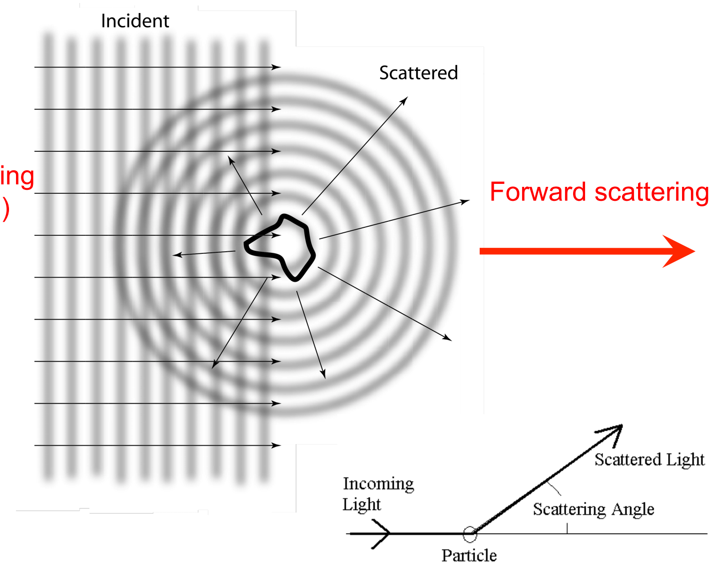
Scattering

Scattering fundamentals

- Scattering can be broadly defined as the *redirection of radiation out of the original direction of propagation*, usually due to interactions with molecules and particles
- Reflection, refraction, diffraction etc. are actually all just forms of scattering
- Matter is composed of discrete electrical charges (atoms and molecules – dipoles)
- Light is an oscillating EM field – excites charges, which radiate EM waves
- These radiated EM waves are *scattered waves*, excited by a source external to the scatterer
- The *superposition of incident and scattered EM waves* is what is observed

Scattering geometry



Backward scattering (backscattering)

Forward scattering

When does scattering matter?

- Scattering can be ignored whenever gains in intensity due to scattering along a line of sight are negligible compared to:
  - Losses due to extinction
  - Gains due to thermal emission
- Usually satisfied in the thermal IR band and for microwave radiation when no precipitation (rain, snow etc.) is present
- Also can be ignored when considering direct radiation from a point source, such as the sun
- In the UV, visible and near-IR bands, scattering is the dominant source of radiation along any line of sight, other than that looking directly at the sun

Radiative transfer with scattering

- Thermal IR and microwave bands:

$$dI = dI_{abs} + dI_{emit} = \beta_a (B - I)\, ds$$

- UV, visible and near-IR bands:

$$dI = dI_{ext} + dI_{emit} + dI_{scat}$$

$$dI = -\beta_e I\, ds + \beta_a B\, ds + \frac{\beta_s}{4\pi} \int_{4\pi} p(\cos\Theta) I(\hat{\Omega}')\, d\omega' ds$$

$\hat{\Omega}'$ = any direction

Types of scattering

- Elastic scattering – the wavelength (frequency) of the scattered light is the same as the incident light (*Rayleigh and Mie scattering*)

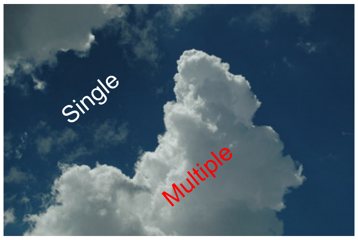
- Inelastic scattering – the scattered radiation has a wavelength different from that of the incident radiation (*Raman scattering, fluorescence*)

- Quasi-elastic scattering – the wavelength (frequency) of the scattered light shifts (e.g., in moving matter due to Doppler effects)

More types of scattering

- Single scattering: photons scattered only once
  - Prevails in optically thin media ($\tau << 1$), since photons have a high probability of exiting the medium (e.g., a thin cloud) before being scattered again
  - Also favored in strongly absorbing media ($\omega << 1$)

- Multiple scattering: prevails in optically thick, strongly scattering and non-absorbing media
  - Photons may be scattered hundreds of times before emerging



Parameters governing scattering

- (1) The wavelength (λ) of the incident radiation

- (2) The size of the scattering particle, usually expressed as the non-dimensional size parameter, x:

$$x = \frac{2\pi r}{\lambda}$$

- **r** is the radius of a spherical particle, **λ** is wavelength

- (3) The particle optical properties relative to the surrounding medium: the complex refractive index

- Scattering regimes:
  - **x << 1** : Rayleigh scattering
  - **x ~ 1** : Mie scattering
  - **x >>1** : Geometric scattering

Atmospheric particles

Type	Size	Number concentration
Gas molecule	~10^{-4} µm	< 3×10^{19} cm^{-3}
Aerosol, Aitken	< 0.1µm	~10^4 cm^{-3}
Aerosol, Large	0.1-1 µm	~10^2 cm^{-3}
Aerosol, Giant	> 1 µm	~10^{-1} cm^{-3}
Cloud droplet	5-50 µm	10^2-10^3 cm^{-3}
Drizzle drop	~100 µm	~10^3 m^{-3}
Ice crystal	10-10^2 µm	10^3-10^5 m^{-3}
Rain drop	0.1-3 mm	10-10^3 m^{-3}
Graupel	0.1-3 mm	1-10^2 m^{-3}
Hailstone	~1 cm	10^{-2}-1 m^{-3}
Insect	~1 cm	<1 m^{-3}
Bird	~10 cm	<10^{-4} m^{-3}
Airplane	~10-100 m	<1 km^{-3}

