FABRICATION AWARE FORM-FINDING

A COMBINED QUASI-RECIPROCAL TIMBER AND DISCONTINUOUS POST-TENSIONED CONCRETE STRUCTURE

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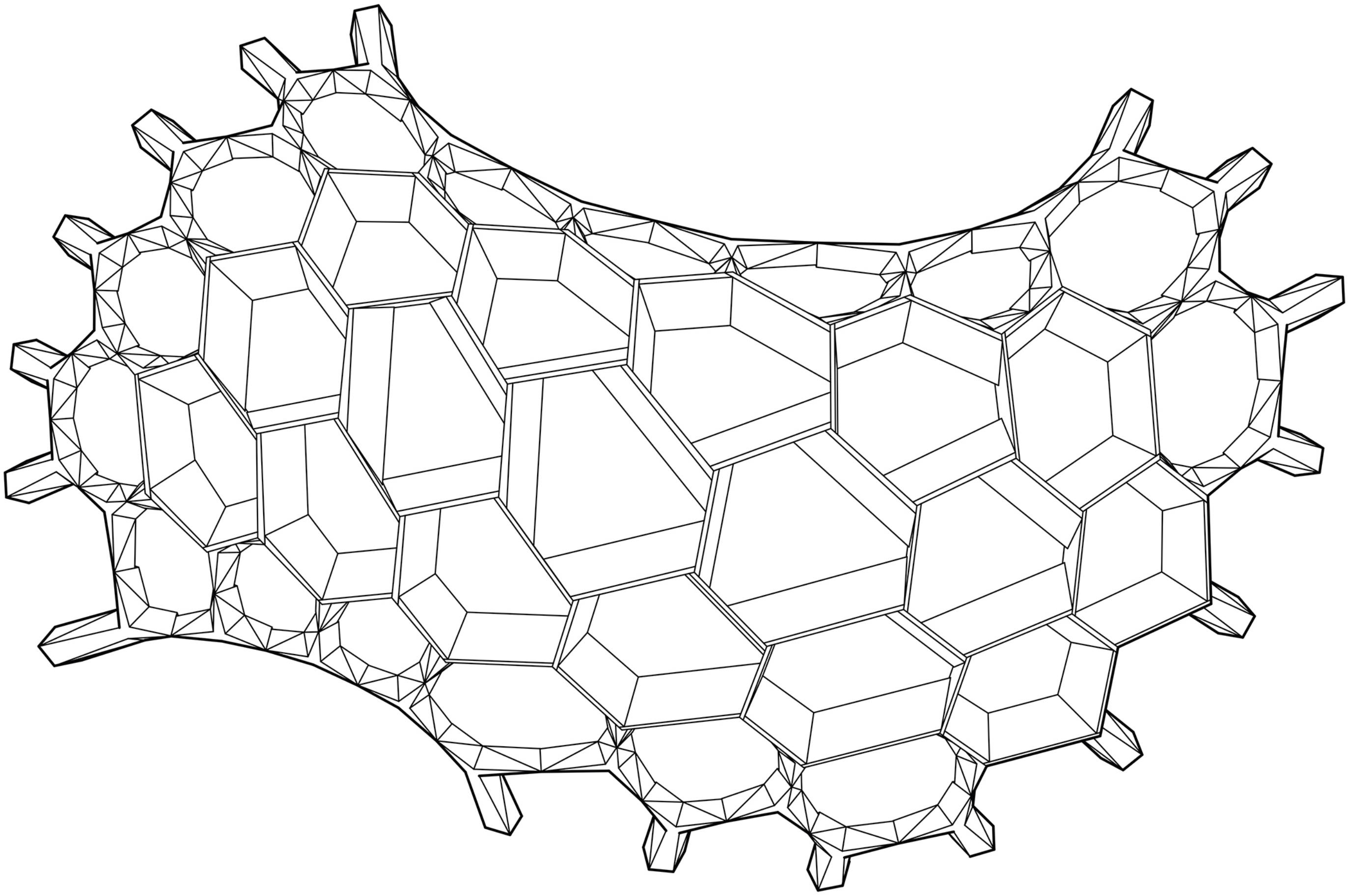


1 Utzon40 Plan (Pigram 2014)

ABSTRACT

This paper describes a design and construction method that combines two distinct material systems with fabrication aware form-finding and file-to-factory workflows. The method enables the fluent creation of complex materially efficient structures comprising high populations of geometrically unique parts. The first material system employs a novel rotated joint design to allow the structural tuning of quasi-reciprocal timber frame elements fabricated from multi-axis machined plywood sheet stock. The second employs discontinuous post-tensioning to assemble unique precast concrete components into load-bearing structures, significantly reducing or eliminating false work during assembly. The method is tested with the construction of a research pavilion.

