MBF1413 | Quantitative Methods

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4: Decision Analysis – Part 1

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1. Problem Formulation

- The first step in the decision analysis process is problem formulation.
- We begin with a verbal statement of the problem.
- We then identify the **decision alternatives**; the uncertain future events, referred to as **chance events**; and the **consequences** associated with each combination of decision alternative and chance event outcome.

1. Problem Formulation

- Let us begin by considering a construction project of the Pittsburgh Development Corporation.
- PDC purchased land that will be the site of a new luxury condominium complex.
- PDC plans to price the individual condominium units between $300,000 and $1,400,000.
- PDC commissioned preliminary architectural drawings for three different projects:
  - one with 30 condominiums,
  - one with 60 condominiums, and
  - one with 90 condominiums.

1. Problem Formulation

- The financial success of the project depends upon the size of the condominium complex and the chance event concerning the demand for the condominiums.

- The statement of the PDC decision problem is to select the size of the new luxury condominium project that will lead to the largest profit given the uncertainty concerning the demand for the condominiums.

1. Problem Formulation

- Given the statement of the problem, it is clear that the decision is to select the best size for the condominium complex.

- PDC has the following three decision alternatives:

$d1$ = a small complex with 30 condominiums

$d2$ = a medium complex with 60 condominiums

$d3$ = a large complex with 90 condominiums

1. Problem Formulation

- A factor in selecting the best decision alternative is the uncertainty associated with the chance event concerning the demand for the condominiums.

- PDC's president considered two possible chance event outcomes:
  - a strong demand and
  - a weak demand.

1. Problem Formulation

- In decision analysis, the possible outcomes for a chance event are referred to as the **states of nature**.

- The states of nature are defined so they are mutually exclusive (no more than one can occur) and collectively exhaustive (at least one must occur); thus one and only one of the possible states of nature will occur.

- For the PDC problem, the chance event concerning the demand for the condominiums has two states of nature:

  s_1 = strong demand for the condominiums

  s_2 = weak demand for the condominiums

1. Problem Formulation

- Management must:
  - first select a decision alternative (complex size);
  - then a state of nature follows (demand for the condominiums); and
  - finally a consequence will occur.
- In this case, the consequence is PDC’s profit.

1. Problem Formulation – *a. Influence Diagrams*

- An **influence diagram** shows the relationships among the decisions, the chance events, and the consequences for a decision problem.

- The **nodes** in an influence diagram represent the decisions, chance events, and consequences.

- Rectangles or squares depict **decision nodes**, circles or ovals depict **chance nodes**, and diamonds depict **consequence nodes**.

- The lines connecting the nodes, referred to as *arcs,* show the direction of influence that the nodes have on one another.

1. Problem Formulation – *a. Influence Diagrams*

- Figure 4.1 shows the influence diagram for the PDC problem:

  - the complex size is the decision node,
  - demand is the chance node, and
  - profit is the consequence node.

- The arcs connecting the nodes show that both the complex size and the demand influence PDC's profit.

