Objectives for Tutorial on Attenuation of Radiation

1. Define attenuation of radiation, half-value layer, and tenth value layer.

2. Explain what happens to intensity of a beam as one removes or adds HVLs or TVLs between source and detector

3. Using graphical analysis, determine HVL or TVL from a semi-logarithmic plot of Intensity vs thickness.

4. State mathematical relationship between μ (the linear absorption coefficient) and the HVL and compare to the mathematical relationship between λ (the decay constant) and the half-life

5. Write the equations for calculations involving intensity changes as a function of the number of HVLs and TVLs

6. Define monochromatic and polychromatic and explain why every HVL is equally thick for monochromatic beams, but not for polychromatic beams.

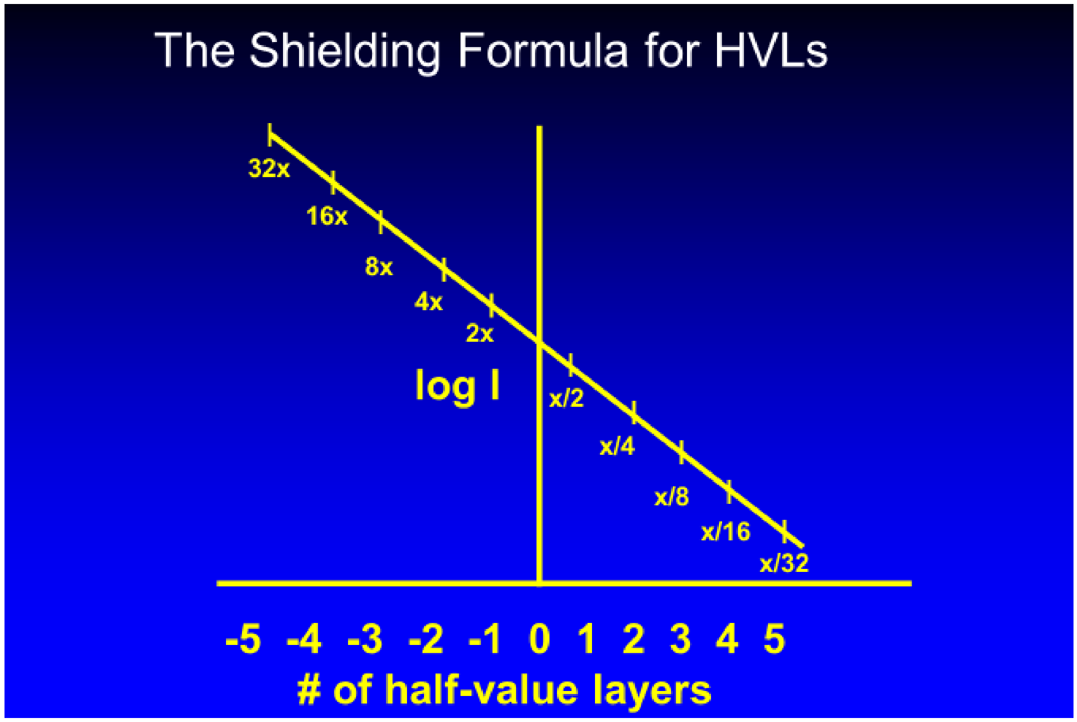
7. Work a variety of math problems involving change of beam intensity with change of number of HVLs or TVLs.

