Fundamental Dimension Limits of Antennas

Ensuring Proper Antenna Dimensions in Mobile Device Designs

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Abstract—The electronics industry has historically decreased the physical dimensions of their product offerings. In the age of wireless products this drive to miniaturize continues. Antennas are critical devices that enable wireless products. Unfortunately, system designers often choose antenna dimensions in an ad hoc manner. Many times the choice of antenna dimensions is driven by convenience rather than through the examination of fundamental electrical limitations of an antenna. In this presentation the fundamental limits and the trade-offs between the physical size of an antenna and its gain, efficiency and bandwidth are examined. Finally, we examine the difficulty experienced in determining the physical dimensions of an antenna when "non-antenna" sections of a device's structure may be radiating.

"It was the IRE (IEEE) that embraced the new field of wireless and radio, which became the fertile field for electronics and later the computer age. But antennas and propagation will always retain their identity, being immune to miniaturization or digitization."

— Harold A. Wheeler

Electrically Small Antennas

Many customers often budget the amount of antenna volume for a given application on an ad hoc basis rather than through the use of electromagnetic analysis. Frequently the volume is driven by customer convenience and is small enough that performance trade-offs are inherent in the antenna solution. Many times the volume allotted may be such that only an electrically small antenna can be used in the application.

Early in a design cycle it is important to determine if the physical volume specified is, in theory, large enough electrically to allow the design of any antenna which can meet the impedance bandwidth requirements specified. There is a fundamental theoretical limit to the bandwidth and radiation efficiency of electrically small antennas. Attempting to circumvent these theoretical limits can divert resources in an unproductive manner to tackle a problem which is insurmountable.

The first work to address the fundamental limits of electrically small antennas was done by Wheeler in 1947.[1] Wheeler defined an electrically small antenna as one whose maximum dimension is less than $\frac{\lambda}{2\pi}$. This relation is often expressed as:

$$ka < 1 \tag{1}$$

$k = \frac{2\pi}{\lambda}$ (radians/meter)

λ=free space wavelength (meters)

a=radius of sphere enclosing the maximum dimension of the antenna (meters)

The situation described by Wheeler is illustrated in Figure 1–1. The electrically small antenna is in free space and may be enclosed in a sphere of radius a. $ka < 1$.

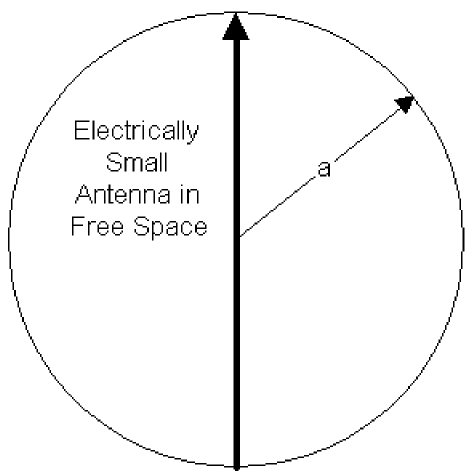

Figure 1–1 Sphere enclosing an electrically small radiating element.

In 1987 the monograph *Small Antennas* by Fujimoto, Henderson, Hirasawa and James summarized the approaches used to design electrically small antennas.[2] They also surveyed refinements concerning the theoretical limits of electrically small antennas. It has been established that for an electrically small antenna, contained within a given volume, the antenna has an inherent *minimum* value of Q. This places a limit on the attainable impedance bandwidth of an Electrically Small Antenna (ESA). The higher the antenna Q the smaller the impedance bandwidth.

The efficiency of an electrically small antenna is determined by the amount of losses in the conductors, dielectrics and other materials out of which the antenna is constructed compared with the radiation loss. This can be expressed as:

$$\eta_a = \frac{R_r}{R_r + R_m} \tag{2}$$

η_a = efficiency of ESA

R_r = Radiation Resistance (Ω)

R_m = Material Loss Resistance (Ω)

The input impedance of an ESA is capacitive and in order to provide the maximum transfer of power into the antenna's driving point a matching network may be required. The efficiency of the antenna and its matching network is expressed as:

$$\eta_s = \eta_a \eta_m \tag{3}$$

η_s = efficiency of system (i.e. antenna and matching network)

η_m = efficiency of matching network

Using common assumptions, the efficiency of the matching network is approximately:

$$\eta_m \approx \frac{\eta_a}{1 + \frac{Q_a}{Q_m}} \tag{4}$$

Q_a = Q of ESA

Q_m = Q of matching network

Often the efficiency of an ESA is obtained from measurement using a "Wheeler Cap." The near field region of an ESA has been shown to reside within a $\frac{\lambda}{2\pi}$ radius. The placement of a $\frac{\lambda}{2\pi}$ radius "Wheeler Cap" on and off of an antenna produces a minimal disturbance of the fields. When the cap covers an antenna, the radiated and reactive power is trapped. Removing the cap allows the radiated power to propagate into free space. The dissipative loss resistance can be separated from the radiation resistance using S-parameter measurements.[3]

In 1996 McLean refined and corrected earlier work on the minimum Q of an ESA.[4] The minimum Q for an electrically small linear antenna in free space is expressed as:

$$Q_L = \frac{1}{k^3a^3} + \frac{1}{ka} \tag{5}$$

The minimum Q for an ESA which is circularly polarized is:

$$Q_{cp} = \frac{1}{2}\left[\frac{1}{k^3a^3} + \frac{2}{ka}\right] \tag{6}$$

Equations (5) and (6) assume a *perfect lossless* matching network.

In Figure 1-2 is a graph of the minimum radiation Q associated with the TE_{01} or TM_{01} spherical mode for a linearly polarized antenna in free space. The exact curve was derived by McLean and should be used in any estimates of minimum Q.

