NATIONAL BUREAU OF STANDARDS REPORT

9921

Progress Report

on

THE CRYSTAL STRUCTURES OF GAYLUSSITE, $CaNa_2(CO_3)_2 \cdot 5H_2O_3$

AND PIRSSONITE, $CaNa_2(CO_3)_2 \cdot 2H_2O$



U.S. DEPARTMENT OF COMMERCE NATIONAL BUREAU OF STANDARDS

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THE CRYSTAL STRUCTURES OF GAYLUSSITE, CaNa₂(CO₃)₂·5H₂O

AND PIRSSONITE, CaNa₂(CO₃)₂ ²H₂O

by

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U.S. DEPARTMENT OF COMMERCE NATIONAL BUREAU OF STANDARDS



The Crystal Structures of Gaylussite, CaNa₂(CO₃)₂·5H₂O and Pirssonite, CaNa₂(CO₃)₂·2H₂O Brian Dickens and Walter E. Brown

Abstract

The crystal structure of synthetic gaylussite has been determined from single-crystal x-ray diffraction data. The unit cell is $a = 14.361 \pm .002$, $b = 7.781 \pm .004$, $c = 11.209 \pm .002Å$, $\beta = 127.84 \pm$.01°, and the space group is C2/c. $R_w = (\Sigma(w||F_0| - |F_c||)^2)/\Sigma(w|F_0|)^2 =$ 0.043, R = 0.054. The hydrogen atoms have been located. Two CO₃ anions are coordinated to a Ca ion and form a dihedral angle of 134.3°. Each CO₃ group is coordinated to four Na ions and four water molecules, but to only one Ca ion. Each Na is coordinated to four CO₃ groups and two water molecules. Two water molecules form hydrogen bonds to neighboring CO₃ anions. The remaining water forms hydrogen bonds with the oxygens of two other water molecules.

The crystal structure of synthetic pirssonite has also been determined from single-crystal x-ray diffraction data. The unit cell is $\underline{a} = 11.340 \pm .004$, $\underline{b} = 20.096 \pm .005$, $\underline{c} = 6.034 \pm .002A$ and the space group is Fdd2. $R_w = 0.029$, R = 0.044. The hydrogen atoms have been located. As in gaylussite, two CO_3 anions are coordinated to a Ca ion but with a dihedral angle of 95.5°. In contrast to gaylussite, the CO_3 anions are also coordinated to a second Ca ion, as well as to four Na ions and two water molecules. Each Na ion is coordinated to four CO_3 anions and loosely to two water molecules. The water molecules complete the coordination of Ca ions and form hydrogen bonds with neighboring CO_3 anions.

INTRODUCTION

In our studies¹ of hydrated salts which have potential importance in biological mineralization, the crystal synthetic synthetic structures of/gaylussite², CaNa₂(CO₃)₂.5H₂O, and/pirssonite³, CaNa₂(CO₃)₂.2H₂O, have been determined and are reported here. Work on both these structures was completed before that of Corazza and Sabelli⁴ on pirssonite came to our attention.

Determination of the Structure of Gaylussite

Crystals of gaylussite were grown in beakers containing 100 ml water, 18 g Na_2CO_3 , 10 g $CaCl_2$ and 800 ppm of sodium polyphosphate, combining the procedures of Bury and Redd⁵ and Brooks, Clark and Thurston.⁶ The initial solid phase was mostly spherulites, probably of $CaCO_3 \cdot H_2O$ or vaterite. On standing, these dissolved and good single crystals of gaylussite were formed.

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A single crystal of gaylussite about 0.2 mm in crosssection ($\mu_{Mo} = 9 \text{ cm}^{-1}$) was sealed in a borate glass capillary to prevent slow dehydration. The cell dimensions^{*} were refined from 25 20 values observed on a diffractometer⁷ to $a = 14.361 \pm .002$, $b = 7.781 \pm .001$, $c = 11.209 \pm .002$ Å, $\beta = 127.84 \pm .01^{\circ}$, assuming λ (Mo K α_1, α_2) = 0.71069 Å, for the cell with space group C2/c or Cc and z = 4. The most obvious cell is body centered, with axial ratios which correspond to those given in Dana². The space groups C2/c and Cc were chosen using the convention adopted in the International Tables for Crystallography⁶.

The intensities of reflections in a hemisphere of the reciprocal lattice were measured on a diffractometer⁷ using Mo-K α radiation, an 0.001 inch Nb filter, and the peak-height method. The peak-to-intensity curve was established from 48 suitable reflections, spread uniformly over the 20 range,

*The uncertainties quoted on cell dimensions are standard errors computed from least squares refinements of the cell dimensions to fit observed 20 values.

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which were measured both by peak heights and by scanning. The data were merged into 2988 unique reflections, of which 2632 were of observable intensity. Since serious errors in peak height measurements are usually caused by measuring the background too near an adjacent peak, by slight misalignment of the crystal, or by absorption, in all of which cases the observed F_{hkl} would be too small, any supposedly genuine and equivalent values of Fhkl which were not within 10 % of one another were not averaged. Instead, the larger of the two was taken as the observed F_{hkl}. The discrepancy value, $\Sigma \|\mathbf{F}_i\| - |\mathbf{F}'_i| / \Sigma |\mathbf{F}_i|$, between reflections accepted as equivalent was 0.045, based on F's. (Although the anisotropy of the mosaic spread of the crystal affects the peak heights, in this case the peak height method provided data of sufficient accuracy.) No corrections for absorption were made.

The subsequent calculations were all performed using the crystallographic computing system (X-ray 63) assembled under the editorship of J. M. Stewart at the University of Maryland. The quasi-unitary structure factors⁹ ($\langle | E^2 | \rangle$ made equal to 1) indicate (see Table 1) that the space group is

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TABLE]
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Quasi-Unitary Structure Factor Statistics for Gaylussite

	< E >	$\langle E_{s} \rangle$	<pre>< E²-1 </pre>	>
All reflections	.816	1.000	.922	
3-dimensional reflections only	.803	.957	.887	
Theoretical, centric Theoretical, acentric	.798 .886	1.000 1.000	.968 .736	
	obs.	cent	ric	acentric
Fraction of E's > 1.0 2.0 3.0) .341) .036) .000		17 46 03	.368 .018 .0001

Number of reflections 2988

Suggested overall temperature factor 1.28

centrosymmetric, C2/c, instead of non-centrosymmetric, Cc. This choice was subsequently verified by the structure determination. The atomic scattering factors used were taken from reference 10, except for those of hydrogen, which were taken from reference 11. The quantity R_w =

 $\Sigma(w \| F_0 \| - \| F_0 \|)^2$ was minimized in the full matrix least squares refinements using a weighting scheme based on the counting statistics.