Refractive indices of substances

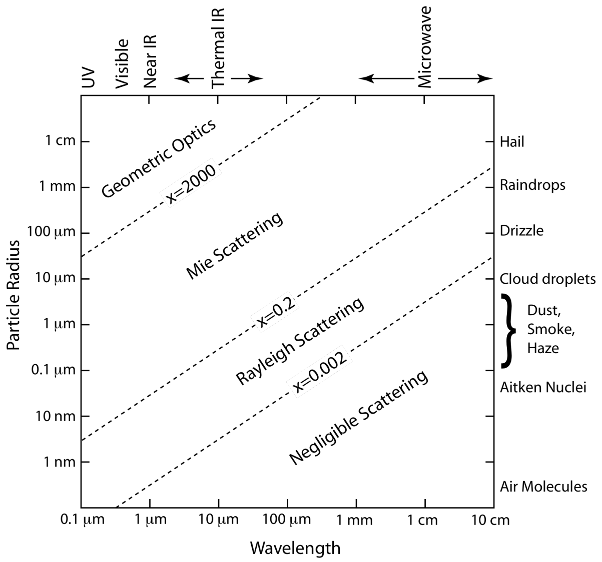
(λ = 589 nm unless indicated)

Substance	n_r	n_i	($n = n_r + i\, n_i$)
Water	1.333	0	
Water (ice)	1.309	0	
NaCl (salt)	1.544	0	
H_2SO_4	1.426	0	
$(NH_4)_2SO_4$	1.521	0	
SiO_2	1.55	0	(λ = 550 nm)
Carbon	1.95	-0.79	(λ = 550 nm)
Mineral dust	1.56	-0.006	(λ = 550 nm)

The most significant absorbing component of atmospheric particles is *elemental carbon (soot)*; reflected in the large value of the imaginary part of the refractive index.

Other common atmospheric particles are purely scattering.

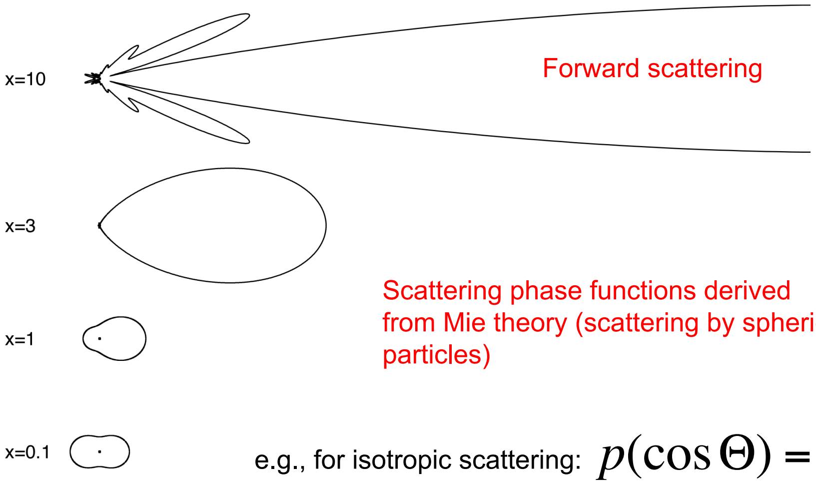
Light scattering regimes



There are many regimes of particle scattering, depending on the particle size, the light wave-length, and the refractive index.

This plot considers only single scattering by spheres. Multiple scattering and scattering by non-spherical objects can get really complex!

Scattering phase functions



Forward scattering

Scattering phase functions derived from Mie theory (scattering by spherical particles)

e.g., for isotropic scattering: $p(\cos\Theta) = 1$

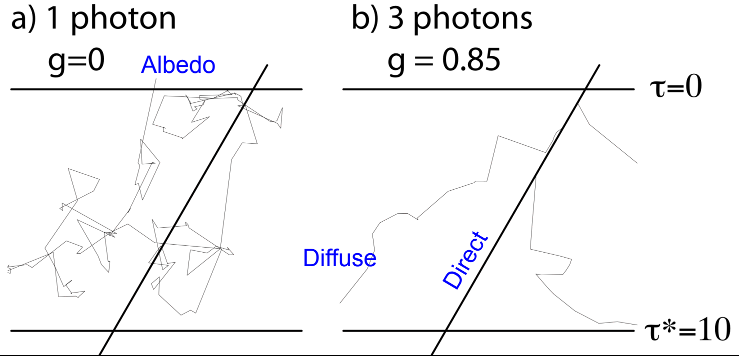
The scattering phase function, or phase function, gives the angular distribution of light intensity scattered by a particle at a given wavelength

Asymmetry parameter

- Considering fluxes, rather than intensities, the important factor is the relative magnitude of scattering in the forward and backward directions

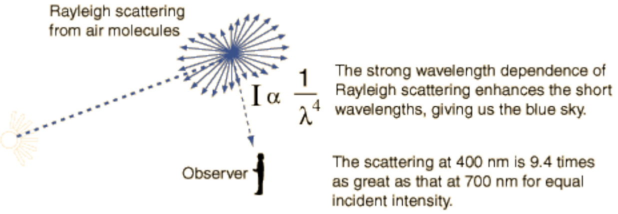
- Asymmetry parameter (g) is the average value of cos Θ for all scattered photons; i.e., $-1 \leq g \leq 1$

- $g > 0$ = forward scattering; $g < 0$ = backscattering; $g = 0$ = isotropic case



Rayleigh scattering

Atmospheric composition: N_2 (78%), O_2 (21%), Ar (1%)
Size of N_2 molecule: 0.31 nm
Size of O_2 molecule: 0.29 nm
Size of Ar molecule: 0.3 nm
Visible wavelengths ~400-700 nm



The strong wavelength dependence of Rayleigh scattering enhances the short wavelengths, giving us the blue sky.

The scattering at 400 nm is 9.4 times as great as that at 700 nm for equal incident intensity.

What color would the sky be if Earth had no atmosphere?