The minimum Q relationship was originally derived for the case of an ESA in free space. In any practical environment an electrically small antenna is near some type of groundplane or other structure. In 2001 Sten et al. evaluated the limits on the fundamental Q of an ESA

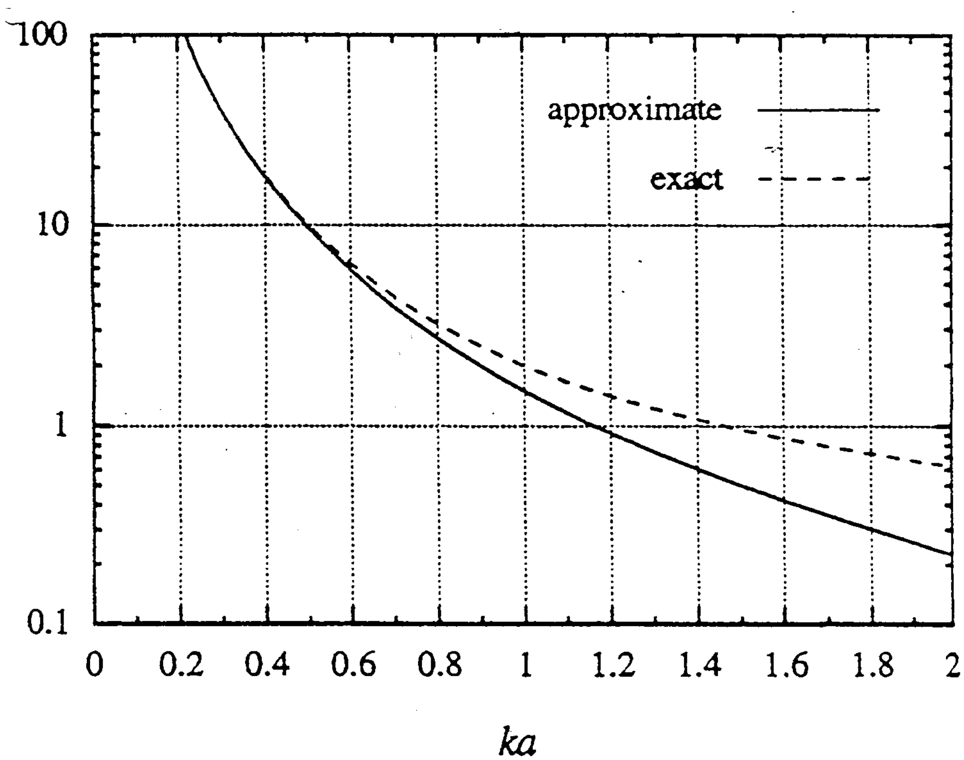

Figure 1–2 Q of an ESA vs. ka. The exact curve was derived by Mclean.[4]

near a groundplane.[5] These relationships provide useful guidelines on theoretical limits to the development of an ESA with a desired impedance bandwidth.

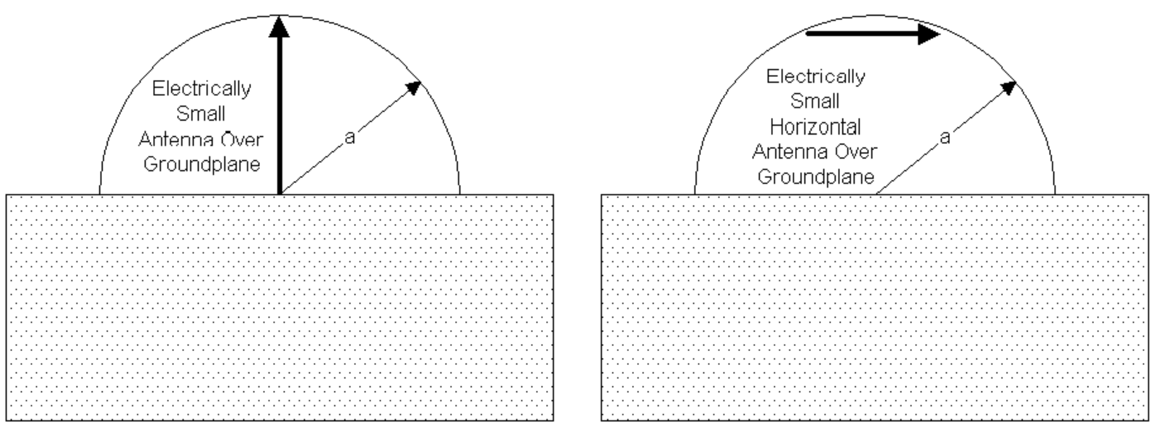

Figure 1–3 Vertical and horizontal ESA's over a large groundplane and their enclosing spheres.

The Q for the case of a horizontal current element and a vertical current element over a groundplane are analyzed as illustrated in Figure 1–3. The formulas for the Q of both instances are found in Sten et. al.

The approximate bandwidth for an RLC type circuit in terms of Q is:

$$BW = \frac{S-1}{Q\sqrt{S}} \tag{7}$$

$S = S : 1$ VSWR

$BW =$ normalized bandwidth

Figure 1–4 presents these results in a graphical form of the maximum (normalized) percent impedance bandwidth for the vertical and horizontal polarization cases with respect to the radius of a sphere which encloses the ESA. In the situation of a vertical ESA over a groundplane we find its Q is equivalent to the free space case. When a horizontal current is over a groundplane the radiation efficiency is reduced. The tangential electric field near a conductor is zero and as a horizontal ESA is brought closer and closer to the conductor surface the radiation decreases, the energy in the stored near fields increases, the Q becomes large, and the bandwidth becomes small. In many practical cases the proximity of a groundplane will decrease the attainable bandwidth of an ESA.

