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## DIG ITAL COMPUTER PROCESSING OF PEACH ORCHARD MULTISPECTRAL AERIAL PHOTOGRAPHY

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## 16 ABSTRACT

Several methods of analysis by digital computer which may be applied to digitized multispectral aerial photography are described, with particular application to peach orchard test sites. This effort was stimulated by the recent premature death of peach trees in the Southeastern United States The techniques discussed are (i) correction of intensity variations by digital filtering, (ix) automatic detection and enumeration of trees in fave size categories, (111) determination of unhealthy foluage by infrared reflectances, and (iv) four band multispectral classification into healthy and declining categories.


## ACKNOWLEDGEMENTS

Gratitude is expressed to personnel of the Southeastern Fruit and Tree Nut Research Station of the United States Department of Agriculture, and of the Environmental Applications Office of NASA/ Marshall Space Flight Center, and to Dr A. D. Bond of Computer Sciences Corporation

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## I. INTRODUCTION

The peach crop is a major contributor to the agricultural output of the Southeastern United States, particularly the state of Georgia. However, the intensive culture of peaches over a long period of time leads to the so-called "peach decline". Over the last few decades, the average productive hfe of peach trees in Georgia has declined from approxımately twenty years to from five to elght years.

Peach decline has been investigated in Georgia since 1929 The premature death of peach trees in the Southeast has been attributed to such disorders as root rot, peachtree borer, nematodes, bacterial canker, fung, cold damage, and virus diseases.

In 1972, between 100,000 and 200,000 trees died, and hence in that year aerial photography of several orchard test sites was obtained. Previously, the occurrence of tree dechne due to the presence of three diseases, three insects, and one mite had been detected by the examination of color infrared aerial photographs obtaned at elevations of $600-4500$ feet. ${ }^{1}$

Color infrared photographs obtained in 1972 were analyzed using the General Electric Image 100 System ${ }^{2}$ By displaying on a color television monitor, categories corresponding to selected density levels on the film are readuly located. With this method, trees in various stages of decline and also different segments of the same tree fall into different categories.

Consequently, analysis by digital computer of multispectral aerial photography was undertaken. The purposes of this study were to determine the feasibilities of orchard inventories (enumeration and sizing of the crowns) and of determining the slate of decline of trees based on an analysis of their multispectral reflectances.

This report presents the results of the analysis of four band multispectral photography by digital computer The major topies presented are (i) digital
correction of intensity variations, (il) enumeration of trees by sizes, and (iil) classufication into healthy and decluming categories.

Thë āerial photography wāa ūndertaken by NASA/MSFC, and ground truth data was furmshed by the USDA Southeastern Fruit and Tree Nut Research Station, Byron, Georgia. The aerial imagery was obtanned over the USDA test sites in Brbb, Crawford and Peach Counties in the state of Georgia The locations of these test sites and the USDA Research Station are shown in Figure 1.1. The photographs used were made by the MSFC I ${ }^{2}$ S four band camera system, which exposes four frames of film size $90 \times 90 \mathrm{~mm}$. through filters which trans mit in the blue, green, red and infrared spectral bands. Digitization of the images and computer analysis were performed at Marshall Space Filght Center by Computer Sciences Corporation.


Fig 1.1 Locations of peach orchard test sites and USDA Research Station

## II. DIGITAL FILTERING TECHNIQUES

### 2.1 Two Dimensional Linear Filtering

The first topıc to be discussed is digital correction of intensity variations, by means of digital filters. This method is feasible as a procedure for removing varıations which have spatial rates of change differing from those of the objects of interest in the scene. This is particularly true of an aerıal photograph of a peach orchard, in which the objects of interest are of a consistent size.

The input/output relation of a linear digital filter is of the form

$$
\mathrm{Z}_{\mathrm{i}}=\sum_{\mathrm{k}=-\mathrm{L}}^{\mathrm{L}} \mathrm{~g}_{\mathrm{k}} \mathrm{X}_{{ }_{\mathrm{i}}+\mathrm{k}}
$$

where $\{X\}$ is the input data sequence and $\{Z\}$ is the output data sequence. It may be seen that an output data value is a sum of the neighboring 2 L input values, plus the input data point, each multiplied by a filter weight $\mathrm{g}_{\mathrm{k}}$ The implementation of a filter consists of selecting the filter weights, followed by evaluating the input/output relation at each data point. Because of the large amount of data in a digitized image, it is essential that the implementations be fast in order to make dıgital processing practically feasible

A very fast implementation is obtaned when the filter weights are given the value one, and the evaluation is done recursively In this case, the filter output is a moving sum over the input data, and in advaneing by one data point, It is necessary to add only one new sample and subtract one sample which has left the filter length of $(2 \mathrm{~L}+1)$ samples The following diagram illustrates:

| $\mathrm{Z}_{\mathrm{I}}-1$ | $1-\mathrm{L}-1$ | $1-\mathrm{L}$ |  |  | $1+\mathrm{L}-1$ |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathrm{Z}_{1}$ |  | $1-\mathrm{L}$ | $\mathrm{I}-\mathrm{L}+1$ |  | $\mathrm{i}+\mathrm{L}-1$ | $\mathrm{i}+\mathrm{L}$ |

$$
Z_{1}=z_{i-1}+X_{i+L}-X_{1-L-1}
$$

Also, in the actual filtering of a digitized image, two dimensional filtering is performed This involves extending the summation over all neighboring points in a region of dimensions ( $2 \mathrm{~L}_{x}+1$ ) $\left(2 \mathrm{~L}_{y}+1\right)$ The output data values will occupy the same magnatude range as the input of the average of the $\left(2 \mathrm{~L}_{\mathrm{x}}+1\right)\left(2 \mathrm{~L}_{\mathrm{y}}+1\right)$ terms is taken.

When the two-dimensional moving averages are applied, data variations which occur over a span of less than $2 \mathrm{~L}+1$ data points tend to be averaged out, while more slowly changing variations remain Hence, the operation has the effect of low pass filtering Denoting the moving average low pass filter operation by LP we can write

$$
Z=L P(X)
$$

Since the inverse of a low pass filter transfer function is that of a high pass filter, a high pass operation may be obtained as

$$
\mathrm{Z}=\mathrm{HP}(\mathrm{X})=[1-\mathrm{LP}](\mathrm{X})=(\mathrm{X})-\mathrm{LP}(\mathrm{X})
$$

This is equivalent to subtracting the moving averages (the slowly varying intensity variations) from each data point (A constant bias may be added to restore the data to its original range).

Band pass filters may be constructed from combinations of low and high pass. A filter which passes in the midrange is given by

$$
\mathrm{BP}=\mathrm{LP}\left(\mathrm{~L}_{1}\right)-\mathrm{LP}\left(\mathrm{~L}_{2}\right)
$$

where $L_{1}>L_{2}$. In this case, $L P\left(L_{1}\right)$ removes the extreme high frequency variations, whle subtracting the output of $L P\left(\mathrm{~L}_{2}\right)$ removes the extreme low frequency variation. A filter which attenuates the midrange is given by

$$
\mathrm{BP}=\mathrm{LP}\left(\mathrm{~L}_{1}\right) \mathrm{HP}\left(\mathrm{~L}_{2}\right) .
$$

In this case, the high and low ranges are passed.

### 2.2 Digatal Filtering of a Digitized Peach Orchard Aernal Photograph

Initial experiments were performed using a green band image obtained at 6000 -feet altitude. A rectangular orchard consisting-of approximately $37 \times 84$ rows of peach trees was dıgitized to 64 levels spanning a range of 0 to 2 D in photographic density. The digitizing spot size was 50 microns, yielding for the digitized scene an array of $850 \times 1800$ pixels

Visual observation of the image indicated large variations in scene intensity, although maintainmg distinct contrast between the tree crowns and background. The histogram of the scene density is given in Figure 2 1. The histogram verifies the wide range of densities present in the image, and lack of uniformly separable intensity ranges corresponding to tree crowns and background


Figure 2.1. Histogram of densities present in the peach orchard image.

As a first step in the processing of the image, it was decided to attempt the removal of the background intensity variations. The possibility of performing this operation arises from the slower rates of spatial change of the background intensity variations compared to the variations due to the tree crowns This fact is illustrated in Figure 2 2, which is a trace of the density variations along a row of trees as the background density varies widely. The ten intensity peaks correspond to ten tree crowns which are dark in appearance on the photograph The overall intensity variation is evident in the plot, with a region of low intensity near pixel 100.

In this case the variations of intensity do not follow a positional dependence which would allow fatting to a function of position. Hence the moving averages (low pass output) must be computed for each point in the digitized image. If the size of the moving average region is chosen correctly, the low pass output will be a measure of the low frequency variations in image density. This output at each point in the image is subtracted from the original data value and a bias is added to restore the overall average density of the output image. As indicated previously, the result of subtracting a low pass filter output from the original data is characteristic of high pass fultering. Figure 2.3 is a trace along the row of pixels shown in Figure 2 2, after the image has been high pass filtered Twenty-five neighboring points were taken as the filter length in each direction, yıelding a total area of 51x51, including the data point itself It can be seen that the high pass output retains the detalled structure of the original data while significantly reducing the slow drift exhibited in the data.

A further refinement was attempted using the high pass filtered data Thus data was low pass filtered, using a fulter length small compared to the dimensions of the tree crowns Thus, in this case, the variations arising from the tree crowns are considered as slowly changing and are relatively unaffected, while smaller patches of vegetation and nonumformity within tree crowns are reduced. The net result of these operations is a band pass filtered image, in


Fig.2.2. Density variations along a row of trees in the original data.


Fig. 2.3. Density variation along a row of trees after high pass filtering.




Fig 2.4. Original data, low pass and band pass outputs
which only those objects which are the size of tree crowns retain their original intensities. Figure 2.4 shows the original data sample and the low pass and band pass outputs.

Examination of the original data scan shows that a separation of tree crowns and background is not possible for any value of threshold. However in the filtered output image all of the tree crowns can be separated from their surrounding background by a density threshold of approximately 25 (on the $0-63$ scale of scanner output). This is further demonstrated by the histogram of the filtered data (Figure 2.5), which has become bi-modal, with a minimum occuring at the value 25 .


Figure 2.5. Histogram of densities in the peach orchard image after digital filtering.


Fig. 2.6. Original image and result of background removal following digital filtering of a peach orchard test site.

Using a threshold of 25 , a digital image can be generated in which the tree crowns remain while the background is set to zero density. However, as the densities of the tree crowns have been altered somewhat by the filtering operation, a more accurate result is obtained if the filtered image is used as a mask to be compared with the original image. In this manner, the background in the original image can be set to zero density. A resulting image obtained in this manner and the original image are shown in Figure 2.6. The histogram of the tree crowns only is shown in Figure 2.7. The symmetry of the distribution indicates the exclusion of the background pixels.


Fig. 2.7. Histogram of tree crown densities [following background removal]

## III. DETECTION TECHNIQUES

### 3.1 Introduction

The problem of determining the positions of specified objects in a digitized image array may be approached by comparing, in some way, a local region in the image with a mathematical template which represents the searched-for objects. A match is considered to have occurred when the similarity between the image and the template reaches a certain level, in terms of a mathematical criterion. The traditional, but time consuming, method is a search for peaks in the crosscorrelation function, at image coordinates $[i, j]$ the sum of the product of each template term with its corresponding image term. For a template T of size mxm , with an image array I, the normalized cross-correlation is given by:

$$
R^{2}(i, j)=\frac{\left[\sum_{k=1}^{m} \sum_{l=1}^{m} T(k, l) I(k, l)\right]^{2}}{\left[\sum_{k=1}^{m} \sum_{l=1}^{m} T^{2}(k, l)\right]\left[\sum_{k=1}^{m} \sum_{l=1}^{m} I^{2}(k, 1)\right]}
$$

However, it may be possible to obtain satisfactory results, with a large decrease in computation time, by using simple differences between the template and the image. ${ }^{3}$

### 3.2 Template Matching with Simple Differences

A test area was chosen from the green band image which had been high pass filtered to remove background variations. The test area of 100 pixels square was relatively small, but was so chosen to allow rapid presentation of results on the output line printer. A line printer plot of the area is shown in Figure 3.1.

The template chosen consisted of a circular region inscribed in a square of eleven pixels. The template elements representing the disk and the border were assigned values approximating the density levels of the tree crowns and background.









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Figure 3.1. Line printer plot of the digitized peach orchard test site.

The template was centered over each pixel in the image, and the differences between each template element and the corresponding image pixels were summed. This sum, obtained at each point in the image, is termed the cumulative error. A printer plot of this error for the test region is given in Figure 3.2. As expected, the errors decrease steadily as the template becomes superimposed on each tree crown. The pixel position for which minimum error occurs will be considered to be the match position.

Plots of the errors obtained along two rows of the image are given in Figure 3.3. The upper plot is the error along line twenty-one of the test site, in an area between rows of trees. Consequently, the matching errors are large and fairly constant. The lower plot is the error along a line of tree crowns, and demonstrates the wide variation in error and in particular the rapid change in the error near the match positions (the minima in the plotted curve).