INTRODUCTION

The influence of computation as an enabler of both the explicit negotiation between multiple constraints and of the efficient production of non-repetitive geometries offers the rejuvenation of traditional material and construction approaches within architecture. This paper describes a method that takes precast concrete and timber frame construction, both mainstays of the construction industry, and uses a computational design approach to deploy them in novel ways.

Each material system has its own set of fabrication constraints with parameters that must be kept within specific ranges for components to be producible. This method deploys an implementation of dynamic relaxation where adaptive member slack-lengths are used to keep individual parts within their fabricable range. In this way local scenarios can negotiate with the structures' overall form to simultaneously achieve multiple objectives.

The first design and fabrication innovation is a variation on a reciprocal frame system deploying multi-axis timber fabrication. A brief history of the use of reciprocal frames in architecture yields a set of defining characteristics that are then used to draw comparisons between this new method and other recent evolutions in the form.

The second design and fabrication innovation is a system for the discontinuous post-tensioning of unique precast concrete components, an extension of a method recently developed and published in this forum by the authors. The system is improved to yield a significant reduction in the demand for falsework as well as via the development of techniques to increase part accuracy. A pavilion constructed for the "What would Utzon do Now?" symposium in Sydney in 2014 serves as a case study used to situate the methods within a multi-material architectural and construction scenario (Figure 1).

FABRICATION AWARE FORM-FINDING

CONTEXT OF FORM FINDING

Like traditional design techniques–cutting, carving, folding, weaving–form-finding techniques harness the positive limitations of a given media material and physical forces resolve formal characteristics in consistent ways. Unlike traditional methods, however, form-finding processes embed a considerable level of material and structural intent within active design modeling processes. Robert Hooke's anecdotal inversion of the suspended chain sets the context for a technique-based approach to computation of funicular (compression only) geometries. The quintessential elaboration of such modeling practices is found in the nested hanging-chain models of Antonio Gaudi used to compute increasingly complex load paths.

DIGITAL FORM FINDING

Gaudi's models can be reproduced digitally through the use of *dynamic relaxation* (Day 1965). "The basis of the method is to trace step-by-step for small time increments, Dt, the motion of each node of a structure until, due to artificial damping, the structure comes to rest in static equilibrium" (Barnes 1999). Killian and Ochsendorf (2005) describe a method for the re-conception of architectural geometry (surfaces, volumes, etc) as a network of weighted *particles* (nodes) tethered to one another by variable *springs* (members). Within such a computational approach, each constituent particle negotiates its immediate neighborhood of connecting *springs* (topology) towards a state of equilibrium or equal residual force (towards a given spring length). Dynamic relaxation (DR) is therefore the iterative application of Hooke's law of elasticity: for elastic deformations of an object, the magnitude of its deformation (extension or compression) is directly proportional to the deforming force or load. Or algebraically: the applied force F equals a constant k multiplied by the displacement (change in length) x, thus: $F = kx$. Paul Bourke (1998) provides an example of one such data structure and its implementation.

EMBEDDING FABRICATION CONSTRAINT

In order to develop an expanded form-finding model, one capable of computing a greater number and quality of design and fabrication constraints, the research draws upon a custom-written implementation of dynamic relaxation (DR). Written in Python and implemented in *Rhinoceros 3D*, the bespoke library employs a node-member data structure that corresponds to the physical topology of connections in a structure.

The elegance of this approach is that it affords an open-ended and tunable approach to overall shape design through simple vector-based operations applied to nodes and members. Constraints necessary to the viability of part fabrication can be used to adaptively vary members' rest lengths, which are then negotiated to achieve multiple outcomes. In the material methods that follow, the critical limit conditions, which, when exceeded triggered a change in member behavior via its rest length, included:

Timber (Refer to Timber: Reticulated Structural Plywood Grid Shell)

> Minimum (450 mm) and maximum part length (1200 mm). These dimensions were governed by the vacuum pods (150 mm x 150 mm) used to fixture the parts during machining and acknowledged the very practical concerns of minimizing the repositioning of parts, maintaining

five-axis clearances and restricting the cantilevering of parts within an acceptable range. The slack length is decreased during the dynamic relaxation process to counter an exceeding of maximum length and vice versa.

Maximum angular deviation between node normals (fifteen degrees). This parameter results from the design detail where each member's top and bottom edges are cut as twisted ruled surfaces to eliminate stepping at the joint. Greater node deviation resulted in very sharp and vulnerable edges. When this value is exceeded, the slack length of the member between the nodes in question is contracted to pull the nodes closer together, thus reducing normal variance.

