# ANALYTICAL AND EXPERIMENTAL DETERMINATION OF LOCALIZED STRUCTURE TO BE USED IN LABORATORY VIBRATION TESTING OF SHELL STRUCTURE-MOUNTED COMPONENTS, SATURN V 

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PROGRESS REPORT COVERING
THE PERIOD MAY 1966 TO NOVEMBER 1966

PROCEDURE FOR DESIGNING A LOCALIZED SHELL AND THE APPLICATION OF THE FINITE DIFFERENCE COMPUTER PROGRAM

Prepared For
George C. Marshall Space Flight Center National Aeronautics and Space Administration Huntsville, Alabama

NASA CONTRACT NAS8-20025


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#### Abstract

The progress report covers the work performed for the contract entitled "Analytical and Experimental Determination of Localized Structure to be Used in Laboratory Vibration Testing of Shell Structure-Mounted Components, SATURN V." The work was carried out during the period of May 1966 to November 1966 inclusive. In the report, the detailed procedure in designing a localized shell is described. Also presented is a computer program using the finite difference method which serves to guide the engineers in designing and predicting the vibration responses of the localized shell.


## FOREWORD

The progress report was prepared by Northrop Corporation, Norair Division, Hawthorne, California, under contract no. NAS8-20025, "Analytical and Experimental Determination of Localized Structure to be Used in Laboratory Vibration Testing of Shell Structure-Mounted Components, SATURN V."

The subject contract is administered under the direction of the Structures Branch, Propulsion and Vehicle Engineering Laboratory, George C. Marshall Space Flight Center of the National Aeronautics and Space Administration by Mr. J. H. Farrow and Mr. R. Jewell, principal and alternate technical representatives, respectively. Mr. L.D. Saint is the program monitor.

The program manager at Northrop Norair is Dr. Chintsun Hwang, M. T. M., Structures and Dynamics Research Branch. Dr's. W.S. Pi, N. M. Bhatia and Mr. J. R. Yamane participate in the project. Mr. P.E. Finwall is responsible for the experimental tasks of the program.

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## INTRODUCTION

During the first year of the contract on localized shell structure to be used in laboratory vibration testing, analytical techniques were used to guide the actual shell design which has been progressing on a trial basis. In the process, it was realized that once a technique has been established, a documented procedure is needed for the test engineers to design and test the localized structures. The present report serves as the document to guide the users in carrying out the design in a rational manner.

The report presents the analytical and experimental techniques in a mixed fashion. They are described in a logical sequence corresponding to the design process. The users are assumed to have available the first year progress report of the same contract which is dated May 1966.

The major supporting computer program for the design procedure is the finite difference program. The basic theory, program mechanization, the program listing and the input-output format are included in the report.

## SHELL RESPONSE AND IMPEDANCE STUDY

In order to design a localized shell structure for laboratory vibration testing purposes, preliminary tests are conducted using the complete shell structure such as the Instrument Unit with proper supporting conditions. Applying a frequency sweep technique, the shell responses and the point impedance function are plotted. Typical response data for a scale model is plotted in Figure 1. Typical impedance data are shown in Figure 2. Similar plots may be obtained for a full scale structure. Note that, in general, each peak response of Figure 1 corresponds approximately to a minimum impedance point of Figure 2. The above response and impedance plots are for a specific driving point. In the present case, the driving point is at the center of the localized shell panel. Considering the nodal line distribution, as well as the possibility of certain non-symmetrical patterns of the natural modes, it is advisable to either move the driving point, or to plot the cross-impedance of the shell structure during the test.


FIGURE 1. MICROPHONE RESPONSE VS FREQUENCY FOR POINT AT CENTER OF INSTRUMENT UNIT $5^{\circ}$ FROM ELECTROMAGNETIC DRIVER

The circumferential harmonic number is obtained by a microphone type pickup traversing along the circumference of the shell structure. For convenience in correlating the test data with the analytical data, it is preferable to test the shell with no components attached.


FIGURE 2. TYPICAL DRIVING POINT IMPEDANCES FOR THE

## I.U. SCALE MODEL

The purpose of the procedure is to design a segmented shell with equivalent dynamic characteristics as the complete shell. Experience has shown that if the segmentation procedure generates a configuration with satisfactory dynamic characteristics for the unloaded shell, then the dynamic response of the same segmented shell with component attachment will be satisfactory. In other words, the segmented shell will simulate the complete shell structure when the components are attached.

Referring to Figure 1, the prominent responses of the complete shell correspond to its natural frequency modes. In order to design a dynamically similar shell segment, the criterion is to retain as many natural frequency modes as possible in the segmented shell. For this purpose, it is necessary to investigate the detailed deformation pattern, the internal stresses, etc. of each major mode. The detailed shell response information collected in this manner is used for segmentation design. In general, the lower the natural frequency, the longer is the characteristic wave length of the deformation. As a result, the corresponding mode response of a specific localized panel is more dependent on the remaining portion of the shell structure. For high frequency modes, the deformation wave lengths are relatively short. The local response of a panel is not overly influenced by the remaining structure. As a result, in designing a localized structure, it takes less elaboration to retain the high frequency modes as compared to the low frequency modes. It is thus advisable to pay more emphasis in retaining the lower frequency modes. For a shell structure of the size of the Instrument Unit, the natural frequency modes under 100 cps are considered most significant and justify special attention.

## SECTION III

## GENERAL STIFFENED SHELL PROGRAM

In order to investigate the shell detailed deformation pattern and the internal stresses, the general stiffened shell computer program may be used. The program can handle an arbitrarily shaped shell of revolution with a number of ring stiffeners. The modal data corresponding to a circumferential harmonic number is printed out in a table together with the natural frequency. The detailed procedure of using the computer program is given in the contract yearly progress report, NOR 66-201, Vo. II, dated May 1966. The discussion here is limited to the application of the computer program for localized shell design purposes.

The computer output consists essentially of a table of eight variables along the meridian of the ring-stiffener shell. The eight variables are:

$$
w_{n}, u_{n}, v_{n}, \beta_{n}, Q_{n}, N_{\varphi n}, N_{n}, M_{\varphi n}
$$

Among the eight variables, $u, v, w$ are the displacement components, $\beta$ is the angle of rotation of the tangent to the meridian in a meridian plane. The subscript $n$ indicates the circumferential harmonic number, or the number of full wave patterns along the circumference of the shell. The four remaining variables $\mathrm{Q}, \mathrm{N}_{\varphi}, \mathrm{N}, \mathrm{M}_{\varphi}$ are the stress variables. The details of the stress variables will be explained later in the section.

Table 1 is an example of the printout data from the general stiffened shell computer program. The data is for the complete Instrument Unit scale model including the supporting shell structures. It represents the first mode corresponding to circumferential harmonic number $\mathrm{n}=4$. The natural frequency is 87.245 cps . The tabulated data is set in eight columns representing the eight variables listed above. Each row of the tabulated data refers to a section of the shell structure from one end to the other as defined by the input data. In Table 1, the data referring to the Instrument Unit proper is listed in rows 11-14. The data is in non-dimensional form. The dimensional values used to normalize the variables are listed directly above the table. The data $w_{\max }$ in inches has been used to normalize the first through third columns w, u,v. The data $\boldsymbol{\beta}_{\max }$ in radian has been used to normalize $\beta$ data in the fourth column. The $N_{\varphi \text { max }}$ data in (lb. /in.) has been used to normalize $Q, N_{\varphi}, N$ data in the fifth through seventh columns. The $M_{\varphi \text { max }}$ data in (lb. in. /in.) has been used to normalize the $\mathbf{M}_{\varphi}$ data in the eighth column. To obtain a dimensional value of the modal data, the nondimensional quantity of Table 1 is to be multiplied by the proper normalizing data $w_{\text {max }}$, $\beta_{\text {max }}$, etc.
$-0.604736 E-02$
$-0.267588 E-03$
$-0.247557 E-01$
$0.274801 E-03$
 $\begin{array}{rrr}0.31217 R F & 00 & -0.7393 \cap 6 E-01 \\ -0.739306 E-01 & 0.194476 E \quad 00 \\ 0.122257 E-01 & -0.147675 E \quad 00 \\ -0.604736 E-02 & -0.267598 E-03\end{array}$
$z$

## $\mathrm{M}-\mathrm{PHI}$

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Among the eight variables, $\mathrm{w}_{\mathrm{n}}, \mathrm{u}_{\mathrm{n}}, \beta_{\mathrm{n}}, \mathrm{Q}_{\mathrm{n}}, \mathrm{N}_{\mathrm{on}_{\mathrm{n}}}, \mathrm{M}_{\varphi \mathrm{n}}$ are in phase to each other, while $v_{n}$ and $N_{n}$ are out of phase. For instance, corresponding to a given meridian location of the shell, the circumferential position with the largest $w_{n}$ also has largest $u_{n}, \beta_{n}, Q_{n}, N_{\varphi n}$, $M_{\varphi n^{\prime}}$. The displacement component $v_{n}$ and the stress component $N_{n}$ vanish at the same position. Moving one quarter wave length along the circumference of the shell, which corresponds to an angle of $\left(\frac{\pi}{2 n}\right)$ radian, $w_{n}, u_{n}, \beta_{n}$, etc vanish, while $v_{n}, N_{n}$ are at their maximum amplitudes. From the normal displacement point of view, the latter circumferential position is the position where a meridian nodal line passes. Altogether, there are ( 2 n ) nodal lines evenly spaced along the circumference of the shell. The nodal line distribution described above has been confirmed in tests with some exceptions. In the exceptional cases, the modal shapes are influenced due to deviation in symmetry of the shell structures or other related reasons. The exceptions include skewed nodal lines or a change of harmonic number from one shell segment to the next where a ring stiffener acts as the barrier.

The four shell stress components $Q_{n}, N_{\varphi n}, N_{n}, M_{\varphi n}$ deserve further explanation. $Q_{n}$ is the amplitude of the modified transverse shear along a meridian. Ignoring the periodic time function $\cos \omega t$, the transverse shear may be expressed below:

$$
\mathbf{Q}=\mathbf{Q}_{\varphi}+\frac{1}{\mathbf{r}} \frac{\partial \mathbf{M}_{\theta \varphi}}{\partial \theta}=\mathbf{Q}_{\mathbf{n}} \cos \mathrm{n} \theta
$$

where $\mathrm{Q}_{\varphi}$ is the transverse shear force, the term $\left(\frac{1}{\mathrm{r}} \frac{\partial \mathrm{M}_{\theta \varphi}}{\partial \theta}\right)$ represents the additional transverse shear force needed at an open edge to compensate for the variation of the twisting moment $\mathrm{M}_{\theta \varphi}$. In general, when a shell is cut by a plane normal to its axis of revolution, the modified transverse shear force $Q_{n} \cos n \theta$ is to be supplied by the supporting system in order to retain the response pattern for the segmented shell. In practice, it is not possible to design a supporting system satisfying all the edge conditions. Tests and analyses have shown that in a localized shell with substantial weight attachment along the edges, the mode shapes and natural frequencies of the original shell may be retained. Corresponding to the retained mode shapes, certain internal stresses may vary substantially from the stresses in the original shell. The reason for the deviation is partially due to the variation in boundary conditions such as the transverse shear $Q_{n}$ described above. On the other hand, the added weights, which are used to retain the modal patterns of the original shell, have substantial inertia during vibration. The inertia forces have a decisive effect on the internal stresses of the localized shell. The inertia effect of the attached weights may be observed from the modal data obtained by the finite difference program described later in the report.

The sixth column of the tabulation gives the in-plane force along the meridian direction:

$$
\mathrm{N}_{\varphi}=\mathrm{N}_{\varphi \mathrm{n}} \cos \mathrm{n} \theta
$$

The seventh column of the tabulation gives the modified in-plane shear force along a section normal to the meridian

$$
\begin{aligned}
\mathrm{N} & =\mathrm{N}_{\mathrm{n}} \sin \mathrm{n} \theta \\
& =\left(\mathrm{N}_{\theta \varphi \mathrm{n}}+\frac{\sin \varphi}{\mathbf{r}} \mathrm{M}_{\theta \varphi \mathrm{n}}\right) \sin \mathrm{n} \theta
\end{aligned}
$$

where the term including $\mathrm{M}_{\theta \varphi \mathrm{n}}$ represents the additional edge shear force needed to form a couple to balance the twisting moment component due to the shell curvature. The eighth column of the tabulation gives the bending moment in the meridian plane

$$
\mathbf{M}_{\varphi}=M_{\varphi \mathrm{n}} \cos \mathrm{n} \theta
$$

The remaining stresses in the shell may be computed using the following formulas:

$$
\begin{aligned}
& \mathrm{N}_{\theta \mathrm{n}}=\nu \mathrm{N}_{\varphi \mathrm{n}}+\frac{\mathrm{Eh}}{\mathrm{r}}\left(\mathrm{w}_{\mathrm{n}} \sin \varphi+\mathrm{u}_{\mathrm{n}} \cos \varphi+\mathrm{n} \mathrm{v}_{\mathrm{n}}\right) \\
& \mathrm{M}_{\theta \mathrm{n}}=\nu \mathrm{M}_{\varphi \mathrm{n}}+\frac{\mathrm{Eh}}{}{ }^{3}\left(\underline{\eta}_{\mathrm{r}}^{2} \mathrm{w}_{\mathrm{n}}+\beta_{\mathrm{n}} \cos \varphi+\frac{\mathrm{n}}{\mathrm{r}} \sin \varphi \mathrm{v}_{\mathrm{n}}\right) \\
& M_{\theta \varphi n}=\frac{D(1-\nu)}{2 r}\left[-2 n \cos \varphi \frac{w_{n}}{r}+n J u_{n}+H \cos \varphi v_{n}-2 n \boldsymbol{\beta}_{n}\right]+\frac{\sin \varphi}{K r} D N_{n} \\
& Q_{\theta n}=-\frac{n}{r} M_{\theta n}+\frac{d}{d s} M_{\theta \varphi n}+2 \frac{\cos \varphi}{r} M_{\theta \varphi n}+\frac{\rho \omega^{2} h^{3}}{12 r}\left(n w_{n}+v_{n} \sin \varphi\right)
\end{aligned}
$$

where

$$
\begin{aligned}
& J=\frac{1}{\mathbf{R}_{\varphi}}+\frac{\sin \varphi}{\mathbf{r}} \\
& \mathbf{H}=\frac{1}{\mathbf{R}_{\varphi}}-\frac{\sin \varphi}{\mathbf{r}}
\end{aligned}
$$

$\rho$ is the material density, $\omega$ is the natural frequency in radian/sec. D is the shell section modulus. The reader is referred to the first year progress report for definitions of the shell geometry.

## SECTION IV

## LOCALIZED SHELL DESIGN

In designing a localized shell, it may be convenient to start with a major modal pattern of the complete shell structure. Along the circumferential direction, cut a section somewhat less than two half-wave lengths of the major mode. (Each half-wave encompasses an angle of $\pi / \mathrm{n}$ radian.) In case there are more than one major mode under consideration, then if physically possible, select the one with the longer wave length. In this manner, the mode(s) with the shorter wave length(s) may be retained through proper design of the localized shell and its supporting system. Along the meridian direction, the localized shell may be cut along some convenient locations. For a thin shell of the cylindrical shape, it has been found that the interaction of the shell stresses along the meridian and the circumferential directions is weak. As long as the two major dimensions of the localized shell assume a reasonable proportion, the meridian length of the localized shell is flexible. For a conical shell section, the situation is somewhat different. In either case, it is best to design a localized shell with ring stiffeners along the two circumferential edges.

The localized shell is supported by a number of cantilever beam type springs. A typical design for a scale model is shown in Figure 3. The spring has a free hinge connection to the shell structure. The formula for computing the spring linear stiffness at the connection point is:

$$
\begin{aligned}
& \mathrm{K}=\frac{3 \mathrm{EI}}{\ell^{3}} \mathrm{lb} . / \mathrm{in} . \text { for a beam with a uniform section } \\
& \mathrm{K}=\frac{\mathrm{E}}{\int_{0}^{\ell} \frac{(\ell-\mathrm{x})^{2}}{\mathrm{I}} \mathrm{dx}} \mathrm{lb} . / \mathrm{in} . \text { for a beam with a variable section }
\end{aligned}
$$

In either case, the length of the cantilever beam is $\ell . x$ is measured from the built-in end. The integration for the variable section beam may be carried out using the area-moment method as shown in the numerical example of Section VI. In designing the spring support, it is advisable to leave the beam length $\ell$ somewhat flexible subject to final adjustment. The number of spring supports used at each edge of the localized shell is arbitrary. At or near the spring supports, it is necessary to attach a number of weights in order to bring the natural frequency of the localized shell down to the level of the original shell. An approximate formula to determine the weight needed for each spring is:

$$
\mathrm{W} \cong \frac{\mathrm{Kg}}{4 \pi^{2} \mathrm{f}^{2}} \mathrm{lb}
$$



FIGURE 3. VIBRATION FIXTURE FOR S-4B INSTRUMENT UNIT SEGMENT
where K is the linear spring constant computed previously, the g is the gravitational constant, $f(c p s)$ is the natural frequency of the major mode of the original shell structure. The above formula is used as a general guide. The actual frequency of the localized shell will be influenced by the shell structure which will be determined by the finite difference computer program described later in the procedure. The weights are usually attached to the edge of the shell near the spring supports. In attaching the weights, consideration should be given to the dynamic stress concentration which may cause damage to the shell structure. For a honeycomb sandwich shell, filler material should be used to replace the honeycomb material along the edge.

The shape of the attached weight is determined essentially through its moment of inertia $I_{\theta}$ about an axis coinciding with the edge of the shell. Again, no precise formula is available. The following formula may be used to determine the approximate sectional dimensions of the attached weight:

$$
\mathrm{M}_{\theta \mathrm{n}} \mathrm{~L} \cong \mathrm{I}_{\theta \mathrm{n}} \beta_{\theta \mathrm{n}} \omega^{2}
$$

where $M_{\theta n}$ is determined by the equation given in Section III based on the general stiffened shell data. $\beta_{\theta \mathrm{n}}$ is the amplitude of the angle of rotation of the shell which may be computed by the following formula

$$
\boldsymbol{\beta}_{\theta \mathrm{n}}=\frac{\mathrm{n} \mathrm{w}_{\mathrm{n}}}{\mathbf{r}}+\frac{\sin \varphi}{\mathbf{r}} \mathrm{v}_{\mathrm{n}}
$$

$L$ is the length of the edge influenced by the attached weight, $\omega$ is the natural frequency in $\mathrm{rad} . / \mathrm{sec}$. For instance, using the data of row 14 of Table 1 and the formula for $\mathrm{M}_{\theta_{\mathrm{n}}}$ in Section III, the following numerical values are computed:

$$
\begin{aligned}
\mathrm{M}_{\theta \mathrm{n}} & =1.818 \times 0.1136 \times 10^{-3}=0.206 \times 10^{-3} \mathrm{lb} . \mathrm{in} . / \mathrm{in} \\
\beta_{\theta \mathrm{n}} & =2.70 \times 10^{-6} \mathrm{rad} \\
\mathrm{~L} & =2.7 \mathrm{in} . \\
\boldsymbol{\omega} & =2 \pi(87.2)=548 \mathrm{rad} . / \mathrm{sec}
\end{aligned}
$$

which yield an approximate value of the moment of inertia of each attached weight:

$$
I_{s}=I_{\theta n}=6.85 \times 10^{-4} \mathrm{lb} . \text { in. sec. }{ }^{2}
$$

Because of fabrication considerations etc., the actual value of $I_{s}$ in the test setup is $5.85 \times 10^{-4} \mathrm{lb}$. in. $\mathrm{sec}^{2}$. Along the circumferential edges, additional weights are attached. These weights serve to pull down the natural frequencies of the localized shell as well as to force a nodal line along the meridan direction for certain low frequency modes. Again, the manner in which the mass and the location of the attachment influence the over all modal pattern is obtained through the finite difference computer program.

## SECTION V

## FINITE DIFFERENCE COMPUTER PROGRAM

The finite difference approach, based on the coupled shell equations of Vlasov is developed for a curved panel cross-stiffened by two sets of orthogonal stiffeners. The panel considered may have a number of rigid or spring supports at arbitrary points along the edges of the panel.

Any number of concentrated weights may be attached to the edges or the internal points of the shell structure. For a component connected to the shell through a number of attachment points, the moment of inertia of the component is considered.

The computer program uses a grid pattern prescribed on the localized shell. The distance between two neighboring grid lines is a constant in each direction. The shell structure data, the spring support data and the attached weight data serve as input to the computer program. The program also interprets the local boundary conditions which are converted into finite difference equations.

All the shell equations are reduced by the program into a matrix form. After a number of internal operations which reduces the size of the eigen-matrix, the natural frequencies and the corresponding modal shapes of the localized shell are obtained using a standard eigenvalue eigen-matrix routine. The program produces the first five (5) natural modes. More modes may be generated as needed. The details of the program and its usage are described in the Appendix.

The modal data generated by the computer program are the basis of the localized shell design. For a given localized shell configuration, the computed data are compared with the test and analytical data of the complete shell structure. If the major modal patterns of the localized shell duplicate those of the complete shell, then the design may be considered satisfactory. Otherwise, the localized shell configuration including the spring supports and the weight attachments may be adjusted in order to reach a better correspondence in the modal patterns. The process may be continued until a satisfactory localized shell design is reached. Typical modal data obtained by this program and the corresponding test data are shown in Figure 4. Experimental results for both mode shape and frequency compared well with the predicted analytical values. Additional deformation data are given in Appendix I.

(a) $\mathrm{f}_{\text {computed }}=213.7 \mathrm{cps}$

(b) $f_{\text {test }}=214 \mathrm{cps}$

FIGURE 4. TYPICAL (a) ANALYTICAL AND (b) TEST MODAL PATTERN OF THE LOCALIZED SHELL STRUCTURE

## USE OF THE SCALE MODEL

The full scale shell structure of SATURN V system is large in size and is expensive. In order to ensure a satisfactory testing technique using localized shell structure, the analytical techniques are developed which are described in the previous sections. Experience indicates that no matter how extensive are the analytical techniques, there will always be some deviation between the predicted analytical data and the test data. Thus, if time and expenses are allowable, it is advisable to use a simple scale model to investigate the dynamic responses of the proposed localized shell design. The work in Northrop has shown that with proper design, the scale model and the corresponding full scale data are quite comparable when proper scale factors are used to interpret the data. The basic technique for a scale model design is given in p. 91, vol. I of Northrop Norair Yearly Progress Report NAS 8-20025. Essentially, the technique is based on the dynamic shell equations which resolve certain important design factors to establish the ratio of the natural frequencies of the scale model and the full scale structure. The same technique is used to correlate the impedance data. For the design of the shell proper, the following equation is used:

$$
\frac{\omega_{\mathrm{S}}^{2}}{\omega_{\mathrm{f}}^{2}}=\frac{\left(\frac{\mathrm{D}}{\mathrm{E}}\right)_{\mathrm{S}}\left(\frac{\mathrm{E}}{\rho_{\mathrm{h}}}\right)_{\mathrm{S}}\left(\frac{1}{\mathrm{~L}^{4}}\right)_{\mathrm{S}}}{\left(\frac{\mathrm{D}}{\mathrm{E}}\right)_{\mathrm{f}}\left(\frac{\mathrm{E}}{\rho_{\mathrm{H}}}\right)_{\mathrm{f}}\left(\frac{1}{\mathrm{~L}^{4}}\right)_{\mathrm{f}}}
$$

In the above equation, $\omega$ is the frequency in rad./sec. Subscripts $\mathrm{s}, \mathrm{f}$ indicate the scale model and the full scale structure respectively. On the righthand side of the equation, $\frac{\mathrm{D}}{\mathrm{E}}$ is a measure of the bending stiffness of the shell. For the Instrument Unit, the scale model is made of a solid sheet, while the actual structure is a honeycomb sandwich. In this case, different formulas are used to compute the shell stiffness $D$. The factor $\frac{E}{\rho_{\mathrm{h}}}$ has a dimension of acceleration. It is a measure of the relative significance of the shell internal forces and the inertia effect of the shell element. For the sandwich shell, the value $h$ is replaced by $H$ which is the compact thickness of the shell. In other words, $H$ is the thickness of the sandwich shell if it is crushed into a solid sheet. The value $\frac{1}{L^{4}}$ is the contribution due to the overall scaling factor, $L$ being a typical overall dimension. As explained in the Yearly Progress Report, the equation applied to the Instrument Unit scale model yields a frequency ratio as shown below:

$$
\frac{\omega_{\mathrm{S}}}{\omega_{\mathrm{f}}}=3.25
$$

The above frequency ratio is based on an overall scaling factor of 6.67. Now that the frequency ratio has been determined, the other design parameters are to be adjusted to reach a consistent design. For instance, the concentrated mass $M$ is scaled according to the following relation:

$$
\frac{\omega_{S}^{2}}{\omega_{f}^{2}}=\frac{D_{S}\left(\frac{1}{M}\right)_{S}\left(\frac{1}{L^{2}}\right)_{S}}{D_{f}\left(\frac{1}{M}\right)_{f}\left(\frac{1}{L^{2}}\right)_{f}}
$$

The same relation may be established for a concentrated force F applied to the structure:

$$
\frac{\omega_{\mathrm{S}}^{2}}{\omega_{\mathrm{f}}^{2}}=-\frac{D_{\mathrm{S}}\left(\frac{1}{\mathrm{~F}}\right)_{\mathrm{S}}\left(\frac{1}{\mathrm{~L}^{2}}\right)_{\mathrm{S}}}{\mathrm{D}_{\mathrm{f}}\left(\frac{1}{\mathrm{~F}}\right)_{\mathrm{f}}\left(\frac{1}{\mathrm{~L}^{2}}\right)_{\mathrm{f}}}
$$

where $F$ is the amplitude of the concentrated force with circular frequency $\omega$. The scale model of the segmented Instrument Unit is shown in Figure 3. Based on the above scaling relations, the design of the corresponding full-scale localized shell is shown in Figure 5. In the following, the determination of the attached weights and the spring supports for the full scale structure are explained.