It is apparent from the preceding figures that an error threshold can be determined such that image points whose corresponding errors exceed the threshold are far from the locations of the tree crowns. This is illustrated by Figure 3.4 which is a printer plot of the errors at only those pixel locations having errors below the threshold. Comparison with Figure 3.1 demonstrates the correlation of these low error locations with the tree crowns in the image. This type of thresholding to indicate the locations of the tree crowns is preferable to simple density thresholding because it includes a dependence on shape and not only density.

### 3.3 Error Evaluation Sequences

The definition of the cumulative error at each pixel location is the sum of the absolute values of the difference between corresponding template and image pixels. However, in the present case, the computations can be formulated to reduce the computation time. This is a result of the circular symmetry of the template, the definition of the template in terms of only two values, and the proportion of background area to tree crown area in the imagery. Several methods of computation were examined in a series of experiments.


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Figure 3.3. Cumulative errors along line 21 and line 11 of the test scene.


Figure 3.4.Cumulative Error after Thresholding

## (i) Natural Sequence

This method is a straightforward summation of the absolute values of the differences for all the terms of the template taken in a raster scan order. The computation time required on the test site was 91 seconds. (All times given are computation times only, and do not include time required for input/output operations. Nachne used was a $7094 / 1$.)
(11) Natural Sequence + Total Error Threshold

Since the only areas of interest are those with errors below the threshold, the accumulation of error may be terminated whenever the threshold is reached. This procedure reduced the computation time to 78 seconds.
(111) Error Sort Method - A

The following two methods are based on sorting of the error terms such that the template evaluation sequence corresponds to the errors obtained, from the highest to the lowest In this case, the template is shifted $\pm 1$ pixel from a match position with itself along rows and columns, the errors in the four cases evaluated, and sorted so that the evaluation sequence determines errors in order of their magnitude ${ }^{4}$ The resulting sequence is given in Figure 35 , and the computation time required was 80 seconds. The method is not advantageous for this particular problem, probably due to the small percentage of evaluations which are performed near match positions
(iv) Error Sort Method - B

This is a more generalized error sort method. The test scene was uscd as a "iraining" area, to determine the average error for each term in the template as the template was placed at each pixel in the scene. The average errors at each point in the template are given in Figure 3.6. The smaller errors in the corners are at the template positions corresponding

| 29 | 35 | 1 | 2 | 37 | 45 | 88 | 4 | 5 | 105 | 107 |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 113 | 7 | 3 | 30 | 31 | 32 | 33 | 34 | 6 | 9 | 36 |
| 11 | 8 | 38 | 39 | 40 | 41 | 42 | 43 | 44 | 10 | 13 |
| 12 | 46 | 47 | 48 | 49 | 50 | 51 | 52 | 53 | 54 | 14 |
| 55 | 56 | 57 | 58 | 59 | 60 | 61 | 62 | 63 | 64 | 65 |
| 66 | 67 | 68 | 69 | 70 | 71 | 72 | 73 | 74 | 75 | 76 |
| 77 | 78 | 79 | 80 | 81 | 82 | 83 | 84 | 85 | 86 | 87 |
| 17 | 89 | 90 | 91 | 92 | 93 | 94 | 95 | 96 | 97 | 20 |
| 15 | 16 | 98 | 99 | 100 | 101 | 102 | 103 | 104 | 18 | 19 |
| 106 | 21 | 22 | 108 | 109 | 110 | 111 | 112 | 23 | 24 | 114 |
| 115 | 116 | 27 | 26 | 117 | 118 | 119 | 27 | 28 | 120 | 121 |

## Figure 35 Evaluation Sequence for Error Sort Method-A (Method No 111)

to the background of the scene, and are smaller since the majority of the scene is background. The evaluation sequence is given in Figure 3 7, and the computation time required was 71 seconds. This method has the disadvantage of being training data dependent, and hence similar, but crrcularly symmetric sequences were tested
(v) Disk - Background Sequence

As a better approximation to the empirical sequence of method No. (iv), a sequence was evaluated which spirals inward over the region of the disk, followed by an outward spiral over the region of the background The computation time was 72 sec onds.

```
7.16 7.23 7.3C 18.15 1&.11 le.Cg l8.C8 18.C8 7.43 7.40 7.36
7.20 7.28 18.1\epsilon 18.11 1&.C7 1&.C5 18.C4 18.C4 18.C6 7.44 7.39
7.26 18.16 1&.1C 18.CE LE.C2 1&.CC 17.99 17.¢9 18.C1 18.04 7.44
18.15 18.10 18.C4 17.G9 17.55 17.93 17.S2 17.53 17.95 17.98 18.03
18.C9 18.C3 17.\varsigma7 17.92 17.&G 17.&7 17.86 17.&7 17.89 17.92 17.98
18.C3 17.97 17.91 17.87 17.&3 17.&1 17.80 17.&1 17.83 17.87 17.93
17.97 17.91 l7.&\epsilon 17.&1 17.7& 17.7t 17.75 17.7\epsilon 17.78 17.82 17.88
17.92 17.87 17.81 17.77 17.73 17.7% 17.71 17.72 17.75 17.79 17.84
    7.€0 17.84 17.7& 17.74 17.71 17.tc 17.68 17.7C 17.72 17.76 7.72
```



```
    7.60 7.67 7.73 17.74 17.71 17.tc 17.68 17.7C 7.&2 7.77 7.71
```

Figure 3.6. Average template errors throughout the test scene

| 121 | 119 | 116 | 4 | ; | 9 | 12 | 11 | 112 | 113 | 115 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| -2) | 117 | $\therefore$ | 6 | 13 | 16 | i 9 | 17 | 24 | 111 | 114 |
| 113 | 1 | 7 | 15 | 24 | 26 | 29 | 27 | 25 | 18 | 110 |
| 3 | 8 | 23 | 28 | 3, | 37 | 4. | 39 | 36 | 30 | 22 |
| 1.$)$ | 21 | 36 | 34 | 46 | 50 | 54 | 32 | 47 | 42 | 31 |
| 73 | 33 | 44 | 53 | 59 | 61 | 65 | 62 | 58 | 49 | 40 |
| 34 | 45 | 55 | 64 | 69 | 74 | 75 | 73 | 68 | 64 | 48 |
| 41 | 51 | 63 | 71 | 77 | 84 | 95 | 82 | 76 | 66 | 56 |
| 437 | 57 | 67 | 77 | 86 | 71 | 45 | $9 \bigcirc$ | 81 | 72 | 103 |
| 117 | 105 | 7 | 2, 1 | 89 | 96 | 97 | 92 | 93 | 99 | 102 |
| 138 | 106 | $1 〕 1$ | 78 | 87 | 93 | 94 | 88 | 98 | 100 | 104 |

Figure 3.7. Evaluation sequence for Error Sort Method-B (Method No.iv]

## (vi) Increasing Radıus Sequence

The sequence in order of increasing radius evaluates the error for all of the template terms representing tree crowns before proceeding to the terms representing background. Since the majority of the scene is not tree crowns, errors on the average accumulate rapidly and this sequence yields a rapid computation, requiring 62 seconds.
(vil) Elimination of Absolute Value Computation
In general, the simple difference between a template term and an image pixel must be the absolute value of the difference to allow for image data values above and below the template values However, in the present case of a double-valued template, the template values may be chosen to insure that all difference terms have the same sign. For example, the tree crowns have high density in the transparencies, and the corresponding template region is set at the maximum image value. This is infact preferable in the present case, since the optimal match is obtained for the densest tree crown and lightest background. Using this procedure, combined with the increasing radius sequence and error thresholding, the computation time drops to 46 seconds.

## (vi1) Recursive Evaluation

The recursive evaluation sequence takes account of the fact that as the template is centered on successive pixels of the image, the majority of the error terms retain their previous values. Hence it is only necessary to consider the new terms appearing at the leading edges of the template regrons (both the leading terms of the whole template and each diskbackground interface), and the terms being dropped at the trailing edges. This method is extremely rapid, requiring 18 seconds, and it is a property of the method that the complete cumulative error is obtained

### 3.4 Error Threshold Sequences

The result shown in Figure 3.4, obtained by thresholding the cumulative error, indicates the locations and approximate sizes of the tree crowns This result is preferable to simple thresholding, in which all pixels above the threshold would be retaned, irregardless of the shapes of the regions retaned A further increase in computation speed is obtained when a constant threshold is replaced by a monotonically increasing threshold sequence In this case the threshold may be exceeded when a smaller number of error terms has been accumulated.

The threshold sequence curve should approximate, in its average slope, the curve of cumulative error versus number of terms. An example is given in Figure 3 . The error at test points $A$ and $B$ accumulates so rapidly that they are discarded as candıdates for match after very little computation. Point C , which is at or near a match, is not discarded llthough the error for all terms is accumulated.


Figure 3 8. Red.ction of Calculations with use of a Monotomeally Increasing Threshold Sequence.

At the point of match, the cumulative error is the sum of the noise due to the fact that the areas being compared are different. If this image noise has a mean $\lambda$ and zero deviation, the ratio of cumulative error to mean of the noise, $E / \lambda$, is unity at each point in the sum, and the cumulative error is linear with the number of terms. However, an increase in the variance of image noise is reflected in an increased rate of accumulation of error, and the threshold curve should take on higher values. Barnea and Silverman ${ }^{3}$ have calculated threshold sequences for various deviations. A set of such curves is given in Figure 3.9. Barnea and Silverman also present calculated curves based on the probability of the cumulative error exceeding the threshold being held constant at each term in the sum. A set of these curves is given in Figure 3.10. Further analysis of this type of computation is given by Ramapriyan. 5

Error sequences were plotted for the peach test site, using an evaluation sequence which starts at the center of the crrcular template and spirals outward The cumulative errors at each term in the sum over a template of size $11 \times 11$ pixels are plotted as the solid lines in Figure 3.11. The upper plot is for a nonmatch position, whule the lower is obtained for the template centered on the upper left tree of the test site The area of the template circular region was nominally 100 pixels, but an actual area of 97 pixels was obtaned, due to the discreteness of pixels. Thus, after 97 terms of the error sum, there is an abrupt change in the template value, and this accounts for the slope change in the error sum plots near the 100 -th term.

From the cumulative error plots, threshold sequences can be obtained such that the error sums at non-match points exceed the threshold terms, while remaning below the threshold near match points. For this test, lunear threshold sequences were chosen, since the error sequences are nearly linear for the initial terms. The threshold sequences shown as dotted lines increase by 8 and 5 for each term in the sum. It may be seen from the plots that, for non-match positions, the imitial term of the error sum will exceed the threshold. The linear threshold curve passing through the orgin does not make allowance for variance in the image norse The smoothness of the actual cumulative error plot justifies


Figure 3.9. Calculated Threshold Sequences Based on Deviation of Image Noise


Figure 3.10. Calculated Threshold Sequences Based on Probabulaty q of Exceeding the Threshold at the n-th Term



Figure 3.11. Cumulative error at a non-match and a match point. The dotted lines are threshold sequences with slopes of 8 and 5 .
this assumption. Sumilarly, it may be seen that, near match points, the error is accumulated for all terms of the template, wathout exceeding the threshold. Hence, an appropriate output function is the number of terms in the error sum at which the threshold is exceeded, or the total number of terms when the threshold is not exceeded. Printer plots of this output are given in Figures 3.12 and 3.13 for threshold sequence slopes of 8 and 5 , respectively. It is apparent that the sums are terminated quickly at non-match points, effecting a large reduction in computation time. The actual computation times were 11 seconds and 77 seconds, respectively The regions in which the error accumulates for a large number of terms approximate the sizes of the trees in Figures 3.12 and 3.13, as do the regions where the error is below a constant threshold in Figure 3.4. However, the result in Figure 3.13 was obtained with a six-fold increase in computation speed.


Figure 3.12 Plot of Error Sequence Terms for Threshold Sequence Slope of 8 Computation time 11 seconds.


Figure 3.13 Plot of Error Sequence Terms for Threshold Sequence Slope of 5. Computation Time 7.7 Seconds

### 3.5 Enumeration, Sizing and Coordınate Determination

### 3.5.1 Technque

Previous sections have demonstrated that simple dufferencing of a circular template and a digitized image of a peach orchard produces an error function with pronounced minima at the locations of tree crowns. The coordinates of a minimum are readily obtained and may be taken as the coordinates of the tree center. However, it is necessary to repeat the process of finding a new minimum value in the vicimity of each tree crown Hence an approximate position of match is found mitially by considering the gradient between the leading and trailing edges of the circular template positions in the data.


Figuce 3.14 Positive and Negative Gradient terms for a Circular Template of Nominal Area 100 Pixels.

The template gradzent is defined as the positive sum of all terms on the leading edge of the circular region, plus the negative sum of all terms on the trailing edge The positive and negative terms for the horizontal gradient of a template with a circular area of 97 pixels are given in Figure 3.14 The horızontal gradients along lines 13 and 23 of the test site are plotted in Figure 3.15. The upper plot is along a line centered on a row of trees, while the lower is between rows.

