physics applications are extremely widespread in manufacturing, medicine, and biology. In this section, we present a few of these applications and the underlying theories supporting them.

Tracing

Radioactive tracers are used to track chemicals participating in various reactions. One of the most valuable uses of radioactive tracers is in medicine. For example, iodine, a nutrient needed by the human body, is obtained largely through the intake of iodized salt and seafood. To evaluate the performance of the thyroid, the patient drinks a very small amount of radioactive sodium iodide containing ¹³¹I, an artificially produced isotope of iodine (the natural, nonradioactive isotope is ¹²⁷I). The amount of iodine in the thyroid gland is determined as a function of time by measuring the radiation intensity at the neck area. How much of the isotope ¹³¹I remains in the thyroid is a measure of how well that gland is functioning.

A second medical application is indicated in Figure 45.18. A solution containing radioactive sodium is injected into a vein in the leg, and the time at which the radioisotope arrives at another part of the body is detected with a radiation counter. The elapsed time is a good indication of the presence or absence of constrictions in the circulatory system.

Tracers are also useful in agricultural research. Suppose the best method of fertilizing a plant is to be determined. A certain element in a fertilizer, such as nitrogen, can be *tagged* (identified) with one of its radioactive isotopes. The fertilizer is then sprayed on one group of plants, sprinkled on the ground for a second group, and raked into the soil for a third. A Geiger counter is then used to track the nitrogen through each of the three groups.

Tracing techniques are as wide ranging as human ingenuity can devise. Today, applications range from checking how teeth absorb fluoride to monitoring how cleansers contaminate food-processing equipment to studying deterioration inside an automobile engine. In this last case, a radioactive material is used in the manufacture of the car's piston rings and the oil is checked for radioactivity to determine the amount of wear on the rings.

Materials Analysis

For centuries, a standard method of identifying the elements in a sample of material has been chemical analysis, which involves determining how the material reacts with various chemicals. A second method is spectral analysis, which works because each element, when excited, emits its own characteristic set of electromagnetic



Figure 45.18 A tracer technique for determining the condition of the human circulatory system.

wavelengths. These methods are now supplemented by a third technique, **neutron** activation analysis. A disadvantage of both chemical and spectral methods is that a fairly large sample of the material must be destroyed for the analysis. In addition, extremely small quantities of an element may go undetected by either method. Neutron activation analysis has an advantage over chemical analysis and spectral analysis in both respects.

When a material is irradiated with neutrons, nuclei in the material absorb the neutrons and are changed to different isotopes, most of which are radioactive. For example, ⁶⁵Cu absorbs a neutron to become ⁶⁶Cu, which undergoes beta decay:

$$^{1}_{0}n + ^{65}_{29}Cu \rightarrow ^{66}_{29}Cu \rightarrow ^{66}_{30}Zn + e^{-} + \overline{\nu}$$

The presence of the copper can be deduced because it is known that ⁶⁶Cu has a half-life of 5.1 min and decays with the emission of beta particles having a maximum energy of 2.63 MeV. Also emitted in the decay of ⁶⁶Cu is a 1.04-MeV gamma ray. By examining the radiation emitted by a substance after it has been exposed to neutron irradiation, one can detect extremely small amounts of an element in that substance.

Neutron activation analysis is used routinely in a number of industries. In commercial aviation, for example, it is used to check airline luggage for hidden explosives. One nonroutine use is of historical interest. Napoleon died on the island of St. Helena in 1821, supposedly of natural causes. Over the years, suspicion has existed that his death was not all that natural. After his death, his head was shaved and locks of his hair were sold as souvenirs. In 1961, the amount of arsenic in a sample of this hair was measured by neutron activation analysis, and an unusually large quantity of arsenic was found. (Activation analysis is so sensitive that very small pieces of a single hair could be analyzed.) Results showed that the arsenic was fed to him irregularly. In fact, the arsenic concentration pattern corresponded to the fluctuations in the severity of Napoleon's illness as determined from historical records.

Art historians use neutron activation analysis to detect forgeries. The pigments used in paints have changed throughout history, and old and new pigments react differently to neutron activation. The method can even reveal hidden works of art behind existing paintings because an older, hidden layer of paint reacts differently than the surface layer to neutron activation.

Radiation Therapy

Radiation causes much damage to rapidly dividing cells. Therefore, it is useful in cancer treatment because tumor cells divide extremely rapidly. Several mechanisms can be used to deliver radiation to a tumor. In Section 42.8, we discussed the use of high-energy x-rays in the treatment of cancerous tissue. Other treatment protocols include the use of narrow beams of radiation from a radioactive source. As an example, Figure 45.19 shows a machine that uses ⁶⁰Co as a source. The ⁶⁰Co isotope emits gamma rays with photon energies higher than 1 MeV.

In other situations, a technique called *brachytherapy* is used. In this treatment plan, thin radioactive needles called *seeds* are implanted in the cancerous tissue. The energy emitted from the seeds is delivered directly to the tumor, reducing the exposure of surrounding tissue to radiation damage. In the case of prostate cancer, the active isotopes used in brachytherapy include ¹²⁵I and ¹⁰³Pd.

Food Preservation

Radiation is finding increasing use as a means of preserving food because exposure to high levels of radiation can destroy or incapacitate bacteria and mold spores (Fig. 45.20). Techniques include exposing foods to gamma rays, high-energy electron beams, and x-rays. Food preserved by such exposure can be placed in a sealed container (to keep out new spoiling agents) and stored for long periods of time. There is little or no evidence of adverse effect on the taste or nutritional value of food from irradiation. The safety of irradiated foods has been endorsed by the World Health



Figure 45.19 This large machine is being set to deliver a dose of radiation from ⁶⁰Co in an effort to destroy a cancerous tumor. Cancer cells are especially susceptible to this type of therapy because they tend to divide more often than cells of healthy tissue nearby.



Organization, the Centers for Disease Control and Prevention, the U.S. Department of Agriculture, and the Food and Drug Administration. Irradiation of food is presently permitted in more than 40 countries. Some estimates place the amount of irradiated food in the world as high as 500 000 metric tons each year.