The attached weights to the scale model Instrument Unit segment weigh $1 / 2 \mathrm{lb}$. each (see Fig. 3). The width of the weights is 1 in . Based on the actual dimensions of the weights, the polar moment of inertia $I_{S}$ of each block along an axis parallel to the edge of attachment is:

$$
I_{S}=5.85 \times 10^{-4} \mathrm{lb} . \text { in. } \mathrm{sec} .^{2}
$$

so that

$$
\mathrm{gI}_{\mathrm{s}}=0.2258 \mathrm{lb} .-\mathrm{in} .^{2}
$$

Scale relation for the mass moment inertia may be based on the following:

$$
\frac{I_{f}}{I_{S}}=\frac{\left(M L^{2}\right)_{f}}{\left(M L^{2}\right)_{S}}
$$

Alternatively, the scale relation may be determined based on edge moment:

$$
\begin{aligned}
\text { Edge moment } & =\frac{1}{\mathrm{~L}} \beta \omega^{2} \\
& \propto \frac{\mathrm{I} \omega^{2} \mathrm{w}}{\mathrm{~L}^{2}} \propto \mathrm{D} \frac{\mathrm{w}}{\mathrm{~L}^{2}}
\end{aligned}
$$


FIGURE 5. VIBRATION JG FOR S4B $45^{\circ}$ INSTRUMENT UNIT SEGMENT
so that

$$
\frac{I_{f}}{I_{S}}=\left(\frac{D}{\omega^{2}}\right)_{f} /\left(\frac{D}{\omega^{2}}\right)_{S}=\frac{\left(M L^{2}\right)_{f}}{\left(M L^{2}\right)_{S}}
$$

For the full scale piece, the attachment weighs 35.25 lb . each. The required ratio between the mass moments of inertia is:

$$
\frac{I_{f}}{I_{s}}=70.5 \times 6.67^{2}=3130
$$

With no consideration to stress concentration, position of edge stiffener, and convenience in fabrication, the dimensions in Figure 6 are first suggested for the full scale piece attachments using steel blocks. Based on the dimensions shown, the moment of inertia is computed:

$$
\mathrm{gI}_{\mathrm{f}}=\frac{70.5}{12}\left|1.265^{2}+\frac{10.3}{9.3}(10.3)^{2}-1^{2}\right|=702 \mathrm{lb} .-\mathrm{in} .^{2}
$$

so that

$$
\frac{\mathrm{gI}_{\mathrm{f}}}{\mathrm{gI}_{\mathrm{s}}}=\frac{702}{.2258}=3110
$$



FIGURE 6. SUGGESTED DIMENSIONS FOR ATTACHED WEIGHTS

As may be observed from Figure 5, the actual dimensions of the attached weights have been modified from the suggested configuration for the four longitudinal edge pieces. The modification was due to design and fabrication considerations.

The scaling formula for the spring supports of the localized shell is:

$$
\frac{\mathrm{K}_{\mathrm{S}}}{\bar{K}_{\mathrm{f}}}=\frac{\left(\omega^{2} \mathrm{M}\right)_{\mathrm{S}}}{\left(\omega^{2} \mathrm{M}_{\mathrm{f}}\right.}=\frac{\left(\mathrm{D} / \mathrm{L}^{2}\right)_{\mathrm{S}}}{\left(\mathrm{D} / \mathrm{L}^{2}\right)_{\mathrm{f}}}=\frac{1}{6.67}
$$

The spring constants $K_{S}$ for the I. U. model are obtained in the following manner for a soft spring and a stiff spring.

Case I SOFT SPRING, $50 \leq\left(\omega_{\mathrm{S}} / 2 \pi\right) \leq 250 \mathrm{CPS}$.


$$
\begin{array}{rl}
b= & 0.375^{\prime \prime} \\
E_{S} & 10.3 \times 10^{6} \mathrm{PSI}
\end{array}
$$

$$
\begin{aligned}
\frac{\mathrm{E}_{\mathrm{S}}}{\mathrm{~K}_{\mathrm{S}}}- & \frac{1}{3} \times 228 \times(0.37 .5)^{2} \\
& +\frac{1}{2} \times 4375 \times 1.5 \times 0.87 .5 \\
& \frac{1}{2} \times 21860 \times 1.5 \times 1.375 \\
& 25,480 \mathrm{IN} . \\
\mathrm{K}_{\mathrm{S}} \quad & 404.2 \mathrm{LB} / \mathrm{IN} .
\end{aligned}
$$

CASE II STIFF SPRING, $\left(\omega_{\mathrm{s}} / 2 \pi\right)>250 \mathrm{CPS}$


$$
\begin{aligned}
& \text { b : 0.375" } \\
& \mathrm{E}_{\mathrm{S}}=10.3 \times 10^{6} \mathrm{PSI} \\
& \frac{\mathrm{E}_{\mathrm{s}}}{\mathrm{~K}_{\mathrm{s}}} \quad \frac{1}{3} \times 1140 \times 1.89^{2} \\
& +\frac{1}{2} \text { X } 1275 \text { X } 1 \text { X } 0.708 \\
& +\frac{1}{2} \text { X } 4670 \text { X } 1 \text { X } 1.042 \\
& 4230 \mathrm{IN}^{-1} \\
& K_{s}=2440 \mathrm{LB} / \mathrm{IN} \text {. }
\end{aligned}
$$

Based on the scaling formula given previously, two spring supports for the full-scale unit Instrument Unit panel are designated below using, carbon steel stock as beam material.

Case I. LOW FREQUENCY REGION $\left(\omega_{\mathrm{f}} / 2 \pi\right) \leq 77$ CPS.


$$
\mathrm{K}_{\mathrm{f}} \quad 2700 \mathrm{LIB} . / \mathrm{IN} . \quad 6.67 \mathrm{~K}_{\mathrm{s}}
$$

CASE II, HIGH FREQUENCY REGION. $\left(\boldsymbol{\omega}_{\mathrm{f}} / 2 \pi\right)>77 \mathrm{CPS}$


The driving point impedance plot of the full scale shell with component attachments is shown in Figure 7. Also plotted is the corresponding impedance data for the scale model test specimen with proper adjustment in the scales used. From the figure. it may be seen that the general impedance pattern is preserved to an agreeable degree through the dynamic scaling technique. In the high frequency region, the deviation is believed due to the high structural damping of the intricately fabricated full scale shell structure as compared to the simple scale model.

The arrowheads shown in Figure 7 indicate the computed eigenfrequencies of the full scale I. U. segment. The frequencies are computed for mode shapes symmetrical with respect to both center lines of the segment using the finite difference program. Among the computed eigenfrequencies, none corresponds to the test frequency for the first low impedance point. Possible reasons of the omission of the frequency may be due to the nonsymmetry of the mode shape, the local vibration coupled to the mechanical excitation device, etc. This point will be further investigated.

## APPENDIX I

## IN LOCALIZED SHELL VIBRATION TESTS

The finite difference approach is developed for a curved panel cross-stiffened by two sets of equidistant stiffeners. The reinforcement, which is assumed to be symmetrical about an axis normal to the middle surface of the shell, is attached to the skin of the cylindrical panel in either circumferential or axial direction. The panel to be considered may have a number of rigid or spring supports at discrete points along the edge of the panel.
A. BASIC EQUATIONS

1. Strain-Displacement Relations

The extensional strain-displacement relations for the middle surface of a cylindrical shell are given by

$$
\begin{align*}
\boldsymbol{\epsilon}_{\mathbf{x}} & =\frac{\partial \mathbf{u}}{\partial \mathbf{x}}  \tag{1}\\
\boldsymbol{\epsilon}_{\varphi} & =\frac{1}{\mathrm{a}}\left(\frac{\partial \mathrm{v}}{\partial \varphi}+\mathrm{w}\right)  \tag{2}\\
\boldsymbol{\gamma}_{\mathbf{x} \varphi} & =-\frac{1}{a} \frac{\partial \mathbf{u}}{\partial \varphi}+\frac{\partial v}{\partial \mathbf{x}} \tag{3}
\end{align*}
$$

where
$\mathrm{u}, \mathrm{v}, \mathrm{w}=$ displacement components in the axial, circumferential and radial directions.

$$
\mathbf{x}=\text { distance along axial direction }
$$

$\varphi=$ cylindrical angle (radian)
$\mathrm{a}=$ radius of cylinder (see Figure 8)


FIGURE 8. ELEMENT OF A CYLINDRICAL SHELL

The changes of curvature of the cylindrical panel may be represented as follows:

$$
\begin{align*}
& \mathbf{k}_{\mathrm{x}}=-\frac{\partial^{2} \mathrm{w}}{\partial \mathbf{x}^{2}}  \tag{4}\\
& \mathrm{k}_{\varphi}=-\frac{1}{a^{2}}\left(\frac{\partial^{2} \mathbf{w}}{\partial \varphi^{2}}-\frac{\partial \mathrm{v}}{\partial \varphi}\right)  \tag{5}\\
& \mathbf{k}_{\mathbf{x} \varphi}=-\frac{1}{\mathrm{a}}\left(\frac{\partial^{2} \mathbf{w}}{\partial \mathbf{x} \partial \varphi}-\frac{\partial \mathbf{v}}{\partial \mathbf{x}}\right) \tag{6}
\end{align*}
$$

For a cylindrical panel, experience has shown that the effect of the v-displacement on the bending moments is negligible. With this simplification, the following expressions are obtained:

$$
\begin{align*}
\mathrm{k}_{\mathrm{x}} & =-\frac{\partial^{2} \mathrm{w}}{\partial \mathrm{x}^{2}}  \tag{4}\\
\mathbf{k}_{\varphi} & =-\frac{1}{\mathrm{a}^{2}} \frac{\partial^{2} \mathrm{w}}{\partial \varphi^{2}} \\
\mathrm{k}_{\mathrm{x} \varphi} & =-\frac{1}{\mathrm{a}} \frac{\partial^{2} \mathrm{w}}{\partial \mathbf{x} \partial \varphi} \tag{6'}
\end{align*}
$$

## 2. Stress-Strain Relations

For a cylindrical panel cross-stiffened by two sets of equidistant stiffeners, it is assumed that the reinforcement is symmetrical about an axis normal to the middle surface of the panel. Using the above assumption, the following approximate stress-strain relations (Ref. 1) may be established:

$$
\begin{align*}
& N_{X}=\frac{E A_{S x}}{a \Delta \varphi} \epsilon_{x}+\frac{E h}{1-\boldsymbol{\nu}^{2}}\left(\boldsymbol{\epsilon}_{\mathrm{x}}+\boldsymbol{\nu} \boldsymbol{\epsilon}_{\varphi}\right)  \tag{7}\\
& \mathrm{N}_{\varphi}=\frac{\mathrm{EA} \mathrm{~S}_{\varphi}}{\Delta \mathrm{x}} \boldsymbol{\epsilon}_{\varphi}+\frac{\mathrm{Eh}}{1-\nu^{2}}\left(\epsilon_{\varphi}+\nu \epsilon_{\mathrm{x}}\right)  \tag{8}\\
& \mathrm{N}_{\mathrm{X} \varphi}=\mathrm{N}_{\varphi_{\mathbf{X}}} \simeq \operatorname{Gh} \gamma_{\mathrm{X} \varphi}  \tag{9}\\
& M_{x}=\frac{E k_{x}}{a \Delta \varphi} \int_{A_{S x}} z_{x}^{2} d A+\frac{E}{1-\nu^{2}}\left(k_{x}+\nu k \varphi\right) \int_{\text {panel }} z_{x}^{2} d z  \tag{10}\\
& \mathbf{M}_{\varphi}=\frac{\mathbf{E} \mathbf{k}_{\varphi}}{\Delta \mathbf{x}} \int_{\mathbf{A}_{\mathbf{S} \varphi}} \mathbf{z}_{\varphi}^{2} \mathrm{dA}+\frac{\mathbf{E}}{1-\nu^{2}}\left(\mathbf{k}_{\varphi}+\nu \mathbf{k}_{\mathbf{x}}\right) \int_{\text {panel }} \mathbf{z}_{\varphi}{ }^{2} \mathrm{dz}  \tag{11}\\
& M_{\mathbf{x} \varphi}=-M_{\varphi \mathbf{x}}=-\frac{E h^{3}}{12(1+\nu)} \quad k_{\mathbf{x} \varphi} \tag{12}
\end{align*}
$$

Ref. 1 S. Timoshenko, S. Woinowsky-Krieger, "Theory of Plates and Shells," pp. 364-370.
where
$A_{S X}, A_{S \varphi}=$ cross sectional area of stiffener*
$I_{S X}, I_{S \varphi}=$ moment of inertia of stiffener about its own centroid axis*
$z_{x^{\prime}}, z_{\varphi}=$ distance measured from the neutral axis of shell-stiffener combination (see Figure 9)


FIGURE 9. CROSS-SECTIONAL VIEW OF A SHELL-STIFFENER COMBINATION
$h$ = thickness of a solid panel or the total facing thickness of a honeycomb sandwich panel
E = Young's modulus
$\nu=$ Poisson's ratio
The following sectional rigidity properties are defined:

$$
\begin{align*}
& K=\frac{E h}{1-\nu^{2}} \\
& \mathbf{K}_{\mathbf{x}}=\mathbf{K}+\frac{\mathbf{E} \mathrm{A}_{\mathbf{S x}}}{\mathbf{a \Delta \varphi}} \\
& \mathbf{K}_{\varphi}=\mathbf{K}+\frac{\mathbf{E ~ A} \mathbf{S}_{\mathbf{S}}}{\Delta \mathbf{x}} \\
& \frac{\mathrm{Eh}^{3}}{12\left(1-\nu^{2}\right)} \quad \text { for solid panel } \\
& \text { D }=  \tag{13}\\
& \frac{\operatorname{Eh}(\mathrm{h}+2 \mathrm{c})^{2}}{16\left(1-\nu^{2}\right)} \quad \text { for honeycomb sandwich panel } \\
& \mathrm{D}_{\mathrm{x}}^{\prime}=\mathrm{D}+\mathrm{Kz}_{\mathrm{cx}}{ }^{2} \\
& \mathrm{D}_{\varphi}^{\prime}=\mathrm{D}+\mathrm{Kz}_{\mathbf{c} \varphi}^{2} \\
& D_{x}=D_{x}{ }^{\prime}+\frac{E}{a \Delta \varphi}\left\{I_{s x}+A_{s x}\left(z_{s x}-z_{c x}\right)^{2}\right] \\
& \mathrm{D}_{\varphi}=\mathrm{D}_{\varphi}{ }^{\prime}+\frac{\mathrm{E}}{\Delta \mathbf{x}}\left[\mathrm{I}_{\mathbf{S} \varphi}+\mathrm{A}_{\mathbf{S} \varphi}\left(\mathrm{z}_{\mathbf{S} \varphi}-\mathrm{z}_{\mathbf{c} \varphi}\right)^{2}\right]
\end{align*}
$$

[^2]where
$\mathrm{z}_{\mathrm{SX}}, \mathrm{z}_{\mathrm{S} \varphi}=$ distances from $\mathrm{c} . \mathrm{g} . \mathrm{s}$ of the stiffeners to the middle surface of the panel
$\mathbf{z}_{\mathbf{c x}} \mathbf{z}_{\mathbf{c} \varphi}=$ distances between the centroids of the stiffeners to neutral axis of shellstiffener combination (see Figure 9).
c = core thickness of a honeycomb sandwich panel
Using the newly defined rigidity expressions, the stress-strain relations may be rewritten as:
\[

$$
\begin{align*}
& \mathbf{N}_{\mathbf{x}}=\mathbf{K}_{\mathbf{x}} \boldsymbol{\epsilon}_{\mathbf{x}}+\boldsymbol{\nu} \mathbf{K} \boldsymbol{\epsilon}_{\varphi}  \tag{7'}\\
& \mathbf{N}_{\varphi}=\mathbf{K}_{\varphi} \boldsymbol{\epsilon}_{\varphi}+\boldsymbol{\nu} \mathbf{K} \boldsymbol{\epsilon}_{\mathbf{x}} \\
& \mathbf{N}_{\mathbf{x} \varphi}=\mathbf{N}_{\varphi \mathbf{x}}=-\frac{(1-\boldsymbol{1})}{2} \mathbf{K} \gamma_{\mathbf{x} \varphi} \\
& \mathbf{M}_{\mathbf{x}}=\mathbf{D}_{\mathbf{x}} \mathrm{k}_{\mathbf{x}}+\nu \mathrm{D}_{\mathbf{x}}^{\prime} \mathrm{k}_{\varphi} \\
& \mathbf{M}_{\varphi}=\mathrm{D}_{\varphi} \mathrm{k}_{\varphi}+\nu \mathrm{D}_{\varphi}^{\prime} \mathbf{k}_{\mathbf{x}}  \tag{11'}\\
& \mathbf{M}_{\mathbf{x} \varphi}=-\mathbf{M}_{\varphi \mathbf{x}}=-\mathbf{D}(1-\nu) \mathrm{k}_{\mathbf{x} \varphi} \tag{12'}
\end{align*}
$$
\]

Figure 10 shows the notation for coordinates, displacements, force and moment components for a cylindrical shell element.


$$
\mathrm{D}_{\mathrm{x}} \frac{\partial^{4} \mathrm{w}}{\partial \mathrm{x}^{4}}+2 \mathrm{H} \frac{1}{\mathrm{a}^{2}} \frac{\partial^{4} \mathrm{w}}{\partial \mathrm{x}^{2} \partial \varphi^{2}}+\mathrm{D}_{\varphi} \frac{1}{\mathbf{a}^{4}} \frac{\partial^{4} \mathrm{w}}{\partial \omega^{4}}+\frac{1}{\mathrm{a}} \mathrm{~N}_{\varphi}=\mathrm{q}
$$

where $2 \mathrm{H}=\nu\left(\mathrm{D}_{\mathrm{x}}^{\prime}+\mathrm{D}_{\varphi}^{\prime}\right)+2 \mathrm{D}(1-\nu)$

## 4. Stress Function and Compatibility Condition

A stress function $\Phi$ is introduced in this analysis in order to reduce the number of unknowns from three ( $u, v$, and $w$ ) to two ( $w$ and $\phi$ ). The membrane forces $N_{x}, N_{\varphi}$ and $\mathrm{N}_{\mathrm{X} \varphi}$ can be expressed in terms of the stress function $\Phi$ :

$$
\begin{align*}
& \mathbf{N}_{\mathbf{x}}=-\frac{\partial^{2} \boldsymbol{\phi}}{\mathbf{a}^{2} \partial \varphi^{2}}  \tag{16}\\
& \mathbf{N}_{\varphi}=-\frac{\partial^{2} \boldsymbol{\phi}}{\partial \mathbf{x}^{2}}  \tag{17}\\
& \mathbf{N}_{\mathbf{X} \varphi}=\mathbf{N}_{\varphi \mathbf{X}}=\frac{\partial^{2} \boldsymbol{\phi}}{\mathbf{a} \partial \mathbf{x} \partial \varphi} \tag{18}
\end{align*}
$$

In order to use the stress function approach, an equation of compatibility is needed to define
$\Phi$. This is accomplished by combining equations (1)-(3) to obtain the following relation:

$$
\begin{equation*}
\frac{\partial^{2} \epsilon_{\varphi}}{\partial \mathbf{x}^{2}}+\frac{1}{a^{2}} \frac{\partial^{2} \epsilon_{\mathrm{X}}}{\partial \varphi^{2}}-\frac{1}{\mathrm{a}} \frac{\partial^{2 \gamma} \mathrm{X} \cdot \varphi}{\partial \mathrm{X} \partial \varphi}-\frac{1}{\mathrm{a}} \frac{\partial^{2} \mathrm{w}}{\partial \mathbf{x}^{2}}=0 \tag{19}
\end{equation*}
$$

Using equations ( $7^{\prime}$ )-( $9^{\prime}$ ) and (16)-(18), the strain components $\epsilon_{\mathrm{X}}, \epsilon_{\varphi}$ and $\gamma_{\mathrm{X} \varphi}$ may be expressed as:

$$
\begin{align*}
& \epsilon_{\varphi}=-\frac{1}{K_{X} K_{\varphi,}-\nu^{2} K^{2}}\left[K_{X} \frac{\partial^{2} \phi}{\partial \mathrm{x}^{2}}-\nu K \frac{\partial^{2} \phi}{\mathrm{a}^{2} \partial \varphi^{2}}\right]  \tag{20}\\
& \epsilon_{\mathbf{X}}=-\frac{1}{\mathrm{~K}_{\mathbf{X}} \mathrm{K}_{\varphi}-\nu^{2} \mathrm{~K}^{2}}\left[\mathrm{~K}_{\varphi} \frac{\partial^{2} \phi}{2{ }_{\mathrm{a}}{ }^{2} \varphi^{2}}-\nu \mathrm{K} \frac{\partial^{2} \phi}{\partial \mathbf{x}^{2}}\right]  \tag{21}\\
& \gamma_{\mathrm{X} \varphi}=\frac{2}{(1-\nu) \mathrm{K}} \frac{\partial^{2} \boldsymbol{\phi}}{\mathrm{a} \partial \mathrm{x} \partial \varphi} \tag{22}
\end{align*}
$$

Substituting equations (20)-(22) into equation (19) yields the compatibility condition:

$$
\begin{equation*}
\mathrm{K}_{\mathrm{X}} \frac{\partial^{4} \phi}{\partial \mathrm{X}^{4}}+\frac{\mathrm{K}_{\mathrm{X}} \mathrm{~K}_{\varphi}-\nu \mathrm{K}^{2}}{(1-\nu) \mathrm{K}^{2}} \frac{2}{\mathrm{a}^{2}} \frac{\partial^{4} \phi}{\partial \mathrm{x}^{2} \partial \varphi^{2}}+\mathrm{K}_{\varphi} \frac{\partial^{4} \phi}{\mathrm{a}^{4} \partial \varphi^{4}}+\frac{\mathrm{K}_{\mathrm{x}} \mathrm{~K}_{\varphi}-\nu^{2} \mathrm{~K}^{2}}{\mathrm{a}} \frac{\partial^{2} \mathrm{w}}{\partial \mathrm{x}^{2}}=0 \tag{23}
\end{equation*}
$$

The equilibrium equation (14') can be rewritten in the form of

$$
\begin{equation*}
\mathrm{D}_{\mathbf{x}} \frac{\partial^{4} \mathrm{w}}{\partial \mathbf{x}^{4}}+2 \mathrm{H} \frac{1}{\mathrm{a}^{2}}-\frac{\partial^{4} \mathrm{w}}{\partial \mathrm{x}^{2} \partial \varphi^{2}}+\mathrm{D}_{\varphi} \frac{1}{\mathrm{a}^{4}}-\frac{\partial^{4} \mathrm{w}}{\partial \varphi^{4}}-\frac{1}{\mathrm{a}} \frac{\partial^{2} \phi}{\partial \mathrm{x}^{2}}=\mathrm{q} \tag{24}
\end{equation*}
$$

Equations (23) and (24) are the two governing differential equations mechanized in the computer program using the finite difference approach. It can be shown that for the special case of a cylindrical shell without any stiffener, the above equations are identical to the coupled shell equations of Vlasov applied to the cylindrical panel. The Vlasov's equations are:

$$
\left\{\begin{array}{l}
\mathrm{D} \nabla^{2} \nabla^{2} \mathrm{w}-\frac{1}{\mathrm{a}} \frac{\partial^{2} \phi}{\partial \mathrm{x}^{2}}=\mathrm{q} \\
\nabla^{2} \nabla^{2} \phi+\frac{E h}{\mathrm{a}} \frac{\partial^{2} \mathrm{w}}{\partial \mathrm{x}^{2}}=0
\end{array}\right.
$$

where $\nabla^{2}$ is the Laplace operator.