- Scattering of visible light off air molecules is Rayleigh Scattering
- Involves particles much smaller than the wavelength of incident light
- Responsible for the blue color of clear sky

Rayleigh scattering phase function

- **E** is the orientation of the electric field vector in the incident wave

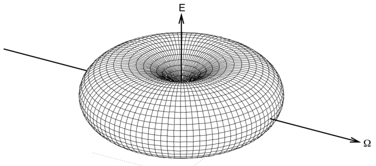
$$I = I_0 \frac{8\pi^4 N\alpha^2}{\lambda^4 R^2}(1 + \cos^2\theta)$$

Scattering at right angles is half the forward intensity for Rayleigh scattering

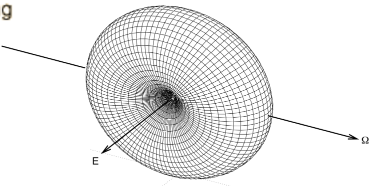
N = # of scatterers

α = polarizability

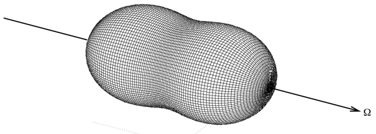
R = distance from scatterer



Vertically polarized



Horizontally polarized



Unpolarized

- Recall that scattered skylight is 100% polarized when viewing the sky at a 90º angle from the sun
- Polarizability: ease with which electrons and nuclei can be displaced from their average positions

Variation in sky brightness

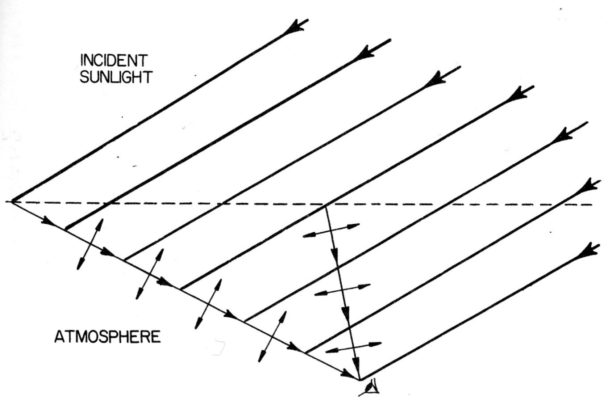


Figure 20.3 Path lengths in the atmosphere. An observer receives light scattered by all the molecules and particles along the line of sight. Paths near the horizon are longer than those near the zenith, hence the horizon sky is brighter. From *The Physics Teacher*, C. F. Bohren and A. B. Fraser, May 1985.

- The horizon sky is usually brighter than the zenith sky
- This is a result of single scattering (zenith) vs. multiple scattering (horizon)

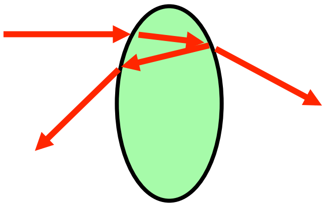
Scattering from particles is much stronger than that from molecules.

They're bigger, so they scatter more.

For large particles, we must first consider the fine-scale scattering from the surface microstructure and then integrate over the larger scale structure.

If the surface isn't smooth, the scattering is incoherent.

If the surfaces are smooth,
then we use Snell's Law
and angle-of-incidence-
equals-angle-of-reflection.



Then we add up all the
waves resulting from all
the input waves, taking into account their coherence, too (Mie theory)

Scattering by a dipole array

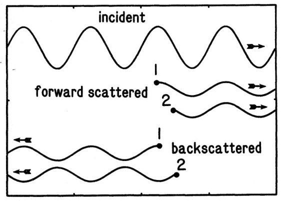


Figure 18.3 Excited by an incident wave, two dipoles scatter waves in all directions. When these two waves are added together, the resultant wave depends not only on the separation of the dipoles, but on the direction of scattering as well. In the forward direction, the two waves are exactly in phase, regardless of the separation of the dipoles. This is not true for any other direction. For the example chosen here (two dipoles one-quarter wavelength apart), the scattered waves are exactly out of phase in the backward direction. Figure courtesy of Roger Johnston.

Bohren 2001, chapter 18

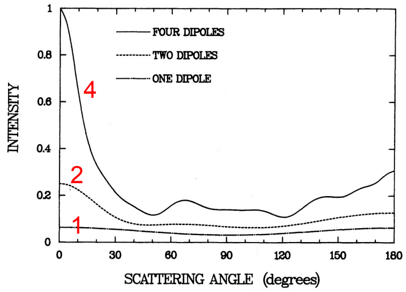
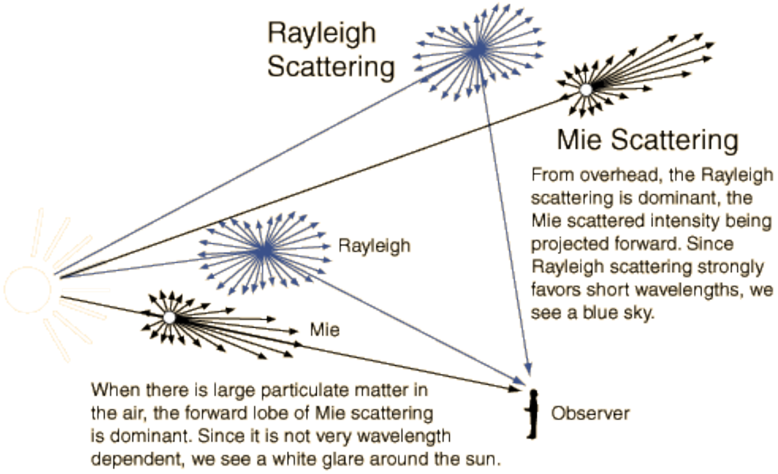


Figure 18.4 The greater the number of dipoles in an array, the more they collectively scatter toward the forward direction. This is evident with only a few dipoles. For the example shown here, all of the dipoles lie on the same line, are separated by one wavelength, and interact with one another. The scattered intensity has been averaged over all orientations of the line of dipoles. Figure courtesy of Shermila Brito Singham.