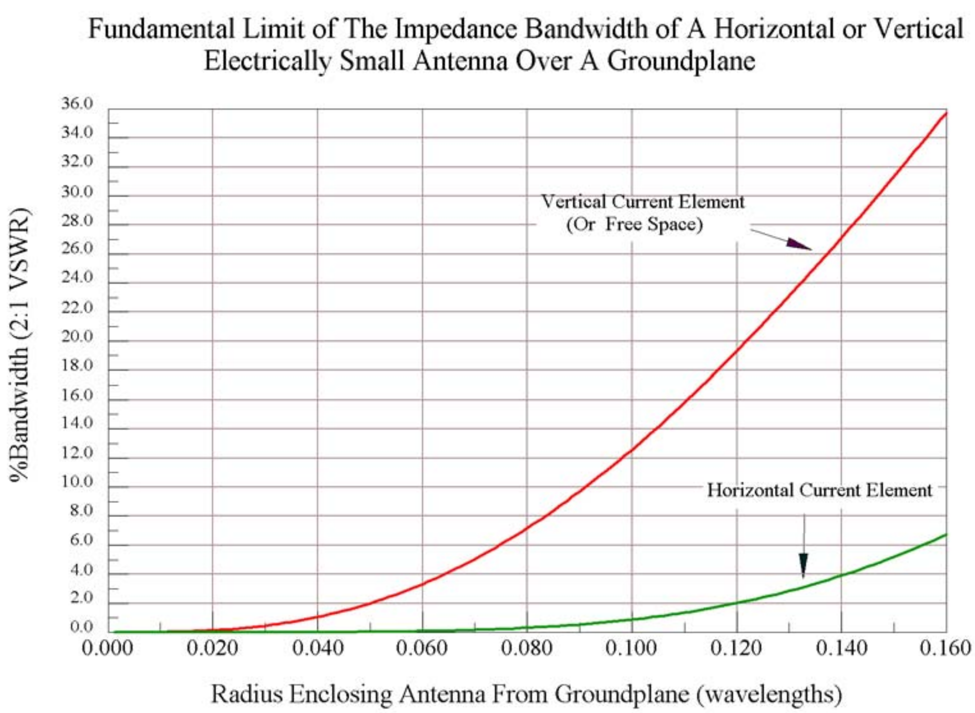

Figure 1–4

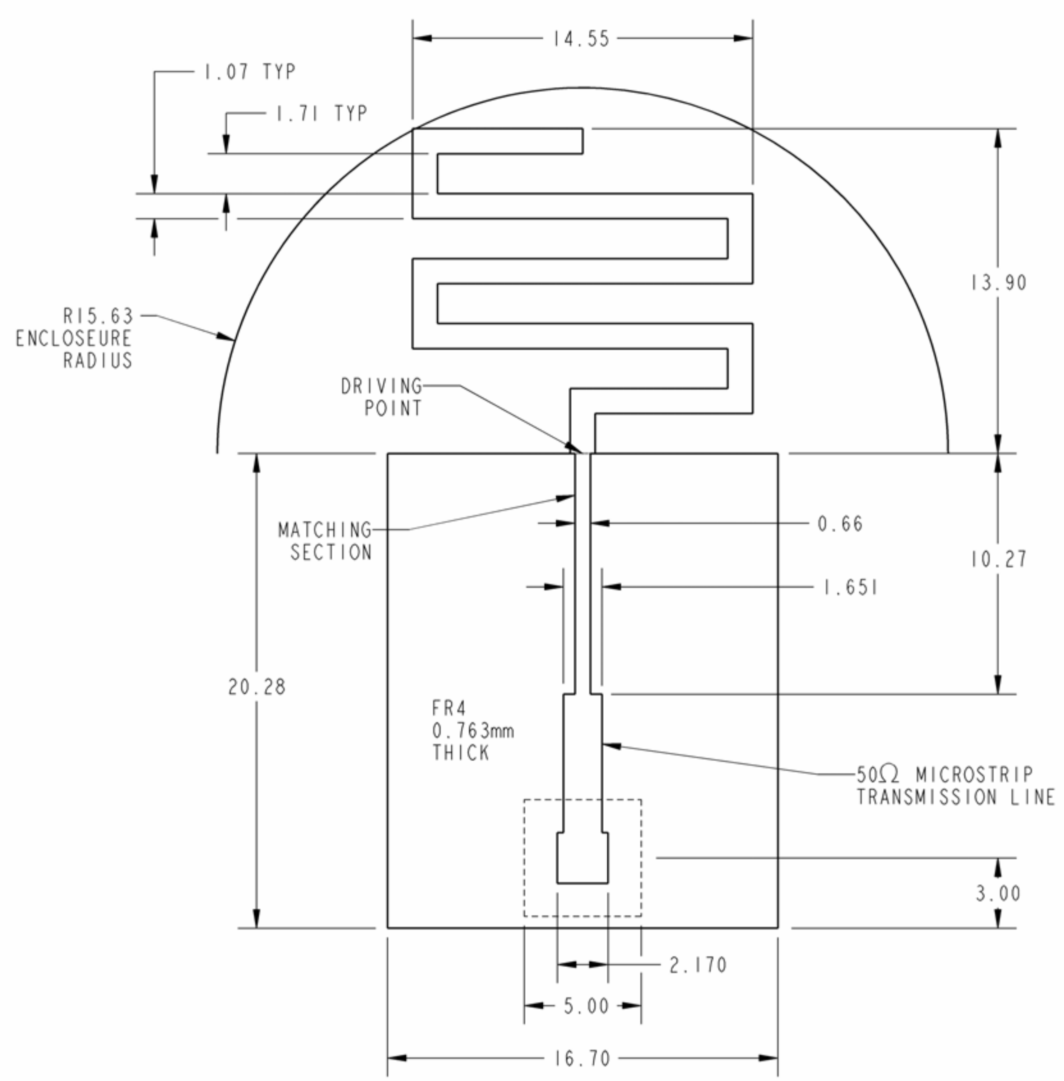

Figure 1–5 Dimensions of a 1.575 GHz Meanderline ESA. All dimensions are in mm. The antenna has a meanderline ESA which is matched with an electrically small section of microstrip transmission line ($\lambda/10$). A 50 Ω microstrip transmission line is probe fed to a coaxial line. (x axis to right, y axis up, z axis out of page)