As a consequence of the facts that (1) the leading edge of the template defines the image points contributing to the positive terms in the gradient, and (ii) the tree crowns have dıgitized values of greater magnitude than the background, the gradient values are positive as the template approaches the position of a tree crown in a raster type scan. The value of the gradient falls rapidly through zero as the template passes over the center of a tree crown. By detecting a simılar passage through zero in the orthogonal direction of scan, it is possible to rapidly obtain approximate coordınates of match

An alternative description of this procedure is obtained by regarding it as a recursive calculation of the sum of the image values corresponding to the circular area of the template disk. The gradient terms at each pixel location are the terms to be added to the preceding sum to obtain the magnitude of the sum at the current pixel location. Thus the gradients are the differential of the sums over circular regions centered at each pixel location, and as such pass through zero at the extremum of the sum.

This method of locating the approximate positions of match was chosen over others (such as total error threshold or monotonic threshold sequence] for the following reasons:
(1) A recursive-type computation is approximately as rapid as a threshold type,
(ii) passage through zero is easily detected,


Figure 3.15 Horizontal Gradients along lines 13 and 23 of the Test Site.
(1ii) the effect of bras in the image density levels on the chorce of threshold values is elimmated,
(iv) differentials involving a small number of terms at the edges of the template disk yıelded inaccurate match positions

After the approximate match position is found by means of gradient sequences, a block of area five pixels by five pixels centered on the approximate position is analyzed in detail. In this region, the total cumulative error between the template and the image is calculated using a recursive method, and the position of minimum error is determined If this error does not exceed a predetermined threshold, this position is accepted as the coordinates of the center of a tree crown In addition, a tree count is readily obtained.

### 3.5.2 Test Site Tree Detection

The peach orchard test site was analyzed in this manner. Figure 3.16 is an intensity plot of the cumulative errors which were computed over the five pixel by five pixel areas centered on the approximate match locations. The intense positions within regions of small error are the final tree coordinate positions, as determined by the monimum total caminulative error. The total error is above the threshold for those five by five squares which are intense at all positions, as in the leftmost three in the second row At these positions, the gradient procedure was sufficiently sensitive to detect the presence of an object, but the size or contrast did not match the template very closely, and hence the cumulative error was large.

The test site was andl'zed using total error thresholds of 1000 and 1500. The computer printouts of tree coordinates and enumerations are given in Figures 3.17 and 318

In the first case, the average total error for the 21 matches was 634 , or 52 per difference term between the template and the image In the second case higher error positions were accepted as matches, and the corresponding figures are 768 or 6.3 . Line printer plots of the tesi sites with the outlines of the

Figure 3.16. Cumulative errors for regions surrounding the approximate match positions.

TREE CODRDINATES

LINE NUMBER
7
10
11
12
28
30
48
49
50
51
5 ？
69
70
71
73
92
96
SAMPLE NUMBER AND TOTAL ERROR
$\begin{array}{rr}104-618 & 126-814 \\ 73-639 & 54-673 \\ 31-515 & 2-57\end{array}$
12－547
$123-837$
73－758 103－661
125－786
$104-575$
$72-557$
$31-571$
52－686
124－664
102－602
$72-621$
53－737
71－403 103－462
$30-603$

## TREE COUNT

あれれれ＊\＃\＃れたあ

| NUMBER | ARFA |
| :---: | :---: |
| ＊\＃\＃\＃\＃＊ | あれ\＃＊ |
| 21 | 97 |

```
ELAPSED TIME = 13.42 SECONDS
```

Figure 3．17－Tree Coordinates and Enumeration using a Threshold of 1000 ．
tree conrdinates


LINE NUMRER
7
10
11
12
28
30
33
48
49
50
51
52
69
70
71
73
74
73
91
92
94
95

SAMPLE NUMBER AND TOTAL ERROR
$104-618$ 126－814
73－639
31－515 54－673
12－547
$123-837$
73－758 103－661
10－1318
$125-786$
104－575
$72-557$
$31-571$
52－686
124－664
102－602．
$72-601$
53－737
30－1470
$11-1244$
$125-1442$
$71-43103-462$
52 -1145
$30-603$

> tree count
> ****れそれあま
> NUMPER AREA

$$
\begin{aligned}
& 26 \quad 97
\end{aligned}
$$

ELAPSEO TIME $=12.30$ SECONDS

Figure 3．18 Tree Coordinates and Enumeration using a Threshold of 1500 ．
detected trees superimposed are given in Figures 3.19 and 3.20. It may be noticed that the higher error threshold results in the inclusion of additional trees which have weak foliage and exhibit less contrast with the background. The apparently paradoxical reduction in computation time as additional trees are detected is due to the fact that an area occupied by a previously detected tree is not searched in succeeding scans, but rather bypassed.

### 3.5.3 Programming Considerations

This section presents a description of the algorithm in greater detail, with particular emphasis on methods of computer programming the algorithm.

In order to rapidly evaluate expressions involving the image pixels corresponding to a template of size nxn pixels, it is necessary to hold n records of the image data in core. If the template is to move to adjacent lines of the digitized image, a corresponding number of additional records should be held in core. In the present case the template is allowed to move over five scan lines of the image, and hence two additional records of data before and after the n records corresponding to the template size are stored.

Now, the simplest method of referring to the various template terms is by numbering them. If the elements of the template are numbered along its columns (consistent with the storage in memory of a two-dimensional array defined in the FORTRAN language), the element $(\mathbf{i}, \mathrm{j})$ is located at location $\mathrm{n}(\mathrm{j}-1)+\mathrm{i}$. The corresponding element of the image data is obtained simply by adding a bias term due to the position of the template along the scan line. Thus, one sequence of location numbers is determined for each evaluation required, such as the gradient terms, cumulative error scans, recursive evaluation sequences.

Three types of evaluation sequences are used. These are:
(i) differential sequences used for recursive evaluation of five sizes of disk and border along rows and columns,


Figure 3.19 Detected Trees using a Threshold of 1000 .


Figure 3.20 Detected Trees using a Threshold of 1500 .
(ii) a sequence over the template in order of increasing radii, used for the initial evaluation of the five disk size cumulative sums,
(iii) a sequence used to flag the area surrounding a match position to suppress re-searching that area.

The initial search for approximate match positions, using the differential of the disk sum, is done using the smallest and the largest disk areas, to ensure detection of all objects of the appropriate size. (If a large mismatch of sizes occurs, the differential sequence does not pass rapidly through zero). A detailed analysis is made if the gradient sequences along both rows and columns passing through a given pixel consist of two positive terms followed by two negative terms, indicating passage through zero. When this occurs, the match coordinates are determined as the best position found in a five pixel by five pixel area surrounding the approximate match position.

The cumulative error which is computed is effectively the difference between the sum of the image values corresponding to the disk area and those of a border area. This is a measurement of the contrast between the circular region and its background. In order to compute this measure over five disk areas without summing over the same area more than once, partial sums are computed over annular rings, whose area is the increment in the disk areas. Then sums of terms over disk regions and border regions of the five specified areas are obtained by summing a small number of the partial sums.

The additional difference sums for the remainder of the five pixel square area are determined recursively, using the sequences of array elements for the new and deleted terms as the templates are moved along rows and columns of the digitized image. If at the position of maximum disk-border contrast a specified threshold is exceeded, the coordinates of the match position and the template number (1-5) are output on the printer and sequential mass storage.

When advancing to the next record of data, it is necessary to rotate the records of data held in a two-dimensional array in core by one row, and then
read the next record into the last row of the core array. (An alternate method, in which the evaluation sequences rather than the imagery data records are rotated, was tested. The execution time of this program was greater by one-third). Similarly, since the match can occur in any of five lines of data, the match parameters must be placed in arrays of five rows, which must be rotated for each record read.

In order to prevent multiple detection of the same match position, and to accelerate the detection process, a specified area surrounding a match position is not searched again. This is accomplished by marking the data around the match in some manner. This region may be rapidly distinguished from the data by the use of negative numbers. If the area to be skipped is large enough that no part of the template will fall on this area, the magnitudes of the elements may be changed from the data values in order to convey useful information, such as the number of pixels to advance along the scan line. This method was used in the $100 \times 100$ pixel test area, where the overall computation time was seen to decrease as a greater number of trees were detected (and hence more area was bypassed in the search). In general, positions of the template near its edges may fall on the marked area, and so the data values are simply negated, so that their magnitudes remain available. With this method, only the midpoint of the template is tested for falling on the marked area, and hence this test is very rapid. However, this method limits the dimensions of the bypassed area to the size of the template. If a larger bypassed area is desired, the corners of the template must be tested for falling on the marked area.

An additional programming consideration which accelerates the execution is the use of a look-ahead test, which cancels the search for passage of the differential through zero if there are not sufficient pixels remaining before collision with a previously detected match.

### 3.5.4 Large Area Test Site

The test site known as Bateman's orchard in Bibb County, Georgia was photographed by the MSFC I ${ }^{2}$ S camera at an altitude of 3000 feet. The blue band image was scanned and digitized to 64 levels in the density range 0 to 2, yielding a digitized image size of 600 lines $X 850$ samples ( 510,000 pixels). The orchard contained 28 X 38 rows of trees, and hence 1064 tree positions. The majority of the trees in the orchard have been examined from the ground, and it was found that 50.5 percent of the original trees remained alive. Extrapolation to the total area of 1064 grid positions indicates 537 live trees present.

The tree detection algorithm was applied to this digitized image, searching for five sizes of trees whose areas ranged from 100 pixels to 300 pixels in increments of 50 . The corresponding diameters ranged from 11 pixels to 19 pixels in increments of 2. The threshold used was 7.0. (A threshold value of 7.0 means that, on the 64 level scale of densities, the average density over the disk region must exceed that over a ring-shaped border region of comparable area by 7.0 ).

In this test site, the tree crowns overlap in many cases, and an important parameter is the allowable separation between match positions, which strongly affects the execution time due to the bypassing of a specified area surrounding previously detected matches. A set of experiments was performed in which the bypassed area varied from 9 pixels on a side to 25 pixels. The results are presented graphically in Figure 3.21. The computation times do not include I/O operations. The machine used was an IBM 360/65. It may be noticed that bypassing larger areas decreases the execution time more rapidly than the number of detected trees decreases.

Sample printouts of the coordinates and parameters of each match position, and of the summary of match sizes and enumeration, are given in Figure 3.22. The five sizes of detected trees, for two values of the area bypassed around previous matches, are indicated on the blue band photograph in Figure 3.23.


Figure 3.21 Computation time and tree count as a function of bypassed area.

cISE nlimber JAFFLL BLNBER, RTLATIVE AREA, CCNTRAST



Figure 3.22 Sample printout of detection algorithm results.


BYPASS SIZE $=$ ?


BYPASS SIZE $=25$

Figure 3.23. Detected trees for two values of the bypassed area

### 3.5.5 Performance Summary

The performance of the algorithm will be discussed with regard to computation speed and detection accuracy.

As stated previously, the computation speed increases with the number of detected trees, due to the bypassing of areas surrounding matches. The present scene is not advantageous in this regard, since one-half of the trees are missing. Also, the detection rate will depend on the resolution at which the imagery is digitized. It is apparent that, for good detection and sizing, the tree diameters should be on the order of several pixels, ranging from eleven to nineteen in the present case. With an IBM $360 / 65$, the digitized image was analyzed at an average rate of approximately 1350 pixels per second. ( 377 seconds were required with bypass size 25.) Expressed in a different fashion, the rate of analysis is 3.8 tree cells per second, where a tree cell is a square region of side the average distance between rows of trees, which is 18.8 pixels.

Using the lower image of Figure 3.23, it was determined manually that, within the 28 by 38 row area of the orchard, fifteen trees were not detected. Subtracting the nineteen trees detected outside of the orchard from the total of 534 detected trees, we obtain a detection accuracy of $515 / 530$ or 97 percent.

## IV. ANALYSIS OF THE INFRARED BAND PHOTOGRAPH

It has been known for some time that healthy vegetation is strongly reflective in the near infrared spectral band, while vegetation under stress from any source is markedly less reflective. This trait has been widely exploited in the determination of disease and insect infected areas of deciduous forests and citrus orchards. Encouraging results in peach orchard photography have been obtained by Georgia Institute of Technology in cooperation with the USDA Southeastern Fruit and Tree Nut Research Station. ${ }^{2}$ In this work, color infrared transparencies were viewed through color separation filters by a television camera. The image was then analyzed by computer, and it was found that orchard sections containing unhealthy trees were discriminated by their infrared reflectances.

Bateman's orchard in Bibb County, Georgia, was the site of a 9-year experiment in various treatment methods. An aerial photograph, with the treatment areas marked, is given in Figure 4-1. Herbicides were used to control weeds in strips lettered "H". Weeds were controlled by disking in strips lettered "D". Blocks of live trees in the disked strips indicate where Fumazone was injected for nematode control. The almost perfect stands within the " H " strips indicate where both herbicides and Fumazone were applied.