- Explains forward scattering by particles of similar size or larger than the wavelength of incident light. The larger the particle, the more it scatters in the forward direction relative to the backward direction.
- For particles (or molecules) much smaller than the wavelength, dipole separation is much smaller than wavelength, so phase differences are small, and scattering is roughly the same in all directions.

Rayleigh and Mie scattering



From overhead, the Rayleigh scattering is dominant, the Mie scattered intensity being projected forward. Since Rayleigh scattering strongly favors short wavelengths, we see a blue sky.

When there is large particulate matter in the air, the forward lobe of Mie scattering is dominant. Since it is not very wavelength dependent, we see a white glare around the sun.

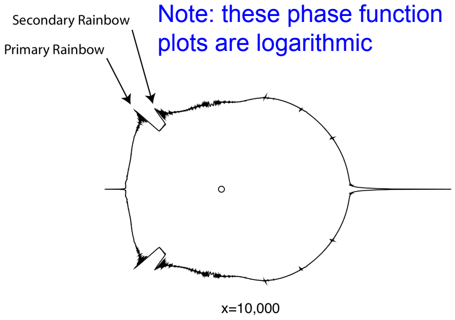
- Scattering determines the brightness and color of the sky

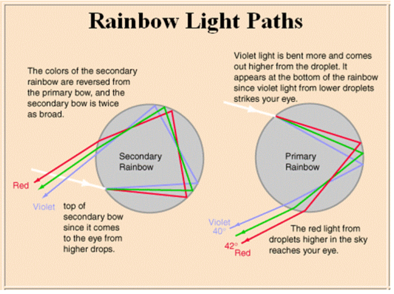
Explain the colors…

http://www.flickriver.com/photos/tags/scattering/interesting/

Optical phenomena

Note: these phase function plots are logarithmic





Rainbow Light Paths

The colors of the secondary rainbow are reversed from the primary bow, and the secondary bow is twice as broad.

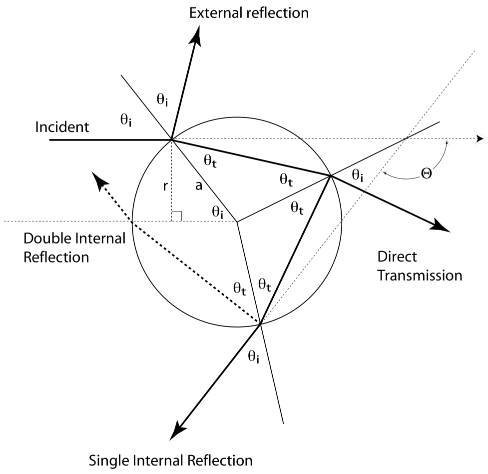
Violet light is bent more and comes out higher from the droplet. It appears at the bottom of the rainbow since violet light from lower droplets strikes your eye.

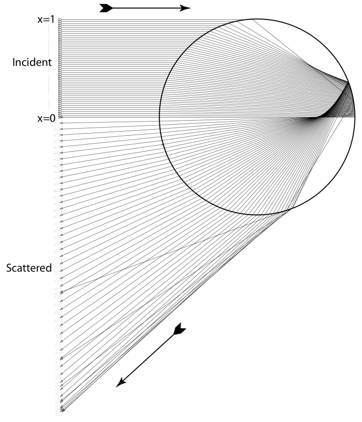
top of secondary bow since it comes to the eye from higher drops.

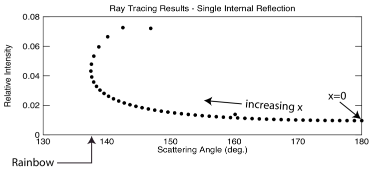
The red light from droplets higher in the sky reaches your eye.

- Rainbow: for large particles (x = 10,0000), the forward and backward peaks in the scattering phase function become very narrow (almost non-existent). Light paths are best predicted using geometric optics and ray tracing
- Primary rainbow: single internal reflection
- Secondary rainbow: double internal reflection

Rainbows

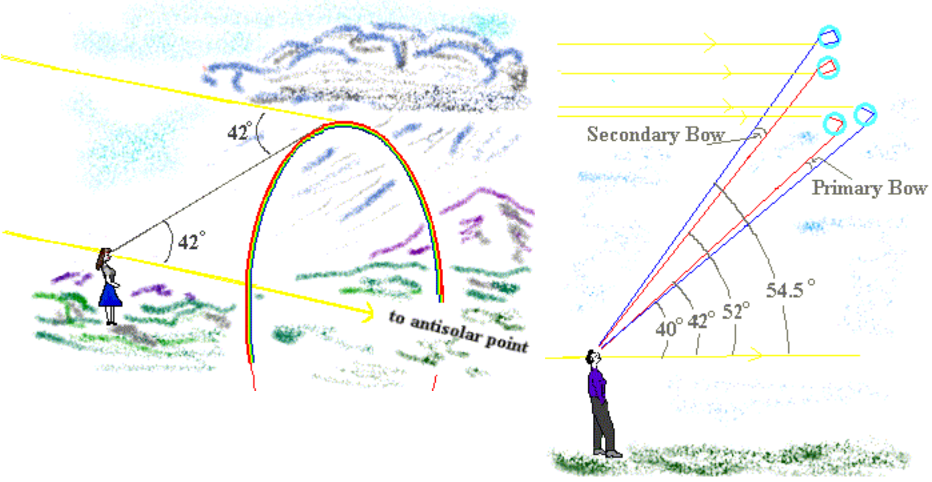






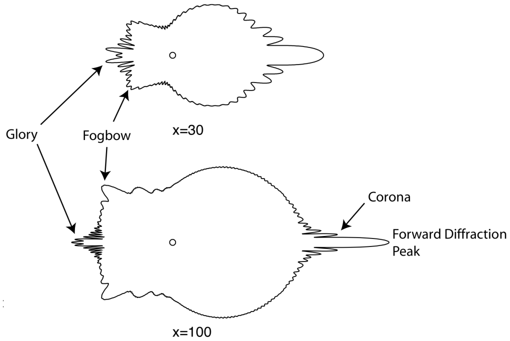
- Rainbows: angular relationships predicted from geometric optics and ray tracing (using Snel's Law)
- Focusing of energy at a particular scattering angle gives the rainbow