Attenuation Of Radiation

The Shielding Formula

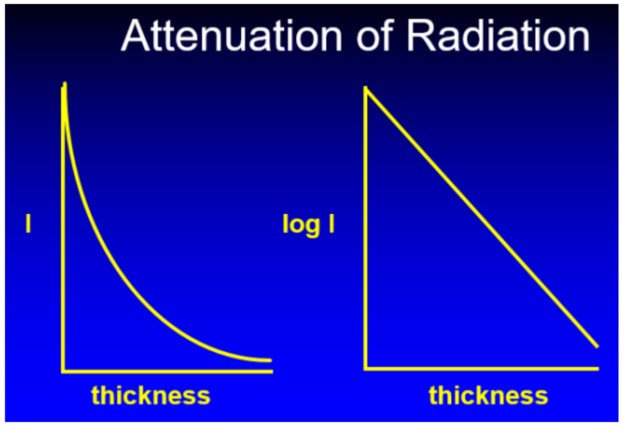
Attenuation of Radiation

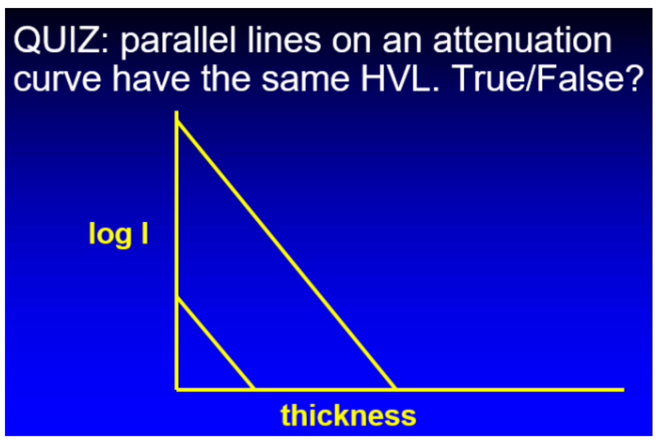
1. *Attenuation* follows an exponential law and is described in terms of the *half-value layer*, the thickness of an absorber that reduces beam intensity to 50% of its original value.
2. *Half-value layer*: Once we know how many half-value layers are in place, we no longer care about the thickness or composition of the absorber.
3. After one half-value layer is placed between source and detector, 1/2 of the beam intensity will remain; after two half-value layers are placed between source and detector, 1/4 of the beam intensity will remain; after three half-value layers are placed between source and detector, 1/8 of the beam intensity will remain; refer to graph below
4. After one half-value layer has been removed from between source and detector, the beam intensity will double; after two half-value layers have been removed from between source and detector, the beam intensity will quadruple; after three half-value layers have been removed from between source and detector, the beam intensity will octople; refer to graph below
5. Graph of radioactivity as a function of absorber thickness

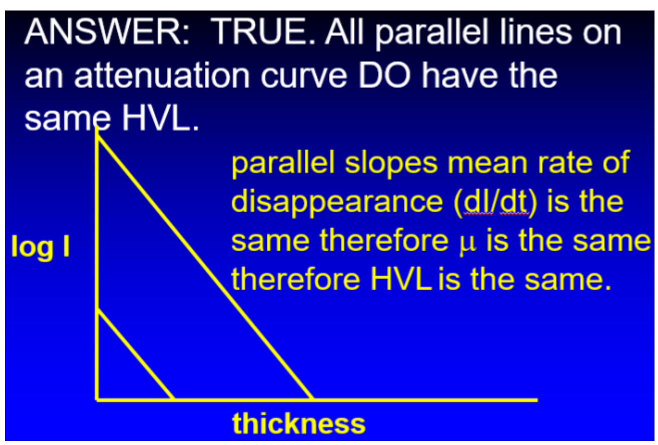


The Shielding Formula for HVLs
32x
16x
8x
4x
2x
log I
x/2
x/4
x/8
x/16
x/32
-5 -4 -3 -2 -1 0 1 2 3 4 5
# of half-value layers

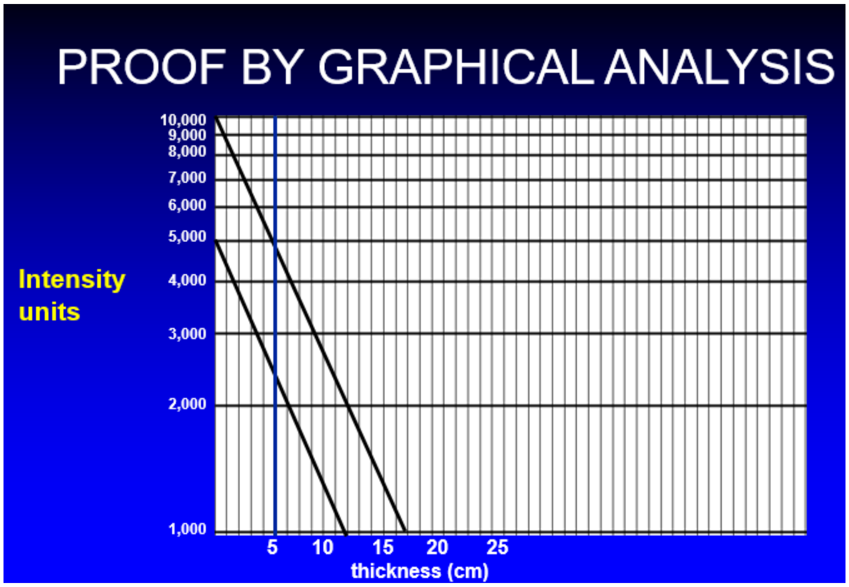
6. Attenuation plotted on both linear paper (left curve) and semilog paper (right curve). Reading values off either curve will yield same values, but attenuation is logarithmic and therefore it is preferable to plot thickness/intensity curves on semilog paper.







7. Proof by graphical analysis



For either line, the HVL = 5 cm.

For the curve at right, at thickness 0, the activity is 10,000 intensity units. If 1 HVL is in place, the reading will drop to is 5,000 intensity units which intersects the curve at 5 cm.

For the curve at left, at thickness 0, the activity is 5,000 intensity units. If 1 HVL is in place, the reading will drop to is 2,500 intensity units which intersects the curve at 5 cm.

Basically, one beam is more intense than the other, but the fraction attenuated per unit of thickness is identical for both and therefore the HVLs must be equal.

QUIZ

Which stops more Tc-99m energy, 1 HVL of Pb or one HVL of Al?

ANSWER: they are the same- by definition a half value layer of any material stops exactly as much as a half-value layer of any other material. For a given radioisotope, regardless of the composition and thickness of the absorber, each half-value layer absorbs 50% of the radiation. Attenuation is therefore directly dependent only upon # of HVLs, not the composition of the absorber.

