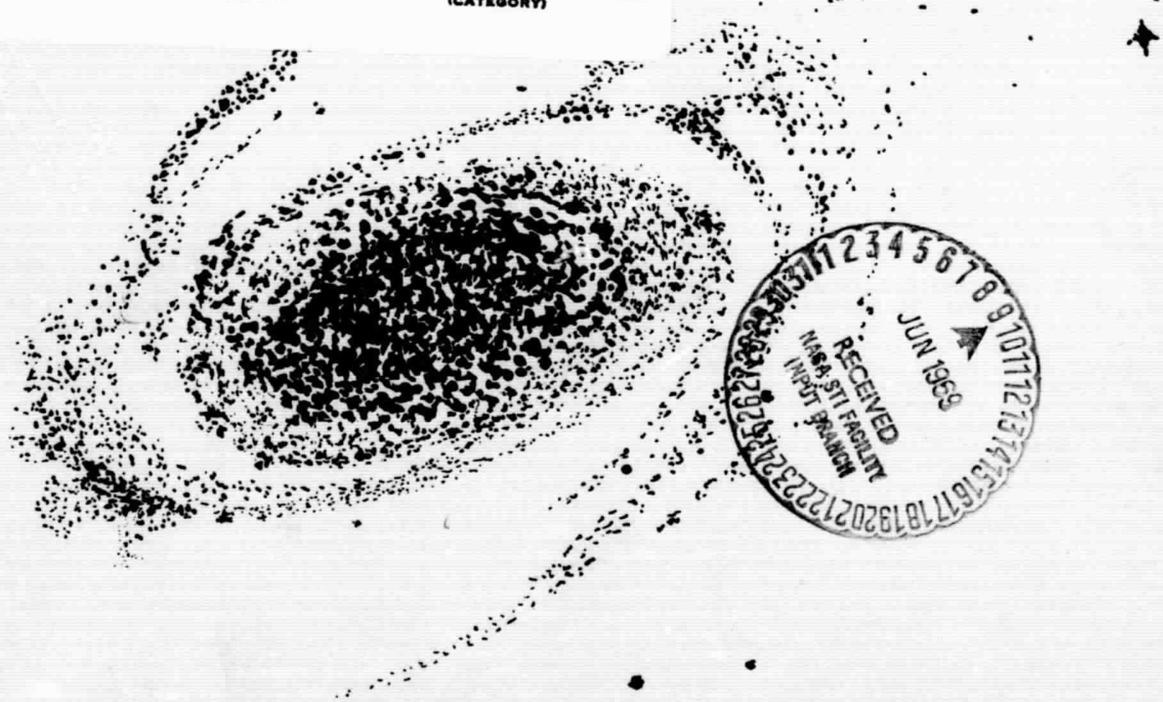
SELENOCENTRIC AND LUNAR TOPOCENTRIC SPHERICAL COORDINATES

B. KOLACZEK



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ABSTRACT

The paper contains a short outline of the Moon's spherical astronomy. The problems of the mean and of the apparent selenocentric and lunar topocentric spherical coordinates is treated deeply. The advantages and disadvantages of the different spherical coordinate systems, such as selenoequatorial, geoequatorial, ecliptic, stellar, and Moon's horizontal, for orientation in space from the Moon are discussed. The necessary formulas are given to calculate the mean and the apparent positions of stars and other celestial bodies in each of the coordinate systems, regarded equally as selenocentric or lunar topocentric. The appendices contain short descriptions of all the phenomena related to the discussed coordinate systems: the Moon's precession and nutation, lunar aberrations and lunar parallaxes; the general aberrational and parallactic formulas are also given.

RÉSUMÉ

Le mémoire contient un court schéma de l'astronomie sphérique de la lune. Nous traitons à fond le problème des coordonnées sphériques sélénocentriques et topocentriques lunaires moyennes et apparentes. En vue d'une orientation dans l'espace à partir de la lune, nous discutons des avantages et désavantages de différents systèmes de coordonnées sphériques, par exemple le système sélénoéquatorial, le système géoéquatorial, le système de l'écliptique, le système stellaire, et le système du plan horizontal de la lune. Nous donnons les formules nécessaires pour calculer les positions moyennes et apparentes des étoiles et autres corps célestes dans chacun des systèmes de coordonnées, considérés à la fois comme sélénocentriques et topocentriques lunaires. Les appendices contiennent de courtes descriptions de tous les phénomènes rattachés aux systèmes de coordonnées discutés: la précession et la nutation de la lune, les aberrations et les parallaxes lunaires; nous donnons aussi les formules générales d'aberrations et de parallaxes.

КОНСПЕКТ

В этой статье приводится краткий очерк сферической астрономии Луны. Проблемы связанные со средними и кажущимися селеноцентрическими и лунными топоцентрическими сферическими координатами детально изучаются. Обсуждаются преимущества и недостатки других сферических координат, таковых как селеноэкваториальных, геоэкваториальных, эклиптических, звездных и лунных горизонтальных, для ориентировки в пространстве от Луны. Приведены необходимые формулы для вычисления средних и кажущихся положений звезд и других небесных тел в каждой системе координат, рассматриваемой одинаково как селеноцентрической или лунной топоцентрической. Приложения содержат краткие описания всех явлений связанных с обсуждаемыми системами координат: лунной прецесии и нутации, лунных абераций и лунных параллаксов; приведены также общие аберационные и параллактические формулы.

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B. Kolaczek

1. INTRODUCTION

The possibility in the near future of man landing on the Moon focuses our attention on the problem of orientation in space from places other than the Earth, e.g., artificial satellites, the Moon, and the planets.

The usefulness, advantages, and disadvantages of the coordinate systems such as the geoequatorial, the ecliptic, and the selenoequatorial, which make possible orientation in space, are different on the Moon than on the Earth because the Moon's position in space and its motions are different from the Earth's.

The selenoequatorial coordinate system (Section 2), in which the Moon's axis of rotation is the basic direction and the lunar equator is the basic plane, is affected by the motions of this axis and this plane, i.e., the lunar precession and the lunar nutation (or physical libration) (Appendices A and B). The selenoequatorial system on the Moon is analogous to the geoequatorial system for terrestrial observers, but the selenoequatorial coordinates of the stars change their values faster than do the geoequatorial coordinates because the lunar precession is about 1360 times faster than the Earth's.

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The small inclination of the lunar equator to the ecliptic, which is approximately equal to 1°32', makes the ecliptic coordinate system (Section 3) much more practical on the Moon than on the Earth. At the same time, the geoequatorial coordinate system (Section 4) becomes, for the same reason, very inconvenient for practical use on the Moon. However, the great number of catalogs, maps, and almanacs giving the coordinates of the stars and other celestial bodies in the geoequatorial system is its great advantage.

The stellar coordinate system, introduced in Section 5, is defined on the basis of the known heliocentric directions to the chosen stars. It does not change its position in space so quickly as the other systems. The coordinates of the stars in this system change their values only because of their proper motions, which are rather small. The comparison of star observations made in remote epochs is also an advantage of this system.

In this work we give all the formulas necessary to calculate the mean and the apparent positions of the stars and other celestial bodies in each of the aforementioned coordinate systems — regarded either as selenocentric or as lunar topocentric.

The translation of any coordinate system from one point in space to another, e.g., from the Earth's center of mass to the center of mass of the Moon or of the Sun, changes the values of the spherical coordinates of fixed points on the celestial sphere. This change is caused by two phenomena: the parallax of the translation of the origin of a coordinate system and the aberration caused by the different motion of this newly translated coordinate system (Appendices C and D).

Thus, the transformation of the mean geocentric or heliocentric coorinates, such as ecliptic, geoequatorial, or selenoequatorial, into mean selenocentric coordinates, and conversely, requires consideration only of the influence of the Moon's monthly or of the Moon's annual parallax (Appendix E). The transformation of the apparent coordinates, however, requires consideration not only of the parallaxes of the translations but also of the influence of the Moon's aberrations (Appendix F). The lunar daily parallax and daily aberration must be added in the transformation of the selenocentric coordinate systems into the lunar topocentric ones.

The lunar horizontal coordinate system (Section 6), defined similarly to the Earth's, has the same meaning on the Moon as the Earth's horizontal system has on the Earth. Hence, the apparent motion of the celestial sphere on the Moon can be described by the same equations as those for the apparent motion of the sphere on the Earth. We need only substitute in these equations the selenoequatorial for the geoequatorial and geohorizontal coordinates.

2. SELENOEQUATORIAL COORDINATE SYSTEM

2.1 Definition; Transformation of Mean Geoequatorial into Mean Selenoequatorial Coordinates

The selenocentric equatorial system* (Jakowkin, Demenko, and Miz, 1964; Gurevich, 1965, 1967), in which the Moon's axis of rotation is the basic axis, has the same meaning on the Moon as the Earth's equatorial system has for terrestrial observers. The selenoequatorial coordinates are:

a^{Ω} ..... Lunar right ascension measured on the lunar equator from its ascending node on the ecliptic. The value of a^{Ω} changes very quickly because of the precession of the Moon's axis of rotation (Appendix A); hence, it is sometimes more convenient to use angle a^{Υ}, which is the lunar right ascension measured first from the vernal equinox on the ecliptic and then from the ascending node on the lunar equator.

The relation between these two lunar right ascensions is given by

$$a^{\Omega} = a^{\Upsilon} - (12 + \psi') \quad , \quad a^{\Upsilon} = a^{\Omega} + 12 + \psi' \quad , \tag{1}$$

where $\psi' = \Omega + \sigma$, and ψ', Ω are the longitudes of the descending node of the lunar equator and of the ascending node of the lunar orbit, respectively; σ denotes the physical libration in the node (Appendix B).

*Hereafter, unless specially noted, we will omit the term selenocentric. We neglect the parallax of the Earth-Moon distance, which is possible in the case of stars, when the mean selenocentric coordinates of the geo-equatorial, selenoequatorial, and ecliptic systems are the same as the mean geocentric ones. This parallax will be described in Appendix D.

d. . . . . Lunar declination, which is the angular distance from the lunar equator measured on a declination circle from 0° to 90°. It has a positive (+) sign on the Northern Hemisphere and a negative (-) sign on the Southern.

The mean selenoequatorial coordinates of a star, a_m^{Ω}, d_m, or a_m^{Υ}, d_m, are the selenocentric equatorial coordinates without the influence of the Moon's monthly aberration and physical libration (Appendices B and F) and can be obtained from the geoequatorial coordinates in two ways: 1. indirect transformation, by the use of the ecliptic coordinates, and 2. direct transformation.

1. First, the mean geoequatorial coordinates α, δ are transformed into the mean ecliptic coordinates λ, β by the well-known formulas

$$\begin{aligned} \sin\beta &= \cos\epsilon \sin\delta - \sin\epsilon \cos\delta \sin\alpha \ , \\ \cos\beta \cos\lambda &= \cos\delta \cos\alpha \ , \\ \cos\beta \sin\lambda &= \sin\epsilon \sin\delta + \cos\epsilon \cos\delta \sin\alpha \ , \end{aligned} \tag{2}$$

where ϵ is the obliquity of the ecliptic.

Next, the transformation formulas of ecliptic coordinates into seleno-equatorial coordinates can easily be obtained from the astronomical triangle on the selenocentric celestial sphere (Figure 1):

$$\begin{aligned} \sin d_m &= \sin\beta \cos I + \cos\beta \sin I \sin(\lambda - \Omega), \\ \cos d_m \sin\left(\Omega - a_m^{\Upsilon}\right) &= \sin\beta \sin I - \cos\beta \cos I \sin(\lambda - \Omega), \\ \cos d_m \cos\left(\Omega - a_m^{\Upsilon}\right) &= \cos\beta \cos(\lambda - \Omega) \ , \end{aligned} \tag{3}$$

where

I = inclination of the lunar equator to the ecliptic,

Ω = ecliptic longitude of the ascending node of the lunar orbit on the ecliptic. (σ is neglected. If we want to consider it, we have to replace Ω by $\Omega + \sigma$.)

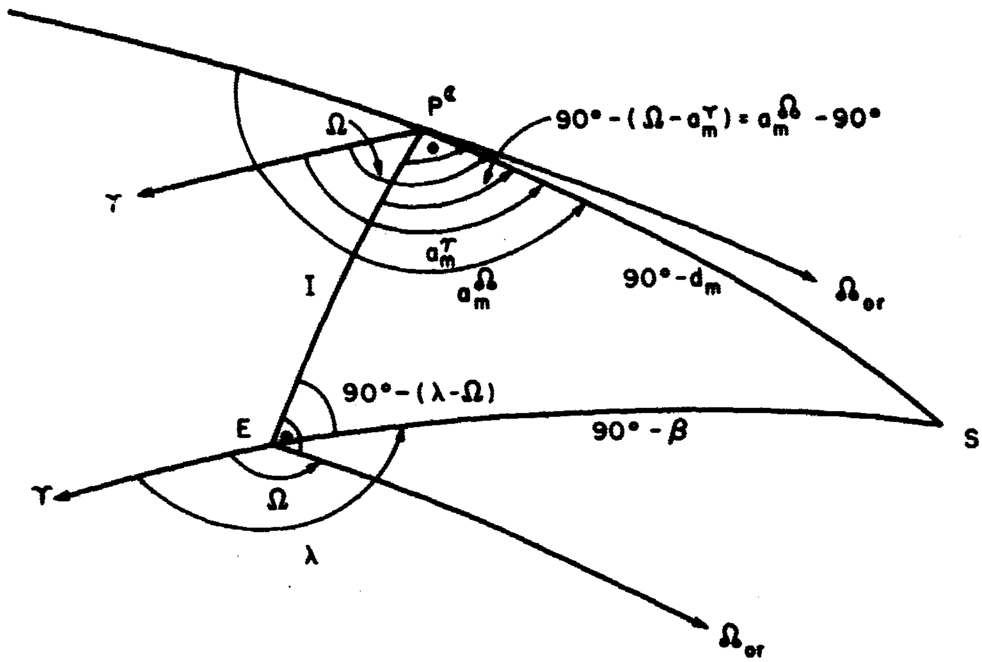

Figure 1. Spherical triangle EP☾S showing the relation of the ecliptic to the selenoequatorial coordinates of a star. In the triangle EP☾S, E = ecliptic pole, P☾ = lunar pole, and S = a star.

The reverse transformation of the mean selenoequatorial into geoequatorial coordinates is given by the formulas:

$$\sin\beta = \cos I \sin d_m + \sin I \cos d_m \sin\left(\Omega - a_m^{\Upsilon}\right) ,$$

$$\cos\beta \sin(\lambda - \Omega) = \sin I \sin d_m - \cos I \cos d_m \sin\left(\Omega - a_m^{\Upsilon}\right) ,$$

$$\cos\beta \cos(\lambda - \Omega) = \cos d_m \cos\left(\Omega - a_m^{\Upsilon}\right) , \qquad (4)$$

and

$$\sin \delta = \cos \epsilon \sin \beta + \sin \epsilon \cos \beta \sin \lambda \quad ,$$

$$\cos \delta \sin \alpha = - \sin \epsilon \sin \beta + \cos \epsilon \cos \beta \sin \lambda \quad ,$$

$$\cos \delta \cos \alpha = \cos \beta \cos \lambda \quad . \tag{5}$$

Equations (3) and (4) for the quantity a_m^{Ω} can be expressed as:

$$\sin d_m = \sin \beta \cos I + \cos \beta \sin I \sin (\lambda - \Omega) \quad ,$$

$$\cos d_m \sin a_m^{\Omega} = \sin \beta \sin I - \cos \beta \cos I \sin (\lambda - \Omega) \quad ,$$

$$\cos d_m \cos a_m^{\Omega} = - \cos \beta \cos (\lambda - \Omega) \quad , \tag{6}$$

and

$$\sin \beta = \cos I \sin d_m + \sin I \cos d_m \sin a_m^{\Omega} \quad ,$$

$$\cos \beta \sin (\lambda - \Omega) = \sin I \sin d_m - \cos I \cos d_m \sin a_m^{\Omega} \quad ,$$

$$\cos \beta \cos (\lambda - \Omega) = - \cos d_m \cos a_m^{\Omega} \quad . \tag{7}$$

2. The second method is the transformation of the geoequatorial into the selenoequatorial coordinates by the use of quantities, given in the almanacs, characterizing the mutual positions of these two systems:

ι = inclination of the mean equator of the Moon to the true equator of the Earth.

Δ = arc of the mean equator of the Moon from its ascending node on the equator of the Earth to its ascending node on the ecliptic of date.

Ω' = arc of the true equator of the Earth from the true equinox of date to the ascending node of the mean equator of the Moon.

According to the notation of triangle $P^{☾}P^{\oplus}S$ (Figure 2), we can write the transformation formulas of these two systems:

$$\sin\delta = \cos\iota\ \sin d + \sin\iota\ \cos d \sin(a^{\Omega} + \Delta) \quad ,$$

$$\cos\delta \sin(\alpha - \Omega') = -\sin d \sin\iota + \cos\iota\ \cos d \sin(a^{\Omega} + \Delta) \quad ,$$

$$\cos\delta \cos(\alpha - \Omega') = \cos d \cos(a^{\Omega} + \Delta) \quad , \tag{8}$$

and, conversely,

$$\sin d = \cos\iota\ \sin\delta - \sin\iota \cos\delta \sin(\alpha - \Omega') \quad ,$$

$$\cos d \sin(a^{\Omega} + \Delta) = \sin\iota\ \sin\delta + \cos\iota \cos\delta \sin(\alpha - \Omega') \quad ,$$

$$\cos d \cos(a^{\Omega} + \Delta) = \cos\delta \cos(\alpha - \Omega') \quad . \tag{9}$$

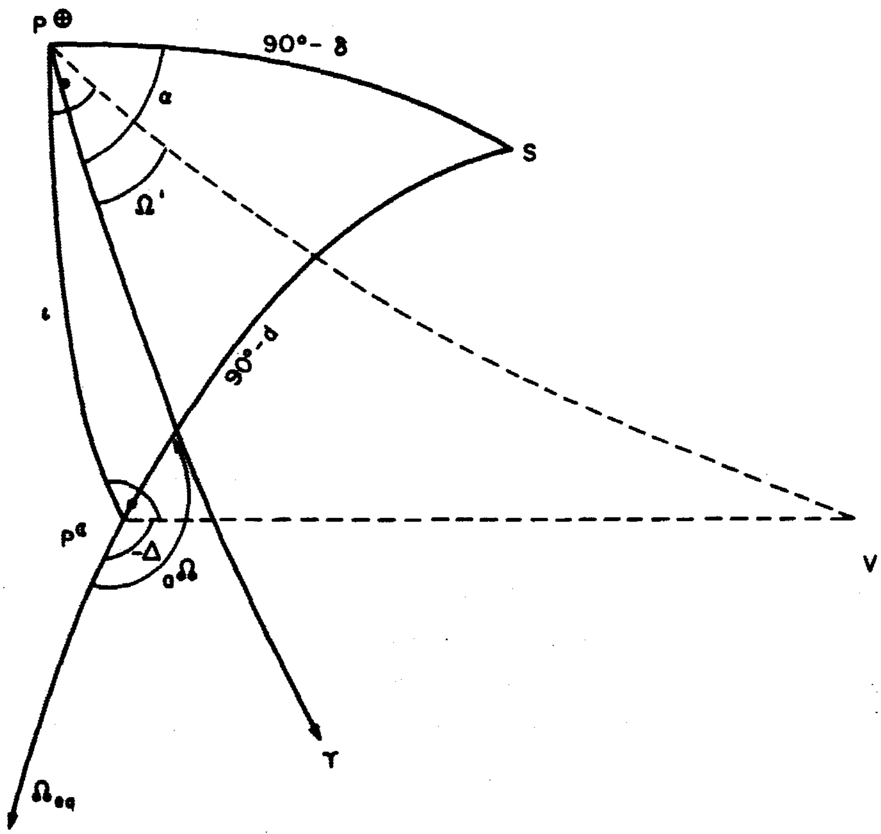

Figure 2. Spherical triangle $P^{☾}P^{\oplus}S$ showing the relation of the geoequatorial to the selenoequatorial coordinates of a star. $\angle P^{☾}P^{\oplus}S = 90° + (\alpha - \Omega')$; $\angle P^{\oplus}P^{☾}S = 90° - (a^{\Omega} + \Delta)$; V = node on the Earth's equator of the lunar equator.

2.2 Transformation of Geo-apparent Geoequatorial into Seleno-apparent Selenoequatorial Coordinates

The transformation of the apparent coordinates is also possible by use of the formulas given above. However, it is necessary to take into account the influence of the lunar monthly aberration and physical libration separately (see Appendices B and F), by replacing I and Ω in equations (4) to (7) by their true values $I + \rho$ and $\Omega + \sigma$, where ρ, σ are the physical librations in the inclination and in the node, respectively. We have another method for this transformation.

The transformation of apparent geoequatorial coordinates into apparent selenocentric selenoequatorial ones can be made in four steps as follows:

1. The apparent geocentric geoequatorial coordinates

$$\alpha_{gapp} = \alpha_0 + V.A._\alpha\, T + V.S._\alpha \frac{T^2}{200} + III_\alpha \left(\frac{T}{100}\right)^3 + (A + A')\, a$$

$$+ (B + B')\, b + Cc + Dd + E + t\,\mu_\alpha + \text{second-order term} \quad ,$$

$$\delta_{gapp} = \delta_0 + V.A._\delta + V.S._\delta \frac{T^2}{200} + III_\delta \left(\frac{T}{100}\right)^3 + (A + A')\, a'$$

$$+ (B + B')\, b' + Cc' + Dd' + t\,\mu_\delta + \text{second-order term} \quad , \qquad (10)$$

are transformed into geo-apparent ecliptic coordinates by formulas (2). The notation in (10) is the usual one, where V.A. is the annual variation; V.S. is the secular variation; A, B, C, D, E are the Besselian Day numbers; $\mu\alpha$, $\mu\delta$ are the proper motion for a star in α, δ; T is the time in tropical centuries; and t is the time in tropical years.

2. The influence of the lunar monthly aberration on the apparent geocentric ecliptic coordinates $\lambda_{gapp}, \beta_{gapp}$ is calculated from formulas (C-7) and (C-8) to give $\lambda_{sapp,2'}$, $\beta_{sapp,2'}$ which are now selenoapparent

3. The $\lambda_{sapp,2'}$, $\beta_{sapp,2}$ are transformed into pseudo-apparent selenoequatorial coordinates $a^{\Omega}_{sapp,2'}$, $d_{sapp,2'}$ by formulas (6). These coordinates can be expressed as

$$a^{\Omega}_{sapp,2'} = \underbrace{a^{\Omega}_{m} + \text{influence of lunar precession}}_{a^{\Omega}_{sapp,1}} + \text{influence of lunar monthly aberration,}$$

$$d_{sapp,2'} = \underbrace{d_{m} + \text{influence of lunar precession}}_{d_{sapp,1}} + \text{influence of lunar monthly aberration.} \tag{11}$$

4. The influence of the Moon's physical libration is calculated from formulas (B-7) to (B-9).

Hence, the apparent selenocentric selenoequatorial coordinates are

$$a^{\Omega}_{sapp} = a^{\Omega}_{sapp,\,2'} + \text{influence of the Moon's physical libration,}$$

$$d_{sapp} = d_{sapp,\,2'} + \text{influence of the Moon's physical libration .} \tag{12}$$

Steps 3 and 4 can be reversed. In this case, it is not important what corrections, aberrational or nutational, we consider first. Hence,

$$a^{\Omega}_{sapp,2} = a^{\Omega}_{sapp,1} + \text{influence of the Moon's physical libration,}$$

$$d_{sapp,2} = d_{sapp,1} + \text{influence of the Moon's physical libration,} \tag{13}$$

and

$$a^{\Omega}_{sapp} = a^{\Omega}_{sapp,2} + \text{influence of lunar monthly aberration,}$$

$$d_{sapp} = d_{sapp,2} + \text{influence of lunar monthly aberration.} \tag{14}$$

The reverse transformation of the coordinates a^{Ω}_{sapp}, d_{sapp} into the coordinates α_{gapp}, δ_{gapp} is made in the following three steps:

1. The influence of the lunar monthly aberration is eliminated from a^{Ω}_{sapp}, d_{sapp} and the coordinates $a^{\Omega}_{sapp,2}$, $d_{sapp,2}$ are obtained from equations (C-6).

2. The influence of the physical libration is eliminated from the coordinates $a^{\Omega}_{sapp,2}$, $d_{sapp,2}$ by equations (B-7) to (B-9).

3. The $a^{\Omega}_{sapp,1}$, $d_{sapp,1}$ are transformed into geo-apparent ecliptic coordinates by formulas (7) and then into geo-apparent geoequatorial coordinates by formulas (5). These coordinates are understood to be geocentric for stars. Steps (1) and (2) can be reversed.

This reverse transformation can also be made by another method:

1. Transform the coordinates a^{Ω}_{sapp}, d_{sapp} by formulas (7), in which I and Ω are replaced by $I + \rho$ and $\Omega + \sigma$ (Appendix B).