The structure was solved from the sharpened Patterson function, calculated from the (E^2-1) coefficients, and from subsequent F_0 Fourier syntheses. It was refined isotropically to $R_w = 0.079$ allowing the scale factor, the positional parameters and the thermal parameters to vary. The structure was then refined anisotropically to $R_w = 0.061$. The hydrogens were located from a difference synthesis. Inclusion of these hydrogens with fixed thermal parameters ($B_{\mu} = 1.0 \text{\AA}^2$) in the refinement decreased R_w to 0.043. The observed and calculated structure factors are given in Table 2.

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Observed	and C	alcula	ated S	tructu	re Fa	ctors	for Ga	ayluss	ite
		Colur	nns ar	e l, 1	OF , 1	lOF			
0.0.L 2 485 -442	-14 245 -260 -12 92 64 -16 271 -285	0 347 -514 1 520 -483 2 49 -87 3 120 208	4 130 138 5 215 -216 6 123 -129	-3 779 -795 -2 77 -67 -1 411 396	0 237 -236 1 37 -32 2 129 -136 356 161	-9 143 176 -4 253 -288 -3 131 -147 -2 135 164	-8 233 -233 -7 238 238 -6 66 -63 -5 227 -233	5 150 131 17,3,L	-4 301 297 -3 830 858 -2 437 473
6 459 477 8 415 413 18 121 125	-4 535 -557 -4 522 545 -2 293 -309	4 152 -145 5 388 -291 6 210 -221	15,1,L	2 240 260 3 148 153 4 286 -388	4 253 -242 5 66 -63 6 306 290	-1 142 139 54 -64	-4 16 • 11 -3 216 -280 -2 139 -132	-19 29+ 39 -18 27+ 29 -17 220 -284	0 276 -280 1 938 962 2 376 -393
12 484 479 14 101 -181 16 249 227	0 71 51 2 288 +213 4 105 112	7 179 183 8 79 83 9 229 -227 18 82 67	-28 42 18 -19 116 -189 -18 240 -7 -17 159 161	5 668 -704 6 138 149 7 121 127 8 531 -557	7 205 198 8 153 -145 9 27e	-16 67 93 -17 76 78	-1 106 -162 0 44 -43 1 36 32 2 758 756	-16 40 29 -15 133 126 -14 279 284 -13 97 -95	3 818 120 4 211 214 5 767 804 6 35 -16
2,8,1	18.8.L	11 09 98 12 52 36	-16 24+ 8 -15 176 -186	9 260 -278 18 33 -22	12.2.L	-16 69 -77 -15 186 -202	3 19 • -1 • 258 -253 5 28 • 1	-12 25• -11 -11 133 -142	7 148 -159 8 22+ 27
-14 164 -175 +12 116 125	-18 231 -230 -16 125 124	7,1,L	-13 47 56 -12 21• 4	12 175 -174 13 59 -55	-10 76 60 -17 09 182	-13 117 109 -12 133 -142	6 285 193 7 195 -106 8 85 -102	-9 101 -116 -8 189 -194	18 88 81 11 260 -16
-8 810 870 -6 569 -621 -4 1742 1739	-12 388 331 -18 289 -384 -8 163 161	-18 106 -93 -17 213 198 -16 42 41	-10 446 -440 -9 78 86 -8 28* 14	4,2,L	-15 141 -146 -14 139 143 -13 41 -31	-10 69 54 -9 153 154 -8 231 -244	9 212 204 18 26 • 37 11 79 -70	-6 242 252 -5 97 -184 -4 126 143	13 146 152 6,4,L
-2 544 -579 8 644 632 2 207 212	-6 361 -367 -6 123 118 -2 308 -330	-15 53 -68 -16 230 -8 -13 680 681	-7 397 -408 -6 67 68 -5 189 192	-17 134 116 -16 111 -99 -15 195 -192	-12 94 -87 -11 184 -191 -10 175 172	-7 150 -175 -6 183 195 -5 88 98	9+3+L	-3 24+ 32 -2 205 224 -1 284 -211	-17 287 -191 -16 39 -37
6 67 75 8 329 325 18 182 -202	8 177 215 2 89 -111	-12 98 -92 -11 105 184 -18 147 157	-4 21 • -6 -3 324 -329 -2 88 -108	-14 538 351 -13 72 72 -12 36 -45	-9 215 233 -8 515 -517 -7 553 -566	-4 55 -58 -3 29+ -6	-18 108 189 -17 260 -9 -16 187 -169	0 26+ 10 1 133 145 2 281 218	-15 233 229 -14 74 59 -13 316 -295
12 403 400 14 77 -94	28+8+L -18 29+ 47	-9 388 388 -8 521 508 -7 339 345	-1 91 90 0 162 -179 1 229 -248	-11 96 -95 -10 190 -13 -9 137 -153	-6 49 -54 -5 221 215 -4 632 -633	-16 77 -93	-13 151 150 -14 182 -79 -13 249 -243	3 99 -111 19,3,L	-12 31 -15 -11 78 82 -18 136 138
4,0+L -16 26+ 32	-16 267 277 -14 370 -381 -12 273 283	-6 541 -529 -5 881 881 -4 111 -98	2 83 -77 3 80 85 4 182 168	-8 51 56 -7 578 -622 -6 718 770	-3 219 -219 -2 176 187 -1 259 266	-15 66 66 -14 52 56 -13 75 -79	-12 418 -396 -11 182 176 -18 241 -248	-19 29+ 25 -18 82 86	-9 308 -315 -8 33 -32 -7 204 280
-14 110 -110 -12 201 205 -10 1115 -1175	-18 136 -137 -0 259 298 -6 88 -91	-3 117 -92 -2 502 -461 -1 723 787	17.1.L	-5 431 470 -6 237 -244 -3 1532 -1454 -2 1672 1798	1 233 -248 2 59 36 3 126 118	-11 225 236 -10 285 305 -9 103 -117	-8 508 -519 -7 67 67 -6 237 236	-16 228 -227 -15 76 -64	-5 499 -502 -4 217 233
-6 761 -628 -4 67 -116 -2 369 -344	-2 183 -286 0 145 182	1 81 -72 2 451 451 3 448 439	-28 290 -31 -19 280 -15 -18 275 -255	-1 36 -13 8 832 -797 1 1239 -1159	6 260 -251 5 63 -30 6 26 -6	-0 109 -103 -7 100 9 -6 244 244	-5 119 -118 -6 678 -676 -3 58 -72	-13 186 -123 -12 128 -130 -11 24• 25	-2 223 -226 -1 405 -489 6 40 62
8 513 -492 2 1281 -1189 4 446 434	22,0,L -18 159 -155	4 268 -268 5 177 -173 6 175 -174	-17 159 172 -16 237 228 -15 61 45	2 380 374 3 98 181 4 36 -8	7 123 114 8 204 -209	-5 46 -43 -6 675 -741 -3 387 367	-2 149 -171 -1 57 74 0 432 -433	-18 100 108 -9 123 -134 -0 112 -116	1 685 689 2 378 388 3 478 -482
6 774 -795 8 543 556 18 129 -136	-16 188 179 -14 48 -31 -12 197 223	7 132 140 8 165 158 9 72 -67	-14 67 -73 -13 321 545 -12 99 -102	5 438 -421 6 127 125 7 198 -197	14.2.L -19 55 -57	-2 347 -373 -1 64 -59 0 224 199	1 172 166 2 19+ 6 3 322 -315	-7 38 24 -6 115 125 -5 277 -288	4 307 -311 5 306 298 6 165 -160
12 87 -81 6.8.L	-10 221 -254 -8 99 119 -6 88 -85	18 257 -246 11 01 95 12 114 116	-11 109 -106 -18 97 89 -9 172 384	9 219 -217 10 282 279	-18 174 169 -17 25• 11 -16 209 -192	1 208 -196 2 52 53 3 126 -143	4 171 -181 5 264 240 6 247 251	-4 218 -239 -3 114 116 -2 189 121	7 47 -39 8 185 112 9 283 285
-18 307 -291 -16 226 202 -14 411 -413	1.1.L	9,1,L -19 148 -133	-7 51 -35 -6 206 -225 -5 303 310	12 305 -300 13 222 -212	-14 215 218 -13 101 104 -12 326 -345	3 275 298 6 141 158 7 388 -402	6 113 -122 9 27 • 2 18 61 31	0 68 -52 1 52 -53	10 80 -67 11 189 -117 12 74 63