1. Problem Formulation – *a. Influence Diagrams*

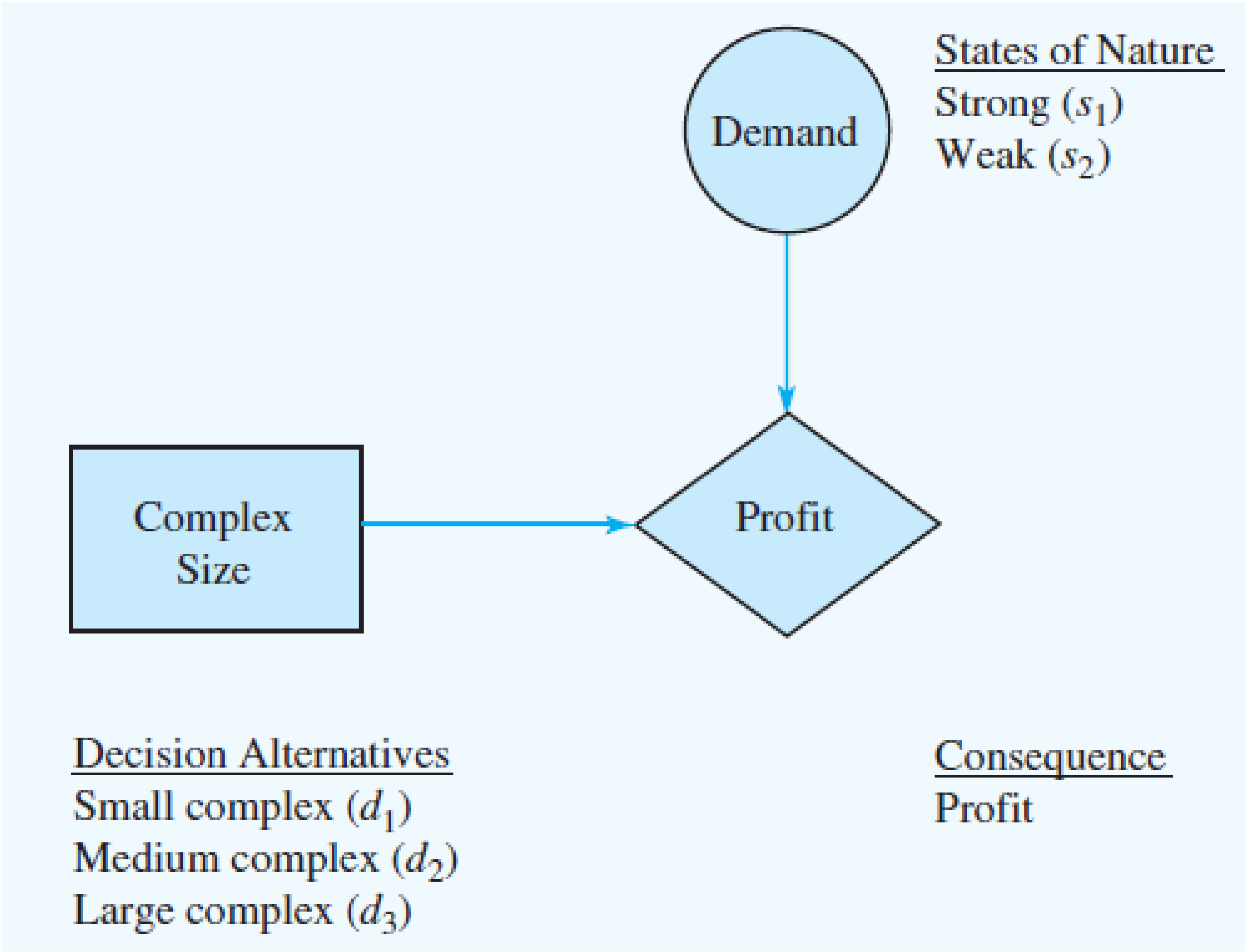

FIGURE 4.1 INFLUENCE DIAGRAM FOR THE PDC PROJECT

1. Problem Formulation – *b. Payoffs*

- Given the:
  - ➢ three decision alternatives and
  - ➢ the two states of nature,
    - ✓ which complex size should PDC choose?

- To answer this question, PDC will need to know the consequence associated with each decision alternative and each state of nature.

1. Problem Formulation – *b. Payoffs*

- In decision analysis, we refer to the consequence resulting from a specific combination of a decision alternative and a state of nature as a **payoff**.

- A table showing payoffs for all combinations of decision alternatives and states of nature is a **payoff table**.

1. Problem Formulation – *b. Payoffs*

- Because PDC wants to select the complex size that provides the largest profit, profit is used as the consequence.

- The payoff table with profits expressed in millions of dollars is shown in Table 4.1.

- Note, for example, that if a medium complex is built and demand turns out to be strong, a profit of $14 million will be realized.

1. Problem Formulation – *b. Payoffs*

	State of Nature	
Decision Alternative	**Strong Demand s_1**	**Weak Demand s_2**
Small complex, d_1	8	7
Medium complex, d_2	14	5
Large complex, d_3	20	−9

TABLE 4.1 PAYOFF TABLE FOR THE PDC CONDOMINIUM PROJECT (PAYOFFS IN $ MILLIONS)

1. Problem Formulation – *b. Payoffs*

- We will use the notation V_{ij} to denote the payoff associated with decision alternative *i* and state of nature *j.*

- Using Table 4.1, V_{31} = 20 indicates a payoff of $20 million occurs if the decision is to build a large complex (d_3) and the strong demand state of nature (s_1) occurs.

- Similarly, V_{32} = -9 indicates a loss of $9 million if the decision is to build a large complex (d_3) and the weak demand state of nature (*s*2) occurs.

1. Problem Formulation – *c. Decision Trees*

- A **decision tree** provides a graphical representation of the decision-making process.

- Figure 4.2 presents a decision tree for the PDC problem.

- The decision tree shows the natural or logical progression that will occur over time. First, PDC must make a decision regarding the size of the condominium complex (d_1, d_2, or d_3).

- Then, after the decision is implemented, either state of nature s_1 or s_2 will occur.