Concrete (Refer to Concrete: Complex Post-Tensioned Assemblies)

Minimum arm length (100 mm) as governed by minimum face size required to fit post-tensioning features in mold.

Maximum arm length (700 mm) as governed by need to produce a mold that can resist hydrostatic pressure and fit within the casting rig.

Minimum angle between arms (40 degrees) as governed by the necessary duct alignments and size requirements for the discontinuous post-tensioning system.

From the perspective of the designer, each of these opportunities can be engaged through multiple channels: originating geometry (mesh or curve network); manipulation through the specification of multiple member types, strengths or resistances (layer types and object properties); varying material weights (use of mesh face areas, curve attributes or node masses); or rather more explicit curation, namely experimentation and play. Thus the designer is free to approach solutions to the control of global shape and fabrication constraint from various positions.

TIMBER: RETICULATED STRUCTURAL PLYWOOD GRID SHELL

DEFINITIONS

The Reciprocal Frame (RF) has a contested definition with many finer points of disagreement in the terms or conditions for qualification. Pugnale, Parigi, Kirkegaard and Sassone, authors of a significant historical survey on structural reciprocity provide three core elements:

> the presence of at least two elements allowing the generation of a certain forced interaction ;
>
> that each element of the composition must support and be supported by another one;
>
> that every supported element must meet its support along the span and never in the vertices (in order to avoid the generation of a space grid with pin-joints). (Pugnale et al. 2011)

In a later paper Parigi and Pugnale add that RFs are characterized by "the use of load bearing elements to compose a spatial configuration wherein they are mutually supported [by] one another" (Parigi 2012). Kohlhammer and Kotnik are more exclusionary, for them an RF is "a structural system formed by a number of short bars that are connected using friction only and span many times the length of the individual bars" (Kohlhammer 2010). Song et al. offer: "[a] reciprocal frame...is a self-supported three-dimensional structure made up of three or more sloping rods, which form a closed circuit, namely an RF-unit." (Song et al. 2013) This definition is useful in that it defines the RF unit as the basic building block allowing RF morphologies to be categorized via two properties: the building block and the topology of any underlying grid. Song's requirement for sloping rods is the result of a denial of notching or mechanical fasteners. Their form-finding algorithms also work to enforce connection co-linearity in order to achieve buildability with non-notched, straight rods. Thus in combination we come to the definition: a Reciprocal Frame is a collection of short members that "mutually support each other" in "closed circuits (RF units)" to create "self-supported" spans "many times the length of the individual bars".

RECIPROCAL FRAMES: HISTORY

Reciprocal frames have a very long history in Architecture. They were used in the 12th century in China and Japan (Kohlhammer 2010) and early European drawings can be found by the medieval French architect Villard de Honnecourt (Figure 2A) and by renaissance figures Sebastiano Serlio (Figure 2B) and Leonardo Da Vinci (Figure 2C). However, their use has remained relatively limited and their academic study has developed only in the last decades (Pugnale et al. 2011).

RELATED WORK

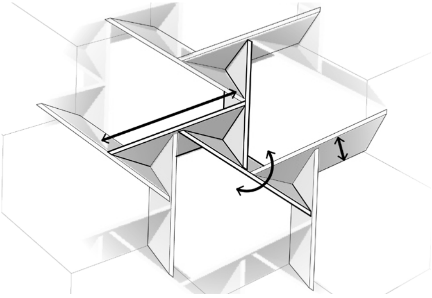
Developments in computational design techniques have led to a renewal of interest in reciprocal frames yielding new discoveries. Douthe and Baverel's novel application of Dynamic Relaxation for RF design, form finding and application to free-form geometries (Baverel 2009) is of particular note. Their efforts are directed at finding valid reciprocal geometries rather than the negotiation of fabrication constraints. Parigi and Pugnale have also made great progress with the discovery of new three-dimensional RF morphologies (Parigi 2012). Related projects contributing novel design of tectonic systems with fabrication workflow innovations and custom cut unique parts are: the Dermoid Pavilion by CITA and RMIT (Burry et al. 2012); a series of experimental structures developed at the ETH Zurich (Thönnissen 2011); and the KREOD Pavilion, designed by Chun Qing Li of Pavilion Architecture with structural engineers Ramboll UK and geometry consultant Evolute.

The latter exhibits a similar tectonic to that presented here, but does not include rotated members, joints closed on one side, or as significant geometric variation (Potman et al. 2014) (Figure 3).