Figure 45.20 The strawberries on the left are untreated and have become moldy. The unspoiled strawberries on the right have been irradiated. The radiation has killed or incapacitated the mold spores that have spoiled the strawberries on the left.

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Summary

The **reproduction constant** *K* is

the average number of neutrons released from each fission event that

cause another event. In a fission

reactor, it is necessary to maintain

 $K \approx 1$. The value of *K* is affected by

mean neutron energy, and probabil-

such factors as reactor geometry,

ity of neutron capture.

Concepts and Principles

The probability that neutrons are captured as they move through matter generally increases with decreasing neutron energy. A **thermal neutron** is a slow-moving neutron that has a high probability of being captured by a nucleus in a **neutron capture event**:

$${}^{1}_{0}\mathbf{n} + {}^{A}_{Z}\mathbf{X} \rightarrow {}^{A+1}_{Z}\mathbf{X}^{*} \rightarrow {}^{A+1}_{Z}\mathbf{X} + \gamma$$
(45.1)

where ${}^{A+1}_{Z}X^*$ is an excited intermediate nucleus that rapidly emits a photon.

Nuclear fission occurs when a very heavy nucleus, such as ²³⁵U, splits into two smaller **fission fragments.** Thermal neutrons can create fission in ²³⁵U:

$${}_{0}^{1}n + {}_{92}^{235}U \rightarrow {}_{92}^{236}U^* \rightarrow X + Y + neutrons$$
 (45.2)

where ²³⁶U* is an intermediate excited state and X and Y are the fission fragments. On average, 2.5 neutrons are released per fission event. The fragments then undergo a series of beta and gamma decays to various stable isotopes. The energy released per fission event is approximately 200 MeV.

In **nuclear fusion,** two light nuclei fuse to form a heavier nucleus and release energy. The major obstacle in obtaining useful energy from fusion is the large Coulomb repulsive force between the charged nuclei at small separation distances. The temperature required to produce fusion is on the order of 10⁸ K, and at this temperature, all matter occurs as a plasma. In a fusion reactor, the plasma temperature must reach the **critical ignition temperature**, the temperature at which the power generated by the fusion reactions exceeds the power lost in the system. The most promising fusion reaction is the D–T reaction, which has a critical ignition temperature of approximately 4.5×10^7 K. Two critical parameters in fusion reactor design are **ion density** *n* and **confinement time** τ , the time interval during which the interacting particles must be maintained at $T > T_{ignit}$. **Lawson's criterion** states that for the D–T reaction, $n\tau \ge 10^{14}$ s/cm³.

Objective Questions

- 1. If the moderator were suddenly removed from a nuclear reactor in an electric generating station, what is the most likely consequence? (a) The reactor would go supercritical, and a runaway reaction would occur. (b) The nuclear reaction would proceed in the same way, but the reactor would overheat. (c) The reactor would become subcritical, and the reaction would die out. (d) No change would occur in the reactor's operation.
- 2. Which particle is most likely to be captured by a ²³⁵U nucleus and cause it to undergo fission? (a) an energetic proton (b) an energetic neutron (c) a slow-moving alpha particle (d) a slow-moving neutron (e) a fast-moving electron
- **3.** In the first nuclear weapon test carried out in New Mexico, the energy released was equivalent to approximately 17 kilotons of TNT. Estimate the mass decrease in the nuclear fuel representing the energy converted from rest energy into other forms in this event. *Note:* One ton of TNT has the energy equivalent of 4.2×10^9 J. (a) 1 µg (b) 1 mg (c) 1 g (d) 1 kg (e) 20 kg
- 4. Which of the following fuel conditions is *not* necessary to operate a self-sustained controlled fusion reactor? (a) The fuel must be at a sufficiently high temperature. (b) The fuel must be radioactive. (c) The fuel must be at a sufficiently high density. (d) The fuel must be confined for a sufficiently long period of time. (e) Conditions (a) through (d) are all necessary.
- 5. In a certain fission reaction, a 235 U nucleus captures a neutron. This process results in the creation of the products 137 I and 96 Y along with how many neutrons? (a) 1 (b) 2 (c) 3 (d) 4 (e) 5
- 6. You may use Figure 44.5 to answer this question. Three nuclear reactions take place, each involving 108 nucleons: (1) eighteen ⁶Li nuclei fuse in pairs to form nine ¹²C nuclei, (2) four nuclei each with 27 nucleons fuse in pairs to form two nuclei with 54 nucleons, and (3) one nucleus with 108 nucleons fissions to form two nuclei with 54 nucleons. Rank these three reactions from the largest positive Q value (representing energy output) to the largest negative value (representing energy input). Also include Q = 0 in your ranking to make clear which of the reactions put out energy and which absorb energy. Note any cases of equality in your ranking.

Conceptual Questions

- **1.** Why is water a better shield against neutrons than lead or steel?
- **2.** If a nucleus captures a slow-moving neutron, the product is left in a highly excited state, with an energy approximately

denotes answer available in Student Solutions Manual/Study Guide

- **7.** In the operation of a Geiger counter, is the amplitude of the current pulse (a) proportional to the kinetic energy of the particle producing the pulse, (b) proportional to the number of particles entering the tube to produce the pulse, (c) proportional to the RBE factor of the type of particle producing the pulse, or (d) independent of all these factors?
- 8. Figure OQ45.8 shows particle tracks in a bubble chamber immersed in a magnetic field. The tracks are generally spirals rather than sections of circles. What is the primary reason for this shape? (a) The magnetic field is not perpendicular to the velocity of the particles. (b) The magnetic field is not uniform in space. (c) The forces on the particles increase with time. (d) The speeds of the particles decrease with time.