Meander Line Antenna Impedance Bandwidth

We will now use a 1.575 GHz meanderline antenna prototype to estimate the best case impedance bandwidth we can expect to obtain for this geometry. In Figure 1–5 the meanderline geometry is defined. The antenna consists of a meanderline element. Its input impedance is matched by a $\lambda/10$ section of microstrip transmission line which then feeds into a 50 Ω feedline. The characteristic impedance of this section was determined using computer optimization to provide enough series inductive reactance to cancel the large capacitive reactance of the meanderline ESA.

We will assume that even though the meanderline resonator and groundplane section are thin, that the minimum Q restrictions for a vertically polarized ESA over an infinite groundplane will approximately apply to this geometry.

The radius of a sphere which can enclose the meanderline antenna assuming an infinite groundplane is $a = 15.63$ mm We calculate the free space wavelength and wavenumber which allows us to evaluate ka.

$$\lambda = \frac{c}{f} = \frac{3.0 \cdot 10^8}{1.575 \cdot 10^9} = 190.48\text{mm}$$

$$k = \frac{2\pi}{\lambda} = 32.987 \cdot 10^{-3}\text{radians/mm}$$

$$ka = 32.987 \cdot 10^{-3} \cdot 15.63 = 0.515$$

We can see that ka is less than one and this 1.575 GHz meanderline antenna is by definition an electrically small antenna. This antenna is known to be linear and polarized vertical to the groundplane so we easily calculate the radiation Q using (5).

$$Q_L = \frac{1}{(0.515)^3} + \frac{1}{0.515} = 9.22$$

We note this value is consistent with Figure 1-2.

Equation (7) provides the normalized bandwidth. We choose a 2:1 VSWR.

$$BW = \frac{1}{Q_L\sqrt{2}} = (0.0291) = 7.66\%$$

Unfortunately this does not match with the actual percent bandwidth of 17.4%. How can one reconcile the measured performance verses the theoretical limit? Have we created an antenna which violates this fundamental limit?

To shed some light on this mystery let's first determine what Q value corresponds to a 17.4% (0.174) impedance bandwidth. We obtain $Q_L = 4.06$ for this bandwidth. We now need to compute what ka value is required to produce a 4.06 value for Q_L. The exact solution requires solving a cubic equation. We can estimate the value of ka from Figure 1–2. It appears $ka \approx 0.72$ which is still electrically small. We know the value of k at 1.575 GHz. The value of radius is:

$$a = 0.72/(32.987 \cdot 10^{-3}\text{radians/ mm}) = 21.83\text{mm}$$

For the case where we have an ESA with vertical polarization over a groundplane, the radius of the antenna appears to be expanded from 15.63 mm to 21.83 mm. The explanation for this is that the radiation of the meanderline structure includes about 6.2 mm of the groundplane. These extra currents may be seen in Figure 1–6 on the upper left and upper right vertical edges of the groundplane. Patches of current are in phase with the four vertical high current radiating sections on the meander line. One can see the horizontal currents on the meander line section cancel. The complement of currents on the groundplane cancel with the currents on the upper microstrip to form a transmission line.