Rainbows



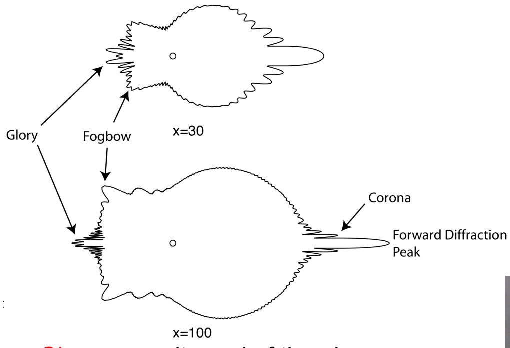
- Rainbows are seen at an angle of 42° above the antisolar point
- So if the sun is too high in the sky (higher than 42°), you don't see them

Optical phenomena



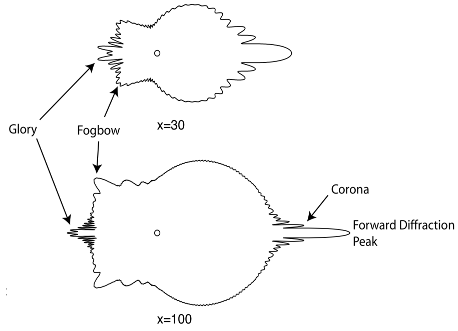
- Fogbow: spikes in scattering phase function present but not sharp as for rainbows. Hence the separation of colors (due to varying refractive index) is not as vivid as a normal rainbow. A whitish ring centered on one's shadow (i.e. opposite the sun) is seen.
- Arises when water droplets have a size characteristic of fog and clouds rather than rain

Optical phenomena



- Glory: opposite end of the phase function from the corona. Seen as a ‘halo’ around one’s shadow when looking at a fog bank with the sun at your back. Also seen from aircraft.
- Glories have vivid colors if the range of drop sizes in the fog is relatively narrow, otherwise they are whitish.

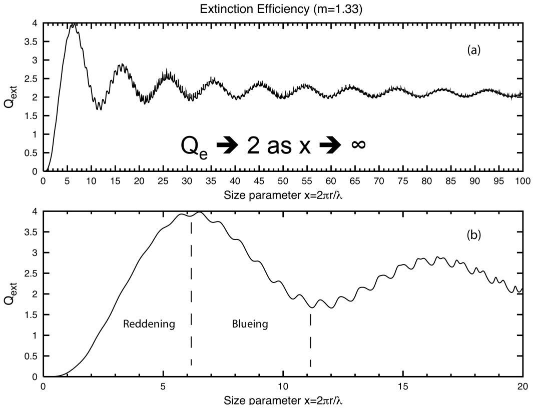
Optical phenomena



- Corona: for intermediate values of the size parameter (x), the forward scattering peak is accompanied by weaker *sidelobes*. If you were to view the sun through a thin cloud composed of identical spherical droplets (with x = 100 or less), you would see closely spaced rings around the light source. The angular position of the rings depends on wavelength, so the rings would be colored. This is a *corona*.
- Because few real clouds have a sufficiently narrow distribution of drop sizes, coronas are usually more diffuse and less brightly colored.
- Also not a good idea to look directly at the sun….

Reddening/Blueing

Non-absorbing sphere with RI (m) = 1.33



$Q_e \rightarrow 2$ as $x \rightarrow \infty$

$$Q_e = \frac{\sigma_e}{\pi r^2}$$

$$\beta_e = \sigma_e N$$

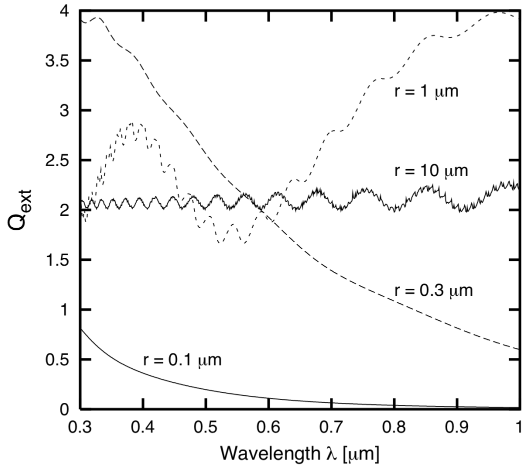
Q_e = extinction efficiency factor
σ_e = extinction cross-section

NB. Q_e can be 2 (for cloud droplets at visible wavelengths) or larger!

- Assume r is constant, so variations in x are due to variations in λ
- Hence increasing x implies decreasing λ, and vice versa.
- For 0 < x < 6, shorter wavelengths attenuated more: reddening (e.g., setting sun)
- For 6 < x < 11, longer wavelengths attenuated more: blueing

Reddening/Blueing

Extinction Efficiency

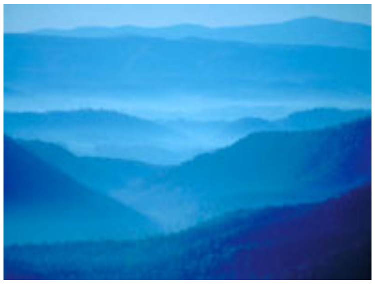


- Extinction efficiency against wavelength for selected water droplet radii
- Haze: 0.1-0.3 µm – classic reddening behavior observed on a hazy day
- Intermediate radius (1 µm) – complex behavior, blue and red light attenuated, with attenuation minimum at 0.5-0.6 µm – would give a green sun at sunset
- For larger radii (10 µm – typical cloud droplet) – no strong wavelength dependence

Once in a blue moon…

Blueing of sunlight or moonlight is only rarely observed as it requires an unusual distribution of aerosol sizes for the blueing to dominate over the reddening by air molecules.