Figure OQ45.8

- **9.** If an alpha particle and an electron have the same kinetic energy, which undergoes the greater deflection when passed through a magnetic field? (a) The alpha particle does. (b) The electron does. (c) They undergo the same deflection. (d) Neither is deflected.
- 10. Working with radioactive materials at a laboratory over one year, (a) Tom received 1 rem of alpha radiation, (b) Karen received 1 rad of fast neutrons, (c) Paul received 1 rad of thermal neutrons as a whole-body dose, and (d) Ingrid received 1 rad of thermal neutrons to her hands only. Rank these four doses according to the likely amount of biological damage from the greatest to the least, noting any cases of equality.

denotes answer available in Student Solutions Manual/Study Guide

8 MeV above the ground state. Explain the source of the excitation energy.

3. Why would a fusion reactor produce less radioactive waste than a fission reactor?

- **4.** Discuss the advantages and disadvantages of fission reactors from the point of view of safety, pollution, and resources. Make a comparison with power generated from the burning of fossil fuels.
- **5.** Discuss the similarities and differences between fusion and fission.
- **6.** Lawson's criterion states that the product of ion density and confinement time must exceed a certain number before a break-even fusion reaction can occur. Why should these two parameters determine the outcome?
- **7.** Discuss the advantages and disadvantages of fusion power from the viewpoint of safety, pollution, and resources.
- 8. The design of a photomultiplier tube (Fig. CQ45.8) might suggest that any number of dynodes may be used to amplify a weak signal. What factors do you suppose would limit the amplification in this device?
- **9.** What factors make a terrestrial fusion reaction difficult to achieve?

Problems

WebAssign The problems found in this chapter may be assigned online in Enhanced WebAssign

- denotes straightforward problem;
 denotes intermediate problem;
 denotes challenging problem
- 1. full solution available in the Student Solutions Manual/Study Guide
- denotes problems most often assigned in Enhanced WebAssign; these provide students with targeted feedback and either a Master It tutorial or a Watch It solution video.

Section 45.1 Interactions Involving Neutrons

Section 45.2 Nuclear Fission

Note: Problem 53 in Chapter 25 and Problems 22 and 74 in Chapter 44 can be assigned with this chapter.

1. Find the energy released in the fission reaction

 $^{1}_{0}n + ^{235}_{92}U \rightarrow ^{98}_{40}Zr + ^{135}_{52}Te + 3(^{1}_{0}n)$

The atomic masses of the fission products are 97.912 735 u for $\frac{98}{40}$ Zr and 134.916 450 u for $\frac{135}{52}$ Te.

- 2. Burning one metric ton (1 000 kg) of coal can yield an energy of 3.30×10^{10} J. Fission of one nucleus of uranium-235 yields an average of approximately 200 MeV. What mass of uranium produces the same energy in fission as burning one metric ton of coal?
- **3.** Strontium-90 is a particularly dangerous fission product of ²³⁵U because it is radioactive and it substitutes for calcium in bones. What other direct fission products would accompany it in the neutron-induced fission of ²³⁵U? *Note:* This reaction may release two, three, or four free neutrons.



Q C	denotes asking for quantitative and conceptual reasoning
S	denotes symbolic reasoning problem
Μ	denotes Master It tutorial available in Enhanced WebAssign
GP	denotes guided problem
haded	denotes "paired problems" that develop reasoning with

- shaded denotes "paired problems" that develop reasoning with symbols and numerical values
- **4.** A typical nuclear fission power plant produces approximately 1.00 GW of electrical power. Assume the plant has an overall efficiency of 40.0% and each fission reaction produces 200 MeV of energy. Calculate the mass of ²³⁵U consumed each day.
- **5.** List the nuclear reactions required to produce ²³³U from ²³²Th under fast neutron bombardment.
- **6.** The following fission reaction is typical of those occurring in a nuclear electric generating station:

 ${}^{1}_{0}n + {}^{235}_{92}U \rightarrow {}^{141}_{56}Ba + {}^{92}_{36}Kr + 3({}^{1}_{0}n)$

(a) Find the energy released in the reaction. The masses of the products are 140.914 411 u for $^{141}_{56}$ Ba and 91.926 156 u for $^{92}_{56}$ Kr. (b) What fraction of the initial rest energy of the system is transformed to other forms?

7. Find the energy released in the fission reaction

 ${}^{1}_{0}n + {}^{235}_{92}U \rightarrow {}^{88}_{38}Sr + {}^{136}_{54}Xe + 12({}^{1}_{0}n)$

8. A 2.00-MeV neutron is emitted in a fission reactor. If it loses half its kinetic energy in each collision with a moderator

atom, how many collisions does it undergo as it becomes a thermal neutron, with energy 0.039 eV?

- 9. M Review. Suppose seawater exerts an average frictional drag force of 1.00×10^5 N on a nuclear-powered ship. The fuel consists of enriched uranium containing 3.40% of the fissionable isotope $\frac{235}{92}$ U, and the ship's reactor has an efficiency of 20.0%. Assuming 200 MeV is released per fission event, how far can the ship travel per kilogram of fuel?
- 10. QC Seawater contains 3.00 mg of uranium per cubic meter. (a) Given that the average ocean depth is about 4.00 km and water covers two-thirds of the Earth's surface, estimate the amount of uranium dissolved in the ocean. (b) About 0.700% of naturally occurring uranium is the fissionable isotope ²³⁵U. Estimate how long the uranium in the oceans could supply the world's energy needs at the current usage of 1.50 × 10¹³ J/s. (c) Where does the dissolved uranium come from? (d) Is it a renewable energy source?.

Section 45.3 Nuclear Reactors

- 11. If the reproduction constant is 1.000 25 for a chain reaction in a fission reactor and the average time interval between successive fissions is 1.20 ms, by what factor does the reaction rate increase in one minute?
- **12.** A large nuclear power reactor produces approximately 3 000 MW of power in its core. Three months after a reactor is shut down, the core power from radioactive by-products is 10.0 MW. Assuming each emission delivers 1.00 MeV of energy to the power, find the activity in becquerels three months after the reactor is shut down.
- **13.** The probability of a nuclear reaction increases dramatically when the incident particle is given energy above the "Coulomb barrier," which is the electric potential energy of the two nuclei when their surfaces barely touch. Compute the Coulomb barrier for the absorption of an alpha particle by a gold nucleus.
- 14. QC S To minimize neutron leakage from a reactor, the ratio of the surface area to the volume should be a minimum. For a given volume V, calculate this ratio for (a) a sphere, (b) a cube, and (c) a parallelepiped of dimensions a × a × 2a. (d) Which of these shapes would have minimum leakage? Which would have maximum leakage? Explain your answers.
- **15.** QC A particle cannot generally be localized to distances much smaller than its de Broglie wavelength. This fact can be taken to mean that a slow neutron appears to be larger to a target particle than does a fast neutron in the sense that the slow neutron has probabilities of being found over a larger volume of space. For a thermal neutron at room temperature of 300 K, find (a) the linear momentum and (b) the de Broglie wavelength. (c) State how this effective size compares with both nuclear and atomic dimensions.