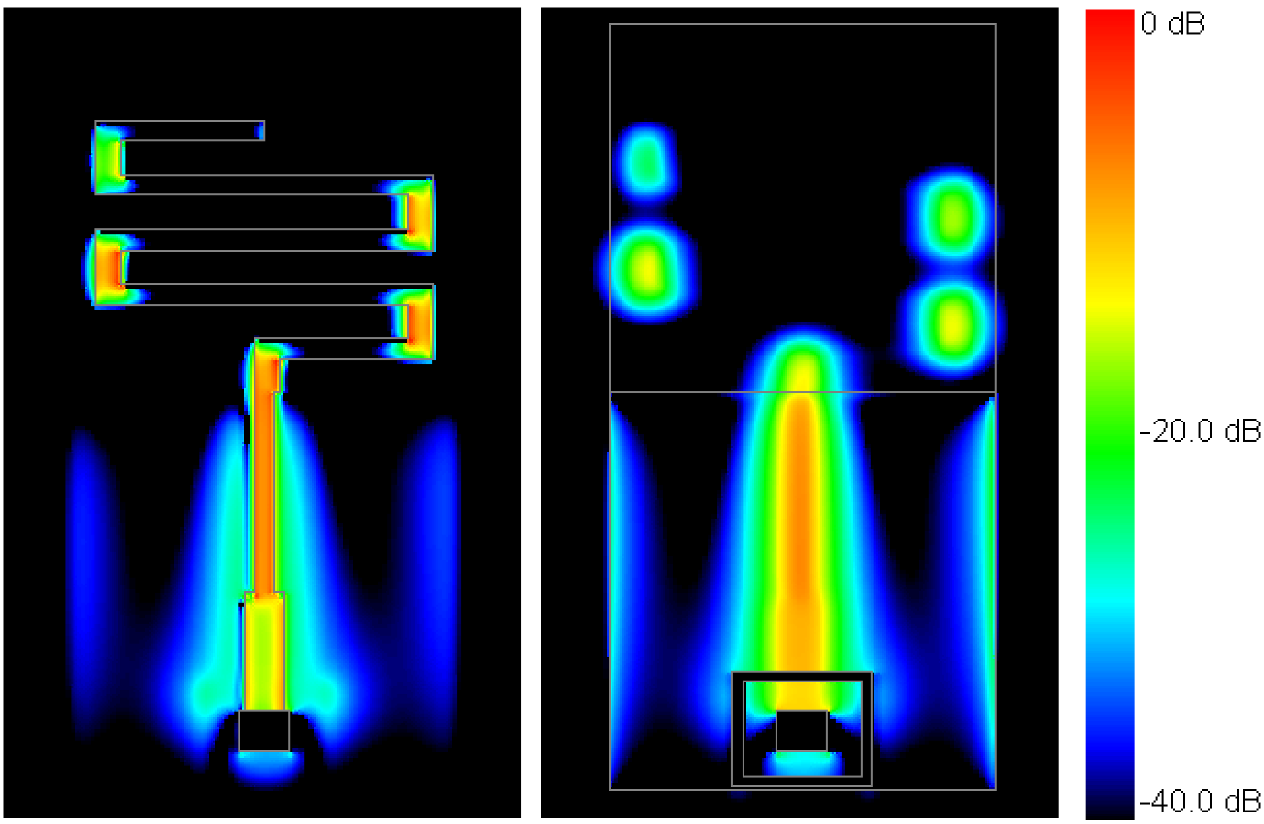

Figure 1–7 FDTD Electric Current Magnitude Plot for the 1.575 GHz Meanderline Prototype. The plot on the left is the current on the upper traces. The plot on the right is of the current on the groundplane. One can see the large magnitude of the currents on the vertical sections of the meanderline. These are essentially the only sources of radiation when the groundplane is enlarged.

Figure 1–6 only provides the direction of electric current on the antenna. Figure 1–7 is a thermal plot of the electric current magnitude normalized to the maximum current in the modeling space. The plot on the left is of the current magnitude on the meanderline resonator and microstrip feed line. One can see the current is concentrated at each of the short vertical sections of the meanderline. The microstrip transmission line current is clearly

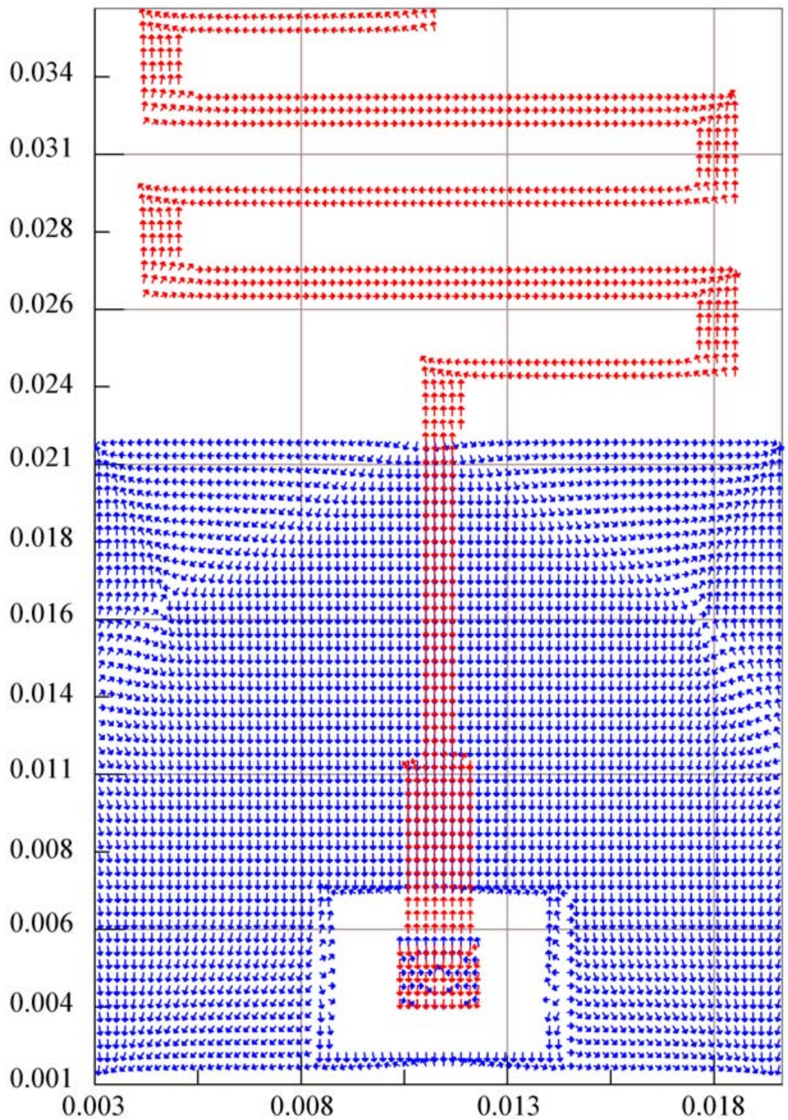

Figure 1–6 FDTD Electric Current Vector Direction Plot for the 1.575 GHz Meanderline Prototype. Red is current on upper traces. Blue is current on lower groundplane. The current on the upper sides of the groundplane contribute to the radiation sphere radius which allows for a larger impedance bandwidth than when the groundplane is large.

seen, as is the current along the vertical edges of the groundplane.

One could be tempted to look at Figure 1–6 and conclude that the groundplane could be terminated just below the currents on each side with no negative effects. When one analyzes the currents on this antenna with FDTD at the 2:1 VSWR band edges, and the center frequency, it may be seen that on the lower frequency limit the currents on the groundplane edges extend almost to the end of the groundplane. The upper limit frequency doesn't appear to drive any significant currents on the groundplane. In order to maintain the large impedance bandwidth we must not truncate the groundplane or it will affect the bandwidth.

