EVALUATING MUTUALLY EXCLUSIVE INVESTMENT ALTERNATIVES: RATE OF RETURN METHODOLOGY RECONCILED WITH NET PRESENT WORTH

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In both transportation planning and investment analysis literature of recent years an occasional inconsistency has been reported between the results of the rate of return approach and the net present worth approach to the evaluation of mutually exclusive investment alternatives. This paper reviews both approaches to investment analysis and proposes a refinement in the rate of return approach. The refinement involves an examination of incremental cash flows even when the alternatives have equal initial investments and/or differing life spans. The refinement is consistent with the intent of the approach as already described in the literature and yields conclusions that are more often identical to the conclusions that result from application of the net present worth approach.

•THIS paper presents a refinement in contemporary rate of return methodology for the evaluation of mutually exclusive investment alternatives so as to bring about more general consistency between the results achieved by it and the conclusions resulting from application of net present worth methodology. A summary of both approaches is given, followed by a review of examples published since 1966 demonstrating that rate of return and net present worth methodologies can lead to different conclusions regarding the relative attractiveness of mutually exclusive alternatives with identical initial investments and/or different life spans. A procedure is then presented and applied to the examples to do away with the reported inconsistency. The ramifications of situations where there are several rates of return are then discussed.

THE METHODOLOGIES

The procedure for selecting the best of several mutually exclusive alternatives using net present worth methodology is universally recognized as involving a determination of the present worth of the cash or benefit flows for each alternative. All present worth calculations are at the investing institution's minimum attractive rate of return (MARR). The present worths of each of the cash flows for a particular alternative are added together to develop the project's net present worth. The alternative with the algebraically largest net present worth (NPW) is then deemed to be the best of the several mutually exclusive alternatives. Summarizing the foregoing, the best alternative, denoted by a^*, is the alternative for which the following inequality is true for all values of a:

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$$\sum_{t=0}^{n_{a*}} \left(\frac{1}{1+i}\right)^t A_{a*,t} \geq \sum_{t=0}^{n_a} \left(\frac{1}{1+i}\right)^t A_{a,t}$$

where

$A_{a,t}$ is the cash or benefit flow for alternative a during time period t ($A_{a,t}$ may be negative),

i is the MARR, and

n_a is the number of time periods involved in the life span for alternative a.

Rate of return methodology for evaluation of mutually exclusive alternatives is somewhat more intricate than the procedure described above. First it is necessary to arrange the alternatives in ascending order of their initial investments. Then the rate of return for each of the alternatives is determined and compared with the minimum attractive rate of return. [Actually it is not necessary to calculate the rate of return for each of the alternatives. If the alternative with the minimum investment has a rate of return in excess of the MARR, all alternatives with larger investments but with rates of return not meeting the MARR criterion will fail to be selected in the analysis of rates of return on incremental investments.] Alternatives whose rates of return are less than the MARR are stricken from the list. The alternative with the smallest investment is then considered as the basis alternative against which the alternative with the next higher investment is compared in order to determine whether the incremental investment and the cash flow following it involve a rate of return in excess of the MARR. If the alternative with the second smallest investment involves an incremental investment whose rate of return is in excess of the MARR, the second alternative replaces the first, and the first is then discarded. Otherwise the second alternative is deleted from the list. In either case the third alternative is then compared with the alternative remaining from the previous comparison in the same manner that the second alternative was compared with the first. The analysis continues iteratively until the list of alternatives is exhausted.

The procedure described in the preceding paragraph essentially follows the procedure described in Grant and Ireson (2, chapter 12) and also in Winfrey (5, chapter 7) for comparing mutually exclusive alternatives using the rate of return approach. It often involves more calculations than does the net present worth approach. Nonetheless the rate of return approach in principle brings the analyst to the same conclusion as the net present worth approach regarding the best of several mutually exclusive investments.