9 121 ~123

-12 144 -141	-15 104 -176	-17 126 176	-3 182 -99		-11 124 125	9 94 90	11.1.1	210 JIL	81415
-8 229 -253	-14 170 -159	-16 219 205	-2 157 -166	-18 28* -8	-9 68 49	10 51 71		-16 111 128	-18 29+ 18
-6 2888 -1888	-13 25+ 42	-15 103 -98	-1 163 175	-17 71 73	-8 68 78	11 47 -53	-19 198 153	-17 72 -83	-17 173 -167
-4 217 -244	-12 285 287	-14 229 -289	8 165 176	+16 26+ -37	-7 393 -391	12 35 -12	-10 61 47	-16 154 -165	-16 25+ 36
-2 1570 -1498	-11 184 -199	-13 76 92	1 49 -43	-15 68 41	-6 197 193	13 240 16	-17 03 -76	-15 44 34	-15 24+ 6
0 258 234	-18 71 -74	-12 269 259	2 126 -131	-14 198 261	-5 158 155	14 27+ 29	-14 147 -149	-14 71 -52	-14 166 -169
2 1205 -1252	-9 19+ -7	-11 109 -119	3 122 133	-13 155 163	-4 444 -464	15 82 -83	-15 148 145	-13 05 -76	-13 282 -276
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12 120 135	-6 582 -506	-4 473 592	-18 28e 68	-8 166 -169	1 126 -126	-15 49 79	-18 108 -108	-8 258 -269	-1 172 162
	-3 582 -567	-5 472 465	-17 254 263	-7 325 -329	2 159 167	-14 387 289	-9 76 64	-7 00 09	-7 16+ -17
8.0.L	-2 533 -534	-4 303 271	-16 97 -87	-6 335 348	3 105 104	-13 94 -105	-8 718 -738	-6 201 281	-6 96 -75
	-1 406 379	-3 216 -216	-15 52 -55	-5 615 642	4 85 -84	-12 61 -75	-7 62 78	~5 27+ -8	-5 943 -999
-18 135 -122	0 785 665	-2 25 1	-14 78 -99	-4 222 213	5 204 -215	-11 21• 4	-6 18+ 3	-4 283 -228	-4 155 -171
-16 163 167	1 658 -648	-1 557 552	-13 317 323	-3 612 -615	6 29 e -9	-1-0 54 52	-5 123 124	-3 43 -35	-3 265 256
-19 229 -244	2 527 521	8 180 -60	-12 125 152	-2 1229 1147	14.2.1	-0 110 -120		-2 290 13	-2 194 -179
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 120
 -101

 200
 -97

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 -374

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 246

 324
 -264

 324
 -364

 324
 -364

 324
 -364

 374
 -355

 115
 -125

 16.0.L
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 246
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-7-

TABLE 2 (continued)

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3 300 -206		912 993	7 109 -104	-12 53 -56	-4 83 76	-1 105 126	-5 152 160	-1 79 01	-10 05 -04	-14 250 21
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6 26 0	2	142 145		-10 270 -46	-1 546 577	20.6.L	-2 94 -95	19.7.4	-13 26+ 10	-11 156 160
7 106 -174		203 207	11+5+L	-5 38 -28	0 62 68		-1 264 280		-12 223 -237	-10 111 114
	2	18+ -13		-8 127 152	1 115 -125	-17 104 102	0 186 -195	-16 92 -95	-11 94 89	-9 65 -57
14.4.4		142 140	-10 200 -31	-7 270 -18	2 192 -105	-16 299 37	50- 10 E	-15 200 -33	~10 239 237	-7 71 50
-10 146 132		49 - 49	-16 102 169	-2 42 -0	4 127 112	-14 53 -63	3 202 218	-13 201 208	-8 188 -174	-6 177 179
-18 147 123		71 62	-15 14 57	-6 211 236	5 24+ -37	-13 113 106	4 150 -171	-12 00 -66	-7 190 196	-5 290 12
-17 173 -170	10	194 -157	-16 110 -119	-3 290 -7	6 123 -118	-12 171 101	5 130 -126	-11 26+ 13	-6 286 285	
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-14 210 0	13	62 63	-11 210 -0		9 56 -61	-9 48 84	8 157 -153	-8 54 -55	-3 129 -137	-12 56 46
-13 08 -71	1.4	198 -195	-10 30 -100	1 315 -338	10.6.1	-7 106 -116	y ya -yu	-0 107 101	-1 151 165	-10 71 -59
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	4 112 113	-3 67 57	-7 282 -280	-13 61 70	-9 127 125	-12 172 -170	0 04 -24	-13 85 -75	-3 104 103
22.4.1	9 92 -92	-2 270 -221	-6 625 -619	-12 24+ 1	-0 112 -123	-11 162 -165	9 40 -42	-12 130 -136	-2 56 53
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TABLE 2 (continued)