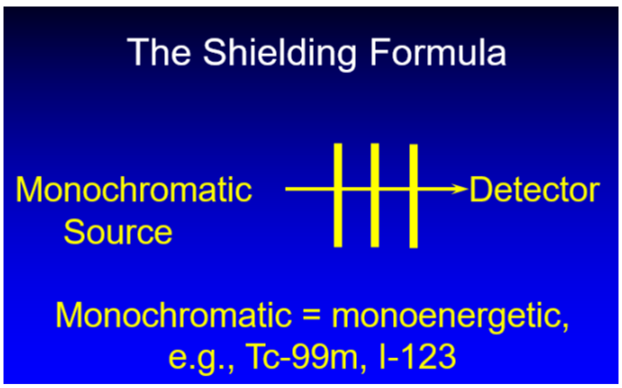
For Tc-99m, one Half-Value Layer of Lead = 0.027 cm and one Half-Value Layer of Water = 4.60 cm. That means that water needs to be 170.37 times as thick as lead to absorb an equal amount of energy from Tc-99m. For other photon energies, the HVLs would vary significantly.

Note the similarity between decay and attenuation:

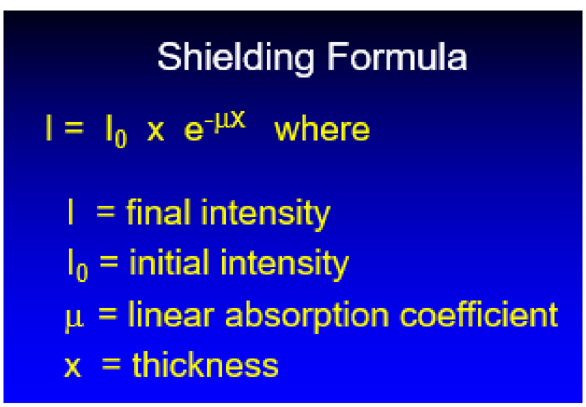
$$\lambda = \frac{0.693}{t_{1/2}} \qquad \mu = \frac{0.693}{t_{1/2}}$$

the mathematical relationship for each is the same; the principal difference is that one is the half-life and the other is the half-thickness. Conceptually, they are identical.

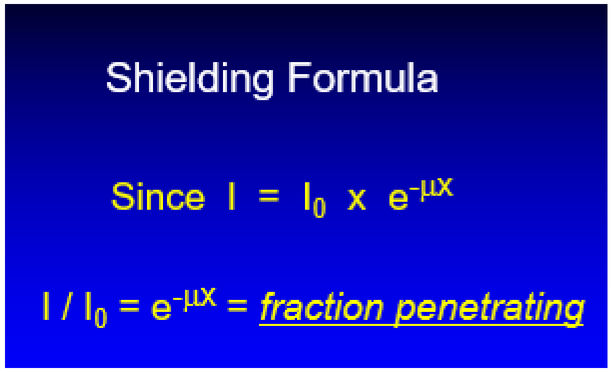
The important thing to note is that, for a monoenergetic beam, each half value layer is identical to every other half-value layer, assuming that they are all composed of the same absorber.



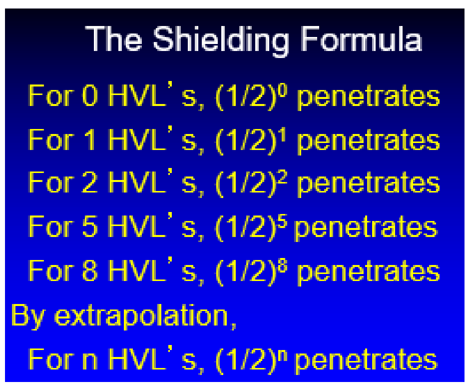
8. The time-honored equation for calculating intensity as a function of thickness is shown below**.**



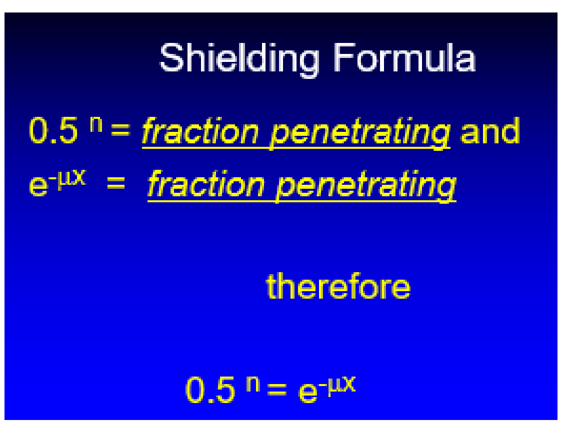
If we solve for $e^{-\mu x}$, we see that it is equal to the fraction penetrating.