2. Eliminate the influence of the lunar monthly aberration by (C-7) to (C-8).

3. Transform the coordinates λ_{gapp}, β_{gapp} into α_{gapp}, δ_{gapp} by formulas (5).

2.3 Calculation of the Mean Selenoequatorial Coordinates

According to Cassini's laws, the Moon's celestial poles and the plane of the lunar equator change their positions in space. They make one revolution about the ecliptic pole axis in approximately 18.6 years. Thus, not only the apparent but also the mean selenoequatorial coordinates a^{Ω}_{m}, d_{m} change their values quickly because of the Moon's precession, which is about 1360 times faster than the Earth's. In this case, the linear interpolation of the mean selenoequatorial coordinates for a period as long as a year is inaccurate, and second-order terms — variatio secularis — must be considered. In a manner similar to the way we calculate the Earth's equatorial coordinates, we can write (see Appendix A):

$$a_m^{\Omega} = a_0^{\Omega} + \left(M_0^d + N_0^d \sin a_0^{\Omega} \tan d_0\right) t$$

$$+ \left[\frac{1}{2}\left(N_0^d\right)^2 \sin 2 a_0^{\Omega} + M_0^d N_0^d \cos a_0^{\Omega} \tan d_0 + \left(N_0^d\right)^2 \sin 2 a_0^{\Omega} \tan^2 d_0\right] \frac{t^2}{2}$$

$$+ \mu_a \tau_a \quad ,$$

$$d_m = d_0 + N_0^d \cos a_0^{\Omega} t - \left[N_0^d M_0^d \sin a_0^{\Omega} + \left(N_0^d\right)^2 \sin^2 a_0^{\Omega} \tan d_0\right] \frac{t^2}{2}$$

$$+ \mu_d \tau_a \quad , \tag{15}$$

where

a_m^{Ω}, d_m = mean selenoequatorial coordinates for a given moment of time t expressed in the number of ephemeris days from the beginning of the tropical year,

a_0^{Ω}, d_0 = mean selenoequatorial coordinates for the beginning of a year,

M_0^d, N_0^d = daily precession in the lunar right ascension and declination, respectively; $M_0^d = - P_0^d \cos I$, $N_0^d = P_0^d \sin I$, where P_0^d is the daily variation in the longitude of the lunar orbital ascending node and $P_0^d = - 0.^{\circ}0529539222$,

τ_a = part of a tropical year,

μ_a, μ_d = proper motion in lunar right ascension and declination.

The linear interpolation of the mean coordinates is possible if they are calculated by the rigorous theoretical formulas for a much shorter period of time, for instance, 10 days.

The approximate formulas, given by Jakowkin et al. (1964), are

$$a_m^{\Omega} = Y^* + (90° - X^*) \quad ,$$

$$\sin d_m = \cos\theta \sin d_0 + \sin\theta \cos d_0 \sin(a_0^{\Omega} - X^*) \quad ; \tag{16}$$

θ, X^*, Y^* are additional angles (see Figure 3) calculated from the formulas

$$\sin\frac{\theta}{2} = \sin I \sin\frac{\psi^{☾}}{2} \quad ,$$

$$\cot X^* = \cos I \tan\frac{\psi^{☾}}{2} \quad ,$$

$$\tan Y^* = \frac{\cos d_0 \cos\left(a_0^{\Omega} - X^*\right)}{\sin\theta \sin d_0 - \cos\theta \cos d_0 \sin\left(a_0^{\Omega} - X^*\right)} \quad , \tag{17}$$

where $\psi^{☾} = P_0^d\,(t_m - t_0)$.

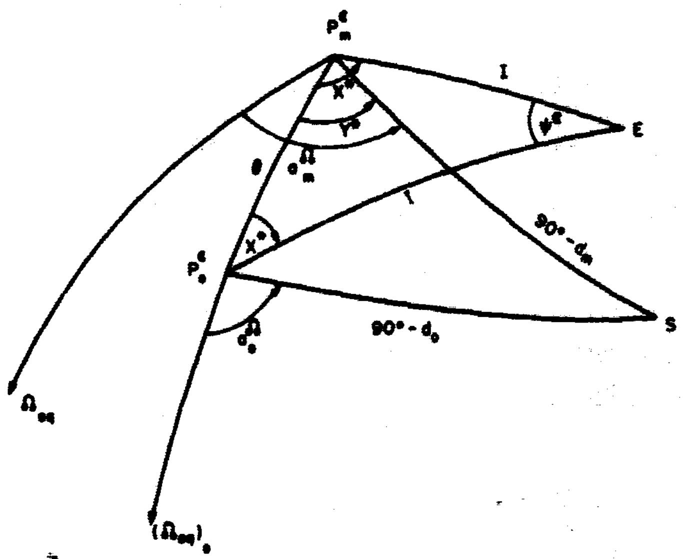

Figure 3. The Moon's precession in selenoequatorial coordinates. E is the ecliptic pole; $P_m^{☾}$ and $P_0^{☾}$ are the lunar celestial poles at moments t_m and t_0, respectively.

2.4 Transformation of Mean into Seleno-apparent Selenoequatorial Coordinates

The equations for this transformation can be given in a form similar to that used for the Earth's equatorial coordinates (Gurevich, 1965):

$$a^{\Omega}_{sapp} = a^{\Omega}_{m} + A^{☾}a^{☾} + B^{☾}b^{☾} + C^{☾}c^{☾} + D^{☾}d^{☾} + \tau_a \mu_a \quad ,$$

$$d_{sapp} = d_m + A^{☾}a'^{☾} + B^{☾}b'^{☾} + C^{☾}c'^{☾} + D^{☾}d'^{☾} + \tau_a \mu_d \quad . \qquad (18)$$

The meaning of these terms is the same as for the Earth's equatorial coordinates.

According to equations (A-6) and (B-8), the influence of the lunar precession and lunar nutation (physical libration) on the selenoequatorial coordinates is expressed as

$$a^{\Omega}_{sapp,3} - a^{\Omega}_{m} = A^{☾}a^{☾} + B^{☾}b^{☾} = \rho \tan d_m \cos a^{\Omega}_{m}$$

$$- \left(P_0^d t + \sigma\right)\left(\cos I - \sin I \tan d_m \sin a^{\Omega}_{m}\right) \quad ,$$

$$d_{sapp,3} - d_m = A^{☾}a'^{☾} + B^{☾}b'^{☾} = -\rho \sin a^{\Omega}_{m} + \left(P_0^d t + \sigma\right)$$

$$\times \sin I \cos a^{\Omega}_{m} \quad , \qquad (19)$$

where $a^{\Omega}_{sapp,3}$, $d_{sapp,3}$ denote the $a^{\Omega}_{sapp,2}$, $d_{sapp,2}$ [equation (13)], but without the influence of the Moon's annual aberration. Thus, we have

$$A^{☾} = -\left(P_0^d t + \sigma\right) \sin I \quad , \qquad B^{☾} = \rho \quad ,$$

$$a^{☾} = +\frac{1}{15}\left(\cot I - \sin a^{\Omega}_{m} \tan d_m\right) \quad ; \qquad b^{☾} = +\frac{1}{15}\left(\cos a^{\Omega}_{m} \tan d_m\right) \quad ,$$

$$a'^{☾} = -\cos a_m \quad , \qquad b'^{☾} = -\sin a^{\Omega}_{m} \qquad (20)$$

where

P_0^d = daily motion of the ascending node of the Moon's orbit on the ecliptic,

t = number of ephemeris days from the epoch t_0 of the mean coordinates a_m^{Ω}, d_m,

I = inclination of the Moon's equator to the ecliptic,

ρ, σ = physical librations in I and in ascending node Ω, respectively.

The appropriate formulas for the influence of the lunar annual aberration on the selenoequatorial coordinates are as follows [(C-14), (C-15)]:

$$a_{sapp}^{\Omega} - a_{sapp,3}^{\Omega} = C^{☾}c^{☾} + D^{☾}d^{☾} = -K \sec d_m \left[\cos(L_{ap} - \Omega + 180°) \sin a_m^{\Omega} - \sin(L_{ap} - \Omega + 180°) \cos a_m^{\Omega} \cos I\right] ,$$

$$d_{sapp} - d_{sapp,3} = C^{☾}c'^{☾} + D^{☾}d'^{☾} = -K \left[\cos(L_{ap} - \Omega + 180°) \cos a_m^{\Omega} \sin d_m + \sin(L_{ap} - \Omega + 180°) \times \left(\cos d_m \sin I + \sin d_m \sin a_m^{\Omega} \cos I\right)\right] . \quad (21)$$

Hence,

$$C^{☾} = -K \sin(L_{ap} - \Omega + 180°) \cos I , \quad D^{☾} = -K \cos(L_{ap} - \Omega + 180°) ,$$

$$c^{☾} = -\frac{1}{15}\left(\cos a_m^{\Omega} \sec d_m\right) , \quad d^{☾} = \frac{1}{15}\left(\sin a_m^{\Omega} \sec d_m\right) ,$$

$$c'^{☾} = \sin a_m^{\Omega} \sin d_m + \tan I \cos d_m , \quad d'^{☾} = \cos a_m^{\Omega} \sin d_m , \quad (22)$$

where

$$L_{ap} = L_{\odot} - 90° + \Delta A \ ,$$

$$K = k\left[1 - \frac{V_1^{☾}}{V^{\oplus}} \cos\left(L_{\odot} - \ell_{☾}\right)\right] \ ,$$

$$\tan \Delta A = \frac{V_1^{☾} \sin\left(L_{\odot} - \ell_{☾}\right)}{V^{\oplus} - V_1^{☾} \cos\left(L_{\odot} - \ell_{☾}\right)} \ , \qquad (23)$$

and

$V^{\oplus}$, $V_1^{☾}$ = velocity of the Earth's and the Moon's orbital motion, respectively,

$L_{\odot}$, $\ell_{☾}$, L_{ap} = longitude of the Sun, the Moon, and the Moon's apex in the motion around the Sun, respectively,

k, K = constants of the Earth's and the lunar annual aberration, respectively.

3. ECLIPTIC COORDINATE SYSTEM

3.1 Introduction

The ecliptic coordinate system was commonly used in astronomy in previous centuries, but was replaced by the geoequatorial system as being more practical on the Earth.

The ecliptic system, however, can be very useful for all astronomical observations that will be made from the Moon's surface. The small inclination of the lunar equator to the ecliptic, which is about $1^\circ.5$, makes this system more convenient for observers on the Moon than it is for terrestrial observers. The star coordinates in this system do not change so rapidly as the selenoequatorial coordinates, and they can be used instead of the latter for a rough orientation on the Moon's celestial sphere.

3.2 Mean Ecliptic Coordinates

The influence of the parallax of the Moon—Earth distance is neglected for the stars (Appendix D); hence, the mean geocentric ecliptic coordinates are the same as the mean selenocentric ecliptic coordinates.

There is a difference, however, in the apparent coordinates caused by the lunar monthly aberration, so we will distinguish between the seleno-apparent ecliptic and the geo-apparent ecliptic coordinates.

The mean selenocentric ecliptic coordinates for some epoch T_0 can be obtained by the transformation of the mean geoequatorial coordinates for this epoch from the well-known equations:

$$\sin\beta = \cos\epsilon \sin\delta - \sin\epsilon \cos\delta \sin\alpha ,$$

$$\cos\beta \cos\lambda = \cos\alpha \cos\delta ,$$

$$\cos\beta \sin\lambda = \sin\epsilon \sin\delta + \cos\epsilon \cos\delta \sin\alpha .$$

Similarly, as in the case of geoequatorial coordinates, we can obtain the mean ecliptic coordinates for another epoch T_1 by using Taylor's series:

$$\lambda_m = \lambda_0 + (T_1 - T_0)\, V.A._\lambda + \frac{(T_1 - T_0)^2}{200}\, V.S._\lambda ,$$

$$\beta_m = \beta_0 + (T_1 - T_0)\, V.A._\beta + \frac{(T_1 - T_0)^2}{200}\, V.S._\beta , \qquad (24)$$

where T_0 is the initial epoch, and $V.A._\lambda$, $V.S._\lambda$, and $V.A._\beta$, $V.S._\beta$ are the annual and secular variations of λ and β, respectively.

The third-order terms in these equations can be omitted when T_0 and T_1 are not very remote, which will apply in the case of first observations made from the Moon. We can calculate the annual and secular variations in two ways:

1. by transforming the known values of the annual and the secular variations in right ascension and declination into the variations in longitude and latitude;

2. by using the appropriate theoretical equations for the precessional variations in longitude and latitude.

1. The first method seems to be easier, especially because the star's proper motion, which is known for many stars only in the geoequatorial coordinate system, must also be transformed.

The differential equations of the transformation are

$$\cos\beta\, d\lambda = \cos\eta \cos\delta\, d\alpha + \sin\eta\, d\delta \quad ,$$

$$d\beta = -\sin\eta \cos\delta\, d\alpha + \cos\eta\, d\delta \quad , \qquad (25)$$

where

$$\sin\eta = \cos\lambda \sec\delta \sin\epsilon = \cos\alpha \sec\beta \sin\epsilon \quad ,$$

$$\cos\eta = \frac{\cos\epsilon}{\cos\delta\cos\beta} - \tan\delta\tan\beta = \frac{\cos\epsilon - \sin\delta\sin\beta}{\cos\delta\cos\beta}$$

or

$$\cos\eta = \sin\lambda\sin\alpha + \cos\lambda\cos\alpha\cos\epsilon \quad ,$$

$$\cos\eta\cos\delta = \cos\epsilon\cos\beta - \sin\epsilon\sin\beta\sin\lambda \quad ,$$

$$\sin\eta\cos\delta = \sin\epsilon\cos\lambda \quad ;$$

η is the position angle of a star in the triangle made by the points S, E, and P (see Figure 4).

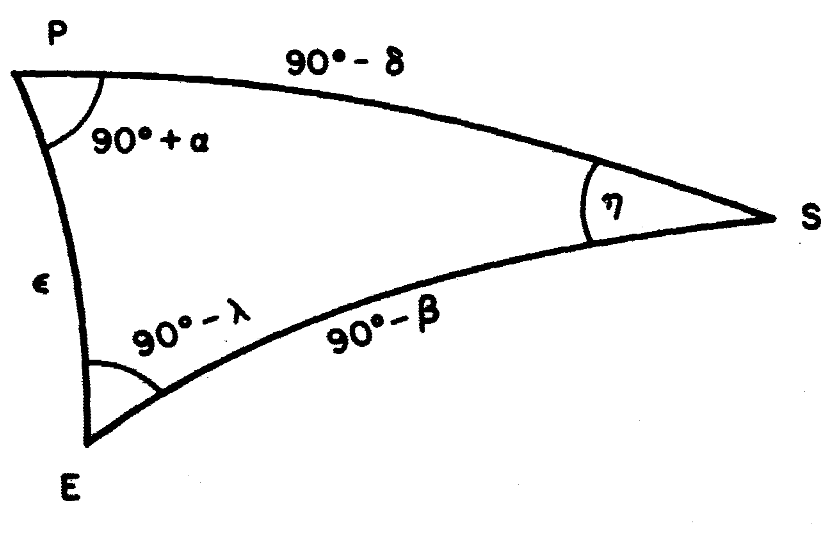

Figure 4. The parallactic angle η of a star S in the spherical triangle SEP. S = star, E = ecliptic pole, and P = Earth's celestial pole.

In equation (25) we must replace $d\lambda$, $d\beta$ and $d\alpha$, $d\delta$ by the annual or the secular variations in the respective coordinates.

2. The annual variations in λ, β for some epoch T_0 can be calculated by the formulas

$$V.A._{\lambda} = \left(\frac{d\lambda}{dt}\right)_0 + \mu_{\lambda} \quad ,$$

$$V.A._{\beta} = \left(\frac{d\beta}{dt}\right)_0 + \mu_{\beta} \quad , \tag{26}$$

where

$$\frac{d\lambda}{dt} = \frac{d\psi}{dt} - \pi \cos(\lambda + N) \tan \beta ,$$

$$\frac{d\beta}{dt} = \pi \sin(\lambda + N) \quad , \tag{27}$$

and μ_{λ}, μ_{β} are the components of the proper motion in ecliptic coordinates. According to Newcomb (1960),

$$\pi = 0''.471\,1 - 0''.000\,7\,T \quad ,$$

$$\frac{d\psi}{dt} = 50''.256\,4 + 0''.022\,2\,T, \text{ and}$$

$$N = 180° - \Pi = 180° - (173°57'.06 + 54'.77\,T) \quad , \tag{28}$$

where T is measured in tropical centuries from 1900.0, π is the annual rate of rotation of the ecliptic, Π is the longitude of the ascending node of the ecliptic of date on the fixed ecliptic of epoch measured along the fixed ecliptic from the fixed mean equinox of epoch, and ψ is the general precession in longitude.

The values of μ_{λ}, μ_{β} have to be transformed from μ_{α}, μ_{δ} by formulas (25). Neglecting the secular variation of the proper motion, we can obtain the secular variations of longitude $V.S._\lambda$ and of latitude $V.S._\beta$ by differentiating equations (27):

$$V.S._\lambda = \frac{d^2\lambda}{dt^2} = \frac{d^2\psi}{dt^2} - \frac{d\pi}{dt} \cos(\lambda + N) \tan\beta + \pi \sin(\lambda + N) \cdot \tan\beta \frac{dN}{dt} \cdot \sin 1' ,$$

$$V.S._\beta = \frac{d^2\beta}{dt^2} = \frac{d\pi}{dt} \sin(\lambda + N) + \pi \cos(\lambda + N) \frac{dN}{dt} , \qquad (29)$$

where

$$\frac{d^2\psi}{dt^2} = 0''.0222, \quad \frac{d\pi}{dt} = -0''.0007 , \quad \frac{dN}{dt} = -54'.77 . \qquad (30)$$

The second-order terms of the precessional motion are small; so, in practice, for short periods of time the following formulas are used (Woolard and Clemence, 1966):

$$\lambda_m = \lambda_0 + a_1 - b_1 \cos(\lambda_0 + c_1) \tan\beta_0 ,$$

$$\beta_m = \beta_0 + b_1 \sin(\lambda_0 + c_1) , \qquad (31)$$

where a_1 is the general precession in longitude, and b_1 is the rotation of the ecliptic calculated from the rate of precession $d\psi/dt$ and rotation of ecliptic π_a, which are taken for the mean point of the considered time interval, $(t_m + t_0)/2$, expressed in years. Hence,

$$a_1 = \left(\frac{d\psi}{dt}\right)_m (t_m - t_0) ;$$

$$b_1 = \pi_m (t_m - t_0) ;$$

$$c_1 = 180° - \left(\Pi_m + \frac{a_1}{2}\right) . \qquad (32)$$

These expressions are equivalent to the first- and second-order terms of Taylor's series.

3.3 Seleno-apparent Ecliptic Coordinates

To calculate the seleno-apparent selenocentric ecliptic coordinates from the mean ones, we must consider the influence of the following phenomena on the ecliptic coordinates: the precession and nutation of the ecliptic plane, the Earth's annual aberration, the lunar monthly aberration, and the proper motion of the star. We have the following formulas:

$$\lambda_{sapp} - \lambda_0 = \left[\left(\frac{d\psi}{dt}\right)_0 - \pi_0 \tan \beta_0 \cos (\lambda_0 + N_0)\right] (t - t_0) \qquad \text{precession}$$

$$+ \Delta\psi \qquad \text{nutation}$$

$$- k \sec \beta_0 \cos (L_\odot - \lambda_0) \qquad \text{Earth's annual aberration}$$

$$+ \mu_\lambda (t - t_0) \qquad \text{proper motion}$$

$$+ (\lambda' - \lambda_0) \ , \qquad \text{lunar monthly aberration} \qquad (33)$$

$$\beta_{sapp} - \beta_0 = \pi_0 (t - t_0) \sin (\lambda_0 + N_0) \qquad \text{precession}$$

$$- k \sin \beta_0 \sin (L_\odot - \lambda_0) \qquad \text{Earth's annual aberration}$$

$$+ \mu_\beta (t - t_0) \qquad \text{proper motion}$$

$$+ (\beta' - \beta_0) \ , \qquad \text{lunar monthly aberration} \qquad (34)$$

where

λ_0, β_0 and λ_{sapp}, β_{sapp} are the mean and the seleno-apparent ecliptic coordinates, respectively;

$\frac{d\psi}{dt}$, π , N can be calculated for $T = t_0$ by formulas (28);

$k = 20''.496$ is the Earth's annual aberration;

$L_\odot$ is the Sun's longitude;

$(t - t_0)$ is expressed in parts of the tropical year;

$\lambda' - \lambda_0$, $\beta' - \beta_0$ can be calculated by formulas (C-7) and (C-8).

The relation between the coordinates λ_{sapp}, β_{sapp} (of stars) and the geo-apparent ecliptic coordinates λ_{gapp}, β_{gapp} is given by the formulas

$$\lambda_{sapp} = \lambda_{gapp} + (\lambda' - \lambda_0) \ ,$$

$$\beta_{sapp} = \beta_{gapp} + (\beta' - \beta_0) \ .$$

If we put

$$S = \pi_0(t - t_0) \sin N_0, \quad s = \tan \beta_0 \sin \lambda_0, \quad s' = \cos \lambda_0 = -z \cot \beta_0$$

$$Z = \pi_0(t - t_0) \cos N_0, \quad z = -\tan \beta_0 \cos \lambda_0, \quad z' = \sin \lambda_0 = s \cot \beta_0,$$

$$C_E = -k \cos L_\odot = C \sec \epsilon \ , \quad c_E = \cos \lambda_0 \sec \beta_0, \quad c'_E = -\sin \lambda_0 \sin \beta_0,$$

$$D_E = -k \sin L_\odot = D, \quad d_E = \sin \lambda_0 \sec \beta_0, \quad d'_E = \cos \lambda_0 \sin \beta_0 \ , \tag{35}$$

then

$$\lambda_{sapp} - \lambda_0 = \left(\frac{d\psi}{dt}\right)_0 (t - t_0) + Ss + Zz + C_E c_E + D_E d_E + \Delta\psi$$

$$+ (\lambda' - \lambda_0) + \mu_\lambda (t - t_0) \ ,$$

$$\beta_{sapp} - \beta_0 = Ss' + Zz' + C_E c_E' + D_E d_E' + (\beta' - \beta_0) + \mu_\beta (t - t_0) \ . \qquad (36)$$

According to (C-7) and (C-8), equations (36) can be written in the form

$$\lambda_{sapp} - \lambda_0 = \left(\frac{d\psi}{dt}\right)_0 (t - t_0) + Ss + Zz + C_E c_E + D_E d_E + P_1 p_1 + Q_1 q_1$$

$$+ \Delta\psi + \mu_\lambda (t - t_0) \ ,$$

$$\beta_{sapp} - \beta_0 = Ss' + Zz' + C_E c_E' + D_E d_E' + P_1 p_1' + Q_1 q_1'$$

$$+ R_1 r_1' + \mu_\beta (t - t_0) \ , \qquad (37)$$

where

$P_1 = - k^m \sin \lambda_{ap} \cos \beta_{ap}$, $Q_1 = - k^m \cos \lambda_{ap} \cos \beta_{ap}$, $R_1 = k^m \sin \beta_{ap}$,

$p_1 = - \sec \beta \cos \lambda$, $q_1 = \sec \beta \sin \lambda$,

$p_1' = \sin \beta \sin \lambda$, $q_1' = \sin \beta \cos \lambda$, $r_1' = \cos \beta$; (38)

λ_{ap}, β_{ap} = the coordinates of the apex of the Moon's motion around the Earth given by (C-1),

k^m = the coefficient of the lunar monthly aberration given by (C-2),

and

$\left(\frac{d\psi}{dt}\right)_0$, π_0 , $\Delta\psi$, $\Pi_0 = 180 - N_0$, and $D_E = D$ are given in the almanacs.

In order to use formulas (37), the quantities $C_E = C \sec \epsilon$, and P_1, Q_1, R_1, S, Z must be given in the almanacs.

3.4 Transformation of the Ecliptic into the Selenoequatorial Coordinate System

The small and nearly constant inclination of the lunar equator to the ecliptic enables us to compute the tables required to transform the coordinates of these two systems immediately. These tables would be obtained from formulas (2) and (6) or from (5) and (7).

The differences between mean selenocentric equatorial and ecliptic coordinates, $a^{\Upsilon}_m - \lambda_m$, $d_m - \beta_m$, are the same for the same arguments $\lambda_m - \Omega$ and β_m, where Ω is the longitude of the Moon's orbital ascending node on the ecliptic.

In the transformation of the apparent coordinates, it is necessary to take into consideration the influence of the physical libration and the lunar monthly aberration. The latter is calculated from formulas (C-6) to (C-8).

Omitting at first the physical libration, and taking as constant the inclination of the lunar equator plane to the ecliptic plane, we can compute the tables required to transform the coordinates from the one to the other system.

The appropriate transformation formulas can be obtained by putting $\Omega = 0$ into formula (3):

$$\sin d_0 = \sin \beta \cos I + \cos \beta \sin I \sin \lambda \quad ,$$

$$\cos d_0 \sin a^{\Upsilon}_0 = - \sin I \sin \beta + \cos \beta \cos I \sin \lambda,$$

$$\cos d_0 \cos a^{\Upsilon}_0 = \cos \beta \cos \lambda \quad . \tag{39}$$

The tables that give the values of $a^{\Upsilon}_0 - \lambda$, $d_0 - \beta$, calculated from formulas (39), would allow immediate transformation of these coordinates for every moment of time and corresponding value of Ω, and for every value of λ, β:

$$a^{\Upsilon} = \lambda + \left(a_0^{\Upsilon} - \lambda\right)_{\lambda - \Omega} \quad ,$$

$$d = \beta + (d_0 - \beta)_{\lambda - \Omega} \quad . \qquad (40)$$

Inverse transformation would be possible by use of formulas (4):

$$\sin \beta = \cos I \sin d_0 - \sin I \cos d_0 \sin a_0^{\Upsilon} \quad ,$$

$$\cos \beta \sin \lambda = \sin I \sin d_0 + \cos I \cos d_0 \sin a_0^{\Upsilon} \quad ,$$

$$\cos \beta \cos \lambda = \cos d_0 \cos a_0^{\Upsilon} \quad , \qquad (41)$$

and

$$\lambda = a^{\Upsilon} + \left(\lambda - a_0^{\Upsilon}\right)_{a^{\Upsilon} - \Omega} , \qquad \beta = d + (\beta - d_0)_{a^{\Upsilon} - \Omega} \quad . \qquad (42)$$

Insufficient knowledge of the value of I can be overcome by the calculation of these tables for two values of I and for an interpolation or extrapolation of the proper values of the differences.

