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# Reduction of out of band radiation using carrier-by-carrier partial response signalling in orthogonal frequency division multiplexing

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**Abstract:** Orthogonal frequency division multiplexing (OFDM) is a mature and one of the most popular multicarrier modulation (MCM) techniques. Also, it is the main candidate for physical layer of cognitive radio (CR) networks. CR is a new method to satisfy ubiquitous demand for wireless services while there is no enough unlicensed spectrum. However, the most important shortcoming of OFDM-based CR systems is the high level of out of band (OOB) components that originate from simple fast Fourier transform (FFT)-based implementation. In this study, the authors propose a novel method to reduce side-lobes of OFDM spectrum. In this method proper carrier-by-carrier partial response signaling is used on the modulated symbols across the time. This method will allow for using other techniques for OOB radiation or peak-to-average power ratio (PAPR) reduction, while does not require high complexity at the transmitter, i.e. just the receiver should have either maximum likelihood sequence detector (MLSD) in case of no pre-coding or a simple slicer in case of pre-coding. Moreover, it will not affect the PAPR but will increase word error rate (WER). Simulation results show that about 7 dB reduction in OOB components can be expected by this method.

## 1 Introduction

Orthogonal frequency division multiplexing (OFDM) is a multicarrier modulation (MCM) technique that has been used in many conventional systems such as wireless local area networks (WLANs), long-term evolution (LTE) systems, digital video broadcast (DVB) and digital audio broadcast. It is a robust method for transmitting and receiving data over frequency selective channels. Because of its excellent performance in multipath fading channel and simple implementation using fast Fourier transform (FFT) algorithm, it has been the most popular MCM method; and it is a mature technology [1]. However, OFDM has relatively large out of band (OOB) components spectrum that may be inconsistent with power spectral density (PSD) specification of WLANs, LTE, DVB and other OFDM-based standards.

In addition, the demand for ubiquitous wireless services has increased rapidly in the past years and this trend is expected to continue in the future [1]. Unfortunately, the vast majority of available spectrum resources have already been licensed and there is a little bandwidth to set up new services. Studies have shown that large per cents of licensed spectra are rarely used [2]. It is the basic idea of cognitive radio (CR) to use this spectrum by unlicensed users under certain conditions.

In CR, unlicensed users are allowed to transmit and receive data over portions of licensed spectra when licensed users or primary users (PUs) are inactive [2, 3]. Because of the discontinuous nature of inactive spectra and the ability of OFDM to utilise such spectrum, it has been the main candidate for the implementation of physical layer of CR networks [1].

However, the most important drawback of OFDM-based CR systems is the large OOB radiation that originates from the high level side-lobes of inverse fast Fourier transform (IFFT) modulated subcarriers [1]. These side-lobes cause unwanted interference among secondary users (SU) and also between SUs and PUs [1]. Therefore the OOB radiation of OFDM has been a considerable issue either in conventional applications or in CR networks. Several methods already exist for OOB radiation reduction in OFDM systems. In [4] and some standards, the insertion of guard bands at the borders of OFDM spectrum has been proposed [5, 6]. The drawback of this approach is the less effective use of the bandwidth. Another common method is time windowing at the transmitter. In this method, the time domain signal is multiplied by a proper window to smooth transition between consecutive OFDM blocks [7]. Various window functions such as raised-cosine, hamming and so on can be used for this purpose. In this approach the effective time duration of transmitted signal is extended and inter-block interference is introduced [7, 8].

A different technique is dual to time windowing and uses pulse shaping filter. In fact, each subcarrier is multiplied by a pulse shaping function which is equal to the convolution of time domain transmitted signal with the impulse response of pulse shaping filter [8]. This approach suffers from high complexity and lack of guard interval [8]. It should be noted that guard interval plays an important role in avoiding inter-block interference between OFDM blocks. Besides, guard interval not only extends the time duration of transmitted signal and reduces side-lobe levels but also increases robustness of system against synchronisation errors [9].

In [10], cancellation carrier insertion is used at the edge of available bandwidth. Proper values for cancellation carriers in each OFDM block have been obtained by solving a convex optimisation problem. This method reduces OOB components considerably but has high complexity, and increases peak-to-average power ratio (PAPR) and word error rate (WER). In [11], the modulated symbols in each OFDM block have been multiplied by proper sequence to reduce OOB components. Proper sequence can be found by solving a convex optimisation problem. In addition to high complexity, this method results in WER loss.

In the adaptive symbol transition method, the transition between consecutive OFDM blocks is smoothed adaptively. Multiple choice sequences method performs a mapping of each transmission sequence into a specific set of sequences. From this set, the sequence that offers the maximum reduction of out-of-band radiation is chosen for the actual transmission. These two approaches are discussed thoroughly in [12, 13], respectively.

In this paper, we propose a novel method to achieve side-lobe suppression. Our proposed method is to introduce proper carrier-by-carrier partial response signalling for spectrum shaping. In this approach, a controlled amount of correlation is introduced [14–16] among modulated symbols on each subcarrier in consecutive blocks. In this method the effective time duration and bandwidth of transmitted signal will remain unchanged. Also, guard interval can be used as before.

It should be noted that the term 'partial response OFDM' that has been used in [17–20] refers to introduced partial response signalling between modulated symbols on various subcarriers in the same OFDM block to achieve spectral efficiency, PAPR reduction, etc. and so it is fundamentally different from our approach.