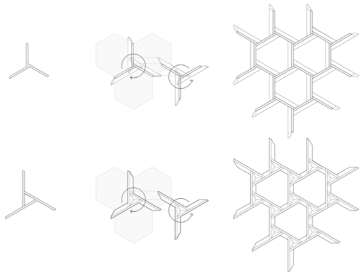
QUASI-RECIPROCAL

The joint system that represents the timber portion of this method shares many – but not all – properties of reciprocal frames. The length of each member is significantly shorter than the overall span. There are more than two members. Each member both supports, and is supported by, two others; there is no structural hierarchy. The system can clearly be understood as the combination of a series of closed circuit joints (RF units or fans), following an underlying hexagonal grid. Observing the bottom of the joint we clearly see the classic three member RF unit. However, if we examine the top of the joint, the tectonic system breaks the requirement that supports should not be at the ends of the members and appears as a network of pin-joints.

Additionally, because in this design members are joined mechanically–clearly breaking Kohlhammer's demand for friction-only connections–there is no requirement for stacking or hanging and therefore no constraint for the co-linearity of each member's connections. This– along with the fact that the members are cut to size and shaped from sheet stock–significantly relaxes the normally tight geometric coupling of engagement length with the curvature across the joint. In most RFs "[s]tacking and weaving the elements naturally shifts the entire frame out of plane, so that it becomes warped in three dimensions. The extent of the out-of-plane shifting depends directly on element thickness and the position of adjacent elements" (Goto et al. 2011). Importantly, in this method, the members are much more freely able to follow the three-dimensional network form resulting from the form-finding negotiations described above. For these reasons, we refer to the frame presented as "quasi-reciprocal"(Figure 4).



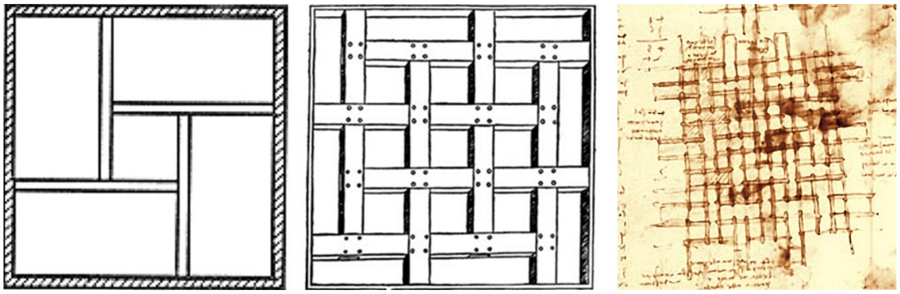
3 Quasi-Reciprocal Frame–the three variable design parameters are length, rotation and depth (Pigram 2014)



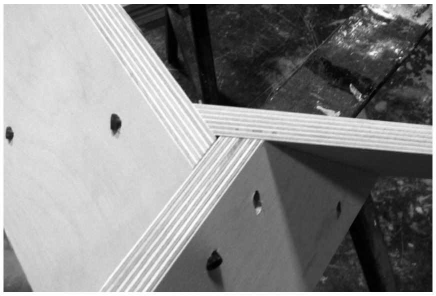
4 The two sides of the Quasi-reciprocal showing contradictory similarities to a standard pin joint (top left) and a classic reciprocal fan (bottom left) (Pigram 2014)

PRODUCTION

The development of a file-to-factory method was key to the viability of producing lightweight structures consisting of non-standardized short members. The unique members are cut out of larger sheets of 18 mm Birch plywood so that all dimensions and cut angles can be varied. Material wastage is minimized though the use of automated part nesting algorithms. A key practical demand was that the real structure must be assembled and disassembled multiple times leading to the adoption of bolts as mechanical fasteners. The nature of the quasi-reciprocal joint–where the connection slopes away

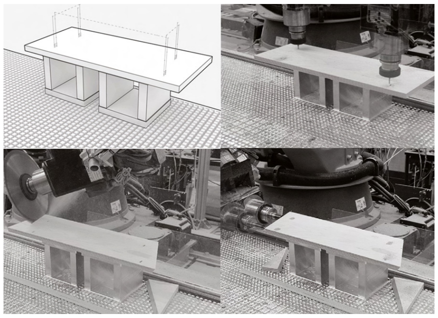


2 From left: Reciprocal frame sketch by Villard de Honnecourt (Bowie 1959); Sketch by Sebastiano Serlio (Hart and Hicks 1996); Da Vinci Self-supporting bar grid reciprocal frame (Anon 1956)



5 Detail of Timber Reciprocal Joint (Pigram 2013).

6 Timber Reciprocal Joint Frame (Wibowo 2014)



7 Timber fabrication sequence -- six-axis robotic arm fabrication method is shown (Pigram 2013)

8 Unique precast concrete components (Kristensen 2014)

9 aPET Mold Components (Kristensen 2014)

from the node–ensures the effective area for bolt placements is maximized, thereby minimizing fatigue and ensuring longevity (Figure 5). Furthermore, the joint rotation provides greater accessibility (tool clearance) to bolt locations, allowing for increased in-plane and radial stiffness by reducing the effective length of each structural member (Figure 6).