The values of $a_0^{\Upsilon} - \lambda$ and $d_0 - \beta$, for $I = 1\overset{\circ}{.}32'$, are given in Tables 1 and 2, respectively.

Table 1. Values of $a_0^{\Upsilon} - \lambda$

λ \ β	0°	30°	60°	85°
0°	0!00	-53!11	-2° 39!22	-17° 00!36
15	-0.31	-51.82	-2 35.98	-17 45.33
30	-0.53	-46.89	-2 21.74	-17 22.57
45	-0.62	-38.60	-1 57.09	-15 26.54
60	-0.47	-27.46	-1 23.56	-11 46.05
90	0	0	0	0

Table 2. Values of $d_0 - \beta$

λ \ β	0°	30°	60°	85°
0°	0'.00	- 0'.71	- 2'.13	-13'.78
15	+23.81	+23.14	+21.79	+ 9.92
30	46.00	45.46	44.36	33.83
45	65.05	64.69	63.95	56.24
60	79.67	79.49	79.11	74.95
90	92.00	92.00	92.00	92.00

The physical libration in latitude can be regarded in the same way as variations in the value of I. The influence of the physical libration in the node can be taken into account in the argument $\lambda - \Omega$ of the tables. The physical libration in the Moon's longitude ought to be regarded as the Moon's time correction.

The approximate transformation of these coordinates can be made by the use of the so-called Woolf networks (Figures 5a, 5b) on the plane of ecliptic meridians $\lambda = \Omega \pm 90°$. We can transform these two systems by placing the network 5b on 5a, or conversely, so that the points $P^{☾}$, $P'^{☾}$ and E, E' of one network coincide with these points on the second network.

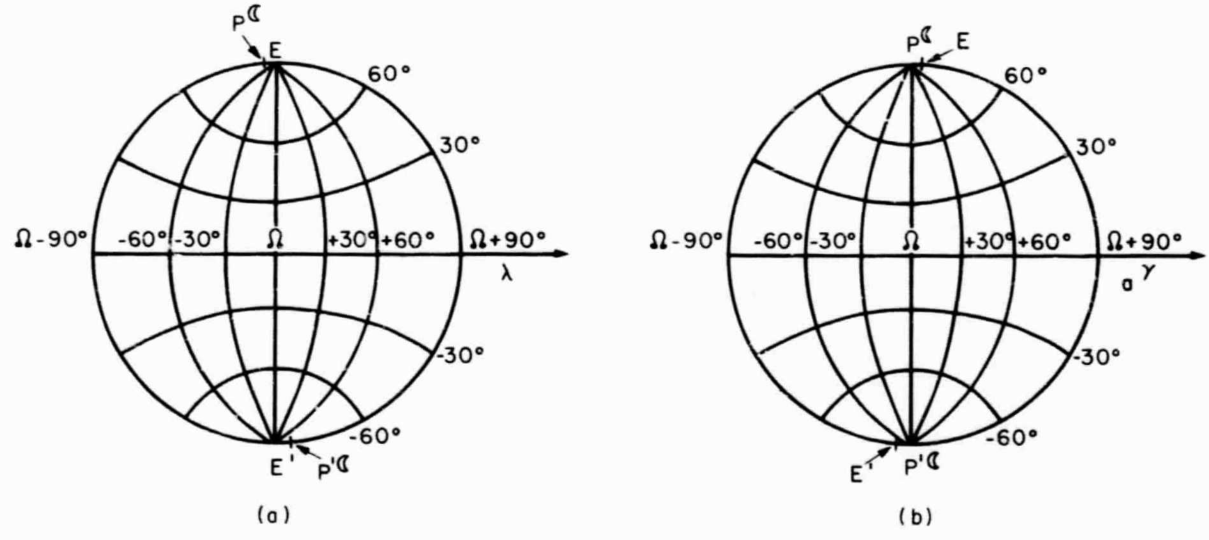

Figure 5. a) Woolf's network in the ecliptic coordinate system for $\lambda = \pm 90°$;
b) Woolf's network in the selenoequatorial coordinate system for $a^{\Upsilon} = \pm 90°$.

4. GEOEQUATORIAL COORDINATE SYSTEM

The geoequatorial coordinate system, although not so convenient for describing the apparent motion of the Moon's celestial sphere, has the advantage that there are many different catalogs of star coordinates and maps or atlases in this system. Hence, it is worthwhile to consider the possibility of using these catalogs or maps for astronomical observations made from the Moon, especially as a first step in this kind of work.

The selenocentric mean geoequatorial coordinates of stars are the same as the geocentric ones (parallax of the Earth-Moon distance is negligible).

The geocentric and selenocentric apparent geoequatorial coordinates are different because of the lunar monthly aberration. This latter influence can be calculated by formulas (C-6).

The parallax of the Moon's orbital radius should be taken into account for all celestial bodies nearer than stars (see Appendix D). Other formulas, such as those for the Earth's precession and nutation or aberration, can be used without change.

5. STELLAR COORDINATE SYSTEM

5.1 Introduction and Definition

The precession of the Earth's equator and of the ecliptic plane -- the basic planes of the commonly used coordinate systems such as the geo-equatorial and the ecliptic – causes constant variations in the coordinates of the fixed point on the celestial sphere.

We sometimes try to avoid this problem by using the fixed equatorial or ecliptic system of a chosen epoch. This system has an unchangeable position in space, but it does not coincide with the real basic planes of date.

On the Moon, for instance, we can use the ecliptic coordinate system for a chosen epoch because the precessional variations are smaller than in the selenoequatorial coordinates. But in this case, the ecliptic at the given epoch differs also from the one at a date. The introduction of the coordinate system whose basic plane is fixed seems to be convenient.

We can therefore define the stellar coordinate system as a heliocentric coordinate system in which the Z axis is directed to the chosen star S_1 and in which the XY plane perpendicular to this direction passes through the center of the Sun. The intersection of the XY plane with the great circle of the heliocentric celestial sphere, which passes through star S_1 and a second chosen star S_2, defines the zero point of the coordinate measured along the great circle lying in the XY plane; this coordinate is called the stellar right ascension, χ. The second coordinate is the stellar declination, $P = 90^\circ - \kappa_0$, and is the angular distance from the XY plane measured along the great circle passing through S_1.

The choice of stars is completely free. For instance, on the Earth we can choose star S_1 in the vicinity of the north celestial pole and S_2 in the vicinity of the vernal equinox of 1950.0. In the case of the Moon, it would be more convenient to choose the star S_1 in the vicinity of the north ecliptic pole and S_2 near the vernal equinox.

5.2 Transformation of Stellar Coordinates into Other Coordinates, and Conversely

The stellar coordinates χ, $P = 90° - \kappa_0$ can be obtained by a transformation of the Earth's equatorial system, or of the ecliptic system. We can find the required formulas immediately by applying the usual formulas of spherical trigonometry to the triangles SQS_1 and S_2QS_1 (Figure 6), neglecting the Earth's annual parallax.

The general transformation formulas are the following:

$$\cos \kappa = \cos \zeta \cos \zeta_1 + \sin \zeta \sin \zeta_1 \cos (\nu_1 - \nu) ,$$

$$\sin \kappa \cos (\chi + \xi) = \cos \zeta \sin \zeta_1 - \cos \zeta_1 \sin \zeta \cos (\nu_1 - \nu) ,$$

$$\sin \kappa \sin (\chi + \xi) = \sin \zeta_1 \sin (\nu_1 - \nu) , \tag{43}$$

where

$$\cos \kappa_0 = \cos \zeta_1 \cos \zeta_2 + \sin \zeta_1 \sin \zeta_2 \cos (\nu_1 - \nu_2) ,$$

$$\sin \kappa_0 \cos \xi = \cos \zeta_2 \sin \zeta_1 - \sin \zeta_2 \cos \zeta_1 \cos (\nu_1 - \nu_2) ,$$

$$\sin \kappa_0 \sin \xi = \sin (\nu_1 - \nu_2) \sin \zeta_2 . \tag{44}$$

Let us put the point Q at the Earth's celestial pole (or at the ecliptic pole or the Moon's celestial pole) and in formulas (43) and (44) replace

ν, ν_1, ν_2 by $\alpha, \alpha_1, \alpha_2$ (or $\lambda, \lambda_1, \lambda_2$; or $a^{\Upsilon}, a_1^{\Upsilon}, a_2^{\Upsilon}$) ,

ζ, ζ_1, ζ_2 by $90° - \delta$, $90° - \delta_1$, $90° - \delta_2$ (or $90° - \beta$, $90° - \beta_1$, $90° - \beta_2$; or $90° - d$, $90° - d_1$, $90° - d_2$) .

We can then obtain the transformation of the Earth's equatorial (or ecliptic, or selenoequatorial) coordinates into stellar coordinates. The formulas for the inverse transformation can easily be found on the basis of Figure 6.

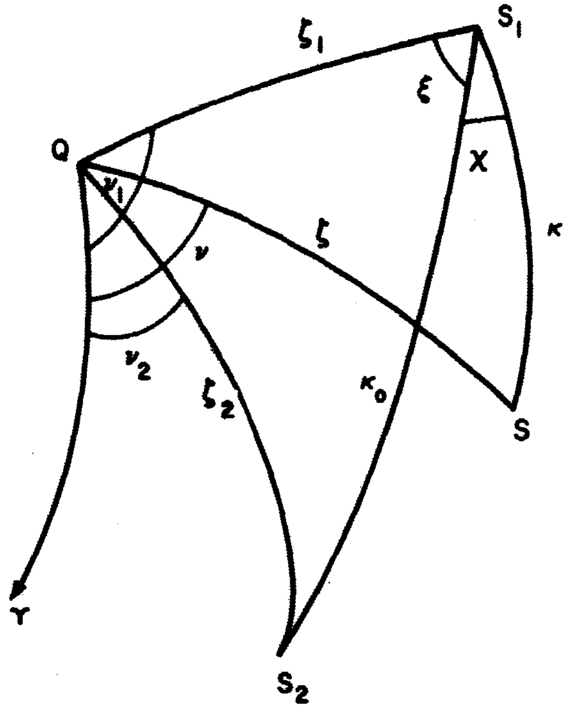

Figure 6. Spherical triangles S_2QS_1 and SQS_1 showing the relation of the stellar coordinates κ, χ to the coordinates of other systems ν, ζ. S = a star, S_1, S_2 = chosen stars, and Q = pole (Earth's celestial or ecliptic, or Moon's celestial).

For the transformation of the heliocentric stellar coordinates of stars into selenocentric coordinates, we must take into account the influence of the aberrations of the Moon's motions and the parallax of this translation. The advantage of this system is the constancy of the coordinates of a fixed point on the celestial sphere. There will be some changes caused by the proper motion of the chosen stars, but, of course, these changes are very small. If the catalogs of stellar coordinates of stars are computed from the known equatorial coordinates of the stars, they could then be used without change for many years.

6. MOON'S HORIZONTAL COORDINATE SYSTEM

6.1 Basic Formulas

The Earth's horizontal coordinates, altitude h, azimuth A, together with the hour angle θ, allow us to describe the apparent motion of the celestial sphere on the Earth.

The Moon's horizontal system, h^L, A^L, and hour angle θ^L, defined in the same way as the Earth's, describe the apparent motion of the Moon's celestial sphere.

The relations of the Moon's horizontal coordinates, altitude $h^L = 90° - z^L$ and azimuth A^L, to the selenoequatorial coordinates and the lunar hour angle θ^L are also the same as in the case of the Earth:

$$\begin{aligned} \cos z^L &= \sin \phi^L \sin d + \cos d \cos \phi^L \cos \theta^L \quad , \\ \sin z^L \sin A^L &= \cos d \sin \theta^L \quad , \\ \sin z^L \cos A^L &= - \sin d \cos \phi^L + \cos d \sin \phi^L \cos \theta^L \quad , \end{aligned} \tag{45}$$

where $\theta^L = \theta^L_{\Upsilon} - a^{\Omega} - (\Omega + 12^h)$, and θ^L_{Υ} is the hour angle of vernal equinox or local lunar sidereal time (see Figure 7).

For the inverse transformation we have

$$\begin{aligned} \sin d &= \sin \phi^L \cos z^L - \cos \phi^L \sin z^L \cos A^L \quad , \\ \cos d \sin \theta^L &= \sin z^L \sin A^L \quad , \\ \cos d \cos \theta^L &= \cos \phi^L \cos z^L + \sin \phi^L \sin z^L \cos A^L \quad . \end{aligned} \tag{46}$$

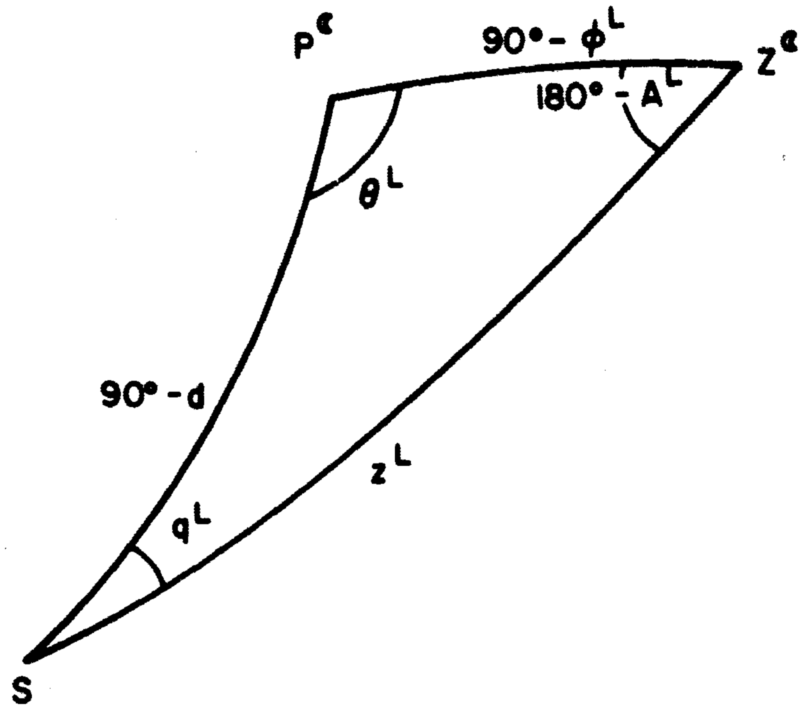

Figure 7. The Moon's astronomical triangle.

It is necessary to remember that, defined in the same way as on the Earth, the lunar hour angle θ^L of a star changes its value 27.3 times more slowly than on the Earth because of the Moon's slower revolution about its axis. So, although the differential formulas of the Moon's horizontal coordinates have the same form as in the case of the Earth,

$$\frac{dz^L}{d\theta^L} = \frac{\cos\phi^L \cos d \sin\theta^L}{\sin z^L} = \cos\phi^L \sin A^L \quad ,$$

$$\frac{dA}{d\theta^L} = \frac{\cos d \cos q^L}{\sin z^L} = \sin\phi^L + \cos\phi^L \cot z^L \cos A^L \quad , \qquad (47)$$

the time changes of these coordinates are also 27.3 times slower than on the Earth. Table 3 gives the maximum values of dz^L in one Earth sidereal second for different latitudes of the Earth and of the Moon.

Table 3. Values of $(dz^L)_{max}$ in one Earth sidereal second

	Earth	Moon
$\phi = 0°$	15''	0.''6
$\phi = 45°$	11	0.4
$\phi = 90°$	0	0.0

All other equations relating horizontal to equatorial coordinates, especially for such phenomena as rising and setting, culmination, etc., are the same on the Moon as on the Earth. It is necessary only to change the Earth's coordinates α, δ and h, A into lunar coordinates a^{Ω}, d and h^L, A^L, respectively.

6.2 Transformation of Ecliptic into Lunar Horizontal Coordinates

The use of the ecliptic coordinate system, as more convenient for astronomical observations from the Moon, will require the transformation of these coordinates into lunar horizontal coordinates.

We obtain these formulas by considering the triangles $P^{☾}EZ^{☾}$ and $SEZ^{☾}$ on the Moon's celestial sphere (Figure 8).

From the triangle $Z^{☾}P^{☾}E$, the zenith distance z_E^L, the azimuth $A_E^L = 180° - Q_{Z^{☾}}$, and the parallactic angle Q_E of the ecliptic pole can be calculated:

$$\cos z_E^L = \sin \phi^L \cos I + \cos \phi^L \sin I \cos Q_{p^{☾}} \quad ,$$

$$\sin z_E^L \sin Q_{Z^{☾}} = \sin I \sin Q_{p^{☾}} \quad ,$$

$$\sin z_E^L \cos Q_{Z^{☾}} = \cos I \cos \phi^L - \sin I \sin \phi^L \cos Q_{p^{☾}}, \text{ and}$$

$$\sin z_E^L \sin Q_E = \cos \phi^L \sin Q_{p^{☾}} \quad . \tag{48}$$

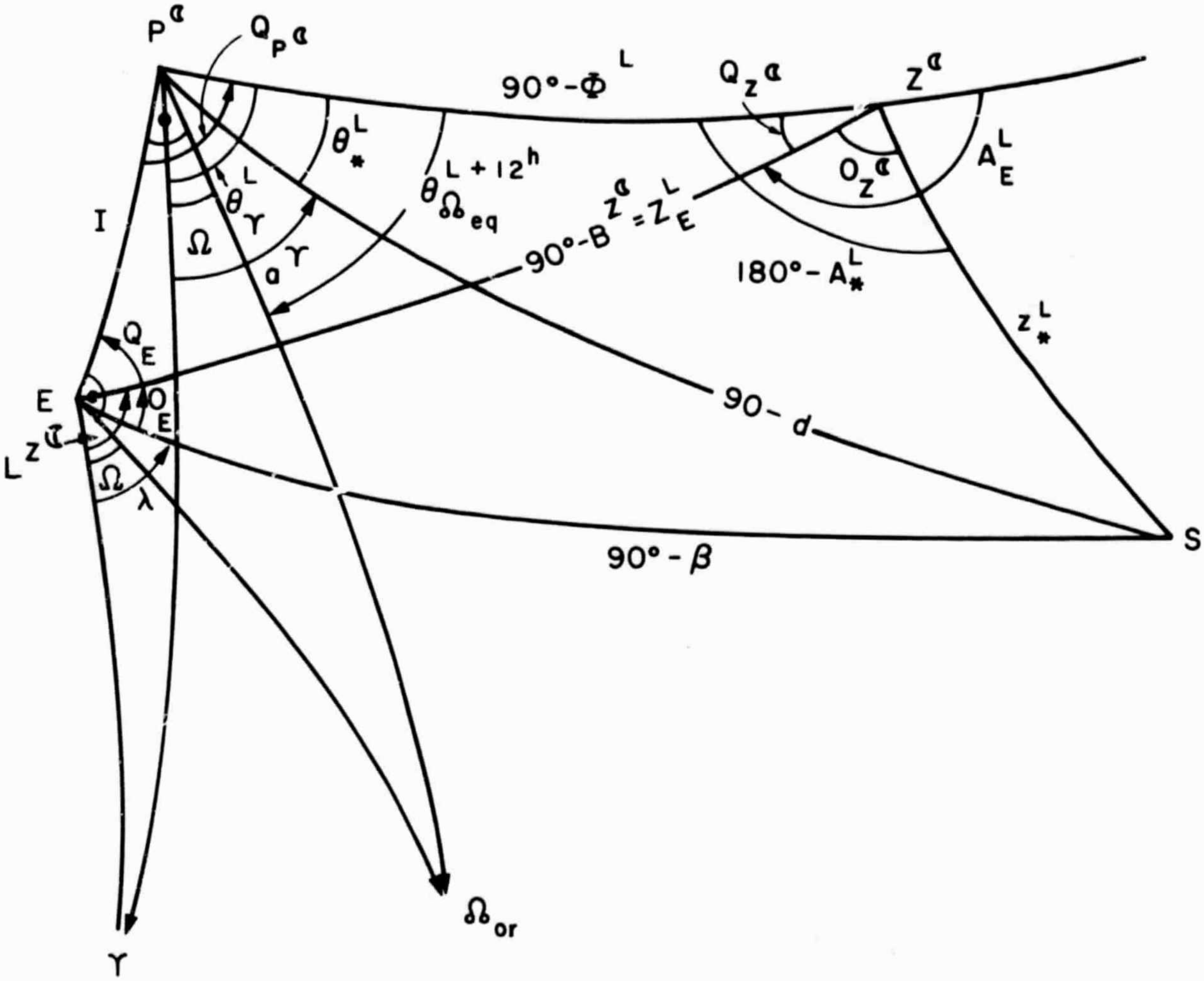

Figure 8. Spherical triangles $P^{☾}EZ^{☾}$ and $SEZ^{☾}$ showing the relation of the ecliptic to the lunar horizontal coordinates. Here $Q_{P^{☾}} = \angle Z^{☾}P^{☾}E = \theta^L_{\Upsilon} + 90° - \Omega$, $Q_{Z^{☾}} = \angle EZ^{☾}P^{☾}$, $Q_E = \angle Z^{☾}EP^{☾}$, $O_{Z^{☾}} = \angle SZ^{☾}E$, and $O_E = \angle Z^{☾}ES = 90° - [Q_E + (\lambda - \Omega)]$.

The quantities z_E^L, $Q_{Z☾}$, Q_E are functions of I (nearly constant), ϕ^L, and angle $Q_{p☾}$. Hence, for given ϕ^L, the values of these quantities can be calculated for every value of $Q_{p☾}$ between 0° and 360°. The angle $Q_{p☾}$ changes its value from 0° to 360° during the month and can be calculated for every moment from known Ω and θ_T^L.

The values of z_E^L and $Q_{Z☾}$ calculated, for example, for latitude $\phi^L = 45°$ are given in Table 4 for different values of $Q_{p☾} = \theta_T^L + 90° - \Omega$.

Table 4. Lunar horizontal coordinates of the north ecliptic pole: z_E^L, $Q_Z = 180° - A_E^L$ for $\phi^L = 45°$

$Q_{p☾}$	z_E^L	$Q_{Z☾}$	$Q_{p☾}$
0°	43° 27!1	0° 00! 0*	360°
30	43 40.6	1 06.5	330
45	43 55.5	1 33.9	315
60	44 13.9	1 53.2	300
90	45 01.2	2 10.2	270
120	45 47.0	1 50.2	240
135	46 05.7	1 30.4	225
150	46 19.3	1 03.6	210
180	46 32.1	0 00.0	180

*These values are plus for column 1 and minus for column 4.

The changes of azimuth $Q_{Z☾}$ will be greater for higher latitudes and smaller for lower latitudes. The ecliptic pole behaves in the same manner for observers on the Moon as Polaris does for observers on the Earth.

Knowing the values of z_E^L, $Q_{Z☾}$, and Q_E, we can transform ecliptic coordinates λ, β into Moon's horizontal coordinates Z^L, A^L. Applying basic spherical trigonometric formulas to the triangle $Z^☾ES$, we can write

$$\cos z^L = \cos z_E^L \sin\beta + \sin z_E^L \cos\beta \cos O_E \quad ,$$

$$\sin z^L \cos O_{Z☾} = \sin\beta \sin z_E^L - \cos\beta \cos z_E^L \cos O_E \quad ,$$

$$\sin z^L \sin O_{Z☾} = \cos\beta \sin O_E \, , \tag{49}$$

where

$$180° - A^L = Q_{Z☾} + O_{Z☾} \quad ,$$

and

$$O_E = 90° - Q_E - (\lambda - \Omega) \quad .$$

Transformation of the ecliptic into the horizontal coordinates can also be made by the formulas of spherical polygonometry introduced by Banachiewicz (1929).

The principal formulas of polygonometry are

$$\{I\} = \{I\}\, p(a_1)\, r(A_1')\, p(a_2)\, r(A_2') \ldots p(a_k)\, r(A_k') \quad , \tag{50}$$

where $\{I\}$, $p(n)$, $r(n)$ are the following cracovians:*

$$\{I\} = \begin{Bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{Bmatrix} \quad , \tag{51}$$

$$p(n) = \begin{Bmatrix} 1 & 0 & 0 \\ 0 & \cos n & -\sin n \\ 0 & \sin n & \cos n \end{Bmatrix} \quad , \tag{52}$$

---

*Cracovians are the matrices for which a different manner of the multiplication is defined: Columns are multiplied by columns instead of rows by columns.

$$r(n) = \begin{Bmatrix} \cos n & -\sin n & 0 \\ \sin n & \cos n & 0 \\ 0 & 0 & 1 \end{Bmatrix} . \qquad (53)$$

The sides of a polygon are denoted by $a_1, a_2, \ldots, a_k$, and the angles by $A'_1 = 180° - A_1$, $A'_2 = 180° - A_2$, $\ldots$, $A'_k = 180° - A_k$ (see Figure 9).