## 5. Boundary Conditions for a Panel with Spring Supports

The cylindrical panel under consideration has spring supports at discrete points along the edges of the panel. These springs are assumed to be restrained against deflection but not restrained against rotation. Lateral deflection at spring supporting point may be either restricted or free. The boundary conditions to be considered are:

Along the edge $\mathrm{x}=\mathrm{constant}$,

$$
\begin{align*}
& \mathbf{M}_{\mathbf{x}}=0  \tag{25}\\
& \mathbf{V}_{\mathbf{x}}=\mathbf{Q}_{\mathbf{x}}-\frac{1}{\mathbf{a}} \frac{\partial \mathbf{M}_{\mathbf{x} \varphi}}{\partial \varphi}= \pm \frac{\overline{\mathrm{K}}_{\mathbf{x}}}{a \Delta \varphi} \mathbf{w}  \tag{26}\\
& \mathbf{N}_{\mathbf{X}}=0 \text { or } u=0  \tag{27a,b}\\
& \mathbf{S}_{\mathbf{x}}=\mathbf{N}_{\mathbf{x} \varphi}+\frac{1}{\mathbf{a}} \mathbf{M}_{\mathbf{x} \varphi}=0 \tag{28}
\end{align*}
$$

Along the edge $\varphi=$ constant,

$$
\begin{align*}
\mathbf{M}_{\varphi} & =0  \tag{29}\\
\mathbf{V}_{\varphi} & =\mathbf{Q}_{\varphi}+\frac{\partial \mathbf{M}_{\varphi \mathbf{x}}}{\partial \mathbf{x}}= \pm \frac{\mathbf{K}_{\varphi}}{\Delta \mathbf{x}} \quad \mathbf{w}  \tag{30}\\
\mathbf{N}_{\varphi} & =0 \text { or } \mathbf{v}=0  \tag{31a,b}\\
\mathbf{N}_{\varphi \mathbf{x}} & =0 \tag{32}
\end{align*}
$$

At the corner point, $\mathbf{x}=$ constant, $\varphi=$ constant,

$$
\begin{equation*}
2 \mathbf{M}_{\mathbf{x} \varphi}=+\overline{\mathbf{K}}_{1} \mathbf{w} \tag{33}
\end{equation*}
$$

where $V_{X}, V_{\varphi}$ are the effective transverse shear forces, $S_{X}$ is the effective in-plane shear force, $\bar{K}_{\mathbf{X}} \overline{\mathrm{K}}_{\varphi}$ and $\overline{\mathrm{K}}_{1}$ are the spring constants for the point supports whose unit is (force/ length). The spring constants equal to zero at a point where the edge is free.

The problem at hand is to determine the natural frequencies and the corresponding mode shapes of the panel. Thus, the loading on the panel is the inertial force:

$$
\begin{equation*}
\mathrm{q}=\overline{\mathrm{n}} \omega^{2} \mathrm{w} \tag{34}
\end{equation*}
$$

where in is the mass per unit area of the panel and $\omega$ is the natural circular frequency of the panel. For an interior point where a weight $W$ is attached, then the additional loading due to this weight is:

$$
\begin{equation*}
q_{a}=\frac{W \omega^{2}}{\operatorname{ga\Delta } \Delta \varphi} \quad w \tag{35}
\end{equation*}
$$

Similarly, for a boundary point where there is an attached weight $W_{x}, W_{0}$ or $W_{1}$, equations (26), (30), and (33) are as shown below:
$\mathrm{x}=\mathrm{constant}$,

$$
V_{x}=+\left(\overline{\mathrm{K}}_{\mathrm{x}}-\frac{\mathrm{W}_{\mathrm{x}}}{\mathrm{~g}} \omega^{2}\right) \frac{\mathrm{w}}{\mathrm{a} \Delta \varphi}
$$

$\rho=$ constant,

$$
\mathrm{V}_{\varphi}= \pm\left(\overline{\mathrm{K}}_{\varphi}-\frac{\mathrm{W}_{\varphi}}{\mathrm{g}} \quad \omega^{2}\right) \frac{\mathrm{w}}{\Delta \mathrm{x}}
$$

$\mathbf{x}=$ constant,$\varphi=$ constant,

$$
2 \mathrm{M}_{\mathrm{x} \varphi}= \pm\left(\overline{\mathrm{K}}_{1}-\frac{\mathrm{W}_{1}}{\mathrm{~g}} \quad \omega^{2}\right) \mathrm{w}
$$

The signs used in equations (26'), (30'), and (33') depend on the relative positions between the boundary lines and the directions of the coordinates. Figure 11 shows the sign convention to be used in the equations corresponding to the coordinates system of Figure 10 .


FIGURE 11. SIGN CONVENTION OF TRANSVERSE FORCES

## B. FINITE DIFFERENCE EXPRESSIONS

1. Grid Setup and Basic Finite Difference Operators

The geometry of the scaled-down Instrument Unit panel under test is shown in Figure 12. Since the panel is symmetrical with respect to both the $\mathrm{x}=\frac{\mathrm{b}}{2}$ and $\varphi=\frac{0}{2}$ axes, onequarter of this panel is considered for the finite difference solution. The quarter panel is shown in shade in Figure 12.


FIGURE 12. GEOMETRY OF SEGMENTED INSTRUMENT UNIT SCALE MODEL

Figure 13 shows the grid pattern with a station numbering system appropriate for finding the modes which are symmetrical about both the $\mathrm{x}=\frac{\mathrm{b}}{2}$ and $\varphi=\frac{\varphi_{0}}{2}$ axes. It should be emphasized that the grid pattern used is not a unique and fixed pattern. Different patterns may be used for various specimens, boundary conditions, etc. The computer program is designed with such a flexibility in mind.

Using the grid pattern of Figure 13, it may be visualized that the deformation function w and the auxiliary function $\Phi$ may be plotted over the grid as two surfaces. The finite difference computer program translates the coupled equations (23) and (24) into linear equations of $w$ and $\phi$ at various grid points. In order to facilitate the translation, the conversion of the partial derivatives into finite difference relations is first explained.

Using a grid with a grid pitch $\Delta \mathrm{x}$ in the x direction and a $\Delta \varphi$ in the $\varphi$ direction, the coefficients of the basic finite difference expressions for the partial derivatives of a function at a point i are as follows:


FIGURE 13. GRID PATTERN USED IN THE FINITE DIFFERENCE PROGRAM

$$
(\Delta \mathrm{x})^{2} \frac{\partial^{2}}{\partial \mathrm{x}^{2}}=\begin{array}{llll}
1 & -2 & 1 \\
& \text { (i) } & \\
\hline
\end{array}
$$

$$
(\Delta \varphi)^{2} \frac{\partial^{2}}{\partial \varphi^{2}}=\begin{gathered}
1 \\
-2 \\
\text { (i) } \\
1 \\
\hline
\end{gathered}
$$

$$
(4 \Delta \mathrm{x} \Delta \varphi) \frac{\partial^{2}}{\partial \mathrm{x} \partial \varphi}=\begin{array}{rrr}
-1 & 0 & 1 \\
0 & 0 & 0 \\
& \text { (i) } & \\
1 & 0 & -1
\end{array}
$$

$$
2(\Delta \mathrm{x})^{3} \frac{\partial^{3}}{\partial \mathrm{x}^{3}}=\begin{array}{|llcll|}
\hline 1 & -2 & \begin{array}{l}
0 \\
(\mathrm{i})
\end{array} & 2 & -1 \\
\hline
\end{array}
$$

$$
2(\Delta \varphi)^{3} \frac{\partial^{3}}{\partial \varphi^{3}}=
$$

$$
\begin{gathered}
0 \\
(\mathrm{i})
\end{gathered}
$$

$$
-2
$$






$$
(\Delta \times \Delta \varphi)^{2} \frac{\partial^{4}}{\partial x^{2} \partial \varphi^{2}}=\quad \begin{array}{ccc}
1 & -2 & 1 \\
-2 & 4 & -2 \\
& \text { (i) } & \\
1 & -2 & 1
\end{array}
$$

2. Equilibrium and Compatibility Equations.

Consider Equation (24); it may be rewritten in the following form:

$$
\left(\frac{\mathrm{a} \Delta \varphi}{\Delta \mathrm{x}}\right)^{4}\left(\frac{\mathrm{D}_{\mathrm{x}}}{\mathrm{D}}\right)(\Delta \mathrm{x})^{4} \frac{\partial^{4} \mathrm{w}}{\partial \mathrm{x}^{4}}+\left(\frac{\mathrm{a} \Delta \varphi}{\Delta \mathrm{x}}\right)^{2}\left(\frac{2 \mathrm{H}}{\mathrm{D}}\right)(\Delta \mathrm{x} \Delta \varphi)^{2} \frac{\partial^{4} \mathrm{w}}{\partial \mathrm{x}^{2} \partial \varphi}+\left(\frac{\mathrm{D}^{2}}{\mathrm{D}}\right)(\Delta \varphi)^{4} \frac{\partial^{4} \mathrm{w}}{\partial \varphi^{4}}
$$

$$
\begin{equation*}
-\left(\frac{\mathrm{a} \Delta \varphi}{\Delta \mathrm{x}}\right)^{2} \frac{\mathrm{a}(\Delta \varphi)^{2}}{\mathrm{D}}(\Delta \mathrm{x})^{2} \frac{\partial^{2} \phi}{\partial \mathrm{x}^{2}}=\frac{\overline{\mathrm{m}}(\mathrm{a} \Delta \varphi)^{4} \omega^{2}}{\mathrm{D}}\left[1+\frac{\mathrm{W}}{\overline{\mathrm{~m}} \mathrm{~g} \Delta \mathrm{xa} \Delta \varphi}\right] \mathrm{w} \tag{36}
\end{equation*}
$$

Using symbols $\lambda=\frac{\mathbf{a} \Delta \varphi}{\Delta \mathbf{x}}, \Omega=\frac{\overline{\mathrm{m}}(\mathrm{a} \Delta \varphi)^{4} \omega^{2}}{\mathrm{D}}$, Equation (36) is:

$$
\begin{gather*}
\lambda^{4} \frac{D_{x}}{\mathrm{D}}(\Delta \mathrm{x})^{4} \frac{\partial^{4} \mathrm{w}}{\partial \mathrm{x}^{4}}+\lambda^{2}\left(\frac{2 \mathrm{H}}{\mathrm{D}}\right)(\Delta \mathrm{x} \Delta \varphi)^{2} \frac{\partial^{4} \mathrm{w}}{\partial \mathrm{x}^{2} \partial \varphi^{2}}+\left(\frac{\mathrm{D} \varphi}{\mathrm{D}}\right)(\Delta \varphi)^{4} \frac{\partial^{4} \mathrm{w}}{\partial \varphi^{4}} \\
-\frac{\lambda^{2} \mathrm{a}(\Delta \varphi)^{2}}{\mathrm{D}}(\Delta \mathrm{x})^{2}-\frac{\partial^{2} \Phi}{\partial \mathrm{x}^{2}}=\Omega\left(1+\frac{\mathrm{w}}{\overline{\mathrm{~m} g \Delta x a \Delta \varphi}}\right) \mathrm{w} \tag{36'}
\end{gather*}
$$

Applying the finite difference operators to replace the partial derivatives, the coefficients for various points on the w-grid and $\Phi$-grid are determined in the following manner. Referring to Equation ( $\mathbf{3 6}^{\prime}$ ), the partial derivatives are translated into finite differences on the $\mathbf{w}$-grid and the $\Phi$-grid corresponding to a grid point marked by $\left(w_{i}\right)$ or $\left(\Phi_{i}\right)$. In the formulation, the inertia term in the equation is omitted for the time being.

$$
\begin{aligned}
& \frac{\mathrm{D}_{\varphi}}{\mathrm{D}} \\
& \frac{2 \mathrm{H}}{\mathrm{D}} \lambda^{2} \\
& -4\left(\frac{H}{\mathrm{D}} \lambda^{2}+\frac{\mathrm{D}}{\mathrm{D}}\right) \\
& \frac{2 \mathrm{H}}{\mathrm{D}} \lambda^{2}
\end{aligned}
$$

$$
\begin{aligned}
& \frac{2 H}{\bar{D}} \lambda^{2} \quad-4\left(\frac{H}{\mathrm{D}} \lambda^{2}+\frac{\mathrm{D} \varphi}{\mathrm{D}}\right) \quad \frac{2 \mathrm{H}}{\mathrm{D}} \lambda^{2} \\
& \frac{D_{\varphi}}{\mathrm{D}}
\end{aligned}
$$

| $-\frac{\lambda^{2}(\mathrm{a} \Delta \varphi)^{2}}{\mathrm{aD}}$ | $2 \frac{\lambda^{2}(\mathrm{a} \Delta \varphi)^{2}}{\mathrm{aD}}$ | $-\frac{\lambda^{2}(\mathrm{a} \Delta \varphi)^{2}}{\mathrm{aD}}$ |
| :---: | :---: | :---: |

The above finite difference operators are applied to all stations on one-quarter of the panel including boundary points (Sta. 1-24). In laying out the coefficients of the finite difference equations in a systematic manner, visualize a square matrix of size ( rxr ), where $r$ is the number of the total unknown w and $\Phi$ values for all the grid points. The columns of the matrix correspond to the coefficients of $w_{1}, w_{2} \ldots \Phi_{1}, \Phi_{2} \ldots$ arranged in that order. For Equation (36') applied to a grid station, a row of matrix is established listing all the coefficients of the finite difference equation as described above. Other rows are added to fill up the (rxr) matrix representing the equilibrium equation, the compatibility equation and the boundary conditions. The detail of the matrix organization will be further explained later in the section.

The finite difference operators for compatibility equation (23) are illustrated below:


$$
\begin{align*}
& \begin{aligned}
& \mathrm{R}(\mathrm{a} \Delta \varphi)^{2} \lambda^{2}-2 \mathrm{R}(\mathrm{a} \Delta \varphi)^{2} \lambda^{2} \\
&\left(\mathrm{w}_{\mathrm{i}}\right)
\end{aligned} \\
& \mathrm{R}(\mathrm{a} \Delta \varphi)^{2} \lambda^{2} \\
& \mathrm{P}=\frac{\mathrm{K}_{\mathrm{x}} \mathrm{~K}_{\varphi}-\nu \mathrm{K}^{2}}{(1-\nu) \mathrm{K}^{2}} \\
& \mathrm{R}=\frac{\mathrm{K}_{\mathrm{x}} \mathrm{~K}_{\varphi}-\nu^{2} \mathrm{~K}^{2}}{\mathrm{aK}}
\end{align*}
$$

where

Experience shows that it is sufficient to apply Equation (23) only to the interior points (sta. 8-12, 14-18, 20-24) in order to fill the final coefficient matrix described above.

## 3. Boundary Conditions.

Equations (25) - (33) are rewritten in terms of $w$ and $\Phi$ for this particular panel as shown below:

$$
\begin{align*}
& {\left[\mathrm{D}_{\mathrm{x}} \frac{\partial^{2} \mathrm{w}}{\partial \mathrm{x}^{2}}+\nu \mathrm{D}_{\mathrm{x}}^{\prime} \frac{\partial^{2} \mathrm{w}}{\mathrm{a}^{2} \partial \varphi^{2}}\right]_{\mathrm{x}=0}=0}  \tag{37}\\
& \left\{D_{x} \frac{\partial^{3} w}{\partial x^{3}}+\frac{1}{a^{2}} \alpha \frac{\partial^{3} w}{\partial \mathrm{x} \partial \varphi} 2+\frac{\bar{K}_{x} w}{a \Delta \varphi}\right\}_{\mathrm{x}=0}=\left[\frac{W_{x} \omega^{2}}{\operatorname{ga\Delta \varphi }} \quad w\right]_{\mathrm{X}=0}  \tag{38}\\
& -\left.\frac{\partial^{2} \phi}{\partial \varphi^{2}}\right|_{x=0}=0 \quad \text { or }\left.\quad u\right|_{x=0}=0  \tag{39a,b}\\
& {\left[\frac{\partial^{2} \boldsymbol{\phi}}{\partial \mathrm{X} \partial \varphi}+\frac{\mathrm{D}(1-\nu)}{\mathrm{a}} \frac{\partial^{2} \mathrm{w}}{\partial \mathrm{X} \partial \varphi}\right]_{\mathrm{x}=0}=0}  \tag{40}\\
& {\left[\mathrm{D}_{\varphi} \frac{\partial^{2} \mathrm{x}}{\mathrm{a}^{2} \partial \varphi^{2}}+\nu \mathrm{D}_{\varphi} \cdot \frac{\partial^{2} \mathrm{w}}{\partial \mathrm{x}^{2}}\right]_{\varphi=0}=0}  \tag{41}\\
& \left\{\mathbf{D}_{\varphi} \frac{\partial^{3} \mathbf{w}}{\mathbf{a}^{3} \partial \varphi^{3}}+\frac{1}{\mathbf{a}} \boldsymbol{\beta} \frac{\partial^{3} \mathbf{w}}{\partial \mathbf{x}^{2} \partial \varphi}+\frac{\overline{\mathrm{K}}_{\varphi} w}{\Delta \mathbf{x}}\right\}_{\varphi=0}=\left[\frac{\mathbf{W}_{\varphi} \omega^{2}}{\mathrm{~g} \Delta \mathbf{x}^{\prime}} \mathrm{w}\right]_{\varphi=0}  \tag{42}\\
& \left.\frac{\partial^{2} \phi}{\partial \mathrm{x}^{2}}\right|_{\varphi=0}=0 \quad \text { or }\left.\quad \mathrm{v}\right|_{\varphi=0}=0  \tag{43a,b}\\
& \left.\frac{\partial^{2} \boldsymbol{\phi}}{\partial \mathbf{X} \partial \varphi}\right|_{\varphi=0}=0 \tag{44}
\end{align*}
$$

where $\quad \alpha=\nu \mathrm{D}_{\mathrm{x}}^{\prime}+2(1-\nu) \mathrm{D}$

$$
\beta=\nu \mathrm{D}_{\varphi}{ }^{\prime}+2(1-\nu) \mathrm{D}
$$

Since only the mode shapes which are symmetric with respect to both $\mathrm{x}=\frac{\mathbf{b}}{2}$ and $v=\frac{\varphi_{\mathrm{O}}}{2}$ axes are considered, the following conditions governing $u, v, \phi$ along the axes of symmetry are made use of:

$$
\left.\mathbf{v}\right|_{\varphi=\frac{\varphi_{0}}{2}} ^{2}=\left.0 \quad \frac{\partial \Phi}{\partial \varphi}\right|_{\varphi=\frac{\varphi_{\mathbf{0}}}{2}}=0
$$

and

$$
\left.u\right|_{x=\frac{b}{2}}=\left.0 \quad \frac{\partial \Phi}{\partial x}\right|_{x=\frac{b}{2}}=0
$$

Equation (2) is substituted into (20) which is then integrated from $\varphi=0$ to $\varphi=\frac{{ }^{\circ} \mathrm{o}}{2}$. The resulting condition for $\mathbf{v}=0$ at $\varphi=0$ is:

$$
\begin{align*}
\left.\mathrm{v}\right|_{\substack{\varphi=0 \\
\mathrm{x}=\mathrm{x}^{*}}} & =\int_{\mathrm{o}}^{\varphi_{\mathrm{o}} / 2}\left[\frac{\mathrm{a}}{\mathrm{~K}_{\mathrm{x}} \mathrm{~K}_{\varphi}-\nu^{2} \mathrm{~K}^{2}}\left(\mathrm{~K}_{\mathrm{x}}-\frac{\partial^{2} \Phi}{\partial \mathrm{x}^{2}}-\nu \mathrm{K} \frac{\partial^{2} \phi}{\mathrm{a}^{2} \partial \varphi^{2}}\right)+\mathrm{w}\right]_{\mathrm{x}=\mathrm{x}^{*}} \mathrm{~d} \varphi \\
& =\int_{0}^{\varphi_{0} / 2}\left[\frac{1}{\mathrm{R}} \frac{\mathrm{~K}_{\mathrm{x}}}{\mathrm{~K}} \frac{\partial^{2} \phi}{\partial \mathrm{x}^{2}}+\mathrm{w}\right]_{\mathrm{x}=\mathrm{x}^{*}} \mathrm{~d} \varphi+\frac{\nu}{\mathrm{a}^{2} \mathrm{R}}\left(\frac{\partial \Phi}{\partial \varphi}\right)_{\mathrm{x}=\mathrm{x}^{*}}=0 \tag{46}
\end{align*}
$$

Similar approach gives:

$$
\begin{equation*}
\left.u\right|_{\substack{\mathrm{x}=0 \\ \varphi=\varphi^{*}}}=\int_{0}^{\mathrm{b} / 2}\left[\frac{1}{\mathrm{a}^{2} \mathrm{R}} \frac{\mathrm{~K}_{\varphi}}{\mathrm{K}}-\frac{\partial^{2} \phi}{\partial \varphi^{2}}\right]_{\varphi=\varphi^{*}} \mathrm{dx}+\frac{\nu}{\mathrm{R}}\left(\frac{\partial \phi}{\partial \mathrm{x}}\right)_{\substack{\mathrm{x}=0 \\ \varphi=\varphi^{*}}}=0 \tag{47}
\end{equation*}
$$

The finite difference operators corresponding to equations (37) - (47) and the stations on which each operator will be applied are given below. The inertia terms are excluded. $\left.\frac{(\Delta \mathrm{x})^{2}}{\mathrm{D}_{\mathrm{x}}} \mathrm{M}_{\mathrm{x}}\right|_{\mathrm{x}=0}=0$. Apply to points along the edge $\mathrm{x}=0$ (Sta. 1-6).

$$
\begin{gather*}
\nu \frac{D_{x}^{\prime}}{\mathrm{D}_{\mathrm{x}}} \frac{1}{\lambda^{2}} \\
-2\left(1+\frac{\mathrm{D}_{\mathrm{x}}^{\prime}}{\mathrm{D}_{\mathrm{x}}^{\prime}} \frac{1}{\lambda^{2}}\right)  \tag{37ㄱ}\\
\left(\mathrm{w}_{\mathrm{i}}\right)
\end{gather*}
$$

$\left.\frac{2(\Delta x)^{3}}{D_{x}} V_{x}\right|_{x=0}=\frac{2(\Delta x)^{3}}{D_{x}} \quad \frac{\bar{K}_{x}}{a \Delta \varphi}$ w. Apply to points along the edge $\mathrm{x}=0$ (Sta. 1-6). Use $\overline{\mathrm{K}}_{\mathrm{X}}=0$ at points where there is no spring support.

$$
\begin{align*}
& -\frac{\alpha}{\mathrm{D}_{\mathrm{x}} \lambda^{2}} \quad 0 \quad \frac{\alpha}{\mathrm{D}_{\mathrm{x}} \lambda^{2}} \\
& \begin{array}{ccc}
-1 & 2\left(1+\frac{\alpha}{D_{x} \lambda^{2}}\right)-\frac{2 \bar{K}_{x}(a \Delta \varphi)^{2}}{D_{x} \lambda^{3}} & -2\left(1+\frac{\alpha}{D_{x} \lambda^{2}}\right) \\
\left(\mathrm{w}_{\mathrm{i}}\right) & 1 \\
-\frac{\alpha}{D_{x} \lambda^{2}} & 0 & \frac{\alpha}{D_{x} \lambda^{2}}
\end{array}
\end{align*}
$$

$\left.(a \Delta \varphi)^{2} N_{x}\right|_{x=0}=0$. Apply to points along the edge $\mathrm{x}=0$ without spring support.

| 1 |
| :---: |
| -2 |
| $\left(\Phi_{\mathbf{i}}\right)$ |
| 1 |

$\left.\frac{2 \mathrm{~K}(\mathrm{a} \Delta \varphi) \lambda \mathrm{R}}{\mathrm{K}_{\varphi}} \mathrm{u}\right|_{\mathrm{x}=0}=0$. Apply to points along the edge $\mathrm{x}=0$ with spring supports. This condition is not used for the panel under consideration because no spring support exists along the edge $\mathrm{x}=0$.