Blue moons have been observed after large volcanic eruptions and forest fires. Blue moons and blue-green suns were seen after the 1883 eruption of Krakatoa (Indonesia)



Blue Ridge Mountains

Trees emit volatile organic compounds that oxidize in the air to form tiny oil droplets. These droplets scatter light to produce a blue hue.

Note that in this case the background is dark – so the color arises from light that *has been scattered*.

Scattering in the Rayleigh regime

$$Q_e = 4x\,\mathrm{Im}\left\{\frac{m^2-1}{m^2+2}\left[1+\frac{x^2}{15}\left(\frac{m^2-1}{m^2+2}\right)\frac{m^4+27m^2+38}{2m^2+3}\right]\right\}+\frac{8}{3}x^4\,\mathrm{Re}\left\{\left(\frac{m^2-1}{m^2+2}\right)^2\right\}$$

$$Q_s = \frac{8}{3}x^4\left|\frac{m^2-1}{m^2+2}\right|^2 \qquad Q_a = 4x\,\mathrm{Im}\left\{\frac{m^2-1}{m^2+2}\right\}$$

Hence, for x << 1, and provided **Im**(m) ≠ 0: $Q_s << Q_a \approx Q_e$

$$\tilde{\omega} \equiv \frac{Q_s}{Q_e} \propto x^3$$

Implications for absorption of thermal IR radiation by atmospheric gases?

Scattering cross-section

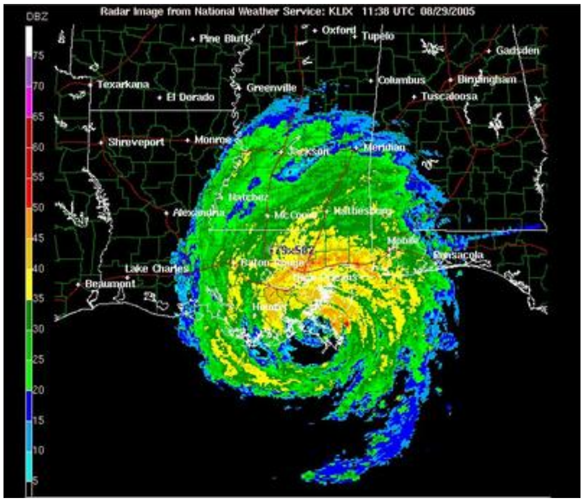
$$Q_s = \frac{\sigma_s}{\pi r^2} \qquad\qquad \beta_s = \sigma_s N$$

- According to Mie theory in the limit of x << 1 (i.e., small particles), the scattering efficiency Q_s of a particle in the Rayleigh regime is proportional to x^4
- Hence, Q_s is proportional to $\left(\frac{r}{\lambda}\right)^4$
- Using the above definition of the scattering efficiency, this implies that the scattering cross-section (σ_s), which is what determines how much radiation is scattered, we have:

$$\sigma_s \propto \frac{r^6}{\lambda^4}$$

- Note that this only applies in the Rayleigh regime, i.e. for x << 1

Radar observations of precipitation



National Weather Radar image of Hurricane Katrina in August 2005

- Allows tracking of severe weather systems in near real-time
- Relies on scattering of microwave radiation (active system) by hydrometeors

Radar observations of precipitation

- Radar transmitter sends out a series of short pulses of microwave radiation, and a receiver measures the backscattered intensity as a function of the time elapsed following each transmitted pulse (Δt).
- The one-way distance *d* to the target is then:

$$d = \frac{c\Delta t}{2}$$

where *c* is the speed of light.

- The backscattered power *P* received by the radar antenna is given by the following proportionality:

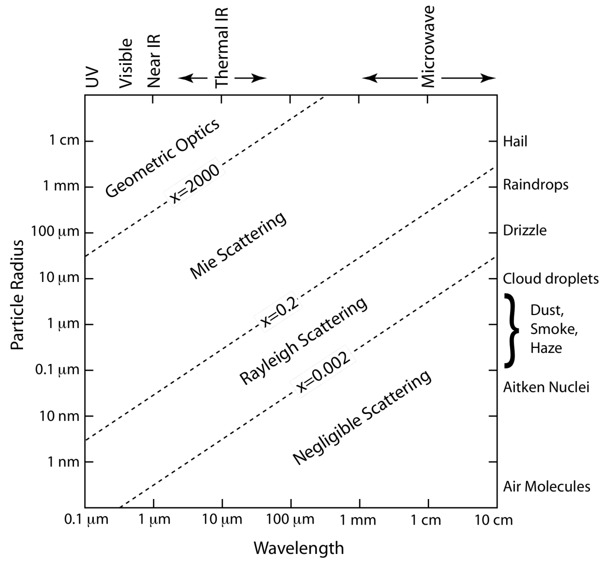
$$P \propto \frac{\eta}{d^2}$$

Where η (*eta)* is the backscatter cross-section per unit volume of air. This is the sum of the backscatter cross-sections (σ_b) of all the particles in the sampled volume of air *V*, divided by *V*:

$$\eta = \frac{1}{V}\sum_i \sigma_{b,i}$$

σ_b is closely related to σ_s, but only accounts for the radiation scattered backwards toward the radar antenna.