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 < -76543-22101234 -6543227654322 -654322 -654322 -654322 -654322 -654322 -654322 -654322 -654322 -654322 -654322 -654322 -654322 -654322 -654322 -76554 -765432 -76554 -72210 -72000 -72200 -72200 -72200 -72200 -7200 -14-12-12-12-12-13-78-78-78-55-28-78-55-28-28-55-28-6 -5 -4 -3 -2 -12 -11 -10 -9 -8 -14 -15 -12 -11 -10 0.14.L 2.10.1 -2 28° 6 -1 127 116 13,11,L -10 139 -120 -9 69 75 -6 45 49 -7 89 -84 -6 73 -74 -5 134 141 51 6.12.L 70 171 27.0 142 27.0 160 48 55 178 79 240 149 228 142 61 111 82 260 126 73 204 274 87 60 102 284 64 51 77 -1 0 1 114 67 127 -102 -57 102 168 185 03 60 216 67 64 27= 215 29+ 0125456789 176 -76 18 149 214 -135 -76 100 78 -7 -7 -5 -3 -2 -1 0 1 2 3 80 144 39 42 65 146 152 25 103 56 91 -71 -126 43 -23 -60 -144 147 57 -105 -39 89 109 -70 -163 24 87 -38 -89 -24 64 -23 -58 -174 -192 80 69 -211 -76 70 25 -207 -3 -9 -8 -7 -8 -3 -4 -3 -2 -1 0 254 125 133 74 46 86 216 216 98 176 93 -1 123 123 -70 -47 63 211 -19 -86 174 61 15,9,L 121 28 121 49 122 40 73 -12 -11 -19 -7 -7 -5 -4 -3 81 101 104 202 185 96 80 179 187 86 50 -91 -22 198 152 -95 100 176 185 61 149 -10 -139 -23 139 -45 2,14,L 54 29+ 29+ 106 26 -137 -33 131 -40 -80 -10 -9 -8 -7 -8 -3 -4 -15 -14 -13 -12 -11 -10 -9 -10 -9 -8 -7 -6 -5 -2 -1 0 40 - 34 - 16

-7b-

The large correlation coefficients are 0.25 between the scale and the Ca $\underline{\beta}_{11}$ anistropic temperature factor, 0.75 between the Ca $\underline{\beta}_{33}$ and $\underline{\beta}_{13}$ anisotropic temperature factors, 0.68 between the Na <u>x</u> and <u>z</u> parameters and 0.78 between the Na $\underline{\beta}_{33}$ and $\underline{\beta}_{13}$ thermal parameters. Most coefficients are, however, much less than 0.04.

The atomic parameters are given in Table 3. The hydrogen positions are those from the final least-squares refinement and are recognized as being only approximate.

Discussion of the Structure of Gaylussite

<u>The calcium environment</u>.--The immediate Ca environment is summarized in Table 4 and in Figure 1. Since the strongest electrostatic attraction in the crystal is between Ca^{2+} and CO_3^{2-} , it is not surprising that both CO_3^{2-} groups are coordinated (<u>via</u> 0(2) and 0(3)) to Ca, which lies on a to be two-fold axis, to form what may be considered/an ion triplet, OCO_2-Ca-O_2CO . The coordination of Ca is completed by four water molecules, 0(5), 0(5), 0(6) and 0(6). The Ca-to-0 distances are within the normal range. The strongest possible electrostatic repulsion in the structure, the Ca-Ca interaction, is minimized by the Ca ions being widely separated from one another (Ca-to-Ca>4.5 Å).

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			Atomic	TABI c Parametei	LE 3 rs of Gaylu	Issite			
Atom	×	А	N	B ₁₁ *	B22	B _{3 3}	B ₁₂	B _{1 3}	B _{2 3}
Ca	• 0000	.1929(1)	.2500	.59(1)	1.27(2)	. 86 (2)	06(3)	•41(1)	16(3)
Na	•0881(1)	.1822(1)	0107(1)	1.56(3)	1.92(3)	1.67(3)	.01(5)	1.12(3)	.11(5)
U	.1853(2)	0286(2)	.3161(2)	.94(6)	1.18(6)	1.18(6)	.96(4)	.65(5)	.67(5)
0(1)	• 2852(1)	0906(2)	.3633(2)	1.26(5)	2.88(6)	1.85 (6)	.01(4)	1.13(5)	01(4)
0(2)	.1033(1)	.0031(2)	.1749(1)	1.07(4)	2.09(5)	.90 (5)	.10(4)	•44(4)	•00(4)
0(3)	.1658(1)	.0095(2)	.4113(1)	.94(4)	1.97(5)	.99 (5)		.67(4)	
0(4)	• 0000	.6791(3)	.2500	1.84(8)	1.90(8)	2.22(9)		1.16(8)	
H(1)**	.05(2)	.60(3)	•27(3)	1.00	1.00	1.00			
0 (5)	•0696(1)	2935 (2)	.5029(2)	1.44(5)	2.02(6)	1.50(5)	.13(5)	.83(5)	06(5)
Н(2)	.10(2)	.21(3)	.55(3)	1.00	1.00	1.00			
H(3)	.11(2)	•38(3)	.54(3)	1.00	1.00	1.00			
0 (6)	.1467(1)	.4048(2)	.3109(2)	1.12(5)	2.60(6)	1.27(5)	47(5)	•76(4)	22(5)
H(4)	.20(2)	.44(3)	•40(3)	1.00	1.00	1.00			
Н(5)	.16(2)	• 39 (3)	•27(3)	1.00	1.00	1.00			
		comp	outed						
*Figure in tl	es in paren he full mat	theses are/ rrix least s	standard er) squares refi	cors in la: nements.	st signific	ant figure	s quoted	and were	obtained
Average Units C	shift/err	or for last	cycle = 0.3	23.					

**All hydrogen thermal parameters were kept constant at B (isotropic) = 1.0 ^2 . The hydrogen positional parameters are only approximate.

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The Calcium Environment in Gaylussite

atoms	distance, Å
Ca,0(2)	2,573(2)*
Ca,0(3)	2.385(1)
Ca,0(5)	2.484(2)
Ca,0(6)	2.420(2)
Ca,0(4)	3.783(2)
Ca.0(4)	3,997(2)

*In all distances and angles quoted in this paper the values computed in parentheses are the/standard errors in the last significant figures.