$\left.(4 \Delta \mathrm{xa} \Delta \varphi) \mathrm{S}_{\mathrm{x}}\right|_{\mathrm{x}=0}=0$. Apply to all points along the edge $\mathrm{x}=0$ except the corner point. For points not on the axis of symmetry (Sta. 2-5)

| -1 | 0 | 1 |
| :---: | :---: | :---: |
| 0 | 0 | 0 |
|  | $\left(\mathrm{w}_{\mathrm{i}}\right)$ |  |
| 1 | 0 | -1 |


| $-\frac{D(1-\nu)}{a}$ | 0 | $\frac{D(1-\nu)}{a}$ |
| :---: | :---: | :---: |
| 0 | 0 | 0 |
|  | $\left(\Phi_{i}\right)$ |  |
| $\frac{D(1-\nu)}{a}$ | 0 | $-\frac{D(1-\nu)}{a}$ |

(40'a)

For points on the axis of symmetry (Sta. 6)

| -1 | 0 | 1 |
| :---: | :---: | :---: |
|  |  |  |
| 1 | 0 | -1 |
|  | $\left(w_{\mathrm{i}}\right)$ |  |


| $-\frac{D(1-\nu)}{a}$ | 0 | $\frac{D(1-\nu)}{a}$ |
| :---: | :---: | :---: |
| $\frac{D(1-\nu)}{a}$ | 0 | $-\frac{D(1-\nu)}{a}$ |

(40'b)
$\left.\frac{(\mathrm{a} \Delta \varphi)^{2}}{\mathrm{D}_{\varphi}} \mathrm{M}_{\varphi}\right|_{\varphi=0}=0$. Apply to points along the edge $\varphi=0$ (Sta. 1, 7, 13, 19).
$\left.\begin{array}{|cc|}\hline \nu \lambda^{2} \frac{\mathrm{D}_{\varphi^{\prime}}}{\mathrm{D}_{\varphi}} & -2\left[1+\nu \lambda^{2} \frac{\mathrm{D}_{\varphi}{ }^{\prime}}{\mathrm{D}_{\varphi}}\right] \\ \left(\mathrm{w}_{\mathrm{i}}\right) \\ 1\end{array} \quad{ }^{2} \lambda^{2} \frac{\mathrm{D}_{\varphi^{\prime}}}{\mathrm{D}_{\varphi}}\right]$
$\left.2 \frac{(\mathrm{a} \Delta \varphi)^{3}}{\mathbf{D}_{\varphi}} \mathrm{V}_{\varphi}\right|_{\varphi=0}=2 \frac{(\mathrm{a} \Delta \varphi)^{3}}{\mathbf{D}_{\varphi}} \frac{\overline{\mathrm{K}}}{\Delta \mathrm{x}} \mathrm{w}$ : Apply to points along the edge $\varphi=0$ (Sta. 1, 7, 13, 19). Use $\overline{\mathrm{K}}_{\varphi}=0$ for stations with no spring support.

| $\frac{\beta \lambda^{2}}{\mathrm{D}_{\varphi}}$ | $-2\left(1+\frac{\beta \lambda^{2}}{\mathrm{D}_{\varphi}}\right)$ | $\frac{\beta \lambda^{2}}{\mathrm{D}_{\varphi}}$ |
| :---: | :---: | :---: |
| 0 | $\frac{-2 \lambda \overline{\mathrm{~K}}_{\varphi}(\mathrm{a} \Delta \varphi)^{2}}{\mathrm{D}_{\varphi}}$ | 0 |
|  | $\left(\mathrm{w}_{\mathrm{i}}\right)$ |  |
| $-\frac{\beta \lambda^{2}}{\mathrm{D}_{\varphi}}$ | $2\left(1+\frac{\beta \lambda^{2}}{\mathrm{D}_{\varphi}}\right)$ | $-\frac{\beta \lambda^{2}}{\mathrm{D}_{\varphi}}$ |
|  | -1 |  |

$\left.(\Delta \mathrm{x})^{2} \mathrm{~N}_{\varphi}\right|_{\varphi=0}=0$. Apply to points along the edge $\varphi=0$ without a spring support.

| 1 | -2 | 1 |
| :--- | :--- | :--- |
|  | $\left(\Phi_{\mathrm{i}}\right)$ |  |

(43')
$\left.2 \frac{\mathrm{~K}}{\mathrm{~K}_{\mathrm{x}}} \operatorname{aR}(\Delta \mathrm{x}) \mathrm{v}\right|_{\varphi=0}=0$. Apply to points along the edge $\varphi=0$ with spring supports.

 point not on axis of symmetry (Sta. 7, 13)

| -1 | 0 | 1 |
| :---: | :---: | :---: |
| 0 | 0 | 0 |
|  | $\left(\boldsymbol{\Phi}_{\mathbf{i}}\right)$ |  |
| 1 | 0 | -1 |

For point on axis of symmetry (Sta. 19)

| -1 | 1 |
| :---: | :---: |
| 0 | 0 |
| $\left(\boldsymbol{\phi}_{\mathrm{i}}\right)$ |  |
| 1 | -1 |

$\left.(4 \Delta \mathrm{xa} \Delta \varphi) 2 \mathrm{M}_{\mathrm{x} \varphi}\right|_{\substack{\mathrm{x}=0 \\ \varphi=0}}=-(4 \Delta \mathrm{x} \Delta \varphi) \overline{\mathrm{K}}_{1} \mathbf{w}$. Apply to the corner point $\mathrm{x}=0, \varphi=0$ (Sta. 1)

| -1 | 0 | 1 |
| :---: | :---: | :---: |
| 0 | $\frac{2 \overline{\mathrm{~K}}_{1}(\mathrm{a} \Delta \varphi)^{2}}{\mathrm{D} \mathrm{\lambda}(1-\nu)}$ | 0 |
|  | $\left(\mathrm{w}_{\mathrm{i}}\right)$ |  |
| 1 | 0 | -1 |

Furthermore, $\Phi$ is an auxiliary stress function whose second derivatives give corresponding stress values. The value of $\Phi$ may be arbitrarily assigned at a certain station without influencing the stress distribution. The condition is represented as:

$$
\begin{equation*}
\phi_{\mathbf{c}}=0 \tag{48}
\end{equation*}
$$

1
${ }_{\left(\boldsymbol{\phi}_{\mathrm{i}}\right)}$
The above condition is applied to station (24).

## 4. Finite Difference Expressions in Matrix Form.

Assembling of all of the equations described previously yields a set of homogeneous equations of the following form:

$$
[\mathrm{A}] \quad\left\{\begin{array}{c}
\left|\mathrm{w}_{\mathrm{a}}\right|  \tag{49}\\
\left|\mathrm{w}_{\mathrm{b}}\right| \\
|\phi|
\end{array}\right\} \quad-\Omega \quad\left[\begin{array}{ccc}
{\left[\mathrm{M}_{1}\right]} & 0 & 0 \\
{\left[\mathrm{M}_{2}\right]} & 0 & 0 \\
0 & 0 & 0
\end{array}\right] \quad\left\{\begin{array}{l}
\left|\mathrm{w}_{\mathrm{a}}\right| \\
\left|\mathrm{w}_{\mathrm{b}}\right| \\
|\boldsymbol{\rho}|
\end{array}\right\}=0
$$

where $|A|=$ the coefficient matrix.
$\left|\mathrm{w}_{\mathrm{a}}\right|=$ the normal deflection matrix for the stations on the quarter panel.
$\left|w_{b}\right|=$ the normal deflection matrix for the exterior stations.
$\left|\mathrm{M}_{1}\right|$ = the interior mass matrix representing the mass of the shell proper.
$\left|\mathrm{M}_{2}\right|=$ the boundary mass matrix representing masses attached at the edges of the panel.

Consider a quarter panel covered by $m$ equidistant grid points along the $x$-direction and n equidistant grid points along the $\varphi$-direction. A total of ( mxn ) grid points are located on the panel. A stiffener with sectional area $A_{s x}$ may be located along any grid line parallel to the x -axis. A stiffener with sectional area $\mathrm{A}_{\mathrm{s} \varphi}$ may be located along the circular grid line parallel to the $\varphi$-direction. The number of stiffeners are not limited. Using the above structural panel, and referring to Equation (49), it may be shown that there are (mn) elements for $\left\{w_{a}\right\}$. There are $(2 m+2 n+1)$ elements for $\left|w_{b}\right|$ corresponding to the exterior stations extending two grid pitches from the panel edges (Figure 13). The auxiliary function $\phi$ is accounted for extending one grid pitch from the edges. In this manner, $[(m+1)(n+1)-1]$ elements are established for $|\boldsymbol{\phi}|$. Adding the numbers of the elements listed above, a total of $[2 m n+3(m+n)+1]$ is counted, which is the order of the square coefficient matrix $[A]$. The details of organizing the $[A]$ matrix is illustrated in the diagram shown on the next page.

In the matrix shown on page 40, the finite difference equations used to generate the rows of the coefficient matrix are listed which are marked with the equation numbers described in the text. The following procedures may be used to generate the basic pattern. The pattern is used as the input to the computer program which computes the elements of A.
a. Draw the grid pattern and the station numbers on a transparent paper.
b. Simplify the finite difference operators by assigning an index number to each coefficient. Make a list of the index numbers and the corresponding coefficients. A typical index table used to generate the $[A]$ matrix for the localized I.U. panel is shown in Table 2 at the end of Section B.
c. Rewrite the finite difference operators by placing corresponding index numbers in position with the grid pattern as shown on the transparent paper.
d. Place the transparent paper over a finite difference operator and keep the center coefficient coinciding with the station for which that operator will be read.
e. Locate the row number as defined in the $[\mathrm{A}]$ matrix diagram shown on page 40. Read each station number on the transparent paper and the corresponding index number on the bottom sheet representing the differential operators of an equation. Enter the index number as an element of $|\mathrm{A}|$ whose column number corresponds to the station number read. Repeat the operation for $w$ and $\Phi$. If a station number appears more than once for a certain operator, assign a new index number to the sum of the coefficients referring to that station. Enter the new index number as an element of $\{A \mid$ which is also included in Table 2.

f. Repeat procedures (d) and (e) until the matrix is filled with the index numbers. In the input format to the computer program, only non-zero elements are included where the index numbers are identified with proper row and column numbers.

| INDEX NUMBERS | MATRIX ELEMENTS | INDEX NUMBERS | MATRLX <br> ELEMENTS | INDEX NUMBERS | MATRIX ELEMENTS | $\begin{aligned} & \text { INDEX } \\ & \text { NUMBERS } \end{aligned}$ | MATRIX ELEMENTS |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| (1) | 1 | (19) | $\nu \lambda^{2} \mathrm{D}_{\varphi}{ }^{\prime} / \mathrm{D}_{\varphi}$ | (36) | -(12) | (57) | 2x(22) |
| (2) | $-4 \lambda^{2}\left(\mathrm{D}_{\mathrm{x}} \lambda^{2}+\mathrm{H}\right) / \mathrm{D}$ | (20) | $-2\left[1+\left(\nu \lambda^{2} \mathrm{D}_{\varphi}{ }^{\prime} / \mathrm{D}_{\varphi}\right)\right]$ | (37) | -(17) | (58) | 2x(24) |
| (3) | $\mathrm{D}_{\mathrm{x}} \lambda^{4} / \mathrm{D}$ | (21) | $-2 \lambda^{2}(\mathrm{a} \Delta \varphi)^{2} \mathrm{R}$ | (38) | -(29) | (59) | 2x(25) |
|  |  |  |  | (39) | 2x(2) | (60) | $2 x(26)$ |
| (4) | $-4\left(H \lambda^{2}+D_{\varphi}\right) / \mathrm{D}$ | (22) | $\lambda^{2}(\mathrm{a} \Delta \varphi)^{2} \mathrm{R}$ | (40) | 2x(3) | (61) | $2 \times(27)$ |
| (5) | $2 \mathrm{H} \lambda^{2} / \mathrm{D}$ | (23) | $\left(6 \mathrm{~K}_{\mathrm{x}} \lambda^{4} / \mathrm{K}\right)+8 \mathrm{P} \lambda^{2}+\left(6 \mathrm{~K}_{\varphi} / \mathrm{K}\right)$ | (41) | 2x(4) | (62) | 2x(28) |
| (6) | $\mathrm{D}_{\varphi} / \mathrm{D}$ | (24) | $-4 \lambda^{2}\left[\left(\mathrm{~K}_{\mathrm{x}} \lambda^{2} / \mathrm{K}\right)+\mathrm{P}\right]$ | (42) | 2x(5) | (63) | 2x(1) |
| (7) | $2 \lambda^{2}(\mathrm{a} \Delta \varphi)^{2} / \mathrm{aD}$ | (25) | $\mathrm{K}_{\mathrm{x}} \lambda^{4} / \mathrm{K}$ | (43) | 2x(6) | (64) | -(30) |
|  |  |  |  | (44) | $(3)+(8)$ | (65) | $2 x(9)$ |
| (8) | $\left(6 D_{x} \lambda^{4}+8 H \lambda^{2}+6 D_{\varphi}\right) / D$ | (26) | $-4\left[\left(\mathrm{~K}_{\varphi} / \mathrm{K}\right)+\mathrm{P} \lambda^{2}\right]$ | (45) | $(6)+(8)$ | (66) | $-(1)$ |
| (9) | $-\lambda^{2}(\mathrm{a} \Delta \varphi)^{2} / \mathrm{aD}$ | (27) | $2 \mathrm{P} \lambda^{2}$ | (46) | (3) $+(6)+(8)$ | (67) | -(11) |
| (10) | $-2 \overline{\mathrm{~K}}_{\mathrm{x}}(\mathrm{a} \Delta \varphi)^{2} / \mathrm{D}_{\mathrm{x}} \lambda^{3}$ | (28) | $\mathrm{K}_{\varphi} / \mathrm{K}$ | (47) | 2x(11) | (68) | $2 \overline{\mathrm{~K}}_{1}(\mathrm{a} \Delta \varphi)^{2} / \mathrm{D} \lambda(1-\nu)$ |
| (11) | $a / \lambda^{2} \mathrm{D}_{\mathrm{x}}$ | (29) | -2 | (48) | -2x(11) | (69) | -(18) |
|  |  | (20) |  | (49) | 4x(5) | (70) | 2x(19) |
| (12) | $-2\left[1+\left(\alpha / \lambda^{2} \mathrm{D}_{\mathrm{x}}\right)\right]$ | (30) | $\mathrm{D}(1-\nu) / \mathrm{a}$ | (50) | 4x(27) | (71) | $-2 \times(29)$ |
| (14) | $\nu \mathrm{D}_{\mathrm{x}}{ }^{\prime} / \lambda^{2} \mathrm{D}_{\mathrm{x}}$ | (31) | $\mathrm{K}(\mathrm{a} \Delta \varphi)^{2} \mathrm{R} / \mathrm{K}_{\mathrm{x}} \lambda^{2}$ | (51) | 2x(14) |  |  |
| (15) | $-2\left[1+\left(\nu \mathrm{D}_{\mathrm{x}}^{\prime} / \lambda^{2} \mathrm{D}_{\mathrm{x}}\right)\right]$ | (32) | - 4 | (52) | $(25)+(23)$ |  |  |
| (16) | $-2 \lambda \overline{\mathrm{~K}}_{\mathcal{U}}(\mathrm{a} \Delta)^{2} / \mathrm{D}_{\varphi}$ | (33) | $-\nu \mathrm{K} / \lambda^{2} \mathrm{~K}_{\mathrm{x}}$ | (53) | (23) $+(28)$ |  |  |
| (17) | $\beta \lambda^{2} / D_{\varphi}$ | (34) | $-4+\left(\nu \mathrm{K} / \lambda^{2} \mathrm{~K}_{\mathrm{x}}\right)$ | $(54)$ $(55)$ | $\begin{aligned} & 2 x(17) \\ & -2 x(17) \end{aligned}$ |  |  |
| (18) | $-2\left[1+\left(\beta \lambda^{2} / \mathrm{D}_{\varphi}\right)\right]$ | (35) | $2 \mathrm{~K}(\mathrm{a} \Delta \varphi)^{2} \mathrm{R} / \mathrm{K}_{\mathrm{x}} \lambda^{2}$ | (56) | $(23)+(25)+(28)$ |  |  |

TABLE 2. TYPICAL INDEX TABLE OF THE FINITE DIFFERENCE PROGRAM

The interior mass matrix $\left|M_{1}\right|$ is a square matrix of order mn. The diagonal terms are generated as follows:

$$
\begin{align*}
& \left.\left(\mathrm{M}_{1}\right)_{\mathrm{ii}}=1+\frac{\mathrm{W}_{\mathrm{i}}+\rho \mathrm{g}\left[\mathrm{~A}_{\mathrm{s} \varphi_{\mathrm{i}}} \mathrm{a} \Delta \varphi+\mathrm{A}_{\mathrm{sx}}^{\mathrm{i}}\right.}{} \Delta \mathrm{x}\right]_{\text {for interior points }}  \tag{50}\\
& \left(\mathrm{M}_{1}\right)_{\mathrm{ii}}=1 \quad \begin{array}{l}
\text { for points on the boundaries }
\end{array} \tag{51}
\end{align*}
$$

where the subscript $i$ indicates the station number and $W_{i}$ is the concentrated weight attached to that station.

The off diagonal terms of $M_{1}$ matrix appear only for a component connected to the quarter panel through more than one attachment point. All values of $\left(M_{1}\right)_{i, j}, i \neq j$ must be precalculated as input to the computer program. Considering the loaded I. U. segment (Figure 14) as an example, the inertia forces acting on the attachment points $A$ and $B$ due to


FIGURE 14. LOADED INSTRUMENT UNIT SEGMENT
a component with mass $M_{o}$ and moment of inertia $\theta$ with respect to the axis through centroid in x -direction may be expressed in the following forms.

$$
\begin{align*}
\mathrm{F}_{\mathrm{A}} & =\frac{M_{0}}{4} \omega^{2}\left(\frac{{ }^{w} A+{ }^{w_{B}}}{2}\right)-\frac{\theta}{2 d} \omega^{2} \frac{{ }^{w_{B}}-w_{A}}{d} \\
& =\frac{\omega^{2}}{8}\left[\left(M_{0}+\frac{4 \theta}{d^{2}}\right) w_{A}+\left(M_{0}-\frac{4 \theta}{d^{2}}\right) w_{B}\right]  \tag{52}\\
F_{B} & =\frac{\omega^{2}}{8}\left[\left(M_{0}-\frac{4 \theta}{d^{2}}\right) w_{A}+\left(M_{0}+\frac{4 \theta}{d^{2}}\right) w_{B}\right] \tag{53}
\end{align*}
$$

where $d$ is the distance between $A$ and $B$ in circumferential direction. If each attachment point coincides with a grid point, it follows that

$$
\left.\begin{array}{rl}
\Delta\left(M_{1}\right)_{A, A}=\Delta\left(M_{1}\right)_{B, B} & =\frac{1}{8 \overline{\mathrm{~m}} \Delta \mathrm{xa} \Delta \varphi}\left(M_{0}+\frac{4 \theta}{d^{2}}\right) \\
\left(M_{1}\right)_{A, B} & =\left(M_{1}\right)_{B, A} \tag{55}
\end{array}\right) \frac{1}{8 \overline{\mathrm{~m}} \Delta \mathrm{xa} \Delta \varphi}\left(\mathrm{M}_{0}-\frac{4 \theta}{\mathrm{~d}^{2}}\right) .
$$

The values computed from Equation (54) represent the concentrated weights attached to stations A and B. The values are to be added to the corresponding elements of Equation (50).

If the attachment points are off the grid points, similar approach can be used to evaluate the corresponding elements of $\mathrm{M}_{1}$. Numerical example is given later in Appendix II.