This paper is organised as follows: in Section 2, the analytic expression of OFDM spectrum is derived in detail. Section 3 includes investigation of our proposed method. Simulation results are given in Section 4 to illustrate the effectiveness of the new method. PAPR and WER analyses are drawn in Section 5. Finally, our conclusion is explained in Section 6.

## 2 PSD of OFDM signal

In this section, the PSD of OFDM signal will be drawn analytically. The OOB components are considered as PSD components that are out of the permitted bandwidth. Basically the finite symbol duration of OFDM symbol results in these OOB components; however, other reasons such as phase transition between successive blocks may contribute to this phenomenon.

In OFDM, the baseband signal can be represented as

$$x(t) = \sum_{l=1}^{N} \sum_{k=-\infty}^{\infty} d_{k,l} w(t - kT_s) \exp(j2\pi f_l(t - kT_s - T_{GI})) \tag{1}$$

where $l$ and $k$ are the subcarrier and time indexes, complex number $d_{k,l}$ is the modulated symbol on $l$th subcarrier at $k$th time interval, $w(t)$ is the window function, for example rectangular, raised-cosine and so on, $f_l$ is the $l$th subcarrier frequency that equals to $l/T_{FFT}$, $T_{FFT}$ is the pure block time duration, $T_s$ is the total block time duration, $T_{GI}$ is the guard interval duration and finally $N$ is the number of subcarriers. Clearly, we have $T_s = T_{FFT} + T_{GI}$.

By defining

$$g_l(t) = \sum_{k=-\infty}^{\infty} d_{k,l} w(t - kT_s) \exp(j2\pi f_l(-T_{GI} - kT_s)) \tag{2}$$

the baseband signal can be written as

$$x(t) = \sum_{l=1}^{N} g_l(t) \exp(j2\pi l \Delta f t) \tag{3}$$

where $\Delta f = 1/T_{FFT}$.



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In general form, a linear filter may be used to implement partial response signalling [14, 15] and make intentional correlation of length $L$ among modulated symbols on each subcarrier as shown in Fig. 1.

Indeed, each subcarrier is treated by such filters individually and a carrier-by-carrier partial response signalling scheme is organised as shown in Fig. 2. Then we can write

$$a_{k,l} = \sum_{n=0}^{L} \alpha_n^l d_{k-n,l} \tag{4}$$

where $\{\alpha_n^l\}$ are the linear filter coefficients for $l$th subcarrier. Then, the baseband signal becomes

$$x(t) = \sum_{l=1}^{N} b_l(t)\exp(\mathrm{j}2\pi l\Delta f t) \tag{5}$$

where $b_l(t)$ is defined as

$$b_l(t) = \sum_{k=-\infty}^{\infty} a_{k,l} w(t-kT_s)\exp(\mathrm{j}2\pi f_l(-T_{\mathrm{GI}}-kT_s)) \tag{6}$$

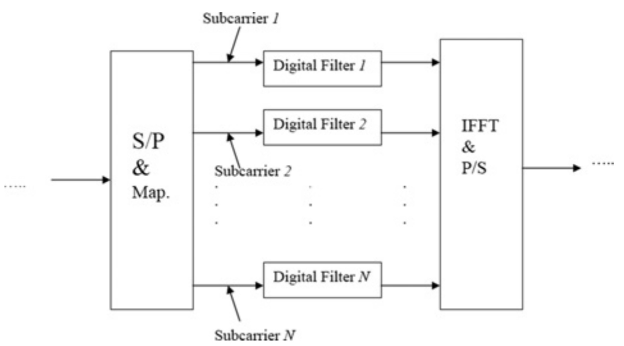


**Figure 1** *Implementing partial response signalling in OFDM carriers using N linear filters*

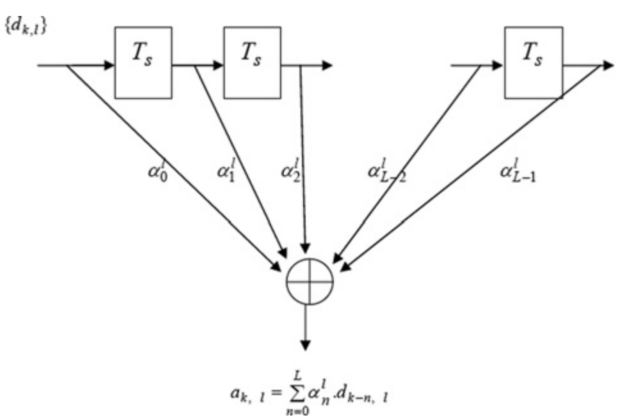


**Figure 2** *General partial response signalling for lth subcarrier*

In [21], it is shown that the PSD of the random process $b_l(t)$ by assumption of uncorrelated data sequence, $d_{k,l}$ (output of source coder or scramble), is given by

$$B_l(f) = \frac{1}{T_s}|W(f)|^2\left|\sum_{n=0}^{L}\alpha_n^l \exp(-\mathrm{j}2\pi f n T_s)\right|^2 \tag{7}$$

where $W(f)$ is the Fourier transform of window function, $w(t)$.

