Homework 12. Additional question

(1) Prove each of the following statements.

    a. Every subset of a nowhere dense subset is nowhere dense.

    b. If A is nowhere dense, then so is $A + c = \{x + c \,|\, x \in A\}$.

    c. If A is nowhere dense, then so is $cA = \{cx \,|\, x \in A\}$.

    d. If every point of A is isolated, then the set A must be nowhere dense.

(2) Show that if A is of the first category and $B \subset A$, then B is of the first category. (Hint: use (1) **a.** above)

(3) Let $\mathbb{Q} = \{r_k : k = 1, 2 \cdots \}$ and define $A = \mathbb{R} \setminus \cup_k \left(r_k - \frac{1}{2^k}, r_k + \frac{1}{2^k} \right)$.

    a. Show that A is closed (Hint: write A as an intersection of closed sets).

    b. Show that A is nowhere dense.

    Remark: Later we will show that A is uncountable and therefore must have points which are not isolated.