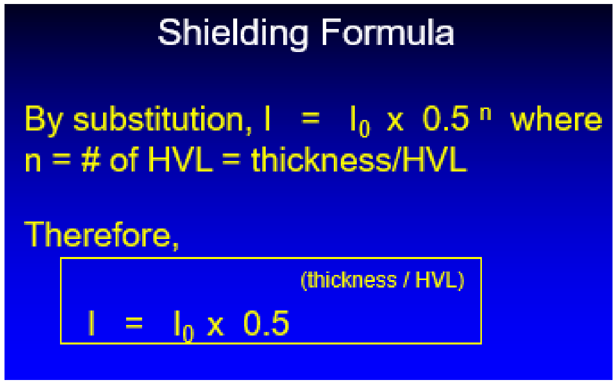
Using a logical approach to understanding attenuation, as displayed below, we see that the fraction penetrating is also equal to 0.5^n

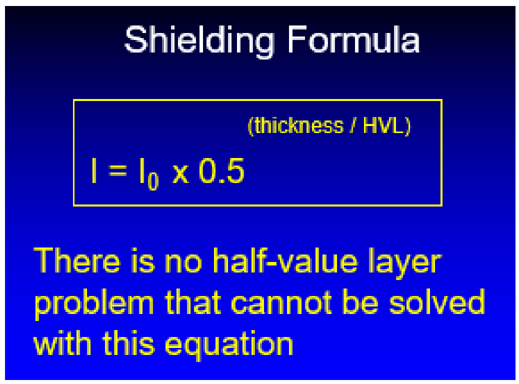


Therefore, as shown below, $e^{-\mu x} = 0.5^n$ where n = number of HVLs.



By substitution,

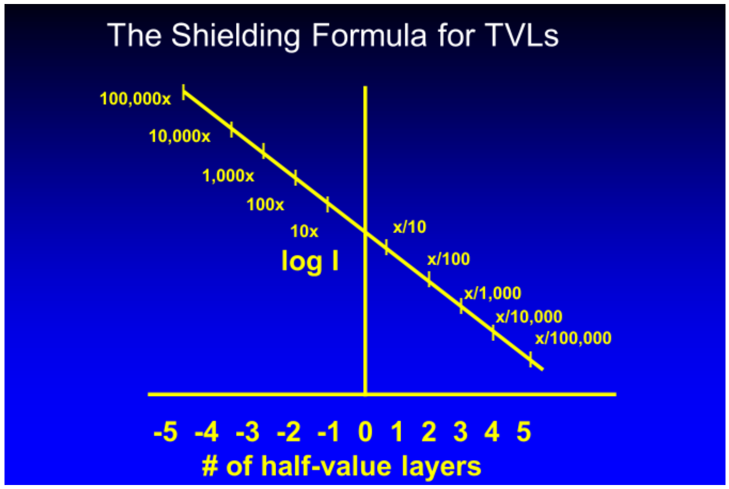




TENTH VALUE LAYERS

1. While we do not have to deal with a tenth life, physicists commonly use tenth value layers (TVL).
2. We define a TVL as the thickness of an absorber that reduces the intensity of a beam to 1/10 its initial value.
3. Once we know how many tenth-value layers are in place, we no longer care about the thickness or composition of the absorber.
4. If one tenth value layer is placed between source and detector, beam intensity drops to 1/10; for 2 tenth value layers, beam intensity drops to 1/100; for 3 tenth value layers, beam intensity drops to 1/1000, etc.
5. If one tenth value layer is removed from between source and detector, beam intensity increases by 10x; if two tenth value layers are removed from between source and detector, beam intensity increases by 100x; if three tenth value layers are removed from between source and detector, beam intensity increases by 1000x
6. <u>Attenuation</u> is directly dependent only upon # of TVLs, not the composition of the absorber

7. Graphical representation of attenuation as a function of thickness.



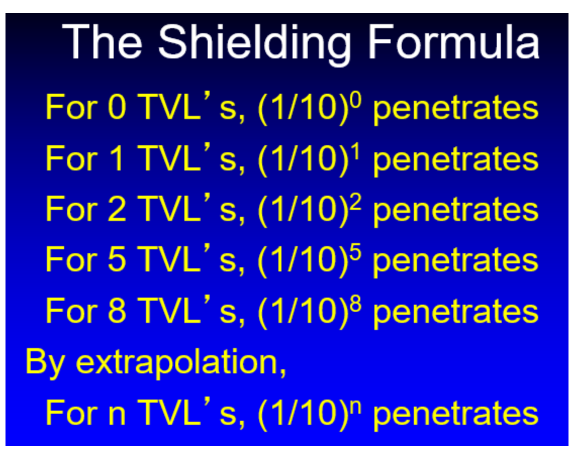
QUIZ: Which stops more Tc-99m energy?

1 HVL of Pb or one TVL of Al?

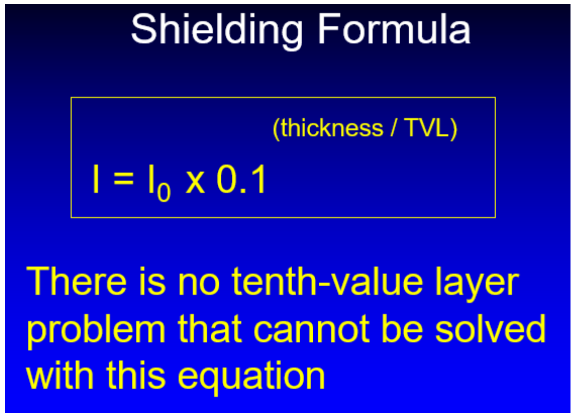
ANSWER

one TVL of Al. A TVL of anything will attenuate more radiation than a HVL of anything else.