1. Problem Formulation – *c. Decision Trees*

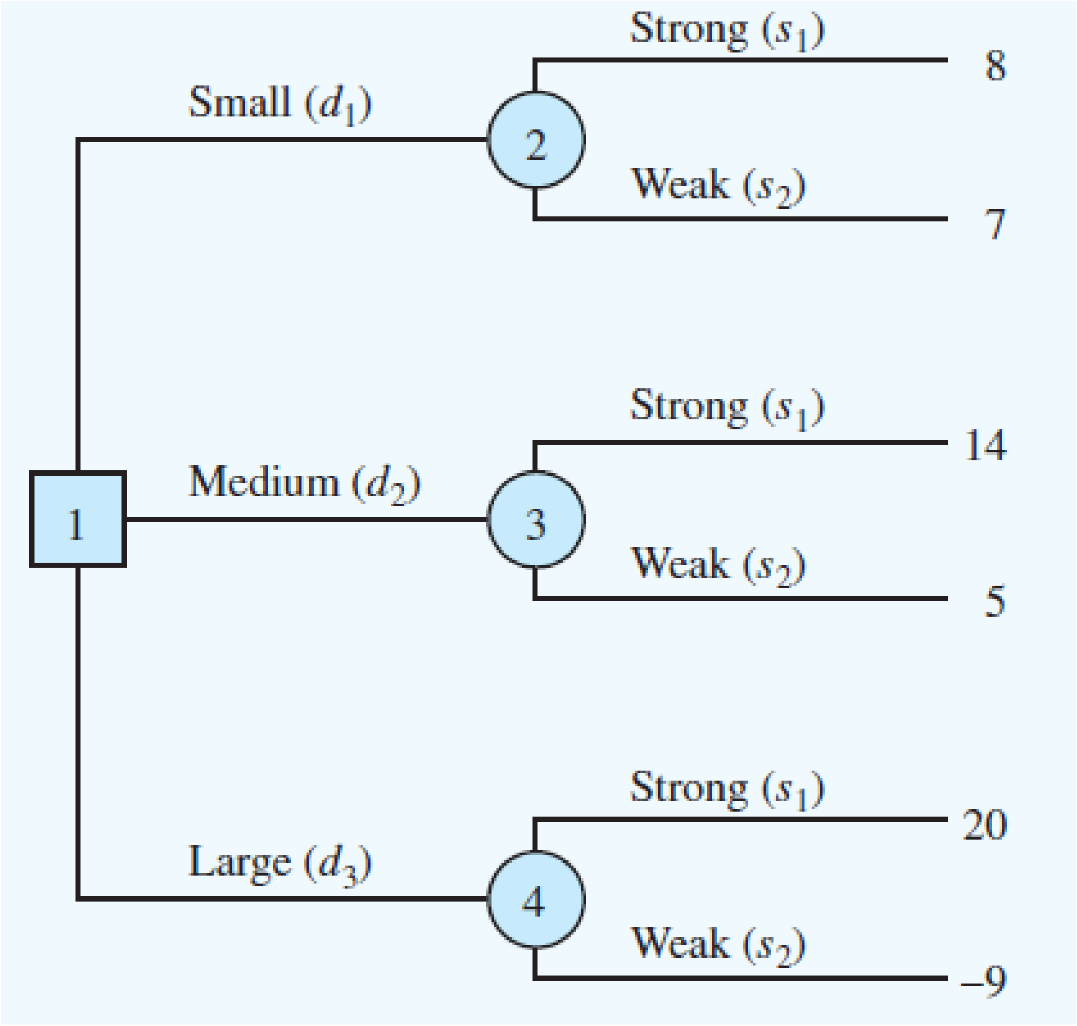

FIGURE 4.2 DECISION TREE FOR THE PDC CONDOMINIUM PROJECT (PAYOFFS IN $ MILLIONS)

1. Problem Formulation – *c. Decision Trees*

- The number at each endpoint of the tree indicates the payoff associated with a particular sequence.
- For example, the topmost payoff of 8 indicates that an $8 million profit is anticipated if PDC constructs a small condominium complex (d_1) and demand turns out to be strong (s_1).
- The next payoff of 7 indicates an anticipated profit of $7 million if PDC constructs a small condominium complex (d_1) and demand turns out to be weak (s_2).
- Thus, the decision tree provides a graphical depiction of the sequences of decision alternatives and states of nature that provide the six possible payoffs for PDC.

1. Problem Formulation – *c. Decision Trees*

- The decision tree in Figure 4.2 shows four nodes, numbered 1 - 4.
- Squares are used to depict decision nodes and circles are used to depict chance nodes.
- Thus, node 1 is a decision node, and nodes 2, 3, and 4 are chance nodes.
- The **branches** connect the nodes; those leaving the decision node correspond to the decision alternatives.
- The branches leaving each chance node correspond to the states of nature.
- The payoffs are shown at the end of the states-of-nature branches.