Therefore because of uncorrelated modulated symbols on various subcarriers at $k$th block for any $k$, the PSD of OFDM signal becomes (without loss of generality it is assumed that $E\{|d_{k,l}|^2\} = 1$)

$$X(f) = \sum_{l=1}^{N} B_l(f - l\Delta f) \tag{8}$$

It is realised from (7) and (8) that the PSD of OFDM signal depends mainly on the window function, number of subcarriers and $\{\alpha_n^l\}$, the coefficients of linear filter used to introduce partial response signalling on $l$th carrier. In this paper, we use these coefficients to shape the spectrum and reduce the OOB components, which carry energy outside of the permitted bandwidth of OFDM signal.

If the common raised-cosine window is used

$$\begin{cases} \sin^2\left(\frac{\pi}{2}\left(5+\frac{t}{T_{\mathrm{R}}}\right)\right) & \text{for} \quad \frac{-T_{\mathrm{R}}}{2} \le t \le \frac{T_{\mathrm{R}}}{2} \\ 1 & \text{for} \quad \frac{T_{\mathrm{R}}}{2} \le t \le T_s - \frac{T_{\mathrm{R}}}{2} \\ \sin^2\left(\frac{\pi}{2}\left(5+\frac{t-T_{\mathrm{R}}}{T_{\mathrm{R}}}\right)\right) & \text{for} \quad T_s - \frac{T_{\mathrm{R}}}{2} \le t \le T_s + \frac{T_{\mathrm{R}}}{2} \end{cases} \tag{9}$$

where $T_{\mathrm{R}}$ is the transition time and for $W(f)$

$$W(f) = T_s \operatorname{sinc}(T_s f)\frac{\cos(\pi T_{\mathrm{R}} f)}{1-4T_{\mathrm{R}}^2 f^2}\exp(-\mathrm{j}\pi T_s f) \tag{10}$$

by choosing $T_{\mathrm{R}}$, various windows can be synthesised. Choosing $T_{\mathrm{R}} = 0$ results in rectangular window and by increasing $T_{\mathrm{R}}$, the OOB components are reduced, while effective time duration of signal increases [9]. In the following, it is assumed that the rectangular window is used. Therefore from (7), (8) and (10) $X(f)$ becomes

$$X(f) = T_s \sum_{l=1}^{N}\left[|\operatorname{sinc}(T_s[f-l\Delta f])|^2 \times \left|\sum_{n=0}^{L}\alpha_n^l \exp(-\mathrm{j}2\pi[f-l\Delta f]nT_s)\right|^2\right] \tag{11}$$

To obtain conventional OFDM signal PSD from (11), assuming there is no partial response signalling ($L = 0$,


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$\alpha_0^l = 1, \forall l$), yields

$$X(f) = T_s \sum_{l=1}^{N} \text{sinc}^2(T_s[f - l\Delta f]) \qquad (12)$$

It should be noted that if the assumption of uncorrelated data sequence is not valid, (7) must be modified as

$$B_l(f) = \frac{1}{T_s}|W(f)|^2 \left| \sum_{n=0}^{L} \alpha_n^l \exp(-\text{j}2\pi f n T_s) \right|^2 \times \left[ \sum_{m=-\infty}^{\infty} E\{d_{k,l} d_{k+m,l}^*\} \exp(-\text{j}2\pi f m T_s) \right] \qquad (13)$$

in this situation if modulated symbols on various subcarriers at $k$th block for any $k$ are uncorrelated, PSD of the OFDM signal can be found using (13) and (8). In the following, without loss of generality, we assume that data sequence is uncorrelated.

In the next section, we investigate how a proper choice of $\{\alpha_n^l\}$ can result in lower OOB in OFDM spectra.

## 3 Investigation of the proposed method

In this section the advantage of introducing proper partial response signalling between modulated symbols on each subcarrier, or proper carrier-by-carrier partial response signalling, is investigated. Considering (7) and (8), proper choice of this controlling term

$$\left| \sum_{n=0}^{L} \alpha_n^l \exp(-\text{j}2\pi[f - l\Delta f] n T_s) \right|^2 \qquad (14)$$

in (11) can be used for shaping each subcarrier's spectrum separately, and hence overall PSD shaping will be possible for OOB component reduction. As mentioned in the previous section, assume that the data sequence, $d_{k,l}$, is uncorrelated. Otherwise, each subcarrier spectrum is expressed as (13) and our discussion still will be valid.

Now, we rewrite the PSD of OFDM signal as

$$X(f) = T_s \sum_{l=1}^{N} \left[ |\text{sinc}(T_s[f - l\Delta f])|^2 \times \left| \sum_{n=0}^{L} \alpha_n^l \exp(-\text{j}2\pi[f - l\Delta f] n T_s) \right|^2 \right] \qquad (15)$$

Fig. 3 shows schematic of the carrier-by-carrier partial response signalling method and intentionally introduced correlation between modulated symbols on each subcarrier across the time.