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The carbonate group. -- The CO, group is essentially planar with an average C-to-O distance of 1.288Å. Its dimensions and environment are summarized in Table 5 and Figure 2. Oxygen 0(1) is bonded electrostatically to Na (2.314A) and is hydrogen bonded to water oxygens 0(5) (2.847Å) via H(3) $(\sim 2.0\text{Å})$ and 0(6) (2.667Å) via H(5) $(\sim 2.0\text{Å})$. Oxygen 0(2)is bonded electrostatically to Ca (2.573Å), Na (2.400Å), and Na (2.610\AA) and is hydrogen bonded to O(5) (2.852\AA) via H(2)(~2.1Å). Oxygen 0(3) is electrostatically bonded to Ca (2.385Å) and Na (2.331Å) and is hydrogen bonded to 0(6) (2.666Å) via H(4) (~1.8Å). The observed C-to-0 bond distances correlate qualitatively with the oxygen environments. Oxygens 0(2) and O(3), which are under strong anisotropic cationic attraction, have longer bond distances to the carbon than does 0(1). Similarly, the O(2)-C-O(3) bond angle, 118.1°, is less than 120° because coordination with the Ca pulls these oxygens together.

The two CO_3 groups coordinated to Ca are also coordinated to two Na ions (Figure 1) which cause a dihedral angle of 134.3° between the planes of the two CO_3 groups instead of the expected 180°. This coordination is instrumental in making the O(2)-to-O(3) vector in one CO_3 group very nearly parallel to the O(2')-to-O(3') vector in the other CO_3 group.

The Carbonate Anion and Environment in Gaylussite

Atoms	distance, Å
C,0(1) C,0(2) C,0(3) 0(1),0(2) 0(1),0(3) 0(2),0(3)	1.280(3) 1.291(2) 1.293(3) 2.247(2) 2.229(3) 2.216(2)
Coordinated atoms	angle, deg
0(1),C,O(2) 0(1),C,O(3) 0(2),C,O(3)	121.8(2) 120.1(2) 118.1(2)
0(1) Environment	
Atoms	distance, Å
0(1),0(5) 0(1),H(3) 0(1),Na 0(1),0(6) 0(1),H(5)	2.847(2) 2.05(2) 2.341(2) 2.667(3) 2.02(4)
0(2) Environment	
Atoms	distance, Å
0(2),Ca 0(2),Na 0(2),Na' 0(2),0(5) 0(2),0(6) 0(2),0(4) 0(2),H(2)	2.573(2) 2.400(2) 2.610(2) 2.852(2) 3.364(2) 3.285(2) 2.11(3)

TABLE 5 (continued)

0(3) Environment

Atoms	distance, Å
0(3),Ca	2.385(1)
0(3),0(5)	3.096(3)
0(3),0(4)	3.204(2)
0(3),Na	2,331(2)
0(3),0(6)	2,666(2)
0(3).H(4)	1.83(2)

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The Ca ion is 0.69Å below the intersection of the planes of these CO_3 groups.

<u>The Na environment</u>.--The sodium ion is coordinated (Table 6 and Figure 3) approximately octahedrally by O(1), O(2), O(2'), O(3), all in different CO_3 groups, and by the O(4) and O(5) waters. The repulsion arising from the Ca-to-Na closest approach of 3.626Å is reduced by their sharing O(2)and O(3') (Figure 1). These atoms are in the two CO_3 groups bonded to the Ca. The next closest Ca-to-Na approach, 3.831Å, is stabilized by the intervening pair O(2) and O(5'). The Na-to-Na approaches are 3.903Å and 4.305Å, and the small stabilization required is provided by the intervening octahedral edges O(2)-O(5) and O(2)-O(2').

The water environments.--The water environments are summarized in Table 7 and Figure 4. The O(6) water is bonded to O(1) of one CO₃ group <u>via</u> a hydrogen bond H(5) to O(1), where H(5) to O(1) ~2.0Å, O(6) to O(1) = 2.667Å, and to O(3) of the next CO₃ group (generated by the c glide) by a hydrogen bond H(4) to O(3), where H(4) to O(3) ~1.8Å, O(6) to O(3) = 2.666Å. O(6) is in the coordination octahedron of the Ca

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The Sodium Environment in Gaylussite

Atoms	distance, Ă
Na,0(2')	2.400(2)
Na,0(2)	2.610(2)
Na,0(3')	2.331(2)
Na,0(4)	2.408(1)
Na,0(5)	2.475(2)
$Na_{0}(1)$	2,341(2)

-

The Water Environments in Gaylussite

1)	the	0(4)	water	environme	nt	H(1	.)	41	H(1)
<u> </u>	CIIC	0(1)	Watter		110		01	-1	
		Atom	S	Ċ	list	anc	e,	Å	
	0	(1) म	(1)		0	921	2)		
	0				0.	.02(5)		
	0	(4), 0	(6)		2.	.774	(2)		
	0	(4),Na	a		2.	408	(1)		
	Н	(1), 0	(6)		1.	.96 (3)		
	ы	(1) บ	(1)		1	1/(1		
	11	(1),11	(1)		e	, T 4 (4)		
	Cooi	rdina atoms	ted	ē	angl	Le,	deg	ſ	
	Н 0 0	(1),0 (6),0 (4),H	(4),H(1 (4),O(0 (1),O(0	L') 5') 5)	88 79 174	3.(3 9.37 1.(2	8) 7 (8) 2)		

2)	the	0(5)	water	environment	н(3)	́н(2)
<u> </u>		Atom	is	dist	ance,	Å
	0 0 0 0 1 1 1 1 1 1 1 1	(5),H (5),C (5),O (5),O (5),N (2),H (2),O (3),O	(2) (3) a (1) (2) a (3) (2) (1)	0 0 2 2 2 2 1 2 1 2 2 2	.81(3) .81(2) .484(2) .847(2) .852(2) .475(2) .35(4) .11(3) .05(2)	
	Cooi	rdina atoms	ted	ang	le, dec	J
	Н О О	(2),0 (1),0 (5),H (5),H	(5),H(3 (5),O(2 (2),O(2 (3),O(2	3) 12 2) 12 2) 12 1) 12	13.(2) 22.36(6 52.(4) 72.(4)	5)

TABLE 7 (continued)