FABRICATION SEQUENCE

Each part stems from a blank, pre-cut from flat plywood stock. The blank is placed on a riser pod over a vacuum table and undergoes three operations via a five-axis CNC workcell. Tool changes are performed automatically and each part takes around five minutes to fabricate. The first tool operation mills two small slots to accept a concealed square threaded fastener. The second tool operation utilizes a 300 mm saw blade fitted into a spindle. The saw was chosen due to the high feed (cut) rates achievable with lower process forces than a router. Low reaction forces are an important requirement, as the small surface area of parts makes vacuum-fixturing difficult. Thus the saw replaces all routing operations to create the complex edge bevels. Of note, counter to intuition, it is also possible to perform variably swept edge (twisting) cuts using the saw. The final tool operation is to drill through the ends of each member to create the bolt holes (Figure 7). This fully automated fabrication sequence was originally developed on a six-axis industrial robotic arm, however, to increase part accuracies and to reduce construction tolerances, production has since shifted to a five-axis CNC gantry-style router.

CONCRETE: COMPLEX POST-TENSIONED ASSEMBLIES

PREVIOUS RESEARCH INTO CUSTOMIZABLE CONCRETE

The method described here expands upon a larger body of research into concrete and masonry structures featuring high part populations by the authors. By investigating material properties, fabrication technologies and construction techniques, the research focuses on the development of new and efficient methods for producing unique concrete elements (Figure 8).

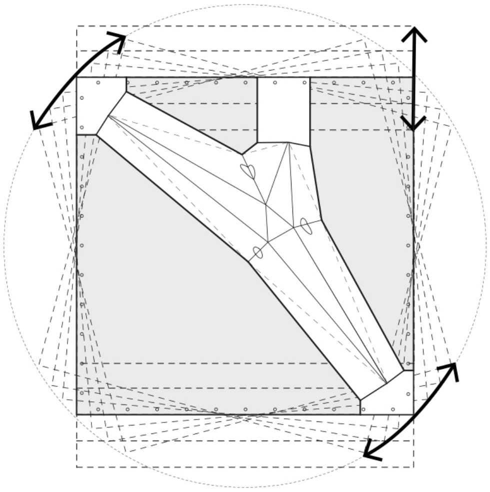
Key to this method and to its earlier manifestation (Larsen 2012) is the design and realization of three-dimensional casting molds produced from flat sheet stock by means of folding following dashed-cut lines. The mold material utilized is amorphous Polyethylene Terephthalate [aPET] plastic, normally used for packaging and soda bottles. It is easily recycled, evaporating only CO2 and water. More importantly, it has a molecular structure that allows for use and reuse without decomposition. In terms of the cradle-to-cradle design strategy, it is a technical nutrient and could remain in a closed recycling process (Braungart 2010). Cuts and fold lines were here produced via a laser cutter but a CNC knife cutter could equally have been utilized (Figure 9).

In 2012 the casting method was exemplified through the construction of a temporary pavilion–PreVault–and two key areas for improvement were identified (Larsen 2012):

a) The minimization of part inaccuracies incurred during the casting process stemming from the twisting of the folded part molds during concrete placement or compaction

b) The reduction of construction falsework required in the construction of complex masonry assemblies, specifically compression-only vault structures

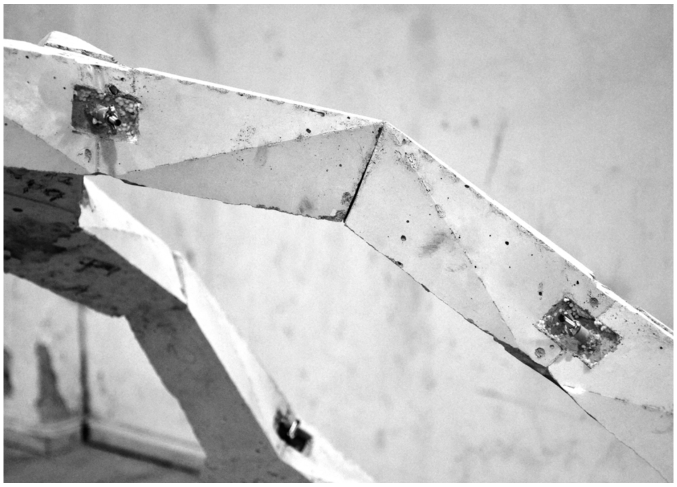
10 Timber casting rigs with alignment dowel holes for increased precision (Bamborough 2014)