Considering (15), in general, each subcarrier may be processed based on its own partial response signalling pattern depending on $\alpha_n^l$. In this case, more degrees of freedom are available and better spectral performance is achievable. But, it results in high complexity because each subcarrier must be treated individually for symbol modulation and detection. In the following, assume that all

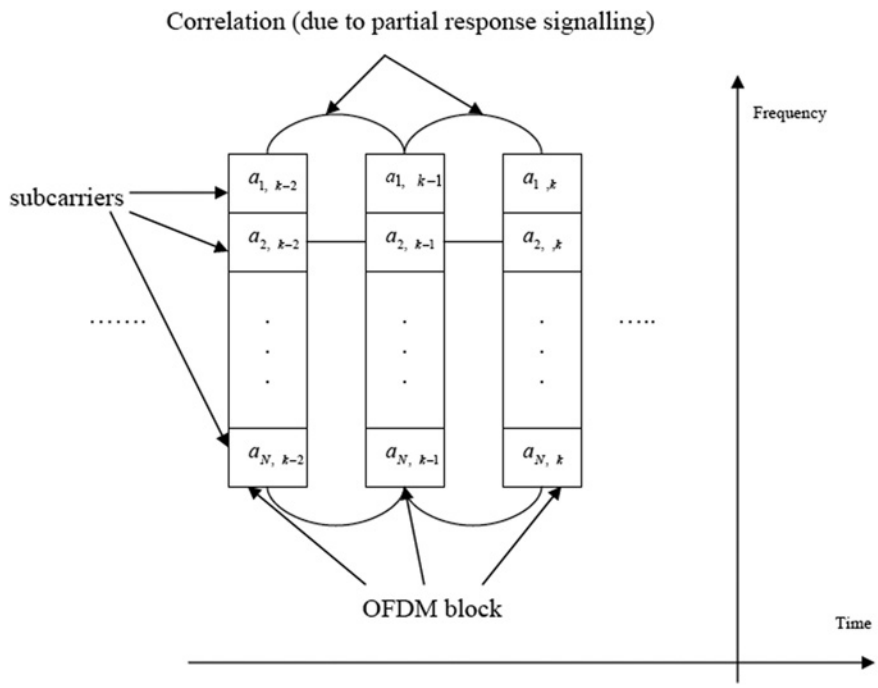


**Figure 3** *Schematic of the carrier-by-carrier partial response signalling method*



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subcarriers have the same partial response signalling pattern. It means

$$\alpha_n^l = \alpha_n, \quad l = 1, 2, \ldots, N \tag{16}$$

Then, according to (15), PSD of OFDM signal becomes

$$X(f) = T_s \sum_{l=1}^{N} \left[ |\text{sinc}(T_s[f - l\Delta f])|^2 \times \left| \sum_{n=0}^{L} \alpha_n \exp(-j2\pi[f - l\Delta f]nT_s) \right|^2 \right] \tag{17}$$

In (17), the partial response signalling pattern should be selected for shaping the PSD and specially OOB components reduction. Clearly, large $L$ gives more degrees of freedom, but leads to more states for receiver slicer. It should be noted that nine patterns of more common partial response signalling schemes are introduced in [14, 15]. All of those schemes require three or five level slicers in the receiver for binary phase shift keying (BPSK) modulation [14, 15]. But, for M-array modulations, the number of slicer levels grows exponentially. Hence $L$ is dictated to the transmitter based on acceptable complexity for the receiver.

In general, in the receiver, the effect of partial response signalling on each subcarrier symbol could be removed by means of maximum likelihood sequence detector (MLSD). In other words, this controlled correlation might be assumed as known channel impulse response and thus detection could be implemented by MLSD methods such as Viterbi algorithm.

It should be noted that not every partial response signalling scheme can improve OOB radiation and thus in this context 'proper partial response signalling' term has been used. In fact, the controlling term in (14) is discrete Fourier transform of the partial response signalling sequence $\{\alpha_n^l\}$, regardless of $l\Delta f$. Therefore it is reasonable that sequences with low-pass frequency characteristic are selected to reduce OOB components; because the PSD of conventional OFDM system is multiplied by the Fourier transform of the chosen sequence. In the next section, we introduce two common examples of these proper schemes.

Although symbols with proper partial response signalling pattern will result in lower OOB radiation compared to the case of no partial response signalling, it will increase the number of equivalent components in the signal space, which leads to increased WER because of more neighbourhoods for each component. In addition, in the proposed method, error propagation may occur because of transmitting symbols by partial response signalling (some controlled correlation) [14, 15]. These effects can be reduced by pre-coding schemes [14, 15]. The general form of pre-coding for M-array modulation used in partial response signalling with integer coefficient

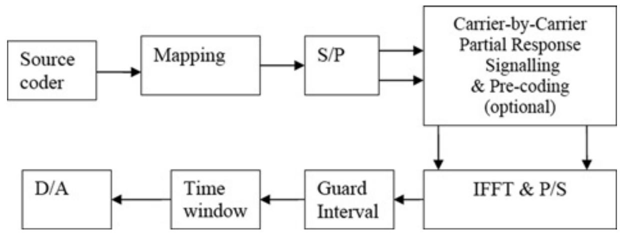


**Figure 4** *Carrier-by-carrier partial response signalling OFDM transmitter block diagram*

could be written as

$$\alpha_0 d_{k,l} = c_{k,l} - \sum_{i=1}^{L} \alpha_i d_{k-i,l} \bmod M \tag{18}$$

where $c_{k,l}$ is the original uncorrelated source output and $d_{k,l}$ is the pre-coded output. The necessary and sufficient condition for uniquely retrieving $c_{k,l}$ from $d_{k,l}$ is that $\alpha_0$ and $M$ be relatively prime [16]. In this condition

$$c_{k,l} = \sum_{i=0}^{L} \alpha_i d_{k-i,l} \bmod M \tag{19}$$

For example in the duobinary system, pre-coding can be implemented as [14, 15, 22]

$$d_{k,l} = c_{k,l} \oplus d_{k-1,l} \tag{20}$$

where $\oplus$ denotes XOR. It can be inferred from (18) and (19) that pre-coding scheme imposes more complexity on transceiver especially for non-binary cases that 'mod $M$' calculations are not as simple as binary case. In the binary case pre-coding could be easily implemented using logical devices, even for large $L$.