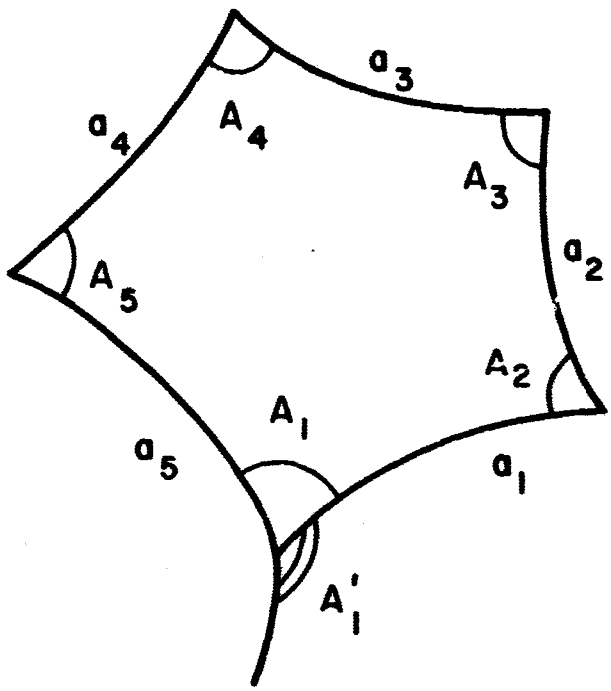

Figure 9. Spherical polygon.

Applying formulas (50) to the quadrangle $SZ^{☾}P^{☾}E$ (Figure 8), we can write

$$\{I\} \cdot r\,(180° - O_S) \cdot p(z^L) \cdot r(A^L) \cdot p(90° - \phi^L) \cdot r(90° + \Omega - \theta_T^L)$$

$$\cdot\ p(I) \cdot r(90° + \lambda - \Omega) \cdot p(90° - \beta) = \{I\} \quad , \qquad (54)$$

and

$$r(O_S - 180°) \cdot p(z^L) \cdot r(A^L) = p(90° - \beta) \cdot r(\Omega - \lambda - 90°) \cdot p(-I)$$

$$\cdot\ r(\theta_T^L - \Omega - 90°) \cdot p(\phi^L - 90°) \quad . \qquad (55)$$

A comparison of the third columns of the cracovians of both sides of equation (55) gives us the required transformation

$$\begin{Bmatrix} \sin z^L \sin A^L \\ \sin z^L \cos A^L \\ \cos z^L \end{Bmatrix} = \begin{Bmatrix} 0 \\ -\sin\beta \\ \cos\beta \end{Bmatrix} \cdot r(\Omega - \lambda - 90°) \cdot p(-I)$$

$$\cdot\ r(\theta_T^L - \Omega - 90°) \cdot p(\phi^L - 90°) \quad . \qquad (56)$$

Formulas (54), written in the form

$$\{I\} \cdot p(z^L) \cdot r(A^L) \cdot p(90° - \phi^L) \cdot r(90° + \Omega - \theta_T^L) \cdot p(I)$$

$$\cdot\ r(90° + \lambda - \Omega) \cdot p(90° - \beta)$$

$$\cdot\ r(180° - O_S) = \{I\} \quad , \qquad (57)$$

give us the inverse transformation

$$r(180° - O_S) \cdot p(\beta - 90°) \cdot r(\Omega - \lambda - 90°)$$

$$= p(-z^L) \cdot r(A^L) \cdot p(90° - \phi^L) \cdot r(90° + \Omega - \theta_T^L) \cdot p(I) \quad . \qquad (58)$$

6.3 Apparent Motion of the Moon's Celestial Sphere

The apparent motion of the Moon's celestial sphere is about 27.3 times slower than the Earth's, but during one long revolution there occur all phenomena that are observed on the Earth's celestial sphere during 1 day, such as setting and rising, culminations, elongations, and transits of the prime vertical. These phenomena can be described on the Moon by the Moon's horizontal coordinates A^L, h^L and lunar hour angle θ^L.

All the formulas for the Moon's horizontal coordinates and the lunar hour angle of a star in the above-mentioned phenomena have the same form as the appropriate formulas for the Earth's horizontal coordinates. They are given in Table 5.

The description of the motion of the Moon's celestial sphere in ecliptic coordinates is more difficult. It is easy to write the formulas given in Table 5 as a function of the horizontal coordinates of the ecliptic pole, z_E^L, $O_{Z☾}$, and the angle O_E. We can do that by replacing

$$
\begin{array}{lll}
90° - \phi^L & \text{by} & z_E^L , \\
180° - A^L & \text{by} & O_{Z☾} , \quad \text{and} \\
\theta^L & \text{by} & O_E .
\end{array}
$$

The quantities z_E^L, $O_{Z☾}$, O_E are time dependent; hence we must use the method of successive approximation: first, with some approximate values of z_E^L, $O_{Z☾}$, O_E we calculate the hour angle of a star, and then we repeat the calculation with the time determined in the first step.

Table 6 gives the selenoequatorial, the ecliptic, the Moon's horizontal, and the geoequatorial coordinates of the special points on the Moon's celestial sphere.

Table 5. Moon's horizontal coordinates and lunar hour angles for special points on the Moon's celestial sphere

Name of point or phenomenon	A^L	z^L	θ^L	Remarks
Z	indefinite	0°	0°	
S	0°	90°	0°	
W	90°	90°	90°	
N	180°	90°	180°	
E	270°	90°	270°	
Upper culmination	0° 180°	$\phi^L - d$ $d - \phi^L$	0° 0°	all stars
Lower culmination	180°	$180° - (\phi^L + d)$	180°	all stars
Setting and rising	$\cos A^L = - \dfrac{\sin d}{\cos \phi^L}$	90°	$\cos \theta^L = - \tan \phi^L \tan d$	only stars with declination $90° - \phi^L \geq d \geq \phi^L - 90°$
Elongation	$\sin A^L = \dfrac{\cos d}{\cos \phi^L}$	$\cos z^L = \dfrac{\sin \phi^L}{\sin d}$	$\cos \theta^L = \dfrac{\tan \phi^L}{\tan d}$	stars with $d > \phi^L$
Prime vertical	± 90°	$\cos z^L = \dfrac{\sin d}{\sin \phi^L}$	$\cos \theta^L = \dfrac{\tan d}{\tan \phi^L}$	stars with $d < \phi^L$

Table 6. The selenocentric coordinates of the special points on the Moon's celestial sphere

Point on the Moon's celestial sphere	Selenoequatorial coordinates a^{Ω}	Selenoequatorial coordinates d	Ecliptic coordinates λ	Ecliptic coordinates β	Moon's horizontal coordinates A^L	Moon's horizontal coordinates z^L	Geoequatorial coordinates a	Geoequatorial coordinates δ
$z^{☾}$	θ^L_{ω}	ϕ^L	[5]	[5]	undefined	90°	[10]	[10]
$p^{☾}$	undefined	+90°	90° + Ω	90° - I	180°	ϕ^L	[11]	$\{90° - \epsilon - I < \lvert\delta\rvert < 90° - \epsilon + I;\ \lvert\delta\rvert = 90° - \epsilon\}$ [11]
$\nu^{☾}$	undefined	-90°	270° + Ω	-90° + I	under horizon		[11]	
E	+90°	90° - I	undefined	+90°	$\{\sin A_{max} = \sin I \sec \phi^L$; formulas (48)$\}$	$\{\phi^L - I < h_E < \phi^L + I$; formulas (48)$\}$	270°	90° - ε
E'	270°	-90° + I	undefined	-90°			90°	-90° + ε
T	$\tan a^{\Omega} = 180° - \tan\Omega \cos I$	$\{\tan d = -\tan I \sin a^{\Omega}\}$	0°	0°	{changeable 0° 360°} [6]	$\{h < 90° - \phi^L + I;\ h > \phi^L - I - 90°\}$ [6]	0	0
≏	$\tan a^{\Omega} = -\tan\Omega \cos I$		180°	0°			180°	0
Ω or [1]	180°	0°	Ω	0°	{changeable 0° - 360°} [7]	$\{h < 90° - \phi^L;\ h > \phi^L - 90°\}$ [7]	$\tan a_{\Omega} = \cos\epsilon \tan\Omega$	$\tan\delta_{\Omega} = \tan\epsilon \sin a_{\Omega}$
☋ or [2]	0°	0°	Ω + 180°	0°			$\tan a_{☋} = \cos\epsilon \tan ☋$	$\tan\delta_{☋} = \tan\epsilon \sin a_{☋}$
p●	[3]	$\{90° - \epsilon - I < \lvert d\rvert < 90° - \epsilon + I\}$ [3]	6^h	90° - ε	[8]	[8]	undefined	+90°
p'●	[3]		18^h	-90° + ε	[8].	[8]	undefined	-90°
●	[4]	$\{\lvert d_{●}\rvert < i + I\}$ [4]	$\lambda_{☾} + 180°$	$-\beta_{☾}$	[9]	[9]	$a_{☾} + 180°$	$-\delta_{☾}$

In Table 6 the numbers in square brackets denote the following:

[1] Ascending node of the lunar crbit or descending node of the lunar equator (neglecting physical libration).

[2] Descending node of the lunar orbit or ascending node of the lunar equator (neglecting physical libration).

[3] Formulas (6) with known $\lambda_{P\oplus}$, $\beta_{P\oplus}$,

$$\sin d_{P\oplus} = \cos \epsilon \cos I + \sin \epsilon \sin I \cos \Omega \quad ,$$

$$\cos d_{P\oplus} \sin a^{\Omega}_{P\oplus} = \cos \epsilon \sin I - \sin \epsilon \cos I \cos \Omega \quad ,$$

$$\cos d_{P\oplus} \cos a^{\Omega}_{P\oplus} = - \sin \epsilon \sin \Omega \quad .$$

Similar formulas can be written for point $P'^{\oplus}$.

[4] Formulas (6) with known $\lambda_{\oplus}$, $\beta_{\oplus}$,

$$-\sin d_{\oplus} = \sin \beta_{☾} \cos I + \cos \beta_{☾} \sin I \sin (\lambda_{☾} - \Omega) \quad ,$$

$$\cos d_{\oplus} \sin a^{\Omega}_{\oplus} = - \sin \beta_{☾} \sin I + \cos \beta_{☾} \cos I \sin (\lambda_{☾} - \Omega) \quad ,$$

$$\cos d_{\oplus} \cos a^{\Omega}_{\oplus} = \cos \beta_{☾} \cos (\lambda_{☾} - \Omega) \quad .$$

[5] Formulas (D-10).

[6] Formulas (45) with known values of a^{Ω}, d of points ♈ and ♎ .

[7] Formulas (45) with known values of a^{Ω}, d of point Ω,

$$\tan A_{\Omega}^{L} = \tan \theta_{\Omega}^{L} \operatorname{cosec} \phi^{L} \quad ,$$

$$\cos A_{\Omega}^{L} \tan z_{\Omega}^{L} = \tan \phi^{L} \quad .$$

Similar formulas can be written for point ℧.

[8] Formulas (48) and (49) with known $\lambda_{P\oplus}$, $\beta_{P\oplus}$.

[9] Formulas (48) and (49) with known $\lambda_{\oplus}$, $\beta_{\oplus}$, or from the triangle $\oplus P^{☾} Z^{☾}$ (see Figure 10),

$$\cos z_{\oplus}^{L} = \sin \phi^{L} \sin b_{\oplus} + \cos \phi^{L} \cos b_{\oplus} \cos (\lambda^{L} - \ell_{\oplus}) \quad ,$$

$$\sin z_{\oplus}^{L} \cos A_{\oplus}^{L} = - \sin b_{\oplus} \cos \phi^{L} + \cos b_{\oplus} \sin \phi^{L} \cos (\lambda^{L} - \ell_{\oplus}) \quad ,$$

$$\sin z_{\oplus}^{L} \sin A_{\oplus}^{L} = \cos b_{\oplus} \sin (\lambda^{L} - \ell_{\oplus}) \quad ,$$

and $\ell_{\oplus}$, $b_{\oplus}$ are given by equation (D-23).

[10] Formulas (8) with known selenoequatorial coordinates of $Z^{☾}$, or formulas (D-10) and (5).

[11] Formulas (5) with known $\lambda_{P☾}$, $\beta_{P☾}$,

$$\sin \delta_{P☾} = \cos \epsilon \cos I + \sin \epsilon \sin I \cos \Omega \quad ,$$

$$\cos \delta_{P☾} \sin \alpha_{P☾} = - \sin \epsilon \cos I + \cos \epsilon \sin I \cos \Omega \quad ,$$

$$\cos \delta_{P☾} \cos \alpha_{P☾} = - \sin I \sin \Omega \quad .$$

Similar formulas can be written for the point $P'^{☾}$.

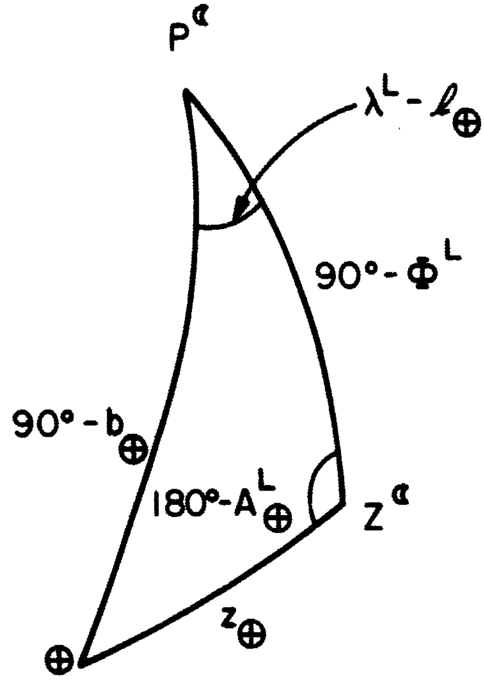

Figure 10. The Moon's astronomical triangle ⊕P☾Z☾.

7. SELENO-RECTANGULAR COORDINATE SYSTEMS

In many astronomical problems, three-dimensional rectangular coordinates are more convenient than spherical ones, and sometimes they are necessary to determine exact positions in space—for example, of planets and of artificial satellites.

The rectangular or polar coordinate system can be associated with any spherical system: selenoequatorial, geoequatorial, ecliptic, or horizontal. In the case of the Moon, the origin of these coordinate systems will be in the center of its mass (selenocentric coordinate systems) or in a point on its surface (lunar topocentric coordinate systems).

The rectangular coordinates x, y, z of a point P are related to its polar coordinates ρ, u, v (Figure 11) by the following expressions:

$$x = \rho_P \cos u \cos v \quad ,$$
$$y = \rho_P \sin u \cos v \quad ,$$
$$z = \rho_P \sin v \quad . \qquad (59)$$

The factors of ρ_P in (59) are the direction cosines and can be used in place of u and v to represent the direction of the point.

When we consider the selenoequatorial rectangular coordinates, the geoequatorial rectangular coordinates, or the ecliptic coordinates, we replace u, v in (59) by a^{Ω}, d; α, δ; or λ, β, respectively.

The transformation of one rectangular coordinate system into another without a translation of the origin of the system requires only the rotation of the system. For example, the transformation of the ecliptic coordinate into the selenoequatorial system requires two rotations. The first is the rotation about the ecliptic pole axis (z axis) through $\Omega + 180°$. The coordinates of the point P in this new system will be x′, y′, z′; here

$$x' = -x_{ecl} \cos \Omega - y \sin \Omega \ ,$$
$$y' = x_{ecl} \sin \Omega - y \cos \Omega \ ,$$
$$z' = z_{ecl} \ .$$

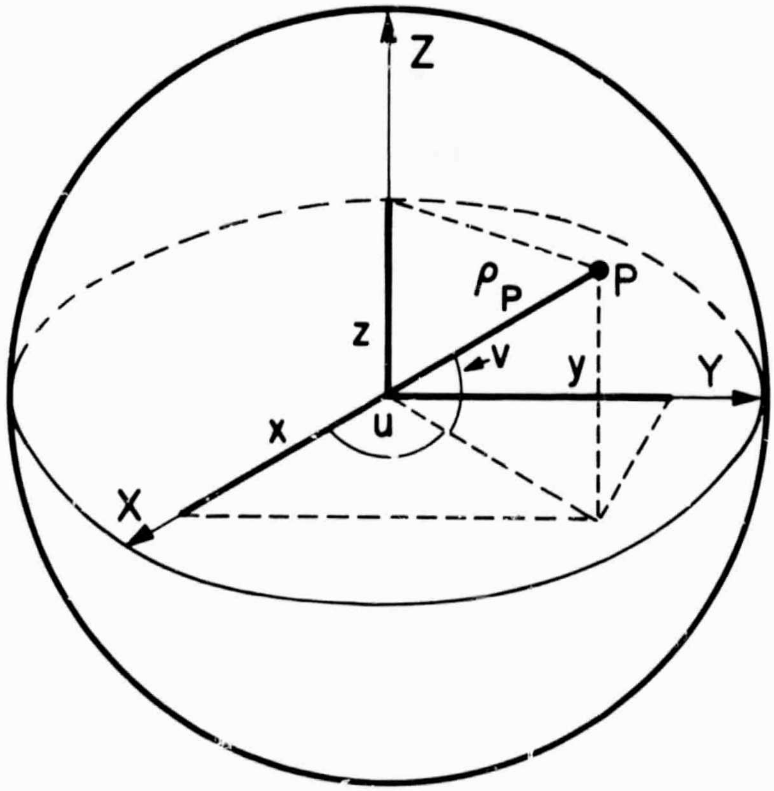

Figure 11. The relation of the rectangular coordinates of the point P(x, y, z) to its polar coordinates (ρ_P, u, v).

The second rotation is that about the line of the lunar orbital nodes through I; and the selenoequatorial coordinates of the point P are

$$x_{sel} = x' \quad ,$$

$$y_{sel} = y' \cos I + z' \sin I \quad ,$$

$$z_{sel} = -y' \sin I + z' \cos I \quad .$$

Using the rotational cracovians (52) and (53), we can write the above transformation in the form

$$\begin{Bmatrix} x_{sel} \\ y_{sel} \\ z_{sel} \end{Bmatrix} = \begin{Bmatrix} x_{ecl} \\ y_{ecl} \\ z_{ecl} \end{Bmatrix} \cdot r(\Omega + 180°) \cdot p(I) \quad , \tag{60}$$

and the reverse transformation in the form

$$\begin{Bmatrix} x_{ecl} \\ y_{ecl} \\ z_{ecl} \end{Bmatrix} = \begin{Bmatrix} x_{sel} \\ y_{sel} \\ z_{sel} \end{Bmatrix} \cdot p(-I) \cdot r[-(180° + \Omega)] \quad . \tag{61}$$

With the use of the same symbols, the transformation of the geoequatorial into the ecliptic system is

$$\begin{Bmatrix} x_{ecl} \\ y_{ecl} \\ z_{ecl} \end{Bmatrix} = \begin{Bmatrix} x_{geo} \\ y_{geo} \\ z_{geo} \end{Bmatrix} \cdot p(-\epsilon) \quad ;$$

the transformation of the geoequatorial into the selenoequatorial, and the reverse transformation, are

$$\begin{Bmatrix} x_{sel} \\ y_{sel} \\ z_{sel} \end{Bmatrix} = \begin{Bmatrix} x_{geo} \\ y_{geo} \\ z_{geo} \end{Bmatrix} \cdot p(-\epsilon) \cdot r(\Omega + 180°) \cdot p(I) \quad , \tag{62}$$

and

$$\begin{Bmatrix} x_{geo} \\ y_{geo} \\ z_{geo} \end{Bmatrix} = \begin{Bmatrix} x_{sel} \\ y_{sel} \\ z_{sel} \end{Bmatrix} \cdot p(-I) \cdot r(180° - \Omega) \cdot p(\epsilon) \quad . \tag{63}$$

Expressions (62) and (63) can be written for a^{Υ}, d in the following form:

$$\begin{Bmatrix} x_{sel} \\ y_{sel} \\ z_{sel} \end{Bmatrix} = \begin{Bmatrix} x_{geo} \\ y_{geo} \\ z_{geo} \end{Bmatrix} \cdot \underbrace{p(-\epsilon)\, r(\Omega + 180°) \cdot p(I) \cdot r(180° - \Omega)}_{\{L_1\}} \quad , \tag{64}$$

$$\begin{Bmatrix} x_{geo} \\ y_{geo} \\ z_{geo} \end{Bmatrix} = \begin{Bmatrix} x_{sel} \\ y_{sel} \\ z_{sel} \end{Bmatrix} \cdot r(-180° + \Omega) \cdot p(-I) \cdot r(-180° - \Omega) \cdot p(\epsilon) \quad . \tag{65}$$

In the cracovian

$$\{L_1\} = \begin{Bmatrix} L_{11} & L_{21} & L_{31} \\ L_{12} & L_{22} & L_{32} \\ L_{13} & L_{23} & L_{33} \end{Bmatrix} \quad , \tag{66}$$

which is used in lunar research; L_{ij} are the direction cosines of the axes of the geoequatorial coordinate system $X_\oplus$, $Y_\oplus$, $Z_\oplus$ and the selenoequatorial one $X_☾$, $Y_☾$, $Z_☾$, as shown by

	$X_\oplus$	$Y_\oplus$	$Z_\oplus$
$X_☾$	L_{11}	L_{21}	L_{31}
$Y_☾$	L_{12}	L_{22}	L_{32}
$Z_☾$	L_{13}	L_{23}	L_{33}

The tables for L_{ij} were published by Banachiewich (1929) for $I = 1°36'06''$ (Hayn's value) and $\epsilon = 23°27'08''.26$.

The transformation of selenoequatorial coordinates into the Moon's horizontal coordinates can be made by the formulas:

$$\begin{Bmatrix} x_{shor} \\ y_{shor} \\ z_{shor} \end{Bmatrix} = \begin{Bmatrix} \rho_P \sin z^L \cos A^L \\ \rho_P \sin z^L \sin A^L \\ \rho_P \cos z^L \end{Bmatrix} = \begin{Bmatrix} x_{sel} \\ y_{sel} \\ z_{sel} \end{Bmatrix} \cdot r\left(\theta^L_{\Omega_{eq}}\right) \cdot q(90° - \phi^L), \quad (67)$$

where

$$\begin{Bmatrix} x_{sel} \\ y_{sel} \\ z_{sel} \end{Bmatrix} = \begin{Bmatrix} \rho_P \cos d \cos a^{\Omega} \\ \rho_P \cos d \sin a^{\Omega} \\ \rho_P \sin d \end{Bmatrix},$$

$$q(90^\circ - \phi^L) = \begin{Bmatrix} \cos(90^\circ - \phi^L) & 0 & \sin(90^\circ - \phi^L) \\ 0 & 1 & 0 \\ -\sin(90^\circ - \phi^L) & 0 & \cos(90^\circ - \phi^L) \end{Bmatrix} , \tag{68}$$

and ρ_P is the distance of the body from the origin of the coordinate system.

The inverse transformation is

$$\begin{Bmatrix} x_{sel} \\ y_{sel} \\ z_{sel} \end{Bmatrix} = \begin{Bmatrix} x_{shor} \\ y_{shor} \\ z_{shor} \end{Bmatrix} \cdot q(\phi - 90^\circ) \cdot r\left(-\theta^L_{\Omega_{eq}}\right) . \tag{69}$$

Formulas (67) and (60) give us the transformation of the ecliptic coordinates into lunar horizontal coordinates:

$$\begin{Bmatrix} x_{shor} \\ y_{shor} \\ z_{shor} \end{Bmatrix} = \begin{Bmatrix} x_{ecl} \\ y_{ecl} \\ z_{ecl} \end{Bmatrix} \cdot r(\Omega + 180^\circ) \cdot p(I) \cdot r\left(\theta^L_{\Omega_{eq}}\right) \cdot q(90^\circ - \phi^L) . \tag{70}$$

The inverse transformation is obtained from (69) and (61):

$$\begin{Bmatrix} x_{ecl} \\ y_{ecl} \\ z_{ecl} \end{Bmatrix} = \begin{Bmatrix} x_{shor} \\ y_{shor} \\ z_{shor} \end{Bmatrix} \cdot q(\phi^L - 90^\circ) \cdot r\left(-\theta^L_{\Omega_{eq}}\right) \cdot p(I) \cdot r[-(\Omega + 180^\circ)] . \tag{71}$$

Similarly, formulas (67) and (62) give us the transformation of the lunar horizontal coordinates into the geoequatorial coordinates. The inverse transformation is obtained from (69) and (63).

8. CONCLUSIONS

Insufficient knowledge of the Moon's physical libration and the inclination of the lunar equatorial plane to the ecliptic limits the accuracy of calculated selenoequatorial coordinates of stars and of other celestial bodies. At present, we can expect the accuracy of these coordinates to be of the order of 1 to 2 arcmin.

Thus, the ecliptic coordinate system appears to be most convenient for space orientation on the Moon. The accuracy of the determined coordinates in this system is high; the precessional motion is smaller than in other coordinate systems (except the stellar); and the small inclination of the ecliptic coordinate system to the selenoequatorial coordinate system permits the use of ecliptic coordinates instead of selenoequatorial ones for a rough orientation.

The practical use of the ecliptic coordinate system requires the preparation of catalogs of ecliptic coordinates of stars, tables for the transformation of ecliptic into selenoequatorial coordinates and vice versa, as well as a special lunar almanac (Gurevich, 1967) giving the current values of different quantities such as the coordinates of the apexes of the Moon's motions, the Moon's orbital nodes, the Moon's physical libration, the apparent-selenocentric ecliptic coordinates of the Sun, of the Earth, and of the planets, and the parallaxes of these bodies as seen from the Moon.

APPENDIX A

PRECESSION OF THE MOON

According to Cassini's laws:

1. The Moon rotates eastward, about a fixed axis, with uniform angular velocity and a period equal to the sidereal period of the Moon's revolution around the Earth.

2. The inclination I of the Moon's equator to the ecliptic is constant and is approximately $1°32'.1$.

3. The ascending node of the lunar orbit on the ecliptic coincides with the descending node of the lunar equator on the ecliptic; therefore, the poles of the Moon's equator, of the ecliptic, and of the Moon's orbit lie, in that order, on one great circle.