The infrared band image taken by the MSFC $I^{2}$ S camera was scanned and digitized to 64 levels in the density range 0 to 2 . In order to establish the existence of variations in infrared reflectance as a function of tree vigor, the digitized image was thresholded at several closely spaced density levels. The digitized image was then rewritten on a film writer with the regions having densities less than the thresholds being blacked out. The results for four thresholds are given in Figure 4-2. The healthiest trees, having the highest reflectance, will be blacked out at the lowest thresholds. It can be seen that only the healthiest trees have densities of 25 or less. Setting the threshold at 28 includes nearly every tree within the perfect stands treated by herbicides and Fumazone, while excluding


Figure 4-1. Bateman's Orchard Showing Treated Areas

## INFRARED BAND



THRESHOLD $=28$


THRESHOLD $=25$


THRESHOLD $=31$

Figure 4-2. Infrared Photograph of Bateman's Orchard Showing Four Density Thresholds


Figure 4-3. Histograms of Declining and Healthy Trees

## V. ANALYSIS OF MULTIBAND AERIAL PHOTOGRAPHY

### 5.1 INTRODUCTION

The photography used in this study was obtamed with an International Imaging Systems ( $I^{2}$ S) Mark I four-band multispectral camera, using Kodak type 2424 black and white infrared film and type 2420 duplicating film. The four spectral bands were defined by filters, whose peak transmissions occur at wavelengths $0.430,0.530,0.635$, and 0.800 microns. The transmittance curves are shown in Figure 5-1.

### 5.2 IMAGE REGISTRATION

The four photographic transparencies were digitized separately and, hence, require digital registration of the imagery. This was accomplished by choosing five template areas in one photograph and searching for match positions in the other three images by means of a sequential similarity detection algorithm. ${ }^{3}$ An error threshold sequence was calculated based on a 10 percent probabslity of exceeding the threshold at the n-th term of the error summation (see Figure 3-10). The templates from the furst mage were then used to search for matches in each of the three remaining images and the match positions were taken as those with minimum cumulative error. ${ }^{4}$ The four digitized files of data were then truncated by the appropriate numbers of pixels to produce registered files. These are then merged so that the four density levels from corresponding locations of the imagery form a four-dimensional feature vector. These steps are illustrated in Figure 5-2.

### 5.3 TRAINING SAMPLE SELECTION

The data values for all four spectral bands were then selected from the template regions corresponding to 50 declinmg and 30 healthy trees. The total numbers of samples obtained were 9906 for the decluning and 7718 for the healthy. This is a prohibituve number of samples when using a nonparametric method of


- Figure 5-1. Filter Transmittances of the Multispectral Camera

multi bano mage set to be separated separated images to be visually CHECKED FOR REGISTRATION AND PREPARED FOR MOUNTING ON THE MICRODENSITOUETER


INDIVIDUAL IHAGES TO BE SCANNED ARD DIGITIZED.
PRIMARY IMAGE FILES TO BE CREATED.

individual feature component flles

Feature vector file to be created by rerging INDIVIDUAL COMPONENT FILES

Figure 5-2. Summary of Preprocessing Requarements
determining a set of classification functions. The number of training samples was reduced by two methods.

The simplest method consusts of sampling the data values at equal intervals such that a tractable number (e.g., 500) of samples is obtained. A second method consists of obtaining the four-dimensıonal histogram of the data in which each cell of the histogram represents a unique set of the four data values forming a feature vector. ${ }^{6}$ The result is 8655 cells for the declining tree data and 6856 cells for the healthy. A reduction in the number of cells to 993 (healthy) and 737 (declining) was then accomplished by eliminatung all cells having a frequency of occurrence of one. This procedure is physically acceptable as these cells are at the extreme edges of the distributions and probably represent mixtures with soil data values, since the irregular shapes of the tree crowns permit such a mixture within the circular template regions.

Succeeding calculations of the classification functions were coded to use the four data components and the frequencies of each cell and thus avoid the repetation of calculations for feature vectors which are identical. These two methods will be referred to as the sampling method and the cell method in the following sections.

### 5.4 SUPERVISED CLASSIFICATIONS

The two supervised classification methods which were applied are maxamum lukelihood and sequential linear.

The maximum likelihood algorithm is based on the assumption that the samples within a given class are distributed accordmg to a normal (Gaussian) multuvariate density function.

$$
P=\frac{1}{\sqrt{(2 \pi)^{\mathrm{n}} \mathrm{D}}} \mathrm{e}^{\frac{-(X-\mathrm{m})^{\mathrm{T}} \mathrm{~K}^{-1}(X-\mathrm{M})}{2}}
$$

where $M$ is the mean vector, $K$ is the covariance matrix, and $D$ is the determinant of K .

Such a distribution is completely specified in terms of a mean vector and a covariance matrix for each class, which are determined from the tramung samples. In the classification phase, the probability of an unknown feature vector belonging to each class is calculated, and assignment is made to the class for which the probabulity is the greatest.

Linear discrimmant functions are also determined for each class, using the training samples. If the linear discriminant function corresponding to a certain class is evaluated by substatuting an unknown feature vector, assignment is made to that class when the result is positive. If the assignment is not made, the data sample belongs to one of the remaining classes, and successive discriminant functions are evaluated. This effectively means that the multiclass problem is treated as a series of two-class problems, in both the training phase (determination of linear discriminant functions) and the classufication phase (assignment of data samples to classes). Visualizing the data from different classes occupying different regions in the spectral measurement space, one can recognize that the outer clusters can be separated from the mner clusters and that this process can be executed sequentially. Thus it is necessary to order the classes of data such that each can be separated from the remaining classes by a linear function (a hyperplane in the spectral measurement space). This is accomplished by computing for each spectral band the totals of the separations between all pairs of points in dufferent classes (interclass) and within each class (intraclass). The optimum separation is obtained when the interclass distance is maximized and the intraclass distance is minimized. The coefficients of the discriminant functions may then be determined by maximizing a criterion such as the distance of the tramng samples from the duscriminant hyperplane. ${ }^{7}$ The processing flow is shown in Figure 5-3. Applications of these algorithms have been discussed in greater detall in other publications. 8,9


Figure 5-3. Sequential Linear Classifier System

Training samples derived by the sampling and the cells methods were used in maximum likelihood and sequential linear classifiers. The digitized scene, consisting of 510,000 pixels, was classified on a pixel-by-pixel basis into three classes--healthy trees, declining trees, and soil. These results were then further analyzed to obtain the classification of each tree. The tree classes were taken as the majority class within the area of each tree as determined by the match coordinates and the five sizes of templates.

The two classification maps are shown in Figure 5-4, obtained using the cells method of training samples selection and data from the four spectral bands. The classifications of the previously detected trees are shown in Figure 5-5. An example of the output of the tree classification algorithm is given in Figure 5-6. The classification results are given in Table $5-1$. When using the infrared band photographs only, the two classification algorithms give similar results. The density levels assigned to the three classes are as follows:

| LINEAR: | $0-30$ healthy | $31-38$ declining | $39-63$ soil |
| :--- | :--- | :--- | :--- |
| GAUSSIAN: | $0-30$ healthy | $31-37$ declining | $38-63$ soil |

### 5.5 PERFORMANCE SUMMARY

The goal of the multiband classification procedure is to determine those trees which are remaining healthy and those which are in decline. One problem in this procedure is the variation within a single tree crown. However, if the classification is sufficiently accurate, this may indicate damage in some limbs of the tree. An attempt was made to overcome this problem by classifying trees as healthy or declining according to the majority of the pixels. It may be seen from Table 5-1 that this procedure results in a classification of the 80 ground survey trees with an accuracy of approximately 70 percent. Additional ground survey information included the statistics of 989 trees, but not the status of individual trees. Of 499 live trees, the ground survey indicated 46.69 percent as healthy and 53.31 percent as declining. The computer classifications also indicate a majority of trees in the declining category.


SEQUENTIAL LINEAR CLASSIFICATION


Figure 5-4. Classification maps obtained by two methods.


SEQUENTIAL LINEAR CLASSIFICATIUN


MAXIMUM LIKELIHOOD CLASSIFICATIUN

Figure 5-5. Classification of the detected trees.

| $\begin{aligned} & \text { CIASS } \\ & \phi \Delta \# * * \end{aligned}$ | SIRE NUMBER | SAMPLES | $\begin{aligned} & \text { PERCENT } \\ & \text { **かか力 } \end{aligned}$ |
| :---: | :---: | :---: | :---: |
| DFCL INE | 1 | 31 | 5.31 |
| DFCLINE | $?$ | 55 | 10.30 |
| DECLINF | ？ | 100 | 18.73 |
| JFCLINE | 4 | 70 | 13.11 |
| DFCLINE | 5 | 27 | 5.06 |
|  |  | $283$ | $53.00$ |
|  |  | －－－－ | －－－－－－－ |
| HEALTHY | 1 | 6 | 1.12 |
| HFALTHY | 2 | 13 | 2.43 |
| HEALTHY | 3 | 63 | 12.73 |
| HEAL THY | 4 | 91 | 15.17 |
| HEAL THY | 5 | 33 | 15.54 |
|  |  | 251 | 47.00 |
| SOIL | 1 | 0 | 0.0 |
| 5012 | 2 | 0 | 0.0 |
| SOIL | 3 | 0 | 0.0 |
| SOIL | 4 | 0 | 0.0 |
| SOIL | 5 | $\bigcirc$ | $n .0$ |
|  |  | －－－－ | 0.0 |
|  |  |  |  |

Figure 5－6．Output of the tree classification algorithm．

Table 5-1. Classification Results

| Classification Method | Training Data Accuracy By Pixels | Training Data Accuracy By Trees | Orchard Classifications (Percentages) |  |  |  |  |  | Classification <br> Time <br> (Seconds) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | By Pixels |  |  | By Trees |  |  |  |
|  |  |  | Healthy | Declining | Soil | Healthy | Declining | Soil |  |
| Sampling Method |  |  |  |  |  |  |  |  |  |
| Maximum <br> Likelihood | 73.7 | 76.25 | 18.62 | 31.14 | 50.23 | 37.39 | 62.43 | 0.18 | 564 |
| Linear | 70.9 | 67.5 | 14.88 | 33.29 | 51.83 | 47.01 | 52.27 | 0.73 | 126 |
| Cells Method |  |  |  |  |  |  |  |  |  |
| Maximum Likelihood | 79.1 | 77.5 | 16.48 | 43.04 | 40.48 | 47.00 | 53.00 | 0.0 | 524 |
| Linear | 76.8 | 67.5 | 11.24 | 26.73 | 62.02 | 27.15 | 71.35 | 1.50 | 71 |
| Infrared <br> Maximum <br> Likelihood | 62.3 | 68.75 | 17.09 | 32.40 | 50.51 | 41.01 | 42.13 | 16.85 | 183 |
| Infrared Linear | 63.2 | 80.0 | 17.09 | 39.17 | 43.73 | 39.70 | 51.12 | 9.18 | 47 |
| A verage | 71.0 | 72.9 | 15.90 | 34.30 | 49.80 | 39.88 | 55.38 | 4.74 |  |
| Ground Survey |  |  |  |  |  | 46.69 | 53.31 |  |  |

The infrared band classifications were performed since it is known that decline would be indicated primarily in the infrared spectral range. However, the accuracy is diminished in this case. This is probably an indication of the fact that the spectral band being used is in the very near infrared. It is anticipated that better results could be obtained with better infrared response in the detector used.

## REFERENCES

1. J.A. Payne, W.G. Hart, M.R. Davis, L.S. Jones, D.J. Weaver and B.D. Horton, "Detection of Peach and Pecan Pests and Diseases with Color Infrared Aerial Photograph," Proc. Third Biennial Workshop_on Color Aerial Photography in the Plant Sciences, 1971.
2. G. William Spann, "A New Method for Studying Peach Tree Decline," USDA Seminar on Infrared Photography of Peach Short Life Sites, 1973 (unpublished).
3. D. S. Barnea and H. F. Silverman, "A Class of Algorithms for Fast Digital Image Registration," IEEE Trans. Computers, Vol. C-21, February, 1972.
4. The FORTRAN programs required were designed and coded by H. K. Ramapriyan.
5. H. K. Ramapriyan, "A Multilevel Approach to Sequential Detection of Pictorial Features," IEEE Trans. Computers, Vol. C-25, January 1976.
6. The FORTRAN programs required were designed and coded by B. V. Dasarathy.
7. Ho, Y. C. and Kashyap, R. L., "A Class of Iterative Procedures for Linear Inequalities, " J. Siam on Control, Vol. 4, 1966.
8. Bond, A. D. and Atkinson, R. J., "An Integrated Feature Selection and Supervised Learning Scheme for Fast Computer Classification of MultiSpectral Data, " Proc. Earth Resources Observation and Information Analysis System Conference, U. of Tennessee Space Institute, March 1972.
9. Bond, A. D., Ramapriyan, H. K., Atkinson, R. J., Hodges, B. C., and Thomas, D. T., "Image Analysis Techniques Associated with Automatic Data Base Generation, " AIAA Computer Network Systems Conference, April, 1973.

## APPENDIX A

A FORTRAN listing of the complete detection package follows. The core required on the IBM $360 / 65$, when using standard FORTRAN input and output tapes, is 148 K bytes [ $\mathrm{K}=1024$ ].