INCONSISTENCIES IN CONCLUSIONS ARISING FROM APPLICATION OF RATE OF RETURN AND NET PRESENT WORTH METHODOLOGIES

Grant and Ireson's statement of the rate of return methodology for evaluation of mutually exclusive investment alternatives is not specifically addressed to situations involving alternatives with differing life spans and/or equal initial investments. With respect to alternatives with differing life spans and different initial investments, there is no reason to suspect that Grant and Ireson intended the incremental analysis to be pursued any differently than as described in the preceding section. Winfrey's approach is very similar to that used by Grant and Ireson. The three authors in their two books also discuss the advisability of generally using a single analysis period for comparing mutually exclusive alternatives with different life spans. Neither of the two books specifically shows how to apply rate of return methodology in the comparison of mutually exclusive alternatives with equal investments. It appears, though, from the methodology's general application that some analysis of incremental cash flows should be made.

A review of several examples published since 1966 to demonstrate that net present worth methodology and rate of return methodology can lead to conflicting conclusions indicates that these examples invariably involve alternatives having identical initial investments and/or different life spans. Furthermore, the conclusion that the two methodologies can lead to inconsistent decisions is generally based on analysis that does not involve reviews of incremental cash flows.

Wohl and Martin in their 1967 Highway Research Board publication (6) as well as in their text published during the same year (7) present three illustrations, each involving two mutually exclusive alternatives and each demonstrating that the net present worth and rate of return methodologies lead to different results. Their first illustration includes two alternatives whose investments are unequal and whose life spans also are unequal. The rate of return for the one alternative is greater than that for the remaining alternative, but the net present worths calculated at the minimum attractive rate of return are in opposite order. Their second illustration involves alternatives with equal investments but different life spans. Again the ordering of the alternatives' rates of return is different from the ordering of their net present worths calculated at the minimum attractive rate of return. Wohl and Martin's third and final example involves two alternatives whose lives are equal and whose investments are equal but whose rates of return are in one order and net present worths are in another. Their third illustration is taken from Bierman and Smidt (1) and will be discussed at greater length later in this paper.

An illustration involving mutually exclusive alternatives is also given by de Neufville and Stafford (3). Their example involves alternatives with equal investments, but the life span for one alternative is one time unit and the life span for the other is two time units. As for each of the three Wohl and Martin examples, the order of the rates of return is opposite to the order of the net present worths.

For each of the illustrations cited above, Wohl and Martin as well as de Neufville and Stafford choose to elect the selections given by application of net present worth methodology. Their rationale is essentially that rate of return methodology implicitly assumes that positive cash flows are immediately invested at an interest rate equal to the rate of return, whereas in fact the positive cash flows are reinvested at the minimum attractive rate of return. There can be no arguing with their selections, for it is generally agreed that the very concept of the minimum attractive rate of return requires that positive cash flows that are reinvested for the long term earn interest at the MARR.

RESOLUTION OF THE INCONSISTENCY

It appears that the basic cause of the inconsistency between conclusions associated with net present worth and rate of return methodologies is essentially that the incremental analysis required by the rate of return methodology has not been completed in illustrations that strive to point out weaknesses in the rate of return methodology.

Bierman and Smidt (1) illustrate the necessity for completing incremental analysis when rate of return methodology is applied. Their illustration involves two alternatives whose net present worths and rates of return suggest different decisions. The cash flow streams are shown in Figure 1 and are respectively labeled Y and Z. [In all the cash flow diagrams shown in this paper the downward pointing arrows indicate cash outlays and the upward pointing arrows indicate cash receipts. In all of these diagrams the abscissa represents time.] With respect to alternatives with identical investments, Bierman and Smidt indicate that such a case seems different from the usual situation in which initial investments are not identical but that "... the difference is superficial" (1, p. 42, 2nd Ed.; p. 41, 3rd Ed.). They go on to determine the incremental cash flow shown in Figure 1 of this paper by the plot for (Y-Z) and then find that the incremental outlay associated with Y relative to Z does indeed produce a rate of return that exceeds the minimum attractive rate of return, although alternative Y has a lower rate of return than does alternative Z. Bierman and Smidt's conclusion then is that alternative Y is the better alternative regardless of whether the selection is made by rate of return methodology or net present worth methodology.