1. Problem Formulation – *c. Decision Trees*

- We now turn to the question: How can the decision maker use the information in the payoff table or the decision tree to select the best decision alternative?

- Several approaches may be used, as discussed in Section 2.

2. Decision Making Without Probabilities – *a. Optimistic Approach*

- The **optimistic approach** evaluates each decision alternative in terms of the *best* payoff that can occur.
- The decision alternative that is recommended is the one that provides the best possible payoff.
- For a problem in which maximum profit is desired, as in the PDC problem, the optimistic approach would lead the decision maker to choose the alternative corresponding to the largest profit.
- For problems involving minimization, this approach leads to choosing the alternative with the smallest payoff.

2. Decision Making Without Probabilities – *a. Optimistic Approach*

- To illustrate the optimistic approach, we use it to develop a recommendation for the PDC problem.
- First, we determine the maximum payoff for each decision alternative; then we select the decision alternative that provides the overall maximum payoff.
- These steps systematically identify the decision alternative that provides the largest possible profit.
- Table 4.2 illustrates these steps.
- Because 20, corresponding to d_3, is the largest payoff, the decision to construct the large condominium complex is the recommended decision alternative using the optimistic approach.

2. Decision Making Without Probabilities – *a. Optimistic Approach*

Decision Alternative	Maximum Payoff	
Small complex, d_1	8	
Medium complex, d_2	14	
Large complex, d_3	20	← Maximum of the maximum payoff values

TABLE 4.2 MAXIMUM PAYOFF FOR EACH PDC DECISION ALTERNATIVE

2. Decision Making Without Probabilities – *b. Conservative Approach*

- The **conservative approach** evaluates each decision alternative in terms of the *worst* payoff that can occur.

- The decision alternative recommended is the one that provides the best of the worst possible payoffs.

- For a problem in which the output measure is profit, as in the PDC problem, the conservative approach would lead the decision maker to choose the alternative that maximizes the minimum possible profit that could be obtained.
- For problems involving minimization, this approach identifies the alternative that will minimize the maximum payoff.

2. Decision Making Without Probabilities –
b. Conservative Approach

- To illustrate the conservative approach, we use it to develop a recommendation for the PDC problem.
- First, we identify the minimum payoff for each of the decision alternatives; then we select the decision alternative that maximizes the minimum payoff.
- Table 4.3 illustrates these steps for the PDC problem.

Decision Alternative	Minimum Payoff	
Small complex, d_1	7	← Maximum of the minimum payoff values
Medium complex, d_2	5	
Large complex, d_3	−9	

TABLE 4.3 MINIMUM PAYOFF FOR EACH PDC DECISION ALTERNATIVE

2. Decision Making Without Probabilities – *b. Conservative Approach*

- Because 7, corresponding to d_1, yields the maximum of the minimum payoffs, the decision alternative of a small condominium complex is recommended.

- This decision approach is considered conservative because it identifies the worst possible payoffs and then recommends the decision alternative that avoids the possibility of extremely “bad” payoffs.
- In the conservative approach, PDC is guaranteed a profit of at least $7 million.
- Although PDC may make more, it *cannot* make less than $7 million.

2. Decision Making Without Probabilities – *c. Minimax Regret Approach*

- In decision analysis, **regret** is the difference between
- the payoff associated with a particular decision alternative and the payoff associated with the decision that would yield the most desirable payoff for a given state of nature.

- Thus, regret represents how much potential payoff one would forgo by selecting a particular decision alternative given that a specific state of nature will occur.

- This is why regret is often referred to as **opportunity loss**.

2. Decision Making Without Probabilities – *c. Minimax Regret Approach*

- As its name implies, under the **minimax regret approach** to decision making one would choose the decision alternative that minimizes the maximum state of regret that could occur over all possible states of nature.

- This approach is neither purely optimistic nor purely conservative.

2. Decision Making Without Probabilities – *c. Minimax Regret Approach*

- Let us illustrate the minimax regret approach by showing how it can be used to select a decision alternative for the PDC problem.

- Suppose that PDC constructs a small condominium complex (d_1) and demand turns out to be strong (s_1).

- Table 4.1 showed that the resulting profit for PDC would be $8 million.

- However, given that the strong demand state of nature (s_1) has occurred, we realize that the decision to construct a large condominium complex (d_3), yielding a profit of $20 million, would have been the best decision.