Based on the following logical argument,



The Shielding Formula for TVLs is



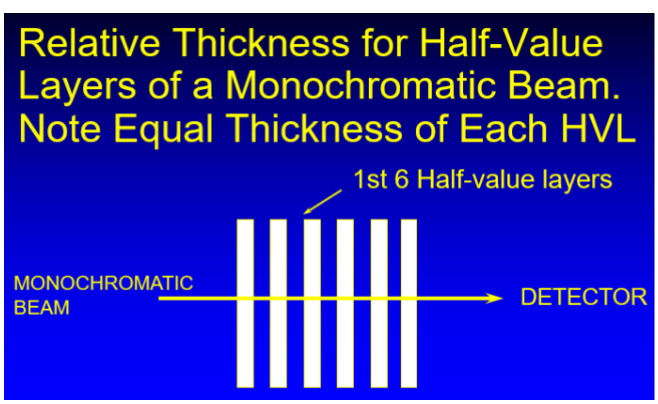
Therefore, when using HVLs,

$$I = I_0 \times 0.5^{(\text{thickness / HVL})}$$

And when using TVLs,

$$I = I_0 \times 0.1^{(\text{thickness / TVL})}$$

Relative Thickness for Half-Value Layers of a Monochromatic Beam. Note Equal Thickness of Each HVL



Comparative chart: HVLs and TVLs

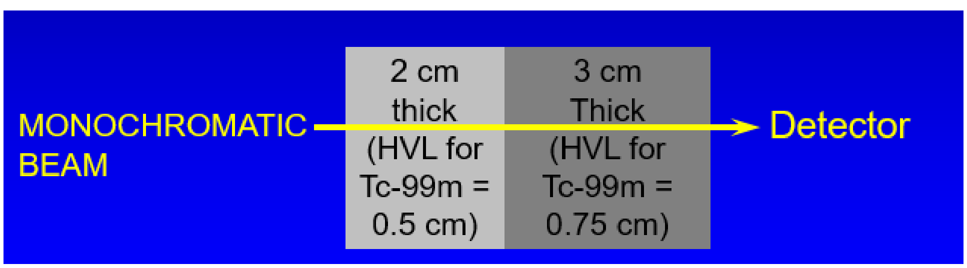
Intensity

Layers	HVL	TVL	LINEAR
0	1,000	1,000	1,000
1	500	100	500
2	250	10	0
3	125	1	----
4	62.5	0.1	----

Problem 1. Based on information in the chart above, 4 TVLs are how many times as effective as 4 HVLs?

Answer: effectiveness of 4 TVLs compared to 4 HVLs = 62.5/0.1 = 625.

Problem 2 What % of the energy of a beam can penetrate the 2 different bonded absorbers in the diagram below?



Answer:

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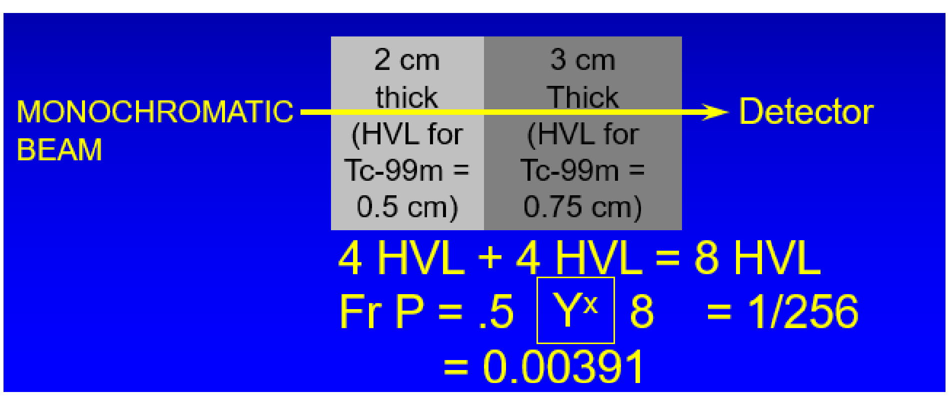
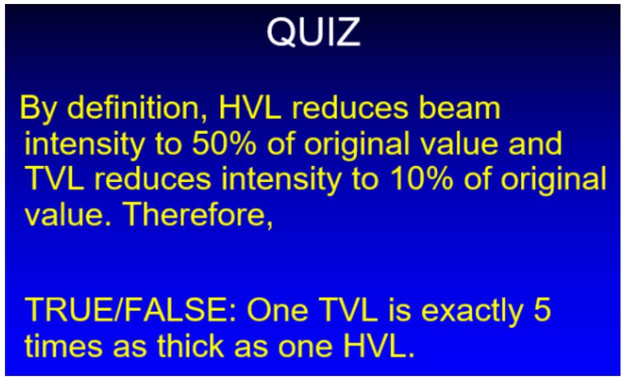
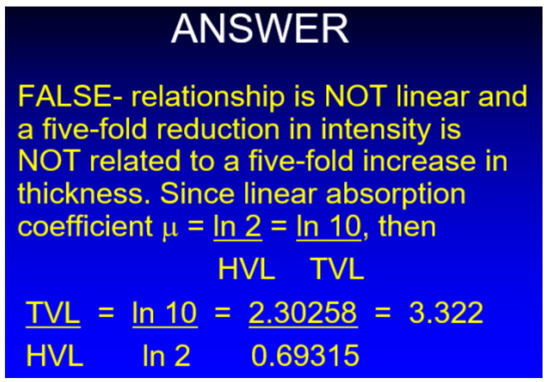