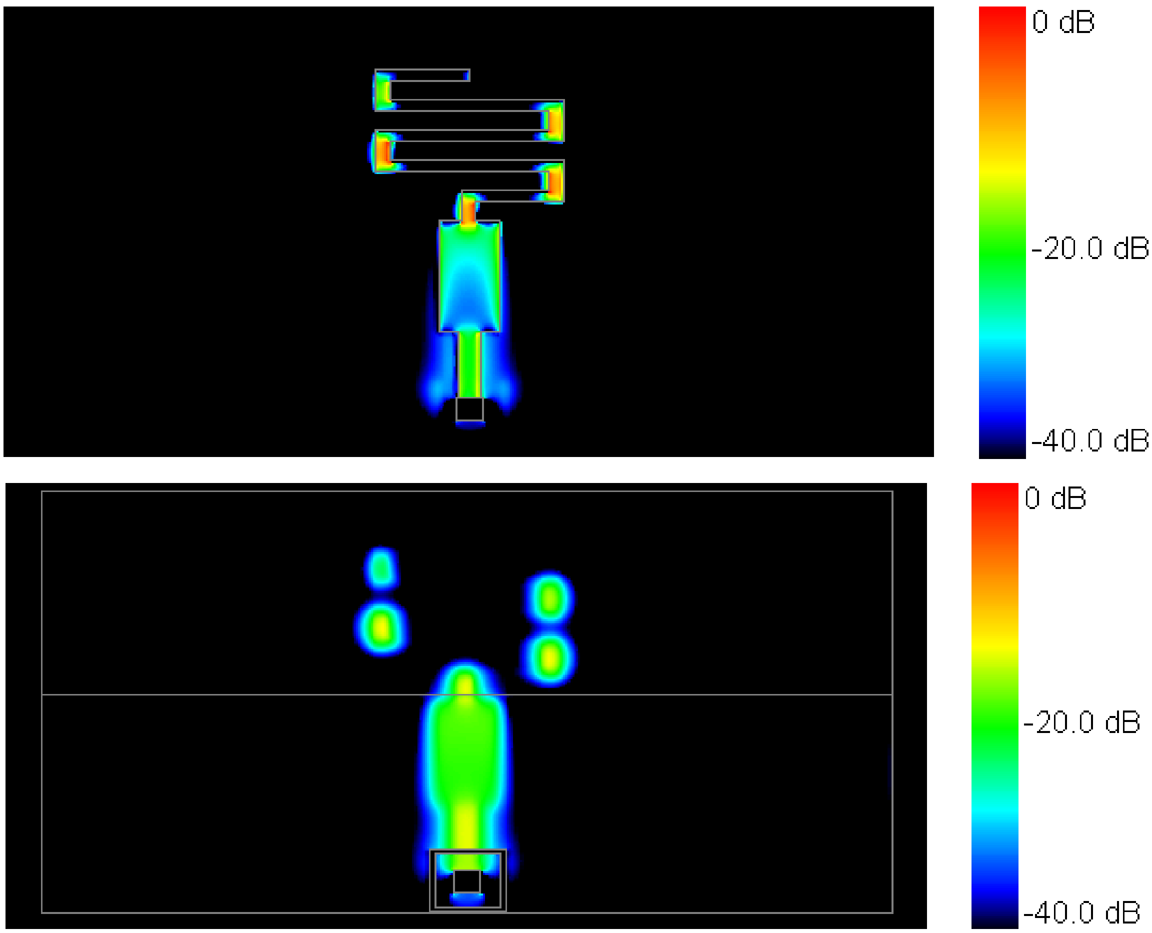

Figure 1–8 FDTD Electric Current Magnitude Plot for the 1.575 GHz Meanderline Prototype with 25 mm of groundplane added to each side. The upper plot is the current on the upper traces. The plot below it is of the current on the groundplane. The vertical current on either side of the groundplane as seen in Figure 1–8 vanishes and doesn't contribute to radiation

If one increases the width of the meanderline antenna groundplane, the impedance bandwidth will decrease until it reaches a limit. When the bandwidth limit is reached, the dimensions of the groundplane have become large enough so that the vertical currents on the meanderline do not drive currents along the edges of the groundplane. FDTD analysis confirms this occurs. The results are seen in Figure 1–8. One can see by comparison with

Figure 1–7 no significant currents exist on the edges when the groundplane is widened. The width of the electrically small matching section had to be increased to cancel the increased capacitive reactance of the meanderline antenna driving point as the antenna's Q increased.

When the groundplane width is increased, the bandwidth of the element decreases to 5.19% bandwidth. This value is lower than our computed estimate of 7.66%. Meeting the bandwidth limit in practice proves elusive. Recent work by Thiele et al suggests the theoretical limit as expressed by McLean is based on a current distribution which may be unobtainable in practice.[6]

Figure 1–10 shows the computed impedance bandwidth change for the baseline antenna of Figure 1-5 and after 25 mm of extra groundplane are added to each side. The reduction in impedance bandwidth is clearly illustrated.

A pair of antennas were constructed using the dimensions obtained with FDTD. Figure 1–11 shows the measured impedance bandwidth change for the baseline antenna and with 25 mm of extra groundplane. We note the measurements correlate very well with the predicted FDTD analysis. The measured antennas had a slightly higher resonant frequency than the analysis predicted.

Meander Line Antenna Radiation Patterns

The antenna patterns computed using FDTD are essentially equivalent for the small and large groundplane (2.0 dB directivity). The FDTD modeling allows for "perfect" feeding of the antenna which minimizes any perturbation from a coaxial feedline.