2. Decision Making Without Probabilities – *c. Minimax Regret Approach*

	State of Nature	
Decision Alternative	**Strong Demand s_1**	**Weak Demand s_2**
Small complex, d_1	8	7
Medium complex, d_2	14	5
Large complex, d_3	20	−9

TABLE 4.1 PAYOFF TABLE FOR THE PDC CONDOMINIUM PROJECT (PAYOFFS IN $ MILLIONS)

2. Decision Making Without Probabilities – *c. Minimax Regret Approach*

- The difference between the payoff for the best decision alternative ($20 million) and the payoff for the decision to construct a small condominium complex ($8 million) is the regret or opportunity loss associated with decision alternative d_1 when state of nature s_1 occurs; thus, for this case, the opportunity loss or regret is:

$20 million - $8 million = $12 million.

- Similarly, if PDC makes the decision to construct a medium condominium complex (d_2) and the strong demand state of nature (s_1) occurs, the opportunity loss, or regret, associated with d_2 would be:

$20 million - $14 million = $6 million.

2. Decision Making Without Probabilities – *c. Minimax Regret Approach*

- In general, the following expression represents the opportunity loss, or regret:

$$R_{ij} = |V_j^* - V_{ij}| \qquad \textbf{(4.1)}$$

where

R_{ij} = the regret associated with decision alternative d_i and state of nature s_j

V_j^* = the payoff value corresponding to the best decision for the state of nature s_j

V_{ij} = the payoff corresponding to decision alternative d_i and state of nature s_j

2. Decision Making Without Probabilities – *c. Minimax Regret Approach*

- Using equation (4.1) and the payoffs in Table 4.1, we can compute the regret associated with each combination of decision alternative d_i and state of nature s_j.

- Because the PDC problem is a maximization problem, V_j^* will be the largest entry in column j of the payoff table.

- Thus, to compute the regret, we simply subtract each entry in a column from the largest entry in the column.

- Table 4.4 shows the opportunity loss, or regret, table for the PDC problem.

2. Decision Making Without Probabilities – *c. Minimax Regret Approach*

Decision Alternative	State of Nature: Strong Demand s_1	Weak Demand s_2
Small complex, d_1	12	0
Medium complex, d_2	6	2
Large complex, d_3	0	16

TABLE 4.4 OPPORTUNITY LOSS, OR REGRET, TABLE FOR THE PDC CONDOMINIUM PROJECT ($ MILLIONS)

2. Decision Making Without Probabilities – *c. Minimax Regret Approach*

- The next step in applying the minimax regret approach is to list the maximum regret for each decision alternative; Table 4.5 shows the results for the PDC problem.

- Selecting the decision alternative with the *minimum* of the *maximum* regret values—hence, the name *minimax regret*—yields the minimax regret decision.

- For the PDC problem, the alternative to construct the medium condominium complex, with a corresponding maximum regret of $6 million, is the recommended minimax regret decision.

2. Decision Making Without Probabilities – *c. Minimax Regret Approach*

Decision Alternative	Maximum Regret	
Small complex, d_1	12	
Medium complex, d_2	6	← Minimum of the maximum regret
Large complex, d_3	16	

TABLE 4.5 MAXIMUM REGRET FOR EACH PDC DECISION ALTERNATIVE

3. Decision Making With Probabilities

- In many decision-making situations, we can obtain probability assessments for the states of nature.
- When such probabilities are available, we can use the **expected value approach** to identify the best decision alternative.
- Let us first define the expected value of a decision alternative and then apply it to the PDC problem.
- Let

$$N = \text{the number of states of nature}$$
$$P(s_j) = \text{the probability of state of nature } s_j$$

3. Decision Making With Probabilities

- Because one and only one of the *N* states of nature can occur, the probabilities must satisfy two conditions:

$$P(s_j) \geq 0 \qquad \text{for all states of nature} \tag{4.2}$$

$$\sum_{j=1}^{N} P(s_j) = P(s_1) + P(s_2) + \cdots + P(s_N) = 1 \tag{4.3}$$

- The expected value (EV) of decision alternative di is defined as follows:

$$\text{EV}(d_i) = \sum_{j=1}^{N} P(s_j)V_{ij} \tag{4.4}$$

where

R_{ij} = the regret associated with decision alternative d_i and state of nature s_j

V_{ij} = the payoff corresponding to decision alternative d_i and state of nature s_j

3. Decision Making With Probabilities

- In words, the expected value of a decision alternative is the sum of weighted payoffs for the decision alternative.

- The weight for a payoff is the probability of the associated state of nature and therefore the probability that the payoff will occur.

- Let us return to the PDC problem to see how the expected value approach can be applied.