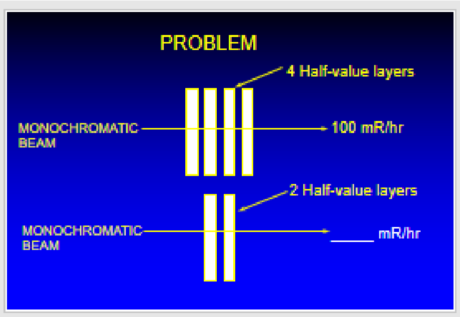
Problem 3.

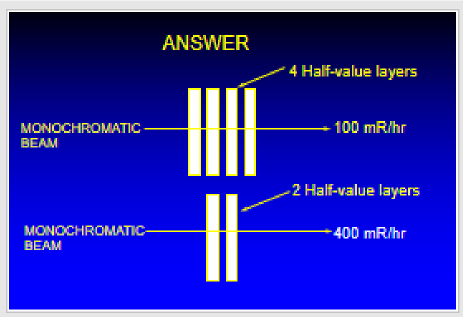


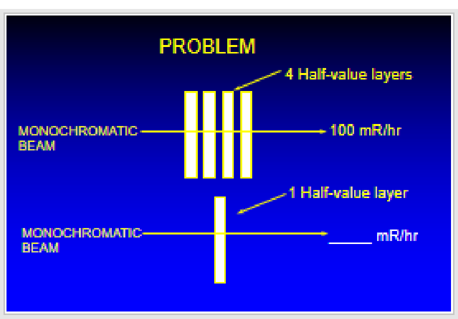
Answer:

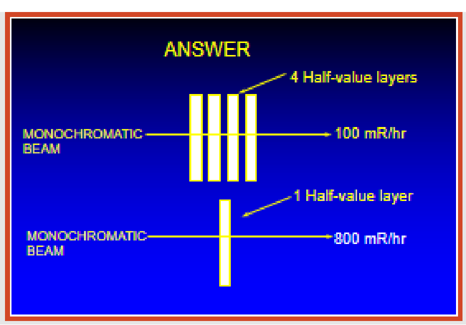


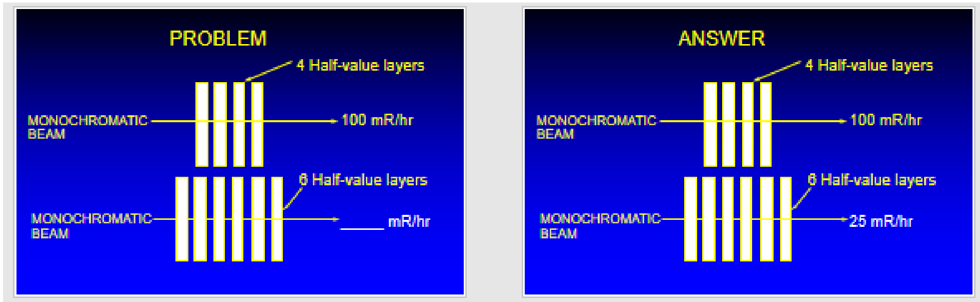
Pictorial Problems and Answers: Half Value Layers