- **16.** Why is the following situation impossible? An engineer working on nuclear power makes a breakthrough so that he is able to control what daughter nuclei are created in a fission reaction. By carefully controlling the process, he is able to restrict the fission reactions to just this single possibility: the uranium-235 nucleus absorbs a slow neutron and splits into lanthanum-141 and bromine-94. Using this breakthrough, he is able to design and build a successful nuclear reactor in which only this single process occurs.
- 17. GP QIC According to one estimate, there are 4.40×10^{6} metric tons of world uranium reserves extractable at \$130/kg or less. We wish to determine if these reserves are sufficient to supply all the world's energy needs. About 0.700% of naturally occurring uranium is the fissionable isotope ²³⁵U. (a) Calculate the mass of ²³⁵U in the reserve in grams. (b) Find the number of moles of ²³⁵U in the reserve. (c) Find the number of ²³⁵U nuclei in the reserve. (d) Assuming 200 MeV is obtained from each fission reaction and all this energy is captured, calculate the total energy in joules that can be extracted from the reserve. (e) Assuming the rate of world power consumption remains constant at 1.5×10^{13} J/s, how many years could the uranium reserve provide for all the world's energy needs? (f) What conclusion can be drawn?

Section 45.4 Nuclear Fusion

- 18. An all-electric home uses 2 000 kWh of electric energy per month. Assuming all energy released from fusion could be captured, how many fusion events described by the reaction ²₁H + ³₁H → ⁴₂He + ¹₀n would be required to keep this home running for one year?
- 19. When a star has exhausted its hydrogen fuel, it may fuse other nuclear fuels. At temperatures above 1.00 × 10⁸ K, helium fusion can occur. Consider the following processes.
 (a) Two alpha particles fuse to produce a nucleus *A* and a gamma ray. What is nucleus *A*? (b) Nucleus *A* from part (a) absorbs an alpha particle to produce nucleus *B* and a gamma ray. What is nucleus *B*? (c) Find the total energy released in the sequence of reactions given in parts (a) and (b).
- 20. Find the energy released in the fusion reaction

$$H + {}^{2}_{1}H \rightarrow {}^{3}_{2}He + \gamma$$

- 21. (a) Consider a fusion generator built to create 3.00 GW of power. Determine the rate of fuel burning in grams per hour if the D–T reaction is used. (b) Do the same for the D–D reaction, assuming the reaction products are split evenly between (n, ³He) and (p, ³H).
- 22. QC Two nuclei having atomic numbers Z_1 and Z_2 approach each other with a total energy *E*. (a) When they are far apart, they interact only by electric repulsion. If they approach to a distance of 1.00×10^{-14} m, the nuclear force suddenly takes over to make them fuse. Find the minimum value of *E*, in terms of Z_1 and Z_2 , required to produce

fusion. (b) State how *E* depends on the atomic numbers. (c) If $Z_1 + Z_2$ is to have a certain target value such as 60, would it be energetically favorable to take $Z_1 = 1$ and $Z_2 = 59$, or $Z_1 = Z_2 = 30$, or some other choice? Explain your answer. (d) Evaluate from your expression the minimum energy for fusion for the D–D and D–T reactions (the first and third reactions in Eq. 45.4).

- 23. Of all the hydrogen in the oceans, 0.030 0% of the mass is deuterium. The oceans have a volume of 317 million mi³.
 (a) If nuclear fusion were controlled and all the deuterium in the oceans were fused to ⁴/₂He, how many joules of energy would be released? (b) What If? World power consumption is approximately 1.50 × 10¹³ W. If consumption were 100 times greater, how many years would the energy calculated in part (a) last?
- **24. QC Review.** Consider the deuterium–tritium fusion reaction with the tritium nucleus at rest:

$$^{2}_{1}H + ^{3}_{1}H \rightarrow ^{4}_{2}He + ^{1}_{0}n$$

(a) Suppose the reactant nuclei will spontaneously fuse if their surfaces touch. From Equation 44.1, determine the required distance of closest approach between their centers. (b) What is the electric potential energy (in electron volts) at this distance? (c) Suppose the deuteron is fired straight at an originally stationary tritium nucleus with just enough energy to reach the required distance of closest approach. What is the common speed of the deuterium and tritium nuclei, in terms of the initial deuteron speed v_i , as they touch? (d) Use energy methods to find the minimum initial deuteron energy required to achieve fusion. (e) Why does the fusion reaction actually occur at much lower deuteron energies than the energy calculated in part (d)?

- 25. M To understand why plasma containment is necessary, consider the rate at which an unconfined plasma would be lost. (a) Estimate the rms speed of deuterons in a plasma at a temperature of 4.00 × 10⁸ K. (b) What If? Estimate the order of magnitude of the time interval during which such a plasma would remain in a 10.0-cm cube if no steps were taken to contain it.
- **26.** It has been suggested that fusion reactors are safe from explosion because the plasma never contains enough energy to do much damage. (a) In 1992, the TFTR reactor, with a plasma volume of approximately 50.0 m³, achieved an ion temperature of 4.00×10^8 K, an ion density of 2.00×10^{13} cm⁻³, and a confinement time of 1.40 s. Calculate the amount of energy stored in the plasma of the TFTR reactor. (b) How many kilograms of water at 27.0°C could be boiled away by this much energy?
- 27. Review. To confine a stable plasma, the magnetic energy density in the magnetic field (Eq. 32.14) must exceed the pressure $2nk_{\rm B}T$ of the plasma by a factor of at least 10. In this problem, assume a confinement time $\tau = 1.00$ s. (a) Using Lawson's criterion, determine the ion density required for the D–T reaction. (b) From the ignition-temperature criterion, determine the required plasma pressure. (c) Deter-

mine the magnitude of the magnetic field required to contain the plasma.