3. Decision Making With Probabilities

- PDC is optimistic about the potential for the luxury high-rise condominium complex.

- Suppose that this optimism leads to an initial subjective probability assessment of:
  - 0.8 that demand will be strong (s_1) and
  - 0.2 that demand will be weak (s_2).

3. Decision Making With Probabilities

- Thus, $P(s_1)$ 0.8 and $P(s_2)$ 0.2.

- Using the payoff values in Table 4.1 and equation (4.4), we compute the expected value for each of the three decision alternatives as follows:

$$\begin{aligned}
\text{EV}(d_1) &= 0.8(8) + 0.2(7) &&= 7.8 \\
\text{EV}(d_2) &= 0.8(14) + 0.2(5) &&= 12.2 \\
\text{EV}(d_3) &= 0.8(20) + 0.2(-9) &&= 14.2
\end{aligned}$$

- Thus, using the expected value approach, we find that the large condominium complex, with an expected value of \$14.2 million, is the recommended decision.

3. Decision Making With Probabilities

- The calculations required to identify the decision alternative with the best expected value can be conveniently carried out on a decision tree.

- Figure 4.3 shows the decision tree for the PDC problem with state-of-nature branch probabilities.

- Working backward through the decision tree, we first compute the expected value at each chance node.
- That is, at each chance node, we weight each possible payoff by its probability of occurrence.
- By doing so, we obtain the expected values for nodes 2, 3, and 4, as shown in Figure 4.4.