Based on the above discussion, the total block diagram of the transmitter can be illustrated as in Fig. 4.

Receiver block diagram is shown in Fig. 5.

# 4 Simulation results

In this section we assume an OFDM system with $N = 64$ subcarriers based on IEEE 802.11a standard and we will use 500 consecutive blocks for simulation [6]. Each subcarrier is modulated using BPSK modulation. It is assumed that permitted normalised bandwidth is in the [0.2, 1.8] interval.

Fig. 6 shows the resulting PSD when all of the subcarriers have the same partial response signalling pattern with conventional OFDM where $L = 1$. The values of $\alpha_0$, $\alpha_1$ have been selected based on well-known duobinary system which has low-pass frequency characteristics [14, 15, 22], $\alpha_0 = 1$, $\alpha_1 = 1$. In this case, the transmitted symbols are


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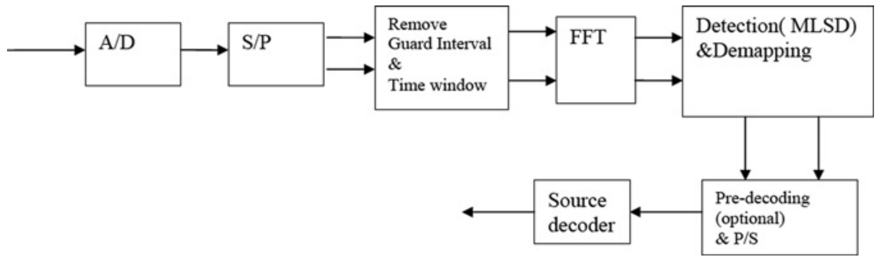


**Figure 5** *Carrier-by-carrier partial response signalling OFDM receiver block diagram*

$\{-2, 0, 2\}$ and analytic PSD using (17) becomes

$$X_{\text{Duobinary}}(f) = T_s \sum_{l=1}^{N} [|\text{sinc}(T_s[f - l\Delta f])|^2 \times |1 + \exp(-j2\pi T_s[f - l\Delta f])|^2] = T_s \sum_{l=1}^{N} [|\text{sinc}(T_s[f - l\Delta f])|^2 \times (2 + 2\cos(2\pi T_s[f - l\Delta f]))] \tag{21}$$

If no pre-coding is used in the transmitter, the receiver should detect the modulated symbols on each subcarrier by means of MLSD after performing FFT. But, by using the pre-coding of (20), receiver can be implemented easily based on the absolute value of received noisy symbol after performing FFT, $y_{k,l}$ [14, 15]

$$\begin{cases} c_{k,l} = 0 & \text{if} \quad y_{k,l} = \pm 2 \\ c_{k,l} = 1 & \text{if} \quad y_{k,l} = 0 \end{cases} \Rightarrow \begin{cases} c_{k,l} = 0 & \text{if} \quad |y_{k,l}| \geq 1 \\ c_{k,l} = 1 & \text{if} \quad |y_{k,l}| \leq 1 \end{cases} \tag{22}$$

Reduced OOB components in duobinary carrier-by-carrier partial response signalling OFDM are observable as depicted in Fig. 6. Explicit ripples in the available bandwidth in Fig. 6 are due to introduced inter-block-interference (because of partial response signalling) similar to frequency selectivity at multipath channel.

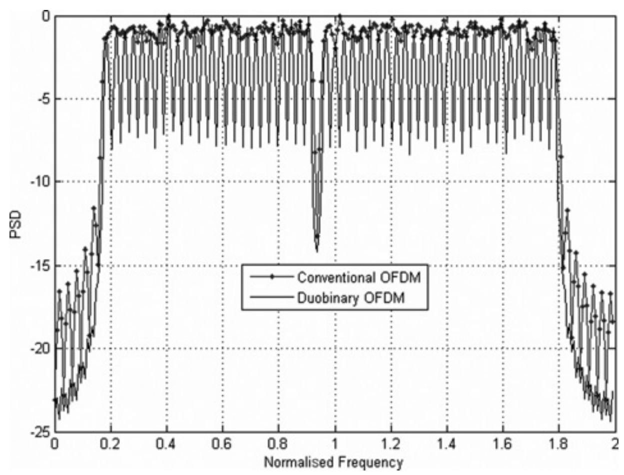


**Figure 6** *PSD comparison of conventional and duobinary carrier-by-carrier partial response signalling OFDM for 802.11a*

Now, let $L = 2$ and choose a class-2 partial response signalling which is another low-pass frequency characteristic sequence [14, 15, 22]. In this case the coefficients are $\{\alpha_0 = 1,\ \alpha_1 = 2,\ \alpha_2 = 1\}$ and five levels will be produced as $\{-4,\ -2,\ 0,\ 2,\ 4\}$ [14, 15, 22]. A typical receiver in this case will use MLSD after performing FFT. In this case, based on (17), analytic PSD expression can be written as

$$X_{\text{class}-2}(f) = T_s \sum_{l=1}^{N} [|\text{sinc}(T_s[f - l\Delta f])|^2 \times |1 + 2\exp(-j2\pi T_s[f - l\Delta f]) + \exp(-j4\pi T_s[f - l\Delta f])|^2] \tag{23}$$