In practice, the gain of an electrically small antenna is bounded. This limitation has been expressed by Harrington as:[7]

$$G = (ka)^2 + 2(ka)$$

When applied to the meanderline antenna, the maximum attainable gain for the antenna on a large groundplane ($a = 15.63$ mm) is 1.13 dBi, when the groundplane is reduced ($a = 21.83$ mm) we have a maximum possible gain of 2.9 dBi.

Meanderline antennas were fabricated and found to match in at 1.655 GHz (4.83% from 1.575 GHz). When measured, the maximum gain of the meanderline antenna with a large groundplane is 0.3 dBi. The measured gain value of the antenna with a smaller groundplane is 0.5 dBi. The smaller groundplane meanderline antenna generated more current along the coaxial cable which connects the antenna to the ESA than the wider antenna. This makes measuring the small groundplane antenna in isolation difficult and adds loss. This measurement problem has been noted and discussed by Staub et al.[8] An electrically small antenna

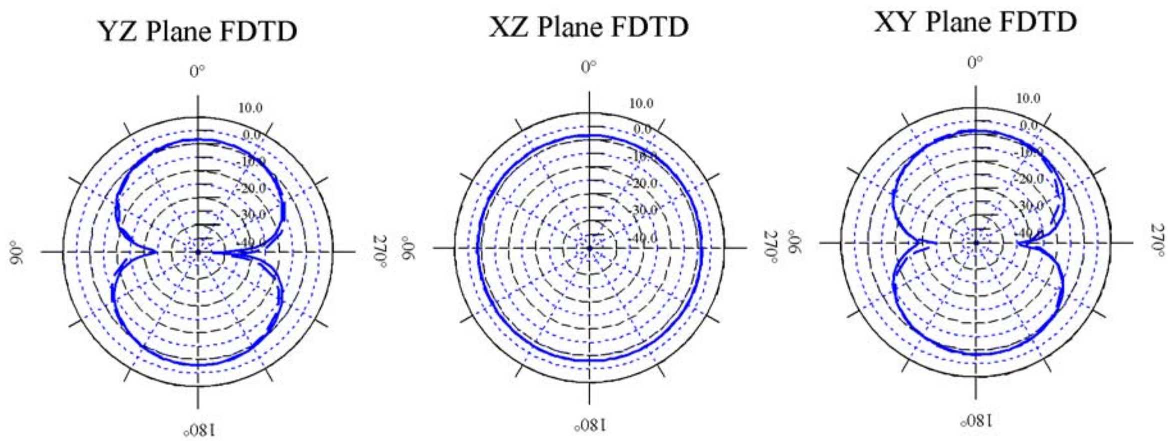

Figure 1–9a Computed radiation patterns of the baseline (narrow groundplane) antenna (dashed lines) and the antenna with 25 mm added (solid lines) using FDTD

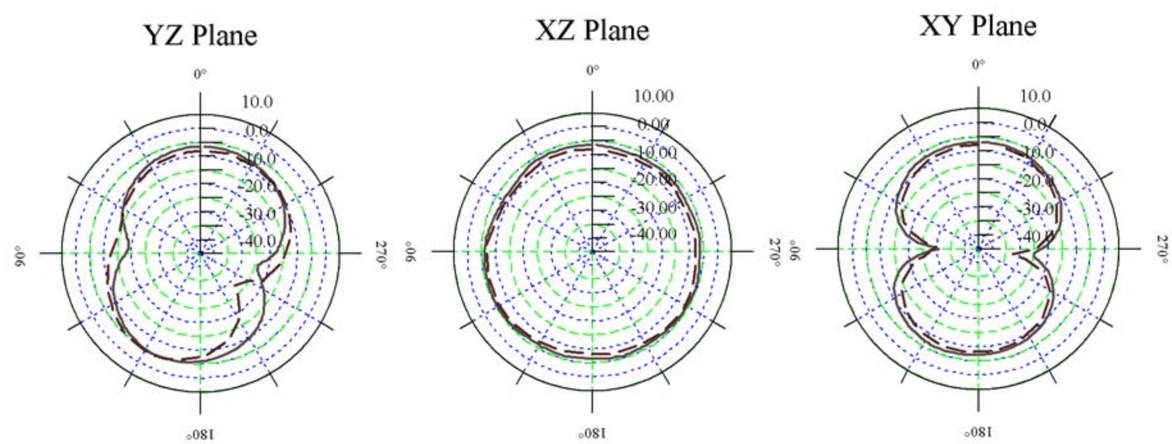

Figure 1–9b The measured radiation patterns of the baseline (narrow groundplane) antenna (dashed lines) and the antenna with 25 mm added (solid lines)

has a combination of balanced and unbalanced mode which makes pattern measurement particularly problematic when using a coaxial (unbalanced) cable to feed the ESA.

Figure 1–9a shows antenna radiation patterns predicted by FDTD analysis. The patterns with and without a large groundplane are very similar. Figure 1–9b has the measured antenna radiation patterns of the meanderline antenna with narrow and wide groundplanes. The YZ plane cut is directly through the connecting cable. One can see the electrically smallest antenna is most perturbed by the cable connection. The other two planes have patterns for each of the two antennas which are very similar.