3. Decision Making With Probabilities

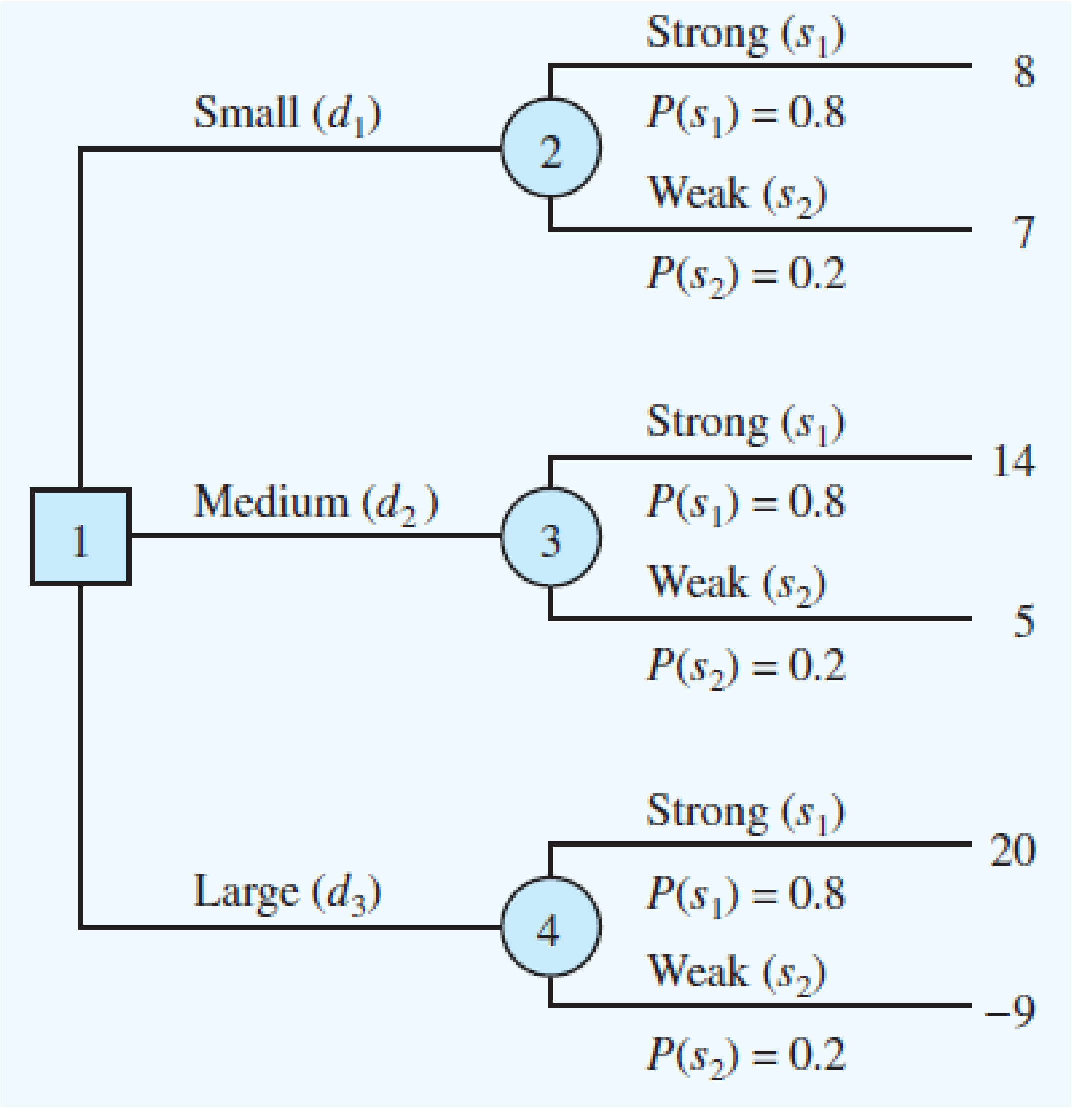

FIGURE 4.3 PDC DECISION TREE WITH STATE-OF-NATURE BRANCH PROBABILITIES

3. Decision Making With Probabilities

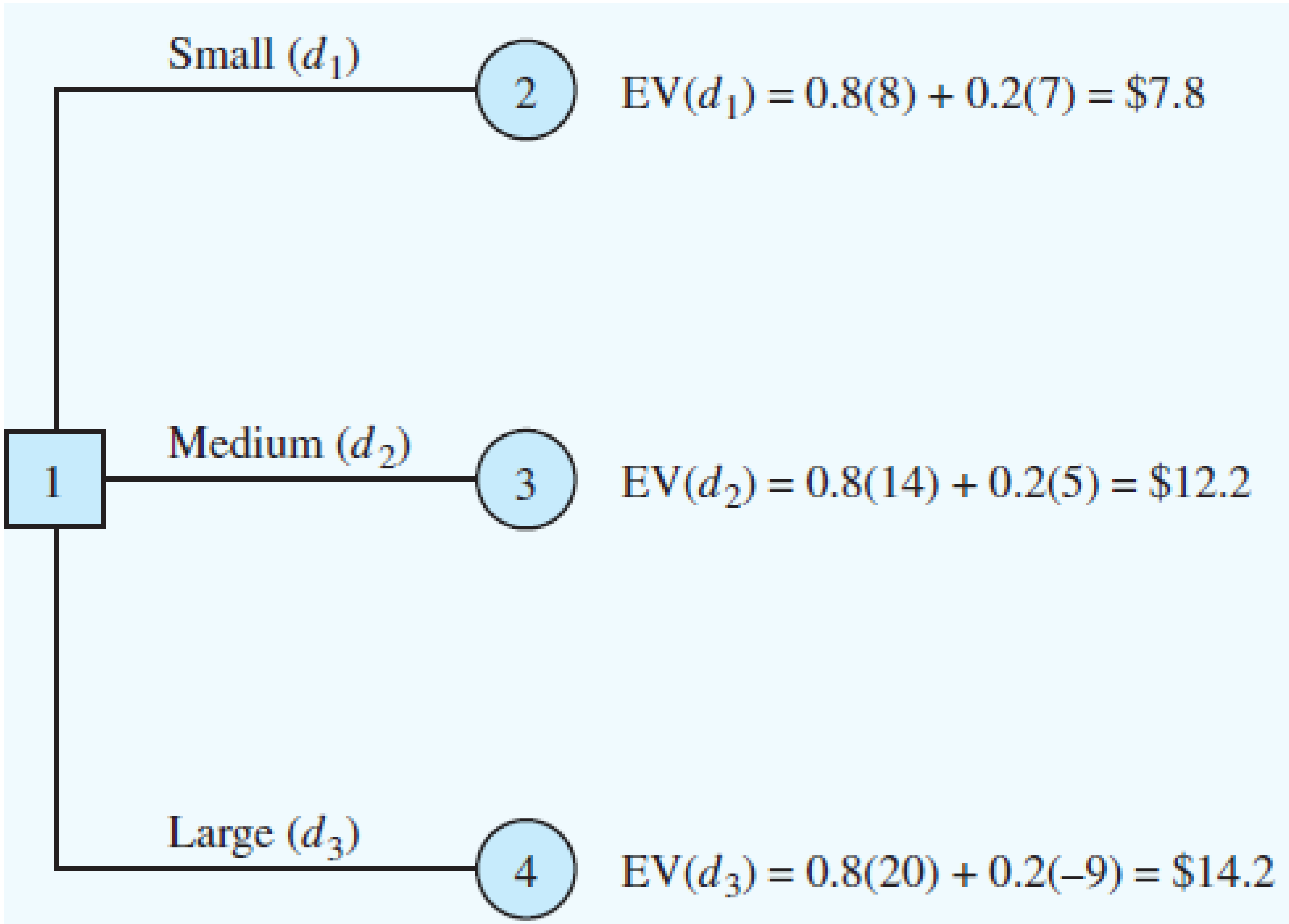

FIGURE 4.4 APPLYING THE EXPECTED VALUE APPROACH USING A DECISION TREE

3. Decision Making With Probabilities

- Because the decision maker controls the branch leaving decision node 1 and because we are trying to maximize the expected profit, the best decision alternative at node 1 is d_3.