DIMEASICN S(24,350). TMPLT(25,25)
LOGICAL TMPIT
DATA NR, IAREA1, MGYP, THRESH, NRECS,NSAMP $/ 25,50,7,7,0,60 C, 8501$
c
C NR - LENGTH OF A TEMPLATE SIDE
C IAREA1 - IEMPLATE AREA INCREMENT
C MBYP - LENGTH DF A SYPASSEL AREA SIDE
C THRESH - IHRESHCL O CN AVERAGE DISK AND BJRDER DENSITY DIFEERENCE
NFECS - NUMBER IF RECDRDS TO BE READ
C MSAMP - $\triangle U M B E R ~ J F ~ S A N P L E S ~ P E R ~ R E C O K D ~$
C
CALL DSECNC (S, TMPLI, NR, IAREAI, MBYP)
C
REWINO 10
REWIND 11
CALL DETECT (S, S. NK, NR+4, IAREA I. THRESH. NRECS, NSAMP) END FILE 11
STAP
END


```
            J1= \1 + l
            ST(JI)=II**2 + JJ**2
    22 LSFO(11) = ( (:R+4)*(|)a-1) + No+ +2
    CALL SCRTSL (ST, ISEU, 1, NR*NR)
c
C COMPUTE SEQUENCE USED FOR DEFINING REGION SURRUUNDING A MATCH
    MID=(NR+1)/2
    MRAD = MOYP/2
    IRECI = MID - 2
    IREC2 = M10 + MRAD + 2
    ISAMP1 = MID - MRAD
    ISAMP2 = MID + MRAD + 1
    MS = n
    D] 10 IW=IREC1,IREC2
    DC 10 Jm=1 SAMP1, ISANP2
    MS =MS + 1
        10MSEQ(MS)=(NR+4)*(JW-1)+1W+2
C
C COGPLTE DISK ANO BORDER SEQUENCES AND AREAS
    D? <0 I =1,11
    KAKEA(I)=(I+1)*IAREAI
    60 (ALL TENPLT (TMPLT, KAREA(I), NR)
    DUS SSILE=1,5
    ASILE(NSILE,1)= KAREA(NSIZE)
    5 ASILE(NSILE,2) = KAREA(2ANSIZE+1) - KAREA(NSIZE)
        RE TURN
        END
```

```
c
            SUBRCUTINE TEMPLT (TMPLTE IAREA, NR)
            LOGICAL TMPLT(NR,NR)
c
            K2 = IAREA / 3.14159265
            IAREA = ?
            DO 1100 I = 1,NR
            AI = I- (NR+1)/2
            DD 1100 J=1,NR
            AJ = N - (NR+1)/2
            TMPLT(I,J) =.FALSE.
            IF (AI**2+AJ**2.GT,R2) CD TO 1100
            IAREA = IAREA + 1
            TMPLTU.JI = TRUE.
    11ON CDNTINUE
            RETURN
            END
```

```
C
    SUBRIUUINE DETECT IS, SI, NR, NRI, IAREA1, THRESH, NRECS, NSAMPI
    c
c
    DETECT CIRCULAR REGICNS BY CINTKAST COMPAKISON
    INTEGER S(INR1,NSAMP), ST(1), SAMP (5,20), SILE(5,20), DELH: DELV,
        .HSN(25,10), HSD(25,10), VSN(25,10), VSD(25,10)
                        OIMENSICW NSET(4). NIESI(4). NSEW(5). NHAT(5), DIFF(5,20). NSIZE
        .(5), NS(17), ISEG(625), KAREA(11), ASIZE(5,2), MSEQ(500)
        DAIA NIESI 12*1, 2*-1/, NSET / 2, 0: 2, 1/
        CONMON /SEUNCE/ INS,HSN,VSN,HSD,VSD,ISEG,KAREA,ASIZE,MS,MSEQ
c
    100 FORMAT (!HI,2JX,'C!JQRDINATES OF MATCH POSITIONS'/21X,30('#')//5X,
        .'LIAE NUMQER',1Ox, 'SAMPLE NUMBER. KELATIVE AREA, CONIRAST'//1
    101 FORMAT (I1),3X,8(IG,I3,F6.21)
    102 FGRMAT (13x,8116,13,F6.21)
    103 FGRMAT {1H1,3JX, 'COUNT OF MATCH POSITIUNS'/31X,2410*'///21X,'NUMBE
        -R NIMINAL GREA + UIAMETER ACTUAL PIXEL AREA + DIAMETEK'/21X,
        .6(1*'),3x,12('*4),3x,8('*4),3x,17('*1),3x,8('*1))
    104 FIRMAT (/I 26.112.F13.1.2115)
    C
        PRINT 1CO
        MIORCW = N2/2 + 1
        MID = (NR+4)*(MICRIN-1) +MIDROW + 2
        NREC2 = NRECS - NR1 + MIDROW - 1
        NSAMP2 = NSAMP - MIDRDN
    C
    INILLALILE SILE COUNTER, MATCHES PER RECORD COUNTER. AND ARRAYS
    c
        0.2 2 I =1.5
        NSILE(I)=0
        2 NMAI(I)= 2
        DO 5 Jin=1,NR1
        5 READ (12) (S(JH.IEL). IEL=1,NSAMP)
    C
        OO1 NREC=M1DKDW.NREC2
    C
        IEL = MIDRINW
        JS = 0
        HSEQ(1)=0
        NSEQ(5) = ?
    C
    5330 CENTINLE
        IEL = IEL + 1
            IF (IEL.GT.NSAMP2) GO TO 1111
            JS = 1S + NR1
C
    IF (STIJS+YILI.GE.2) GD TC 80
    NSEL(1) = )
    NSEC(5)=?
    GO TO 5330
c
C CANCEL SEARCH IF INSUFFICIENT PIXELS AHEAD TO SATISFY DIFFERENCE SEQ.
        80 NPIX = 3-MAXO (NSEQ(1). NSEQ(5))
        1SI = 1S +NFIXNNR1
        IF (ST|.|SI+MINI.OF.OI GOT TM &a
```

```
\(c\)
C SEARCH FOR NAXIMUM IN SUM OVER CIRCULAR REGION BY SEARCHING FOR
c DIFEERENCE SEQUENCE WLTH SIGNS + + - -
c
    \(8800 \quad 40 \quad \mathrm{~N}=1,5,4\)
    \(N^{3}=N S(N)\)
    DELH \(=\) C
    DD \(45 \mathrm{JSEQ}=1\), N3
```



```
    NSEQ(N) = NSEQ(N) + 1
    IF (DELH*NTEST(NSEQ(N)).LT.O) GO TD 41
    IF (NSEGINI.EQ.4) GO TO 1250
    GO TO 40
        \(41 \operatorname{NSEG}(N)=\operatorname{NSET}(N S E Q(N))\)
        40 CONTINLE
            GO TO 5330
\(c\)
C COMPUTE DIFFERENCE SEQUENCE ALONG COLUMNS
    1250 NSEQ(1) \(=0\)
            \(\operatorname{NSEQ}(5)=0\)
            \(002100 \mathrm{~N}=1,5,4\)
            \(151=15-2 * N R 1-1\)
            \(\mathrm{N} 3=\mathrm{NS}(\mathrm{N})\)
            DO 2C1C \(K=1,4\)
            \(D E L V=\) ?
            DO \(35 \quad\) ISED \(=1, \mathrm{~N}_{3}\)
            35 DELV = DELV + IABS(ST(JSI +VSN(JSEW,N))I-IABS(ST(JSSI +VSC(JSEQ,N)))
            IF CDELVANTEST(K).LT.O) CO IO 2100
    \(2010 \mathrm{JSI}=\mathrm{JSI}+1\)
            CALL MATCH (ST, NRL, NMAT, SAMP, SIZE, DIFF, JHRESH, IEL, JS)
            GO TO 5330
    \(21 C 0\) CONTINUE
            G] TQ 5330
    1111 CONTINLE
c
c ROTATE ARRAY S BY ONE ROW
C
            DD \(\in \quad J_{h}=2, N B I\)
            \(00 \in I E L=1\), NSAMP
        \(6 S(J n-1, I E L)=S(J W E I E L)\)
    \(c\)
    C DUTPLI LISLS OF MAICH CUORDINATES. READ NEXT DALA RECORD.
    c
```



```
    \(N H=\) NMAT(1)
    NMAICH \(=\) NYAICH \(+N^{M}\)
    OO \(2121 \mathrm{I}=1, \mathrm{~N}: 4\)
    2121 NSIZEISILEI1.11.) = NSIZE(SIZE(1.1)) +1
            IF (NM.GT.3) GO TO 800
            PRIAT 101, NREC, (SAMP(1, I), SILE(1, I), DIFF(1, I), \(I=1, N M)\)
            GO TO 85
        800 PRINT 101, NREC, \(\operatorname{SSAMP(1,11,~SIZE(1,I)\text {.DIFFU,11,}1=1,81)}\)
            PRINT 102, (SAMP(1,I), SIZE(1,I), DIFF(1,I), I=9,NM)
        35 WRIIE ( 111 NREC. NM. (SAMP(1.1), SILE(1, H. \(I=1, N M)\)
    2000 CENTINLE
            READ (10) (SSNR1. IEL), IEL \(=1\) NNSAMP)
    r
```

IF (NMATII).EQ.0) GO TO 120
NM $=$ NNAT(I)
DO 110 IEL $=1, \mathrm{NM}$
$\operatorname{SAMP}(I-1, I E L)=\operatorname{SAMP}(I, I E L)$
SILEII-1,IEL) $=$ SILE(I,IEL)
110 DIFF(I-1,IEL) = DIFF(I,IEL)
120 NMAT(I-1) = NMAT(I)
NMAT(5) $=0$
1 CONTINUE
C
PRINT 103
NMATCH $=$ ?
DO $10 \quad A=1.5$
NAREA $=(N+1) *$ IAREA1
$D=2.2$ © SWKT(NAREA/3.141592051
ISILE $=\operatorname{ASILE}(N, 1)$
$I D=20 I N T(D / 2 \cdot 0+1.01-1$
PRINT IC4, NSIZE(N), NAREA, D, ISIZE, ID
10 NMAICH $=$ NMATCH + NSILE (N)
PRINT 1O4, NMATCH
RFTLRN
END

```
c
SUBROUIINE MAICH (ST,NR1,NMAL,SAMP,SIZE,DLFF.IHRESH.LEL,JS)
COMPLTE CENTRAST FOR 5 SILES OF DISK OVER A 5 x 5 PIXEL AREA
INTEGER ST(1), SAMP(5,1), SILE(5,1), SUM(11), DISK, BORD, DSAVE,
. BSAVE, [DISK, HSN(25,10), VSN(25,10), HSD(25,10), VSD(25,10)
DIMEASICN NMGTU1). DIFF(5.11. NSI101, ISEO(625), KAREAC111, ASILE
    . (5,2), MSEa(500)
    COMMCN /SEOHCE/ NS,HSN,VSN,HSD,VSD,ISEL,KAREA,ASIZE,MS,MSEO
c
c
    COMPUTE SLMS JF DATA values over anNuLar RINGS
        JS1 = 1S - 4*NR1 - 2
        KEL = KAREA(11)
        k=1
        Sum(1) = 0
        DO 3n5 JSED=1,KEL
        IF (JSEG.NE.KAREA(K)+1) GC TO 305
        K = K + 1
        SUM(K) = n
        305SLM(K) = SUM(K) + LABSISTIJSI + ISEQ(JSEQ)/1)
C
c LCOP TVER = SIZES OF DISK AND 5 X 5 PIXEL AREA
C
        AO=C.O
        OO 18 NSIZE=1,5
        JS = JS - 5NNRI
        ND = NS(NSIZE)
        NB=NS(NSLIZE+5)
    c
        DO 18 KHCS =1,5
        JS = JS + NR1
        DO 18 KYER=1,5
        JS1 = JS + KVER - 3
        IF IKYER.NE.1) GC TC 200
            IF (KHCR.NE.1) GC TO 2O1
    c
    C COMPUIE DISK AND bORDER SUMS AT FIRST POSITIUN AND SAVE FOR RECURSIVE
    c CALCULATICNS
    c
    OISK=0
    BTRD = n
    OO 21 N=1.NSILF
    DISK = DISK + SUM(N)
        21 BDRD = BORD + SUN(NSIZE+NI
        BORD = BORD + SUN(2*NSILE+1)
        DSAVE = DISK
        BSAVE = BCRD
        GO In 5210
    c COMPUTE RECURSIVE SUM ALONG RONS
    C
        201 DDISK = 0
            OD 34 JSEQ=1,ND
            34 DDISK = DOLSK + IABSISTINSI+HSNCISER,NSULEII)- IABSIST(JSI+HSD
            .11SFi,M,C12F|11
```

```
    35BחRD = BOK) + IABSIST(JSI+HSNIJSEQ,NSI LE+5)I) - IABS(ST(JSI +HSD
        .(JSE(N,NSIZE+5)))
C
C SUBTRACT SUM OF TERMS ADDED TO PREVIOUS DISK SUM
    BORD = BORC - DOISK
    OSAVE = OISK
    BSAVE = BORD
    GO TO $210
C
C COMPUIE RECURSIVE SLM ALONG COLUMNS
C
    200.DOISK =C
        DO 32 JSEQ =1,ND
    32 DOISK = ODISK_ IABSIST(ISItVSNCISEQ,NSIVE)I) - IABSCSIIISITVSD
        .(JSEQ,NSIZE)))
            DISK = DISK + UDISK
            DD 33 JSEN=1,NB
    33 OORD = GORD + IAES(STIJSI +VSN(JSEQ,NSILE+5))) - IABS(ST(JSI+VSO
        .(JSEQ,NSILE+51)1)
    BMRD = BORC - ODISK
C
C FIND MAXINUM OF DLFFERENCE BETWEEN AVERAGE DISK TERM AND AVERAGE
C BORDER TERM
    9210 CONTINUE
        UIF = OISK/ASILIEINSIZE,1) - BURD/ASIZELNSIZE,2)
        IF (OIF.LT.ACI GO TO 18
        AO=DIE
        K1 = KVER
        K2 = KHOR
        K3 = JS1
        K4 = NSILE 
    18 CONTINLE
C STORF MATCH PARAMETERS IF CONTRAST IS GREATER THAN THRESHOLD
    IF (AD.LT.THRESH) RETURN
    MMAI(K)L=NMAI(K)) +1
    NM=NNAT(N1)
    SAMP(K1,LM1 = IEL - 5 +K2
    SIZE(KI,NM)=K4
    DIFF(K1,AN)=A\cap
C IF MATCH FOUND, SET CENTRAL REGIUN TO NEGALIVE UATA VALUES FOR
C SKIPPING ALGDNITHM
    UO 10 JSEQ =1,MS
    lJ=K? + MSEW(JSEQ)
    10 STIJJ) = ISIGN(ST(JJ),-1)
    RETURN
    END
```

```
C
C
    SORT ARRAY AI AND ARRANGE AFRAY AL CORRESPONDINGLY
C DO FROM ELEMENTS II TO JJ
            DIMENSICN A1(1), AZ(1), IU(16), IL(16)
            INTEGFR A1, A2, I1, I2, II1, IT2
            LDGICAL*1 SL
            SL =. TRUE.
            GO TO 1
            ENTRY SORTLS (A1, AL, II, JJ)
            SL = .FALSE.
    1ND =JJ - UI + 1
            M=1
            I= II
            J= JJ
    5 IF(I.GE.J) GOTO }7
    10 K=I
            IJ=(J+I)/2
            T1 = Al(IJ)
            T2=A己(IJ)
            IF (Al(I).LE.T1) GO TO 20
            Al(IN) = A1(1)
            A2(IJ)=A2(I)
            A1(I) = T1
            A2(I)=T2
            I1=A1(1JI
            T2 = A2(IJ)
        20 L = = 
            IF (A1(J).GE.T1) GO TO 40
            Al(1J)=A1(J)
            Az(IJ) = A2(J)
            A1(1)=I1
            A2(J)=T2
            I1=A1(1J)
            T2 = A2(IJ)
            IF (Al(I).LE.II) GO IN 40
            Al(Ij)=41(I)
            A2(IJ)=A)(I)
            41(I)=T1
            A2(I)= = 2
            T1 = A1(IJ)
            T2=A2(11)
            GOTO 4C
            30. A:(L) = A1(K)
            AZ(L) = 42(K)
            A1(K)=TT1
            A2(K) = TT2
            40 L=L-1
            IF (A1(L).GT.TI) GO TO 40
            T1 = A1(L)
            TT2 = A2(L)
50 K=k+1
            IF (alif) IT Tl| i.1 IT an
```