By using some trigonometric equalities, the PSD becomes

$$X_{\text{class}-2}(f) = T_s \sum_{l=1}^{N} [|\text{sinc}(T_s[f - l\Delta f])|^2 \times (6 + 8\cos(2\pi T_s[f - l\Delta f]) + 2\cos(4\pi T_s[f - l\Delta f]))] \tag{24}$$

In Fig. 7, a subcarrier and overall PSD comparison of conventional OFDM, duobinary carrier-by-carrier partial response signalling OFDM, and a class-2 carrier-by-carrier partial response signalling OFDM are illustrated. It is clear from Fig. 7 that larger $L$ results in lower OOB components while it imposes more complexity. Also, as we expect, more ripples exist for larger $L$ because of more (controlled) correlation.

Fig. 8 shows the PSD of conventional OFDM, duobinary and a class-2 carrier-by-carrier partial response signalling OFDM systems for a hypothetical CR physical layer with three available frequency notches.

The OOB reduction of carrier-by-carrier partial response signalling OFDM and less interference by PUs in CR application can be realised clearly from Fig. 8. As




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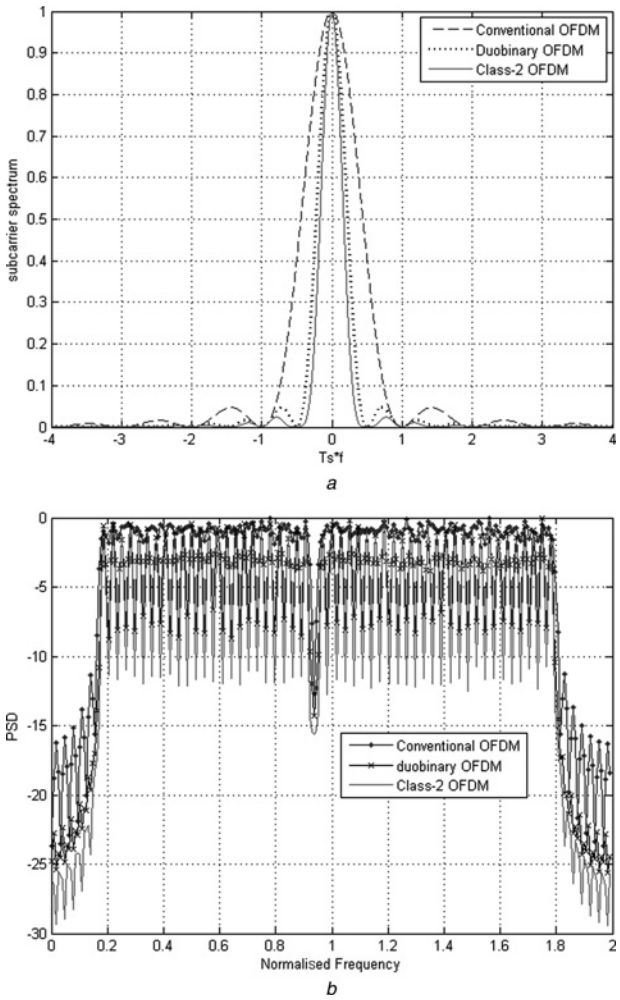


**Figure 7** *Comparison of conventional, duobinary and a class-2 carrier-by-carrier partial response signalling OFDM*

*a* A subcarrier PSD
*b* Overall PSD

mentioned above, larger $L$ can make this situation better. It makes carrier-by-carrier partial response signalling OFDM system a more appropriate candidate for physical layer of CR networks than conventional OFDM. In addition, this method allows for using other techniques to reduce OOB, PAPR and so on.

## *4.1 Comparison with other methods*

As mentioned in the introduction, there are several OOB reduction methods. Some of them such as cancellation carrier insertion have high complexity and so they are not suitable for practical applications. On the other hand, some methods such as time windowing, insertion of guard band at the border of spectrum and frequency filtering are very common in various applications and they could be implemented with acceptable complexity. Therefore we compare our proposed method to the latter. The results are shown in Fig. 9 and Table 1. In time windowing, two

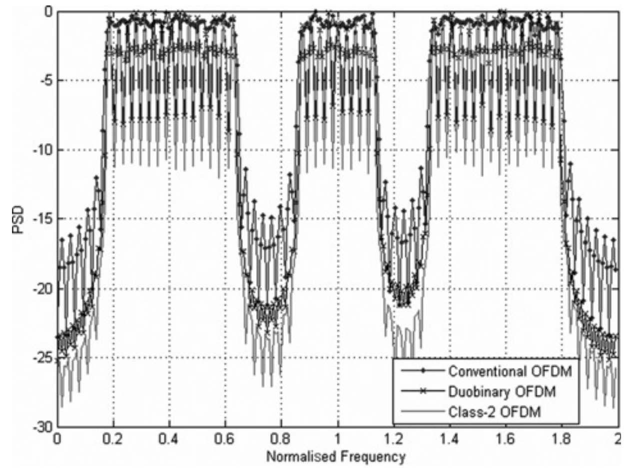


**Figure 8** *PSD comparison of conventional, duobinary and a class-2 carrier-by-carrier partial response signalling OFDM for a hypothetical CR system*