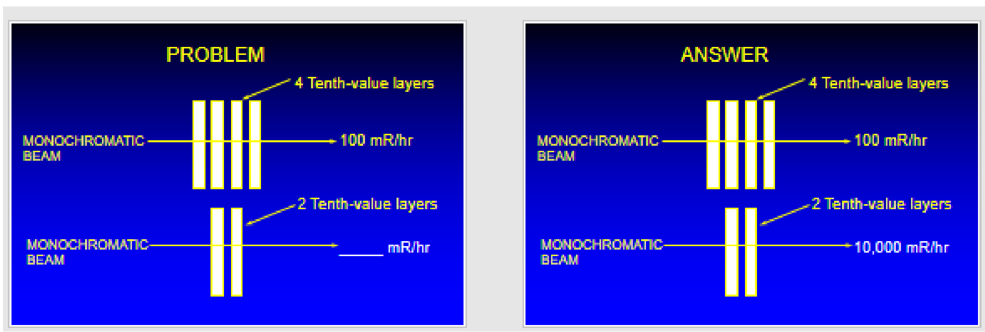


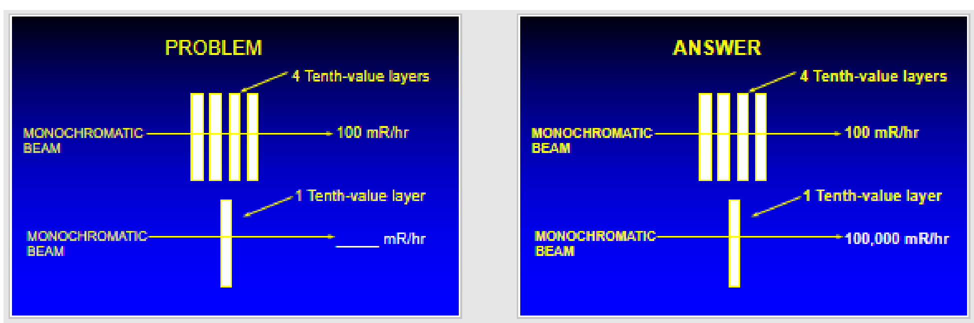


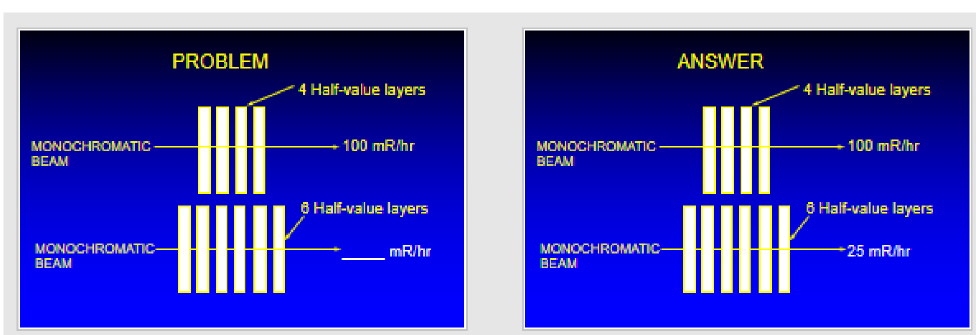




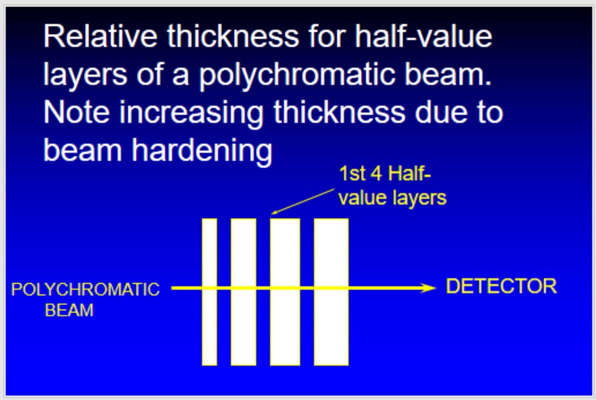
Pictorial Problems and Answers: Tenth Value Layers

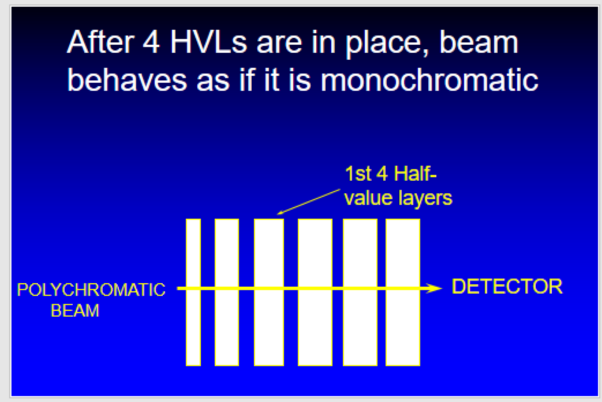


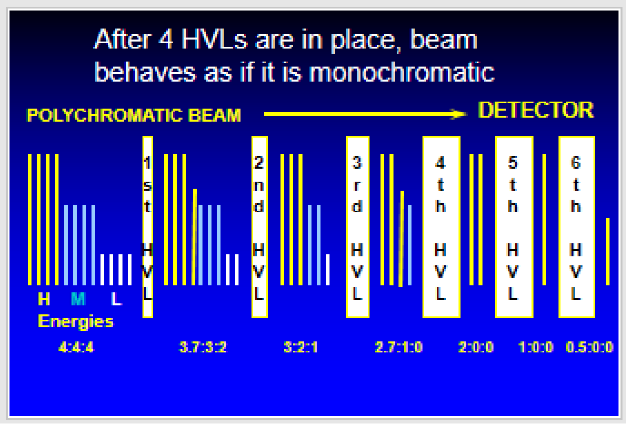




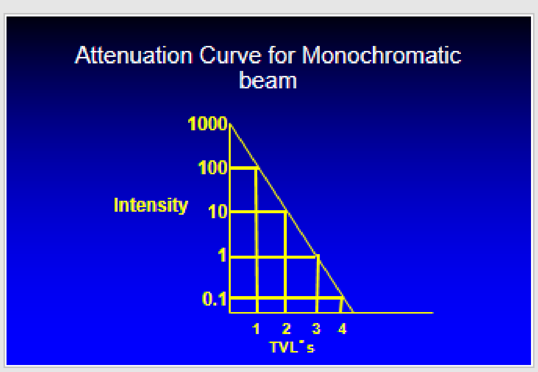
Polychromatic beams contain a mixture of low, medium, and high energies.