2 1 2 $(c) 1$	H(4) $H(5)$
3) the O(6) water	environment 0(6)
Atoms	distance, Å
0(6),H(4)	0.84(2)
O(6),H(5)	0.67(4)
0(6),Ca	2.420(2)
0(6),0(4)	2.774(2)
0(6),H(1)	1.96(3)
0(6),0(3)	2.666(2)
0(6),0(1)	2.667(3)
H(4),H(5)	1.28(4)
H(4),0(3)	1.83(2)
H(5),0(1)	2.02(4)
Coordinated	
atoms	angle, deg
H(4),0(6),H(5 0(3),0(6),0(1 0(6),H(4),0(3 0(6),H(5),0(1	5) 115.(3) 106.87(8) 3) 171.(4) 162.(3)

ion and is the oxygen acceptor of a hydrogen bond from the water molecule 0(4) on the two-fold axis. The distances in these hydrogen bonds are H(1)-to- $0(6) \sim 2.0$ Å, 0(4) - to - 0(6) = 2.774Å. Besides linking two 0(6) waters to each other, the 0(4) water serves to separate two Na ions (Figures 1 and 4). The 0(5) water connects 0(1) in one CO_3 group to 0(2) in the CO_3 group related by a two-fold screw axis. The hydrogen bonds thus used are $0(5) - H(3) \dots 0(1)$, where $H(3) - to - 0(1) \sim 2.0$ Å, 0(5) - to - 0(1) = 2.847Å, and $0(5) - H(2) \dots 0(2)$, where $H(2) - to - 0(2) \sim 2.1$ Å and 0(5) - to - 0(2) = 2.852Å. Also 0(5) is in an edge which is shared between neighboring Na coordination octahedra.

Determination of the Crystal Structure of Pirssonite

We determined the crystal structure of **synthetic** pirssonite, CaNa₂(CO₃)₂·2H₂O, before the recent work of Corazza and Sabelli⁴ came to our attention. There are enough differences in the two procedures to warrant a brief description of our determination. They used a mineral specimen ground to a sphere of 0.616 mm diameter, measured 481 reflections of observable intensity from integrated films with a microdensitometer, corrected for absorption, and refined using block-diagonal least squares. In the present work, good crystals of synthetic pirssonite were grown at 50°C from a solution of 27% by weight of Na₂CO₃ and 5% NaOH in water in contact with powdered calcite.⁵ A small crystal, maximum dimension 0.20 mm ($\mu_{MO} = 10.6 \text{ cm}^{-1}$) was selected. The cell dimensions* were determined to be $\underline{a} = 11.340 \pm .004$, $\underline{b} = 20.096 \pm .005$ and $\underline{c} = 6.034 \pm .002$ Å from 20 values of axial reflections observed on a diffractometer. Evans^{1 2} reported the dimensions $\underline{a} = 11.32 \pm .02$, $\underline{b} = 20.06 \pm$.02 and $\underline{c} = 6.00 \pm .02$ Å, and that the space group is Fdd2. Our values, which were calculated using the weighted mean value for M₀K_{Q1}, Q₂ radiation ($\lambda = .71069$ Å are systematically larger than these reported by Evans.

Over 2200 reflections from two octants of the reciprocal lattice were measured on a diffractometer⁷ with the peak height procedure used for gaylussite. These data were merged into a unique set of 1141 reflections, 1079 of which were of observable intensity. The R value between reflections accepted as equivalent was 0.027 based on F's. The quasiunitary structure-factor statistics are given in Table 8 and confirm that the space group is acentric. Weights based on the counting statistics were used in the full-matrix least-squares refinements. The atomic scattering

^{*}The uncertainties quoted on cell dimensions are estimates based on experience with the technique used for measurement and in the authors' opinion may be treated as standard errors.

Quasi-Unitary Structure-Factor Statistics for Pirssonite

	< e >		$\langle E_{5} \rangle$	$\langle E^2-1 \rangle$
all reflections	.879		1.000	.731
3-dimensional reflections only	.875		.968	.694
Theoretical centric Theoretical acentric	.798 .886		1.000 1.000	.968 .736
		obs.	centric	acentric
Fraction of E's >	1.0	.394	.317	.368
	2.0	.014	.046	.018
	3.0	.0002	.003	.0001

Number of reflections 1140

Suggested overall temperature factor 0.65

factors used for gaylussite were also used for pir^ssonite. No corrections for absorption were made.

All atoms other than the water molecule were found from the sharpened Patterson map. The oxygen of the water molecule was found from an F_0 Fourier synthesis. The structure was refined isotropically to $R_* = 0.059$ and then anisotropically to $R_* = 0.042$. These hydrogens were found from a difference synthesis and were included with fixed thermal parameters $(B_{\mu} = 1.0 Å^2)$. The structure was then refined anisotropically to $R_* = 0.029$, R = 0.044. The largest correlation coefficients are about 0.2 and are between the scale factor and some of the anisotropic temperature factors, between some of the anisotropic factors themselves, and between the x and y parameters of most atoms. Most of the remaining coefficients are less than 0.05. The observed and calculated structure factors are given in Table 9.

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TABLE 9

Observed and Calculated Structure Factors for Pirssonite

16 13 20 22 24 26	19 159 81 154 90	11 161 2 79 14 79	500 300 500	6 3 10 12 14 16 20 22	26 101 30 52 88 43 96	27 103 18 63 50 90 39 81 56 84	372 525 957 51 493 15 978 27	7 9 11 13 15 17 19 21 23	7 4 9 8 7 8 8 0 3 8 5 1	73 101 78 43 74 46	71 278 854 980 243 707 52 821	5 7 9 11 13 15 17	97 69 69 89 70 16• 15	97 65 37 34 81 70 19	53 849 262 933 129 778 21	22 2 6 8 10	160 16 23 76 32 139 26	21 , K , 2 89 19 75 30 139 16	0 981 35 572 972 861	2 6 8 10	16 86 15° 45 16° 135	, K + 6 0 6 1 3 6 4 3 1 27 , K + L	29 575 12 417 981	11 13 15 17	77 77 39 53 17 42	73 74 38 44 •K,3 39	944 64 843 302 942	2 % 6 6 10 12 14	33 106 23 79 23 79 47 76	34 109 24 79 23 82 42 71	467 970 108 93 679 0 133 34	1 3 5 7 9	19, 59 81 63 63 53	56 71 35 55 47	L 981 734 306 978
0 2 6 8 10 12 14 16 18	127 60 52 31 157 17 79 33 66 20	138 65 33 153 11 82 32 83 22	993 777 725 143 993 866 592 37 316	0 2 6 8 10 12	156 35 42 45 173 26 27	157 34 38 40 164 25 77 17	953 177 49 819 950 467 38 137	1 3 7 9 11 13 15	15 128 162 79 21 70 124 98 54	24 121 166 78 15 68 122 95 61	912 139 878 104 715 993 806 152 942	1357	54 70 63 45 16 219 70 20 96	55 69 56 37 K+0 228 71 0 96	890 207 978 88 500 0	14 18 20 0 24 6	33 12° 109 38 16 63 13° 96 32	35 11 98 30 	957 498 39 757 8 208 984 0	L 35 79 11 13 15 17	5 0 2 0 0 7 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0	49 63 28 63 45 45 40	834 94 831 112 874 69 754 143 880 70	3 7 9 11	77 130 39 29 86	72 116 43 28 84	100 871 223 753 93	2	18, 81 13+	74 20	948 892	024	20, 50 33 83	48 23 84	500 0
20 22 24 2	60 154 58 14	60 19 54 54 113	906 911 3 45	16 1 3 5	48 45 90 77	94 5, K, 1 48 88 78	938 220 824 104	14 21 L J.	34 84 15 13* 61	37 78 , K, 5 14 58	66 910 163 783	8 10 12 14 16 18 20	124 33 55 130 91 140 100	128 30 64 14 85 3 98	500 0 0 0	8 10 12 14 16 18	52 31 61 140 86 32	50 25 62 10 77 28	2% 391 37 180 979 380	13570	17 13* 84 117 24 47	, K, 3 79 118 20 43	733 952 104 659 410	2 6 8 10 12 14 16	16 39 30 40 39 101 72 134 23 10 117	, «, 2 119	0 300 0 500 0	6 8 10 1 3 5 7 9 11 13	63 16* 121 19 31 73 71 30 63 72 30	36 9 111 37 71 71 56 68 36	953 580 958 157 896 93 768 80 901 42	6 8 2 4	13* 83 20 67 15*	0 80 98,2 62 11	0 0 12 711