28. QC Another series of nuclear reactions that can produce energy in the interior of stars is the carbon cycle first proposed by Hans Bethe in 1939, leading to his Nobel Prize in Physics in 1967. This cycle is most efficient when the central temperature in a star is above 1.6×10^7 K. Because the temperature at the center of the Sun is only 1.5×10^7 K, the following cycle produces less than 10% of the Sun's energy. (a) A high-energy proton is absorbed by ¹²C. Another nucleus, A, is produced in the reaction, along with a gamma ray. Identify nucleus A. (b) Nucleus A decays through positron emission to form nucleus B. Identify nucleus B. (c) Nucleus B absorbs a proton to produce nucleus C and a gamma ray. Identify nucleus C. (d) Nucleus *C* absorbs a proton to produce nucleus *D* and a gamma ray. Identify nucleus D. (e) Nucleus D decays through positron emission to produce nucleus E. Identify nucleus E. (f) Nucleus E absorbs a proton to produce nucleus F plus an alpha particle. Identify nucleus F. (g) What is the significance of the final nucleus in the last step of the cycle outlined in part (f)?

Section 45.5 Radiation Damage

- **29. Review.** A particular radioactive source produces 100 mrad of 2.00-MeV gamma rays per hour at a distance of 1.00 m from the source. (a) How long could a person stand at this distance before accumulating an intolerable dose of 1.00 rem? (b) **What If?** Assuming the radioactive source is a point source, at what distance would a person receive a dose of 10.0 mrad/h?
- 30. QC Assume an x-ray technician takes an average of eight x-rays per workday and receives a dose of 5.0 rem/yr as a result. (a) Estimate the dose in rem per x-ray taken.
 (b) Explain how the technician's exposure compares with low-level background radiation.
- **31.** When gamma rays are incident on matter, the intensity of the gamma rays passing through the material varies with depth *x* as $I(x) = I_0 e^{-\mu x}$, where I_0 is the intensity of the radiation at the surface of the material (at x = 0) and μ is the linear absorption coefficient. For 0.400-MeV gamma rays in lead, the linear absorption coefficient is 1.59 cm^{-1} . (a) Determine the "half-thickness" for lead, that is, the thickness of lead that would absorb half the incident gamma rays. (b) What thickness reduces the radiation by a factor of 10^4 ?
- **32. S** When gamma rays are incident on matter, the intensity of the gamma rays passing through the material varies with depth *x* as $I(x) = I_0 e^{-\mu x}$, where I_0 is the intensity of the radiation at the surface of the material (at x = 0) and μ is the linear absorption coefficient. (a) Determine the "half-thickness" for a material with linear absorption coefficient μ , that is, the thickness of the material that would absorb

half the incident gamma rays. (b) What thickness changes the radiation by a factor of f?

- **33. Review.** The danger to the body from a high dose of gamma rays is not due to the amount of energy absorbed; rather, it is due to the ionizing nature of the radiation. As an illustration, calculate the rise in body temperature that results if a "lethal" dose of 1 000 rad is absorbed strictly as internal energy. Take the specific heat of living tissue as 4 186 J/kg · °C.
- **34. Review.** Why is the following situation impossible? A "clever" technician takes his 20-min coffee break and boils some water for his coffee with an x-ray machine. The machine produces 10.0 rad/s, and the temperature of the water in an insulated cup is initially 50.0°C.
- **35.** A small building has become accidentally contaminated with radioactivity. The longest-lived material in the building is strontium-90. ($^{90}_{38}$ Sr has an atomic mass 89.907 7 u, and its half-life is 29.1 yr. It is particularly dangerous because it substitutes for calcium in bones.) Assume the building initially contained 5.00 kg of this substance uniformly distributed throughout the building and the safe level is defined as less than 10.0 decays/min (which is small compared with background radiation). How long will the building be unsafe?
- **36.** Technetium-99 is used in certain medical diagnostic procedures. Assume 1.00×10^{-8} g of ⁹⁹Tc is injected into a 60.0-kg patient and half of the 0.140-MeV gamma rays are absorbed in the body. Determine the total radiation dose received by the patient.
- **37.** To destroy a cancerous tumor, a dose of gamma radiation with a total energy of 2.12 J is to be delivered in 30.0 days from implanted sealed capsules containing palladium-103. Assume this isotope has a half-life of 17.0 d and emits gamma rays of energy 21.0 keV, which are entirely absorbed within the tumor. (a) Find the initial activity of the set of capsules. (b) Find the total mass of radioactive palladium these "seeds" should contain.
- **38.** Strontium-90 from the testing of nuclear bombs can still be found in the atmosphere. Each decay of ⁹⁰Sr releases 1.10 MeV of energy into the bones of a person who has had strontium replace his or her body's calcium. Assume a 70.0-kg person receives 1.00 ng of ⁹⁰Sr from contaminated milk. Take the half-life of ⁹⁰Sr to be 29.1 yr. Calculate the absorbed dose rate (in joules per kilogram) in one year.

Section 45.6 Radiation Detectors

39. M In a Geiger tube, the voltage between the electrodes is typically 1.00 kV and the current pulse discharges a 5.00-pF capacitor. (a) What is the energy amplification of this device for a 0.500-MeV electron? (b) How many electrons participate in the avalanche caused by the single initial electron?

40. Q(**C**) (a) A student wishes to measure the half-life of a radioactive substance using a small sample. Consecutive clicks of her Geiger counter are randomly spaced in time. The counter registers 372 counts during one 5.00-min interval and 337 counts during the next 5.00 min. The average background rate is 15 counts per minute. Find the most probable value for the half-life. (b) Estimate the uncertainty in the half-life determination in part (a). Explain your reasoning.