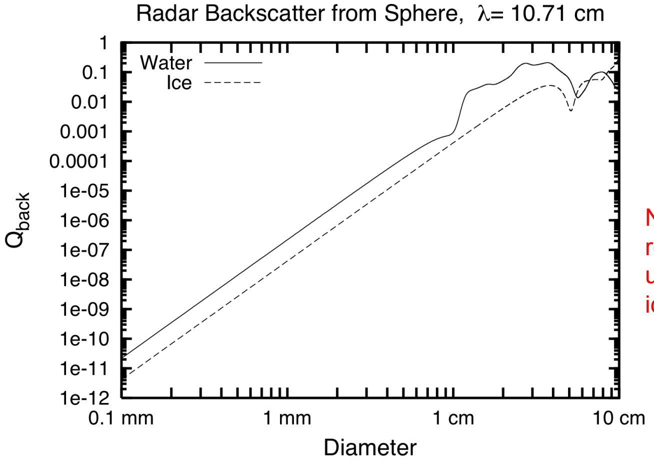
Rayleigh regime for raindrops



The Rayleigh regime for raindrops corresponds to wavelengths of ~10 cm

US Operational Weather Radar network: λ = 10.71 cm

Radar observations of precipitation



Note that Rayleigh relationship holds up to D = ~3 cm for ice (i.e. hailstones)

- Radar backscatter efficiency (Q_{back}) for water and ice spheres at the wavelength of the WSR-88D operational weather radar (wavelength = 10.71 cm)
- Up to Diameters of ~6 mm, the Rayleigh relationship (Q_{back} proportional to r^4) holds
- 6 mm is the rough upper limit of the size of raindrops observed in heavy rain

Radar observations of precipitation

- Because of these relationships:

$$Q_s = \frac{\sigma_s}{\pi r^2} \qquad Q_s \propto \left(\frac{r}{\lambda}\right)^4$$

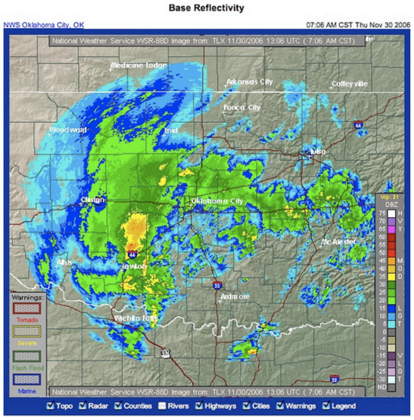
The backscattered power measured by the radar receiver is actually proportional to a reflectivity factor, *Z*:

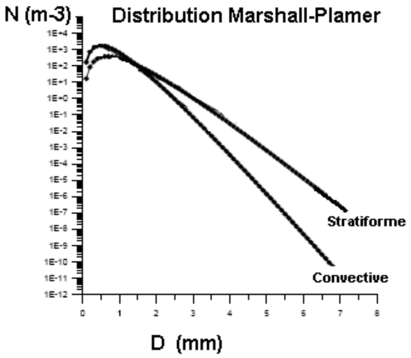
$$Z = \int_0^\infty n(D) D^6 dD$$

where *D* is droplet diameter and n(D) is the droplet size distribution function

- Hence the reflectivity factor is equal to the sum of the sixth powers of the diameters of all the drops in a unit volume of air.
- Most weather radars record and display estimates of Z at each range d.
- Standard units of Z are mm^6 m^{-3} (D in mm), but due to the enormous range of observed values of Z, a non-dimensional logarithmic unit dBZ is used:
- Z [dBZ] = 10 log (Z)

Radar observations of precipitation





- A typical weather radar measures reflectivities ranging from -20 to 70 dBZ
- Because of the D^6 dependence in Z, reflectivity is strongly influenced by the few largest drops in a volume of air – *a single drop of diameter 5 mm reflects more microwave radiation than 15,000 drops of 1 mm diameter*
- Clouds (D of ~20 μm) are invisible to most radars, despite large droplet concentrations

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Radar reflectivity example

- Take a cloud containing 100 cloud droplets per cm^3 with diameter 20 μm
- What is the radar reflectivity factor?

$$Z = \int_0^\infty n(D) D^6 dD = \sum_i N_i D_i^6$$

- Hence we have $N = 100\times10^6$ m^{-3} and $D = 20\times10^{-3}$ mm, so

  $Z = 0.0064\ mm^6m^{-3}$

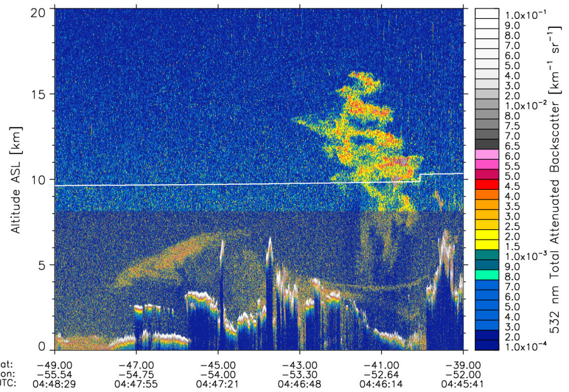
- So Z [dBZ] = -22 dBZ (i.e. very low reflectivity)

The lidar equation

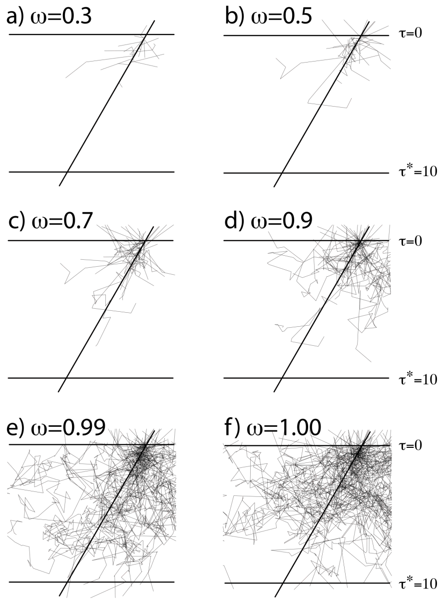
$$P_r(R) = \frac{C}{R^2}\frac{h}{2}\frac{\beta}{4\pi}\exp(-2\int_0^R \sigma_{ext}(r')\,dr')$$