raised-cosine windows are used with $T_R/T_s$ equal to 6% and 11%, respectively. According to Fig. 9 and Table 1, this method has failed to reduce the first peak of OOB components. In addition, larger $T_R/T_s$ will result in better OOB reduction while producing more inter-block interference. In frequency filtering approach, the OOB components are filtered by a FIR filter with length 8 in the transmitter. Similar to time windowing, this method has failed to reduce peak OOB components effectively. It should be noted that although better OOB reduction is possible by frequency filtering, it requires long FIR filter that not only imposes more complexity but also extends blocks in time domain and hence creates more inter-block interference. In the last method, about 10% of subcarriers are set to zero at the border of the spectrum. Therefore the available bandwidth is not used efficiently. Fig. 9 and Table 1 show that our proposed method can reduce the main side-lobe peak more effectively compared to other methods.

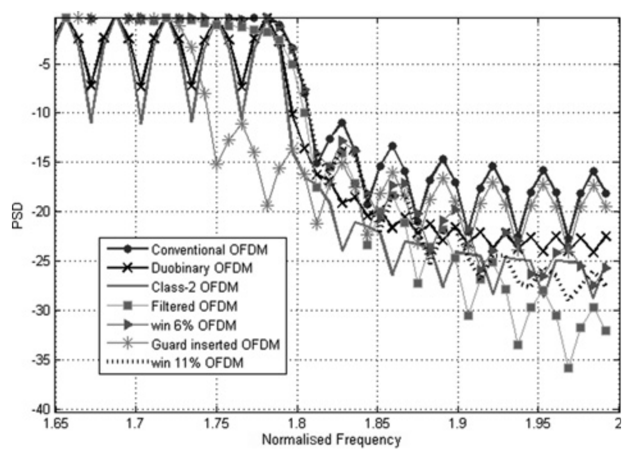


**Figure 9** *PSD comparison of conventional, proposed, windowed, filtered, and guard band inserted OFDM*

**Table 1** Comparison of peak and mean values of OOB components for investigated methods

| Method | Peak OOB, dB | Mean OOB, dB |
|---|---|---|
| Conventional OFDM | −10.74 | −16.96 |
| Duobinary OFDM | −16.71 | −21.11 |
| Class-2 OFDM | −17.95 | −23.72 |
| Time window OFDM (6%) | −12.84 | −20.28 |
| Time window OFDM (11%) | −13.45 | −21.55 |
| Frequency filtered OFDM | −14.15 | −23.92 |
| Guard insertion (10%) | −15.62 | −19.15 |

## 5 PAPR and WER analysis

In this section, we examine our proposed method, carrier-by-carrier partial response signalling OFDM, in the view of PAPR as a challenge for many OFDM systems and WER. Assume that in each OFDM block the carrying symbols, $a_{k,l}$, are independent, hence

$$E\{a_{k,l} \cdot a^*_{k,l+n}\} = \delta(n) \tag{25}$$

Hence, after performing IFFT, the assumption of independency is still valid [17, 23]. Similar to conventional OFDM systems, for large $N$ the complementary cumulative distribution function (CCDF) of PAPR converges to normal distribution [23]

$$\text{prob}\{\text{PAPR} \geq z\} = 1 - (1 - \mathrm{e}^{z})^N \tag{26}$$

Therefore we expect that carrier-by-carrier partial response signalling OFDM will not lead to greater PAPR compared to conventional OFDM systems.

Fig. 10 shows the CCDF of PAPR for conventional OFDM, duobinary and a class-2 carrier-by-carrier partial response signalling OFDM systems. The simulation parameters are the same as in Section 4 and $10^4$ OFDM blocks have been tested. This figure indicates that carrier-by-carrier partial response signalling between consecutive OFDM blocks will not trade off higher PAPR for a better OOB reduction.

Now we investigate our proposed method in the view of WER. As mentioned in Section 3, the introduced correlation between subsequent symbols on each subcarrier will cause error propagation and higher WER in the system. In [15], the lower band and upper band of the WER in AWGN channel for single carrier partial response signalling systems have been calculated as follows [15]

$$P_{\text{e,lower}} \leq P_{\text{e}} \leq \frac{M^{L-1} P_{\text{e,lower}}}{M/M - 1 P_{\text{e,lower}}(M^{L-1} - 1) + 1} \tag{27}$$

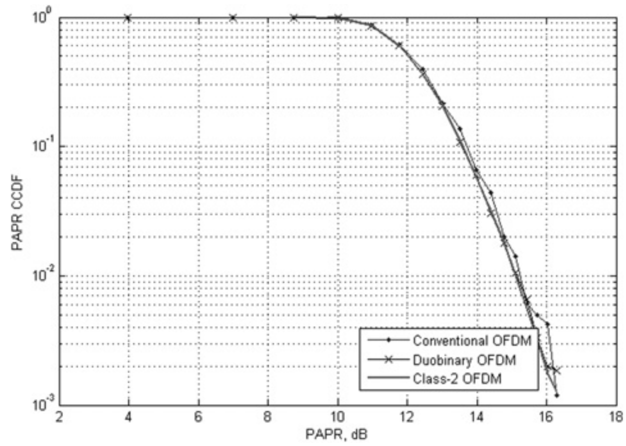


**Figure 10** *CCDF of PAPR for conventional, duobinary and a class-2 carrier-by-carrier partial response signalling OFDM*