$$
I \cup(M)=L
$$

## $\mathrm{I}=\mathrm{K}$

$\mathrm{M}=\mathrm{N}+1$
$\mathrm{COTO} \quad 8 \mathrm{C}$
60. $1(1 M)=K$
$I U(M)=J$
$J=L$
$M=M+1$
6010 8 C
$70 M=M-1$
If MADE. Ol GO TO 75
IF (SL) RETUKN
$N D 2=N D / 2$
$11=1 I$
$12=11$
D] $110 \quad \mathrm{I}=1, \mathrm{ND} 2$
$T 1=-11(11)$
$T 2=A 2(11)$
A1(11) $=21(12)$
A2(11) $=A 2(12)$
Q11121 $=11$
A2 $2121=T 2$
$11=11+1$
$11012=12-1$
REILKN.
75 I = IL(M)
$1=1$ (M)
80 IF(J-I.GE.II) GOTO 10
IFII.EG.IL) GJTO 5
$1=1-1$
ㅇC $1=1+1$
IFII.EG.J) GOTO 70
$I 1=A 1(I+1)$
$T 2=A 2(1+1)$
IF (AILLLLE.TI) 60 TO 90
$\mathrm{k}=\mathrm{I}$
100 A1 $(K+1)=A 1(K)$
$A 2(k+1)=42(k)$
$k=k-1$
IF (T1.LT.A1(K)) GO TO 100
A1 $(K+1)=11$
$A_{2}(k+1)=T 2$
601090

## APPENDIX B

Listings of the following data handling subroutines are given:
SETREE - selects training data from the feature vector file
CELLS

CLTREE - | determines the frequencies and feature vectors in the |
| :--- |
| four-dimensional histogram analysis |

RETREE - reconstructs the tree templates at the match points


```
C
    SFLECT G:NU:!: TAUTH TZLES
```



```
        IHTEGFK SAAP, SILE, S11
        L?GICAL THPLT(<5,25,5)
    10? FUPMAT (1015)
    171 FORAAT (120,' UCCJPIEN CFLLS',I10,1 TUTAL', )
    122 FJXMAT (1x.2UIo)
    106 FMFMAT (516)
    C
    \(20 \quad 20 \mathrm{~N}=1,5\)
    \(R 2=5 C .0 *(i v+1) / 3.14159265\)
    IAREA \(=0\)
    DO \(10 \mathrm{I}=1, \mathrm{NR}\)
    \(A I=1-(N R+1) / 2\)
    DU \(10 \mathrm{~J}=1\), ivर
    \(\Delta J=J-(N R+1) / 2\)
    TIAPLT(I,J,N) \(=\).FALSE.
    IF \((A I * * 2+A J * * 2\). UT. K 2\()\) GC TO 10
    IAREA \(=\) IAKEA +1
    TKPLT(I,J,IV) \(=\). TRUE.
        10 ChivT INUE
        C) PRIAT 1C2, IAREA
    C
    Dij 150 NL \(=1\), NRECS
            KELD (1C) S
        150 WRITE \(190^{\circ}\) NLI S
            DT 260 NC \(=1.2\)
            NTET(NNC) \(=0\)
            \(\mathrm{NT}(\mathrm{NC})=0\)
            \(J C(1)=0\)
    C EXTPACT CATA FIR A TAEL
        250 RTAN (5.1( I.L.N) \(=220)\) LIAF, SANP, SIZE
            MT(VC) \(=\) NiTIVC) + 1
    \(c\)
    \(L 11=\) LILE - w/ 2
    \(N 1=?\)
    REA) (9)'L11) S
    S11 = SAMF - IR/2
    DJ \(\angle 4\) HS \(1=1, \mathrm{~d}\)
    IF (.WIT.T:OPLT(VLI,HS1,SI7F)) Gา T? 44
    \(N_{1}=1 / 1+1\)
    517 6C NF \(1=1.4\)
        (2) \(x(1: F 1, N 1)=S(N F 1, S 11)\)
        \(44511=511+1\)
        40 L11 \(=L 11+1\)
        NTOT \((N C)=\operatorname{VOT}(N C)+N 1\)
C
C DETERMINE CELL NUMBEKS
```

SUBRZUTINE CFLLS (X, NS, MC, JC, KS, N:)
DTMENSITN X(NH, 1), HC(1), JC(1)
LUUICAL \#1 OYTE(4)
FOUIVALFNCF (VS. YTF (1) )

$$
\frac{D C 3 ; N S i=1,15}{0.3142 N F=1, N 1}
$$

140 BYTE (VF) $=X($ VF.NS 1$)$
C CRECK FGR CELL JECUPABICY

$$
D C 36 \mathrm{~K}=1, \mathrm{KS}
$$

$\frac{\text { DC } 36 \mathrm{~K}=1, \mathrm{KS}}{\text { IF (i)CU.EW.JC(K)) 60 TO } 37}$
36 CONTINUE
$\qquad$
C START NEW CELL

$$
\frac{K S=K S+1}{J C(K S)=N C O}
$$

$$
\begin{aligned}
& M C(K S)=0 \\
& K=K S
\end{aligned}
$$

$$
K=k S
$$

$c$

$$
\begin{aligned}
& 37 \begin{array}{l}
\text { HC(K) }=M C(K)+1 \\
\\
\text { QETURN }
\end{array} \text { END }
\end{aligned}
$$

$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$

SURZGUTIVE CLTREE (S, ST, NK, IAREG1, NROE, IVRECS, ISAYP)
C
$\qquad$ CLASSIFY DETECTED TKEES ACLEEDNG TR THF NAJ ITY GF PIXELS
IF MUDE =1, KEAD COUKUINATES EAJM TAPE UUITT 8
IF $A \mathcal{U} D E=2$. KEAD CCUNAJI:ATSS FGOM CAROS
C
DIAE:SIJN ISE $2(5,3 O C)$, NCLASS(3), MCLASS(EN), PCT(50), KS(5, 3)
INTEGER ST(1), S(IN2,1), FIXL(5), SAMF(CO), SIZE(CO), CLASS(3) DUUBLE PAECISIJN CLASSI(3)
DATA CLASS/'DC', 'OK', 'SC'/, CLASSI/' DECLINE', HEALTHY',

.'.5X, 'SAMPLE, SILE, CLASS, PERCENT'//I
101 FIGRMAT (315)
172 FORMAT ('1', $30 X_{1}{ }^{\prime}$ CLASSIFICATICN SUMMARY'/31X,221'*'1)//20X, 'CLASS',
 $.8 x, 7(1 * 1), 10 x, 7(1 * 1)$
103 FURMAT (/A 26, 113,117,F18.2)

105 FERMAT (112,7X,6(I6,13,A6,F4,1))
176 FGRMAT $(19 x, 6(16,13, A 6, F 4,1))$
107 FORMAT $(52 \times, 5(1-1), 11 \times, 7(1-1) / 156, F 18.2 / 52 \times, 5(1-1), 11 \times, 7(1-1) / /)$

## $c$

DETERMINE FIVE TÉMPLATE SEQUENCES
0] $310 \mathrm{NT}=1.5$
$R 2=(N T+1)$ \& 2 IAREA1 / 3.14159265
NSEQ $=0$
DO $20 \quad I=1$, NR
$I I=I-(N K+1) / 2$
DD $20 \mathrm{~J}=1$, NR
$J J=J-(N R+1) / 2$
IF (11 $\ddagger * 2+J J * * 2.6 T, R 2) \quad G 0$ TO 20
NSEO $=$ NSEU +1
ISFQ $(N T, N S E Q)=N R *(J-1)+1$
20 CONTI:UUE
310 PIXL $(N T)=$ NSEQ
C
C IIITIALIZE AKRAYS, READ INITIAL CDURDINATES
PKINT 100
DO $1 \mathrm{C} L=1.5$
Di: $10 \quad \mathrm{~N}=1,3$
$10 \mathrm{KS}(\mathrm{L}, 1 \mathrm{~V})=0$
DC $22 \quad J n=1$, NB
22 REAC ( 10 ) (S(J..IEL), IEL=1, NSAMP)
$4 T=1$
IF (NCDE.EW.1) READ ( 8 ) NEEC, NT, (SAMP(I), SIZE(I), I=1,NT)
IF (1, D) E.E.2) READ $(5,1 \cap 1)$ MREC, SAAP(1), SIZE(1)
MIDRCN $=12 / 2+1$
NREC2 $=$ NRECS - MR + MIDPDh -1
C
UO 132 NREC $=11$ IDRCA, IVREC2
IF (NREC.NF.NikEC) forg TO 55
50 DO 75 : $1 T=1$ eiv
$c$
$\frac{C}{C}$ COUNT NUMBEK LF R IXELS ASSIGNED IJ EACH CLASS
DO. $74 \quad N=1,3$

```
    74 !(CLASS(:) = 0
    TAFFA = ? X XL(SILE(#T))
    JS = 隹(JMIP(aI)-1-NK/2)
    D) 76 &SF=1.14रह%
    JJ = JS + ISEU(SILE(!:T),:SEO)
    IC = ST(JJ)
        75.NCLASSIICI = NCLASS(IC) + 1
    C
    C nETER:IINE CLASSHAVIISNAJPITV RF PIYELS
    C
    MAX=0
    DO 7C N=1.3
    IF (N:CLASS(iv).LT,MAX) GO TO 70
    YAX = NCLASS(IN)
    NC = N
        70 CONTINUE
    c
    KS(SILE(MT),NC) = NS(SILE(MT),NC) + 1
    HCLASS(:TT) = NC
    PCT(IAT) = MAX*1C0.0/ IAREA
    75 CTVTINUE
    IF (MDDE.EG.1) WRITE (11) MREC, NT, (SAMP(I), SILE(I), MCLASS(I),
        -I = I,NT)
            IF (NT.GT.6) GO TO }8
            PKINT 1O5, NKEC, (SAMP(I),SIZEII),CLASS(MCLASS(I)),P(T(I), I=1,NT)
            GD TO }3
        30 PRINT 105. NREE. (SAKP(I),SIZE(I),CLQSSIPCLQSS(I)),PCT(I), I=1,0)
            PRINT 1CO, (SA:AP(I),SIZE(I),CLASS(MCLASS(I)),PCT(I), I=7,NT)
    C
        85NT=1
            IF (M IDE.EW.1) READ (G,FNT=3त) MFEC, NT, (SAMP(I),SIZE(I), I= 1,NT)
            IF (AJJE.EU.2) REAO (5,1\cap1,FAD=30) MREC, SAMP(1), SILE(1)
            IF (MREC.EU.NREC) GU TO 50
C
        95 DO 6 Jw=2,NR
            DO 6 ISLE1.NSAMP
        6 S(Jm-1,IEL)=S(Jn,IEL)
            READ (10) (SIIR,IEL), IFL=1,NSA:AP)
        130 CONTINUE
    C
        30 NTOT = 0
            O) 210 NC=1,3
            NCLASS(NC)=0
    2.2 DCLISS(.CC)= N(LASS(.C) + &S(1,NC)
    \angle10 OT7T = NTJT +NCLASS(NC)
    C
        PK1IT 102
        DN 11CO NC = 1.,3
            DO 1110 N=1.5
            PCT(ivi) = 100.0 KS(1.|.C) / 1.TMI
1110 PKI':T 103. CLASSII:NC),N, KS(I.,ICC), PCT(NC)
    PCT(NC) = 1OU.0 % NCLASS(NC) / NTOT
112n POI'IT 1C7, NGLLASS(INL), PCT(I.C)
    DRINT 104. NTJT
    RETURN
    FNO
```