Section 45.7 Uses of Radiation

- **41. M** When gamma rays are incident on matter, the intensity of the gamma rays passing through the material varies with depth x as $I(x) = I_0 e^{-\mu x}$, where I_0 is the intensity of the radiation at the surface of the material (at x = 0) and μ is the linear absorption coefficient. For low-energy gamma rays in steel, take the absorption coefficient to be 0.720 mm⁻¹. (a) Determine the "half-thickness" for steel, that is, the thickness of steel that would absorb half the incident gamma rays. (b) In a steel mill, the thickness of sheet steel passing into a roller is measured by monitoring the intensity of gamma radiation reaching a detector below the rapidly moving metal from a small source immediately above the metal. If the thickness of the sheet changes from 0.800 mm to 0.700 mm, by what percentage does the gamma-ray intensity change?
- **42. S** A method called *neutron activation analysis* can be used for chemical analysis at the level of isotopes. When a sample is irradiated by neutrons, radioactive atoms are produced continuously and then decay according to their characteristic half-lives. (a) Assume one species of radioactive nuclei is produced at a constant rate R and its decay is described by the conventional radioactive decay law. Assuming irradiation begins at time t = 0, show that the number of radioactive atoms accumulated at time t is

$$N = \frac{R}{\lambda} (1 - e^{-\lambda t})$$

(b) What is the maximum number of radioactive atoms that can be produced?

43. You want to find out how many atoms of the isotope ⁶⁵Cu are in a small sample of material. You bombard the sample with neutrons to ensure that on the order of 1% of these copper nuclei absorb a neutron. After activation, you turn off the neutron flux and then use a highly efficient detector to monitor the gamma radiation that comes out of the sample. Assume half of the ⁶⁶Cu nuclei emit a 1.04-MeV gamma ray in their decay. (The other half of the activated nuclei decay directly to the ground state of ⁶⁶Ni.) If after 10 min (two half-lives) you have detected 1.00 × 10⁴ MeV of photon energy at 1.04 MeV, (a) approximately how many ⁶⁵Cu atoms are in the sample? (b) Assume the sample contains natural copper. Refer to the isotopic abundances listed in Table 44.2 and estimate the total mass of copper in the sample.

Additional Problems

44. QC A fusion reaction that has been considered as a source of energy is the absorption of a proton by a boron-11 nucleus to produce three alpha particles:

$${}^{1}_{1}H + {}^{11}_{5}B \rightarrow 3({}^{4}_{9}He)$$

This reaction is an attractive possibility because boron is easily obtained from the Earth's crust. A disadvantage is that the protons and boron nuclei must have large kinetic energies for the reaction to take place. This requirement contrasts with the initiation of uranium fission by slow neutrons. (a) How much energy is released in each reaction? (b) Why must the reactant particles have high kinetic energies?

45. Review. A very slow neutron (with speed approximately equal to zero) can initiate the reaction

$$^{1}_{0}n + ^{10}_{5}B \rightarrow ^{7}_{3}Li + ^{4}_{9}He$$

The alpha particle moves away with speed 9.25×10^6 m/s. Calculate the kinetic energy of the lithium nucleus. Use nonrelativistic equations.

- 46. Review. The first nuclear bomb was a fissioning mass of plutonium-239 that exploded in the Trinity test before dawn on July 16, 1945, at Alamogordo, New Mexico. Enrico Fermi was 14 km away, lying on the ground facing away from the bomb. After the whole sky had flashed with unbelievable brightness, Fermi stood up and began dropping bits of paper to the ground. They first fell at his feet in the calm and silent air. As the shock wave passed, about 40 s after the explosion, the paper then in flight jumped approximately 2.5 m away from ground zero. (a) Equation 17.10 describes the relationship between the pressure amplitude $\Delta P_{\rm max}$ of a sinusoidal air compression wave and its displacement amplitude s_{max} . The compression pulse produced by the bomb explosion was not a sinusoidal wave, but let's use the same equation to compute an estimate for the pressure amplitude, taking $\omega \sim 1 \text{ s}^{-1}$ as an estimate for the angular frequency at which the pulse ramps up and down. (b) Find the change in volume ΔV of a sphere of radius 14 km when its radius increases by 2.5 m. (c) The energy carried by the blast wave is the work done by one layer of air on the next as the wave crest passes. An extension of the logic used to derive Equation 20.8 shows that this work is given by $(\Delta P_{\max})(\Delta V)$. Compute an estimate for this energy. (d) Assume the blast wave carried on the order of one-tenth of the explosion's energy. Make an order-of-magnitude estimate of the bomb yield. (e) One ton of exploding TNT releases 4.2 GJ of energy. What was the order of magnitude of the energy of the Trinity test in equivalent tons of TNT? Fermi's immediate knowledge of the bomb yield agreed with that determined days later by analysis of elaborate measurements.
- **47. Review.** Consider a nucleus at rest, which then spontaneously splits into two fragments of masses m_1 and m_2 .

(a) Show that the fraction of the total kinetic energy carried by fragment m_1 is

$$\frac{K_1}{K_{\text{tot}}} = \frac{m_2}{m_1 + m_2}$$

and the fraction carried by m_2 is

$$\frac{K_2}{K_{\text{tot}}} = \frac{m_1}{m_1 + m_2}$$

assuming relativistic corrections can be ignored. A stationary $^{236}_{92}$ U nucleus fissions spontaneously into two primary fragments, $^{87}_{35}$ Br and $^{149}_{97}$ La. (b) Calculate the disintegration energy. The required atomic masses are 86.920 711 u for $^{87}_{35}$ Br, 148.934 370 u for $^{149}_{57}$ La, and 236.045 562 u for $^{236}_{92}$ U. (c) How is the disintegration energy split between the two primary fragments? (d) Calculate the speed of each fragment immediately after the fission.

- **48.** A fission reactor is hit by a missile, and 5.00×10^{6} Ci of 90 Sr, with half-life 29.1 yr, evaporates into the air. The strontium falls out over an area of 10^{4} km². After what time interval will the activity of the 90 Sr reach the agriculturally "safe" level of 2.00 μ Ci/m²?
- 49. The alpha-emitter plutonium-238 (²³⁸₉₄Pu, atomic mass 238.049 560 u, half-life 87.7 yr) was used in a nuclear energy source on the Apollo Lunar Surface Experiments Package (Fig. P45.49). The energy source, called the Radioisotope Thermoelectric Generator, is the small gray object to the left of the gold-shrouded Central Station in



Figure P45.49

the photograph. Assume the source contains 3.80 kg of ²³⁸Pu and the efficiency for conversion of radioactive decay energy to energy transferred by electrical transmission is 3.20%. Determine the initial power output of the source.