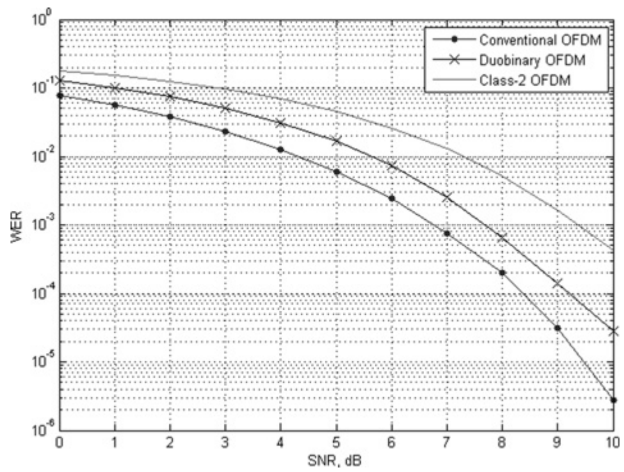


**Figure 11** *WER comparison for conventional, duobinary and a class-2 carrier-by-carrier partial response signalling OFDM*

where

$$P_{\text{e,lower}} = 2\left(1 - \frac{1}{M}\right) Q(\alpha_0/\sigma) \tag{28}$$



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and $P_e$ is the WER, $M$ is the modulation alphabet size, $\sigma^2$ is the variance of the Gaussian noise and $Q(x)$ is defined as

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty \exp(-u^2/2)\, du \tag{29}$$

Although (27) and (28) in [14, 15] have been drawn under the implicit assumption of decision feedback detector, they can be useful for overall WER behaviour of the system.

In [16], for single carrier systems, the exact WER has been calculated for MLSD in doubinary case for both non pre-coded data sequence and pre-coded data sequence according to (18). If a real M-array modulation is used, we will have

$$P_{e,\mathrm{MLSD}} = 2M(M-1)Q(d) \tag{30}$$

where $P_{e,\mathrm{MLSD}}$ is WER in non-pre-coded data sequence system, $d^2 = A^2/2\sigma^2$, and $A$ is the smallest distance between modulated symbols in the constellation. Similarly, the WER in the pre-coded data sequence, $P_{e,p\_\mathrm{MLSD}}$, becomes

$$P_{e,p\_\mathrm{MLSD}} = 4(M-1)Q(d) \tag{31}$$

It could be realised from (30) and (31) that $P_{e,p\_\mathrm{MLSD}} \le P_{e,\mathrm{MLSD}}$, and the equality holds only for $M = 2$. It means that for $M > 2$, pre-coding will prevent error propagation and hence it will improve WER performance. Although in binary modulation, $M = 2$, pre-coding will yield no WER gain, it results in very simple detection according to (22).

In Fig. 11, the simulated WER for the same OFDM system of previous section is shown. It is reasonable that larger $L$ would result in higher WER because of more correlation between symbols. Table 2 shows the approximated signal-to-noise ratio (SNR) loss because of partial response signalling compared to conventional OFDM system for the two investigated cases.

Again it should be emphasised that carrier-by-carrier partial response signalling OFDM method can be generally implemented with acceptable complexity by means of a simple digital filter in the transmitter and MLSD in the receiver. Note that in duobinary case with pre-coding and BPSK modulation, receiver can be implemented by a simple slicer rather than MLSD. Furthermore, it will not limit the system. In other words, in the proposed method, many other techniques for OOB radiation reduction can be used such as windowing, cancellation carrier insertion, in addition to PAPR reduction techniques. Also, guard interval can be added to signal as before to improve inter-block-interference robustness and obtain better synchronisation performance. In addition, larger $L$ will decrease OOB components in the cost of more complexity (especially in the receiver) and higher WER.

**Table 2** Approximated SNR loss at WER $= 10^{-3}$ for two investigated systems compared to conventional OFDM

| Method | Duobinary OFDM | Class-2 OFDM |
|---|---|---|
| approximated SNR loss at WER $= 10^{-3}$ (dB) | 0.9 | 1.7 |

## 6 Conclusion

The reduction of OOB components of an OFDM spectrum is a challenge both in conventional applications and in CR networks. Our contribution in this paper is to introduce proper carrier-by-carrier partial response signalling between consecutive OFDM blocks to reduce OOB radiation. This method neither increases effective time duration of signal nor decreases the bandwidth efficiency. Also, it is compatible with many other techniques and can be implemented in many existing systems by means of digital filtering in the transmitter and MLSD, or in special case by a simple slicer, in the receiver. However, error propagation (and thus higher WER) may occur because of introduced controlled correlation; it can be reduced by means of pre-coding [14, 15]. In addition, in the proposed system many other techniques for OOB radiation or PAPR reduction can be used. Furthermore, guard interval can be added to signal as before. Simulation results show that this method can reduce OOB radiation effectively by an acceptable additional complexity while it increases WER and does not affect the PAPR level. Also, we showed introducing correlation between more modulated symbols leads to better OOB radiation reduction, more complexity, higher WER and the same PAPR level.

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