It is unfortunate that Wohl and Martin in both of their 1967 publications did not fully adhere to Bierman and Smidt's suggestions regarding the necessity for analysis of incremental cash flows. If they had done so their conclusions regarding the Bierman and Smidt example in Figure 1 would of course have been consistent with Bierman and Smidt's own conclusions presented in the text adjoining the table in which Bierman and Smidt partially summarize their example. Furthermore, the other two examples presented by Wohl and Martin, when subjected to similar incremental analysis as shown

Figure 1. Comparison of mutually exclusive alternatives Y and Z when minimum attractive rate of return is 5.0 percent [source: Bierman and Smidt (1, p. 42, 2nd Ed.; p. 41, 3rd Ed.); referenced in part: Wohl and Martin (6, pp. 46-48, and 7, pp. 241-243)]. Note: In this and following figures, NPW = net present worth at time zero; R of R = rate of return.

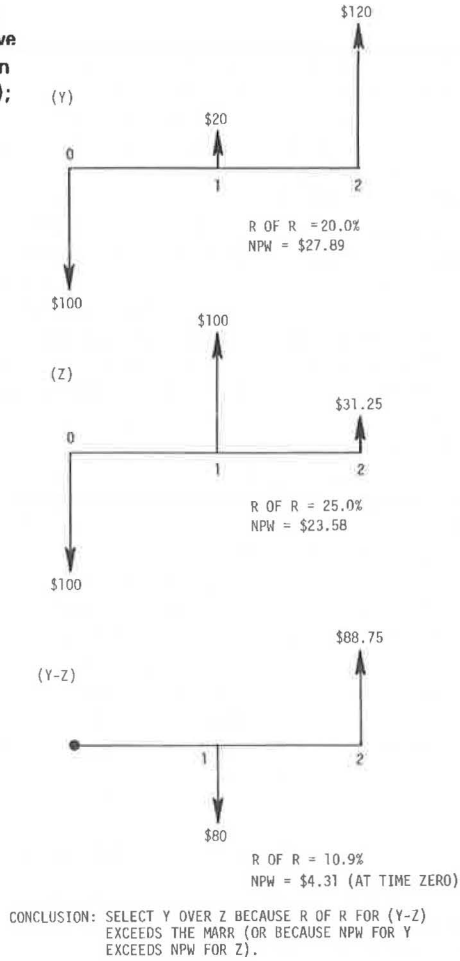


Figure 2. Comparison of mutually exclusive alternatives A and B when minimum attractive rate of return is 6.0 percent [source for cash flows A and B: Wohl and Martin (6, p. 45, and 7, p. 238)].