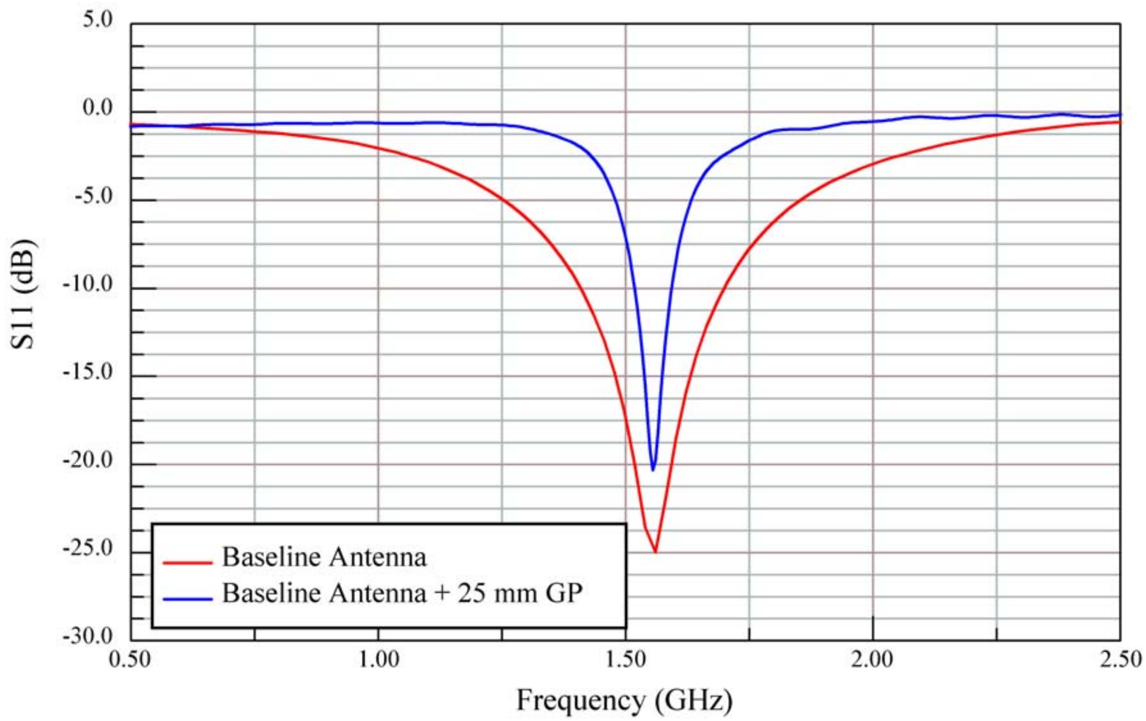

Figure 1–10 S_{11} dB of the baseline (narrow) antenna and the antenna with 25 mm added to each side as predicted with FDTD

Conclusion

In a number of applications such as wireless PCMCIA cards, ESA's have been implemented that have adequate impedance bandwidth, and are well matched. Later a groundplane change considered to be minor would be implemented by a customer on a board turn and the antenna would no longer be matched. In some cases after matching the impedance bandwidth would decrease or in some cases increase. Fundamental limitations on antenna size versus impedance bandwidth brings order to what can appear to be mysterious changes in antenna performance.

When possible, an electrically large antenna should be implemented. This allows for the possibility of a large impedance bandwidth. A large impedance bandwidth often allows the electrically large antenna to continue functioning even when loaded by objects in its environment. Electrically large antennas generally have higher efficiencies than ESAs. When a design doesn't allow for a full size antenna, it is imperative to understand the trade-offs involved when using an ESA to realize a successful design.

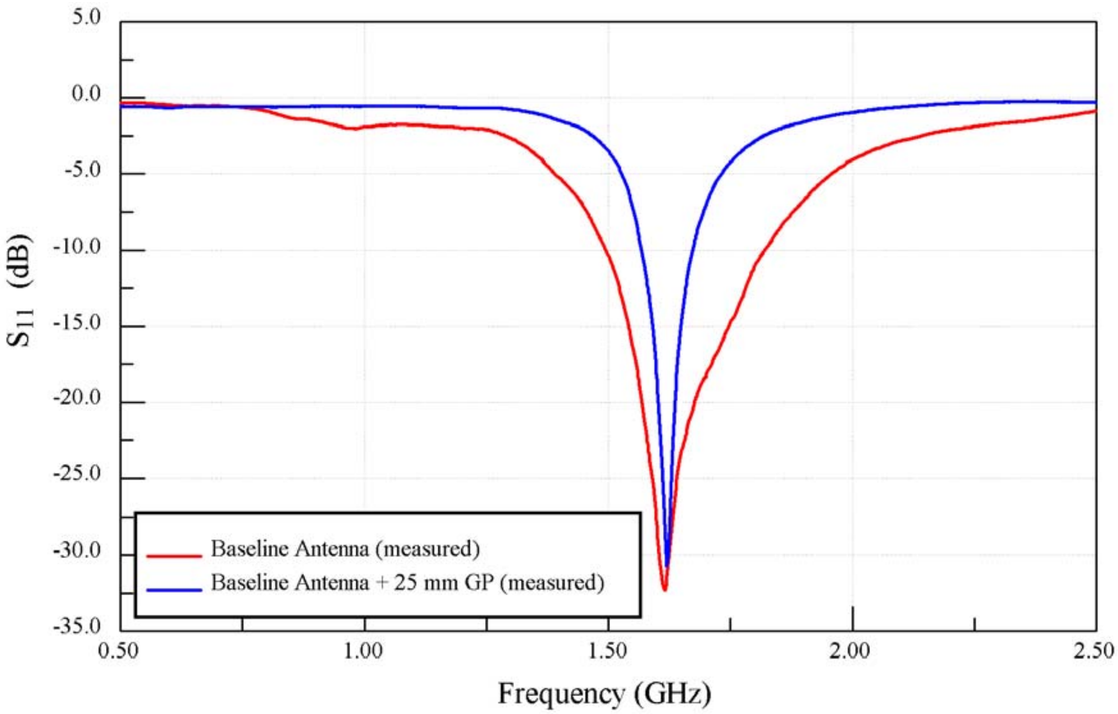

Figure 1–11 S_{11} dB of the baseline (narrow) antenna and the antenna with 25 mm added to each side (measured)

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