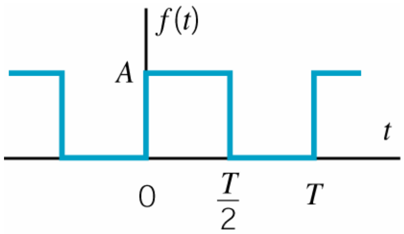
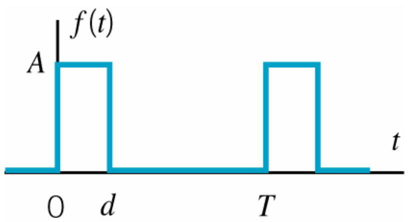
Table 15.4-1 The Fourier Series of Selected Waveforms.

Function	Trigonometric Fourier Series



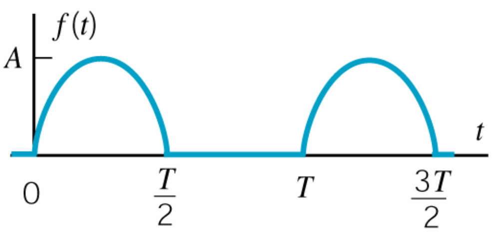
Square wave: $\omega_0 = \dfrac{2\pi}{T}$

$$f(t) = \frac{A}{2} + \frac{4A}{\pi}\sum_{n=1}^{\infty}\frac{\sin\big((2n-1)\omega_0 t\big)}{2n-1}$$



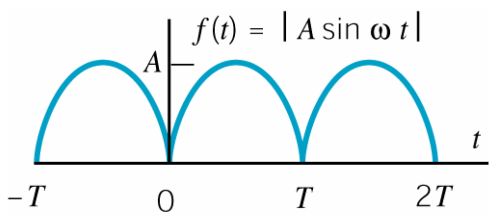
Pulse wave: $\omega_0 = \dfrac{2\pi}{T}$

$$f(t) = \frac{Ad}{2} + \frac{2Ad}{\pi}\sum_{n=1}^{\infty}\frac{\sin\left(\frac{n\pi d}{T}\right)}{\frac{n\pi d}{T}}\cos(n\omega_0 t)$$



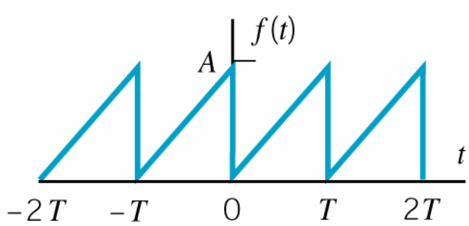
Half wave rectified sine wave: $\omega_0 = \dfrac{2\pi}{T}$

$$f(t) = \frac{A}{\pi} + \frac{A}{2}\sin\omega_0 t - \frac{2A}{\pi}\sum_{n=1}^{\infty}\frac{\cos(2n\omega_0 t)}{4n^2-1}$$



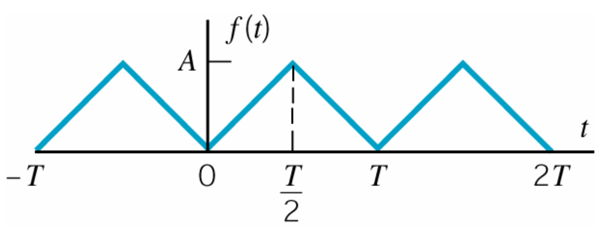
Full wave rectified sine wave: $\omega_0 = \dfrac{2\pi}{T}$

$$f(t) = \frac{2A}{\pi} - \frac{4A}{\pi}\sum_{n=1}^{\infty}\frac{\cos(n\omega_0 t)}{4n^2-1}$$



Sawtooth wave: $\omega_0 = \dfrac{2\pi}{T}$

$$f(t) = \frac{A}{2} + \frac{A}{\pi}\sum_{n=1}^{\infty}\frac{\sin(n\omega_0 t)}{n}$$



Triangle wave: $\omega_0 = \dfrac{2\pi}{T}$

$$f(t) = \frac{A}{2} - \frac{4A}{\pi^2}\sum_{n=1}^{\infty}\frac{\cos\big((2n-1)\omega_0 t\big)}{(2n-1)^2}$$

15.4 Fourier Series of Selected Waveforms

Table 15.4-1 provides the trigonometric Fourier series for several frequently encountered waveforms. Each of the waveforms in Table is represented using two parameters: *A* is the amplitude of the wave form and *T* is the period of the waveform.