11 An algorithm seeks for the optimal positioning in the casting frame and adds stabilizing features (Larsen 2014)

ADDRESSING CASTING TOLERANCES

In order to minimize distortions of the aPET molds during casting, a timber casting rig was developed. The rig, a simple three by three bay frame constructed from 90 x 45 mm timber sections, incorporates laser-cut positioning templates featuring regular 6 mm alignment holes (Figure 10). The optimal positioning of each folded plastic mold within the casting rig is determined via algorithms that test a series of possible part orientations and placements (Figure 11). Once a suitable placement is established, flaps with precise hole-patterns are created as part of the unrolled mold. These ensure each part is accurately placed and remains securely fixed, flat and resistant to rotations and distortions. The multi-bay frame allows multiple molds to be set up and poured into from the same concrete mix. Casting tolerances were reduced from plus or minus 5 mm to plus or minus 1.5 mm due to the casting rig (Figure 12).



12 Mock-up demonstrating joint precision achieved through use of casting rig (Pedersen 2014)

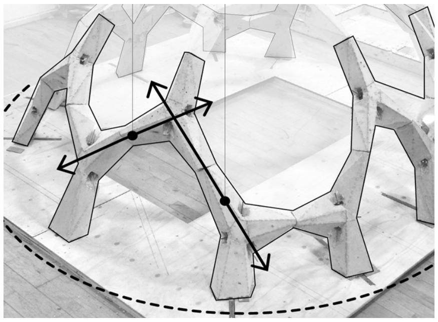
DEVELOPMENT OF A POST-TENSIONED SYSTEM

Given the relative thinness of the structure (effective structural depth), it is extremely important that the built structure matches the computationally found compression-only form so that all load paths remain within the sectional profile of the elements. Problematically, masonry structures are not stable in an incomplete form. Vaults (excluding *Catalan* vaults) require considerable falsework to ensure the temporary support and exact positioning of each component during assembly. Many, if not all, of the falsework elements will be unique (Figure 13) thereby contributing to the complexity and expense of the build!

In a simple but effective method for the elimination of construction falsework, the precast elements presented here are tied together in pairs in a discontinuous manner across each joint

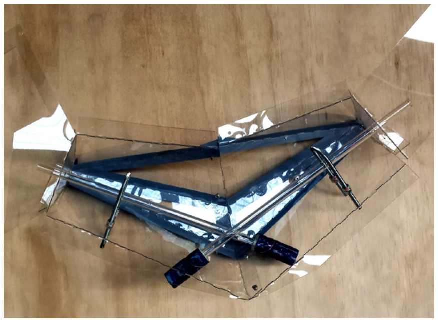
13 The Pre-Vault Pavilion 2012 demanded extensive scaffolding (Pedersen 2012)

through the introduction of a threaded steel rod secured at either end by a nut and washer assembly (Figure 14). In this way, the steel post-tensioning device can also provide support to each element during assembly. This differs greatly from traditional formats of post-tensioning which typically include continuous steel tendons or staged tendons that can only be tightened once an entire array of parts are in place. As with all modes of post-tensioning, the system provides increased orders of structural performance and safety (redundancy). The latter is seen as necessary given that the structural optimization employed here was limited to the dominant gravity case (dead load) and does not include dynamic or non-uniform live loading.


14 Ring forces enacting on the concrete structure once a level is fully assembled (Pedersen 2014)

POST-TENSIONED SYSTEM: MOLD DESIGN

The inclusion of the post-tensioning system added significant complexity to the folded mold design. Each component typically featured three ducts, one for each arm. Given the relatively small cross-section of the parts and the need for the ducts to cross-over within the component, it became necessary to order the arms in such a way that any conflicts could be avoided and sufficient concrete cover could be ensured. To avoid eccentricity across the joint, the post-tensioning rod passed through the center of the end of each arm (Figure 15).


15 The completed APET mold with plexi conduits for post-tension construction system (Kristensen 2014)

POST-TENSIONED SYSTEM: TESTING THE CONSTRUCTION SEQUENCE

The post-tensioning method allows for the assembly of components to be undertaken in a concentric or row-by-row manner as the tensile member (threaded rod) permits each element to temporarily cantilever in space. Minor propping is still required on downward parts to ensure the bearing surface of each component does not crush due to temporary misalignment (Figures 16) and (Figure 17).