The boundary mass matrix $\left[\mathrm{M}_{2} \mid\right.$ is a rectangular matrix of size $(2 \mathrm{~m}+2 \mathrm{n}+1) \mathrm{x}(\mathrm{mn})$. It is composed of the inertia terms for the masses attached to the edge of the panel which is reflected in the transverse shear equations. Referring to the grid pattern and number systems of Figure 13, the elements of $\mathrm{M}_{2}$ are defined as follows:

$$
\begin{align*}
& \left(M_{2}\right)_{i, j}=0 \text { except }  \tag{56}\\
& \left(M_{2}\right)_{i, i}=-\frac{2 D\left|(1 / 2) \rho g\left(A_{s \varphi_{i}} a \Delta \varphi+A_{s_{x}} \Delta x\right)+W_{x_{i}}\right|}{D_{x_{i}} \lambda^{3}(a \Delta \varphi)^{2} \bar{m} g} \quad i=2,3, \ldots n  \tag{57}\\
& \left(\mathrm{M}_{2}\right)_{\mathrm{n}+1,1}=\frac{2\left\{\frac{\rho \mathrm{~g}}{4}\left|\mathrm{~A}_{\mathrm{s} \varphi_{1}} \mathrm{a} \Delta \varphi+\mathrm{A}_{\mathrm{Sx}_{1}} \Delta \mathrm{x}\right|+\mathrm{W}_{\mathrm{x}_{1} \mid}\right.}{\lambda(1-v)(\mathrm{a} \Delta \varphi)^{2} \overline{\mathrm{~m} g}} \tag{58}
\end{align*}
$$

## C. THE EIGENVALUE-EIGENVECTOR PROBLEM

In the previous section, a procedure is described to generate the index numbers representing the elements of $[A],\left[M_{1}\right]$ and $\left[M_{2}\right]$. The index number pattern is used as the input to the subject computer program. Based on the input, the computer program generates the matrices according to the local shell and stiffener data which is also part of the input information. The final matrix equation generated by the computer is of the following form:

$$
\left[\begin{array}{lll}
\mathrm{A}_{11} & \mathrm{~A}_{12} & \mathrm{~A}_{13}  \tag{60}\\
\mathrm{~A}_{21} & \mathrm{~A}_{22} & 0 \\
\mathrm{~A}_{31} & \mathrm{~A}_{32} & \mathrm{~A}_{33}
\end{array}\right] \quad\left\{\begin{array}{c}
\mathrm{w}_{\mathrm{a}} \\
\mathrm{w}_{\mathrm{b}} \\
\Phi
\end{array}\right\}-\Omega\left\{\begin{array}{cc}
\mathrm{M}_{1} \mathrm{w}_{\mathrm{a}} \\
\mathrm{M}_{2} \mathrm{w}_{\mathrm{a}} \\
0
\end{array}\right\}=0
$$

$$
\begin{align*}
\mathrm{A}_{11} \mathrm{w}_{\mathrm{a}}+\mathrm{A}_{12} \mathrm{w}_{\mathrm{b}}+\mathrm{A}_{13} \Phi-\Omega \mathrm{M}_{1} \mathrm{w}_{\mathrm{a}} & =0  \tag{61}\\
\mathrm{~A}_{21} \mathrm{w}_{\mathrm{a}}+\mathrm{A}_{22} \mathrm{w}_{\mathrm{b}}-\Omega \mathrm{M}_{2} \mathrm{w}_{\mathrm{a}} & =0  \tag{62}\\
\mathrm{~A}_{31} \mathrm{w}_{\mathrm{a}}+\mathrm{A}_{32} \mathrm{w}_{\mathrm{b}}+\mathrm{A}_{33} \Phi & =0 \tag{63}
\end{align*}
$$

where

$$
\begin{aligned}
& A_{11}=\text { square matrix (mn) } x(m n) \\
& \mathrm{A}_{12}=\text { rectangular matrix }(\mathrm{mn}) \times(2 m+2 n+1) \\
& A_{13}=\operatorname{rectangular} \text { matrix }(m n) x(m n+m+n) \\
& A_{21}=\text { rectangular matrix }(2 m+2 n+1) x(m n) \\
& A_{22}=\text { square matrix }(2 m+2 n+1) x(2 m+2 n+1) \\
& A_{31}=\text { rectangular matrix }(m n+m+n) x(m n) \\
& \mathrm{A}_{32}=\text { rectangular matrix }(\mathrm{mn}+\mathrm{m}+\mathrm{n}) \times(2 \mathrm{~m}+2 \mathrm{n}+1) \\
& \mathrm{A}_{33}=\text { square matrix }(\mathrm{mn}+\mathrm{m}+\mathrm{n}) \times(\mathrm{mn}+\mathrm{m}+\mathrm{n})
\end{aligned}
$$

Equation (60) represents an eigenvalue-eigenvector problem. $\Omega$ is the eigenvalue and $\mathrm{w}_{\mathrm{a}}$ the corresponding eigenvectors. Specifically, matrix Equation (61) represents the dynamic equations of equilibrium for the interior and boundary points of the system. Equation (62) represents the boundary conditions involving bending moments and transverse shears. Equation (63) represents the compatibility equations and the boundary conditions involving the inplane forces. The column of $w_{b}$ pertains to the deflections at the exterior points of the system. $\Phi$ represents the stress function at all stations concerned.

Equations (62) and (63) are used to eliminate $\mathrm{w}_{\mathrm{b}}$ and $\boldsymbol{\phi}$ from Equation (61). The end result is a matrix equation suitable for eigenvalue-eigenvector determination by the "MITERS" routine which forms part of the computer program.

From Equation (62),

$$
\begin{equation*}
\mathrm{w}_{\mathrm{b}}=\mathrm{A}_{22}{ }^{-1}\left(\Omega \mathrm{M}_{2}-\mathrm{A}_{21}\right) \mathrm{w}_{\mathrm{a}} \tag{64}
\end{equation*}
$$

From Equation (63),

$$
\begin{equation*}
\Phi=-\mathrm{A}_{33}{ }^{-1}\left(\mathrm{~A}_{31} \mathrm{w}_{\mathrm{a}}+\mathrm{A}_{32} \mathrm{w}_{\mathrm{b}}\right) \tag{65}
\end{equation*}
$$

Substituting equation (64) into (65) yields:

$$
\begin{equation*}
\Phi=-\mathrm{A}_{33}{ }^{-1}\left[\mathrm{~A}_{31}+\mathrm{A}_{32} \mathrm{~A}_{22}{ }^{-1}\left(\Omega \mathrm{M}_{2}-\mathrm{A}_{21}\right)\right] \mathrm{w}_{\mathrm{a}} \tag{66}
\end{equation*}
$$

Substituting equations (64) and (66) into (61), the standard format suitable for the "MITERS" routine is obtained:

$$
\begin{equation*}
[\overline{\mathrm{A}}]\left|\mathrm{w}_{\mathrm{a}}\right|-\frac{1}{\bar{\Omega}}\left|\mathrm{w}_{\mathrm{a}}\right|=0 \tag{67}
\end{equation*}
$$

where

$$
\begin{gather*}
{[\bar{A}]=\left[A_{11}-A_{12} A_{22}{ }^{-1} A_{21}-A_{13} A_{33}{ }^{-1}\left(A_{31}-A_{32} A_{22}{ }^{-1} A_{21}\right)\right]^{-1}} \\
{\left[M_{1}-\left(A_{12}-A_{13} A_{33}{ }^{-1} A_{32}\right) A_{22}{ }^{-1} M_{2}\right]} \tag{68}
\end{gather*}
$$

The computer program yields the eigenvalues and the eigenvectors starting with the largest. value of $\frac{1}{\Omega}$. In other words, the lowest frequency and the corresponding modal data are generated first, to be followed by modal data corresponding to gradually increasing frequencies. After one eigenvalue and the corresponding mode shape are obtained, Equation (66) is used to compute the corresponding $\phi$ values as partial output of the computer program.

The above writeup describes the general scheme used in mechanizing the computer program. Results obtained so far have been promising as compared to the test data. The complete program, the work instructions and the sample data are given in Appendix II.

## D. SEGMENTED INSTRUMENT UNIT DATA

Four (4) sets of analytical data were acquired from the finite difference computer program for the unloaded and loaded Instrument Unit segments. The computed natural frequencies and corresponding mode shapes of the first five modes of the unloaded Instrument Unit scale model segment supported by stiff springs (supporting configuration II) are plotted in Figure 4 and Figures 15 through 18. Figures 19 through 23 give the analytical data of the unloaded I. U. scale model supported by soft springs (supporting configuration I). Modal data of loaded scale model supported by stiff springs, and loaded full scale Instrument Unit segment supported by soft springs are shown in Figures $24-28$ and Figures $29-33$ respectively. Figures 34 and 35 are the plots of in-plane stress distribution and magnitude and direction of the top surface principal stresses corresponding to the mode given in Figure 4. The in-plane stress distribution corresponding to the first mode of loaded full scale I.U. segment is shown in Figure 36.

Testing was undertaken on the segmented I. U. scale model supported by stiff springs. Test data compared favorably with the analytical data for the first two modes in both unloaded and loaded configurations. The experimental frequencies and corresponding nodal lines are shown in the bottom portion of figures 4,15 , and 24 . The comparison between the theoretical predicted frequencies and the test frequencies corresponding to the minimum impedance points for the loaded I. U. segment is given in Figure 7.

(a) ANALYTICAL DATA $\mathrm{f}=289.7 \mathrm{CPS}$

(b) EXPERIMENTAL DATA $\mathrm{f}=264 \mathrm{CPS}$

FIGURE 15. SECOND MODAL DATA OF SEGMENTED INSTRUMENT UNIT SCALE MODEL, SUPPORT CONFIGURATION (II)


FIGURE 16. THIRD MODAL DATA OF SEGMENTED INSTRUMENT UNIT SCALE MODEL, SUPPORT CONFIGURATION (II)


FIGURE 17. FOURTH MODAL DATA OF SEGMENTED INSTRUMENT UNIT SCALE MODEL, SUPPORT CONFIGURATION (II)


FIGURE 18. FIFTH MODAL DATA OF SEGMENTED INSTRUMENT UNIT SCALE MODEL, SUPPORT CONFIGURATION (II)


FIGURE 19. FIRST MODAL DATA OF SEGMENTED INSTRUMENT UNIT SCALE MODEL, SUPPORT CONFIGURATION (I)


FIGURE 20. SECOND MODAL DATA OF SEGMENTED INSTRUMENT UNIT SCALE MODEL, SUPPORT CONFIGURATION (I)


FIGURE 21. THIRD MODAL DATA OF SEGMENTED INSTRUMENT UNIT SCALE MODEL, SUPPORT CONFIGURATION (I)


FIGURE 22. FOURTH MODAL DATA OF SEGMENTED INSTRUMENT UNIT SCALE MODEL, SUPPORT CONFIGURATION (I)


FIGURE 23. FIFTH MODAL DATA OF SEGMENTED INSTRUMENT UNIT SCALE MODEL, SUPPORT CONFIGURATION (I)

(a) ANALYTICAL DATA $\mathrm{f}=172.8 \mathrm{CPS}$

(b) EXPERIMENTAL DATA $\mathrm{f}=172 \mathrm{CPS}$

FIGURE 24. FIRST MODAL DATA OF SEGMENTED INSTRUMENT UNIT SCALE MODEL WITH SIMULATED COMPONENTS ATTACHED, SUPPORT CONFIGURATION (II)


FIGURE 25. SECOND MODAL DATA OF SEGMENTED INSTRUMENT UNIT SCALE MODEL WITH SIMULATED COMPONENTS ATTACHED, SUPPORT CONFIGURATION (II)


FIGURE 26. THIRD MODAL DATA OF SEGMENTED INSTRUMENT UNIT SCALE MODEL WITH SIMULATED COMPONENTS ATTACHED, SUPPORT CONFIGURATION (II)


FIGURE 27. FOURTH MODAL DATA OF SEGMENTED INSTRUMENT UNIT SCALE MODEL WITH SIMULATED COMPONENTS ATTACHED, SUPPORT CONFIGURATION (II)


FIGURE 28. FIFTH MODAL DATA OF SEGMENTED INSTRUMENT UNIT SCALE MODFL WITH SIMULATED COMPONENTS ATTACHED, SUPPORT CONFIGURATION (II)


FIGURE 29. FIRST MODAL DATA OF INSTRUMENT UNIT SEGMENT WITH COMPONENTS ATTACHED, SUPPORT CONFIGURATION (I)


FIGURE 30. SECOND MODAL DATA OF INSTRUMENT UNIT SEGMENT WITH COMPONENTS ATTACHED, SUPPORT CONFIGURATION (I)


FIGURE 31. THIRD MODAL DATA OF INSTRUMENT UNIT SEGMENT WITH COMPONENTS ATTACHED. SUPPORT CONFIGURATION (I)


FIGURE 32. FOURTH MODAL DATA OF INSTRUMENT UNIT SEGMENT WITH COMPONENTS ATTACHED, SUPPORT CONFIGURATION (I)


FIGURE 33. FIFTH MODAL DATA OF INSTRUMENT UNIT SEGMENT WITH COMPONENTS ATTACHED, SUPPORT CONFIGURATION (I)


FIGURE 34. IN-PLANE STRESS DISTRIBUTION CORRESPONDING TO THE FIRST MODE OF SEGMENTED INSTRUMENT UNIT SCALE MODEL, SUPPORT CONFIGURATION (II)


FIGURE 35. MAGNITUDE AND DIRECTION OF TOP SURFACE PRINCIPAL STRESSES CORRESPONDING TO THE FIRST MODE OF SEGMENTED INSTRUMENT

UNIT SCALE MODEL. SUPPORT CONFIGURATION (II)


## APPENDIX II

## USER INFORMATION OF THE FINITE DIFFERENCE COMPUTER PROGRAM

The program gives the analytical predictions of natural vibration modes and frequencies of a curved panel or a curved sandwiched panel with arbitrary boundary and supporting conditions. Coefficient matrix derived from the finite difference expression for equilibrium equations, compatibility equations, and boundary conditions is organized and read into the program as input data.

The complete program consists of three CHAIN's. The first chain is used to compute all necessary data and set up a table called "E table" - that has all the elements needed in the coefficient matrix $A$. The second chain is mechanized to transfer these elements to proper locations in the matrix and to reduce the matrix into an eigenmatrix. The third chain is used to compute the eigenvalues and eigenvectors, and the associated stress function.

## INPUT DATA

The program is set up for the grid pattern shown in Figure 13.

1. Solid Shell

| Symbols used <br> in Analysis | Fortran <br> Coding | Definitions |
| :---: | :---: | :--- |
| a | A | Radius of curvature (in) |
| b | B | Length of a panel in x-direction (in) |
| $\varphi_{\mathrm{o}}$ | PHIO | Circumferential angle in $\varphi$-direction (rad) |
| $\nu$ | PNU | Poisson's ratio |
| E | E | Young's modulus of elasticity (lb/in ${ }^{2}$ ) |
| $\rho$ | RHO | Mass density per unit volume (lb- $\sec ^{2} / \mathrm{in}^{4}$ ) |
| h | H | Shell thickness (in) |
| n | N | Number of grid points in $\varphi$-direction $=6$ |
| m | M | Number of grid points in x-direction $=4$ |
|  | NMODE | Number of modes sought |
|  | IOPT | $=1$ for solid shell |


| Symbols used in Analysis | Fortran Coding | Definitions |
| :---: | :---: | :---: |
| $\mathbf{z}_{\mathbf{S X}}, \mathbf{z}_{\mathbf{s} \varphi}$ | SZX, SZP | Distances from c.g. of the stiffeners to the middle surface of the panel in x-direction and $\varphi$-direction, (in.) numbers of values to be read in are controlled by an index, NEXT; their locations in the array by J1, J2 and K2. |
| $\mathbf{A}_{\mathbf{S X}}, \mathrm{A}_{\mathbf{S} \varphi}$ | SAX, SAP | Cross sectional area of stiffener* in $x$ direction and $\varphi$-direction (in ${ }^{2}$ ). Controlling indices are J1, J2, K2 and NEXT. |
| $\mathrm{I}_{\mathbf{S x}}, \mathrm{I}_{\mathbf{S} \varphi}$ | SIX, SIP | Moment of inertial of stiffener, * in xdirection and $\varphi$-direction respectively, about its own centroidal axis (in ${ }^{4}$ ). Controlling indices are J1, J2, K2 and NEXT. |
| $\overline{\mathrm{K}}_{\mathrm{x}}$ | BARKX | Spring constant of a point-support along the axis $\mathrm{x}=0$, ( $\mathrm{lbs} / \mathrm{in}$ ) <br> Controlling indices are I1, 12, and NXT1. |
| $\overline{\mathbf{K}}_{\varphi}$ | BARKP | Same as above along the axis $\varphi=0$, ( $\mathrm{lbs} / \mathrm{in}$ ) Controlling indices are I3, I4, and NXT2. |
| $\mathrm{W}_{\mathrm{x}}$ | wx | Weights along the boundary $\mathrm{x}=0$, (lbs) Controlling indices are I5, 16, and NXT3. |
| $\mathrm{W}_{\varphi}$ | WP | Weights along the boundary $\varphi=0$, (lbs) Controlling indices are I5, I6, and NXT4. |
| W | wT | Weights at interior points of the panel (lbs) Controlling index is JWT |
| i, j | IM1, IM2 | Row and column number, respectively, of off-diagonal term of mass (internal) matrix. |
| $\left(M_{1}\right)_{i, ~}^{\text {j }}$, $\mathrm{i} \neq \mathrm{j}$ | AM3 | Actual element that corresponds to IM1 and IM2 in the (internal) mass matrix. <br> Number of this off-diagonal terms is limited to six. |
|  | IETBL, NEC | Row number and column number, respectively of the matrix element ( F ) table. The E table is generated according to the definition of Table 2 for all grid points. A particular element of the table is to be transferred to a particular location in the coefficient matrix $A$. |
|  | NZRO | Number of non-zero elements in a row of coefficient matrix. |
|  | NAC | Column number of the non-zero element in the coefficient matrix. |

NOTE: For a stiffener which is placed along one boundary of the panel, double the values of the corresponding cross-sectional area and the moment of inertia as input.

## 2. Sandwiched Shell

The definitions made for the solid shell apply to the sandwiched shell with the following exceptions and additional definitions.

Symbols used in Analysis

| h | H |
| :---: | :---: |
| c | CORE |
| m | BARM |

IOPT

Fortran
Coding Definition

Total thickness of outer and inner facings.
Thickness of core.
Mass per unit area of the panel.
$=2$. For sandwiched panel.
KEY PUNCH FORM - GEN $=$ RAL PURPOSE

KEY PUNCH FORM - GENERAL PURPOSE




The following listing contains the complete main program and its subroutines with the exception of a subroutine named "MITER"*. The MITER subroutine is a standard eigenvalue, eigenvector routine. Its physical package of the subroutine follows CHAIN 3. DATA are also included here, and the technique for their arrangements is stated in the next section of the appendix.

[^3]




$$
4_{5}^{5}
$$ READ INPUT T FORMAT ( 8 H ** FORMAT (// 9H WRITE OUTPUT WRITE OUTPU
FORMAT // $9 H$ FORMAT CUTRTPUT
 FORMAT (// 9H WRITE OUTPUT WRITE OUTPUT FORMAT (// 9 H
WRITE OUTPUT WRITE OUTPUT FORMAT (// 9 H WRITE OUTPUT WRITE OUTPUT
FORMAT $/ / / 9 H$ OORMAT (6E18.5)
$H 2=H * H$ $\mathrm{H}_{3}=\mathrm{H} 2 * \mathrm{H}$
IAPE
\[

$$
\begin{aligned}
& \text { APE } 5,32,(W T(I), I=1, J W T)) \\
& \text { TAPE } 6,330
\end{aligned}
$$
\] FORMAT (// 9H ** WT **)


PNU2 $=$ PNU*PNU




110 CONTINUE


|  | DO $120 \mathrm{I}=1$, NM |
| :---: | :---: |
|  | IF(I-N1) 130.130.125 |
| 125 | AMI(I,I) $=1 .+(W T(I)+R G *(S A P(I) * A D P+S A X(I) * D E L X)) / D \times A P$ |
|  | GO TO 120 |
| 130 | AM1 (I,I) $=1$. |
| 120 | CONTINUE |
|  | $\mathrm{K}=1$ |
|  | DO $140 \mathrm{I}=1 . \mathrm{M1}$ |
|  | $A M 1(K, K)=1$. |
|  | $\mathrm{K}=\mathrm{K}+\mathrm{N} 1$ |
| 140 | CONTINUE |
|  | READ INPUT TAPE 5.403.(IM1(IJ), IM2 (IJ).IJ=1,6) |
|  | READ INPUT TAPE 5, 32, (AM3(K), K=1,6) |
| 403 | FORMAT (1213) |
|  | DO $414 \mathrm{~K}=1.6$ |
|  | $\mathrm{I}=\mathrm{IM} 1(\mathrm{~K})$ |
|  | J=IM2 (K) |
| 414 | AM1 $(1, J)=A M 3(K)$ |

$M 2=2 *(N+M+2)+1$
DO $142 \mathrm{I}=1$. M2 142 AM2 $(I, J)=0$.


$\xrightarrow{\sim}$



## $\stackrel{-}{\circ}$

$=+($ BARKX $(1) * A D P 2) /(D * B L A M *(1 .-P N U) * 0.5)$




$00610 \mathrm{~J}=1$,NR


TAPE 6.630

| 615 WRITE OUTPUT TAPE 6.630 |  |
| :---: | :---: |
| 630 | FORMAT(1H1 16H** A31 MATRIX **//) |
|  | DO $640 \mathrm{I}=1.34$ |
|  | WRITE OUTPUT TAPE 6.560.I |
| 640 | WRITE OUTPUT TAPE 6.565.(A31(I,J).J=1,24) |
|  | WRITE OUTPUT TAPE 6.650 |
| 650 | FORMAT (1H1 16H** A32 MATRIX **//) |
|  | DO $660 \mathrm{I}=1.34$ |
|  | WRITE CUTPUT TAPE 6.560.I |
| 660 | WRITE OUTPUT TAPE 6.565.(A32(I.J).J=1,21) |
|  | WRITE OUTPUT TAPE 6,670 |
| 670 | FORMAT(1H1 16H** A33 MATRIX **//) |
|  | DO 680 $\mathrm{I}=1.34$ |
|  | WRITE OUTPUT TAPE 6.560.I |
| 680 | WRITE OUTPUT TAPE 6.565,(A33(I,J),J=1,34) |
|  | DO $522 \mathrm{I}=1.21$ |
| 522 | B1(I, 1) $=1$. |
|  | CALL MXIV1 (A22, E2, B1,1.DETER) |
|  | WRITE OUTPUT TAPE 6.576,DETER |
| 576 | FORMAT (///10X.14H DETERMINANT =E15.6) |
|  | CALL MLTMX4 (34.21,A32,21.21,A22,EXTRA,A32) |
| 535 | FORMAT (2413) |
|  | CALL MLTMX1 (34.21.A32.21.24.A21.EXTRA.TEMP1) |
|  | DO $690 \mathrm{I}=1.34$ |
| 690 | $\mathrm{B1}(\mathrm{I}, 1)=1$. |
|  | CALL MXIV2 (A33,34,R1,1,DETR) |
|  | WRITE OUTPUT TAPE 6.576, DETR |
|  | DO $700 \mathrm{I}=1,34$ |
|  | DO $700 \mathrm{~J}=1.24$ |
| 700 | TEMP1(I,J) $=$ A31(I, J)-TEMP1 $(1 . J)$ |
|  | CALL MLTMX2 (34.34,A33,34,24,TEMP1,EXTRA,TEMP1) |


CALL MLTMX5 $(24,21, A 12,21,21, A 22, E X T R A, A 12)$
CALL MLTX12 $(24,21, A 12,21,24, A 21, E X T R A, A 31)$
CALL MLTMX6 $(24,34, A 13.34 .24, \operatorname{TEMP1,EXTRA,EL)}$
D0 $772 I=1,24$
DO 772 J=1.24
A11(I,J) $=A 11(I, N)-A 31(I, J)-E L(I, J)$
CALL MLTMX8 (24.É1.A12.21.24.AM2.EXTRA.EL)
 DO $773 \mathrm{I}=1.24$



| 0 |
| :--- |
| $\mathbf{N}$ |
|  |

**FOR MULTIPLICATION OF PEAL MATRICES
AB $=A * B$
DIMENSION $A(24,21) \cdot B(21,24), A B(34,24), A B F(34,24)$


DIMENSION $A(34,21), B(21,24), A B(34,24), \operatorname{ABF}(34,34)$

SUBROUTINE MLTX10 (N1,M1,A,N2,M2,B,AB,ABF)
DIMENSION $A(24,34), B(34,34), A B(34,24), \operatorname{ABF}(24,34)$
$\begin{array}{lll}D O & 1 & I=1, N 1 \\ D O & 1 & J=1, M 2\end{array}$



## 


$\sum_{i=1}^{\sum \sum}$

直 $\frac{1}{2} \frac{1}{2} \frac{1}{\Sigma} \frac{1}{2} \frac{1}{2} \frac{1}{\Sigma} \frac{1}{\Sigma} \frac{1}{\Sigma}$
$A B(I, J)=0$.