c C RFCQNSTRUCT TRE TREE TENPLATE AT THF MATCH PCINTS c

INTRGER S(NR, 1LL_ STII), ISEQ(5,30न), PIXL(5), SAMP(50), SIZE(50), -Class(50)
LEGICAL MATCHS
DETERMIVE FIVE TEYPLAIE SECUENCES
DO $2 N T=1,5$
$R_{2}=(N T+1)$ IAKEA1 / 3.14159265
VSEO $=0$
$0011=1, N R$
$4 I=1-(N R+1) / 2$
DO $1 \quad J=1, N R$
$A J=J-(N R+1) / 2$
If ( 4 I $* * 2+4 J * 2 . G T . R 2) \mathrm{GO} T O 1$
NSEO $=$ NSEO +1
ISEQ( $\mathrm{H}_{\mathrm{T}}$, NSEQ $)=\mathrm{NR} *(\mathrm{~J}-1)+\mathrm{I}$
1 continue
2 PIXL(NT) = NSE日
$c$
c
READ INITIAL COORDINATES, SET BACKGROUND TO CLASS '3'
READ (8) MREC, NT, (SAMP(I), SILE(I), CLASS(I), I = 1, NT)
MATCHS $=$, TRUE,
MIDREK $=N R / 2+1$
NREC? $=$ NKECS $-N K+$ MIDRON - 1
DO $7 \mathrm{~J}=1$, NR
007 IEL=1,NSAMP
$7 S(J, I E L)=3$
$c$
c WRite output file of reccistructed trees
DJ 10 NREC $=$ MIDROW, NREC 2
if (NREC.NE MREC) IO TO 95
DT 180 MT $=1, \mathrm{NT}$
IAREA $=$ PI XL(SIZEE MT) )
$J S=V R *(S A M P(11)-1-N R / 2)$
20 7E USEU $=1$. 1 AREA
$J J=J S+$ ISEU(SILE (AT), NSEO)
75 STIJN) $=\operatorname{CLASS}(M T)$
130 CONTINUE
C IF (MATCHS) KEAD $(8, E N O=30)$ MKEC, NT, (SAAP(I), SIZE(I), CLAJS(I),
$.1=1, N(T)$
G. Tu 95

32 MATCHS $=$. FALSE,
c
95 DU $\in-J W=2, N K$
Di) 6 IEL=1, V SAMP

- 6-S $\left(J_{n}-1,1 E L\right)=S(J d, I E L)$

OH 8 IEL $=1$, NSAMP
8 S(NR. $1 E L$ ) $=3$
10 WRITE (11) (S(1,IEL), IEL=1,NSAMP)
$c$
DO $200 \mathrm{~J}=1 . \mathrm{GK}$
200 .nRITE (11). (S(Jd.IEL), IEL=1,NSAMP)
KETURN
END

## APPENDIX C

The following subroutines were used in performing the supervised classifications:

SUBLPC - Compute Gaussian statistics of training samples
PTEST - Test maximum likelihood classification of training samples
PCLASS - Handle I/O for classification of the data
MALICA - Maximum likelihood classifier
EFFECT - Determine class to be tested for linear classifier using training samples
SNOPAL - Compute discriminant function coefficients for one class
NTEST - Test linear classification of training samples
NCLASS - Handle I/O for classification of the data
NOPACA - Nonparametric (linear) classifier

## SUBROUTINE SUOLRE (LC, MC, HS, CLASS, S, EM, EK, B, NN, MMI

SUPERVISED BAICH LEARNING TF PARAMETERS
DIMEFSIOY MC(11), NS(GM,3), EM(NN,MM), EK(NN,NN, NMI, B(MM)
DUURLE PRECISIC: CLASS(:A), S(NN, XN), DET


- 'MEAL VECTuKS':10X, 'Civa \&IAMC MATRICES')
4554 FUKMAT ( $=15,2,15 \times, 16$ F7. 21$)$
4557 ELIRMAT (/2うX, 'OETERMINAVT = 1,1PE10.3)
4577 FDRMAT (//I2U.A1U.IG,' SAMPLES')
c compute mean vectors and covariance matrices
NRITE 16,4550$)$
0: $1110 \quad 13=1,414$
NC11 $=\operatorname{NS}(13,6)$
$N C 12=\operatorname{NS}(13,3)$
D2 $900011=1$, ..
EM(IL, I3) $=6.2$
D5 $4000 \times 51=$ NC11.11612

$\operatorname{sCO} \operatorname{sm}(11.13)=E: 4(11,13) / \operatorname{NS}(13,1)$
$c$
on $9200 \quad 11=1$, wit
D- $9200 \quad 13=1,11$
$\mathrm{F}_{\mathrm{K}}(11,14,1=1=0.0$
DO O 120 NS $1=1611,1+C 12$


EK(11, I2,I三) =EK(11,12,13) / (1:S(13,1)-1)
$s(11,12)=\operatorname{Ex}(11,12,13)$
s(12.11) $=s(11,12)$
gzNO CRNTITUE
c
WRITE 16,4577$)$ (3, CLASS(IS), NS $(13,1)$
$00.5008 \quad \mathrm{IL}=1$.NN
5RO3 PKIMT 4554, EM(12,13), (EKM11, 12,13), $12=1,11)$
$c$
$c$
$c$
c
CALL CASIIV (S. Sive UET)
D $1000 \quad 14=1,6 \mathrm{~N}$
DC $1030 \quad 1<=1$, iN .
1CO EK(11,12,13) $=$ S(11,12)

1110 WRITE $(6,4557)$ DET
REIUR 4
END

```
C
    SUEPLUTINF PTEST (LC, NC, NS._CLASS, FM, EK, E, NN, NN)
C CLASSIEIES KILMN DATA SAMPLES - PAFAPETRIC CLASSIFICATIGN
            DIMENSILN N(C(1), :S(:*,こ), X(zこ), FP(1), 「K(1), ?(1), KS(22)
            OLLLRLE PRECLSIL:N CLASS(SM)
                    LGICDL* LC(:, 1)
            104 FI,RMAT (// 3UX,1SHAVENAGL A(CURACY =,F6.1,8H PEK(E5VT/3?X,32(1H*))
```




```
            - PEKCENT NU:ADER MF SAMPLFS CLASSIFIED AS'/6X,'CLASS SAM
            .PLES (CRKELT CNNRECT',1,AY/(43X,1OAG))
            2009 FURMAT (/14,A9,17,110,F12,1,1C19,(/42x,1019))
C
    PRINT 2COB, CLAJS
    TF = C.0
    00 1301 NC = 1,MM
    DC 1221 NN=1.MM
1221 KS(H%) =0
    ivSC = NS(NC,1)
    iNC1)=NS(NC,C)
    NC12 = NS(iNC,3)
C
    DJ 14CO NS 1=NC11,NC1L
    DC 15\capO NF=1,NN
1500 X(IVF) = LC(INF,NS1)
    CALL MALICA (X, ICLASS, KS, EM, EK, R, 1, NN, MM)
    1400 KS(ICLASS)= KS(ICLASS) + MC(NSI) - 1
    EFF=100.0 * KS(ITC) / NSC
    PQIN!T 2O\capG. NC, CLASS(NC), NSC, KS(NC), EFF, (KS(NC1),NCl=1,MM)
1301 TE = TE + EFF
    AVF=TE/ FLJAT(MM)
    PRINT 104, AVE
    RETURN
    END
```

c
SURRJUTINE PCLASS (S,MCLASS,CLASS,EM,EK, B, NRECS,NSAMP, NA,MM)

## c

c
CLASSIFIES UVKNJIN DATA SAMPLES - PARAMETRIC CLASSIFICATION
OIHENSIJN S(NV, NSAMP , MCLASS(NSAMP), EM(1), EK(1), B(1), KS(20)
OCUBLE PRECISION CLASS(1.M)
2010 FUBMAT $\left(111 / 30 \times 27(1 * 1) / 30 x_{0} 1\right.$ * RESULTS OF CLASSIFICATION $\# 1 / 30 \mathrm{X}$,


2011 FORMAT (/118,A9,120,F21.2)
$2 C 12$ FERMAT //14X,13HTOTAL SAMPLES, 120/14X,13(1H*))
c
DD $1415 \mathrm{NC}=1, \mathrm{MM}$
$1415 \mathrm{KS}(\mathrm{NC})=0$
DO_12 NREC $=1$, NRECS
(ALL MALICA (S, MCLASS, KS, EM, EK, B, NSAMP, NN, MM)
c
12 WRITE (111 MCLASS
NTOT $=$ NSAMP * NRECS
PRINT 2010
On $1301 \mathrm{irC}=1, \mathrm{M} / \mathrm{M}$
$P C T=100.0 *$ KS(NC) / NTOT

PKINT 2012. NTOT
RETURN
END

DD 2260 NS $1=1$, NSS
GMAX $=$ GMAXX
DC $1900 \quad I=1 . \mathrm{MM}$
$G=B(I)$
D) $63 \mathrm{CO} \quad 11=1, \mathrm{NN}$
DX(II) $=X 1(I I, N S I)-E M(I I, I)$
SUM $=0.0$
DO $6200 \mathrm{JJ}=1$, I I
$A=D \times(J J): E K(J 」, 1 L, 1)$
6200 SUM $=$ SUM $-A$
$63 C 0 G=G+($ SUH $+0.5 * \mathrm{~A}) *[\times(11)$
IF (G.LT.GMAX) GO TO 1900
KMAX $\operatorname{KNS1)=I}$
GMAX $=6$
1900 CONTINUE
$2000 \mathrm{KS}(K M 4 X(N S 1))=K S(K M A X(N S 1))+1$
RETURIY
END

```
c
SLRQCUTINF EFFECT (LC, NC, IIS, CLASS,NUC, DS, NVM, MN, N:11)
c
EFFECTIVE FIGUKE QF IERIT FEATUKF SELECTICN CKITFRION
c
```



```
        LCGICAL*1 LC(%AN.1)
        OCUALF PRECISING CLASS(PN)
    1CO FORMAT ('1'/20X,'EFFECTIVE FIGURFS OF NERIT'/20x,26('%')//)
    143 FOKMAT (15,AG, 5x,16F7.4/(19X,16F7.4))
    ISC FRRMAT //LCX,'CDMSINED FIGLKES CF MERITT/LCX,2E(1*1)/)
    151 FORIAAT (125,49,F10.5,5X,1315)
c
c CJMPUTES INTER-CLASS AND INTRA-CLASS DISTANCES
C
        PRINT 100
        IF (NW1.NE.1) GO TO 2000
        OC 1005 NF1=1,NN
        DO 2 [1 = 1,MM
        NC11 = NS(11,2)
        NC12 = NS(I1.3)
        CO 2 I2=1,I1
        NC21 = NS(12,2)
            NC22 = NS(12,3)
            DE(I1.I2.NF1) = 0.0
            DO 3 LK1 = NC11,NC12
            IF (I1.FO.I2) NC22 = LK1 - 1
            OD 3 LK2 = NC21,NC22
            3 DF(I1,I2,NF1) = DÉ(I1,I2,NF1) + IABS(LC(NF1,LK1)-LC(NF1,LK2))
            *#AC(LK1)#MC(LK2)
            DE(I2,I1,iNF1)= DE(I1,I2,NFI)
        10J5 CONTINUE
            DD 1109 NC=1.MM
    1109 MOC(NC) = NC
    C
    C CIMPUTES THE NQRMALILED FIGURE DF MFRIT DF ALL REMAINING PATTERN
    C CLASSES ALONG EACH OF THE FEATURE DIRECTIUNS
    C
    2CDO DO 1000 J3=Nwil.MM
        CFM(J3) = 1.0
            I3 = MjC(J3)
            Ai3 = N5(13,1)
            DD 3CCO I=1.NN
            FCMIN = 1.0 E 50
            SUM1 = C.O
            SUM2 = 0.0
    C COMPUTE SUM1 - TOTAL OF INTERCLASS DISTANCES FROM CLASS I3 TO ALL
    REMAININS CLASSES
    D2 6- J4 =Nm1,MM
    IF (J4.EQ.J3) GŨ TU 6
    14. = MOC(J4)
    A14 = NS(14.1)
    NST =NST + NS(14,1)
    SUM1 = SUM 1 + DE(I3,I4,I)
    c. COMPUTE SUM2 - TLTAL DF DISTANCES AMTNG ALL REMAINING CLASSES,
    C EQUIVALENT TO INLNACLASS DISTALGE LF ALL EESAIUI:GG CLASSES CONSIDERED
```

```
C AS TNE CLASS
```