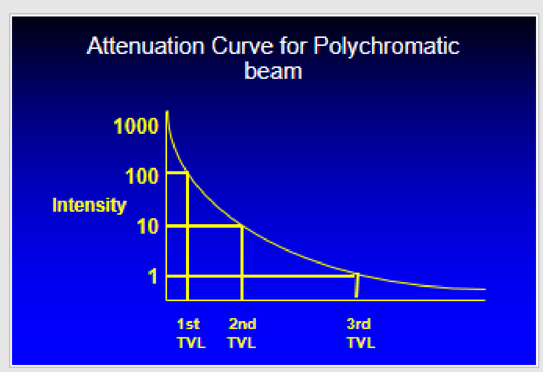






Attenuation Curves for Monochromatic and Polychromatic beams





Note that for monochromatic photons, all TVLs are equally thick; for polychromatic photons, layer 3 is thicker than layer 2, which is thicker than layer 1,

PROBLEM 1. If the HVL of lead for Tc-99m is 0.2 mm and a barrier is 0.3 cm thick, how many HVL's are present? What fraction of the beam can penetrate this barrier?

ANSWER # of HVL's = thickness/HVL

= 0.3 cm/0.02 cm = 3 mm/0.2 mm so

# of HVL's = 15

Fraction penetrating = 0.5 Y^x 15 = 0.00003

PROBLEM 2. If I_0 is 100 mR/hr, the HVL for a particular isotope is 4.5 mm, and the thickness of the barrier is 1 cm, what will be the intensity after the beam has passed through the barrier?

ANSWER

$I = I_0 \times 0.5^{(\text{thickness/HVL})}$

$= 100 \text{ IU} \times 0.5^{(1.0/0.45)} = 21.4 \text{ IU}$

Keystrokes: 100 x 0.5 Y^x (1.0 ÷ 0.45) =

PROBLEM 3. If the thickness of an absorber is 1.5 cm and 36.45% of a beam *is attenuated by* the absorber, what is the half-value layer?

ANSWER

Fraction Penetrating = 0.5^n

$0.6355 = 0.5^{(1.5\text{cm/HVL})}$

log 0.6355 =1.5 cm/HVL x log 0.5 and

HVL = 1.5 cm x log 0.5/log 0.6355

HVL = 2.29 cm

Keystrokes: 1.5 x 0.5 **log** ÷ 0.6355 **log** =

PROBLEM 4. If the tenth value layer (TVL) for a particular absorber/isotope system is 0.3 cm and the thickness of the absorber is 1.2 cm, what fraction of the original intensity will penetrate the absorber?

ANSWER:

Fraction penetrating = 0.1 $Y^{x\ (1.2/3)}$ = 0.0001

PROBLEM 5. If the thickness of an absorber is 1.5 cm and 36.45% of a beam ***is attenuated by*** the absorber, what is the tenth-value layer?

ANSWER:

Fraction Penetrating = 0.1^n

0.6355 = $0.1^{(1.5cm/TVL)}$

log 0.6355 =1.5 cm/TVL x log 0.1 and

TVL = 1.5 cm x log 0.1/log 0.6355

TVL = 7.62 cm

Keystrokes: 1.5 x 0.1 **log** ÷ 0.6355 **log** =

PROBLEM 6. The attenuation coefficient of an absorber is 0.0693 cm^{-1}. The original intensity was 100 mR/hr. What would be the intensity if a 10 cm barrier is placed between source and detector?

ANSWER: Since $\mu = 0.693/t_{1/2}$ then

$t_{1/2} = 0.693/\mu$

$= 0.693/0.0693\ cm^{-1} = 10$ cm

Therefore, 1 HVL is in place and

I = 100 mR/hr x 0.5 = 50 mR/hr

PROBLEM 7. A source has an intensity of 4 mR/hr. A barrier 4 cm thick is placed between source and detector; the reading drops to 0.4 mR/hr. What is the tenth-value layer of the absorber?

a. 0.4 cm
b. 1.2 cm
c. 4 cm
d. 20 cm

ANSWER: Correct answer is "c".

Logically, 4 mR/hr>0.4 mR/hr implies 1 TVL
so TVL has to be 4 cm.

Mathematically,

$I = I_0 \times 0.1^{\text{\# of TVL's}}$
$= I_0 \times 0.1^{\text{thickness/TVL}}$ therefore
$0.4 \text{ mR/hr} = 4.0 \text{ mR/hr} \times 0.1^{4\text{cm/TVL}}$ and
$0.1 = 0.1^{4\text{cm/TVL}}$. Taking log of both sides,
log 0.1 = (4 cm/TVL) x log 0.1. Canceling and rearranging,
TVL = 4 cm

PROBLEM 8. The first half-value layer (HVL) for a polychromatic beam is 3 mm of Al. The second HVL is _______________ the first.

a) Thinner than
b) Equal to
c) Thicker than
d) 1.44 x

ANSWER. Correct answer is "c". For a polychromatic beam, the first half-value layer reduces the original intensity reading to ½ its value. In doing so, it eliminates essentially all the low energy emissions, but only a small fraction of the high energy emissions. The net effect is to raise the average energy of the beam. This is called *beam hardening*. The 2nd half-value layer therefore has to be thicker than the first since the average energy of the beam has increased