## AB ( $1, \mathrm{~J}=0$. <br> $\operatorname{ABF}(I, J)=A B(I, J)$ $\operatorname{RETURN}$ END SUBROUTINE MLTMX8

**FOR MULTIPLICATION OF REAL MATRICES
AB $=A * B$
DIMENSION A $(24,21) \cdot B(21,24), A B(34,24), \operatorname{ABF}(24,80)$
SUBROUTINE MLTMXㄱ (N1,M1,A,N2,M2,B,AB,ABF)
**FOR MULTIPLICATION OF REAL MATRICES DIMENSION $A(24,24), B(24,34), A B(34,24), A B F(24,80)$ $A B=A * B$


$\operatorname{ABF}(I, J)=A B(I, J)$
RETURN
END
SUBR
DIMENSION $A(24,34), B(34,24), A B(34,24), A B F(24,80)$
$D O 1 \quad I=1, N 1$
$D O 1 \quad J=1, M 2$
$A B(I, J)=0$,
$D O 1 \quad K=1, M 1$
PRODCT=A(I,K)*B(K,J)
$A B(I, J)=A B(I, J)+P R O D C T$
$D O 2 \quad I=1, N 1$
$D O 2 \quad J=1, M 2$
ABF $I, J)=A B(I, J)$
RETURN
END
SUBROUTINE MLTMX5 (N1,M1,A,N2,M2,B,AB,ARF)
DIMENSION A $(24,21) \cdot B(21,21) \cdot A B(34,24) \cdot \operatorname{ABF}(24,21)$ DO $1 \quad I=1, N 1$
DO $1 \mathrm{~J}=1 . \mathrm{M} 2$
$A B(I, J)=0$.
DO $1 \mathrm{~K}=1$. M1
$\operatorname{PRODCT}=A(I, K) * B(K, J)$
$A B(I, J)=A B(I, J)+P R O D C T$
$A B(I, J)=A B(I, J)+P R O D C T$
$\begin{array}{ll}D O & I=1 / N 1 \\ 0 & J=1 . M 2\end{array}$
$\operatorname{ABF}(I, J)=A B(I, J)$ RETURN
SUBROUTINE MLTMX4 (N1,M1,A,N2,M2,B,AB,ABF)
**FOR MULTIPLICATION OF REAL MATRICES
$\xrightarrow{\circ}$ $\sim$
uuuu

## 른 <br>  <br>  <br> $\sum_{\sum}^{\sum} \sum_{\sum} \sum_{i} \frac{1}{2}$


uu $\stackrel{\circ}{7}$
$\rightarrow \rightarrow \sim$

$* * F O R$ MULTIPLICATION OF REAL MATRICES
$A B=A * B$
 DO 1 I=1•N1
DO $1 \mathrm{~J}=1, \mathrm{M} 2$
$A B(I, J)=0$.
DO $1 \mathrm{~K}=1$. M 1
PRODCT=A $(I, K) * B(K, J)$
$A B(I, J)=A B(I, J)+P R O D C T$
AB(I:J) $=A B(1, J)+P$ RODCT
DO $2 \mathrm{I}=1 \cdot \mathrm{~N} 1$
DO $2 \mathrm{~J}=1 \cdot \mathrm{M} 2$
$\operatorname{ABF}(I, J)=\operatorname{AB}(I, J)$
RETURN


[^4]$000{ }^{\infty}$
$4 \quad \mathrm{~N}$

DIMENSION A 34,34 ), $B(34,24), A B(34,24), \operatorname{ARF}(34,24)$

> SUBROUTINE MLTMX2 (N1,M1,A,N2,N2,B,AB,ABF) **FOR MULTIPLICATION OF REAL MATRICES
> (
> $A B=A * 3$ REAL MATRICES

uvue $\begin{array}{r}\text { a } \\ \hline\end{array}$

## 直 $\frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2}$ <br> 

DO $1 K=1 \cdot M 1$

> PRODCT=A(I K)*P(K*J)
$A B(I \quad J)=A B(I \quad J)+P R O D C T$ 00 $T=1, N 1$
> $\operatorname{ABF}(I, J)=A B(I, J)$
> $\operatorname{ABF}(I, J)=A B(I ; J)$
$R E T U R N$
> END

> SUBROUTINE MLTMX1 (N1,M1,A,N2,M2,B,AB,ABF)
**FOR MULTIPLICATION OF REAL MATRICES
DIMENSION A 34,21$) \cdot B(21,24), A B(34,24) \cdot \operatorname{ABF}(34,24)$ DO $1 \quad I=1, N 1$
$D 01 J=1 \cdot M 2$
$A B(I, J)=0$
$D O 1 \quad K=1, M 1$
$P R O D C T=A(I, K) * B(K, J)$
$A B(I, J)=A B(I \cdot J)+P R O D C T$
$D O 2 \quad I=1, N 1$
$D O 2, J=1, M 2$
$A R F(I, J)=A B(I, J)$
$R E T U R N$
$E N D$
$\rightarrow \quad N$

| C | SUBROUTINE MXIV1 (A.N.B.M.DETERM) |
| :---: | :---: |
|  | MATRIX INVERSION WITH ACCOMPANYING SOLUTION OF LINEAR EQUATIONS |
|  | DIMENSION IPIVOT 21$), A(21,21), B(21,1), \operatorname{INDEX}(21,2)$, PIVOT (21) |
|  | EQUIVALENCE (IROW,JROW), (ICOLUM.JCOLUM), (AMAX, T, SWAP) |
| C |  |
| C | INITIALIZATION |
| C |  |
| 10 | DETERM $=1.0$ |
| 15 | DO $20 \mathrm{~J}=1 . \mathrm{N}$ |
| 20 | IPIVOT (J) $=0$ |
| 30 | DO $550 \mathrm{I}=1$, N |
| C |  |
| C | SEARCH FOR PIVOT ELEMENT |
| C |  |
| 40 | AMAX $=0.0$ |
| 45 | DO $105 \mathrm{~J}=1 . \mathrm{N}$ |
| 50 | IF (IPIVOT(J)-1) 60. 105, 60 |
| 60 | DO $100 \mathrm{~K}=1 . \mathrm{N}$ |
| 70 | IF (IPIVOT(K)-1) 80, 100, 740 |
| 80 | IF (ABSF (AMAX)-ABSF (A $(J, K))$ ) 85, 100. 100 |
| 85 | IROW=J |
| 90 | I COLUM $=K$ |
| 95 | AMAX $=A(J, K)$ |
| 100 | CONTINUE |
| 105 | CONTINUE |
| 110 | IPIVOT (ICOLUM $)=$ IPIVOT ( ICOLUM $)+1$ |
| C |  |
| C | INTERCHANGE ROWS TO PUT PIVOT ELEMENT ON DIAGONAL |
| C |  |
| 130 | IF (IROW-ICOLUM) 140, 260, 140 |
| 140 | DETER $=-$ DETERM |
| 150 | DO $200 \mathrm{~L}=1, \mathrm{~N}$ |
| 160 | SWAP=A (IROW $L$ L) |
| 170 | A(IROW,L) $=$ A (ICOLUM,L) |
| 200 | A (ICOLUM,L) $=$ SWAP |
| 205 | IF (M) 260, 260, 210 |
| 210 | DO 250 L=1. M |
| 220 | SWAP=B(IROW L ) |
| 230 | $B(I R O W, L)=B(I C O L U M, L)$ |
| 250 | $B(I C O L U M \cdot L)=S W A P$ |


260 INDEX $(I, 1)=I R O W$
270 INDEX $(I, 2)=I C O L$
310 PIVOT（I）＝A（ICOLU
320 DETERM＝DETERM＊PI






| 40 | AMAX $=0.0$ |
| :---: | :---: |
| 45 | DO $105 \mathrm{~J}=1, \mathrm{~N}$ |
| 50 | IF (IPIVOT(J)-1) 60, 105, 60 |
| 60 | $00100 \mathrm{~K}=1, \mathrm{~N}$ |
| 70 | IF (IPIVOT $(\mathrm{K})-1$ ) 80, 100, 740 |
| 80 | IF (ABSF (AMAX)-ABSF(A(J.K))) 85. 100, 100 |
| 85 | IROW=J |
| 90 | ICOLUM=K |
| 95 | AMAX $=$ A $(J, K)$ |
| 100 | CONTINUE |
| 105 | CONTINUE |
| 110 | $\operatorname{IPIVOT}($ ICOLUM $)=\operatorname{IPIVOT}(\operatorname{ICOLUM})+1$ |
| C |  |
| C | INTERCHANGE ROWS TO PUT PIVOT ELEMENT ON DIAGONAL |
| C |  |
| 130 | IF (IROW-ICOLUM) 140, 260, 140 |
| 140 | DETERM $=-$ DETERM |
| 150 | CO $200 \mathrm{~L}=1$, N |
| 160 | SWAP $=A($ IROW $\cdot L)$ |
| 170 | A (IROW,L) $=A($ ICOLUM,L) |
| 200 | A (ICOLUM,L) $=$ SWAP |
| 205 | IF (M) 260, 260, 210 |





## CALL EXIT CONV $=D /(A D P 2 * A D P 2 * S M A S)$ <br> GO TO 35

CONV $=D /(A D P 2 * A D P 2 * B A R M)$
IF（NC－2）1261，1265，1261
NMODE＝NMODE＊2
1234 I＝1，NMODE
$\operatorname{EIGVAL}(I)=1 \cdot / E I G V A L(I)$
$\operatorname{FREQ}(I)=\operatorname{SQRTF}(A B S F(E I G V$
$\operatorname{FREQ}(\mathrm{I})=\operatorname{SQRTF}(\operatorname{ABSF}(E I G V A L .(I)) * \operatorname{CONV}) /(2 . * P I)$ CONTINUE

$2 \varepsilon$
$+\varepsilon$
1234
1235
1236 $\cup$


[^5]





| $m$ | $\vec{\sim}$ | $\underset{\sim}{\sim}$ | $\underset{M}{\infty}$ | $\begin{gathered} m \\ 0 \end{gathered}$ | $\stackrel{60}{9}$ | $\stackrel{M}{ \pm}$ | $\underset{\sim}{N}$ | $\stackrel{M}{\sim}$ | $0$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\cdots$ | 0 | $\Omega$ | $\cdots$ | N | $\pm$ | 18 | N | N | $1)$ | ONNNN |
| O | $\underset{-1}{\infty}$ | $\underset{\sim}{\mathrm{r}}$ | $\stackrel{M}{N}$ | $\hat{N}$ | $\stackrel{M}{J}$ | $\underset{N}{N}$ | $\vec{N}$ | $\underset{\sim}{N}$ | ̇N | $\begin{array}{lll} \sigma & 1 \\ 0 & 0 & 0 \\ 0 & 0 \end{array}$ |
| U | $N$ | N | $\square$ | $a$ | 0 | N | $\square$ | ！ | $N$ | $N \ln$ |
| $\stackrel{n}{-1}$ | $12$ | $\underset{\sim}{0}$ | $\underset{\sim}{\infty}$ | $\vec{n}$ | $\underset{\sim}{\sim}$ | O | $\underset{\sim}{0}$ | $\vec{N}$ | $\stackrel{M}{N}$ | $\begin{array}{llll} \text { Mon } \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{array}$ |
| N | $1 \times$ | ） | $N$ | $\cdots$ | n | 10 | 0 | 0 | $n$ | $\sigma \pm 000$ |
| $\pm$ | $\underset{\sim}{J}$ | $0$ | $\underset{\sim}{N}$ | $\cdots$ | $\vec{F}$ | $a$ | $\stackrel{\sim}{f}$ | $\underset{\sim}{\propto}$ | $\underset{N}{N}$ | ヘにいのテ いますNN゚ |
| 10 | 0 | 0 | $n$ | $m$ | in | 0 | $\pm$ | $\pm$ | $\pm$ | N00ささ |
| $\stackrel{M}{7}$ | $\underset{F}{-1}$ | $\underset{\sim}{N}$ | $\underset{\sim}{0}$ | $\underset{\sim}{J}$ | $\underset{N}{O}$ | $\underset{\sim}{0}$ | $\underset{\sim}{0}$ | $\underset{\sim}{\sim}$ | $\underset{\sim}{\infty}$ | \#J NNM N |
| 0 | $\pm$ | $\pm$ | $\pm$ | $N$ | $\sim$ | $\pm$ | $\stackrel{\rightharpoonup}{7}$ | $\underset{\ddagger}{ \pm}$ | $\stackrel{0}{\mathbf{J}}$ | $\underset{\sim}{\sim} \underset{\sim}{\sim}+\infty \infty$ |
| $0$ | $0$ | $\underset{-1}{ }$ | $\underset{\sim}{\sim}$ | $\underset{-1}{\infty}$ | $\cdots$ | 10 | $2$ | $\underset{-1}{0}$ | $\underset{\sim}{r}$ | $\underset{\sim}{N} \underset{\sim}{M} \underset{\sim}{N}$ |
| $\pm$ | $\infty$ | $\infty$ | $\stackrel{10}{7}$ | $\underset{\sim}{\sim}$ | 0 | $\begin{aligned} & \text { 士 } \end{aligned}$ | $\pm$ | Ј | $\pm$ | $\underset{于}{ \pm} \infty \infty \pm \pm$ |
| $\sigma$ | $\sigma$ | $0$ | $\vec{F}$ | $\underset{\sim}{r}$ | $\stackrel{1}{7}$ | ت | $\underset{-1}{ \pm}$ | $\xrightarrow[\sim]{1}$ | $\underset{-1}{6}$ | CNON |
| $\infty$ | $\pm$ | $\pm$ | $\pm$ | $\infty$ | Ј | $\pm$ | 0 | 0 | 0 | $\underset{J}{-1} \pm \pm 000$ |
| $\infty$ | $\propto$ | 0 | $\xrightarrow{\circ}$ | $\sim$ | J | $\cdots$ | $\underset{-1}{m}$ | J | $\underset{\sim}{n}$ | Hoono |
| $\cdots \pm 0$ | 0 | 0 |  | $\underset{\exists}{7}$ | J | 10 | $\Omega$ | $\bigcirc$ | 10 | $\underset{\sim}{M} \underset{\sim}{\sim} \underset{\Im}{\sim} \underset{J}{N}$ |
| $\kappa_{\Omega} r \bar{j}$ |  |  |  | $\mp$ | $\cdots$ | 0 | $\underset{\sim}{E}$ | $\underset{\sim}{7}$ | $\underset{\sim}{n}$ | $\therefore 9$ |
| ๑介 |  |  | 5， | $\underset{\sim}{M}$ |  | $\sim$ | $N$ | $\sim$ | $N$ | $\begin{array}{r} \sim \sim \sigma \sigma \sigma \\ \text { JMMM } \end{array}$ |
| $\begin{aligned} & N M \\ & \vdots \end{aligned}$ |  |  | 0 | $10$ |  | $\infty$ | $a$ |  |  | Nまさ』ロ |
| の～の | N | $N$ | $\sim$ |  |  | $\square$ | $\square$ | ก |  |  |
| $\underset{J}{0} \sim$ |  |  |  |  |  |  |  |  |  |  |
| $\bigcirc$ |  | 0 |  | $\underset{J}{\sim}$ |  |  | M | $\cdots$ |  | $\begin{array}{r} \operatorname{Mog} 00 \\ \pm 9 \end{array}$ |
| $\stackrel{m}{4}=$ |  | $\cdots$ | J |  |  |  |  |  |  | $\leftrightarrow N \infty 0$ |
| $\pm \underset{\rightarrow c}{0}$ | $0$ | $\begin{aligned} & 0 \\ & - \end{aligned}$ | $\underset{\sim}{n}$ | $\underset{\sim}{\sim}$ | 18 | n | n | $\stackrel{1}{7}$ | $\pm$ |  |

Some of the preliminary computations for the arrangement of input data are shown here, together with the definitions illustrated in figures. Appendix I supplements further information needed for the use of the programs.

The sample run provided here is the segment of the full scale Instrument Unit with sandwiched panel. Its idealized configuration is shown in Figure 37. The unit mass of the panel is

$$
\begin{aligned}
\text { BARM } & =0.2588 \times 10^{-3} \times(0.02+0.03)+\frac{3.1}{12 \times 144} \times 0.9 \times \frac{1}{386} \\
& =0.1712 \times 10^{-4}\left(\mathrm{lb}-\mathrm{sec}^{2} / \mathrm{in}^{2}\right)
\end{aligned}
$$

A, B, PHIO, PNU, E, RHO, N, M, NMODE, CORE, and IOPT are defined in the list of definitions and also in Figure 37. Referring to Figure 38, the distance from c.g. 's of the stiffener to the center line of the panel is assumed to be small.

$$
\mathbf{z}_{\mathbf{S} \varphi}=\mathbf{S Z P} \approx \mathbf{0}
$$

Along the edge parallel to the $x$-direction, similar stiffening effect is assumed due to the filler material used at the boundary to replace the honeycomb material.

The cross-sectional area and moment of inertia of the stiffener are calculated as follows:

$$
\begin{aligned}
A_{S \varphi}= & (2)(1.0)(0.125)+(1.25)(0.1625)=0.453 \mathrm{in} .2 \\
I_{S \varphi}= & 2\left[\left(\frac{1}{12}\right)(1)(0.125)^{3}+(0.125)(1)\left(0.3875^{2}\right)\right]+ \\
& \left(\frac{1}{12}\right)(0.1625)(1.25)^{3}=0.0643 \mathrm{in} .4
\end{aligned}
$$

Here, symmetry about the center line of panel is assumed. Since the stiffeners are placed along the boundaries, the values calculated above ( $A_{S}$ and $I_{S}$ ) are doubled for use in the computer program:

$$
\begin{aligned}
& A_{S X}=A_{S \varphi}=0.906 \mathrm{in} .^{2} \\
& I_{S X}=I_{s \varphi}=0.1286 \mathrm{in} .{ }^{4} \\
& \mathrm{z}_{\mathrm{SX}} \approx 0
\end{aligned}
$$

The Instrument Unit segment is supported by six cantilever beam type springs as shown in Figure 5. For a quarter panel, two springs are assumed acting on stations 7 and 19 (see


FIGURE 37. FULL SCALE INSTRUMENT UNIT LOCALIZED SHELL


FIGURE 38. EDGE STIFFENER CONFIGURATION

Figure 37). The length of the cantilever supports is 9 inches. The corresponding spring constant, $\overline{\mathrm{K}}_{\varphi}$, is

$$
\begin{aligned}
& \overline{\mathrm{K}}_{\varphi}(7)=\overline{\mathrm{K}}_{\varphi}(19)=\frac{3 \mathrm{EI}}{\ell^{3}}=\frac{3 \times 30 \times 10^{6} \times 2.5 \times 0.625^{3}}{12 \times 9^{3}}=6,270 \mathrm{lb} . / \mathrm{in} . \\
& \overline{\mathrm{K}}_{\mathrm{X}}=0 .
\end{aligned}
$$

For convenience in computation, the weights attached near the cantilever supports (see Figure 5) are assumed evenly distributed over the springs (Figure 37).

$$
\mathrm{W}_{\varphi}(7)=\mathrm{W}_{\varphi}(19)=\frac{1}{3} \times 70.5=23.5 \mathrm{ib}
$$

Along the circumferential edges, boundary weights are attached at stations 2 and 3 :

$$
\mathrm{W}_{\mathrm{x}}(2)=\mathrm{W}_{\mathrm{x}}(3)=\frac{1}{2} \times 35.2=17.6 \mathrm{lb} .
$$

The component-weight is assumed to be symmetrical with respect to both center lines of the I. U. segment, as shown in Figure 39. Considering the mass and mass moment of inertia of the component, if the attachment points $\mathrm{A}, \mathrm{B}$ are both coinciding with the grid points, Eqs. (54) (55) may be used to compute its mass matrix elements. In the present case, point A falls on grid point 9. Point B is located between grid points 11, 12. The off-diagonal terms of $M_{1}$ matrix, and the equivalent concentrated weights used at the points of attachment, are calculated as follows (see Figure 39).


FIGURE 39. INERTIA FORCE DISTRIBUTION DUE TO A RIGID COMPONENT

$$
\begin{aligned}
\mathrm{M}_{\mathrm{o}}= & \text { mass of the component }=\frac{90}{\mathrm{~g}}\left(\mathrm{lb} .-\mathrm{sec} .^{2} / \mathrm{in} .\right) \\
\theta= & \begin{array}{l}
\text { moment of inertia of the component about the } \\
\\
\text { axis through centroid in x-direction }
\end{array} \\
\approx & 3.5 \text { (in. -lb. -sec. }{ }^{2} \text { ) } \\
\mathrm{w}_{\mathrm{A}}= & \text { deflection at station } 9=\mathrm{w}_{9} \\
\mathrm{w}_{\mathrm{B}}= & \frac{1}{3}\left(\mathrm{w}_{11}+2 \mathrm{w}_{12}\right) \text { by assuming that } \mathrm{w}_{\mathrm{B}} \text { is inneariy } \\
& \text { proportional with } \mathrm{w}_{11} \text { and } \mathrm{w}_{12}
\end{aligned}
$$

$w_{C}=$ normal deflection at the centroid of the component
$=\frac{1}{26}\left(12 \mathrm{w}_{\mathrm{A}}+14 \mathrm{w}_{\mathrm{B}}\right)$
$=\stackrel{1}{26}\left(12 \mathrm{w}_{9}+\frac{14}{3} \mathrm{w}_{11}+\frac{28}{3} \mathrm{w}_{12}\right)$
$\mathrm{F}_{\mathrm{A}}=\frac{1}{2}\left(\begin{array}{lll}12 \\ 26 & \mathrm{M}_{\mathrm{o}} \omega^{2} \mathrm{w}_{\mathrm{c}}-\frac{1}{26} \ominus \omega^{2} \quad \frac{\mathrm{dw}}{\mathrm{ad} \varphi}\end{array}\right)$
$=$ inertia force acting at A due to $\mathrm{M}_{\mathrm{o}}$ and $\boldsymbol{\theta}$

$$
\begin{aligned}
\mathrm{F}_{\mathrm{B}} & =\frac{1}{2}\left(\frac{14}{26} \mathrm{M}_{\mathrm{o}} \omega^{2} \mathrm{w}_{\mathrm{c}}+\frac{1}{26} \omega^{2} \frac{\mathrm{~d} w_{\mathrm{c}}}{\mathrm{ad} \varphi}\right) \\
& =\text { inertia force acting at } B \text { due to } M_{\mathrm{o}} \text { and } \\
\mathrm{F}_{\mathrm{C}} & =\mathrm{M}_{\mathrm{o}} \omega^{2} \mathrm{w}_{\mathrm{c}} \\
\mathrm{M}_{\mathrm{C}} & =\omega^{2} \frac{\mathrm{dw}}{\mathrm{ad} \varphi}
\end{aligned}
$$

The inertia forces applied on the panel at points 9,11 and 12 are

$$
\begin{aligned}
\mathrm{F}_{9} & =\mathrm{F}_{\mathrm{A}}=\omega^{2}\left(27.5 \mathrm{w}_{9}+8.8 \mathrm{w}_{11}+17.6 \mathrm{w}_{12}\right) \times 10^{-3} \mathrm{lb} \\
\mathrm{~F}_{11} & =\frac{1}{3} \quad \mathrm{~F}_{\mathrm{B}}=\omega^{2}\left(8.8 \mathrm{w}_{9}+4.05 \mathrm{w}_{11}+8.1 \mathrm{w}_{12}\right) \times 10^{-3} \mathrm{lb} \\
\mathrm{~F}_{12} & =(2)\left(\frac{2}{3}\right) \mathrm{F}_{\mathrm{B}}=\omega^{2}\left(35.2 \mathrm{w}_{9}+16.2 \mathrm{w}_{11}+32.4 \mathrm{w}_{12}\right) \mathrm{lb} .
\end{aligned}
$$

Since the inertia force of the panel, as shown in the diagonal of $M_{1}$ matrix, is unity per unit area, the above inertia forces must be normalized accordingly:

$$
\begin{aligned}
& \left(\mathrm{M}_{1}\right)_{9,11}=\frac{\omega^{2} \times 10^{-3}}{\overline{\mathrm{~m}}(\Delta \mathrm{x})(\mathrm{a} \Delta \varphi) \omega^{2}} \times 8.8=8.4 \\
& \left(\mathrm{M}_{1}\right)_{9,12}=\frac{\omega^{2} \times 10^{-3}}{\overline{\mathrm{~m}}(\Delta \mathrm{x})(\mathrm{a} \Delta \varphi) \omega^{2}} \times 17.6=16.8
\end{aligned}
$$