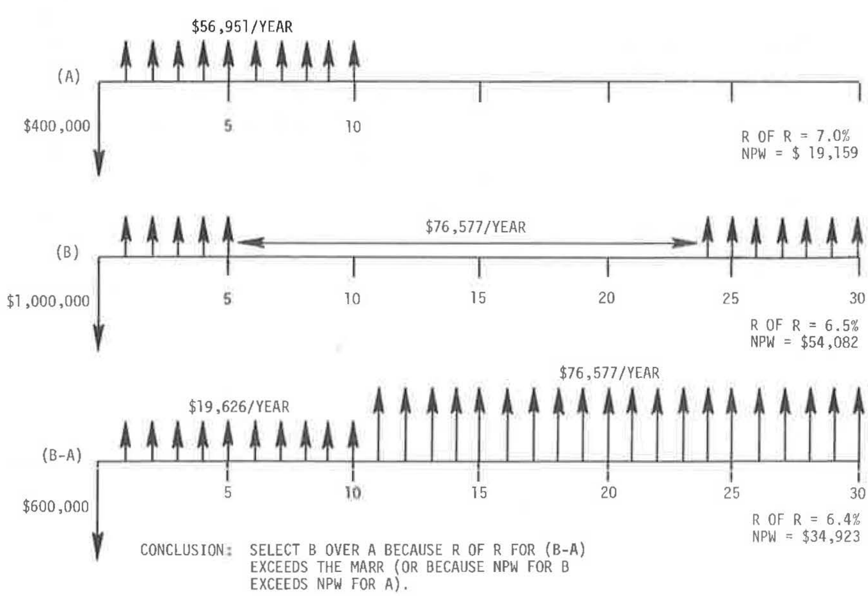


Figure 3. Comparison of mutually exclusive alternatives B and C when minimum attractive rate of return is 6.0 percent [source for cash flows B and C: Wohl and Martin (6, p. 45, and 7, p. 239)].

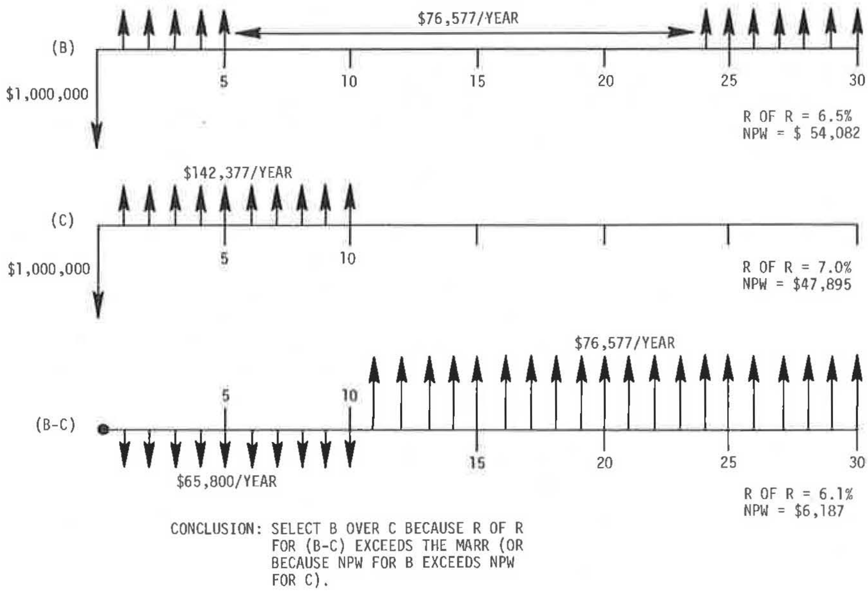
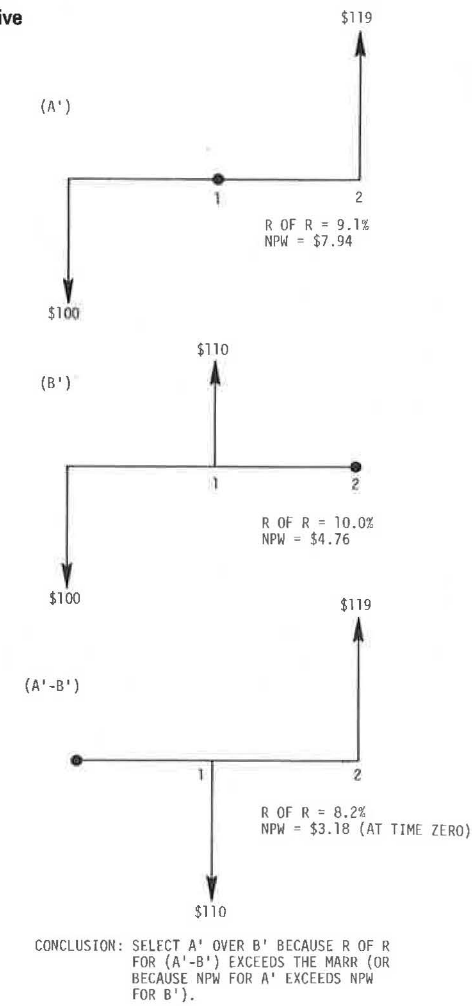


Figure 4. Comparison of mutually exclusive alternatives A′ and B′ when minimum attractive rate of return is 5.0 percent [source for cash flows A′ and B′: de Neufville and Stafford (3, p. 186)].



in Figures 2 and 3, result in conclusions that are totally consistent with those reached using net present worth methodology. The same conclusion can be made with regard to the de Neufville and Stafford example as a result of analysis that is shown in Figure 4.