Figure 15.4-1 shows a voltage waveform that is similar to, but not exactly the same as, a waveform in Table 15.4-1. To obtain a Fourier series for the voltage waveform, we select the Fourier series of the similar waveform from Table 15.4-1 and then do four things:

1. Set the value of *A* equal to the amplitude of the voltage waveform.
2. Add a constant to the Fourier series of the voltage waveform to adjust its average value.
3. Set the value of *T* equal to the period of the voltage waveform.
4. Replace *t* by $t - t_o$ when the voltage waveform is delayed by time t_o with respect to the waveform form Table 15.4-1. After some algebra, the delay can be represented as a phase shift in the Fourier series of the voltage waveform.

Example 15.4-1:
Determine the Fourier series of the voltage waveform shown in Figure 15.4-1.

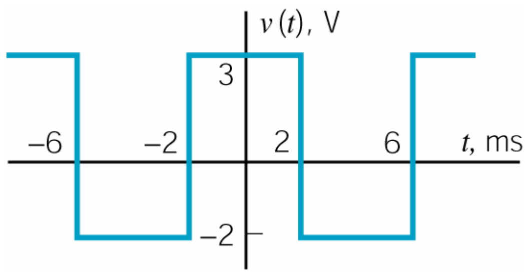


Figure 15.4-1 A voltage waveform.

Solution:
The voltage waveform is similar to the square wave in Table 15.4-1. The Fourier series of the square is

$$f(t) = \frac{A}{2} + \frac{4A}{\pi} \sum_{n=1}^{\infty} \frac{\sin\left(\left(2n-1\right)\omega_0 t\right)}{2n-1}$$

Step 1: The amplitude of the voltage waveform is $3-(-2)=5$ V. After setting $A=5$, the Fourier series becomes

$$2.5 + \frac{20}{\pi} \sum_{n=1}^{\infty} \frac{\sin\left(\left(2n-1\right)\omega_0 t\right)}{2n-1}$$

Step 2: The average value of the Fourier series is 2.5, the value of the constant term. The average value of the voltage waveform is $\left(3+\left(-2\right)\right)/2=0.5\text{ V.}$ We change the constant term of the Fourier series from 2.5 to 0.5 to adjust its average value. This is equivalent to subtracting 2 from the Fourier series, corresponding to shifting the waveform downward by 2 V.

$$0.5+\frac{20}{\pi}\sum_{n=1}^{\infty}\frac{\sin\left(\left(2n-1\right)\omega_0\, t\right)}{2n-1}$$

Step 3: The period of the voltage waveform is $T=6-\left(-2\right)=8\text{ ms.}$ The corresponding fundamental frequency is

$$\omega_0=\frac{2\pi}{0.008}=250\pi\ \text{ rad/s}$$

After setting $\omega_0=250\pi\ \text{ rad/s}$, the Fourier series becomes

$$0.5+\frac{20}{\pi}\sum_{n=1}^{\infty}\frac{\sin\left(\left(2n-1\right)250\pi\, t\right)}{2n-1}$$

Step 4: The square wave in Table 15.4-1 has a rising edge at time 0. The corresponding rising edge of the voltage waveform occurs at -2 ms. The voltage waveform is advanced by 2 ms or, equivalently, delayed by –2 ms. Consequently, we replace *t* by $t-\left(-0.002\right)=t+0.002$ in the Fourier series. We notice that

$$\sin\left(\left(2n-1\right)250\pi\left(t+0.002\right)\right)=\sin\left(\left(2n-1\right)\left(250\pi\, t+\frac{\pi}{2}\right)\right)=\sin\left(\left(2n-1\right)\left(250\pi\, t+90°\right)\right)$$

After replacing *t* by $t+0.002$, the Fourier series becomes

$$v\left(t\right)=0.5+\frac{20}{\pi}\sum_{n=1}^{\infty}\frac{\sin\left(\left(2n-1\right)\left(250\pi\, t+90°\right)\right)}{2n-1}$$