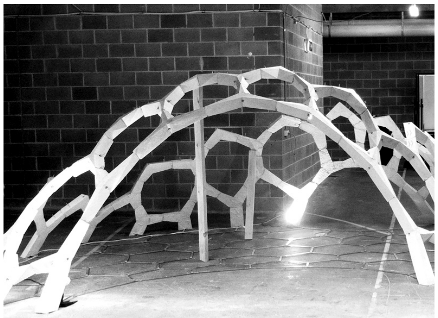

16 Construction sequence occurs in a concentric or row-by-row fashion (Pedersen 2014)

CASE STUDY: UTZON/40 PAVILION

BACKGROUND

In affiliation with the ‘What would Utzon do Now?’ symposium held in Sydney in March, 2014, the authors were afforded an opportunity to construct a pavilion celebrating the anniversary of Jorn Utzon’s magnificent Sydney Opera House. The purpose of the project was to demonstrate how the methods outlined above could permit a challenging of the economies of repetition so famously necessary to the realization of his scheme forty years ago (Mikami 2001).

MULTI-MATERIAL STRUCTURES: THE DESIGN OF THE UTZON/40 PAVILION


17 Post-tensioned concrete elements after assembly with only minor propping. Note the cantilevered elements as the geometry is incomplete (Pedersen, 2014)

Architecture operates at the complex intersection of multiple materials, scales and trades. Acknowledging the limitation of any small-scale pavilion to model a comprehensive architectural scenario–structure, enclosure, multiple materials and trades–the design of the Utzon/40 pavilion nevertheless seeks an integrated multi-material tectonic approach. The pavilion consists of sixty precast concrete elements and seventy machined timber pieces (Figure 18) and (Figure 22). In contrast to the establishment of a typical material hierarchy, that is structure in one material and cladding or secondary structure in another, the project investigates a hybrid approach where both materials act as primary structure. Due to its weight and ability to receive higher compressive loads, the concrete components are placed in the lower parts of the construction. The lightweight timber structure allows for larger spans, and is therefore used in the upper or enclosing parts of the grid shell. The concrete visually “grounds” the pavilion while the twisting-nature of the timber frame affords a striking play of light and shadow. The concrete components are well suited to interfacing with the foundations that would be present in permanent constructions and that would likely themselves also be concrete. Additionally, keeping the lower elements concrete allows all timber to avoid the potential dampness of the ground.

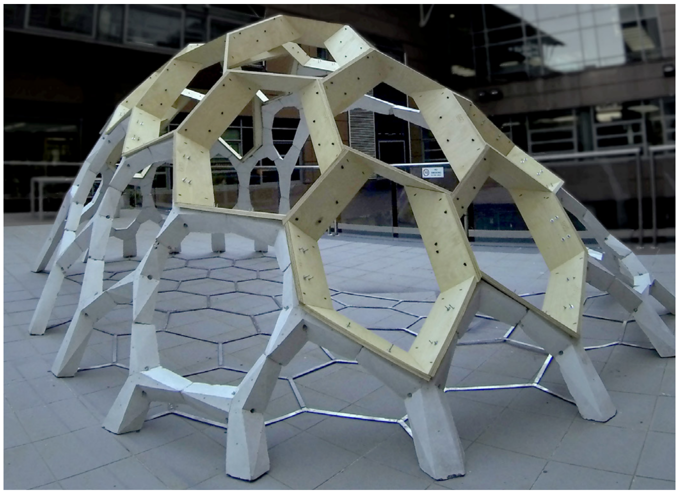
INTEGRATION OF TIMBER AND CONCRETE MATERIAL SYSTEMS

Matched timber and concrete parts are mechanically fastened with a U-shaped threaded rod cast into each concrete element. The corresponding timber part features two holes to ensure accurate placement and alignment. The script responsible for generating the casting molds–i.e. unrolled concrete geometries–is capable of detecting which elements interface with timber and adjusts their geometry in two key ways: firstly, the resulting element features only two arms; secondly, the otherwise triangulated and faceted faces of the concrete mold, desired for increase mold stiffness, are forced planar to ensure a flat face-to-face connection. Here the script gleans information from the interfacing timber part, specifically the angle of rotation in order to modify the mold pattern (Figures 19, 20).