The Moon's axis of rotation and the plane of the Moon's equator make one revolution about the axis of the ecliptic poles in approximately 18.6 years. The angle of the precession cone of the Moon's polar axis is equal to the inclination of the lunar equator to the ecliptic. Hence, the Moon's precession is approximately 1360 times faster than that of the Earth.

The longitude of the mean ascending node of the lunar orbit on the ecliptic measured from the mean equinox of date is expressed by

$$\Omega = 259°10'59''.79 - 5^{r}134°08'31''.23\ T + 7''.48\ T^2 + 0''.008\ T^3$$

$$= 259°.183\,275 - 0°.052\,953\,922\,2\ d + 0°.002\,078\ T^2 + 0°.000\,002\ T^3, \quad \text{(A-1)}$$

where T is measured in Julian centuries from 1900 January 0.5, i.e., from J.D. 2 415 020.0, and d is the number of ephemeris days from epoch.

By differentiating equation (A-1), we can calculate the daily variation in this longitude:

$$\frac{d\Omega}{dT} = -0.^{\circ}052\ 953\ 922\ 2 + 0.^{\circ}004\ 156\ T \quad . \tag{A-2}$$

Thus, the daily precession of the ascending node of the lunar orbit is

$$P_0^d = -0.^{\circ}052\ 953\ 922\ 2 \quad . \tag{A-3}$$

The daily precessional motions in the lunar right ascension and declination a^{Ω}, d measured from the ascending node of the lunar equator on the ecliptic are

$$M_0^d = -P_0^d \cos I \quad , \quad N_0^d = +P_0^d \sin I \quad , \tag{A-4}$$

where $I = 1^{\circ}32'.1$.

The precessional variations of the selenoequatorial coordinates can easily be obtained by differentiation of equation (6). Thus,

$$\frac{da^{\Omega}}{dI} = \tan d \cos a^{\Omega} \quad , \quad \frac{da^{\Omega}}{d\Omega} = -\cos I + \sin I \tan d \sin a^{\Omega} \quad ,$$

$$\frac{dd}{dI} = -\sin a^{\Omega} \quad , \quad \frac{dd}{d\Omega} = \sin I \cos a^{\Omega} \quad . \tag{A-5}$$

Replacing dI and $d\Omega$ by their precessional variations,

$$dI = 0 \quad \text{and} \quad d\Omega = P_0^d t \quad ,$$

and taking into account formulas (A-4), we obtain the influence of the precession in selenoequatorial coordinates:

$$a'^{\Omega} - a_0^{\Omega} = \left(M_0^d + N_0^d \tan d_0 \sin a_0^{\Omega} \right) t \quad ,$$

$$d' - d_0 = N_0^d \, t \cos a_0^{\Omega} \quad , \qquad \text{(A-6)}$$

where t is a number of ephemeris days from the epoch t_0 of the mean coordinates a_0^{Ω}, d_0 .

APPENDIX B

PHYSICAL LIBRATION OF THE MOON

The Moon's rotation about the center of its axis is described approximately by Cassini's three empirical laws. The different values of the principal moments of inertia cause some oscillation about the Moon's mean position as described by Cassini's laws.

A full description of the actual rotation of the Moon is given by the angles

$$\phi = 180^\circ + (\ell_{☾} + \tau) - \psi', \qquad \theta = I + \rho, \qquad \psi' = \Omega + \sigma \quad , \qquad \text{(B-1)}$$

where

ϕ	=	angular distance of the positive part of the Moon's first radius directed toward the Earth from the descending node of the lunar equator,
ψ'	=	longitude of the descending node of the lunar equator,
θ	=	inclination of the lunar equator to the ecliptic,
$\ell_{☾}$	=	mean longitude of the Moon,
I	=	mean inclination of the lunar equator to the ecliptic, and
τ, ρ, σ	=	physical libration in longitude $\ell_{☾}$, in inclination I, and in node Ω, respectively.

The quantities τ, ρ, σ can be written in the form (Koziel, 1962)

$\tau \; = \; A^* \sin (a^* + 3t \sqrt{0.985\,3\, M\gamma})$, free physical libration

$$\left.\begin{array}{l}
- 12''.9 \sin g \\
- 0''.3 \sin 2g \\
- 65''.2 \sin (-g') \\
+ 9''.7 \sin (2\omega) \\
- 1''.4 \sin (-2g' - 2\omega') \\
+ 2''.5 \sin (-g' + \omega - \omega') \\
- 0''.6 \sin (g' + 2\omega - 2\omega') \\
- 7''.3 \sin (-2g' + 2\omega - 2\omega') \\
- 3''.0 \sin (g - 2g' + 2\omega - 2\omega') \\
- 0''.4 \sin (2g - 2g' + 2\omega - 2\omega') \\
+ 7''.6 \sin \Omega
\end{array}\right\} \begin{array}{l}\text{forced}\\ \text{physical}\\ \text{libration}\end{array} \qquad \text{(B-2)}$$

$$\rho = \left.\begin{array}{l}
- B^{*} \cos (b^{*} - 146''.6t) \\
+ 0.662C^{*} \cos (c^{*} + 50''.8t - g - \omega) \\
+ 1.662C^{*} \cos (c^{*} + 50''.8t + g + \omega)
\end{array}\right\} \begin{array}{l}\text{free}\\ \text{physical}\\ \text{libration}\end{array}$$

$$\left.\begin{array}{l}
- 106'' \cos g \\
+ 35'' \cos (g + 2\omega) \\
- 11'' \cos (2g + 2\omega) \\
- 3'' \cos (2g' + 2\omega') \\
- 2'' \cos (g - 2g' + 2\omega - 2\omega')
\end{array}\right\} \begin{array}{l}\text{forced}\\ \text{physical}\\ \text{libration}\end{array} \qquad \text{(B-3)}$$

$$I\sigma = \left.\begin{array}{l}
I\tau + B^{*} \sin (b^{*} - 146''.6t) \\
- 0.662 C^{*} \sin (c^{*} + 50''.8t - g - \omega) \\
+ 1.662 C^{*} \sin (c^{*} + 50''.8t + g + \omega)
\end{array}\right\} \begin{array}{l}\text{free}\\ \text{physical}\\ \text{libration}\end{array}$$

$$\left.\begin{array}{l}
- 108'' \sin g \\
+ 35'' \sin (g + 2\omega) \\
- 11'' \sin (2g + 2\omega) \\
- 3'' \sin (2g' + 2\omega') \\
- 2'' \sin (g - 2g' + 2\omega - 2\omega')
\end{array}\right\} \begin{array}{l}\text{forced}\\ \text{physical}\\ \text{libration}\end{array} \qquad \text{(B-4)}$$

Here,

$A^*, a^*; B^*, b^*; C^*, c^*$ = the constants of the free libration in longitude, inclination, and node, respectively;

g, g' = mean anomaly of the Moon and Sun, respectively;

ω, ω' = angular distance of the Moon's and the Sun's perigee from the ascending node of the Moon's orbit, respectively;

t = time expressed in mean days;

$M' = 3M$, where M is the Earth's mass;

γ = one of the three quantities (α, β, γ) connected with the Moon's principal moments of inertia (A, B, C) by the formulas

$$\alpha = \frac{C - B}{A}, \qquad \beta = \frac{C - A}{B}, \qquad \gamma = \frac{B - A}{C} \quad . \tag{B-5}$$

In the last formulas, A is the moment about the axis directed earthward and C is the moment about the Moon's rotational axis. The coefficients of (B-2) to (B-4) are given for a value of the mechanical ellipticity of the Moon,

$$f = \frac{\alpha}{\beta} = \frac{B}{A} \frac{(C - B)}{(C - A)} = 0.73 \quad . \tag{B-6}$$

The quantity f characterizes the ratio of the Moon's principal moments of inertia as well as the Moon's nutation.

The influence of the physical libration in selenoequatorial coordinates a^{Ω}, d can be calculated by the formulas (Gurevich, 1965)

$$a'^{\Omega} - a^{\Omega} = I_a^{\Omega} + II_a^{\Omega} \quad , \qquad d' - d = I_d + II_d \quad . \tag{B-7}$$

The first-order terms I_a^{Ω}, I_d can be obtained by differentiation of equation (6) (see A-5) and replacement of $d\Omega$ and dI by the physical libration in the node σ and in the inclination ρ, respectively:

$$I_d = -\rho \sin a^{\Omega}_{sapp,1} + \sigma \cos a^{\Omega}_{sapp,1} \sin I \quad ,$$

$$I^{\Omega}_a = \rho \tan d_{sapp,1} \cos a^{\Omega}_{sapp,1} - \sigma(\cos I - \sin I \tan d_{sapp,1} \sin a^{\Omega}_{sapp,1}) .$$

(B-8)

The physical libration in the Moon's longitude τ ought to be taken into account as the correction of the Moon's time.

The second-order terms are the Fabritius terms

$$II^{\Omega}_a = \tan d_{sapp,1} I^{\Omega}_a I_d \quad ,$$

$$II_d = -0.5 \sin d_{sapp,1} \cos d_{sapp,1} I^2_d \quad . \qquad \text{(B-9)}$$

The influence of the physical libration in selenoequatorial coordinates can be taken into consideration also if we replace in the transformation formulas (3), (4) or (6), (7) the values of I and Ω by $I + \rho$ and $\Omega + \sigma$. The physical libration in the Moon's longitude τ can also be treated as the correction to the Moon's time.

But now, while the physical libration is not known accurately, it is better to regard this influence separately from the transformation formulas (3), (4) or (6), (7) by the use of formulas (B-7) to (B-9).

APPENDIX C

ABERRATION OF THE MOON'S MOTIONS

The Moon is involved simultaneously in several motions: rotation about its axis, revolution around the Earth, revolution with the Earth around the Sun, and motion with the Sun in space.

All these motions cause the phenomenon of aberration, which changes the positions of the celestial bodies on the celestial sphere. The lunar daily aberration is caused by the Moon's rotation about its axis; the lunar monthly aberration, by its revolution around the Earth; and the lunar annual aberration, by its motion around the Sun.

C.1 Lunar Daily Aberration

The small size of the Moon ($r_{☾} = 1738$ km) and the slow rotation around its axis are the reasons for the small linear velocity of the points on the Moon's surface. The maximum velocity on the lunar equator is $V_0^{☾} = 4.6$ m $\sec^{-1}$.

The coefficient of the lunar daily aberration,

$$k^d = \frac{V_0^{☾}}{c} 206265 < 0.''001 ,$$

is so small that its influence can be neglected.

C.2 Lunar Monthly Aberration

The average velocity of the Moon's orbital motion around the Earth is 1.023 km $\sec^{-1}$. The coordinates of the apex of the Moon's orbital motion, given by Gurevich (1965), with the simplifying assumption that angles $\ell_{☾}$, M, Γ', Ω are constant after projection onto the plane of the Moon's orbit, are

$$\lambda_{ap} = l_{☾} + 90° - X, \qquad \beta_{ap} = i \sin(\lambda_{ap} - \Omega) \quad ,$$

and

$$\tan X = \frac{e \sin M}{1 + e \cos M} \quad , \tag{C-1}$$

where

e, M = eccentricity of the lunar orbit ($e_{max} = 0.07$) and the Moon's anomaly, respectively,

(β_{ap}) max = $\pm i = \pm 5°$; i is the inclination of the Moon's orbit to the ecliptic; and

Ω = the longitude of the ascending node of the Moon's orbit on the ecliptic.

Coefficients of the lunar monthly aberration can be calculated (Gurevich, 1965) by the formulas

$$k^m = k^m_{av} \frac{1 + 2e \cos M + e^2}{1 - e^2} \quad . \tag{C-2}$$

The mean value of $k^m_{av} = 206\,265\,(V_1^{☾})_{av}/c = 0''.70$, with $c = 299\,792.5$ km sec^{-1}. The coefficient k^m changes its value within the limits $0''.65 < k^m < 0''.75$ because of changes of the Moon's orbital velocity,

$$0.94 \text{ km sec}^{-1} < V_1^{☾} < 1.09 \text{ km sec}^{-1} \quad .$$

On the basis of the known ecliptic coordinates of the apex of the Moon's orbital motion, its selenoequatorial and geoequatorial coordinates can be determined.

The geoequatorial coordinates can be obtained by the transformation of the known ecliptic coordinates λ_{ap} , β_{ap} from formulas (5):

$$\sin \delta_{ap} = \cos \epsilon \ \sin \beta_{ap} + \sin \epsilon \ \cos \beta_{ap} \sin \lambda_{ap} \quad ,$$

$$\cos \delta_{ap} \sin \alpha_{ap} = - \sin \epsilon \sin \beta_{ap} + \cos \epsilon \ \cos \beta_{ap} \sin \lambda_{ap} \quad ,$$

$$\cos \delta_{ap} \cos \alpha_{ap} = \cos \beta_{ap} \cos \lambda_{ap} \quad ; \qquad \text{(C-3)}$$

or approximately, if we put $\beta_{ap} \approx 0$:

$$\sin \delta_{ap} \approx \sin \lambda_{ap} \sin \epsilon \quad ,$$

$$\tan \delta_{ap} = \tan \epsilon \ \operatorname{cosec} \alpha_{ap}$$

$$\tan \alpha_{ap} \approx \tan \lambda_{ap} \cos \epsilon \quad . \qquad \text{(C-4)}$$

The selenoequatorial coordinates of this apex can be expressed in ecliptic coordinates by the formulas that are obtained from the triangles $C\,\Omega_{or}K$ and $K\,\Omega_{or}L$ (see Figure C-1):

$$\sin C\Omega_{or} = \frac{\sin \beta_{ap}}{\sin i} \quad ,$$

$$\sin d_{ap} = \sin C\Omega_{or} \ \sin (i + I) = \sin \beta_{ap} \frac{\sin (i + I)}{\sin i} \quad ,$$

$$d_{ap} \approx \beta_{ap} \frac{\sin (i + I)}{\sin i} \approx 1.3 \ \beta_{ap} \quad ,$$

$$\sin (a^{\Omega}_{ap} - \Omega - 12^{h}) = \tan d_{ap} \cot (i + I) \quad . \qquad \text{(C-5)}$$

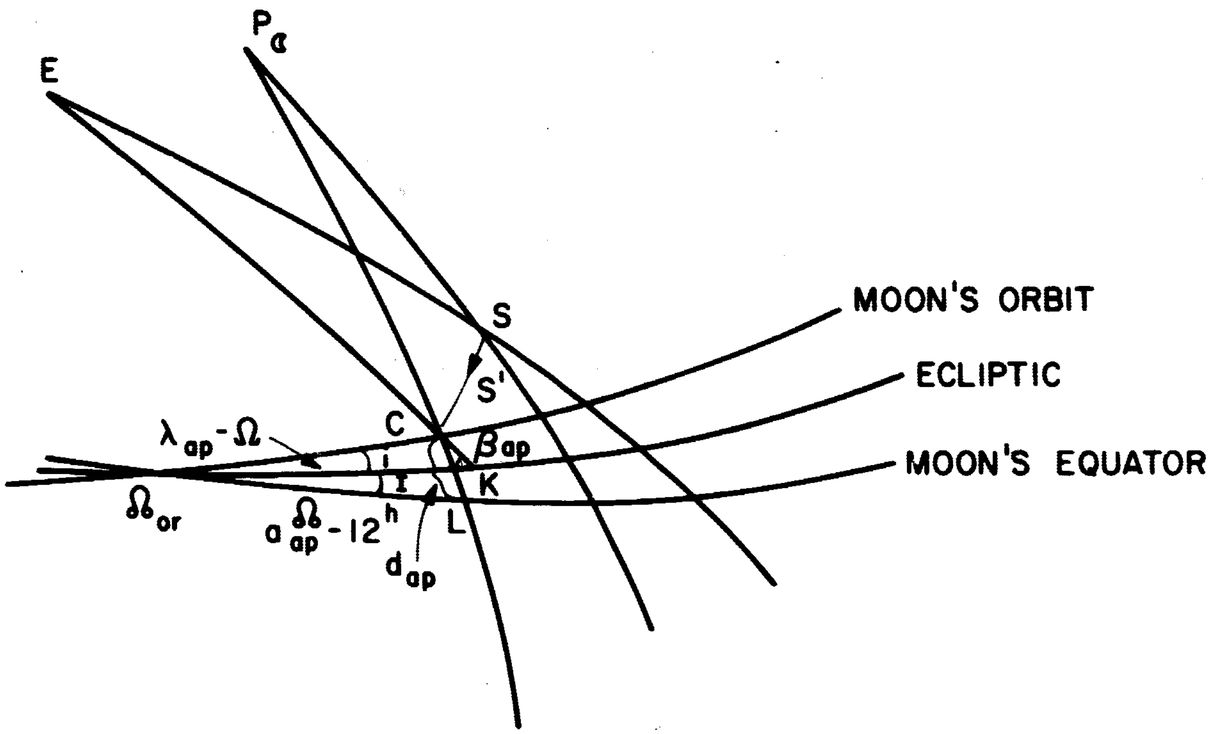

Figure C-1. The ecliptic and selenoequatorial coordinates of the apex C of the Moon's orbital motion. S, S′ are the true and the apparent positions of a star, respectively; P☾, E are the poles of the Moon's equator and of the ecliptic, respectively.

The influence of the lunar monthly aberration in spherical coordinates is different in the different systems.

We can compute this influence in each of the spherical coordinate systems by using the general formulas for the influence of an aberration in spherical coordinates u, v (Appendix F),

$$u - u' = k \cos v_{ap} \sin (u - u_{ap}) \sec v \quad ,$$

$$v - v' = k \cos v_{ap} \sin v \cos (u - u_{ap}) - \sin v_{ap} \cos v \quad , \qquad \text{(C-6)}$$

where u_{ap}, v_{ap} are the proper spherical coordinates of an apex and k is a coefficient of an aberration. The approximate substitutions are given in Table C-1.

Table C-1. The coordinates of the apex of the Moon's orbital motion and the notation of the true and apparent (at this motion) spherical coordinates in the different coordinate systems

Name of the cooodinate system	True coordinates of a body		Apparent coordinates of a body		Coordinates* of the apex of the Moon's orbital motion	
	u	v	u'	v'	u_{ap}	v_{ap}
Selenoequatorial	a^{Ω}	d	a'^{Ω}	d'	a^{Ω}_{ap}	d_{ap}
Ecliptic	λ	β	λ'	β'	λ_{ap}	β_{ap}
Geoequatorial	α	δ	α'	δ'	α_{ap}	δ_{ap}

*Given by the formulas (C-1) and (C-3) to (C-5).

Gurevich (1965) gives the formulas for the influence of the lunar monthly aberration in ecliptic coordinates as:

$$\lambda' - \lambda = P_1 p_1 + Q_1 q_1 ,$$

$$\beta' - \beta = P_1 p_1' + Q_1 q_1' + R_1 r_1' , \qquad \text{(C-7)}$$

with the following notation:

$$P_1 = - k^m \sin \lambda_{ap} \cos \beta_{ap}, \; Q_1 = - k^m \cos \lambda_{ap} \cos \beta_{ap}, \; R_1 = k^m \sin \beta_{ap} ,$$

$$p_1 = - \sec \beta \cos \lambda, \; q_1 = \sec \beta \sin \lambda ,$$

$$p_1' = \sin \beta \sin \lambda, \; q_1' = \sin \beta \cos \lambda, \; r_1' = \cos \beta . \qquad \text{(C-8)}$$

C. 3 Lunar Annual Aberration

We can assume that the Moon's apex in its motion around the Sun lies on the ecliptic plane. The latitude of this apex can be calculated by the following formula (see Figure C-2):

$$\tan B_{ap} = \frac{V_1^{☾}}{V^{\oplus}} \sin [i \sin (\lambda_{ap} - \Omega)] \quad , \qquad \text{(C-9)}$$

where

$V^{\oplus}$, $V_1^{☾}$ = velocity of the Earth's and of the Moon's orbital motions, respectively; $V^{\oplus} = 29.75$ km sec^{-1}, $V_1^{☾} = 1.023$ km sec^{-1},

i = inclination of the Moon's orbit to the ecliptic, $i = 5°.15$.

λ_{ap} ≈ $\ell_{☾} + 90°$, longitude of the apex of the Moon's orbital motion.

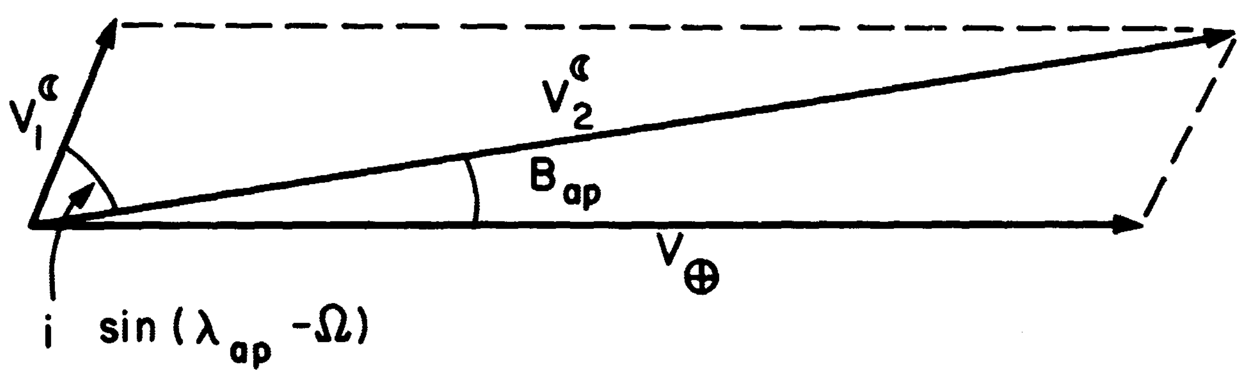

Figure C-2. The projection of the velocities of the Moon's and of the Earth's orbital motions on the plane perpendicular to the ecliptic. $V_2^{☾}$ is the velocity of the Moon's motion around the Sun.

Because the angles B_{ap} and i are small and $V_1^{☾}/V^{\oplus} = 0.03$, we can write

$$B_{ap} = 0.03\, i \sin(\lambda_{ap} - \Omega) < 0.^{\circ}15 \quad . \qquad \text{(C-10)}$$

The angle B_{ap} is so small that it can be neglected. Hence, the ecliptic coordinates of the apex of the Moon's motion around the Sun, L_{ap} and B_{ap}, as well as the velocity of this motion, $V_2^{☾}$ (Jakowkin et al., 1964), are the following (Figure C-3):

$$L_{ap} = L_{\odot} - 90^{\circ} + \Delta A \quad ,$$

$$B_{ap} = 0 \quad ,$$

$$V_2^{☾} = \sqrt{(V^{\oplus})^2 + (V_1^{☾})^2 - 2\, V_1^{☾}\, V^{\oplus} \cos(L_{\odot} - \ell_{☾})} \quad , \qquad \text{(C-11)}$$

where

$$\tan \Delta A = \frac{V_1^{☾} \sin(L_{\odot} - \ell_{☾})}{V^{\oplus} - V_1^{☾} \cos(L_{\odot} - \ell_{☾})} \quad ,$$

and $L_{\odot}, \ell_{☾}$ are the longitudes of the Sun and of the Moon, respectively. If we assume $V^{\oplus}/V_1^{☾}$ to be so small that we can neglect the second and higher order terms, then we can write

$$\Delta A = \frac{V_1^{☾}}{V^{\oplus}} \sin(L_{\odot} - \ell_{☾}) = 1.^{\circ}9 \sin(L_{\odot} - \ell_{☾}) \quad ,$$

$$V_2^{☾} = V_{\oplus}^{☾}\left[1 - \frac{V_1^{☾}}{V^{\oplus}} \cos(L_{\odot} - \ell_{☾})\right] \quad ,$$

$$L_{ap} = L_{\odot} - 90^{\circ} + \Delta A \quad . \qquad \text{(C-12)}$$

The limits of $V_2^{☾}$ are:

$$28.7 \text{ km sec}^{-1} < V_2^{☾} < 30.8 \text{ km sec}^{-1} \quad .$$

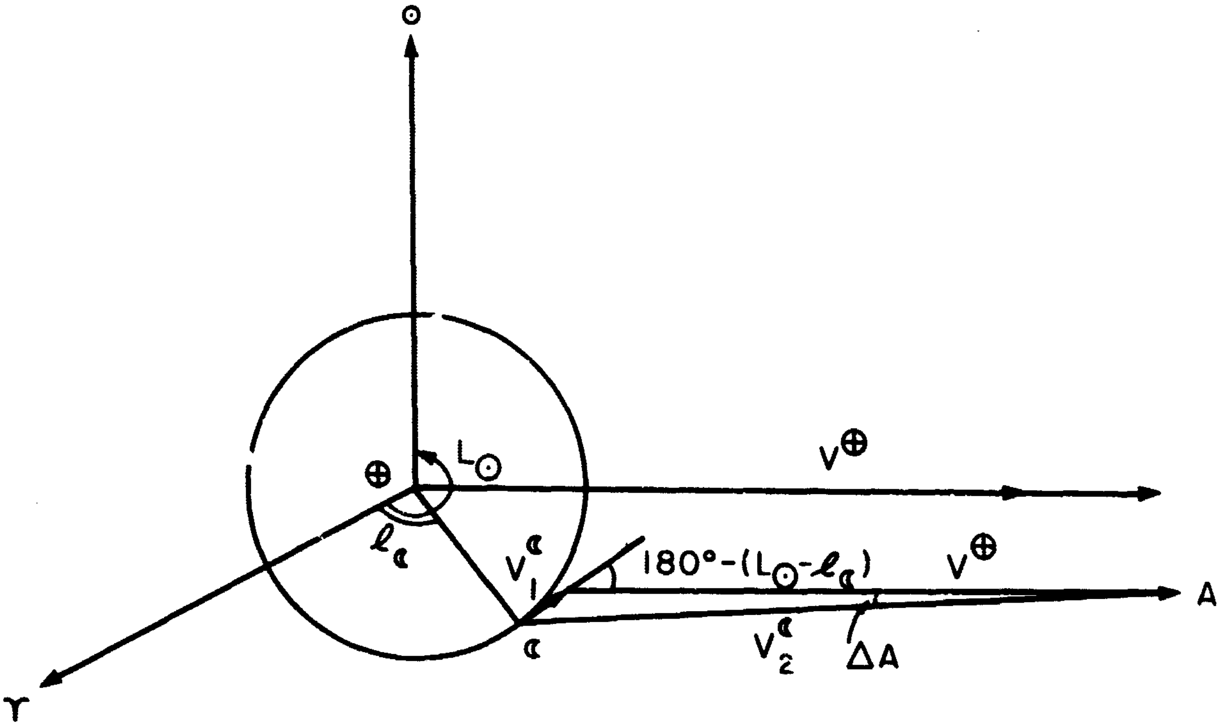

Figure C-3. The projection of the velocities of the Earth's and of the Moon's orbital motion on the Moon's orbital plane.