P_r = received power
R = range
C = lidar ‘constant’ (transmitted power, receiver cross-section, etc.)
h = pulse duration
$\beta/4\pi$ = backscatter factor (km^{-1} sr^{-1})
σ_{ext} = extinction coefficient

CALIPSO space-borne lidar



Direct and diffuse radiation



- Random paths of 100 photons in a plane-parallel, isotropically scattering layer for variable single scatter albedo (ω)

- Some photons are transmitted directly through the cloud

- Radiation re-emerging from the top of the layer (e.g., a cloud) determines the albedo

- Radiation that emerges from the cloud base after scattering is diffuse radiation

- The remaining photons are absorbed and their energy goes into warming the cloud

Visibility

Determined by the *visual contrast between the brightness of an object and its surroundings*.

Atmospheric scattering reduces contrast by adding a source of radiation to the line-of-sight that is independent of the brightness of the target. This source is integrated along the line-of-sight, and so is greater for a longer path.
The distance at which the contrast of an object is reduced to the minimum required for visual detection defines the visibility.

Visibility

- Visibility depends on the relative difference (or contrast) between the light intensity from an object and from the intervening atmosphere.
- A simple analysis expresses visibility as a Beer's Law problem:

$$C(x) = \exp(-\beta_e x)$$

- Where C(x) is the contrast, decreasing exponentially with distance from the object. β_e is the extinction coefficient of the intervening atmosphere.

- The lowest visually perceptible brightness contrast is called the *threshold contrast*, and is typically about 2% (C(x) = 0.02). Hence, at the threshold contrast:

$$x_v = \frac{3.912}{\beta_e}$$

Koschmeider equation

Optical thickness?

- Where β_e and x have similar units (m^{-1} and m)

Visibility

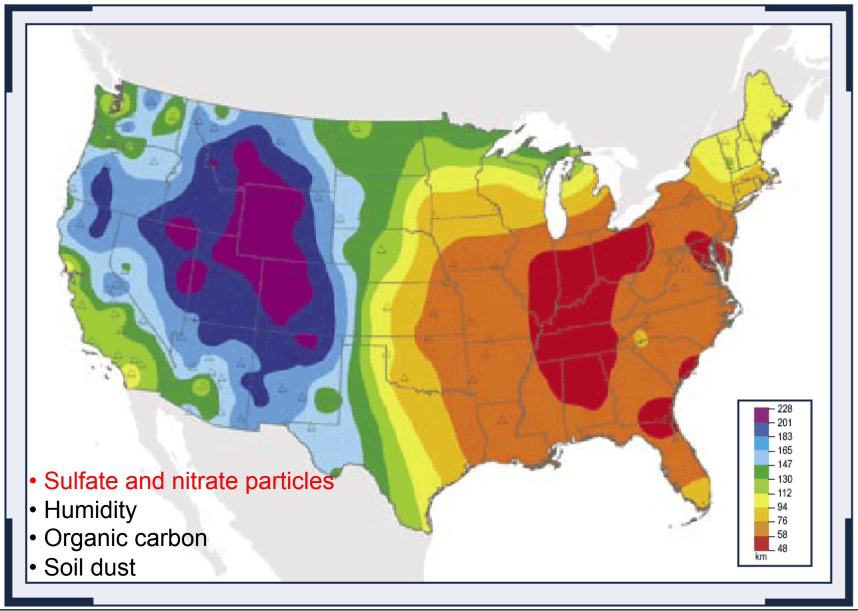
- In the absence of aerosols, extinction is due purely to Rayleigh scattering
- At sea level the Rayleigh atmosphere has an extinction coefficient β_e of ~13.2×10^{-6} m^{-1} at a wavelength of 520 nm
- This gives a visual range in the cleanest possible atmosphere of ~296 km
- Note that Rayleigh scattering is proportional to air density and decreases with altitude
- Mie scattering by aerosol particles comparable in size to visible wavelengths is responsible for most visibility reduction, and dominates in urban areas
- Note that this simple analysis of visibility neglects the reflective properties of the object, the direction of incident sunlight, the scattering phase function (which varies with aerosol type), etc.

$$x_v \approx \frac{1}{\beta_e} \ln\left[\frac{200\mu}{\omega\, p(\cos\theta)} + 1\right]$$

μ = cos (solar zenith angle)
ω = single scattering albedo
$p(\cos\theta)$ = phase function

USA visibility

Annual Average Standard Visual Range in the Contiguous United States, 2004



- Sulfate and nitrate particles
- Humidity
- Organic carbon
- Soil dust

Visibility

- A general term for light scattered by molecules and particles along a line of sight is airlight
- Airlight initially increases linearly with optical thickness (more scattering), but the increase slows down as *multiple scattering* comes into play
- A threshold contrast of 2% (0.02) corresponds to an optical thickness of ~3.9.
- Mie scattering by aerosol particles comparable in size to visible wavelengths (0.1-1 µm) is responsible for most visibility reduction, and dominates in urban areas
- Scattering by air molecules usually has a minor influence on urban visibility
- Particle absorption is ~5-10% of extinction in remote areas and up to 50% in urban areas (*carbon*)
- **Nitrogen dioxide (NO_2)** is the only light absorbing gas present in significant quantities in the troposphere
- NO_2 is strongly blue-absorbing, and hence colors plumes red, brown or yellow