TABLE 9 (continued)

The atomic parameters obtained by us, as well as those of Corazza and Sabelli,⁴ are given in Table 10. The agreement between the two sets of parameters is excellent. All positional parameters for atoms other than hydrogens are within 2^{σ} when our estimated standard errors are used; four of the 18 parameters differ by more than 2σ when the estimates of Corazza and Sabelli⁴ are used. Their estimates of errors, which are about 60% as large as ours, were derived from the block-diagonal least-squares approximation using only 481 observed reflections and are probably too small. The placement of the hydrogen atoms from our refinements is recognized as being only approximate. Corazza and Sabelli apparently assumed that the O(4)-H(1)....O(1) and O(4)-H(2).....0(1') hydrogen bonds are linear in placing the hydrogen atoms. This is reasonable since the 0(1)-0(4)-0(1') angle is 108.7 ± .4°.

Atomic Parameters of Pirrsonite

$B_{2,3}$		I		.00(2)		.00(4)		.02(3)		•00(3)		02(3)		.07(3)						
B ₁₃		1		.10(19)		41(36)		43(34)		.23(26)		.30(28)		.63(29)				1		
B _{1 2}		.00(1)		.00(2)		.00(2)		04(2)		01(2)		.01(2)		.06(2)		1				
B33		.50(8)		.82(16)		.55(30)		1.24(31)		.73(27)		1.05(33)		1.62(36)		1.0		1.0		
B B		. 63 (8)		2.89(23)		1.05(36)		.81(26)		1.45(29)		1.29(27)		1.99(34)		1.0		1.0		
B ₁₁ **		•73(8)	.52(2)	1.09(16)	1.51(4)	.84(33)	.63(1)	I.76(29)	. 98 (6)	.87(25)	.94(6)	1.08(27)	1.05(7)	1.26(28)	1.39(7)	1.0	5.00	1.0	5.00	
Ŋ		• 0000	• 0000	0073(12)	0073(7)	0072(24)	0090(13)	0121(18)	0119(10)	<pre>。1 291 (15)</pre>	.1294(10)	1420(15)	1419(9)	.0645(18)	•0625(10)	•07(3)	•034	.20(3)	.220	
Λ		• 0000	• 0000	.1118(3)	.1124(1)	.1355(5)	.1362(2)	.1975(4)	.1982(2)	.1161(4)	.1161(2)	.0935(4)	.0934(2)	. 2432 (4)	.2437(2)	•27(1)	. 265	。21(1)	.227	
×		• 0000	0000	.5653(4)	.5653(3)	.0841(8)	.0834(5)	.1169(7)	•1171(4)	.0052(6)	.0052(4)	.1297(6)	.1291(4)	.6093(7)	.6088(5)	.56(1)	.527	.62(1)	.611	
Atom	B* C+S*		Ga		Na		U	1)	0(3)	2)	0(1)	3)	0(2)	4)	0(4)	1)***	H(2)	2)	H(1)	-
	Å	U C		Na		υ) ()		0)0) ()) Н)н(-

*D+B, this work; C+S Corazza and Sabelli⁴

**Values of B given by Corazza and Sabelli

The hydrogen ***All hydrogen thermal parameters were kept constant at B (isotropic) = 1.0A². parameters are only approximate. Average shift/error for last cycle = .22. Discussion of the Structure of Pirssonite

The calcium environment -- As in gaylussite and CaCO₃. 6H₂O, ¹³ the Ca ions lie on two-fold axes. Their immediate environment in pirssonite is summarized in Table 11 and Figure 5. In these tables we have numbered the atoms in the CO₃ group to correspond to gaylussite. The correspondence between our numbering scheme and that of Corazza and Sabelli is shown in Table 10. The strong electrostatic attraction between Ca²⁺ and CO₃²⁻ forces the formation of OCO₂-Ca-O₂CO ion triplets as in gaylussite, but, since there are only two water molecules present, further coordination of CO₃²⁻groups to neighboring triplets is necessary to complete the Ca environment. This is a step towards the coordination in calcite and aragonite where each oxygen in a CO₂ group is and three respectively coordinated to two/different Ca ions/and the coordination of Ca is octahedral. In calcite and aragonite, no CO3 group has two oxygens bonded to the same Ca. In gaylussite and pirssonite, however, the Ca coordination comprises two CO3 edges and four other oxygens. In pirssonite only two of the latter are water molecules; the other two are CO3 apexes (Figure 5). The two CO₃ groups coordinated to Ca by their

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Ca Environment in Pirssonite

Atoms	distance, Å
Ca,0(1) Ca,0(2)	2.428(9) 2.461(7)
Ca,0(3)	2.536(7)
Ca,0(4)	2.483(10)

edges are at a dihedral angle of 95.5° to one another, and have twisted around to allow the close approach to Ca of the apexes of the other CO₃ groups. The Ca lies 0.1Å away from the intersection of the planes of the CO₃ groups coordinated edgewise.