Assuming the above-mentioned approximation, we can calculate the coefficient of the lunar annual aberration $K = 206265\ V_2^{☾}/c$ from

$$K = k\left[1 - \frac{V_1^{☾}}{V^{\oplus}} \cos\,(L_{\odot} - \ell_{☾})\right] \quad , \qquad \text{(C-13)}$$

where k denotes the coefficient of the Earth's annual aberration, and

$$k = 20''.496 \ .$$

The limits of the variation of K are

$$19''.7 < K < 21''.2 \ .$$

The influence of the lunar annual aberration has to be taken into account only in the case of the transformation of mean selenoequatorial coordinates into the apparent ones. Whenever we transform the geo-apparent geoequatorial coordinates or the ecliptic ones into seleno-apparent selenoequatorial coordinates, it is necessary to take into consideration the lunar monthly aberration only.

The formulas for the influence of the lunar annual aberration in selenoequatorial coordinates are (Jakowkin <u>et al.</u>, 1964)

$$a^{\Omega} - a'^{\Omega} = K \sec d \left[\cos (L_{ap} - \Omega + 180°) \sin a^{\Omega} - \sin (L_{ap} - \Omega + 180°) \cos a^{\Omega} \cos I\right] ,$$

$$d - d' = K \left[\cos (L_{ap} - \Omega + 180°) \cos a^{\Omega} \sin d + \sin (L_{ap} - \Omega + 180°) \times (\cos d \sin I + \sin d \sin a^{\Omega} \cos I)\right] , \qquad \text{(C-14)}$$

where a^{Ω}, d and a'^{Ω}, d' denote the mean and the apparent coordinates, respectively. We obtain approximate formulas by putting $\Delta A = 0$, $L_{ap} = L_{\odot} - 90°$, $V_1^{☾}/V^{\oplus} = 0$, and $I = 0$. Hence, we have

$$a^{\Omega} - a'^{\Omega} = - k \sec d \cos (L_{\odot} - a^{\Omega} - \Omega) ,$$

$$d - d' = - k \sin d \sin (L_{\odot} - a^{\Omega} - \Omega) . \qquad \text{(C-15)}$$

It is easy to write the formulas for the influence of the lunar annual aberration in ecliptic and geoequatorial coordinates if we assume the same approximation as before, namely, $B_{ap} = 0$. The formulas for ecliptic coordinates can be obtained from those for the influence of the Earth's annual aberration if we change the longitude of the apex from $L_{\odot} - 90°$ to L_{ap} and if we change the coefficients of the aberrations from k to K. Hence, we have

$$\lambda - \lambda' = - K \sec \beta \sin (L_{ap} - \lambda) ,$$

$$\beta - \beta' = K \sin \beta \cos (L_{ap} - \lambda) . \qquad \text{(C-16)}$$

To calculate the influence of the lunar annual aberration in geoequatorial coordinates, it is necessary first to find the geoequatorial coordinates of the apex, A_{ap}, D_{ap}, of the Moon's motion around the Sun. If we assume $B_{ap} = 0$ and $L_{ap} = L_{\odot} - 90 + \Delta A$, the geoequatorial coordinates of the apex are

$$\tan A_{ap} = \cos \epsilon \tan L_{ap} \quad ,$$

$$\tan D_{ap} = \tan \epsilon \sin A_{ap} \quad . \tag{C-17}$$

Next, from Figure C-4, we can write $\delta - \delta' = SS' \cos S' SS_0$; $(\alpha - \alpha') \cos \delta = SS' \sin S' SS_0$; $SS' = K_1 \sin SA_p$.

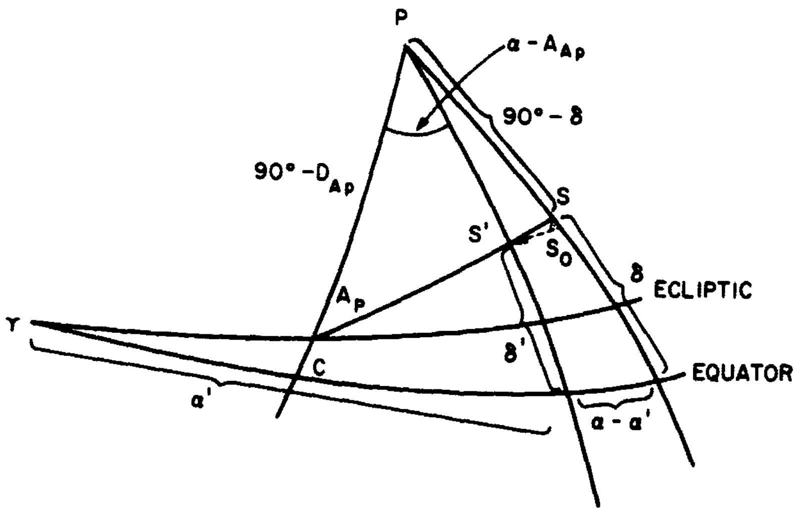

Figure C-4. The lunar annual aberration of a star.

Hence,

$$\sin SA_p \cos S' SS_0 = - \sin D_{ap} \cos \delta + \cos D_{ap} \sin \delta \cos (\alpha - A_{ap})$$

$$\sin SA_p \sin S' SS_0 = \cos D_{ap} \sin (\alpha - A_{ap}) \quad ,$$

and

$$(\alpha - \alpha') = K_1 \cos D_{ap} \sin(\alpha - A_{ap}) \sec\delta \quad ,$$

$$(\delta - \delta') = K_1 [\cos D_{ap} \sin\delta \cos(\alpha - A_{ap}) - \sin D_{ap} \cos\delta] \quad , \qquad \text{(C-18)}$$

where α, δ and α', δ' denote the mean and the apparent coordinates, respectively.

APPENDIX D

PARALLAX OF THE RADIUS OF THE MOON, THE RADIUS OF ITS ORBIT, AND THE MOON–SUN DISTANCE

D.1 Introduction

The changes of the directions of the radius vector of a celestial body (or artificial satellite) caused by the translation of the origin of the coordinate system from the center of the Moon's mass to a point on its surface is called the lunar daily parallax.

Taking into account the Moon's orbital motion and its motion around the Sun (jointly with the Earth), we can consider the lunar monthly parallax — the parallax of the Moon–Earth distance —and the lunar annual parallax— the parallax of the Moon–Sun distance.

The mean diameter of the Moon's globe as well as the radius of the lunar orbit are small in comparison with the distances of stars, so we can neglect the lunar daily and monthly parallaxes of stars.

The monthly parallax of a star is defined as

$$p_m^{☾} = \frac{R_0}{\rho_{☾} \sin 1''} , \qquad \text{(D-1)}$$

where $\rho_{☾}$ is a selenocentric star distance, and R_0 = 384,400 km is the mean Moon–Earth distance. This parallax for the nearest 15 stars, for which 1.3 parsec < $\rho_{☾}$ < 3.5 parsec, is of the order of 0".01 to 0".02. For all other stars, this parallax is < 0".01 and generally of the order of several thousandths of a second, or smaller.

D.2 The Lunar Daily Parallax

The lunar daily parallax, or the parallax of the Moon's radius for the nearer celestial bodies such as the Sun and the planets and for the Moon's artificial satellites, is quite large. In the same way that we define the Earth's horizontal equatorial parallax, we define the mean lunar horizontal parallax as

$$\sin p_d^{☾} = \frac{r_{☾}}{\rho_{☾}} \qquad \text{(D-2)}$$

or

$$p_d^{☾} \approx \frac{r_{☾}}{\rho_{☾} \sin 1''} \quad , \qquad \text{(D-3)}$$

where $r_{☾}$ is the mean equatorial radius of the Moon and $\rho_{☾}$ is the selenocentric distance of the considered body. Putting for $r_{☾}$ the value 1738 km, we can calculate the mean lunar horizontal parallax for different distances of lunar artificial satellites (Table D-1) and for the extreme distances of the planets (Table D-2). The parallax for small distances of the order of several times the Moon's radius, which are the distances of the lunar artificial satellites, is calculated by equation (D-2), in which $\rho_{☾} = r_{☾} + H$, and H is a height above the Moon's surface.

Table D-1. The lunar daily parallaxes of the distances of the lunar artificial satellites

$\rho_{☾}$ (in units of $r_{☾}$)	$H = \rho_{☾} - r_{☾}$ (in units of $r_{☾}$)	$p_d^{☾}$
1	0	90°
1.5	0.5	42°
5	4	11°
6	5	10°
10	9	6°
20	19	3°

Table D-2. The lunar daily parallaxes of the Sun and the planets

Name	a^* (in units of 10^6 km)	Extreme values of $\rho_{☾}$ (in units of 10^6 km)	$p_d^{☾}$
Moon	0.4 †	0.364 400 to 0.406 730	$881''.4$ to $983''.8$
Earth	149.5	149.1 to 149.9	$2''.39_2$ to $2''.40_5$
Mercury	57.9	72 to 220	$1''.5$ to $5''.0$
Venus	108.3	42 to 257	$1''.5$ to $8''.5$
Mars	228.1	57 to 398	$1''.0$ to $6''.5$
Jupiter	778.6	629 to 928	$\sim 0''.5$
Saturn	1 430.1	128 0 to 158 0	$\sim 0''.25$
Uranus	2 876.5	302 7 to 272 7	$\sim 0''.1$
Neptune	4 506.6	465 6 to 435 6	$\sim 0''.1$
Pluto	5 914.8	606 5 to 576 4	$\sim 0''.05$

*a is the semimajor axis of the orbit of the planet around the Sun.

†a is the semimajor axis of the Moon's orbit around the Earth.

The distances of points on the Moon's surface from its center of mass vary from one point to another, so the lunar horizontal parallax also changes its value. The lunar horizontal parallax for the point whose selenocentric distance is r is

$$\sin \pi_d^{☾} = \frac{r}{\rho_{☾}} , \quad \text{or} \quad \pi_d^{☾} = \frac{r}{\rho_{☾} \sin 1''} . \tag{D-4}$$

The parallax $\pi_d^{☾}$ can be expressed by the mean lunar horizontal parallax $p_d^{☾}$,

$$\sin \pi_d^{☾} = \frac{r_{☾}}{\rho_{☾}} \frac{r}{r_{☾}} = \sin p_d^{☾} \, r_r \quad , \tag{D-5}$$

where r_r is a radius vector of a point on the Moon's surface expressed in units of the mean Moon's radius $r_{☾}$.

The shape of the Moon is not well known. However, it is very close to a sphere and we can expect that the differences in the mean radius of the different parts of the Moon are not greater than several kilometers. The heights of the Moon's mountains are of the same order. Hence, these small differences of the selenocentric distances of the point on the Moon's surface have to be taken into account in the parallax calculation only for such near bodies as the Earth, Sun, Mercury, Venus, and Mars.

Differentiating formula (D-5), we obtain

$$\cos \pi_d^{☾} \, d\, \pi_d^{☾} = \sin p_d^{☾} \, dr_r \quad ,$$

and putting

$$dr_r \approx \frac{10\,\mathrm{km}}{1\,738} = 6 \cdot 10^{-3}, \quad \cos \pi_d^{☾} = 1 \, , \quad \sin p_d^{☾} = p_d^{☾} \, ,$$

we have

$$d\pi_d^{☾} = 6 \times 10^{-3} \, p_d^{☾} \quad . \tag{D-6}$$

Taking into consideration the parallaxes given in Tables D-1 and D-2, we can easily see that for all bodies more distant than Mars the influence of $dr_r = 10\,\mathrm{km}$ is $< 0''.01$ and for other planets is $< 0''.003$.

Changing the place of observations from the center of the Moon's mass to its surface, or vice versa, causes the translation of the coordinate system; we can obtain the proper transformation by using the rectangular coordinate system

$$\begin{Bmatrix} x \\ y \\ z \end{Bmatrix} = \begin{Bmatrix} x' \\ y' \\ z' \end{Bmatrix} + \begin{Bmatrix} X_0 \\ Y_0 \\ Z_0 \end{Bmatrix} , \qquad \text{(D-7)}$$

where

x, y, z = selenocentric rectangular coordinates,

x', y', z' = lunar topocentric coordinates,

X_0, Y_0, Z_0 = selenocentric coordinates of the origin of the lunar topocentric coordinates, which are expressed by the selenographic latitude and longitude of the Moon.

These rectangular coordinates, expressed by the polar coordinates ρ_p, u, v [see (E-2) and (E-3)], are

$$\begin{Bmatrix} x \\ y \\ z \end{Bmatrix} = \begin{Bmatrix} \rho_P \cos u \cos v \\ \rho_P \sin u \sin v \\ \rho_P \sin v \end{Bmatrix} , \quad \begin{Bmatrix} x' \\ y' \\ z' \end{Bmatrix} = \begin{Bmatrix} \rho'_P \cos u' \cos v' \\ \rho'_P \sin u' \cos v' \\ \rho_P \sin v' \end{Bmatrix} , \qquad \text{(D-8)}$$

$$\begin{Bmatrix} X_0 \\ Y_0 \\ Z_0 \end{Bmatrix} = \begin{Bmatrix} \rho_0 \cos u_0 \cos v_0 \\ \rho_0 \sin u_0 \cos v_0 \\ \rho_0 \sin v_0 \end{Bmatrix} . \qquad \text{(D-9)}$$

Substitutions for selenocentric coordinates (ρ_P, u, v), topocentric coordinates (ρ'_P, u', v'), and selenocentric coordinates of the origin of the topocentric coordinate system (ρ_0, u_0, v_0) are given for different spherical systems in Table D-3. If we neglect the difference between selenodetic and selenocentric latitudes $(\phi^L_{sel} - \phi^L)$, which in the case of the Moon is small, we obtain $\rho_0 = r$, u_0 is undefined, $v_0 = 90°$, $X_0 = 0$, $Y_0 = 0$, and $Z_0 = r$.

Table D-3. The selenocentric coordinates of a point on the Moon's surface and the notation of the selenocentric and lunar topocentric coordinates in the different coordinate systems

Coordinate system	Selenocentric coordinates			Topocentric coordinates			Selenocentric coordinates of topocentric origin		
	ρ_P	u	v	ρ'_P	u'	v'	ρ_0	u_0	v_0
Selenoequatorial	ρ_P	a^{Ω}	d	ρ'_P	a'^{Ω}	d'	r	θ^L_{Ω}	ϕ^L
Lunar-horizontal	ρ_P	A^L	$90° - z^L$	ρ'_P	A'^L	$90° - z'^L$	r	$\lambda^{L^{eq}}$	$90° - \left(\phi^L_{sel} - \phi^L\right)$
Ecliptic	ρ_P	λ	β	ρ'_P	λ'	β'	r	$L^{Z☾}$	$B^{Z☾}$
Geoequatorial	ρ_P	α	δ	ρ'_P	α'	δ'	r	$\alpha^{Z☾}$	$\delta^{Z☾}$

The coordinates $L^{Z☾}$, $B^{Z☾}$ and $\alpha^{Z☾}$, $\delta^{Z☾}$ are, respectively, the ecliptic and the geoequatorial coordinates of the zenith of the observer. The ecliptic coordinates $L^{Z☾}$, $B^{Z☾}$ can be determined from the triangle $Z^☾P^☾E$ (Figure 8, Section 6.2):

$$\sin B^{Z☾} = \cos I \sin \phi^L + \cos \phi^L \sin I \sin \theta^L_{\Omega_{eq}} ,$$

$$\cos B^{Z☾} \sin (L^{Z☾} - \Omega) = \sin \phi^L \sin I - \cos \phi^L \cos I \sin \theta^L_{\Omega_{eq}} ,$$

$$\cos B^{Z☾} \cos (L^{Z☾} - \Omega) = - \cos \phi^L \cos \theta^L_{\Omega_{eq}} . \qquad \text{(D-10)}$$

The geoequatorial coordinates $\alpha^{Z☾}$, $\delta^{Z☾}$ can be obtained from the ecliptic coordinates by transformation (5) (Section 2.1).

Hence, for example, in the selenoequatorial coordinate system the formulas are

$$\begin{Bmatrix} x \\ y \\ z \end{Bmatrix} = \begin{Bmatrix} \rho_P \cos a^{\Omega} \cos d \\ \rho_P \sin a^{\Omega} \cos d \\ \rho_P \sin d \end{Bmatrix} , \quad \begin{Bmatrix} x' \\ y' \\ z' \end{Bmatrix} = \begin{Bmatrix} \rho'_P \cos a'^{\Omega} \cos d' \\ \rho'_P \sin a'^{\Omega} \cos d' \\ \rho'_P \sin d' \end{Bmatrix} , \quad \text{(D-11)}$$

$$\begin{Bmatrix} X_0 \\ Y_0 \\ Z_0 \end{Bmatrix} = \begin{Bmatrix} r \cos \theta^L_{\Omega_{eq}} \cos \phi^L \\ r \sin \theta^L_{\Omega_{eq}} \cos \phi^L \\ r \sin \phi^L \end{Bmatrix} , \quad \text{(D-12)}$$

where $\theta^L_{\Omega_{eq}}$ denotes the hour angle of the ascending node of the lunar equator on the ecliptic, and ρ_0 is equal to the Moon's radius r.

In the system of the coordinates in which the x axis is directed to the Earth (first radius), the y axis is directed 90° to the west (Moon's) of the x axis, and the z axis is the Moon's axis of rotation, the coordinates X_0, Y_0, Z_0 are the following:

$$\begin{Bmatrix} X_0 \\ Y_0 \\ Z_0 \end{Bmatrix} = \begin{Bmatrix} r \cos \lambda^L \cos \phi^L \\ r \sin \lambda^L \cos \phi^L \\ r \sin \phi^L \end{Bmatrix} , \quad \text{(D-13)}$$

where λ^L is the selenographic longitude measured along the lunar equator from the meridian of the first radius to the Moon's west direction, from 0° to 360°.

The coordinates x, y, z and x′, y′, z′ in this system are

$$\begin{Bmatrix} x \\ y \\ z \end{Bmatrix} = \begin{Bmatrix} \rho_P \cos(\theta^{L,0}_{\Omega_{eq}} - a^{\Omega}) \cos d \\ \rho_P \sin(\theta^{L,0}_{\Omega_{eq}} - a^{\Omega}) \cos d \\ \rho_P \sin d \end{Bmatrix} , \begin{Bmatrix} x' \\ y' \\ z' \end{Bmatrix} = \begin{Bmatrix} \rho'_P \cos(\theta'^{L,0}_{\Omega_{eq}} - a'^{\Omega}) \cos d' \\ \rho'_P \sin(\theta'^{L,0}_{\Omega_{eq}} - a'^{\Omega}) \cos d' \\ \rho'_P \sin d \end{Bmatrix} , \tag{D-14}$$

where $\theta^{L,0}_{\Omega_{eq}}$ denotes the lunar hour angle of the ascending node of the lunar equator measured from the lunar meridian of the first radius.

The hour angles $\theta^{L}_{\Omega_{eq}}$, $\theta^{L,0}_{\Omega_{eq}}$ change their values continuously. They can be expressed by other known values such as the hour angle of the vernal equinox (the Moon's sidereal time), the longitude of the ascending node of the Moon's orbit, and the Moon's longitude.

Generally, we can write

$$\theta^{L}_{\Omega_{eq}} = \theta^{L}_{\Upsilon} - (\Omega - 180°) \quad ,$$

$$\theta^{L}_{*} = \theta^{L}_{\Omega_{eq}} - a^{\Omega}_{*} = \theta^{L}_{\Upsilon} - a^{\Upsilon}_{*} \quad . \tag{D-15}$$

For example, for the situation shown in Figure D-1, we can write

$$\theta^{L,0}_{\Omega_{eq}} = \theta^{L,0}_{\Upsilon} - (\Omega - 180°) = \theta^{L,P}_{\Upsilon} + \lambda^{L}_{P} - (\Omega - 180°) \quad ,$$

$$\theta^{L,P}_{*} = \theta^{L,P}_{\Omega_{eq}} - a^{\Omega}_{*} = \theta^{L,P}_{\Upsilon} - a^{\Upsilon}_{*} \quad .$$

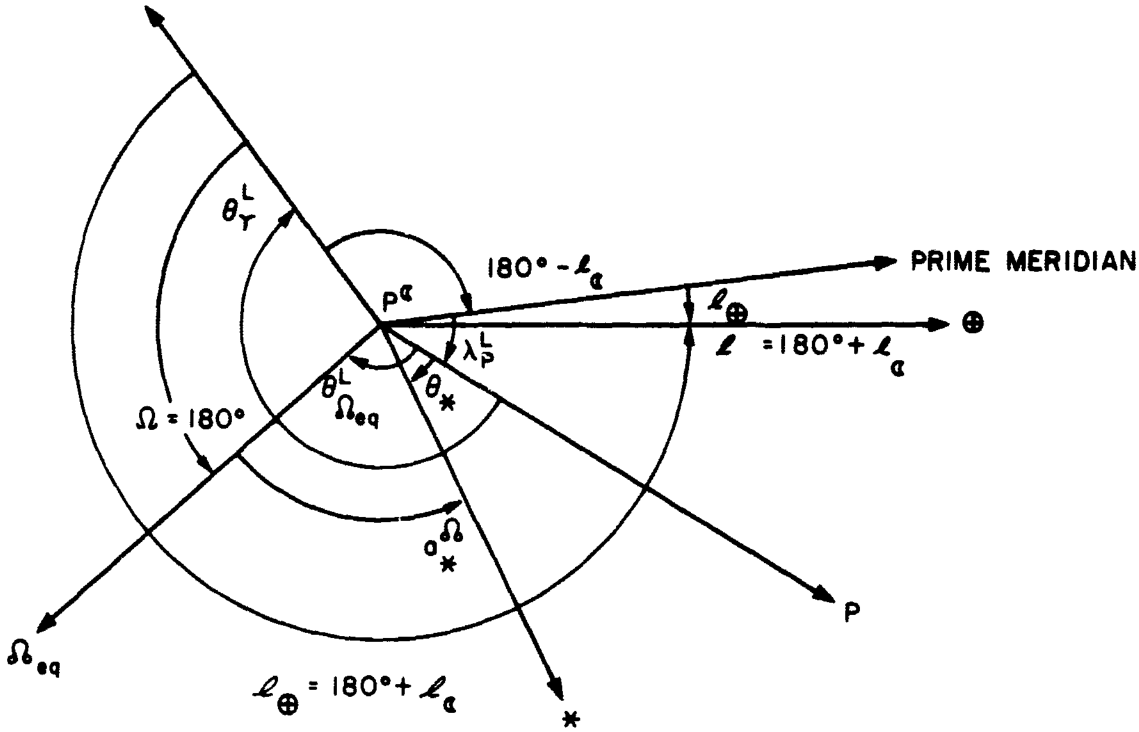

Figure D-1. The relation between lunar hour angles of different points on the Moon's celestial sphere.

The formulas for the influence of the selenocentric parallax in different spherical coordinates can be written on the basis of the general parallactic formulas (E-6) to (E-11):

$$\tan (u - u') = - \frac{m_1 \sin (u - u_0)}{1 - m_1 \cos (u - u_0)} \quad ,$$

$$\tan (v - v') = - \frac{n_1 \sin (v - \gamma_p)}{1 - n_1 \cos (v - \gamma_p)} \quad , \qquad \text{(D-16)}$$

where

$$m_1 = \frac{r}{\rho_p} \frac{\cos v_0}{\cos v} \quad , \quad n_1 = \frac{r}{\rho_p} \frac{\sin v_0}{\sin \gamma_p} \quad , \qquad \text{(D-17)}$$

$$\tan \gamma_p = \tan v_0 \frac{\cos [0.5 (u' - u)]}{\cos [u_0 - 0.5 (u + u')]} ; \qquad \text{(D-18)}$$

and in this case,

u, v = selenocentric spherical coordinates;

u', v' = topocentric spherical coordinates;

u_0, v_0 = selenocentric coordinates of the observation site;

r = selenocentric distance of the observation site (radius of the Moon);

ρ_p = selenocentric distance of a considered body.

The proper substitutions for the spherical coordinates u, v; u', v'; u_0, v_0 in the different coordinate systems are given in Table D-3.