- Thus, the decision tree analysis leads to a recommendation of d_3, with an expected value of $14.2 million.

- Note that this recommendation is also obtained with the expected value approach in conjunction with the payoff table.

3. Decision Making With Probabilities – *Expected Value of Perfect Information*

- Suppose that PDC has the opportunity to conduct a market research study that would help evaluate buyer interest in the condominium project and provide information that management could use to improve the probability assessments for the states of nature.
- Assume that the study could provide *perfect information* regarding the states of nature.
- To make use of this perfect information, we will develop a decision strategy that PDC should follow once it knows which state of nature will occur.
- A decision strategy is simply a decision rule that specifies the decision alternative to be selected after new information becomes available.

3. Decision Making With Probabilities – *Expected Value of Perfect Information*

- To help determine the decision strategy for PDC, we reproduced PDC's payoff table as Table 4.6.

- Note that, if PDC knew for sure that state of nature s_1 would occur, the best decision alternative would be d_3, with a payoff of $20 million.

- Similarly, if PDC knew for sure that state of nature s_2 would occur, the best decision alternative would be d_1, with a payoff of $7 million.

3. Decision Making With Probabilities – *Expected Value of Perfect Information*

	State of Nature	
Decision Alternative	Strong Demand s_1	Weak Demand s_2
Small complex, d_1	8	7
Medium complex, d_2	14	5
Large complex, d_3	20	−9

TABLE 4.6 PAYOFF TABLE FOR THE PDC CONDOMINIUM PROJECT ($ MILLIONS)

3. Decision Making With Probabilities – *Expected Value of Perfect Information*

- Thus, we can state PDC's optimal decision strategy when the perfect information becomes available as follows:

If s_1, select d_3 and receive a payoff of \$20 million.

If s_2, select d_1 and receive a payoff of \$7 million.

3. Decision Making With Probabilities – *Expected Value of Perfect Information*

- What is the expected value for this decision strategy?

- To compute the expected value with perfect information, we return to the original probabilities for the states of nature:

  $P(s_1) = 0.8$ and $P(s_2) = 0.2$.

- Thus, there is a 0.8 probability that the perfect information will indicate state of nature s_1, and the resulting decision alternative d_3 will provide a \$20 million profit.

- Similarly, with a 0.2 probability for state of nature s_2, the optimal decision alternative d_1 will provide a \$7 million profit.

3. Decision Making With Probabilities – *Expected Value of Perfect Information*

- Thus, from equation (4.4) the expected value of the decision strategy that uses perfect information is

$$0.8(20) + 0.2(7) = 17.4.$$

- We refer to the expected value of $17.4 million as the *expected value with perfect information* (EVwPI).

3. Decision Making With Probabilities – *Expected Value of Perfect Information*

- Earlier in this section we showed that the recommended decision using the expected value approach is decision alternative *d*3, with an expected value of $14.2 million.

- Because this decision recommendation and expected value computation were made without the benefit of perfect information, $14.2 million is referred to as the *expected value without perfect information* (EVwoPI).

3. Decision Making With Probabilities – *Expected Value of Perfect Information*

- The expected value with perfect information is $17.4 million, and the expected value without perfect information is $14.2; therefore, the expected value of the perfect information (EVPI) is

$17.4 - $14.2 = $3.2 million.

- In other words, $3.2 million represents the additional expected value that can be obtained if perfect information were available about the states of nature.

3. Decision Making With Probabilities – *Expected Value of Perfect Information*

- In general, the **expected value of perfect information (EVPI)** is computed as follows:

$$\text{EVPI} = |\text{EVwPI} - \text{EVwoPI}| \quad (4.5)$$

- where

EVPI = expected value of perfect information

EVwPI = expected value *with* perfect information about the states of nature

EVwoPI = expected value *without* perfect information about the states of nature

3. Decision Making With Probabilities –
Expected Value of Perfect Information

- Note the role of the absolute value in equation (4.5).

- For minimization problems, the expected value with perfect information is always less than or equal to the expected value without perfect information.

- In this case, EVPI is the magnitude of the difference between EVwPI and EVwoPI, or the absolute value of the difference as shown in equation (4.5).