The equivalent concentrated weight attached at station 9 is

$$
\begin{aligned}
\mathrm{WT}(9) & =10^{-3} \times 27.5 \times \mathrm{g}=10.6 \mathrm{lb} \\
\left(\mathrm{M}_{1}\right)_{11,9} & =\frac{\omega^{2} \times 10^{-3}}{\overline{\mathrm{~m}}(\Delta \mathrm{x})(\mathrm{a} \Delta \varphi) \omega^{2}} \times 8.8=8.4 \\
\left(\mathrm{M}_{1}\right)_{11,12} & =\frac{\omega^{2} \times 10^{-3}}{\overline{\mathrm{~m}}(\Delta \mathrm{x})(\mathrm{a} \Delta \varphi) \omega^{2}} \times 8.1=7.7 \\
\mathrm{WT}(11) & =10^{-3} \times 4.05 \mathrm{~g}=1.56 \mathrm{lb} \\
\left(\mathrm{M}_{1}\right)_{12,9} & =\frac{\omega^{2} \times 10^{-3}}{\overline{\mathrm{~m}}(\Delta \mathrm{x})(\mathrm{a} \Delta \varphi) \omega^{2}} \times 35.2=33.6 \\
\left(\mathrm{M}_{1}\right)_{12,11} & =\frac{\omega^{2} \times 10^{-3}}{\overline{\mathrm{~m}}(\Delta \mathrm{x})(\mathrm{a} \Delta \varphi) \omega^{2}} \times 16.2=15.4 \\
\mathrm{WT}(12) & =10^{-3} \times 32.4 \times \mathrm{g}=12.5 \mathrm{lb}
\end{aligned}
$$

The non-zero elements of the $\mathrm{M}_{1}$ matrix are computed as follows:

FINITE DIFFERENCE METHOD FOR SANDWICHED SHELL, CORE THICKNESS $=0.90000 \mathrm{E} 00 \mathrm{MBAR}=0.17120 \mathrm{E}-04$

FINITE DIFFERENCE METHOD FOR SANDWICHED SHELL, CORE THICKNESS $=0.90000 \mathrm{E} 00 \mathrm{MBAR}=0.17120 \mathrm{E}-04$ $\quad \stackrel{N}{0}$


|  | $\therefore 0^{\circ} 0^{\circ}$ | $\dot{\circ}$ |
| :---: | :---: | :---: |



#  





FINITE DIFFERENCE METHOD FOR SANDWICHED SHELL, CORE THICKNESS $=0.90000 \mathrm{E} 00 \mathrm{MBAR}=0.17120 \mathrm{E}-04$

$\therefore \dot{O} \dot{O} \dot{O} \dot{O} \dot{O}$







FINITE DIFFERENCE METHOD J FOR SANDWICHED SHELL, CORE THICKNESS $=0.90000 \mathrm{E} 00 \mathrm{MBAR}=0.17120 \mathrm{E}-04$











mass 2 matrix - boundary
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0. ROW NC.
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 Rum No. $\begin{gathered}8 \\ 0 . \\ 0 . \\ 0 . \\ \\ 0 .\end{gathered}$ RUn Nu. 9

FINITE DIFFERENCE METHOD FOR SANDWICHED SHELL, CORE THICKNESS $=0.90000 \mathrm{E} 00 \mathrm{MBAR}=0.17120 \mathrm{E}-04$


INPUT DATA



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$\therefore \dot{\circ} 00^{\circ}$




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FINITE DIF FERENCE METHOD FOR SANDWICHED SHELL, CORE THICKNESS $=0.90000 \mathrm{E} 00 \mathrm{MBAR}=0.17120 \mathrm{E}-04$






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＊＊INPUT

















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| $0.49 \angle 3 E$ | 01 |
| $0.0386 E$ | 01 |
| $-0.1036 E-00$ |  |
| $-0.3117 E$ | 02 |
| $0.2317 E$ | 02 |
| $0.2392 E$ | 07 |
| $-0.3330 E-04$ |  |


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| $-0.1918 E$ | 04 |
| $0.4923 E$ | 01 |
| $0.8380 E$ | 01 |
| $-0.1036 E$ | 00 |
| $-0.3117 E$ | 02 |
| $0.2317 E$ | 02 |
| $0.2392 E$ | 07 |
| $-0.3336 E$ | 04 |


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$\stackrel{3}{\alpha}$



 $\dot{\circ} \dot{\circ}$ óó $\dot{\circ} \dot{\circ}$ $\dot{\circ} \dot{\circ}$ óó $\dot{0} \dot{\circ}$


FINITE DIFFERENCE METHOD FOR SANDWICHED SHELL, CORE THICKNESS $=0.90000 \mathrm{E} 00 \mathrm{MBAR}=0.17120 \mathrm{E}-04$ oo o io oi tojo
 -0.4923E 01
0.
0.
0.
00.
0.
0.
00
0.
0.
 $\therefore \dot{0} 0 \dot{0} 0$














** a 22 matrix **


 $\therefore \quad \circ$ 0.
0.
0.
0. $\therefore \circ^{\circ}$
 0.
0.
0.
0.
0.
$0.1000 E \quad 01$
$\therefore \circ$ $\therefore \dot{\circ} \dot{0} \quad \dot{\circ}$ 0. 0.
0. $\therefore \dot{\circ}$ $\dot{\circ} \quad \dot{\circ} \quad \dot{\circ}$
 0.
$0.1000 E \quad 01$
0.0
0.
0.
0.
0.
0.
0.
0.
0.
0.
0.
0.
0.
0.
0.
$0.1000 E$
0.1 0.1000 E 01 000 $\therefore 00$ 0.
0.
0.
0. $\therefore 0^{\circ}$
 $\therefore \dot{0}$ $\frac{0}{o}$




** a31 matrix **


FINITE DIFFERENCE METHOD FOR SANDWICHED SHELL, CORE THICKNESS $=0.90000 \mathrm{E} 00 \mathrm{MBAR}=0.17120 \mathrm{E}-04$





FINITE DIFFERENCE METHOD FOR SANDWICHED SHELL, CORE THICKNESS $=0.90000 \mathrm{E} 00 \mathrm{MBAR}=0.17120 \mathrm{E}-04$

$$
\begin{aligned}
& \therefore \circ \stackrel{\stackrel{\circ}{\circ}}{\stackrel{( }{\mu}} \\
& \begin{array}{l}
0 . \\
0 . \\
0 . \\
0 . \\
0 . \\
0 . \\
0 .
\end{array} \\
& \begin{array}{l}
0 . \\
0 . \\
0 . \\
0 . \\
0 . \\
0.30
\end{array} \\
& 0.3027 E 06 \\
& \begin{array}{lll}
0 . & 0 . \\
0 . & 0 . \\
0 . & 0 . \\
0 . & & 0 . \\
0 . & 0 . & \\
0.3027 E & 06 & 0.3027 E \\
0 . & 06
\end{array}
\end{aligned}
$$

$$
\begin{aligned}
& \text { ○○் ㅇ́ó }
\end{aligned}
$$







 ** a32 Matrix **




** INPUT DATA **






FINITE DIFFERENCE METHOD FOR SANDWICHED SHELL, CORE THICKNESS $=0.90000 \mathrm{E} 00 \mathrm{MBAR}=0.17120 \mathrm{E}-04$
$\dot{\circ} \dot{\circ} \dot{\circ}$
$\dot{\circ} \dot{0} \dot{0}$
$\dot{\circ} \dot{\circ} \dot{\circ}$

$\dot{\circ} \dot{\circ} \dot{0} \dot{\circ} \dot{\circ}$
$\dot{\circ} \dot{\circ} \dot{0} \dot{0} \dot{\circ}$









| FINITE DIF FERENCE METHOD FOR SANDWICHED SHELL, CORE THICKNESS $=0.90000 \mathrm{E}$ ( $00 \mathrm{MBAR}=0.17120 \mathrm{E}-04$$* *$ INPUT DATA $* *$ |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| ** a33 matrix ** |  |  |  |  |  |  |  |  |
| ** ROW 1** |  |  |  |  |  |  |  |  |
| 0.5792E 01 | -0.4513E 02 | 0.5792 El | 0. | 0. | 0. | -0.1558E 02 | 0.7948 E | 02 |
| -0.1558t 02 | 0.1000 El | 0. | 0. | 0.5792 E 01 | -0.4513E 02 | 0.5792 El |  |  |
| 0. | 0. | 0. | 0.8386 El | 0. | 0. | 0. | 0. |  |
| 0. | 0.8386 E 01 | 0. | 0 . | 0 . | 0. | 0 . | 0.1000 E | 01 |
| 0. | 0. |  |  |  |  |  |  |  |
| ** RCW 2** |  |  |  |  |  |  |  |  |
| 0. | 0.5792 E 01 | -0.4513E 02 | 0.5792 E 01 | 0. | 0. | $0.1000 \mathrm{E}^{01}$ | -0.1558E | 02 |
| 0.7548 O | -0.1558E 02 | 0.1000 E O1 | 0. | 0. | 0.5792 E 01 | -0.4513E 02 | 0.5792 E |  |
| 0. | 0. | 0. | 0 . | 0.8386 El | 0. | 0. | 0. |  |
| 0. | 0. | 0.8386 E 01 | 0 . | 0. | 0. | 0. | 0. |  |
| 0. | 0. |  |  |  |  |  |  |  |
| ** RCW 3** |  |  |  |  |  |  |  |  |
| 0. | 0. | 0.5792 E 01 | -0.4513E 02 | 0.5792 E 01 | 0. | 0. | 0.1000 E | 01 |
| -0.1558E O2 | 0.7948 E 02 | -0.1558E 02 | 0.1000 El | 0. | 0. | 0.5792 El | -0.4513E |  |
| 0.5792 El | 0. | 0. | 0. | 0. | 0.8386 E 01 | 0. | 0. |  |
| 0. | 0. | 0. | 0.8386 E 01 | 0. | 0. | 0. | 0. |  |
| 0. | 0. |  |  |  |  |  |  |  |
| ** RCW 4** |  |  |  |  |  |  |  |  |
| 0. | 0. | 0. | 0.5792 El | -0.4513E 02 | 0.5792 E 01 | 0. | 0. |  |
| 0.1000 E 01 | -0.1558E 02 | 0.8048 E 02 | -0.1558E 02 | 0. | 0. | 0. | 0.5792 E | 01 |
| -0.4513E 02 | 0.5792 El | 0. | 0. | 0. | 0. | $0.8386 E 01$ | 0. |  |
| 0. | 0. | 0. | 0. | 0.8386 El | 0. | 0 . | 0. |  |
| 0. | 0. |  |  |  |  |  |  |  |
| ** ROW 5** |  |  |  |  |  |  |  |  |
| 0. | 0. | 0. | 0. | 0.1158 E 02 | -0.4513E 02 | 0. | 0. |  |
| 0. | 0.2000 E 01 | -0.3117E 02 | 0.7948 E 02 | 0. | 0. | 0. | 0. |  |
| 0.1158 E 02 | -0.4513E 02 | 0. | 0. | 0. | 0. | 0. | 0.8386 E | 01 |
| 0. | 0. | 0. | 0. | 0. | 0.8386 E 01 | 0. | 0. |  |
| 0. | 0. |  |  |  |  |  |  |  |
| ** RUW 6** |  |  |  |  |  |  |  |  |
| $0 \cdot 0$ | 0.8386 E 01 | 0. | 0. | 0. | 0. | 0.5792801 | -0.4513E | 02 |
| 0.5752E 01 | 0. | 0. | 0. | -0.1558E 02 | $0.8787 E 02$ | -0.1558E 02 | 0.1000 E |  |
| 0. | 0. | 0.5792 El | -0.4513E 02 | 0.5792 Cl | 0. | 0. | 0. |  |
| 0. | 0. | 0. | 0. | 0. | 0. | 0 . | 0 . |  |
| C. 1000E OI | 0 . |  |  |  |  |  |  |  |
| ** RCw 7** |  |  |  |  |  |  |  |  |
| 0. | 0.1 | 0.8386 E 01 | 0. | 0. | 0. | 0. | 0.5792 E |  |
| -0.4513E 02 | 0.5792 E 01 | 0. | 0. | $0.1000 E^{01}$ | -0.1558E 02 | 0.8787 E 02 | -0.1558E |  |
| 0.1000 O 01 | 0. | 0. | 0.5792 El | -0.4513E 02 | 0.5792 El | 0. | 0. |  |
| $\bigcirc$. | 0. | 0. | 0 . | 0 . | 0. | 0 . | 0. |  |
| 3. | 0. |  |  |  |  |  |  |  |

FINITE DIFFERENCE METHOD FOR SANDWICHED SHELL，CORE THICKNESS $=0.90000 \mathrm{E} 00 \mathrm{MBAR}=0.17120 \mathrm{E}-04$

| N | NOO | $\because 0$ | ～ | N | NOO | No |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\underset{\underset{\infty}{\infty}}{\underset{\infty}{\omega}}$ | $\begin{aligned} & \text { 山己 } \\ & \underset{\sim}{\sim} \\ & \text { ñ } \\ & \text { ñ in } \end{aligned}$ |  | $\begin{aligned} & \underset{\sim}{\sim} \\ & \stackrel{0}{\boldsymbol{o}} \end{aligned}$ | $\begin{aligned} & \underset{\infty}{\infty} \\ & \stackrel{\sim}{n} \\ & = \end{aligned}$ | $\begin{aligned} & \text { w u } \\ & \text { No } \\ & \text { م。 } \end{aligned}$ |  |
| $0^{\circ} 0^{\circ}$ | $0^{\circ} 0^{\circ} 0^{\circ}$ | －000 | －000 | －000 | $\dot{\circ} \dot{0} \dot{\circ}$ | $\bigcirc 0^{\circ}$ |


| NOO | －10 | ～ | ～ | NO－ | N～0 | N |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\begin{aligned} & \text { 山⿱丷天心} \\ & \stackrel{\sim}{\circ} \\ & \\ & \end{aligned}$ | $\begin{aligned} & \underset{\sim}{\infty} \\ & \stackrel{\sim}{7} \end{aligned}$ | $\begin{aligned} & \text { 山̈ } \\ & \stackrel{N}{n} \end{aligned}$ | $\begin{aligned} & w{ }_{n}^{4} \\ & \text { Nö } \end{aligned}$ |  | 山 |
| -0; Ó |  | $\bigcirc 0^{\circ} 0^{\circ}$ | $\therefore 0^{\circ} 0^{\circ}$ | $\dot{0} 0$ | $\therefore 000$ | $\bigcirc 0^{\circ} 0^{\circ}$ |


| －\％ | $\stackrel{\square}{0}$ | $\stackrel{\rightharpoonup}{0}$ | Nor | No | ～ | N |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $$ | $\begin{aligned} & 山_{0} \\ & \infty \\ & \infty \\ & \infty \\ & \infty \end{aligned}$ |  | $\begin{aligned} & \mathbf{w}_{\infty}^{\infty}{ }_{n}^{\infty} \\ & n_{n}^{n} \\ & \sim \end{aligned}$ | $\begin{aligned} & \stackrel{\mu}{\infty} \\ & \stackrel{+}{+} \\ & \stackrel{\rightharpoonup}{\sim} \end{aligned}$ | $$ |
| $\therefore 0^{\circ} 00$ | $\bigcirc 0^{\circ} 0^{\circ}$ | $\bigcirc 0^{\circ \circ}$ | $\dot{\circ}$ | $\dot{0} 0 \dot{0}$ | $\bigcirc 0^{\circ} 0^{\circ}$ | $\bigcirc 0^{\circ} 0^{\circ}$ |

## ＊＊INPUT DATA＊＊

| － | $\stackrel{\rightharpoonup}{0}$ |  | N～0 | N | N | $\stackrel{\square}{0}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $\bigcirc 0^{\circ} 0^{\circ}$ |  |  | $\therefore \stackrel{\substack{u \\ i n \\ n}}{\substack{n \\ 0}}$ |  |
| － | $\bigcirc$ | ～ | N | ～ | $\vec{O}$ |  |
|  | $\begin{gathered} \stackrel{山}{\sim} \\ \stackrel{\sim}{n} \\ \circ \circ \circ \circ \end{gathered}$ | $\begin{gathered} \stackrel{\sim}{m} \\ \stackrel{\sim}{n} \\ \stackrel{\sim}{n} \\ 0 \\ 1 \end{gathered}$ |  | $\therefore \stackrel{\substack{w \\ \sim \\ \sim}}{\substack{w \\ \hline}}$ |  | $\bigcirc 0^{\circ} 0^{\circ}$ |
| $\overrightarrow{0}$ | ～ | $\sim$ | N | 0 |  | N |
| $\begin{gathered} \underset{\sim}{\sim} \\ \underset{\sim}{n} \end{gathered}$ | $\stackrel{\sim}{\sim}$ | $\begin{aligned} & \underset{\infty}{\infty} \\ & \stackrel{n}{a} \end{aligned}$ | $\begin{aligned} & \underset{\infty}{\infty} \\ & \stackrel{n}{n} \\ & \stackrel{n}{n} \end{aligned}$ | － |  | $\underset{\sim}{\text { ¢ }}$ |
| $00^{\circ} 0^{\circ}$ | $\bigcirc 0^{\circ} 0^{\circ}$ | －0．00 | $\therefore 0^{\circ}{ }^{\circ}$ | $\therefore 0^{\circ} 0^{\circ}$ | $\bigcirc 0^{\circ} 0^{\circ}$ | $\bigcirc 0^{\circ} 0^{\circ}$ |



$0.17120 \mathrm{E}-04$












** INPUT DATA **

$-0.2000 E 01$
0.
0.
0.
$\stackrel{\rightharpoonup}{0}$

$\begin{array}{lll}0.1000 E & 01 \\ 0 . \\ 0 . \\ 0 . \\ 0 & \\ -0.2000 E & 01 \\ 0 . \\ 0 . \\ 0 .\end{array}$
O.1000E 01
0.0
0.
0.

0.
0.
0.
0.
$0.1000 E \quad 01$
0.
0.
0.
0
$-0.2000 E \quad$
01
0.
0.
0.
0.1000 E 01
00.
0.0
0.
$0 \circ \circ^{\circ \circ}$
$\dot{\circ O \dot{O}} \dot{\circ} \dot{0} 0 \dot{\circ}$
0.
0.
0.
0.

 -0.1000E 01

-0.0ㅇ

FINITE DIFFERENCE METHOD FOR SANDWICHED SHELL, CORE THICKNESS $=0.90000 \mathrm{E} 00 \mathrm{MBAR}=0.17120 \mathrm{E}-04$



or oo-0.2000E 01
0.2000 E
01
0.
0.

$$
\dot{\circ O O} \quad \dot{\circ} \dot{0} \dot{0}
$$

$$
\dot{\circ \circ \circ \circ}
$$

$$
\dot{\circ} \dot{\circ} \dot{\circ}
$$



$$
\therefore \circ \circ \circ \circ
$$

$$
\dot{0} \dot{0} \quad \dot{0} \dot{\circ} \dot{0}
$$



 0.1000 E
0. .

0. 0. 0. 0. 0.2000 E OI

-0.2000 E OI
0.
0.



 $\qquad$ $\dot{\circ} \dot{\circ} \dot{\circ} \dot{\circ}$

 -0.3962E-01

FINITE DIF FERENCE METHOD FOR SANDWICHED SHELL，CORE THICKNESS $=0.90000 \mathrm{E} 00 \mathrm{MBAR}=0.17120 \mathrm{E}-04$

$-0.8107 E \quad 02$
0.
0.
$0.5792 E \quad 01$
0.
0.
0.
$0.8386 E$
0.
0.
0.
0.
0.
0.
0.
0.
0.
0.
$0.1334 E$



|  | $\stackrel{\square}{0}$ | O | N | NO－ | －20 |  | $\overrightarrow{0}$ | － | Nö |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\begin{aligned} & \underset{\sim}{w} \\ & \underset{\sim}{\sim} \\ & \underset{\sim}{n} \end{aligned}$ | $\begin{aligned} & 山_{\mathbf{D}}^{\infty} \\ & \sim \\ & \sim \end{aligned}$ | $\begin{aligned} & \text { 山 } \\ & \stackrel{\text { ث }}{\circ} \end{aligned}$ | $\begin{aligned} & w 山 \\ & \infty \\ & \sim \\ & \sim \\ & \sim \\ & \sim \end{aligned}$ |  |  | $\begin{aligned} & \text { w } \\ & \infty \\ & \infty \\ & \infty \\ & \infty \end{aligned}$ | $\stackrel{\underset{\sim}{\sim}}{\stackrel{\sim}{\sim}}$ |  |
| －000 | $\bigcirc 0^{\circ} 0^{\circ}$ | i000 | $\bigcirc 0^{\circ} 0^{\circ}$ | ¢00＇ | $\therefore 0$ | $\bigcirc 0^{\circ} 0^{\circ}$ | $\bigcirc 0^{\circ} 0^{\circ}$ | －0． | io |



| N | N | ¢\％O | －0\％ | $\stackrel{\square}{0}$ | $\checkmark$ | $\square$ | N－ | －70 | \％ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| w $\underset{\sim}{\infty}$ $\underset{\sim}{\infty}$ | $\begin{aligned} & \underset{\sim}{\sim} \\ & \stackrel{\rightharpoonup}{*} \\ & \underset{\sim}{2} \end{aligned}$ | $\begin{aligned} & \underset{\sim}{\infty} \underset{\sim}{\sim} \\ & \underset{\sim}{\sim} \\ & \underset{\sim}{\sim} \end{aligned}$ | $\underset{\sim}{\underset{\sim}{u}} \underset{\sim}{\sim}$ | $\begin{aligned} & \underset{\sim}{\sim} \\ & \underset{\sim}{n} \\ & \text { in } \end{aligned}$ | $\begin{aligned} & \text { 山 } \\ & \stackrel{0}{\infty} \\ & \underset{\infty}{\infty} \end{aligned}$ | $\begin{aligned} & \underset{\sim}{\sim} \\ & \underset{\sim}{\sim} \\ & i \end{aligned}$ |  |  | $\begin{aligned} & w \\ & \infty \\ & \stackrel{\rightharpoonup}{w} \\ & \stackrel{\rightharpoonup}{\sim} \end{aligned}$ |
| ¢00 | $00^{\circ}{ }^{\circ}$ | $\dot{0} 0 \dot{0}$ | $\dot{0} 0$ | $\bigcirc 0^{\circ}{ }^{\circ}$ | $\bigcirc 0^{\circ} 0^{\circ}$ | －00 | $\dot{0} 00^{\circ}$ | $\dot{0} 0_{1}^{\circ}$ | $\bigcirc 0^{\circ}$ |