In the selenoequatorial coordinate system, these formulas are expressed as

$$\tan (a'^{\Omega} - a^{\Omega}) = \frac{m_d^{☾} \sin \theta^L}{1 - m_d^{☾} \cos \theta^L} ,$$

$$\tan (d' - d) = \frac{n_d^{☾} \sin (d - \gamma_d^{☾})}{1 - n_d^{☾} \cos (d - \gamma_d^{☾})} ,$$

where

$$m_d^{☾} = \frac{r_r \sin p_d^{☾} \cos \phi^L}{\cos d} , \qquad n_d^{☾} = \frac{r_r \sin p_d^{☾} \sin \phi^L}{\sin \gamma_d^{☾}} ,$$

$$\tan \gamma_d^{☾} = \tan \phi^L \frac{\cos [0.5 (a'^{\Omega} - a^{\Omega})]}{\cos [\theta^L_{\Omega_{eq}} - 0.5 (a^{\Omega} + a'^{\Omega})]} . \qquad \text{(D-19)}$$

In the case of lunar horizontal coordinates, if we put $(\phi^L_{sel} - \phi^L) = 0$, we obtain $A^L = A'^L$ and $z^L - z'^L = - \pi^{☾}_d \sin z^L$.

D. 3 Lunar Monthly Parallax

The lunar monthly parallax can be treated as the greater lunar diurnal parallax. Instead of the translation of the origin of a coordinate system from the center of the Moon to its surface, there is the translation of the coordinate system from the center of the Moon's mass to the center of the Earth's mass.

We define the mean lunar monthly parallax by the formula

$$\sin p^{☾}_m = \frac{R_0}{\rho_{☾}} \quad , \tag{D-20}$$

where $\rho_{☾}$ is the selenocentric distance of a body. Expressing the Earth–Moon distance $\Delta_{\oplus☾}$ in the unit of the mean distance R_0 = 384,000 km,

$$R_r = \frac{\Delta_{\oplus☾}}{R_0} \quad ,$$

we can define the lunar monthly parallax $\pi^{☾}_m$ by the mean parallax,

$$\sin \pi^{☾}_m = \frac{\Delta_{\oplus☾} R_0}{R_0 \rho_{☾}} = R_r \sin p^{☾}_m \quad . \tag{D-21}$$

The mean lunar monthly parallax for stars is small, of the order of several thousandths of a second or smaller.

The values of $p_m^{☾}$ for the Sun and the planets are $R_0/r_{☾} = 221.174$ times greater than the lunar daily parallaxes given in Table D-2. The approximate values of $p_m^{☾}$ for the Sun and the planets, excluding the Earth, are given in Table D-4.

Table D-4. The approximate values of the lunar monthly parallaxes of the Sun and of the planets

Name	$p_m^{☾}$
Sun	~ 9'
Mercury	6' to 17'
Venus	5' to 32'
Mars	3' to 23'
Jupiter	1!5 to 2'
Saturn	~ 1'
Uranus	~ 0!5
Neptune	< 0!5
Pluto	< 0!5

Hence, the general formulas for the translation of the rectangular coordinate system can be written in the same form as in the case of the lunar diurnal parallax [see transformation (D-7)] :

$$\begin{Bmatrix} x \\ y \\ z \end{Bmatrix} = \begin{Bmatrix} x'' \\ y'' \\ z'' \end{Bmatrix} + \begin{Bmatrix} X_{0\oplus} \\ Y_{0\oplus} \\ Z_{0\oplus} \end{Bmatrix} , \qquad \text{(D-22)}$$

where

x, y, z = selenocentric coordinates,

x'', y'', z'' = geocentric coordinates,

$X_{0\oplus}\ Y_{0\oplus}\ Z_{0\oplus}$ = selenocentric coordinates of the Earth's center of mass.

Generally, these rectangular coordinates of a point can be expressed by the different polar coordinates [see (D-8) and (D-9)] ; but the topocentric coordinates (ρ', u', v') will be replaced by the geocentric coordinates (ρ'', u'', v''), and the selenocentric coordinates of the point on the Moon's surface will be replaced by the selenocentric coordinates of the Earth $(\Delta_{\oplus☾}, u_{\oplus}, v_{\oplus})$.

The appropriate substitutions for the spherical coordinates u, v; u'', v''; and $u_{\oplus}, v_{\oplus}$ are given in Table D-5 for different coordinate systems.

Table D-5. The selenocentric coordinates of the Earth and the notation of the selenocentric and geocentric coordinates in different systems

Coordinate system	Selenocentric coordinates		Geocentric coordinates		Selenocentric coordinates of the Earth	
	u	v	u''	v''	$u_{\oplus}$	$v_{\oplus}$
Selenoequatorial	$\theta^{L,0}_{\Omega_{eq}} - a^{\Omega}$	d	$\theta''^{L,0}_{\Omega_{eq}} - a''^{\Omega}$	d''	$\ell_{\oplus}$	$b_{\oplus}$
	a^{Ω}	d	a''^{Ω}	d''	$\theta^{L,0}_{\Omega_{eq}} - \ell_{\oplus}$	$b_{\oplus}$
Ecliptic	λ	β	λ''	β''	$180^{\circ} + \ell_{☾}$	$-\beta_{☾}$
Geoequatorial	α	δ	α''	δ''	$180^{\circ} + \alpha_{☾}$	$-\alpha_{☾}$

Here, $b_\oplus$, $\ell_\oplus$ are the selenocentric selenographic latitude and longitude of the center of the Earth (or the sub-Earth point on the Moon's surface), which are given in the almanacs and are calculated by the known formulas of the geocentric Moon's optical libration (Arthur, 1960):

$$\cos(☾ + \ell_\oplus - \Omega)\cos b_\oplus = \cos(\ell_☾ - \Omega - N)\cos\beta_☾ \quad ,$$

$$\sin(☾ + \ell_\oplus - \Omega)\cos b_\oplus = \sin(\ell_☾ - \Omega - N)\cos\beta_☾ \cos I - \sin\beta_☾ \sin I \quad ,$$

$$\sin b_\oplus = -\sin(\ell_☾ - \Omega - N)\cos\beta_☾ \sin I - \sin\beta_☾ \cos I \quad , \qquad \text{(D-23)}$$

where $\ell_☾$, $\beta_☾$ are the Moon's true geocentric coordinates, Ω is the longitude of the mean ascending node of the lunar orbit on the ecliptic, I is the inclination of the Moon's equator to the ecliptic, N is the nutation in the longitude, and ☾ is the mean geocentric longitude of the Moon.

The appropriate formulas in the selenoequatorial coordinate system are

$$\begin{Bmatrix} x \\ y \\ z \end{Bmatrix} = \begin{Bmatrix} \rho_☾ \cos a^\Omega \cos d \\ \rho_☾ \sin a^\Omega \cos d \\ \rho_☾ \sin d \end{Bmatrix} \quad , \quad \begin{Bmatrix} x'' \\ y'' \\ z'' \end{Bmatrix} = \begin{Bmatrix} \rho''_\oplus \cos a''^\Omega \cos d'' \\ \rho''_\oplus \sin a''^\Omega \cos d'' \\ \rho''_\oplus \sin d'' \end{Bmatrix} \quad ,$$

$$\begin{Bmatrix} X_{0\oplus} \\ Y_{0\oplus} \\ Z_{0\oplus} \end{Bmatrix} = \begin{Bmatrix} \Delta_{\oplus☾} \cos\left(\theta^{L,0}_{\Omega eq} - \ell_\oplus\right)\cos b_\oplus \\ \Delta_{\oplus☾} \sin\left(\theta^{L,0}_{\Omega eq} - \ell_\oplus\right)\cos b_\oplus \\ \Delta_{\oplus☾} \sin b_\oplus \end{Bmatrix} \quad , \qquad \text{(D-24)}$$

or

$$\begin{Bmatrix} x \\ y \\ z \end{Bmatrix} = \begin{Bmatrix} \rho_{☾} \cos\left(\theta^{L,0}_{\Omega_{eq}} - a^{\Omega}\right) \cos d \\ \rho_{☾} \sin\left(\theta^{L,0}_{\Omega_{eq}} - a^{\Omega}\right) \cos d \\ \rho_{☾} \sin d \end{Bmatrix} , \quad \begin{Bmatrix} x'' \\ y'' \\ z'' \end{Bmatrix} = \begin{Bmatrix} \rho_{\oplus} \cos\left(\theta''^{L,0}_{\Omega_{eq}} - a''^{\Omega}\right) \cos d'' \\ \rho_{\oplus} \sin\left(\theta''^{L,0}_{\Omega_{eq}} - a''^{\Omega}\right) \cos d'' \\ \rho_{\oplus} \sin d'' \end{Bmatrix}$$

$$\begin{Bmatrix} X_{0\oplus} \\ Y_{0\oplus} \\ Z_{0\oplus} \end{Bmatrix} = \begin{Bmatrix} \Delta_{\oplus☾} \cos \ell_{\oplus} \cos b_{\oplus} \\ \Delta_{\oplus☾} \sin \ell_{\oplus} \cos b_{\oplus} \\ \Delta_{\oplus☾} \sin b_{\oplus} \end{Bmatrix} . \qquad \text{(D-25)}$$

Similarly, as in the case of the selenocentric parallax, the influence of the lunar monthly parallax in spherical coordinates can be expressed by the general formulas for the parallax (D-16) to (D-18) with the substitutions given in Table D-5.

The influence of the lunar monthly parallax in selenoequatorial coordinates a^{Ω}, d is expressed by

$$\tan\left(a''^{\Omega} - a^{\Omega}\right) = \frac{m^{☾}_{m} \sin\left(\theta^{L,0}_{\Omega_{eq}} - \ell_{\oplus} - a^{\Omega}\right)}{1 - m^{☾}_{m} \cos\left(\theta^{L,0}_{\Omega_{eq}} - \ell_{\oplus} - a^{\Omega}\right)} ,$$

$$\tan\left(d'' - d\right) = \frac{n^{☾}_{m} \sin\left(d - \gamma^{☾}_{m}\right)}{1 - n^{☾}_{m} \cos\left(d - \gamma^{☾}_{m}\right)} , \qquad \text{(D-26)}$$

where

$$m_m^{☾} = \frac{\sin \pi_m^{☾} \cos b_{\oplus}}{\cos d} \quad , \qquad n_m^{☾} \frac{\sin \pi_m^{☾} \sin b_{\oplus}}{\sin \gamma_m^{☾}} \quad ,$$

$$\tan \gamma_m^{☾} = \tan b_{\oplus} \frac{\cos\left[0.5\,(a''^{\Omega} - a^{\Omega})\right]}{\cos\left[\theta_{\Omega eq}^{L,0} - \ell_{\oplus} - 0.5\,(a^{\Omega} + a''^{\Omega})\right]} \quad ,$$

$$\sin \pi_m^{☾} = \frac{\Delta_{\oplus ☾}}{\rho_{☾}} \quad . \qquad \text{(D-27)}$$

Knowledge of the lunar monthly parallax in geoequatorial and ecliptic coordinates can be very useful in practice because the coordinates of the Sun and of the planets are given in these systems in astronomical almanacs. So we will be interested in the corrections that allow us to calculate the selenocentric coordinates of the Sun, the planets, etc., from the geocentric coordinates.

In this case, we can treat the lunar monthly parallax as the larger Earth's geocentric parallax and introduce into the formulas for the Earth's geocentric parallax the proper substitutions that are shown in Table D-6.

Table D-6. Transformation of the formulas for the Earth's geocentric parallax into the formulas for the lunar monthly parallax

Earth's geocentric parallax	Lunar monthly parallax
Radius vector of the Earth's surface point	Radius vector of the Moon's mass center, $\Delta_{\oplus ☾}$
Earth's topocentric distance	Selenocentric distance, $\rho_{☾}$
Earth's topocentric coordinates: $\lambda', \beta'; \alpha', \delta'$	Selenocentric coordinates: $\lambda, \beta; \alpha, \delta$
Coordinates of the zenith: $h = S - \alpha, \phi'; \lambda^{zenith}, \beta^{zenith}$	Geocentric coordinates of the Moon: $\alpha_{☾}, \delta_{☾}; \ell_{☾}, \beta_{☾}$

Hence, the influence of the lunar monthly parallax in ecliptic coordinates is expressed by

$$\tan(\lambda_{sel} - \lambda_{geo}) = \frac{-m'_e \sin(\ell_☾ - \lambda_{geo})}{1 - m'_e \cos(\ell_☾ - \lambda_{geo})} ,$$

$$\tan(\beta_{sel} - \beta_{geo}) = \frac{-n'_e \sin(\gamma'_e - \beta_{geo})}{1 - n'_e \cos(\gamma'_e - \beta_{geo})} , \tag{D-28}$$

where

$$m'_e = \frac{\Delta_{\oplus☾} \cos\beta_☾}{\rho_{geo} \cos\beta_{geo}} , \quad n'_e = \frac{\Delta_{\oplus☾} \sin\beta_☾}{\rho_{geo} \sin\gamma'_e} ,$$

$$\tan\gamma'_e = \frac{\tan\beta_☾ \cos[0.5(\lambda_{sel} - \lambda_{geo})]}{\cos[\ell_☾ - 0.5(\lambda_{sel} + \lambda_{geo})]} ; \tag{D-29}$$

and in geoequatorial coordinates by

$$\tan(\alpha_{sel} - \alpha_{geo}) = -\frac{m' \sin(\alpha_☾ - \alpha_{geo})}{1 - m \cos(\alpha_☾ - \alpha_{geo})} ,$$

$$\tan(\delta_{sel} - \delta_{geo}) = -\frac{n' \sin(\gamma'_g - \delta_{geo})}{1 - n \cos(\gamma'_g - \delta_{geo})} , \tag{D-30}$$

where

$$m' = \frac{\Delta_{\oplus☾} \cos \delta_{☾}}{\rho_{geo} \cos \delta_{geo}} \quad , \quad n' = \frac{\Delta_{\oplus☾} \sin \delta_{☾}}{\rho_{geo} \sin \gamma'_g} \quad ,$$

$$\tan \gamma'_g = \frac{\tan \delta_{☾} \cos[0.5\,(\alpha_{sel} - \alpha_{geo})]}{\cos[\alpha_{☾} - 0.5\,(\alpha_{sel} + \alpha_{geo})]} \quad .$$

In these formulas, λ_{geo}, β_{geo}; α_{geo}, δ_{geo} denote the geocentric coordinates, and λ, β; α, δ the selenocentric coordinates.

D.4 Lunar Annual Parallax

Earth astronomical almanacs very often give not only the geocentric coordinates of planets but also the heliocentric ones, mainly the ecliptic. It would be useful to be able to calculate the selenocentric coordinates directly from the heliocentric coordinates without calculating the geocentric ones.

The translation of the celestial body's heliocentric rectangular coordinates (X^0, Y^0, Z^0) of any system into the selenocentric ones (x, y, z) can be written in the form

$$\begin{Bmatrix} X^0 \\ Y^0 \\ Z^0 \end{Bmatrix} = \begin{Bmatrix} x \\ y \\ z \end{Bmatrix} + \begin{Bmatrix} X^0_{☾} \\ Y^0_{☾} \\ Z^0_{☾} \end{Bmatrix} \quad , \tag{D-31}$$

where $X^0_{☾}$, $Y^0_{☾}$, $Z^0_{☾}$ are the heliocentric rectangular coordinates of the Moon, which are not given in the almanacs. The almanacs give the geocentric rectangular coordinates of the Sun ($X_{\odot}$, $Y_{\odot}$, $Z_{\odot}$), which can be easily changed into heliocentric rectangular coordinates of the Earth:

$$X^0_{\oplus} = -X_{\odot}, \qquad Y^0_{\oplus} = -Y_{\odot}, \qquad Z^0_{\oplus} = -Z_{\odot} \quad .$$

If we know the geocentric rectangular coordinates of the Moon ($X_{☾}, Y_{☾}, Z_{☾}$), which will be useful to have in the Moon's almanac, we can calculate the $X_{☾}^0, Y_{☾}^0, Z_{☾}^0$:

$$\begin{pmatrix} X_{☾}^0 \\ Y_{☾}^0 \\ Z_{☾}^0 \end{pmatrix} = \begin{pmatrix} X_{\oplus}^0 + X_{☾} \\ Y_{\oplus}^0 + Y_{☾} \\ Z_{\oplus}^0 + Z_{☾} \end{pmatrix} \quad . \tag{D-32}$$

Hence, the heliocentric distance of the Moon can also be calculated:

$$\Delta_{☾\odot} = \left(X_{☾}^{0^2} + Y_{☾}^{0^2} + Z_{☾}^{0^2} \right)^{1/2} \quad . \tag{D-33}$$

APPENDIX E

GENERAL PARALLACTIC FORMULAS IN RECTANGULAR AND SPHERICAL COORDINATES

The general formulas for a parallax caused by a translation of the origin of any coordinate system can be written if we know the length and direction of the translation.

The general relations between the rectangular coordinates of a body in the two systems, initially X, Y, Z and translated X′, Y′, Z′, are the following:

$$\begin{Bmatrix} x \\ y \\ z \end{Bmatrix} = \begin{Bmatrix} x' \\ y' \\ z' \end{Bmatrix} + \begin{Bmatrix} X_0 \\ Y_0 \\ Z_0 \end{Bmatrix} . \qquad \text{(E-1)}$$

Introducing the polar coordinate system ρ, u, v, we can write the rectangular coordinates in the form

$$\begin{Bmatrix} x \\ y \\ z \end{Bmatrix} = \begin{Bmatrix} \rho_P \cos u \cos v \\ \rho_P \sin u \cos v \\ \rho_P \sin u \end{Bmatrix} , \quad \begin{Bmatrix} x' \\ y' \\ z' \end{Bmatrix} = \begin{Bmatrix} \rho'_P \cos u' \cos v' \\ \rho'_P \sin u' \cos v' \\ \rho'_P \sin u' \end{Bmatrix} , \qquad \text{(E-2)}$$

$$\begin{Bmatrix} X_0 \\ Y_0 \\ Z_0 \end{Bmatrix} = \begin{Bmatrix} \Delta \cos u_0 \cos v_0 \\ \Delta \sin u_0 \cos v_0 \\ \Delta \sin u_0 \end{Bmatrix} . \qquad \text{(E-3)}$$

The notation used above is illustrated in Figure E-1.

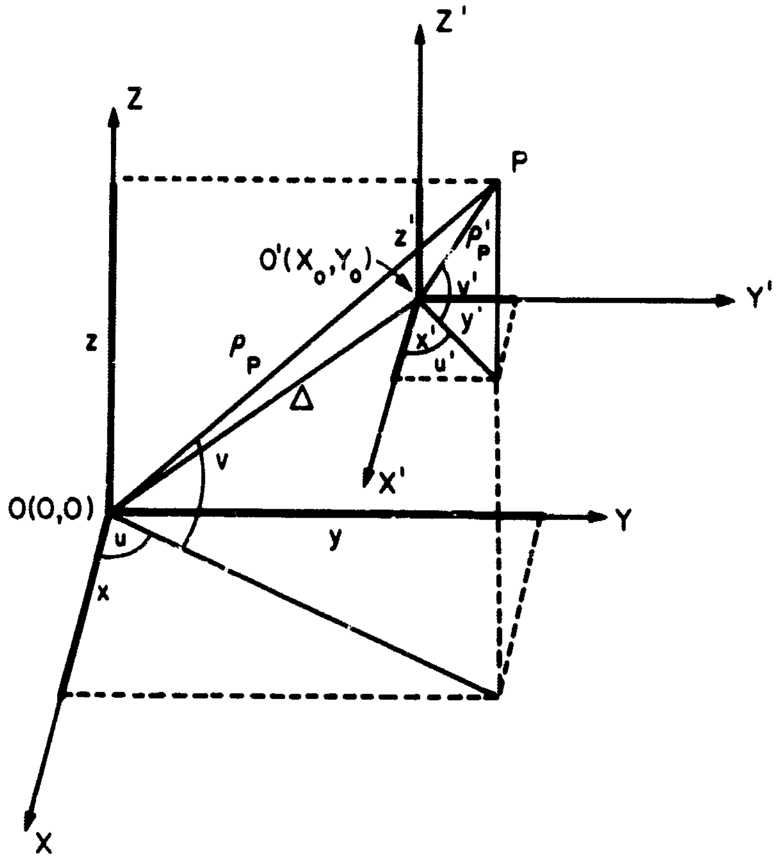

Figure E-1. Translation of the rectangular coordinate system.

Introducing equations (E-2) and (E-3) into (E-1) gives us

$$\rho'_P \cos u' \cos v' = \rho_P \cos u \cos v - \Delta \cos u_0 \cos v_0 \quad ,$$

$$\rho'_P \sin u' \cos v' = \rho_P \sin u \cos v - \Delta \sin u_0 \cos v_0 \quad ,$$

$$\rho_P \sin u' = \rho_P \sin u - \Delta \sin u_0 \quad , \qquad \text{(E-4)}$$

which can easily be transformed into

$$\rho'_P \mathrm{ccs}\ v' \sin (u' - u) = \Delta \sin (u - u_0) \cos v_0 \quad ,$$

$$\rho'_P \cos v' \cos (u' - u) = \rho_P \cos v - \Delta \cos (u - u_0) \cos v_0 \quad ,$$

$$\rho'_P \sin v' = \rho_P \sin v - \Delta \sin v_0 \quad . \qquad \text{(E-5)}$$

The first two equations of (E-5) yield

$$\tan(u' - u) = \frac{m_1 \sin(u - u_0)}{1 - m_1 \cos(u - u_0)} \quad , \qquad \text{(E-6)}$$

where

$$m_1 = \frac{\Delta}{\rho_p} \frac{\cos v_0}{\cos v} \quad . \qquad \text{(E-7)}$$

Multiplying the first equation of (E-5) by $\sin[0.5(u' - u)]$ and the second by $\cos[0.5(u' - u)]$ and adding them, we obtain

$$\rho'_p \cos v' = \rho_p \cos v - \Delta \cot \gamma_p \sin v_0 \quad , \qquad \text{(E-8)}$$

where

$$\tan \gamma_p = \tan v_0 \frac{\cos[0.5(u' - u)]}{\cos[u_0 - 0.5(u + u')]} \quad . \qquad \text{(E-9)}$$

Equation (E-8), together with the third equation of (E-5), yields

$$\tan(v' - v) = \frac{n_1 \sin(v - \gamma_p)}{1 - n_1 \cos(v - \gamma_p)} \quad , \qquad \text{(E-10)}$$

where

$$n_1 = \frac{\Delta}{\rho_p} \frac{\sin v_0}{\sin \gamma_p} \quad . \qquad \text{(E-11)}$$

The formulas (E-6), (E-7), and (E-9) to (E-11) are the general formulas for the influence of a parallax in spherical coordinates u, v.

APPENDIX F

GENERAL FORMULAS FOR THE INFLUENCE OF AN ABERRATION IN SPHERICAL COORDINATES

The basis for our calculations is given in Figure F-1.

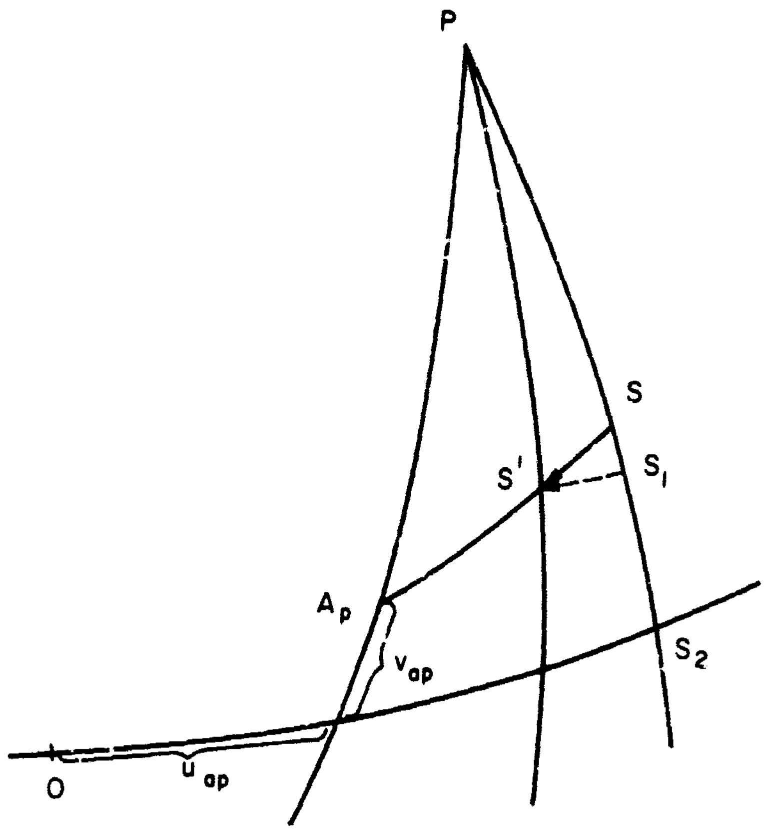

Figure F-1. Aberrational displacement of a star.

$Ap\ (u_{ap}, v_{ap})$	=	apex of the considered motion of the coordinate system,
$S\ (u, v)$, $S'\ (u', v')$	=	true and apparent positions of a star, respectively,
P	=	pole of the coordinate system,
OS_2	=	equator of the system,
0	=	zero point of the first spherical coordinate u.

According to Figure F-1, we have

$$SS' = k_0 \sin SA_p ,$$
$$(u - u') \cos v = SS' \sin \angle A_pSS_1 ,$$
$$v - v' = SS' \cos \angle A_pSS_1 . \qquad \text{(F-1)}$$

In these formulas, the coefficient of an aberration k_0 is

$$k_0 = \frac{V}{c \sin 1''} ,$$

where V is the velocity of the considered motion of the coordinate system, and c is the velocity of light, 299,792 km sec^{-1}. On the basis of the triangle A_pPS,

$$u - u' = k_0 \cos v_{ap} \sin (u - u_{ap}) \sec v ,$$

$$v - v' = k_0 \cos v_{ap} \sin v \cos (u - u_{ap}) - \sin v_{ap} \cos v , \qquad \text{(F-2)}$$

which are the general formulas for the influence of an aberration in spherical coordinates.