- 50. QC The half-life of tritium is 12.3 yr. (a) If the TFTR fusion reactor contained 50.0 m³ of tritium at a density equal to 2.00×10^{14} ions/cm³, how many curies of tritium were in the plasma? (b) State how this value compares with a fission inventory (the estimated supply of fissionable material) of 4.00×10^{10} Ci.
- **51. Review.** A nuclear power plant operates by using the energy released in nuclear fission to convert 20°C water into 400°C steam. How much water could theoretically be converted to steam by the complete fissioning of 1.00 g of ²³⁵U at 200 MeV/fission?
- 52. S Review. A nuclear power plant operates by using the energy released in nuclear fission to convert liquid water at T_c into steam at T_h . How much water could theoretically be converted to steam by the complete fissioning of a mass m of ²³⁵U if the energy released per fission event is E?
- 53. QIC Consider a 1.00-kg sample of natural uranium composed primarily of ²³⁸U, a smaller amount (0.720% by mass) of ²³⁵U, and a trace (0.005 00%) of ²³⁴U, which has a half-life of 2.44 × 10⁵ yr. (a) Find the activity in curies due to each of the isotopes. (b) What fraction of the total activity is due to each isotope? (c) Explain whether the activity of this sample is dangerous.
- 54. QC Approximately 1 of every 3 300 water molecules contains one deuterium atom. (a) If all the deuterium nuclei in 1 L of water are fused in pairs according to the D–D fusion reaction ${}^{2}\text{H} + {}^{2}\text{H} \rightarrow {}^{3}\text{He} + n + 3.27$ MeV, how much energy in joules is liberated? (b) What If? Burning gaso-line produces approximately 3.40×10^{7} J/L. State how the energy obtainable from the fusion of the deuterium in 1 L of water compares with the energy liberated from the burning of 1 L of gasoline.
- 55. Carbon detonations are powerful nuclear reactions that temporarily tear apart the cores inside massive stars late in their lives. These blasts are produced by carbon fusion, which requires a temperature of approximately 6×10^8 K to overcome the strong Coulomb repulsion between carbon nuclei. (a) Estimate the repulsive energy barrier to fusion, using the temperature required for carbon fusion. (In other words, what is the average kinetic energy of a carbon nucleus at 6×10^8 K?) (b) Calculate the energy (in MeV) released in each of these "carbon-burning" reactions:

$${}^{12}C + {}^{12}C \rightarrow {}^{20}Ne + {}^{4}He$$
$${}^{12}C + {}^{12}C \rightarrow {}^{24}Mg + \gamma$$

(c) Calculate the energy in kilowatt-hours given off when 2.00 kg of carbon completely fuse according to the first reaction.

- 56. A sealed capsule containing the radiopharmaceutical phosphorus-32, an e⁻ emitter, is implanted into a patient's tumor. The average kinetic energy of the beta particles is 700 keV. The initial activity is 5.22 MBq. Assume the beta particles are completely absorbed in 100 g of tissue. Determine the absorbed dose during a 10.0-day period.
- 57. A certain nuclear plant generates internal energy at a rate of 3.065 GW and transfers energy out of the plant by electrical transmission at a rate of 1.000 GW. Of the waste energy, 3.0% is ejected to the atmosphere and the remainder is passed into a river. A state law requires that the river water be warmed by no more than 3.50° C when it is returned to the river. (a) Determine the amount of cooling water necessary (in kilograms per hour and cubic meters per hour) to cool the plant. (b) Assume fission generates 7.80×10^{10} J/g of 235 U. Determine the rate of fuel burning (in kilograms per hour) of 235 U.
- 58. The Sun radiates energy at the rate of 3.85 × 10²⁶ W. Suppose the net reaction 4(¹₁H) + 2(⁻⁰₋₁e) → ⁴₂He + 2ν + γ accounts for all the energy released. Calculate the number of protons fused per second.

59. Consider the two nuclear reactions

$$\begin{array}{rrrr} A + B & \rightarrow & C + E \\ C + D & \rightarrow & F + G \end{array}$$

(a) Show that the net disintegration energy for these two reactions $(Q_{\text{net}} = Q_{\text{I}} + Q_{\text{II}})$ is identical to the disintegration energy for the net reaction

$$A + B + D \rightarrow E + F + G$$

(b) One chain of reactions in the proton–proton cycle in the Sun's core is

$${}^{1}_{1}H + {}^{1}_{1}H \rightarrow {}^{2}_{1}H + {}^{0}_{1}e + \nu$$

$${}^{0}_{1}e + {}^{0}_{-1}e \rightarrow 2\gamma$$

$${}^{1}_{1}H + {}^{2}_{1}H \rightarrow {}^{3}_{2}He + \gamma$$

$${}^{1}_{1}H + {}^{3}_{2}He \rightarrow {}^{4}_{2}He + {}^{0}_{1}e + \nu$$

$${}^{0}_{1}e + {}^{0}_{-1}e \rightarrow 2\gamma$$

Based on part (a), what is Q_{net} for this sequence?

60. QC Natural uranium must be processed to produce uranium enriched in ²³⁵U for weapons and power plants. The processing yields a large quantity of nearly pure ²³⁸U as a by-product, called "depleted uranium." Because of its high mass density, ²³⁸U is used in armor-piercing artillery shells.
(a) Find the edge dimension of a 70.0-kg cube of ²³⁸U (ρ = 19.1 × 10³ kg/m³). (b) The isotope ²³⁸U has a long half-life of 4.47 × 10⁹ yr. As soon as one nucleus decays, a relatively rapid series of 14 steps begins that together constitute the net reaction

$${}^{238}_{92}\text{U} \rightarrow 8({}^{4}_{2}\text{He}) + 6({}^{0}_{-1}\text{e}) + {}^{206}_{82}\text{Pb} + 6\overline{\nu} + Q_{\text{net}}$$

Find the net decay energy. (Refer to Table 44.2.) (c) Argue that a radioactive sample with decay rate R and decay

energy *Q* has power output P = QR. (d) Consider an artillery shell with a jacket of 70.0 kg of ²³⁸U. Find its power output due to the radioactivity of the uranium and its daughters. Assume the shell is old enough that the daughters have reached steady-state amounts. Express the power in joules per year. (e) **What If?** A 17-year-old soldier of mass 70.0 kg works in an arsenal where many such artillery shells are stored. Assume his radiation exposure is limited to 5.00 rem per year. Find the rate in joules per year at which he can absorb energy of radiation. Assume an average RBE factor of 1.10.