    \(007 \quad J 5=1.1, \mathrm{~J} 4\)
        IF (J5.EA.J3) UJT」 7
        \(I 5=4 \operatorname{ll}(J 5)\)
    
7 CLNTIIUE
$S 1=D E(13,14,1) /(A 13 * A 14)$
$S 2=C E(I 3,1 s, 1)$ ( $A 1 j \neq(A I S-1.0))+D E(I 4, I 4, I) /(A I 4 *(\bar{A} I 4-1.0))$
$F=S 115<$
IF (F.IT.FCHI, $)$ FCiMI: $=F$
6 CONTIMUE
C
$\triangle N S T=1 ; S T$
SUM1 = SUT1 $/(A I 3 * A, S T)$

FC(I) = FCMIN \% JJ.1 / SUW2
FC(I) $=$ Exp $(-1.0 / r C(I))$
C CFNPUTE CFM, CUIPII:ED FIGURF TFF NERIT, AND QRDER BY CFM TO CETERMINE
C THE NLST SEPAINAOLE LLASS
C 3 (CO CFM $(J 3)=$ CFM(J) $)=F \bar{F}(I)$
$C F M(J 3)=C F B(J 3) * *(1 . C / A . N)$
WRITE $(6.143)$ 13. CLASS(13), (FC(ivF), NF=1,NN)
1 COO CONTINUE
(ALL SURTLS (CFB, MUC, NWI, MM)
C
WRITE (6.150)
$00 \quad 1251 \quad \mathrm{NC} 1=\mathrm{Na} 1$, MM
$N C=M O C(i v C 1)$
1251 WRITE (6.151) NC. CLASS(NC), CFM(NC1)
CALL SORTSL (MUC, MUC, Ninl +1, AM)
RETURN
END

```
C
        SUBRDUTINE SNOPAL ILC,MC,NS,CLASS,h,MCC,S,Y,B,NW1,NN,MM,NN1,MM1)
C SUPERVISED NIN-PARAMETRIC LFARNING
C
    UIMFNSION HC(1), NS(INM,3), W(:H1,:N1), MCC(MM), Y(1), S(1)
        LCGICAL*1 LC(iv.f.1)
        CRUPLE PRECISIJNCLISSSNN), S(NN1,NM1), RFT
        LCGICAL TEST
        CATA HI /lCO/
        99 FOKMAT (/1PGE15.4)
    100 FERMAT (18,111,I8,4X,1P7E14.3/(31X,1P7E14.3))
    101 FURHAT (/113,A1C,10X,1P7L14,3/(33x,1P7E14,3))
```



```
        .RATIUN NO.',5x,'ERRUKS',10X,'LINEAF. DISCRININAINT COEFFICIENTS'/
        .AL2.' CTHER'//
        216 FRRMAT ('I'/10X, 'URUEREO CLASSES', 20X,'ELENENTS CF THE DISCRIMINAN
        .T VE(TOR'/10X, 15('*1),?OX,25('*1)/)
        220 FGRMAT (/7X,'TUTAL ERRURS',I5)
C
    INITIALIZE W, Y, AND O ARRAYS
    NK = MOC(NW1)
        NSN1 = NS(NW,2)
        NSW2 = NS(NW,3)
        NW2 = NW1 + 1
        DELTA = iN/4C0.0
        DC 1112 NFA=1,NN1
    1112 W(NW 1.NFA) = 0.0
    J=0
    NST2 = 0
    DC 1110 NC1 = N%1,MM
    NC= NCC(NC1)
    NST2 = NST2 + NS(NC,1)
    NS11= NS(NC.2)
    NS12 = NS(NC,3)
    DL 1110 NS =NS11,NS12
    J=J+1
    Y(J) =-1.0
1110 B(J)}=1.
C
C CEMPUTE INVEKSE UF AITRANSP(SE) A WHERE AA' IS AUGNENTED MATRIX UF SAKPLES
    On 130 I=1.NN
        CC 1 jO j=1 iNNL
        S(I, J)=0.0
        CO 131 NCI=N%1.MM
        \C= MGC(IVC1)
        N511 = NS(NC,2)
        NS12=NS(:C,3)
        CO 131 N'SI=NSS11,NS1L
        K1 = LC(I,NS1)
        IF (J,NE,Nivi) Kl=KI#LC(J,NS1)
    131S(I,J) 三S(I,J) + MC(NS1)*K1.
    130 S(J.I) = S(I.J)
        S(HN1,NN1) = VST2
        CALL CASINV (S, NN1, DET)
    C DG VI ITFRATI JNS SF THE HU-KASHYAP ALGORITHM, UNLESS ALL EOEFFICIENTS
    C CHANGE BY LESS THAN DELIA = NN/4_PEKCENT
```

```
    r
    PRIVT 1C2. N.., CLASS(inn), (LASS(Nn)
    DN 1r 4C 1. [: }x=1..
    TEST = .TRUE.
    C
    0.j 1101 1=1,N*N
    nO=n(in 1.1)
    J = 0
    D.] 2101 i:S1=ivjn1,i,5m2
        J = J + 1
        \Delta2 =S(I,NN1)
        0) 140 K=1,\mp@code{N.}
    14OA2=A2+S(I,K)#LC(K.NS1)
```



```
        DI 20On NC 1=6w+2,M:1
        INC = MJC(NC1)
        NS11 = NS(NC.2)
        NS12 = NS(NC,3)
        0J 2000 NS 1=, S 11.NS1.2
        J=J + 1
        \Delta2 =S(I,I.N1)
        Di) 141 K=1, Niv
        141 A2 = A2 + S(1,K)*LC(K,NS1)
        2~00 W(NN1,1) = (NW1,1)-1C(NS1)=A2%4BS(Y(J))
        IF (ABS!a(lim1,I)-a0).GT.ARS(DELTA*NO)) TFST=,FALSE.
    11כ1 CONTINUE
C CJMPUTE NEW UISCRIMIVANT VALUES AND CLASSIFICATION ERRIRS
        NERR1 = 0
        I}=
        OO 1CC4 NSI=NSW1,NSW2
        I=I+1
        IF (Y(I).CT.0.0) B(I) = E(I) + 2.0*Y(I)
        Y(I) = W(N*I,NN1)
        DO 1141 NF < =1,NN
    1141Y(1)=Y(1) +N(NW1,HF2)*LC(NF2,NS1)
        IF IY(TI.LE.O.O) I.ERF1 = NFFR1 + NC(NST)
        1Ci& Y(I) = Y(i) - O(I)
        \thereforeEF-2 = U
        DO 1CC5 liC I=,N2.IMM
        NC = NC(INCl)
        NS1i = NS(NC,2)
        NS12 = NS(NC,3)
        OO 1CC5 NS =NS&1,NSIL
        I=1 1 1
        IF (Y(I).GT.0.0) 8(1) = R(I) + 2.0&Y(I)
        V(I) = W(NW1,NN1)
        DC 145 NF2=1,N'N
        145Y(I)=Y(I) + W(NW1,i,F2)*LC(IVFE,NSI)
            IF (Y(1\.6T.0.0) NEKRC = :IFFRC + "(1,51)
    -1:05-v(I) =-v(I) - E(I)
            PRINT 1CO, INUEX, NFNKL1, JFRR2, (S(I:V1,NFA), NFA=1,NN1)
            IF (TESTI GJ TJ 1210
            1^\angleO CONTINUE
    1010 NERK = NENK1 + NERR2
    PRINT < 2C, NENK
    C
```

    IF (iNHI NE. A1:1) KETUKN
    $c$
SUENDUTIME NTEST (LC, NC, NS, CLASS, K, RLC, YN, YM)
c
C CLASSIFIES RNGNN DATA SAAPLFS - WONPARAIETKIC CLASSIFICATIUN
DI'ENSIEN. $\therefore C(1), N S(N, 2), x(2 C), \ldots(1), 1, C(1), k s(2 C)$
LRGIC4L*1 LC(iviv.1)
DIUBLE PXECLDI : CLAこS(N:M)



-PLFS CORKECT CUARFCT', 1 AAG/(43x,1OA9)
2009 FCRHAT (/14.2ヲ.17.111,F12.1.1(19/(41x,1019))
$\mathrm{NH}_{1}=\mathrm{NN}+1$
$1 / 141=\operatorname{RM}-1$
PRINT 2CO3. CLASS
$T E=0.0$
DC $1301 \quad N C=1 . M 4$
DO $1109 \quad 1=1, \mathrm{MM}$
$1109 \mathrm{KS}(1)=0$
NSC $=$ NS (INC. 1$)$
NC11 $=$ NS(NC, 2)
NC12 $=$ NS(NC,3)
$c$
DO 14CC NS $=$ NCIII,NC12
DO $1500 \mathrm{NF}=1, \mathrm{NN}$
$1500 \times(N F)=L C(N F \cdot N S 1)$
CALL NGPACA $(X$, ICLASS, W, NOC, 1, NN, NN1, MM1)
1400 KSIICLASS) $=$ KS(ICLASS) + MC(NSI)
$c$
EFF $=100.0$ K KS(NC) / NSC
PRINT 2(O9, NC. CLASS(NC), NSC, KS(NC), EFF, $(K S(N C 1), N C 1=1, M M)$
$1301 \mathrm{TE}=\mathrm{TE}+\mathrm{EFF}$
$\triangle V E=T E / F L J A T(M M)$
PRINT 104. AVE
RETURN
fNo

    \(C\)
    
    SUBRCLTINE NLLASS (S, HCLASS, CLASS, W, MOC, NRECS, NSAMP, NN, MM)
    C
    C NONPARANETRIC CLASSIFICATICN
    C CLASSIFIES DATA SAIFPLES AI:D STCRES CLASSIFICATIUN RESULTS CN TAPE
    c
            DIMFNSIEV S(INN,NSAMP), MCLASS(NSAMP), W(1), MOC(1), KS(20)
            DRUBLF PRECISION CLASS(IAN)
        2010 FDRYAT ('1'/30X,29('*1)/30X, " RESULTS OF CLASSIFICATICN *'/30X,
    

2011 FDRMAT $1 / 117.49,120, F 22.21$
2C12 FERMAT (/1 $3 X, 1$ OHTUTAL SAMPLES,120/13X,13(1H*))
C
$\mathrm{NN} 1=\mathrm{NN}+1$
$M M 1=M N-1$
DC $1 C C 9 \quad N C=1,1 \mathrm{M}$
$1009 \mathrm{KS}(\mathrm{NC})=0$
$c$
CD 12 NREC $=1$, NRECS
READ (10) $S$
CALL NCPACA (S, MCLASS, KS, W, MOC, NSAMP, NN, NN1, MM1)
12 WRITE (11) MCLASS
C
NTOT $=$ NSAMP * NRECS
PRINT 2010
DO 1221 NC $=1, \mathrm{MM}$
$P C T=100.0$ \% KS(NC) / NTUT
1221 PRINT 2011. NC, CLASS(NC), KS(NC), PCT
PRINT 2012. NTOT
RETURN
END

C NDN-PARAMETRIC CLASSIFICATICN CF A STRING OF NSS FEATURE VECTORS
C USING PDE-LEARNED LINFAR GIJCRIMIAVANT FUNCTIGNS
C

## DIMENSICN X(NV, $\left.{ }^{\prime} S S\right)$, Ih(NSS). W(NiN1,MM1), MOC(1), KS(1)

$c$

## 0020 NSI $=1.1 .5 S$

$D 0 \quad 1 \quad N_{n}=1,141$
$G=h(N N 1, \operatorname{Ninl})$
DI 2 AF $1=1, \mathrm{M}$
$2 u=G+n(+F(, i n 1) \neq x(N=1, N S 1)$
IF (G.GT.C.O) GTT T 3
1 COMTINUE
$\mathrm{Nw}(\operatorname{NS} 1)=\operatorname{MuC}(\operatorname{Na} 1+1)$
GC TC 2 C
3 NW $(M S 1)=\operatorname{MiCl}_{1}(\ln 1)$
20 KS(Nin(NS1)) $=\operatorname{KS}(\operatorname{tin}(\mathrm{ivS}$ (1) $)+1$
RFTIRN
END