APPENDIX G

TABLE OF CONSTANTS

G.1 The IAU System of Astronomical Constants

G.1.1 Defining constants

1.	Number of ephemeris seconds in one tropical year (1900)	$s = 31\ 556\ 925.974\ 7$
2.	Gaussian gravitational constant, defining A.U.	$k = 0.017\ 202\ 098\ 95$

G.1.2 Primary constants

1.	Measure of 1 A.U. in meters	$A = 149\ 600 \times 10^6$
2.	Velocity of light in meters per second	$c = 299\ 792.5 \times 10^3$
3.	Equatorial radius for Earth in meters	$a_e = 6\ 378\ 160$
4.	Dynamical form-factor for Earth	$J_2 = 0.001\ 082\ 7$
5.	Geocentric gravitational constant (units: $m^3\ sec^{-2}$)	$GE = 398\ 603 \times 10^9$
6.	Ratio of the masses of the Moon and Earth	$\mu = 1/81.30$
7.	Sidereal mean motion of Moon in radians per second (1900)	$n_{☾}^{*} = 2.661\ 699\ 489 \times 10^{-6}$
8.	General precession in longitude per tropical century (1900)	$p = 5\ 025.''64$
9.	Obliquity of the ecliptic (1900)	$\epsilon = 23°27'08.''26$
10.	Constant of nutation (1900)	$N = 9.''210$

G.1.3 Auxiliary constants and factors

1.	k/86400, for use when the unit of time is 1 sec	$k' = 1.990\ 983\ 675 \times 10^{-7}$
2.	Number of seconds of arc in 1 radian	206 264.806
3.	Factor for constant of aberration	$F_1 = 1.000\ 142$
4.	Factor for mean distance of Moon	$F_2 = 0.999\ 093\ 142$
5.	Factor for parallactic inequality	$F_3 = 49\ 853''.2$

G.1.4 Derived constants

1.	Solar parallax	$\arcsin(a_e/A) = \pi_\odot = 8''.794\ 05\ (8''.794)$
2.	Light-time for unit distance	$A/c = \tau_A = 499^s.012 = 1^s/0.002\ 003\ 96$
3.	Constant of aberration	$F_1 k' \tau_A = \kappa = 20''.495\ 8\ (20''.496)$
4.	Flattening factor for Earth	$f = 0.003\ 352\ 9 = 1/298.25$
5.	Heliocentric gravitational constant (units: $m^3 sec^{-2}$)	$A^3 k'^2 = GS = 132\ 718 \times 10^{15}$
6.	Ratio of masses of Sun and Earth	$(GS)/(GE) = S/E = 332\ 958$
7.	Ratio of masses of Sun and Earth + Moon	$S/E(1 + \mu) = 328\ 912$
8.	Perturbed mean distance of Moon in meters	$F_2 \left[GE(1 + \mu)/n_☾^{*2}\right]^{1/3} = a_☾ = 384\ 400 \times 10^3$
9.	Constant of sine parallax for Moon	$a_e/a_☾ = \sin \pi_☾ = 3422''.451$
10.	Constant of lunar inequality	$\frac{\mu}{1+\mu} \frac{a_☾}{A} = L = 6''.439\ 87\ (6''.440)$
11.	Constant of parallactic inequality	$F_3 \frac{1-\mu}{1+\mu} \frac{a_☾}{A} = P_☾ = 124''.986$

G.1.5 System of planetary masses

	Reciprocal mass		Reciprocal mass
Mercury	6 000 000	Jupiter	1 047.355
Venus	408 000	Saturn	3 501.6
Earth + Moon	329 390	Uranus	22 869
Mars	3 093 500	Neptune	19 314
		Pluto	360 000

G.1.6 The true values of the primary constants are believed to lie between the following limits*

A	$= 149\ 597$ to $149\ 601 \times 10^6$ m	μ^{-1}	$= 81.29$ to 81.31
c	$= 299\ 792$ to $299\ 793 \times 10^3$ m sec^{-1}	$n^*_{☾}$	= correct to number of places given
a_c	$= 6\ 378\ 080$ to $6\ 378\ 240$ m	p	$= 5\ 026''.40$ to $5\ 026''.90$
$\mathcal{J}_2$	$= 0.001\ 082\ 4$ to $0.001\ 082\ 9$	ϵ	$= 23°27'08''.16$ to $23°27'08''.36$
GE	$= 398\ 600$ to $398\ 606 \times 10^9$ m^3 sec^{-2}	N	$= 9''.200$ to $9''.210$

G.1.7 Correspondingly, the limits for the derived constants are

π_0	$= 8''.793\ 88$ to $8''.794\ 34$	f^{-1}	$= 298.33$ to 298.20
τ_A	$= 499^s.001$ to $499^s.016$	$a_{☾}$	$= 384\ 399$ to $384\ 401 \times 10^3$ m
κ	$= 20''.495\ 4$ to $20''.496\ 0$	$\sin \pi_{☾}$	$= 3\ 422''.397$ to $3\ 422''.502$
GS	$= 132\ 710$ to $132\ 721 \times 10^{15}$ m^3 sec^{-2}	L	$= 6''.439\ 0$ to $6''.440\ 8$
S/E	$= 332\ 935$ to $332\ 968$	$P_{☾}$	$= 124''.984$ to $124''.989$
$S/E(1+\mu)$	$= 328\ 890$ to $328\ 922$		

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*Given by the Working Group in Joint Discussion of XII General Assembly of the IAU, 1964.

G.2.1 Size, mass, density, gravitational constant, principal moments of inertia, physical libration

Name	Value	Ratio: $\frac{\text{Moon's value}}{\text{Earth's value}}$
Mean radius	$r_{☾} = 1\ 738$ km	0.27 a_e*
Mean surface	$S_{☾} = 37.96 \times 10^6$ km^2	0.17
Mean volume	$V_{☾} = 2.199 \times 10^{25}$ cm^3	0.02
Absolute mass (for G = $6.668 \pm 0.05\ 10^{-8}$ cm^3 g^{-1} sec^{-2})	$m_{☾} = 7.353 \times 10^{25}$ g	0.012
Mean density	$\rho^{☾} = 3.34$ g cm^{-3}	0.60
Mass ratio	$m_{\oplus}/m_{☾} = 81.303$†	—
Moon's gravitational constant	$Gm_{☾} = 4\ 902.66 \pm 0.16$‡	0.012
Gravitational acceleration	$Gm_{☾}/r_{☾}^2 = 162$ cm sec^{-2}	less than 1/6
Velocity of escape	$\left(2Gm_{☾}/r_{☾}\right)^{1/2} = 2.38$ km sec^{-1}	0.21
Angular velocity of rotation	2.67×10^{-6} rad sec^{-1}	0.037

*a_e = 6 378.155 km; it is calculated with the value $GM_{\oplus}$ = 398 601 ± 1 km sec^{-2}, determined by JPL on the basis of Rangers 6 to 8 and with adopted c = 299 792.5 km sec^{-1}.

†The value determined by JPL on the basis of Rangers 6 to 8. The value adopted by IAU is 81.30.

‡Mean value of results determined by JPL on the basis of Rangers 6 to 9 and Mariner 4.

Axes of the Moon's ellipsoid (Potter, 1967)

$$a = 1\,739.23 \pm 0.11 \text{ km} \quad \text{(toward the Earth)}$$

$$b = 1\,735.44 \pm 0.27 \text{ km}$$

$$c = 1\,736.04 \pm 0.29 \text{ km} \quad \text{(Polar)}$$

The flattening of the visible disk $f_{☾} = 1/920$.

Functions of the Moon's principal moments of inertia*

$$\alpha = 398.4 \times 10^{-6}$$

$$\beta = 629.4 \times 10^{-6}$$

$$\gamma = 231.0 \times 10^{-6}$$

$$f = \frac{\alpha}{\beta} = 0.633$$

Moisting A coordinates

$$\lambda = -\,5°9'50'' \pm 4''.5$$

$$\beta = -\,3°10'47'' \pm 4''.4$$

$$h = 932''.28 \pm 0.009$$

Several recent determinations of the value of the mechanical ellipticity f and inclination of lunar equator I† are given on the next page.

*Given by Koziel (1967) for I = 1°32'04''.

†Given in transactions of the IAU XII A, 1964.

Author	Observer	Observations		Method	$I = 1°30'$ $+ \Delta I$	f ($\times 10^{-3}$)
		Place	Time			
Koziel	Hartwig	Strasburg	1877-1879	Heliometer	2'04" ± 7".0	633 ± 1.1
		Dorpat	1883-1885	Heliometer	2'04" ± 7".0	633 ± 1.1
		Bamberg	1890-1912	Heliometer	2'01" ± 7".1 *	633 ± 1.1
	Banachiewicz	Kazan	1910-1915	Heliometer		
Maslowski	Hartwig			Heliometer	1'5[illegible] ± 6".8	6[illegible]7 ± 1.2
Mietelski	Banachiewicz				2'37" ± 10".9	628 ± 1.9
Nefiediew	Nefiediew	Kazan	1938-1945	Heliometer	3'26" ± 14" †	71_0 ± 2.0 †
					3'24" ± 14" ‡	63_0 ± 3.0 ‡
Gorynia	different	Kiev	1950-1959	Photographic	3'19" ± 16"	89_0 ± 1.3
Shakirow	different	Greenwich	1952-1954	Transit	0'54" ± 30"	71_0 ± 0.8

* Jointly with determination of the constants of free physical libration:
A = 18".7 ± 4".7, a = 334°.3 ± 15°.7 (1800).

† With initial value of f = 0.73.

‡ With initial value of f = 0.50.

Coefficients of the development of the forced physical libration in longitude τ, in node σ, and in inclination ρ calculated for the most probable value of $\beta = 0.000\ 63$, for $0.000\ 14 < \gamma < 0.000\ 28$ and $I = 1°32'50''$ (Eckhardt, 1968):

Coefficients for $\gamma = 0.000\ 14$ to $\gamma = 0.000\ 28$

	Delaunay argument l l' F D	0.000 14	0.000 18	0.000 20	0.000 22	0.000 24	0.000 28
τ	0 0 0 2	-0.30	-0.38	-0.43	-0.47	-0.51	-0.60
sine terms	0 0 2 -2	0.98	1.28	1.43	1.58	1.73	2.04
	0 1 -2 2	0.22	0.29	0.33	0.36	0.40	0.47
	0 1 0 0	52.91	69.60	78.23	87.07	96.12	114.89
	0 2 0 0	–	–	0.20	0.22	0.24	0.29
	1 -1 0 -1	-1.49	-1.43	-1.38	-1.37	-1.37	-1.33
	1 -1 0 0	–	–	–	–	–	-0.20
	1 0 -2 0	-0.61	-0.53	-0.49	-0.44	-0.40	-0.32
	1 0 0 -2	2.55	3.26	3.62	3.98	4.33	-5.05
	1 0 0 -1	-1.98	-2.63	-2.97	-3.31	-3.67	-4.43
	1 0 0 0	-10.96	-13.60	-14.91	-16.23	-11.55	-20.19
	1 1 0 -2	–	–	0.20	0.22	0.24	0.28
	2 -2 0 -2	0.84	0.56	0.50	0.46	0.43	0.40
	2 -1 0 -2	-0.52	0.71	0.80	0.91	1.02	1.25
	2 0 -2 0	-0.33	-4.04	-14.80	32.72	10.80	6.13
	2 0 0 -2	5.99	7.76	8.65	9.54	10.45	12.28
	2 0 0 0	-0.27	-0.35	-0.39	-0.43	-0.47	-0.54
I_σ	0 0 0 2	-0.25	-0.26	-0.26	-0.26	-0.26	-0.26
sine terms	0 0 2 -2	-3.14	-3.08	-3.05	-3.02	-2.99	-2.93
	0 0 2 0	-10.26	-10.44	-10.53	-10.61	-10.69	-10.83
	1 0 -2 0	-40.73	-33.37	-29.69	-26.03	-22.35	-15.02
	1 0 0 -2	2.77	2.64	2.57	2.50	2.43	2.30
	1 0 0 0	-112.67	-107.65	-105.15	-102.63	-100.14	-95.13
	1 0 2 -2	0.71	0.61	0.56	0.50	0.45	0.34
	1 0 2 0	-0.87	-0.85	-0.84	-0.84	-0.83	-0.81
	2 0 -2 0	0.42	–	–	–	-0.21	-0.45
	2 0 0 0	-1.10	-1.01	-0.96	-0.91	-0.87	-0.79
ρ	0 0 2 -2	-3.22	-3.15	-3.12	-3.09	-3.05	-2.99
cosine terms	0 0 2 0	-10.63	-10.73	-10.77	-10.81	-10.85	-10.93
	0 1 0 0	0.39	0.32	0.28	0.24	0.20	–
	1 0 -2 0	40.82	33.45	29.77	26.11	22.42	15.09
	1 0 0 -2	-2.28	-2.12	-2.05	-1.97	-1.89	-1.74
	1 0 0 0	-110.55	-105.45	-102.90	-100.26	-97.82	92.73
	1 0 2 -2	0.75	0.64	0.58	0.53	0.48	0.37
	1 0 2 0	-0.77	-0.76	-0.75	-0.74	-0.73	-0.71
	2 0 -2 0	-0.81	-0.47	-0.33	–	–	0.34
	2 0 0 0	-0.49	-0.44	-0.41	-0.39	-0.36	-0.32
I constant term		5562.7	5560.6	5551.5	5558.5	5557.5	5555.4

The coefficients calculated for the most probable value of $\gamma = 0.000\ 22$, for $0.000\ 60 < \beta < 0.000\ 66$ and for $I = 1°32'50''$ (Eckhardt, 1968)*:

Coefficients for $\beta = 0.000\ 60$ to $\beta = 0.000\ 66$

	Delaunay argument l l' F D	$\beta = 0.000\ 60$	$\beta = 0.000\ 63$	$\beta = 0.000\ 66$
τ	0 0 0 2	-0.47	-0.47	-0.47
sine terms	0 0 2 -2	1.59	1.58	1.57
	0 1 -2 2	0.36	0.36	0.36
	0 1 0 0	87.08	87.07	87.06
	0 2 0 0	0.22	0.22	0.22
	1 -1 0 -1	-1.37	-1.37	-1.36
	1 0 -2 0	-0.37	-0.44	-0.53
	1 0 0 -2	3.97	3.98	3.98
	1 0 0 -1	-3.32	-3.31	-3.32
	1 0 0 0	-16.06	-16.23	-16.42
	1 1 0 -2	0.22	0.22	0.22
	2 -2 0 -2	0.45	0.46	0.45
	2 -1 0 -2	0.91	0.91	0.91
	2 0 -2 0	32.98	32.72	31.10
	2 0 0 -2	9.55	9.54	9.54
	2 0 0 0	-0.43	-0.43	-0.43
I_σ	0 0 0 2	-0.25	-0.26	-0.27
sine term	0 0 2 -2	-2.87	-3.02	-3.17
	0 0 2 0	-10.01	-10.61	-11.22
	1 0 -2 0	-22.05	-26.03	-30.22
	1 0 0 -2	2.32	2.50	2.69
	1 0 0 0	-95.69	-102.63	-109.68
	1 0 2 -2	0.44	0.50	0.56
	1 0 2 0	-0.78	-0.84	-0.89
	2 0 0 0	-0.85	-0.91	-0.98
ρ	0 0 2 -2	-2.93	-3.09	-3.24
cosine term	0 0 2 0	-10.18	-10.81	-11.46
	0 1 0 0	0.22	0.24	0.26
	1 0 -2 0	22.13	26.11	30.30
	1 0 0 -2	-1.81	-1.97	-2.13
	1 0 0 0	-93.53	-100.26	-107.28
	1 0 2 -2	0.47	0.53	0.59
	1 0 2 0	-0.69	-0.74	-0.79
	2 0 0 0	-0.36	-0.39	-0.42
I constant term		5220.5	5558.5	5906.1

*The new values of coefficients given in this table are given by Eckhardt in manuscript form.

G.2.2 Orbit

Mean Earth-Moon distance	R_0 = 384 402 ± 1 km
Extreme values of the Earth-Moon distance	364 400 km - 406 730 km
Mean eccentricity of the lunar orbit	e = 0.054 900 489
Extreme values of the eccentricity	0.043 2 - 0.066 6
Mean inclination of the orbit	i = 5°15 = 5°8'43"
Extreme values of the inclination	5°00 - 5°30

The longitude of the mean ascending node of the lunar orbit on the ecliptic measured from the mean equinox of date:

$$\Omega = 259°10'59''.79 - 5^r134°08'31''.23\ T + 7.48\ T^2 + 0.008\ T^3$$
$$= 259°.183\,275 - 0°.052\,953\,922\,2\ d + 0°.002\,078\ T^2 + 0°.000\,002\ T^3 \quad .$$

The mean longitude of the lunar perigee measured in the ecliptic from the mean equinox of date to the mean ascending node of the lunar orbit, and then along the orbit:

$$\Gamma' = 334°19'46''.40 + 11^r109°02'02''.52 - 37''.17\ T^2 - 0''.045\ T^3$$
$$= 334°.329\,556 + 0°.111\,404\,080\,3\ d - 0°.010\,325\ T^2 - 0°.000\,012\ T^3 \quad .$$

The mean longitude of the Moon measured on the ecliptic from the mean equinox of date to the mean ascending node of the lunar orbit and then along the orbit:

$$☾ = 270°26'02''.99 + 1\,336^r\ 307°52'59''.31\ T - 4''.08\ T^2 + 0''.006\,8\ T^3$$
$$= 270°.434\,164 + 13°.176\,396\,526\,8\ d - 0°.001\,133\ T^2 + 0°.000\,001\,9\ T^3 \quad .$$

The mean elongation of the Moon from the Sun:

$$D = 350°44'14''.95 + 1\,236^r\,307°06'51''.18\,T - 5''.17\,T^2 + 0''.0068\,T^3$$

$$= 350°.737\,486 + 12°.190\,749\,191\,4\,d - 0.°001\,436\,T^2 + 0.°000\,001\,9\,T^3,$$

where T is measured in Julian centuries from 1900 January 0.5 E.T. = J.D. 2 415 020.0 and d is the number of ephemeris days from the epoch.

The lengths of the months for the epoch 1900 are

Synodic	$29^d.530\,589$	$29^d 12^h 44^m 02^s.9$
Tropical	27.321 582	27 07 43 04.7
Sidereal	27.321 661	27 07 43 11.5
Anomalistic	27.554 551	27 13 18 33.2
Draconitic	27.212 220	27 05 05 35.8

G.3 The Earth's Constants

G.3.1 Size, mass, density, and gravitational constant

Equatorial radius	$a_e = 6\,378.155$ km*
Flattening	$f_\oplus = 1/298.25$
Polar radius	$a(1 - f_\oplus) = 6\,356.769\,7$ km*
Radius vector	$\rho = 0.998\,327\,07 + 0.001\,676\,44 \cos 2\Phi - 0.000\,003\,52 \cos 4\Phi$

Reduction from geodetic latitude ϕ_g to geocentric latitude ϕ'_g:

$$\phi_g - \phi'_g = 692.74 \sin 2\phi_g - 1''.16 \sin 4\phi_g$$

*Determined by JPL based on Rangers 6 to 8.

Mean volume:

$$1.083 \times 10^{27}\ \text{cm}^3$$

Absolute mass for G = $(6.668 \pm 0.005)\ 10^{-8}\ \text{cm}^3\ \text{g}^{-1}\ \text{sec}^{-2}$:

$$5.978 \times 10^{27}\ \text{g}$$

Mean density:

$$5.54\ \text{g cm}^{-3}$$

Mass ratio:

$$\frac{m_\odot}{m_\oplus} = 332\ 958$$

Normal gravity:

$$g = 978.049\ (1 + 0.005\ 302\ 4 \sin^2 \phi_g - 0.000\ 005\ 9 \sin^2 2\ \phi_g)\text{cm sec}^{-2}$$

Velocity of escape:

$$11.2\ \text{km sec}^{-1}$$

G.3.2 Orbit

Astronomical unit	149 600 000 km
Inclination of the ecliptic for 1900	$\epsilon = 23°27'08''.26$

Mean obliquity of the ecliptic:

$$\epsilon = 23°27'08''.26 - 46''.845\ T - 0''.005\ 9\ T^2 + 0''.001\ 81\ T^3$$

$$= 23°.452\ 294 - 0°.013\ 012\ 5\ T - 0°.000\ 001\ 64\ T^2 + 0°.000\ 000\ 503\ T^3$$

Eccentricity:

$$e_{\oplus} = 0.016\ 751\ 04 - 0.000\ 041\ 80\ T - 0.000\ 000\ 126\ T^2$$

Geometric mean longitude of the Sun, referred to the mean equinox of date:

$$L_{\odot} = 279°41'48''.04 + 129\ 602\ 768''.13\ T + 1''.089\ T^2$$
$$= 279°.696\ 68 + 0°.985\ 647\ 335\ 4\ d + 0°.000\ 303\ T^2$$

Mean longitude of perigee of the Sun, referred to the mean equinox of date:

$$\Gamma_{\odot} = 281°13'15''.0 + 6\ 189''.03\ T + 1''.63\ T^2 + 0''.012\ T^3$$
$$= 281°.220\ 83 + 0°.000\ 047\ 068\ 4\ d + 0°.000\ 453\ T^2 + 0°.000\ 003\ T^3$$

Mean anomaly of the Sun:

$$g_{\odot} = 358°28'33''.0 + 129\ 596\ 579''.10\ T - 0''.54\ T^2 - 0''.012\ T^3$$
$$= 358°.475\ 83 + 0°.985\ 600\ 267\ 0\ d - 0°.000\ 150\ T^2 - 0°.000\ 003\ T^3$$

In the above, T denotes the time measured in Julian centuries of 36 525 ephemeris days from the epoch, and d the time in ephemeris days from epoch 1900 January 0.5 E.T. = J.D. 2 415 020.0.

Length of the years:

Tropical	$365^d.242\ 198\ 79 - 0^d.000\ 006\ 14\ T$ $365^d 05^h 48^m 56^s.0 - 0^s.530\ T$
Sidereal	$365^d.256\ 360\ 42 + 0^d.000\ 000\ 11\ T$ $365^d 06^h 09^m 09^s.5 + 0^s.01\ T$
Anomalistic	$365^d.259\ 641\ 34 + 0^d.000\ 003\ 04\ T$ $365^d 06^h 13^m 53^s.0 + 0^s.26\ T$
Eclipse	$346^d.620\ 031 + 0^d.000\ 032\ T$ $346^d 14^h 52^m 50^s.7 + 2^s.8\ T$

G.4 Constants of Precession

G.4.1 Earth's precession (Newcomb, 1960; International Astronomical Union, 1964)

General precession	$\psi = 50''.256\,4 + 0''.022\,2\ T$
Planetary precession	$\lambda' = 0''.124\,7 - 0''.018\,8\ T$
Lunisolar precession	$\psi_1 = 50''.370\,8 + 0''.005\,0\ T$
Precession in right ascension	$m = 3^s.072\,34 + 0^s.001\,86\ T$
Precession in declination	$n = 20''.046\,8 - 0''.008\,5\ T$

Mean obliquity of the ecliptic:

$$\epsilon = 23°27'08''.26 - 46''.845\ T - 0''.005\,9\ T^2 + 0''.001\,81\ T^3$$

$$= 23°.452\,294 - 0°.013\,012\,5\ T - 0°.000\,001\,64\ T^2 + 0°.000\,000\,503\ T^3$$

Annual rate of rotation of the ecliptic

$$\pi = 0''.471\,1 - 0''.000\,7\ T$$

Longitude of axis of rotation

$$\Pi = 173°57'.06 + 54'.77\ T$$

The position of the ecliptic in terms of its inclination π_1 and node Π_1 on the fixed ecliptic of the epoch is represented by

$$\pi_1 \sin \Pi_1 = + \ 4''.964\ T + 0''.193\,9\ T^2 - 0''.000\,19\ T^3$$

$$\pi_1 \cos \Pi_1 = - 46''.845\ T + 0''.054\,5\ T^2 + 0''.000\,35\ T^3$$

G.4.2 Moon's precession

Daily motion of the ascending node of the lunar orbit on the ecliptic	$P_0^d = -0\overset{\circ}{.}052\ 953\ 922\ 2$
Precession in seleno-right ascension	$M_0^d = +0\overset{\circ}{.}052\ 035\ 235$*
Precession in seleno-declination	$N_0^d = -0\overset{\circ}{.}001\ 394\ 420$*

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* $M_0^d = -P_0^d \cos I$ and $N_0^d = P_0^d \sin I$ are calculated for $I = 1°32'.1$.

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BARBARA KOLACZEK received a Master of Science degree in astronomy from the Jagiellonic University in Cracow, Poland, in 1953, and a doctorate in geodetic astronomy from the Warsaw Technical University in 1964.

From 1953 to 1966, Dr. Kolaczek was in the Geodetic Astronomy Department at the Warsaw Technical University. She also took part in the organization and management of the Latitude Service and in satellite observations at the Jozesfoslaw Observatory, near Warsaw. She returned to the Warsaw Technical University in the fall of 1968, after 2 years at SAO.

While at SAO, Dr. Kolaczek worked on problems of the moon's spherical astronomy and of solar-radiation pressure on artificial satellites.

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This series of Special Reports was instituted under the supervision of Dr. F. L. Whipple, Director of the Astrophysical Observatory of the Smithsonian Institution, shortly after the launching of the first artificial earth satellite on October 4, 1957. Contributions come from the Staff of the Observatory.

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