To conclude then, it must be recognized that analysis of incremental cash flows is mandatory when comparing mutually exclusive alternatives, even when initial investments among alternatives are identical or when project life spans are different. Perhaps this conclusion is slightly more specific than Grant and Ireson's description of rate of return methodology for comparison of mutually exclusive alternatives. Nonetheless it is consistent with their description as well as with Bierman and Smidt's conclusions regarding the example shown in Figure 1. To summarize this conclusion that incremental analysis is mandatory in cases when the initial investments of two alternatives are identical or when their life spans are different, a flow chart has been prepared, shown here as Figure 5. Notice that the second step in this flow chart establishes a procedure for ordering alternatives with identical initial investments and that the fifth step provides a procedure for determining incremental cash flows when the life spans of the two alternatives being compared are not necessarily equal.

MULTIPLE SOLUTIONS FOR THE RATE OF RETURN

Figure 5 emphasizes the necessity of calculating rates of return for each alternative's cash flow as well as each alternative's incremental cash flow over that of the last acceptable alternative. At either stage the analyst will occasionally discover that the solution for the rate of return will not be unique, thus necessitating further work before reaching a decision. The purpose here is not to describe in detail the character of the analysis that is required; rather, it is to recognize the problem and to then point out difficulties in applying both net present worth and rate of return methodology in such cases.

Multiple solutions cannot occur unless there is more than one change in the signs of successive cash flows. For example, for the following cash flow,