The carbonate group. -- The CO₃ group is planar and trigonal within experimental error with an average C-to-0 distance of 1.286Å. Its dimensions and environment are summarized in Figure 6 and Table 12. Oxygen 0(1) is bonded electrostatically to Ca (2.428Å) and forms two hydrogen bonds, $0(1) - t_0 - H(1) \sim 2.2 \text{Å}, 0(1) - t_0 - 0(4) = 2.716 \text{\AA}, \text{ and } 0(1) - t_0 - H(2)$ ~1.8Å, 0(1)-to-0(4) = 2.865Å, with neighboring water molecules. Oxygen 0(1) is too far from Na (2.945Å) for Na to be its primary coordination. Oxygen 0(2) may form a very weak hydrogen bond with H(1) (2.4Å) but its primary coordination is electrostatically to Ca (2.461Å) and to the 'chain' of Na ions (2.299Å, 2.302Å) formed above it by the d glide (Figure 7). Oxygen 0(3) is not hydrogen bonded but is electrostatically bonded to Ca (2.536Å) and to a 'chain' of Na ions (2.351, 2.392Å) formed below it by the d glide. The C-to-O(2) and C+to-0(3) distances are shorter (1.28\AA) and C-to-0(1)

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Carbonate Group Dimensions and Environment in Pirssonite

Atoms	distance, Å
C-0(1)	1.300(12)
C-0(2)	1.276(14)
C-0(3)	1.281(14)
Coordinated atoms	angle, deg
0(1),0(2)	120.5(1.1)
0(1),0(3)	120.1(1.1)
0(2),0(3)	119.4(.9)

0(1) Environment

Atoms

0(1),Ca
0(1),0(4)
0(1),H(2)
0(1),0(4')
0(1),H(1)
0(1),Na

distance, Ă
2.428(9)
2.716(15)
1.8(2)
2.865(11)
2.2(1)
2.945(10)

0

0(2) Environment

Atoms	distance, A
0(2),Ca	2.461(7)
O(2),H(l)	2.4(1)
0(2),0(4)	3.118(11)
0(2),Na	2.299(12)
0(2),Na'	2.302(9)

TABLE 12 (continued)

0(3) Environment

Atoms	distance, Ă
0(3),Ca	2.536(7)
0(3),Na	2.351(12)
0(3),Na'	2.392(9)
0(3),0(1')*	3.023(10)
0(3),0(1")**	3.177(10)

*both coordinated to same Ca **separated by <u>d</u> glide (1.30Å), the reverse of that in gaylussite, and the O(2)-C-O(3) angle is essentially 120°. Presumably this, if a real difference, is due to the moderating influence of the coordinated 'chain' of Na ions, producing forces at right angles to those produced by the Ca ion, and to the fact that O(1) is coordinated to a Ca ion at a Ca-to-O distance slightly less than those of O(2) and O(3).

The Na environment.--The Na ion is coordinated approximately octahedrally. Its coordination is summarized in Table 13 and Figure 7. The repulsion associated with the Na-Na" and Na-Na' closest approaches of 3.255\AA is reduced by the intervening 'octahedral' edges 0(2')-0(4) and 0(2)-0(3'), which are the most closely coordinated oxygens of the Na octahedron. Na, Na' and Na" are all related by the <u>d</u> glide. The water molecules, which complete the coordination of Na, are primarily coordinated to Ca.

The low charges of the Na and the atoms in the water molecule, together with the $\frac{1}{r}$ dependence of electrostatic energy, allow the water molecule to coordinate to Na with the large Na-to-O distance of 2.722Å, where, other things being equal, 2.3Å is expected.

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The Na Environment in Pirssonite

Atoms	distance, Å
Na,0(3)	2.351(12)
Na,0(2)	2.299(12)
Na,0(2')	2.302(9)
Na,0(3')	2.392(9)
Na,0(4)	2.722(10)
Na,0(4')	2.751(10)
Na,Na'	3.255(8)
Na,Na"	3.255(8)

The water environment.---The geometry of the unique water molecule and its environment are summarized in Table 14. Through its two hydrogens, the water molecule links two CO_3 groups together (Figure 6) and is also coordinated electrostatically (2.483Å) to Ca. The water molecule is also coordinated to two Na ions (2.722Å, 2.751Å), and reduces their mutual repulsion. It appears in the coordination of Na twice (Figure 7). In forming hydrogen bonds in the pirssonite structure, the hydrogens of the water define a plane which is approximately perpendicular to the configuration Na-O(4)-Na (angle = 164°), using the two Na's mentioned above.

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TA	BL	E	1	4

H	H ₂
Atoms	O ₄ distance, Å
0(4),H(1)	.8(1)
0(4),H(2)	1.0(2)
H(1),H(2)	1.5(2)
0(4),0(1)	2.716(15)
0(4),0(1')	2.865(11)
0(4),Ca	2.483(10)
0(4),Na	2.722(10)
0(4),Na'	2.751(10)
Coordinated atoms	angle, deg
H(l),0(4),H(2)	118.(15)
O(l),O(4),O(l)	108.7(4)
O(4),H(l),O(l')	147.(13)
O(4),H(2),O(l)	153.(11)

The Water Environment in Pirssonite

References

- 1. W. E. Brown and B. Dickens, submitted to Science.
- 2. C. Palache, H. Berman and C. Frondel, Dana's System of Mineralogy, 7th ed., Vol. II, p. 234, (1951).
- 3. Reference 2, p. 232
- 4. E. Corazza and C. Sabelli, Acta Cryst., 23, 763 (1967).
- 5. C. R. Bury and R. Redd, J. Chem. Soc., 1160 (1933).
- 6. R. Brooks, L. M. Clark and E. F. Thurston, Phil. Trans. Roy. Soc., <u>A243</u>, 145 (1951).
- 7. F. A. Mauer and A. L. Koenig, paper #10, Summer meeting of American Crystallographic Association, August 1967, University of Minnesota, Minneapolis, Minnesota.
- International Tables for Crystallography, The Kynoch Press (1962).
- 9. See C. Dickinson, J. M. Stewart and J. R. Holden, Acta, Cryst., <u>21</u>, 663 (1966) for the method used in estimating the quasi-normalized structure factors.

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- 10. Reference 8, Vol. III, p. 202.
- 11. R. McWeeney, Acta Cryst., <u>4</u>, 513 (1951).
- 12. H. T. Evans, Amer. Min., 33, 261 (1948).
- 13. B. Dickens and W. E. Brown, in preparation.



Fig. 1. The calcium ion environment in gaylussite. Primed atoms are related to unprimed atoms by the twofold axis.





Fig. 2. The carbonate anion environment in gaylussite.





Fig. 3. The sodium ion environment in gaylussite.



Fig. 4. The water environments and hydrogen bonding in gaylussite.





Fig. 5. The calcium ion environment in pirssonite.





Fig. 7. The sodium ion environment in pirssonite.