- 61. Suppose the target in a laser fusion reactor is a sphere of solid hydrogen that has a diameter of 1.50 × 10⁻⁴ m and a density of 0.200 g/cm³. Assume half of the nuclei are ²H and half are ³H. (a) If 1.00% of a 200-kJ laser pulse is delivered to this sphere, what temperature does the sphere reach? (b) If all the hydrogen fuses according to the D–T reaction, how many joules of energy are released?
- **62.** When photons pass through matter, the intensity *I* of the beam (measured in watts per square meter) decreases exponentially according to

$$I = I_0 e^{-\mu x}$$

where *I* is the intensity of the beam that just passed through a thickness *x* of material and I_0 is the intensity of the incident beam. The constant μ is known as the linear absorption coefficient, and its value depends on the absorbing material and the wavelength of the photon beam. This wavelength (or energy) dependence allows us to filter out unwanted wavelengths from a broad-spectrum x-ray beam. (a) Two x-ray beams of wavelengths λ_1 and λ_2 and equal incident intensities pass through the same metal plate. Show that the ratio of the emergent beam intensities is

$$\frac{I_2}{I_1} = e^{-(\mu_2 - \mu_1)x}$$

(b) Compute the ratio of intensities emerging from an aluminum plate 1.00 mm thick if the incident beam contains equal intensities of 50 pm and 100 pm x-rays. The values of μ for aluminum at these two wavelengths are $\mu_1 = 5.40 \text{ cm}^{-1}$ at 50 pm and $\mu_2 = 41.0 \text{ cm}^{-1}$ at 100 pm. (c) Repeat part (b) for an aluminum plate 10.0 mm thick.

63. Assume a deuteron and a triton are at rest when they fuse according to the reaction

$$^{2}_{1}H + ^{3}_{1}H \rightarrow ^{4}_{2}He + ^{1}_{0}n$$

Determine the kinetic energy acquired by the neutron.

64. (a) Calculate the energy (in kilowatt-hours) released if 1.00 kg of ²³⁹Pu undergoes complete fission and the energy released per fission event is 200 MeV. (b) Calculate the energy (in electron volts) released in the deuterium-tritium fusion reaction

$$^{2}_{1}H + ^{3}_{1}H \rightarrow ^{4}_{2}He + ^{1}_{0}n$$

(c) Calculate the energy (in kilowatt-hours) released if 1.00 kg of deuterium undergoes fusion according to this reaction. (d) **What If?** Calculate the energy (in kilowatt-hours) released by the combustion of 1.00 kg of carbon in coal if each $C + O_2 \rightarrow CO_2$ reaction yields 4.20 eV. (e) List advantages and disadvantages of each of these methods of energy generation.

Challenge Problems

- 65. During the manufacture of a steel engine component, radioactive iron (⁵⁹Fe) with a half-life of 45.1 d is included in the total mass of 0.200 kg. The component is placed in a test engine when the activity due to this isotope is 20.0 μ Ci. After a 1 000-h test period, some of the lubricating oil is removed from the engine and found to contain enough ⁵⁹Fe to produce 800 disintegrations/min/L of oil. The total volume of oil in the engine is 6.50 L. Calculate the total mass worn from the engine component per hour of operation.
- 66. Assume a photomultiplier tube has seven dynodes with potentials of 100, 200, 300, ..., 700 V as shown in Figure P45.66. The average energy required to free an electron from the dynode surface is 10.0 eV. Assume only one electron is incident and the tube functions with 100% efficiency. (a) How many electrons are freed at the first dynode at 100 V? (b) How many electrons are collected at the last dynode? (c) What is the energy available to the counter for all the electrons arriving at the last dynode?



67. (a) At time t = 0, a sample of uranium is exposed to a neutron source that causes N_0 nuclei to undergo fission. The sample is in a supercritical state, with a reproduction constant K > 1. A chain reaction occurs that proliferates fission throughout the mass of uranium. The chain reaction can be thought of as a succession of *generations*. The N_0 fissions produced initially are the zeroth generation of fissions. From this generation, N_0K neutrons go off to

produce fission of new uranium nuclei. The N_0K fissions that occur subsequently are the first generation of fissions, and from this generation N_0K^2 neutrons go in search of uranium nuclei in which to cause fission. The subsequent N_0K^2 fissions are the second generation of fissions. This process can continue until all the uranium nuclei have fissioned. Show that the cumulative total of fissions N that have occurred up to and including the *n*th generation after the zeroth generation is given by

$$N = N_0 \left(\frac{K^{n+1} - 1}{K - 1}\right)$$

(b) Consider a hypothetical uranium weapon made from 5.50 kg of isotopically pure ²³⁵U. The chain reaction has a reproduction constant of 1.10 and starts with a zeroth generation of 1.00×10^{20} fissions. The average time inter-

val between one fission generation and the next is 10.0 ns. How long after the zeroth generation does it take the uranium in this weapon to fission completely? (c) Assume the bulk modulus of uranium is 150 GPa. Find the speed of sound in uranium. You may ignore the density difference between ²³⁵U and natural uranium. (d) Find the time interval required for a compressional wave to cross the radius of a 5.50-kg sphere of uranium. This time interval indicates how quickly the motion of explosion begins. (e) Fission must occur in a time interval that is short compared with that in part (d); otherwise, most of the uranium will disperse in small chunks without having fissioned. Can the weapon considered in part (b) release the explosive energy of all its uranium? If so, how much energy does it release in equivalent tons of TNT? Assume one ton of TNT releases 4.20 GJ and each uranium fission releases 200 MeV of energy.