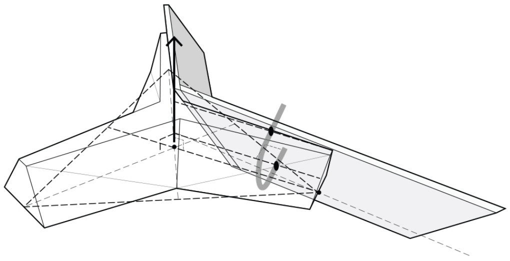
The interface strategy was developed through full-scale mockups, ensuring tolerances remained within practical limits and that any design upgrades, including the embedding of all part numbers, fabrication sequence, part alignment, post-tension rod conduits and fixing methods could be systematically tested at one-to-one (Figure 21).

CONCLUSION

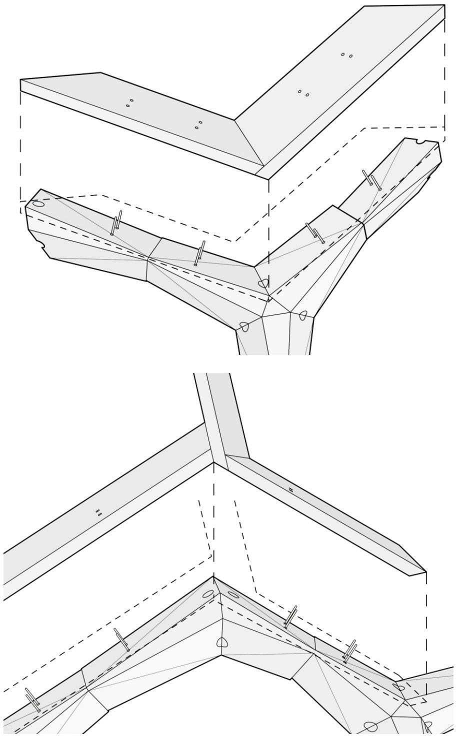
Computational techniques form the basis of a robust design method capable of ensuring structural integrity and fabrication viability within highly complex, mass-customized structures via the simultaneous negotiation of multiple constraints. As the Utzon/40 pavilion demonstrates, the successful application of computational design methods allows the knowledge of material, tectonic, and production possibilities. This knowledge is gained via a rigorous commitment to physical testing, mockups, and prototypes, to ensure critical experience and knowledge of material, tectonic, and production processes are embedded in the generating techniques of design. Thus, the pursuit of computational design methodologies and automated technologies of production can be framed as the search for a new form of design intelligence, one where the decisions of the designer and maker intermingle within an extended context of the traditions of form-finding in architecture: formation embedded design.



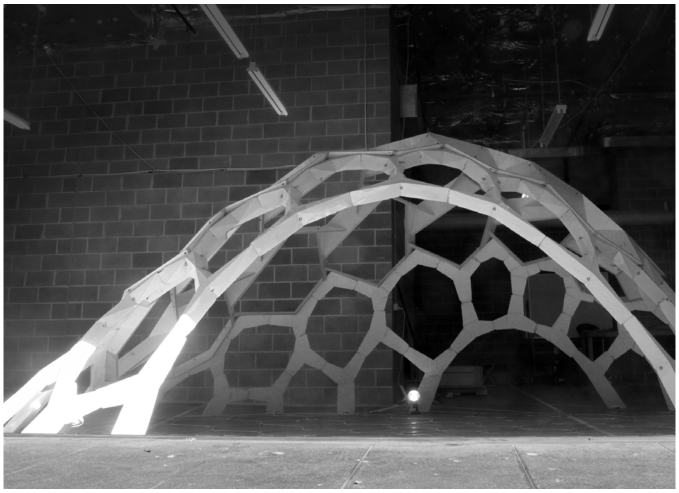
18 Completed Utzon/40 pavilion (Wibowo 2014)



19 Vectors used in both concrete and timber script to generate holes for a U-shaped connector (Pedersen 2014)



20 Concrete components are adjusted (one side flattened) to meet the timber. Bottom: An arm is removed to make room for the timber. Timber member to right of image flips to make room for concrete (Larsen 2014).



21 Mock-up testing timber and concrete interface (Pedersen 2014)

22 Utzon/40 pavilion (Wibowo 2014)

FUTURE WORK

As this paper covers multiple innovations, there are several directions for future work.

For the timber reciprocal frame, despite the successful embedment of fabrication constraints within the generative design model, the integration of Finite Element Analysis of the complex timber joint, rather than simply the centerline, would allow significant enhancement of the optimization beyond compression-only to also include other failure modes and structural actions, for example: bending moments at post-tensioned joints, rotation of nodes due to reciprocal geometry, etc. Thus, the development of a real-time pipeline between common engineering finite element analysis platforms (such as Arup's GSA and Strand7) and the generative design tools described above is an area of significant ongoing interest.

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