Time	Cash Flow
0	-$100
1	+$250
2	-$155

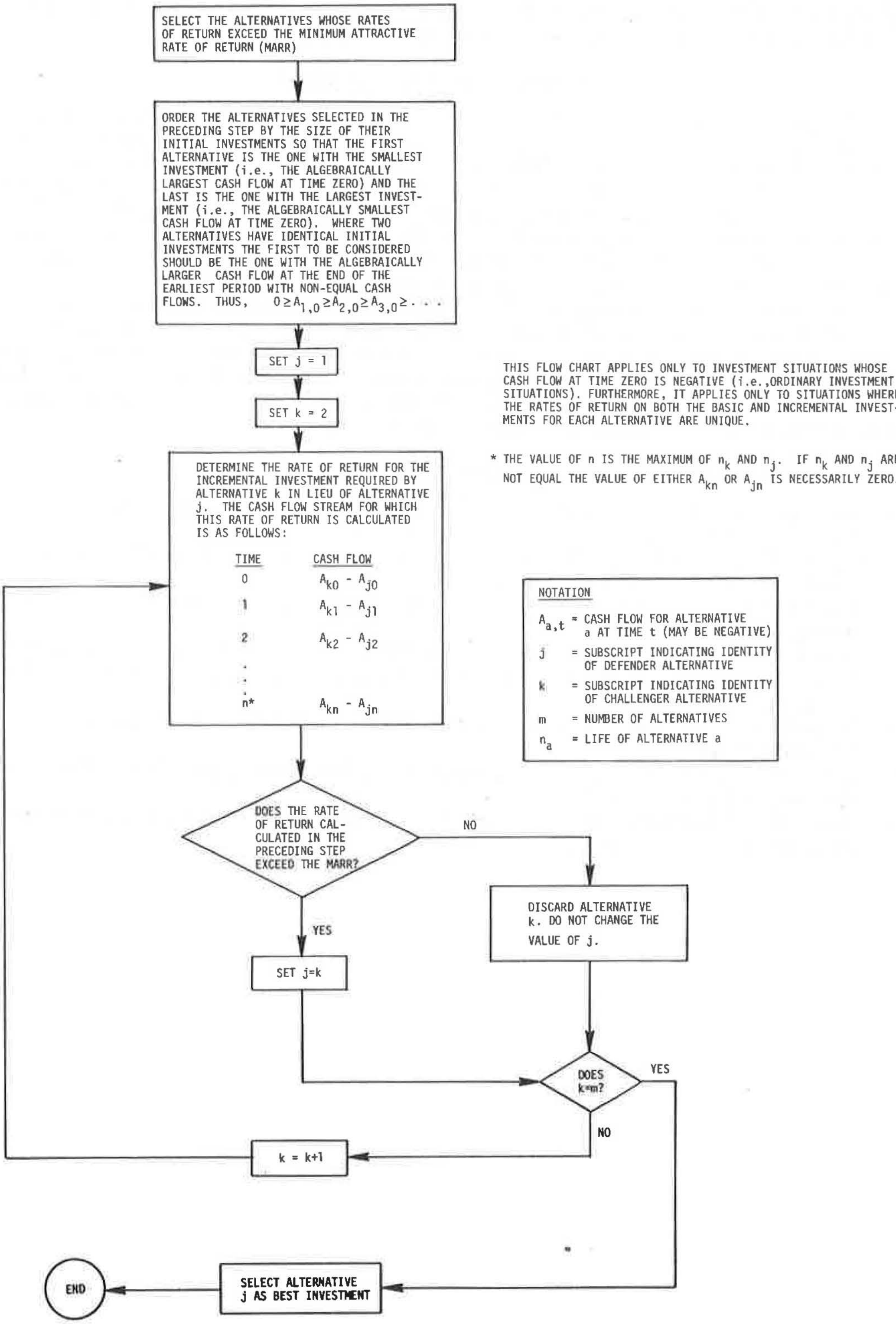
there is a change in sign between the cash flows at times 0 and 1 and again between the cash flows at times 1 and 2, making possible a maximum of two solutions for the rate of return. In this particular case there are two rates of return, whose values are 13.8 percent and 36.2 percent. For all MARR values that are either below 13.8 percent or above 36.2 percent the net present worth is less than zero. But if the MARR is between 13.8 percent and 36.2 percent it will be found to be positive. Consequently an enterprise that ordinarily has a 10 percent MARR can find itself in the curious situation of justifying this project only by increasing its MARR to some rate such as 15 percent or 30 percent. Thus both methodologies are ambiguous for the situation just described.

The resolution of the problem in the case at hand lies in ascertaining the rate of interest that applied to the portion of the funds received at time 1 and that is reinvested to provide for the outlay required at time 2. Often this rate is much less than the MARR and can be as low as the interest rate paid on short-term government securities. If that rate is 0 percent for the situation just described the net cash flow stream becomes

Time	Cash Flow
0	-$100
1	+$ 95

and the project has a negative rate of return and a negative net present worth for all positive MARR's.

Figure 5. Rate of return methodology for selecting the best of several mutually exclusive investment alternatives.



There are other ramifications of this problem that can be discussed, but these will not be reviewed here. The interested reader is encouraged to consult references such as Appendix B of Grant and Ireson (2), Chapter 3 of Bierman and Smidt (1), or an interesting paper by Teichroew, Robichek, and Montalbano (4).

SUMMARY AND CONCLUSIONS

The main point of this paper is that incremental cash flows must always be reviewed if rate of return methodology for the analysis of mutually exclusive alternatives is to yield results that are consistent with those resulting from application of net present worth methodology. Several examples that have been published since 1966 to illustrate an inconsistency between the two methodologies have been reviewed and shown to in fact involve consistent conclusions when the rate of return methodology involves review of incremental cash flows. To outline in detail the steps involved in rate of return methodology, a flow chart has been prepared and included as Figure 5.

In closing, it may be appropriate to note that rate of return methodology often involves a larger number of calculations than does net present worth methodology. Consequently no issue is taken here with the viewpoint that net present worth methodology is often simpler to apply in the evaluation of mutually exclusive alternatives than is the rate of return methodology. The only point here is that the two methodologies when properly defined do in fact yield consistent results.

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