＊＊all matrix＊＊

| ＊＊ | Ruw | 1＊＊ |  |
| :---: | :---: | :---: | :---: |
|  | 0.1 | 443 E |  |
|  | 0. |  |  |
| ＊＊ | ROw | 2＊＊ |  |
|  | －0．2 | 288E |  |
|  | 0.5 | 742E |  |
|  | 0. |  |  |
| ＊＊ | ROW | 3＊ |  |
|  | 0.2 | 824 E |  |
|  | －0．4 | bl3E |  |
|  | 0. |  |  |
| ＊＊ | ROW | 4＊＊ |  |
|  | 0. |  |  |
|  | 0.5 | 792E |  |
|  | 0. |  |  |
| ＊＊ | ROW | b＊ |  |
|  | 0. |  |  |
|  | 0. |  |  |
|  | C． 8 | 3386 E | 01 |
| ＊＊ | RCw | 6＊＊ |  |
|  | 0 ． |  |  |
|  | 0. |  |  |
|  | 0. |  |  |
| ＊＊ | ROW | 7＊＊ |  |
|  | －0．8 | 107E | 02 |
|  | C． 1 | OUOE | 01 |
|  | 0. |  |  |
| \＃＊ | Ruw | 8＊＊ |  |
|  | 0.5 | 792E |  |
|  | －0．1 | 5うdE | 02 |
|  | 0. |  |  |
| ＊＊ | RCW | 9＊ |  |
|  | 0. |  |  |
|  | 0.7 | 948E |  |
|  | 0. |  |  |
| ＊＊ | RCW | 10＊＊ |  |
|  | 0. |  |  |
|  | －0．1 | 558 E | 02 |
|  | 0.5 | 92E |  |


| 0. |  |
| :--- | :--- | :--- |
| $0.5792 E$ | 01 |
| 0. |  |
| 0. |  |
| 0. |  |
| $0.8386 E$ | 01 |
| $0.5792 E$ | 01 |
| 0. |  |
| 0. |  |
| $-0.4513 E$ | 02 |
| $0.1000 E$ | 01 |
| 0. |  |
| $0.5792 E$ | 01 |
| $-0.1558 E$ | 02 |
| 0. |  |
| 0. |  |
| $0.8787 E$ | 02 |
| 0. |  |
| 0. |  |
| $-0.1558 E$ | 02 |
| $0.5792 E$ | 01 |
| 0. |  |
| $0.2000 E$ | 01 |
| $-0.4513 E$ | 02 |
| 0. |  |
| 0. |  |
| 0. |  |
| $0.1677 E$ | 02 |
| 0. |  |
| 0. |  |
| 0. |  |
| $0.1158 E$ | 02 |
| 0. |  |
| 0 |  |







| NO | $\rightarrow{ }_{-}^{\circ}$ |  | - | $\stackrel{-1}{0}$ | Nor | O~0 | N |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\begin{aligned} & \text { u 岗 } \\ & \text { on } \\ & \text { y } \end{aligned}$ |  | $\begin{aligned} & \text { u } \\ & 0 \\ & \infty \\ & \infty \\ & \infty \end{aligned}$ | $\begin{aligned} & \stackrel{\sim}{N} \\ & \stackrel{N}{N} \\ & \stackrel{N}{n} \end{aligned}$ |  | $\begin{aligned} & {\underset{N}{\sim}}_{\sim}^{\sim} \\ & \underset{\sim}{\sim} \\ & \underset{\sim}{n} \end{aligned}$ | $\stackrel{\text { w }}{\sim}$ |  |  |
| $000$ | $\therefore \circ \circ$ | -0ं0́ | -000 | -0. | $\dot{0} \dot{0}$ | -0i | $\therefore 0^{\circ}$ | -0: | $0^{\circ} 0^{\circ}$ |


FINITE DIFFERENCE METHOD FOR SANDWICHED SHELL, CORE THICKNESS $=0.90000 \mathrm{E} 00 \mathrm{MBAR}=0.17120 \mathrm{E}-04$

| ** INPUT DATA ** |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| ** K0h 22** |  |  |  |  |  |  |  |
| 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. |
| 0. | 0.1677 E 02 | 0. | 0. | 0. | 0. | 0.1158 E 02 | -0.9025E 02 |
| 0.1158 t 02 | 0. | 0. | 0.1000E 01 | -0.1558E 02 | 0.7948E 02 | -0.1558E 02 | 0.1000 El |
| ** ROW 23** |  |  |  |  |  |  |  |
| 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. |
| 0. | 0. | $0.1677 E 02$ | 0. | 0. | 0. | 0. | 0.1158 E 02 |
| -0.9025E 02 | 0.1158 E 02 | 0. | 0. | 0.1000 El | -0.1558E 02 | 0.8048 O | -0.1558E 02 |
| ** RUW 24** |  |  |  |  |  |  |  |
| 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. |
| 0. | 0. | 0. | $0.1677 E 02$ | 0. | 0. | 0. | 0. |
| 0.2317 E 02 | -0.9025E 02 | 0. | 0. | 0. | $0.2000 E 01$ | -0.3117E 02 | 0.7948 E 02 |

FINITE DIFFERENCE METHOD FOR SANDWICHED SHELL, CORE THICKNESS $=0.90000 \mathrm{E} 00 \mathrm{MBAR}=0.17120 \mathrm{E}-04$


จั

$\dot{0} \dot{0} \dot{0}$ io $\dot{i}$







ioㅇ
oio jó $0^{\circ \circ}$ 0.8386E 01 ¿우
.:;
** alc Matrix **


FINITE DIFFERENCE METHOD FOR SANDWICHED SHELL, CORE THICKNESS $=0.90000 \mathrm{E} 00 \mathrm{MBAR}=0.17120 \mathrm{E}-04$
 $\dot{\circ} \dot{\circ} \dot{\circ} \dot{\circ} \dot{\circ} \dot{O} \quad \dot{O} \quad \dot{\circ} \dot{\circ} \dot{O} \quad \dot{O} \quad \dot{O} \quad \dot{O}$





© ${ }^{\circ}$ ó
-0'0
$\stackrel{\square}{i}$
oio jóóo joió joió
$\stackrel{+}{d}$
$\stackrel{1}{\otimes}$
$\vdots$
$\vdots$
$\vdots$
$\vdots$
oio

$\therefore \dot{\circ} \dot{\circ}$
oióo oióo jóo jóo oojo




 0.
00
0.
0.
 ooóo ooó 0.
0
0
-0.1
0


ióojo
$0.3836 E-04$
0.
$-0.1918 E-04$。 o̊óoio
** al3 matrix **
** RUW 1 1**
$0.3836 \mathrm{E}-04$
$+0-39161 \cdot 0_{0}$
$\cdot 0$
** ROW 2**

$$
\begin{gathered}
\text { óx } \\
\\
\#
\end{gathered}
$$

FINITE DIFFERENCE METHOD FOR SANDWICHED SHELL, CORE THICKNESS $=0.90000 \mathrm{E} 00 \mathrm{MBAR}=0.17120 \mathrm{E}-04$


FINITE DIFFERENCE METHOD FOR SANDWICHED SHELL, CORE THICKNESS $=0.90000 \mathrm{E} 00 \mathrm{MBAR}=0.17120 \mathrm{E}-04$
-0:0
 1000 0 oo

 $-0.1918 E-04$
00
0.
0.
0.
00
0.
0.
0.
$0.3836 E-04$
00
0.
0.
00
0.
0.
0.
0.
$-0.1918 E-04$
0.
:oo $\dot{0} 0 \dot{0}$








$\dot{\circ} \dot{0} \dot{0} \quad \dot{0} \dot{0} \dot{\circ}$
 $\dot{\circ} \dot{0} \dot{\circ} \dot{\infty} \dot{\infty}$ $=0.90000 \mathrm{E} 00 \mathrm{MBAR}=$
 .o.

FINITE DIFFERENCE METHOD FOR SANDWICHED SHELL，CORE THICKNESS $=0.90000 \mathrm{E} 00 \mathrm{MBAR}=0.17120 \mathrm{E}-04$
$\begin{array}{rrr}-0.5335 E-01 & 0.6062 E 01 & 0.7048 \mathrm{E}-01 \\ 0.3335 \mathrm{E}-01 & 0.7738 \mathrm{E}-02 & -0.2573 \mathrm{E}-01\end{array}$ $-0.2573 E-01$
$-0.1493 E-01$

 $0.1011 \mathrm{E}-00$
$0.1686 \mathrm{E}-01$ $-0.3054 \mathrm{E}-01$ $N$
0
1
$\omega_{0}$
N
0
0
$i$ $0.5275 E-01$
$0.3009 E-02$ -1
1
$\mathbf{1}$
$\vdots$
$\vdots$
$\vdots$
$\dot{0}$
1



 a
1
$u$
$\mathbf{~}$
0
0
$\vdots$
0
$i$ $0.1469 \mathrm{E}-00$
$-0.1394 \mathrm{E}-01$
 30
10
10
$\psi$
0
0
0
0
0
0
0
0
0 3
0
1
1
1
0
0
0
$\infty$
0
0
0
0
0
0
0
1 $-0.1061 \mathrm{E}-01$
$0.7857 \mathrm{E}-01$ 1
0
0
1
0
0
0
0
0
0
 0.6062 E 01
$0.7738 \mathrm{E}-02$
$-0.2535 \mathrm{E}-01$ $0.3091 E 01$
$0.7469 \mathrm{E}-01$
$-0.5300 \mathrm{E}-01$ 0.6211 E 00
$0.9446 \mathrm{E}-01$




 0.4975 E 01
$0.5410 \mathrm{E}-02$
$-0.1834 \mathrm{E}-01$


 -0.1180 E 01
$0.2862 \mathrm{E}-01$ $0.2862 \mathrm{E}-01$
$0.2211 \mathrm{E}-01$
-0.2226 E 01
$-0.7223 \mathrm{E}-01$
$0.9315 \mathrm{E}-01$ －1165E－00
$-0.1165 \mathrm{E}-00$
$0.1040 \mathrm{E}-00$
$-0.1104 \mathrm{E}-01$
$-0.8952 \mathrm{E}-01$
$0.7106 \mathrm{E}-01$
$0.4174 \mathrm{E}-02$


$0.4544 \mathrm{E}-00$
$-0.1325 \mathrm{E}-00$
$0.6111 \mathrm{E}-02$
－0．6111E－02
$-0.3727 \mathrm{E}-01$
$0.3436 \mathrm{E}-01$
$-0.9985 \mathrm{E}-02$
$-0.1103 \mathrm{E}-00$
$0.1255 \mathrm{E}-00$
$-0.7605 \mathrm{E}-02$
 $10-36921^{\circ} 0$
$10-3 \varepsilon \varepsilon 18^{\circ} 0$
$10-3 ゅ 526^{\circ} 0-$
$0.3325 \mathrm{E}-01$
$-0.1405 \mathrm{E}-01$
$0.3520 \mathrm{E}-01$
 $-0.8800 \mathrm{E}-01$
$0.3324 \mathrm{E}-01$ $0.3324 \mathrm{E}-01$ $-0.1659 E-00$
$0.1510 E-01$
$0.3262 E-01$ $-0.9031 \mathrm{E}-01$
$-0.6223 \mathrm{E}-02$
$0.4009 \mathrm{E}-01$
$0.1722 \mathrm{E}-00$
$-0.2619 \mathrm{E}-01$
$0.4003 \mathrm{E}-02$

0.5623 E 00
$-0.4500 \mathrm{E}-01$
$-0.6043 \mathrm{E}-01$
$-0.6418 \mathrm{E}-01$
$0.7006 \mathrm{E}-01$
$0.2255 \mathrm{E}-02$
$-0.1627 \mathrm{E}-00$
$0.3391 \mathrm{E}-01$
$0.3946 \mathrm{E}-01$
 10－369ヶ5．0
 $-0.2310 \mathrm{E}-01$
$0.1246 \mathrm{E}-01$
 $-0.3089 E-01$
$-0.1040 E 01$ -0.1040 E 01
$0.1082 \mathrm{E}-01$ $0.1512 \mathrm{E}-01$
-0.1280 E 01
$0.4396 \mathrm{E}-01$ $0.1338 \mathrm{E}-00$
$-0.2740 \mathrm{E}-01$
$0.3064 \mathrm{E}-01$ $0.2740 \mathrm{E}-00$
0.1828 E 01
$-0.1104 \mathrm{E}-01$ 10
$00-3952 \angle 10^{\circ}$ $-0.5048 \mathrm{E}-01$ $0.1142 \mathrm{E}-00$
0.6644 E 01 0.6644 E 01
$-0.6611 \mathrm{E}-01$ $-0.3342 E-01$
$-0.8590 E 00$




 $0.1591 E-01$
$-0.7794 E-02$

 -0.5554 E 00
$0.1842 \mathrm{E}-00$ $0.6762 E \quad 01$
$-0.7061 E 00$
$0.4061 E-01$



 | -0.2458 E |
| ---: |
| 0.25334 E |
| 01 |
| -0.9244 E | 00

 0.3477 E 01
-0.1026 E 01 $0.4650 \mathrm{E}-00$
$-0.4556 \mathrm{E}-00$ $-0.4556 E-00$
$0.1351 E 01$
 ä
0
1
0
0
0
0
0
0
0
0
0
0 $-0.5341 E-01$
$-0.3227 E-01$



$10-300 \varepsilon 2^{\circ} 0-$
$1039065^{\circ} 0$
$-0.2300 \mathrm{E}-01$
$-0.3024 \mathrm{E}-01$
0.1105 E 02
$-0.7413 \mathrm{E}-02$
$-0.6972 \mathrm{E}-01$
 $10-31 \varepsilon 566^{\circ} 0$
$00-\exists 25$ カカ $20-35555^{\circ} 0$
$10-31 E 56^{\circ} 0$
$00-375 ヶ 力$
 $0.6312 \mathrm{E}-01$
0.1068 E
 -10
08
10
0
0

0
0
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0
0.0


 ． 46 ． 0
0
0
0
0
0
0
0
0
0 $10-39699^{\circ} 0-$
$10-3659 \varepsilon^{\circ} 0$ $-0.3128 \mathrm{E}-00$ C． $9642 \mathrm{E}-01$ 10－389を苼0 -0.4617 E 01
$0.5682 \mathrm{E}-01$
$0.1173 \mathrm{E}-00$ final matrix ＊＊Row 1＊＊
 －0．9111E－01 ＊＊ROW 2＊＊ $-0.3981 \mathrm{E}-00$ RCW 3＊＊ 07
1
4
4
0
0
0
5
5
5
 ROW 4＊＊
 $0.2347 \mathrm{E}-01$
$0.3476 \mathrm{E}-01$




 $0.11825-00$ －0．8435E OC ＊＊RCW 8＊＊ $0.8154 \mathrm{E}-01$ 0.8154600
-0.5436 E 00
$-0.9843 \mathrm{E}-01$ ROW 9 \＃＊＊
$0.2407 \mathrm{E}-01$ 30
0
1
$\omega_{0}$
0
$\pm$
$\pm$
0
0

 ＊＊ROW 11＊＊
FINITE DIFFERENCE METHOD FOR SANDWICHED SHELL, CORE THICKNESS $=0.90000 \mathrm{E} 00 \mathrm{MBAR}=0.17120 \mathrm{E}-04$ $-0.1314 \mathrm{E}-00$ $0.2720 \mathrm{E}-01$
$0.8879 \mathrm{E}-01$ $0.2896 \mathrm{E}-01$
$-0.1828 \mathrm{E}-01$ $-0.9249 \mathrm{E}-02$ $0.1254 \mathrm{E}-00$
$-0.112 \mathrm{E}-01$
$-0.3212 \mathrm{E}-01$ $0.8256 \mathrm{E}-01$
$0.3921 \mathrm{E}-01$
$-0.2819 \mathrm{E}-01$ -0.1398E-01 $0.1063 \mathrm{E}-00$
$0.8432 \mathrm{E}-02$
 $0.5891 \mathrm{E}-01$
$0.6514 \mathrm{E}-01$ $-0.1310 \mathrm{E}-00$
$0.0234 \mathrm{E}-01$
$0.1076 \mathrm{E}-00$ $0.2209 \mathrm{E}-01$
$-0.1735 \mathrm{E}-01$
$-0.8480 \mathrm{E}-02$ $0.1161 \mathrm{E}-00$
$-0.1048 \mathrm{E}-01$
$-0.3204 \mathrm{E}-01$ $0.7879 \mathrm{E}-01$
$0.4178 \mathrm{E}-01$ -0.2811E-01 $-0.1531 \mathrm{E}-01$
$0.1084 \mathrm{E}-00$
$0.9440 \mathrm{E}-02$
 -0.2585 E 01
$-0.1138 \mathrm{E}-00$ $-0.138 \mathrm{E}-00$
$0.1138 \mathrm{E}-00$ 0.3541 E 01
$0.58823-02$
$-0.1615 \mathrm{E}-01$ $0.2031 E 01$
$0.9193 E-01$
$-0.4807 E-01$ $0.3164 \mathrm{E}-00$
$0.1450 \mathrm{E}-00$ -0.1187 E 01
$0.3924 \mathrm{E}-01$
$0.3078 \mathrm{E}-01$ -0.2191 E 01
$-0.6927 \mathrm{E}-01$
$0.1143 \mathrm{E}-00$ $-0.2541 E 01$
$-0.1126 \mathrm{E}-00$
$0.1303 \mathrm{E}-00$ 0.2699 E 01
$0.4835 \mathrm{E}-02$
$-0.1492 \mathrm{E}-01$
 $0.2578 \mathrm{E}-00$
$0.1470 \mathrm{E}-00$
$-0.3342 \mathrm{E}-01$ -0.1192 E 01
$0.4202 \mathrm{E}-01$
$0.3559 \mathrm{E}-01$ $0.3352 \mathrm{E}-00$
$-0.1289 \mathrm{E}-00$ $-0.3118 \mathrm{E}-01$
$0.4241 \mathrm{E}-01$
$-0.8975 \mathrm{E}-02$ $-0.1082 \mathrm{E}-00$
$0.1523 \mathrm{E}-00$
$-0.5155 \mathrm{E}-02$










0.4328E-00

$-0.5550 \mathrm{E}-01$
$0.11162 \mathrm{E}-00$
$0.3532 \mathrm{E}-02$

$-0.1275 E-00$
$0.6901 \mathrm{E}-02$
$0.7348 \mathrm{E}-01$


 $-0.5014 \mathrm{E}-01$
$0.1114 \mathrm{E}-00$
$0.3483 \mathrm{E}-02$
$-0.1644 \mathrm{E}-00$
$0.5267 \mathrm{E}-01$
$0.5181 \mathrm{E}-01$
$-0.1337 E-00$
$0.8461 E-02$
$0.8918 E-01$

 0.6520 E
-01
$-0.6427 \mathrm{E}-01$ $-0.3623 \mathrm{E}-01$
-0.8205 E 00
$0.2244 \mathrm{E}-01$




 0.5804 E 01
$-0.6392 \mathrm{E}-01$
 -0.7789 E 00
$0.2440 \mathrm{E}-01$


 $-0.5147 E$
0.31
$0.3402 E$
01
$-0.1002 E$ $-0.2335 E-00$
$-0.4331 E-00$
-0.2817E 01
$\begin{array}{cc}0.3205 E & 01 \\ -0.7781 E & 00 \\ 0.1202 E & 01\end{array}$
$0.4259 E 01$
$-0.0371 E-00$
$0.1471 E-00$
$\begin{array}{r}0.7662 E 00 \\ 0.6849 \mathrm{E} \quad 00 \\ -0.5098 \mathrm{O} \\ \hline\end{array}$
$\begin{array}{rl}-0.3559 E & 01 \\ 0.2148 E & 01 \\ -0.8761 E & 00\end{array}$






 $-0.6266 \mathrm{E} 01$ $00-35581 \cdot 0$
$10-358 \varepsilon \varepsilon^{\circ} 0$ 0.5807 E 00
$-0.1960 \mathrm{E}-01$
$-0.1847 \mathrm{E}-01$ 0.4681 E 01
$-0.1398 \mathrm{E}-01$
$-0.6416 \mathrm{E}-01$
0.3361 E 01
$0.2864 \mathrm{E}-01$
$-0.5634 \mathrm{E}-01$





 $-0.1684 \mathrm{E}-01$

0.2926 E 01
$0.2504 \mathrm{E}-01$
$-0.5638 \mathrm{E}-01$
-0.1006 E 01
$0.7047 \mathrm{E}-01$
$0.1684 \mathrm{E}-01$ ** ROW 12** $-0.7121 E-01$ ** ROW 2343 Ek ** $0.3268 \mathrm{E}-01$ 0.32615 E 00
-0.815
$-0.3230 \mathrm{E}-01$ ** ROW 14** $0.3831 \mathrm{E}-01$
$-0.7662 \mathrm{E} \quad 00$ $-0.76621 \mathrm{E}-01$
RUW $15 * *$ ** RUN 1573E-02
 $-0.6933 \mathrm{E}-01$
ROW $10 * \%$ ROW $160 * *$
$-0.2981 \mathrm{E}-01$

 -1
0
1
1
0
0
$a$
a
in
0
1 ** ROK 18** $-0.0978 E-01$
$0 . b 107 E 01$ $0.31651 \mathrm{E}-00$
** KCh $19 * *$ $0.7069 E-02$
$-0.886 E 00$
$-0.2909 E-01$ -0.858 E
-0.2909E-01
** RUW 20***
 $0.2458 \mathrm{E}-01$
0.8713 E 00
$0.9575 \mathrm{E}-01$ RCH 21 2**
$0.3494 \mathrm{E}-02$ $0.3494 \mathrm{E}-0$
$0.4281 \mathrm{E}-00$ $0.42815-00$
-0.6876t-01
RUW $22 * *$
FINITE DIFFERENCE METHOD FOR SANDWICHED SHELL, CORE THICKNESS $=0.90000 \mathrm{E} 00 \mathrm{MBAR}=0.17120 \mathrm{E}-04$

\[

\]

* INPUT DATA **
$-0.9876 \mathrm{E}-01$
$0.6170 \mathrm{E}-01$
$0.6990 \mathrm{E}-01$
$-0.1311 \mathrm{E}-00$
$0.3387 \mathrm{E}-01$
$0.1234 \mathrm{E}-00$


$$
\begin{aligned}
& \begin{array}{r}
-0.2180 E 01 \\
-0.6852 \mathrm{E}-01 \\
0.1310 \mathrm{E}-00 \\
-0.2528 \mathrm{E} 01 \\
-0.1125 \mathrm{E}-00 \\
0.1398 \mathrm{E}-00
\end{array} \\
& \begin{array}{rr}
-0.1282 \mathrm{E}-00 & -0.1125 \mathrm{E} \\
0.1891 \mathrm{E}-01 & 0.1398 \mathrm{E}-00
\end{array} \\
& \begin{array}{rr}
0.4189 \mathrm{E}-01 & 0.2484 \mathrm{E}-00 \\
0.3825 \mathrm{E} 01 & -0.3762 \mathrm{E}-01 \\
-0.4768 \mathrm{E}-01 & -0.3334 \mathrm{E}-01 \\
0.2017 \mathrm{E}-01 & 0.3231 \mathrm{E}-00 \\
0.5485 \mathrm{E} 01 & -0.4313 \mathrm{E}-01 \\
-0.6392 \mathrm{E}-01 & -0.5614 \mathrm{E}-01
\end{array} \\
& \begin{array}{l}
01 \\
01 \\
00
\end{array} \\
& 7080
\end{aligned}
$$

$$
\begin{aligned}
& \begin{array}{r}
-0.4782 \mathrm{E} 01 \\
0.4430 \mathrm{E}-01 \\
0.1303 \mathrm{E}-00 \\
-0.6282 \mathrm{E} 01 \\
0.2473 \mathrm{E}-01 \\
0.2153 \mathrm{E}-00
\end{array} \\
& \begin{array}{l}
\text { ** RUW 23** } \\
-0.5892 \mathrm{E}-01 \\
0.3388 \mathrm{E} 01 \\
0.2286 \mathrm{E}-00 \\
\text { ** ROW } 24 * * \\
-0.6930 \mathrm{E}-01 \\
0.4761 E 01 \\
0.2607 E-00
\end{array}
\end{aligned}
$$

$0.17120 \mathrm{E}-04$
THICKNESS $=0.90000 \mathrm{E} 00 \mathrm{MBAR}=$


을
column


$\infty$

FINITE DIFFERENCE METHOD FOR SANDWICHED SHELL, CORE THICKNESS $=0.90000 \mathrm{E} 00 \mathrm{MBAR}=0.17120 \mathrm{E}-04$


FINITE DIFFERENCE METHOD FOR SANDWICHED SHELL，CORE THICKNESS $=0.90000 \mathrm{E} 00 \mathrm{MBAR}=0.17120 \mathrm{E}-04$

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[^0]:    NORTHROP CORPORATION, Norair Division 3901 West Broadway
    Hawthorne, California
    Chintsun Hwang, W.S. Pi (Authors)

[^1]:    

[^2]:    *Note: For a stiffener which is placed along one boundary of the panel, double the values of the corresponding cross sectional area and the moment of inertia as input.

[^3]:    *The writeup and subroutine deck of "MITER" are available at IBM SHARE general program library.

[^4]:    AB(1, $(\mathbb{N})=0$

[^5]:    ＊＊FOR MULTIPLICATION OF REAL MATRICES
    AB $=A * B$
    OIMENSION $A(34,24), B(24,20), A B(34,2), A B F(34)$

    $$
    J=K L
    $$

